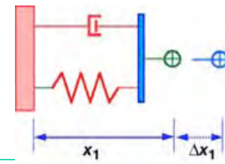
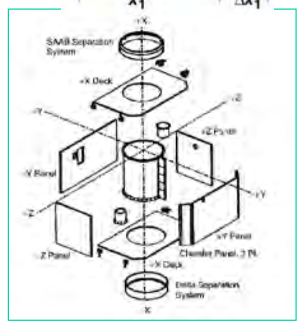
Spacecraft Structures

Space System Design, MAE 342, Princeton University
Robert Stengel

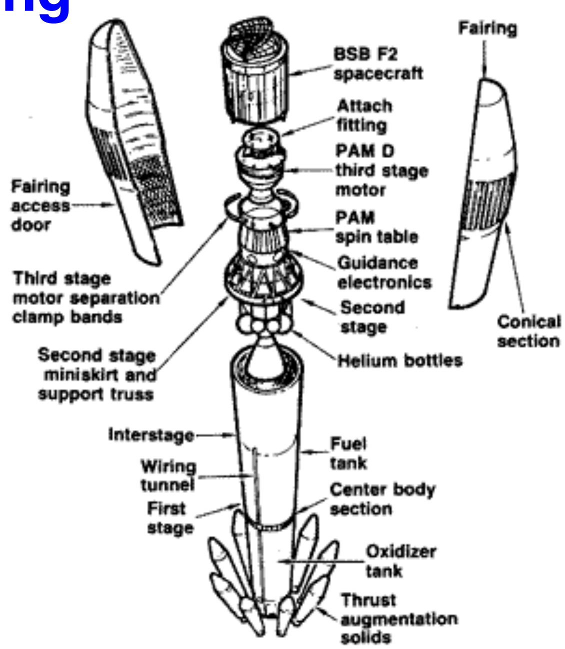
- Discrete (lumped-mass) structures
- Distributed structures
- Buckling
- Fracture and fatigue
- Structural dynamics
- Finite-element analysis





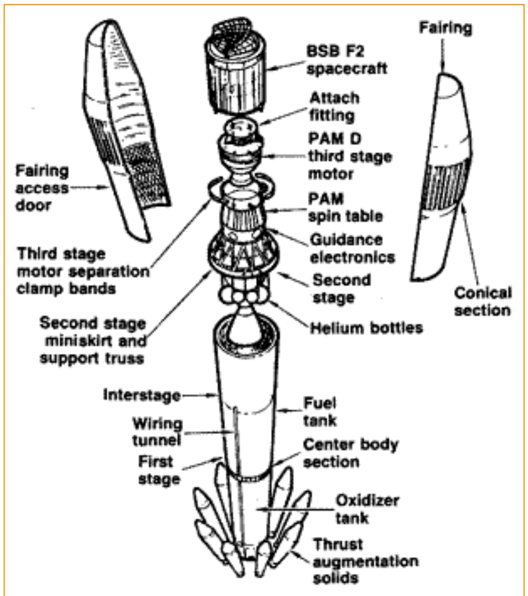
Spacecraft Mounting for Launch

- Spacecraft protected from atmospheric heating and loads by fairing
- Fairing jettisoned when atmospheric effects become negligible
- Spacecraft attached to rocket by adapter, which transfers loads between the two
- Spacecraft (usually) separated from rocket at completion of thrusting
- Clamps and springs for attachment and separation

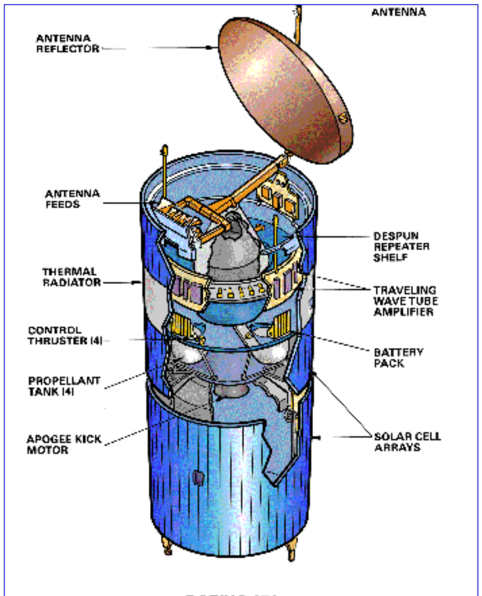


DELTA II LAUNCH VEHICLE

Communications Satellite and Delta II Launcher



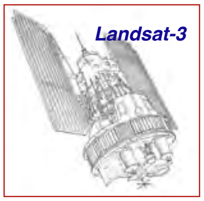
DELTA II LAUNCH VEHICLE



BOEING 376
SPACECRAFT CONFIGURATION

Satellite Systems

- Power and Propulsion
  - –Solar cells
  - –“Kick” motor/ payload assist module (PAM)
  - –Attitude-control/ orbit-adjustment/ station-keeping thrusters
  - –Batteries, fuel cells
  - –Pressurizing bottles
  - –De-orbit/ “graveyard” systems
- Structure
  - –Skin, frames, ribs, stringers, bulkheads
  - –Propellant tanks
  - –Heat/solar/ micrometeoroid shields, insulation
  - –Articulation/ deployment mechanisms
  - –Gravity-gradient tether
  - –Re-entry system (e.g., sample return)
- Electronics
  - –Payload
  - –Control computers
  - –Control sensors and actuators
  - –Control flywheels
  - –Radio transmitters and receivers
  - –Radar transponders
  - –Antennas



Typical Satellite Mass Breakdown

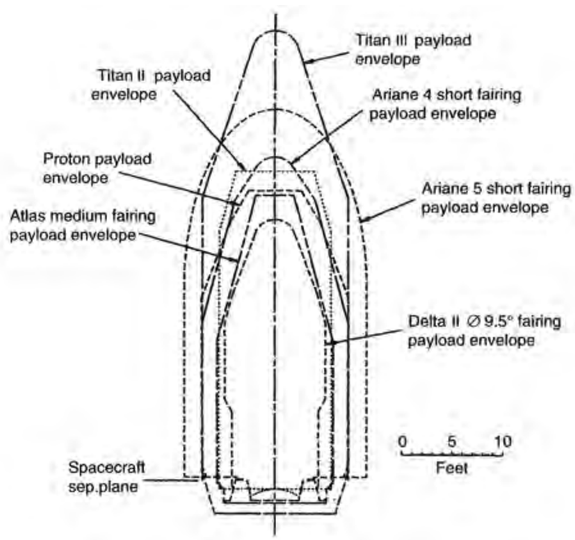
Item	Range (%)
Structure (total)	15–22
Primary structure	12–15
Secondary structure	2–5
Fasteners	1–2
Power	12–30
Thermal control	4–8
Harness	4–10
Avionics	3–7
Guidance & control	5–10
Communication	2–6
Payload	7–55

Pisacane, 2005

Satellite without on-orbit propulsion
"Kick" motor/ PAM can add significant mass
Total mass: from a few kg to > 30,000 kg

Fairing Constraints for Various Launch Vehicles

- **Static envelope**
- **Dynamic envelope accounts for launch vibrations, with sufficient margin for error**
- **Various appendages stowed for launch**
- **Large variation in spacecraft inertial properties when appendages are deployed**



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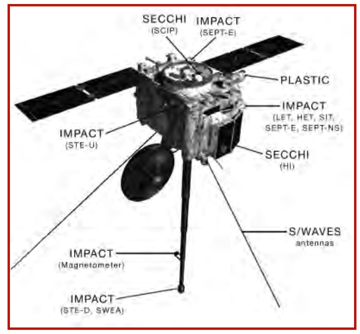
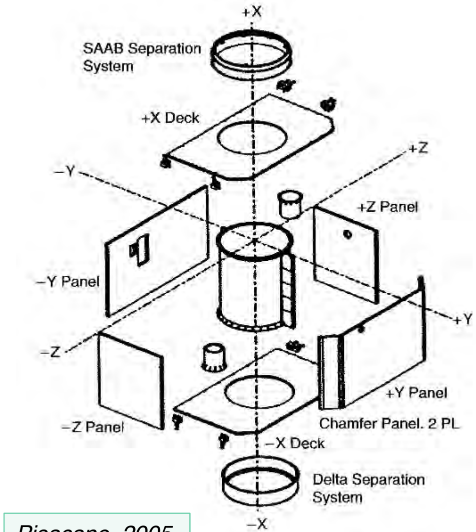
STEREO Spacecraft Primary Structure Configuration

Solar TErrestrial RElations Observatory





Pisacane, 2005

- Spacecraft structure typically consists of
  - Beams
  - Flat and cylindrical panels
  - Cylinders and boxes
- **Primary structure** is the "rigid" skeleton of the spacecraft
- **Secondary structure** may bridge the primary structure to hold components

Upper-Atmosphere Research Satellite *(UARS)* Primary and Secondary Structure

- **Primary Structure** provides
  - Support for 10 scientific instruments
  - Maintains instrument alignment boresights
  - Interfaces to launch vehicle (SSV)
- **Secondary Structure** supports
  - 6 equipment benches
  - 1 optical bench
  - Instrument mounting links
  - Solar array truss
  - Several instruments have kinematic mounts

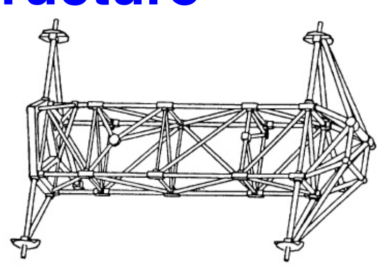


Figure 3-1. The UARS Instrument Module Primary Structure

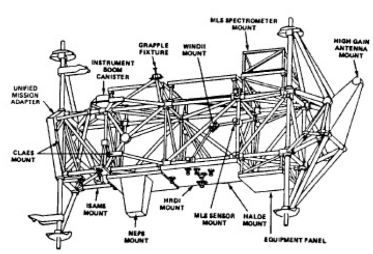
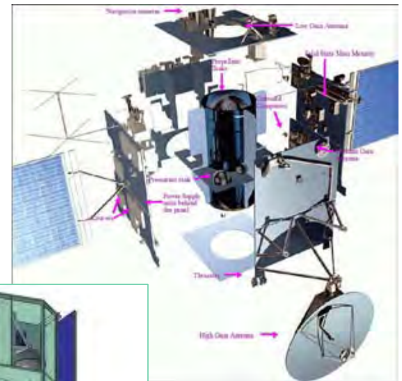


Figure 3-2. The UARS Instrument Module Secondary Structure attached to Primary Structure

Expanded Views of Spacecraft Structures

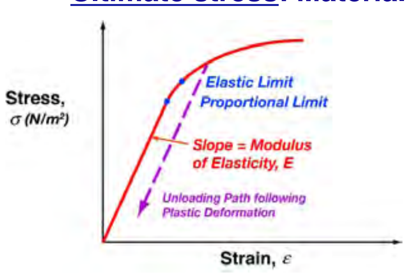


Structural Material Properties

- **Stress, σ: Force per unit area**
- **Strain, ε: Elongation per unit length**

$$\boxed{\sigma = E\,\varepsilon}$$

- **Proportionality factor, E: Modulus of elasticity, or Young's modulus**
- **Strain deformation is reversible below the elastic limit**
- **Elastic limit = yield strength**
- **Proportional limit ill-defined for many materials**
- **<u>Ultimate stress</u>: Material breaks**



Poisson's ratio, ν:

$$\nu = \frac{\varepsilon_{lateral}}{\varepsilon_{axial}}$$

typically 0.1 to 0.35

Thickening under compression
Thinning under tension

Nice explanation at *http://silver.neep.wisc.edu/~lakes/PoissonIntro.html*

Uniform Stress Conditions

Average axial stress, σ

$$\boxed{\sigma = P/A = Load/Cross\ Sectional\ Area}$$

Average axial strain, ε

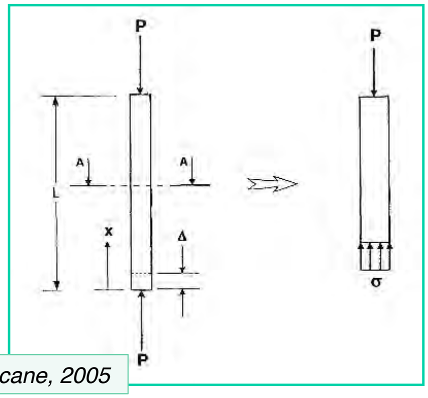
$$\boxed{\varepsilon = \Delta L/L}$$

P: Load, N
A: Cross-sectional area, m^2
L: Length, m

Effective spring constant, k_s

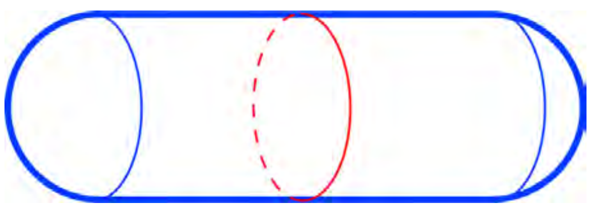
$$\sigma = P/A = E\varepsilon = E\frac{\Delta L}{L}$$

$$P = \frac{AE}{L}\Delta L = k_s \Delta L$$



Pisacane, 2005

Stresses in Pressurized, Thin-Walled Cylindrical Tanks



- **For the cylinder**

$$\sigma_{hoop} = pR/T$$
$$\sigma_{axial} = pR/2T$$
$$\sigma_{radial} \approx \text{negligible}$$

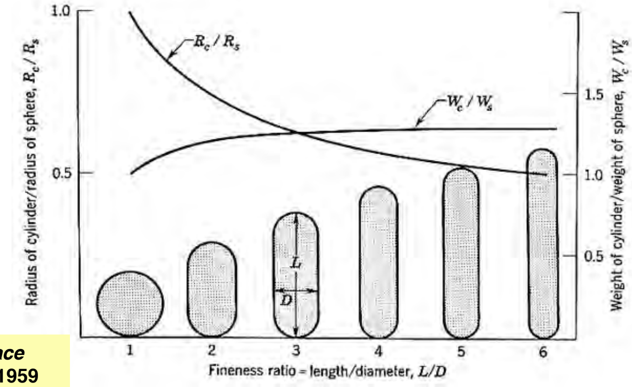
R: radius
T: wall thickness
p: pressure
σ: stress

- **For the spherical end cap**

$$\sigma_{hoop} = \sigma_{axial} = pR/2T$$
$$\sigma_{radial} \approx \text{negligible}$$

Hoop stress is limiting factor

Weight Comparison of Thin-Walled Spherical and Cylindrical Tanks



Sechler, *Space Technology*, 1959

Pressure vessels have same volume and maximum shell stresses due to internal pressure; hydraulic head* is neglected

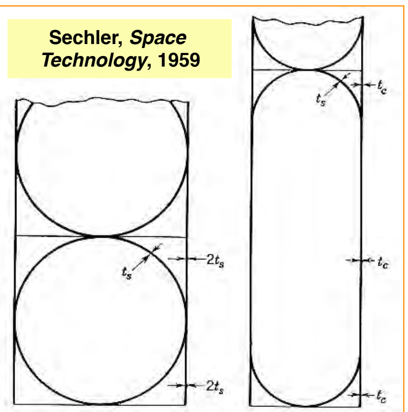
R_c = cylindrical radius

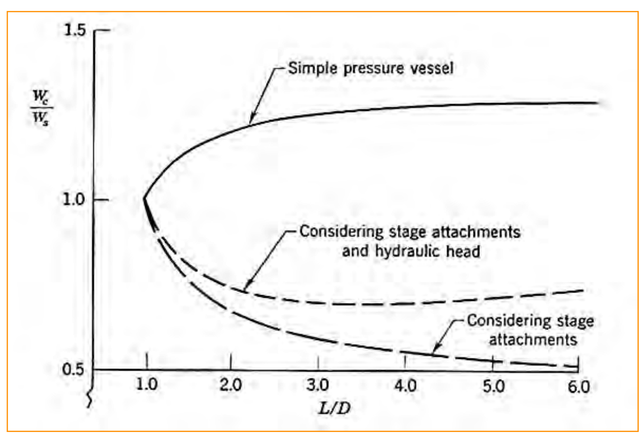
R_s = spherical radius

*** Hydraulic head = Liquid pressure per unit of weight x load factor**

Staged Spherical vs. Cylindrical Tanks

Sechler, *Space Technology*, 1959



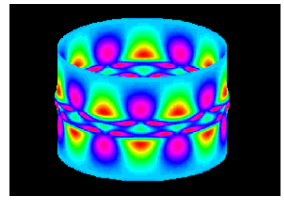


Pressure vessels have same volume and same maximum shell stresses due to internal pressure with and without hydraulic head (with full tanks)

Numerical example for load factor of 2.5

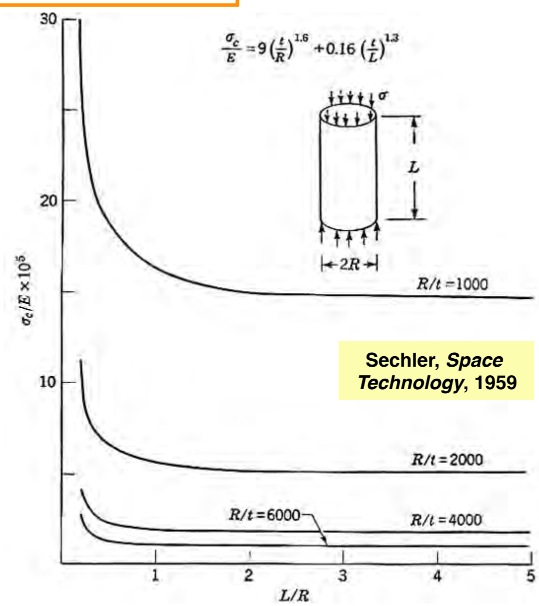
Cylindrical tanks lighter than comparable spherical tanks

Critical Axial Stress in Thin-Walled Cylinders



$$\frac{\sigma_c}{E} = 9\left(\frac{t}{R}\right)^{1.6} + 0.16\left(\frac{t}{L}\right)^{1.3} \quad [\text{no internal pressure}]$$

- Compressive axial stress can lead to buckling failure
- Critical stress, σ_c, can be increased by
  - Increasing E
  - Increasing wall thickness, t
    - solid material
    - honeycomb
  - Adding rings to decrease effective length
  - Adding longitudinal stringers
  - Fixing axial boundary conditions
  - Pressurizing the cylinder



Sechler, *Space Technology*, 1959

SM-65/Mercury Atlas

- Launch vehicle originally designed with balloon propellant tanks to save weight
  - Monocoque design (no internal bracing or stiffening)
  - Stainless steel skin 0.1- to 0.4-in thick
  - Vehicle would collapse without internal pressurization
  - Filled with nitrogen at 5 psi when not fuelled to avoid collapse

Pressure stiffening effect

No internal pressure

$$\frac{\sigma_c}{E} = 9\left(\frac{t}{R}\right)^{1.6} + 0.16\left(\frac{t}{L}\right)^{1.3}$$

Sechler, *Space Technology*, 1959

With internal pressure

$$\sigma_c = \left(K_o + K_p\right)\frac{E\,t}{R}$$

where

$$K_o = 9\left(\frac{t}{R}\right)^{0.6} + 0.16\left(\frac{R}{L}\right)^{1.3}\left(\frac{t}{R}\right)^{0.3}$$

$$K_p = 0.191\left(\frac{p}{E}\right)\left(\frac{R}{t}\right)^2$$

Quasi-Static Loads

Table 8.1 Launch quasi-static loads for Ariane 4

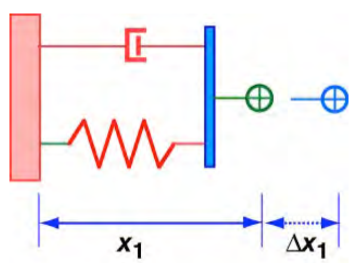
Flight event	Acceleration (g) Q.S.L.	
	Longitudinal	Lateral axis
Maximum dynamic pressure	−3.0	±1.5
Before thrust termination	−5.5	±1.0
During thrust tail-off	+2.5	±1.0

Note: The minus sign with longitude axis values indicates compression.

Fortescue, 2003

Oscillatory Components

Newton's second law leads to a 2nd-order dynamic system for each discrete mass



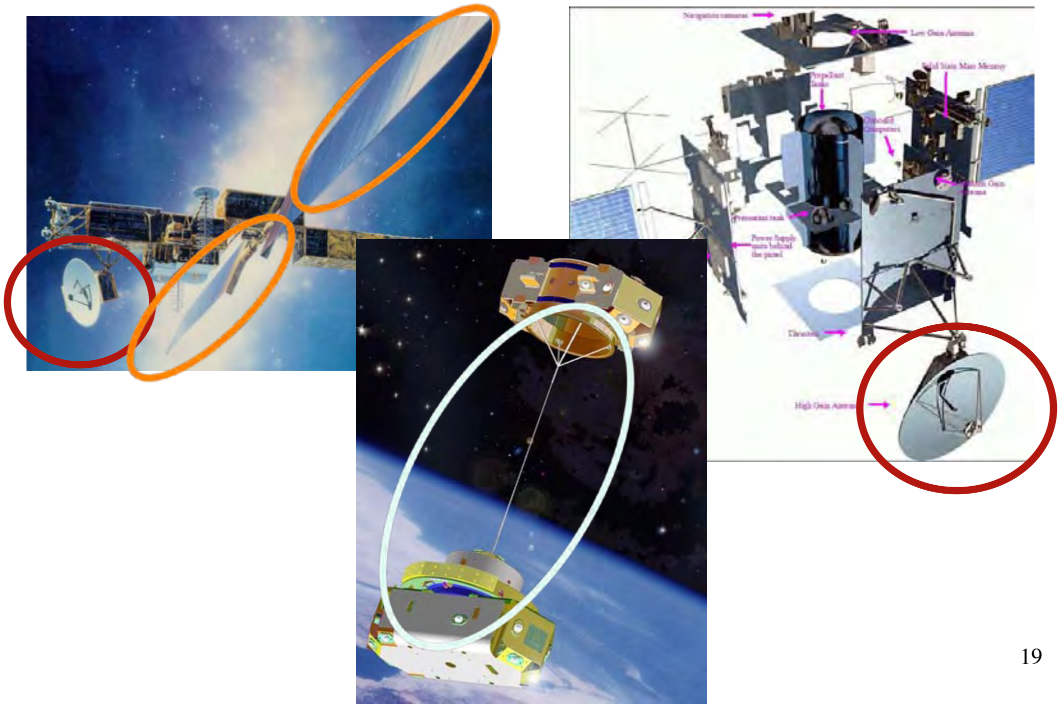
$$\Delta\ddot{x} = f_x/m = \left(-k_d\Delta\dot{x} - k_s\Delta x + \text{forcing function}\right)/m$$

$$\Delta\ddot{x} + \frac{k_d}{m}\Delta\dot{x} + \frac{k_s}{m}\Delta x = \frac{\text{forcing function}}{m}$$

$$\Delta\ddot{x} + 2\zeta\omega_n\Delta\dot{x} + \omega_n{}^2\Delta x = \omega_n{}^2\Delta u$$

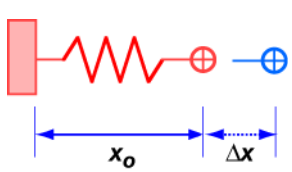
ω_n = natural frequency, rad/s
ζ = damping ratio
Δx = displacement, m
Δu = disturbance or control

Examples of Oscillatory Discrete Components



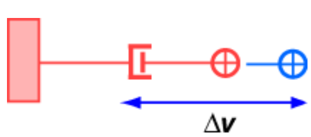
Springs and Dampers

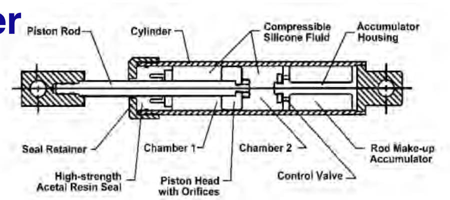
Force due to linear spring



$$f_x = -k_s \Delta x = -k_s\left(x - x_o\right) \quad ; \quad k = \text{spring constant}$$

Force due to linear damper



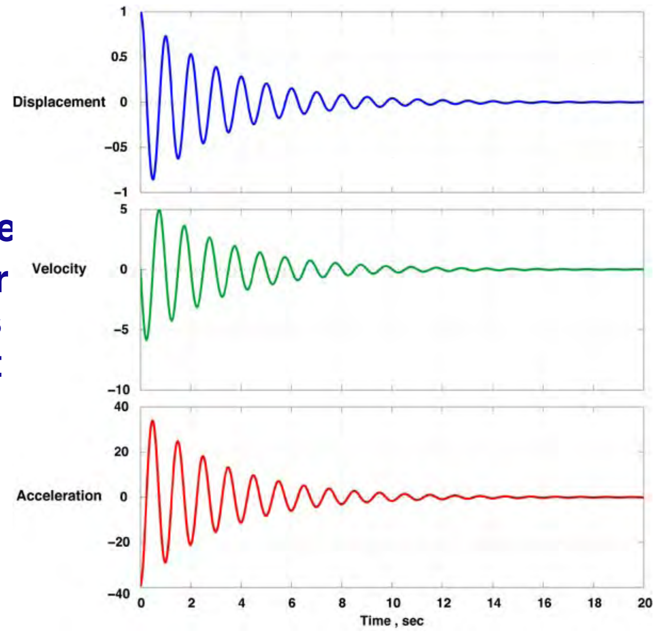


$$f_x = -k_d \Delta \dot{x} = -k_d \Delta v = -k_d\left(v - v_o\right) \quad ; \quad k = \text{damping constant}$$

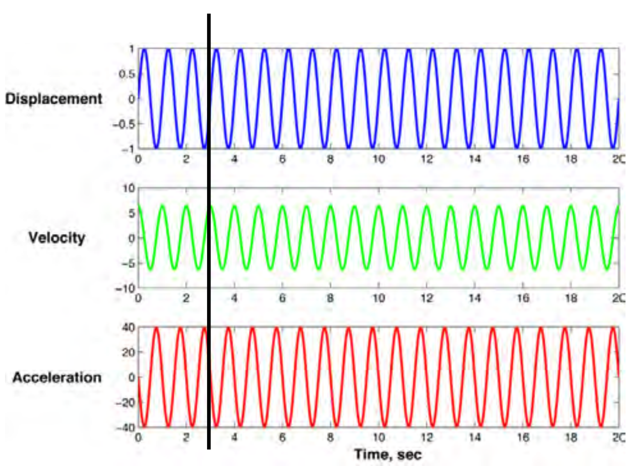
Response to Initial Condition

- Lightly damped system has a decaying, oscillatory transient response
- Forcing by step or impulse produces a similar transient response

$$\omega_n = 6.28 \text{ rad/sec}$$
$$\zeta = 0.05$$



Oscillations



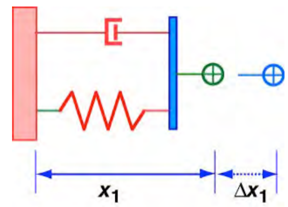
$$\Delta x = A\sin(\omega t)$$

$$\Delta \dot{x} = A\omega\cos(\omega t) = A\omega\sin(\omega t + \pi/2)$$

$$\Delta \ddot{x} = -A\omega^2\sin(\omega t) = A\omega^2\sin(\omega t + \pi)$$

- *Phase angle* of velocity (wrt displacement) is $\pi/2$ rad (or 90°)
- *Phase angle* of acceleration is π rad (or 180°)
- As oscillatory input frequency, ω varies
  - Velocity amplitude is proportional to ω
  - Acceleration amplitude is proportional to ω^2

Response to Oscillatory Input



Compute Laplace transform to find transfer function

$$\mathcal{L}\left[\Delta x(t)\right] = \Delta x(s) = \int_0^\infty \Delta x(t) e^{-st}\, dt,$$

$$s = \sigma + j\omega, \quad (j = i = \sqrt{-1})$$

Neglecting initial conditions

$$\mathcal{L}\left[\Delta \dot{x}(t)\right] = s\Delta x(s)$$

$$\mathcal{L}\left[\Delta \ddot{x}(t)\right] = s^2 \Delta x(s)$$

Transfer Function

$$\mathcal{L}\left(\Delta \ddot{x} + 2\zeta\omega_n \Delta \dot{x} + \omega_n{}^2 \Delta x\right) = \mathcal{L}\left(\omega_n{}^2 \Delta u\right)$$

or

$$\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)\Delta x(s) = \omega_n{}^2 \Delta u(s)$$

Transfer function from input to displacement

$$\frac{\Delta x(s)}{\Delta u(s)} = \frac{\omega_n{}^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)}$$

Transfer Functions of Displacement, Velocity, and Acceleration

- **Transfer function from input to displacement**
- **Input to velocity: multiply by *s***
- **Input to acceleration: multiply by *s*²**

$$\frac{\Delta x(s)}{\Delta u(s)} = \frac{\omega_n^{\,2}}{\left(s^2 + 2\zeta\omega_n s + \omega_n^{\,2}\right)}$$

$$\frac{\Delta \dot{x}(s)}{\Delta u(s)} = \frac{\omega_n^{\,2} s}{\left(s^2 + 2\zeta\omega_n s + \omega_n^{\,2}\right)}$$

$$\frac{\Delta \ddot{x}(s)}{\Delta u(s)} = \frac{\omega_n^{\,2} s^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n^{\,2}\right)}$$

From Transfer Function to Frequency Response

Displacement transfer function

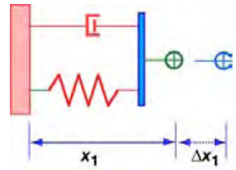
$$\frac{\Delta x(s)}{\Delta u(s)} = \frac{\omega_n^{\,2}}{\left(s^2 + 2\zeta\omega_n s + \omega_n^{\,2}\right)}$$

Displacement frequency response ($s = j\omega$)

$$\frac{\Delta x(j\omega)}{\Delta u(j\omega)} = \frac{\omega_n^{\,2}}{(j\omega)^2 + 2\zeta\omega_n (j\omega) + \omega_n^{\,2}}$$

Real and imaginary components

Frequency Response



ω_n: natural frequency of the system

ω: frequency of a sinusoidal input to the system

$$\frac{\Delta x(j\omega)}{\Delta u(j\omega)} = \frac{\omega_n^{\,2}}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^{\,2}}$$

$$= \frac{\omega_n^{\,2}}{\left(\omega_n^{\,2} - \omega^2\right) + 2\zeta\omega_n(j\omega)} \equiv \frac{\omega_n^{\,2}}{c(\omega) + jd(\omega)}$$

$$= \left[\frac{\omega_n^{\,2}}{c(\omega) + jd(\omega)}\right]\left[\frac{c(\omega) - jd(\omega)}{c(\omega) - jd(\omega)}\right] = \frac{\omega_n^{\,2}\left[c(\omega) - jd(\omega)\right]}{c^2(\omega) + d^2(\omega)}$$

$$\equiv a(\omega) + jb(\omega) \equiv A(\omega)e^{j\varphi(\omega)}$$

Frequency response is a complex function

Real and imaginary components, or

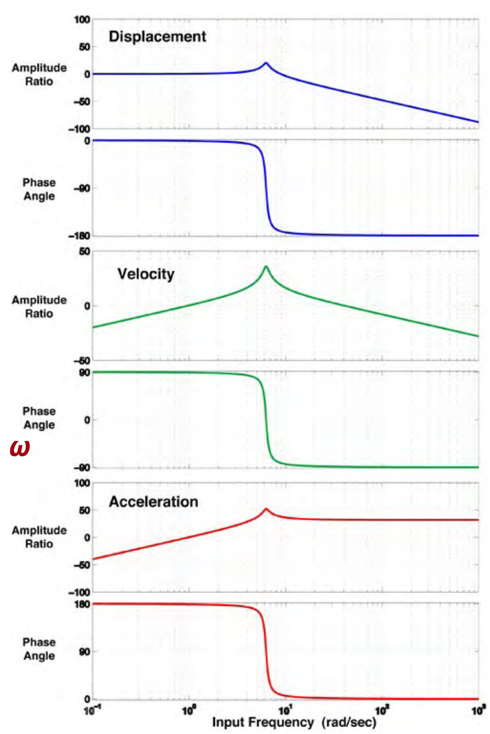
Amplitude and phase angle

Frequency Response of the 2nd-Order System

- Convenient to plot response on logarithmic scale

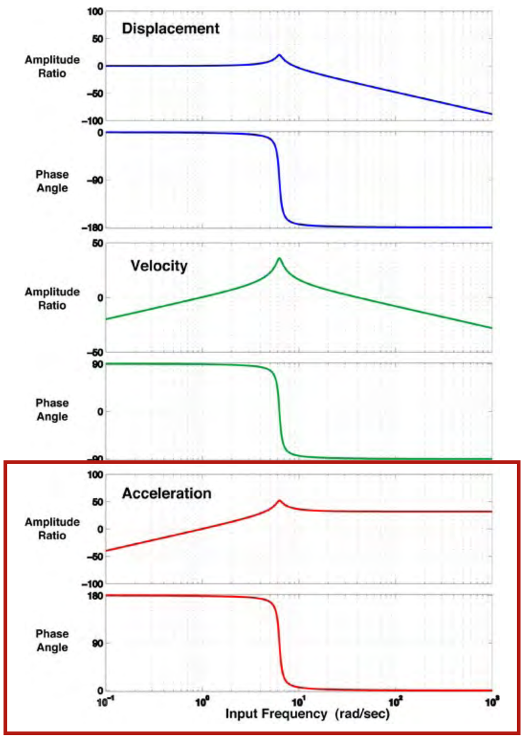
$$\boxed{\ln\left[A(\omega)e^{j\varphi(\omega)}\right] = \ln A(\omega) + j\varphi(\omega)}$$

- Bode plot
  - 20 log(Amplitude Ratio) [dB] vs. log ω
  - Phase angle (deg) vs. log ω
- Natural frequency characterized by
  - Peak (resonance) in amplitude response
  - Sharp drop in phase angle
- Acceleration frequency response has the same peak



Acceleration Response of the 2nd-Order System

- **Important points:**
  - Low-frequency acceleration response is attenuated
  - Sinusoidal inputs at natural frequency resonate, I.e., they are amplified
  - Component natural frequencies should be high enough to minimize likelihood of resonant response



Spacecraft Stiffness* Requirements for Primary Structure

Vehicle	Thrust (Hz)	Lateral (Hz)
Delta II	35	20
Delta III (2 stage)	27	10
Delta IV Med/Med+	27	10
Delta IV Heavy	30	8
Atlas IIAS	15	8
Atlas III	15	8
Atlas V	15	8
Proton	25	10
Pegasus	–	20
Taurus	35–45, >75(1)	25
Titan II	24	10
Ariane 4	31	10
Ariane 5	18(2)	8(3)
Athena 1	30, ≠ 45 – 70(4)	15
Athena 2	30, ≠ 45 – 70(4)	12

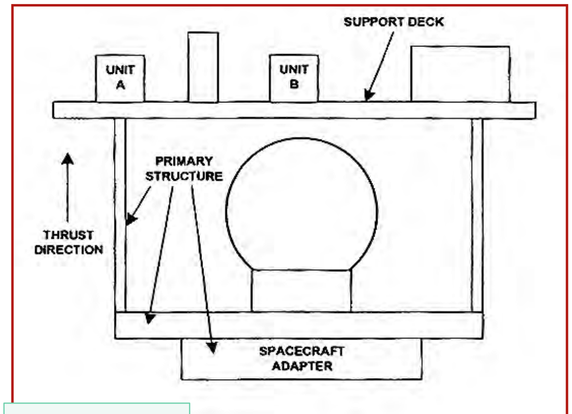
(1) Coupled vehicle/payload system requirement.
(2) Sum of effective mass at a given frequency.
(3) Worst case—payload mass dependent.
(4) ≠ applies to all values within the range.

*** Natural frequency**

Typical Spacecraft Layout

- ***Atlas IIAS*** **launch vehicle**
- **Spacecraft structure meets primary stiffness requirements**
- **What are axial stiffness requirements for Units A and B?**
  - **Support deck natural frequency = 50 Hz**

<u>Octave Rule</u>: Component natural frequency ≥ 2 x natural frequency of supporting structure



Pisacane, 2005

Unit A: 2 x 15 Hz = 30 Hz, supported by primary structure
Unit B: 2 x 50 Hz = 100 Hz, supported by secondary structure

Factors and Margins of Safety

Factor of Safety

Typical values: 1.25 to 1.4

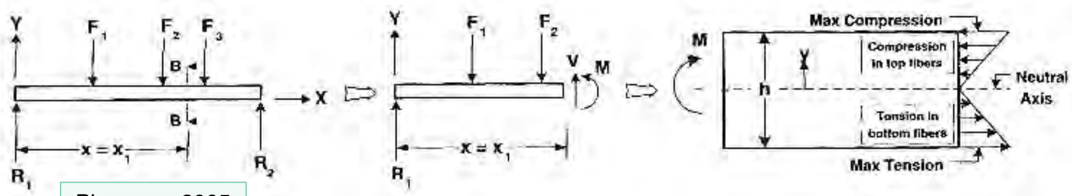
$$\frac{\text{Load (stress) that causes yield or failure}}{\text{Expected service load}}$$

Margin of Safety

"the amount of margin that exists above the material allowables for the applied loading condition (with the factor of safety included)"
Skullney, Ch. 8, Pisacane, 2005

$$\frac{\text{Allowable load (yield stress)}}{\text{Expected limit load (stress)} \times \text{Design factor of safety}} - 1$$

Worst-Case Axial Stress on a Simple Beam



Pisacane, 2005

Axial stress due to bending

$$\sigma = My/I$$

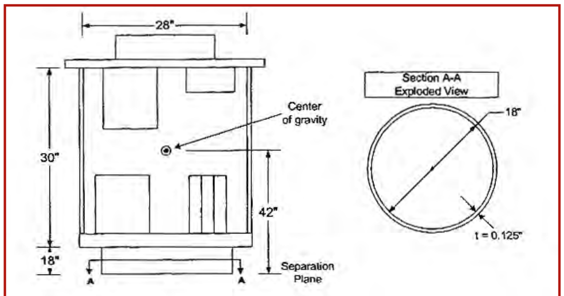
Maximum stress

$$\sigma = \frac{M(h/2)}{I}$$

Worst-case axial stress due to bending and axial force

$$\sigma_{wc} = \pm\left(\frac{P}{A}\right)_{max} \pm \frac{M(h/2)}{I}$$

Stress on Spacecraft Adapter



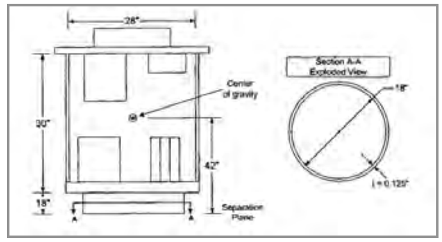
Pisacane, 2005

- **Spacecraft weight = 500 lb**
- ***Atlas IIAS* launch vehicle**
- **Factor of safety = 1.25**
- **Maximum stress on spacecraft adapter?**

Atlas IIAS Limit Loads (g)

Event	Lateral	Axial
Liftoff	± 1.3	$+1.2 \pm 1.1$
Flight winds	$+0.4 \pm 1.6$	$+2.7 \pm 0.8$
BECO (axial)	± 0.5	$+5.0 \pm 0.5$
BECO (lateral)	± 2.0	$+2.5 \pm 1.0$
SECO	± 0.3	$+2.0 \pm 0.4$
MECO (axial)	± 0.3	$+4.5 \pm 1.0$
MECO (lateral)	± 0.6	± 2.0

Example, con't.



Worst-case stress

$$\sigma_{wc} = \pm\left(\frac{P}{A}\right)_{max} \pm \frac{Mc}{I}$$

$$A = 2\pi rt = 7.1 \text{ in}^2$$
$$I = \pi r^3 t = 286 \text{ in}^4$$

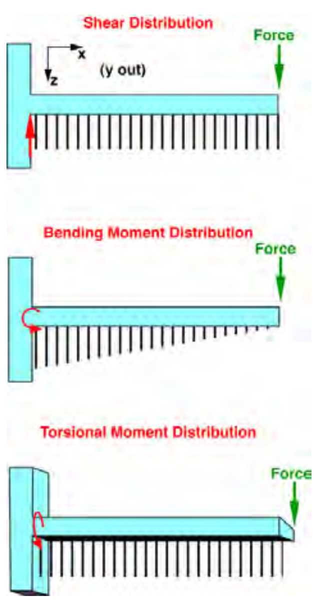
Worst-case axial load at BECO (5±0.5 *g*)

$$\sigma_{wc} = \left[\frac{500\times 5.5}{7.1} + \frac{500\times 0.5\times 42\times 9}{286}\right]\times 1.25 = 897.1 \text{ psi}$$

Worst-case lateral load at BECO (2.5 ± 1 *g*) or Maximum Flight Winds (2.7 ± 0.8 g)

$$\sigma_{wc} = \left[\frac{500\times 3.5}{7.1} + \frac{500\times 2\times 42\times 9}{286}\right]\times 1.25 = 1960 \text{ psi}$$

Force and Moments on a Slender Cantilever (Fixed-Free) Beam



- Idealization of
  - Launch vehicle tied-down to a launch pad
  - Structural member of a payload
- For a point force
  - Force and moment must be opposed at the base
  - Shear distribution is constant
  - Bending moment increases as moment arm increases
  - Torsional moment and moment arm are fixed

Structural Stiffness

- Geometric stiffening property of a structure is portrayed by the area moment of inertia
- For bending about a *y* axis (producing distortion along an *x* axis)

$$\boxed{I_x = \int_{z_{\min}}^{z_{\max}} x(z) z^2 \, dz}$$

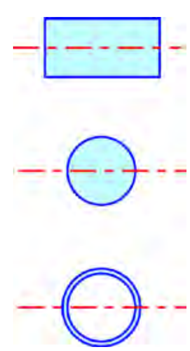
- Area moment of inertia for simple cross-sectional shapes

- Solid rectangle of height, *h*, and width, *w*:  $I_y = wh^3/12$
- Solid circle of radius, *r*:  $I_y = \pi r^4/4$
- Circular cylindrical tube with inner radius, r_i: and outer radius, r_o:  $I_y = \pi\left(r_o^{\,4} - r_i^{\,4}\right)/4$

Bending Stiffness

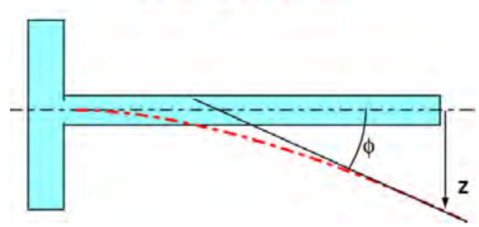
- Neutral axis neither shrinks nor stretches in bending
- For small deflections, the bending radius of curvature of the neutral axis is

$$\boxed{r = \frac{EI}{M}}$$

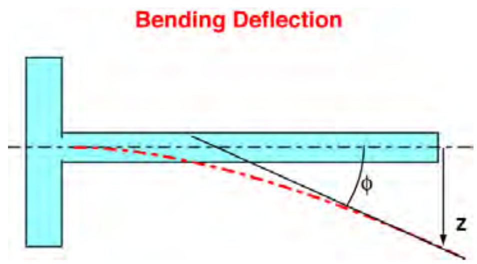


- Deflection at a point characterized by displacement and angle:

Bending Deflection



Bending Deflection



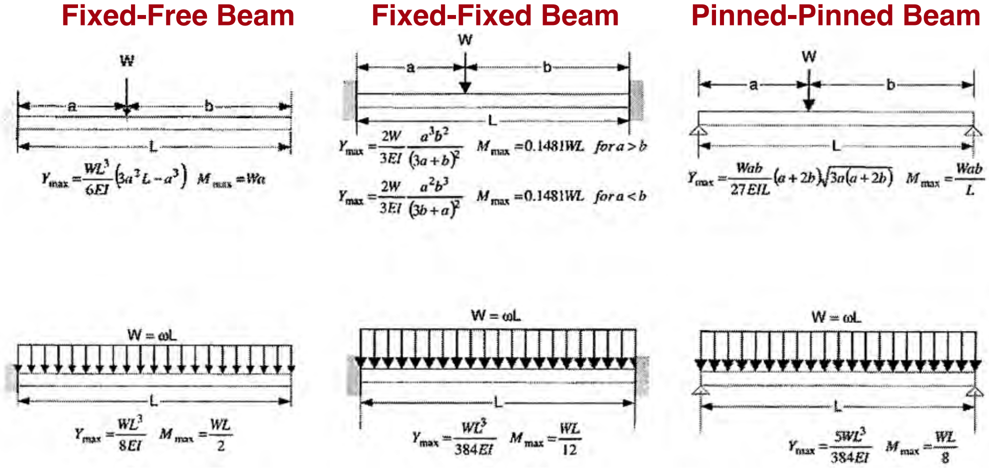
Second derivative of *z* and first derivative of *φ* are inversely proportional to the bending radius:

$$\boxed{\frac{d^2 z}{dx^2} = \frac{d\phi}{dx} = \frac{M_y}{EI_y}}$$

Maximum Deflection and Bending Moment of Beams

(see *Fundamentals of Space Systems* for additional cases)

Fixed-Free Beam **Fixed-Fixed Beam** **Pinned-Pinned Beam**



Y_{max} **= maximum deflection**

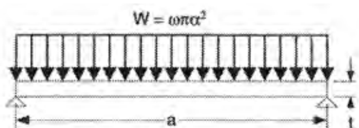
M_{max} **= maximum bending moment**

Pisacane, 2005

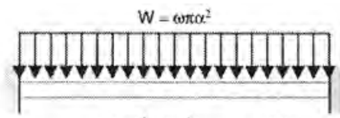
Maximum Deflection and Bending Moment of Plates

(see *Fundamentals of Space Systems* for additional cases)

Circular Plate



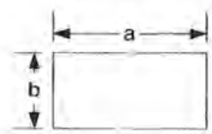
$$Y_{max} = \frac{3W(m-1)(5m+1)a^2}{16\pi E m^2 t^3} \qquad \sigma_{max} = \frac{3W}{8\pi m t^2}(3m+1)$$



$$Y_{max} = \frac{3W(m^2-1)a^2}{16\pi E m^2 t^3} \qquad \sigma_{max} = \frac{3W}{4\pi t^2}$$

$$m = 1/\nu$$

Rectangular Plate



All Edges Supported
Uniform Load over
Entire Surface

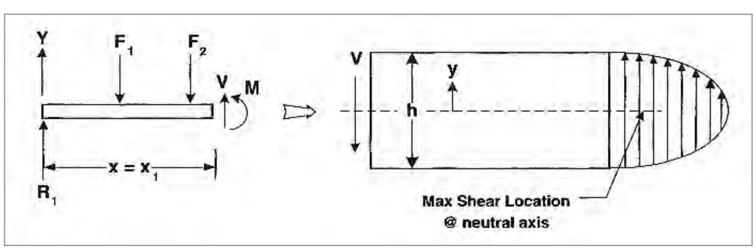
$$Y_{max} = \alpha\frac{\omega b^4}{Et^3} \qquad \sigma_{max} = \beta\frac{\omega b^2}{t^2}$$

a/b	1	1.2	1.4	1.6	1.8	2	3	Inf
β	0.2874	0.3762	0.4530	0.5172	0.5688	0.6102	0.7134	0.75
α	0.0444	0.0616	0.0770	0.0906	0.1017	0.1110	0.1335	0.1421

Pisacane, 2005

Typical Cross-Sectional Shear Stress Distribution for a Uniform Beam

Shear stress due to bending moment is highest at the neutral axis

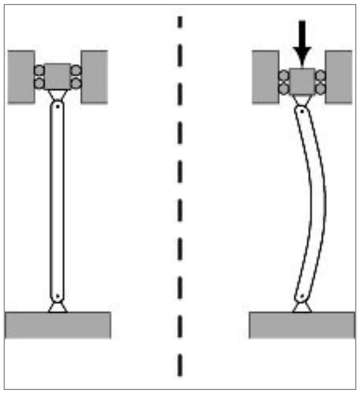


Maximum values for various cross sectons

(see *Fundamentals of Space Systems*)

Pisacane, 2005

Buckling



- Predominant steady stress during launch is compression
- Thin columns, plates, and shells are subject to elastic instability in compression
- Buckling can occur below the material's elastic limit

Critical buckling stress of a column (Euler equation)

$$\sigma_{cr} = \frac{C\pi^2 E}{(L/\rho)^2} = \frac{P}{A}$$

C = function of end "fixity"

E = modulus of elasticity

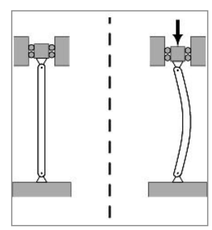
L = column length

$\rho = \sqrt{I/A}$ = radius of gyration

P_{cr} = critical buckling load

A = cross sectional area

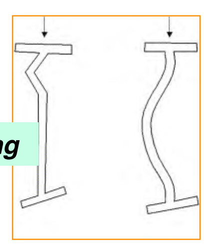
Effect of "Fixity" on Critical Loads for Beam Buckling



1 — Free 2 — Fixed	$P_{cr} = \frac{\pi^2 EI}{4L^2}$
1 — Hinged 2 — Hinged	$P_{cr} = \frac{\pi^2 EI}{L^2}$
1 — Hinged 2 — Fixed	$P_{cr} = \frac{\pi^2 EI}{(0.71L)^2}$
1 — Fixed 2 — Fixed	$P_{cr} = \frac{4\pi^2 EI}{L^2}$

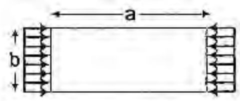
Pisacane, 2005

- Euler equation
  - Slender columns
  - Critical stress below the elastic limit
  - Relatively thick column walls
- Local collapse due to thin walls is called *crippling*



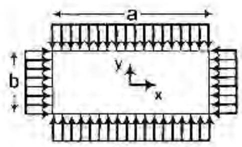
Crippling vs. Buckling

Critical Stress for Plate and Cylinder Buckling



$$\sigma_{cr} = K \frac{E}{1-\nu^2}\left(\frac{t}{b^2}\right)$$

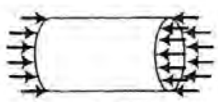
	a/b	0.4	0.8	1.0	1.4	1.8	2.0	2.4	3	∞
All sides SS	K	6.9	3.45	3.3	3.7	3.3	3.3	3.4	3.3	3.3
All sides F	K	–	–	7.7	–	–	6.7	–	6.4	5.7



All sides SS $\quad \sigma_{x,cr}\left(\frac{m^2}{a^2}\right) + \sigma_{y,cr}\left(\frac{n^2}{b^2}\right) = 0.823\frac{E}{1-\nu^2}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)$

All sides F $\quad \sigma_{x,cr} + \frac{a^2}{b^2}\sigma_{y,cr} = 1.1\frac{Et^2a^2}{1-\nu^2}(3/a^4 + 3/b^4 + 2/a^2b^2)$

where m, n refer to the modes in the x & y directions, respectively, and with $m, n = 1,2,3\ldots$



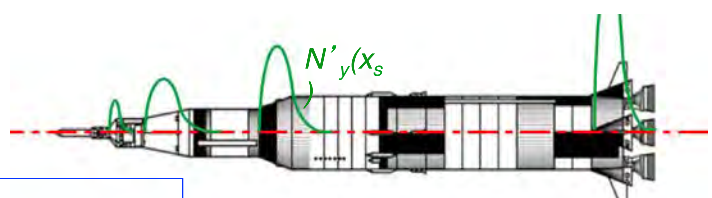
$\phi = \frac{1}{16}\sqrt{\frac{r}{t}}$

for $\frac{r}{t} < 1500$

Ends not constrained	$\sigma_{cr} = \frac{\gamma E}{\sqrt{3(1-\nu^2)}}\frac{t}{r}$
Axial	$\gamma = 1 - 0.901(1-e^{\phi})$
Bending	$\gamma = 1 - 0.731(1-e^{\phi})$

Pisacane, 2005

Bending Moment and Linear Deflection due to a Distributed Normal Force



$$M_y(x) = \int_{x_{\min}}^{x_{\max}} N_y(x)\left(x - x_{cm}\right)dx$$
$$= \int_{x_{\min}}^{x_{\max}}\int_{x_{\min}}^{x_{\max}} N'_y(x)dx\left(x - x_{cm}\right)dx$$

$N'(x) =$ normal force variation with length

Deflection is found by four integrations of the deflection equation

$$\frac{d^2}{dx^2}\left(EI_y\frac{d^2z}{dx^2}\right)\Bigg|_{x=x_s} = N'_y\left(x_s\right)$$

Bending Vibrations of a Free-Free Uniform Beam

$$EI_y \left.\frac{d^4 z}{dx^4}\right|_{x=x_s} = k = -m' \left.\frac{d^2 z}{dt^2}\right|_{x=x_s}$$

$$EI_y = \text{constant}$$
$$m' = \text{mass variation with length (constant)}$$
$$k = \text{effective spring constant}$$

Solution by separation of variables requires that left and right sides equal a constant, *k*

An infinite number of separation constants, k_i, exist

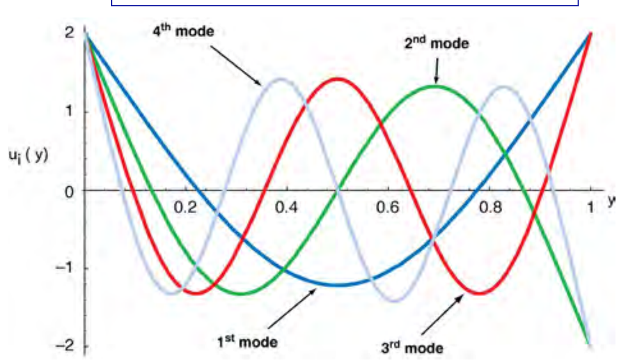
Therefore, there are an infinite number of vibrational response modes

Bending Vibrations of a Free-Free Uniform Beam

$$EI_y \frac{d^4 z}{dx^4} = k_i = -m' \frac{d^2 z}{dt^2}$$

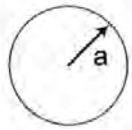
Mode shapes of bending vibrations

- **In figure, ($u = z$, $y = x$)**
- **Left side determines vibrational mode shape**
- **Right side describes oscillation**
- **Natural frequency of each mode proportional to $(k_i)^{1/2}$**



Fundamental Vibrational Frequencies of Circular Plates

***f* = natural frequency of first mode, Hz**



$$f = \frac{\lambda^2}{2\pi a^2}\sqrt{\frac{Eh^3}{12\gamma(1-\nu^2)}}$$

where h = plate thickness, γ = mass/unit area, and ν = Poisson's ratio

Description/End Conditions	λ^2 For Fundamental Frequency
Free edge	5.253
Simply supported (SS) edge	4.977
Clamped edge	10.22

Pisacane, 2005

Vibrational Mode Shapes for the X-30 (NASP) Vehicle

Computational Grid for Finite-Element Model

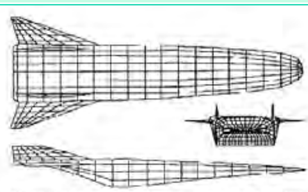


Fig. 1 Three-view finite element representation of subject configuration.

Raney, J. Aircraft, 1995

Shapes of the First Seven Modes

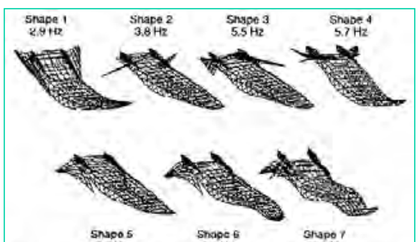


Fig. 2 Structural mode shapes and in-vacuo frequencies included in the aeroelastic model.

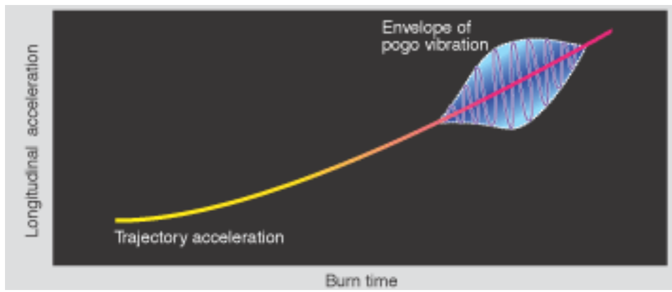
Body elastic deflection distorts the shape of scramjet inlet and exhaust ramps

Aeroelastic-propulsive interactions

Impact on flight dynamics

Pogo Oscillation

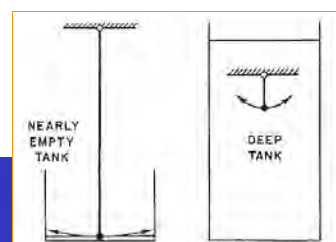
- **Longitudinal resonance of launch vehicle structure**
  - **Flexing of the propellant-feed pipes induces thrust variation in liquid-propellant rocket**
  - **Gas-filled cavities added to the pipes, damping oscillation**
  - **"Organ-pipe" oscillation in Space Shuttle Solid Rocket Booster**
    - **15-Hz resonance in 4-segment motor**
    - **12-Hz resonance in 5-segment motor**

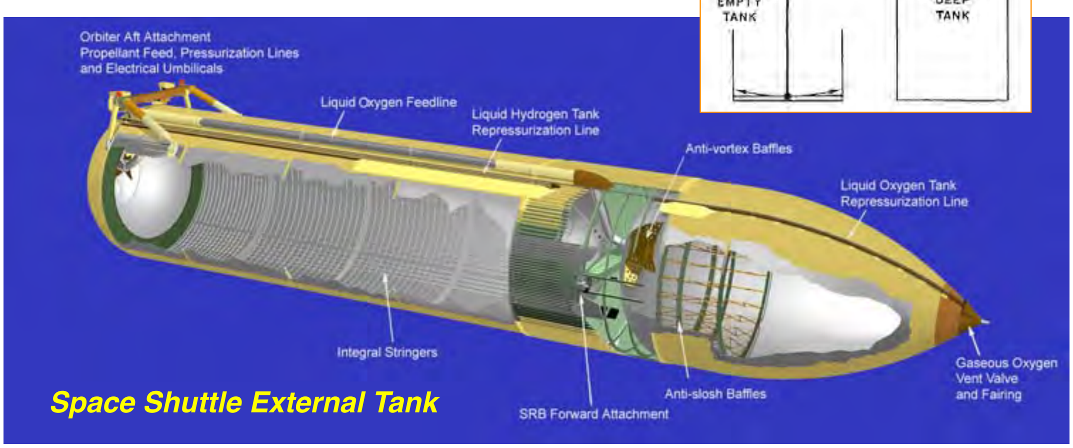


Pogo oscillation http://history.nasa.gov/SP-4205/ch10-6.html

Fuel Slosh

- **Lateral motion of liquid propellant in partially empty tank induces inertial forces**
- **Resonance with flight motions**
- **Problem reduced by baffling**



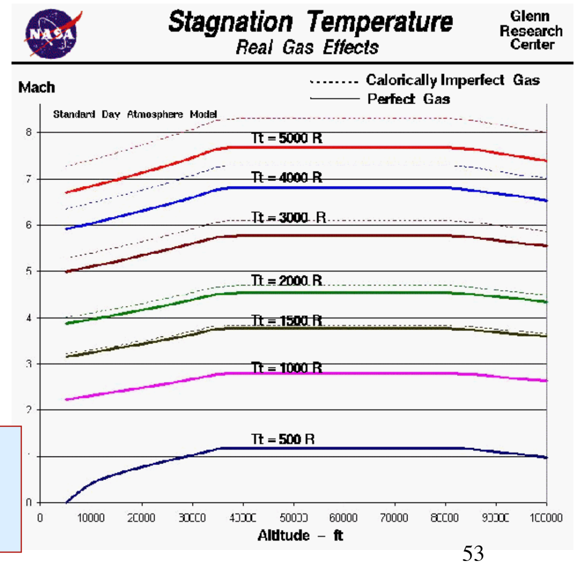


Space Shuttle External Tank

http://en.wikipedia.org/wiki/Space_Shuttle_external_tank

Thermal Stresses

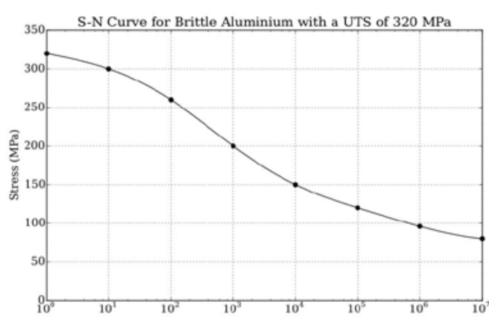
- Direct weakening of material by high temperature, e.g., effect of aerodynamic heating
- Embrittlement of metals at low temperature
- Internal stress caused by differential temperatures, e.g., on common bulkhead between hydrogen and oxygen tanks



Temperature of
Liquid Hydrogen: 20.3 K (–253°C)
Liquid Oxygen: 50.5 K (–223°C)

Fracture and Fatigue Failure from Repeated/Oscillatory Loading

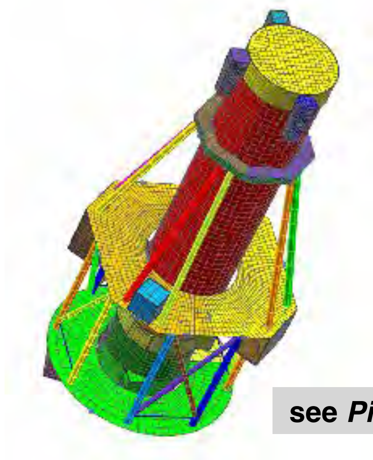
- Cyclic loading produces cracks
- Fatigue life: # of loading cycles before failure occurs

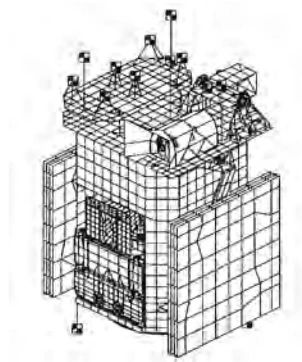


Miner’s rule, Paris’s law, Goodman relation, …

Finite-Element Structural Model

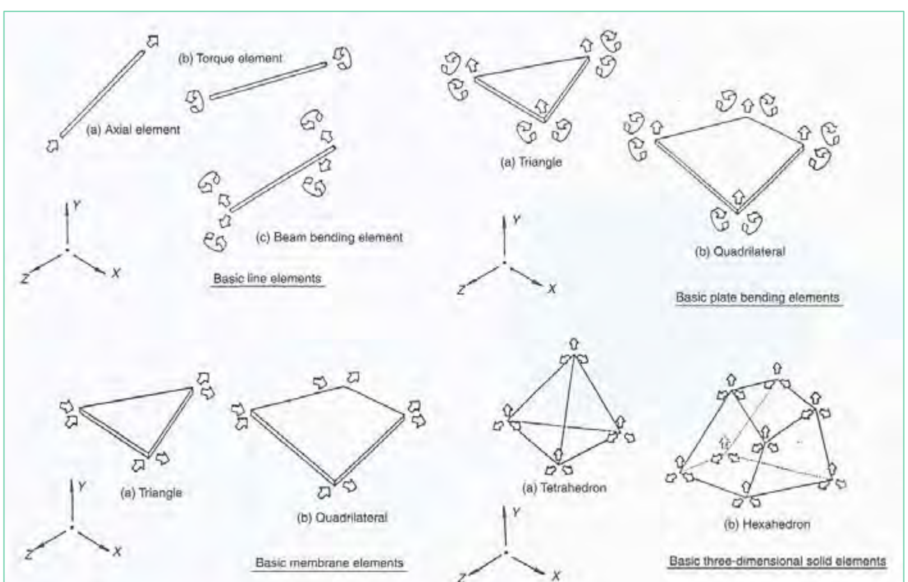
- Grid of elements, each with
  - Mass, damping, and elastic properties
  - 6 degrees of freedom at each node
- Static and dynamic analysis





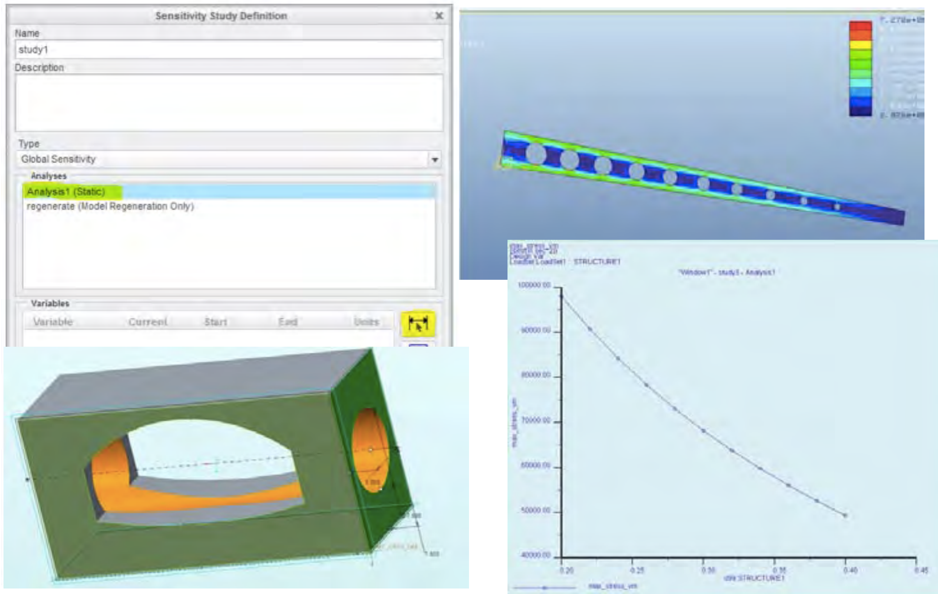
see *Pisacane (Skullney),* Sec 8.15

Types of Finite Element

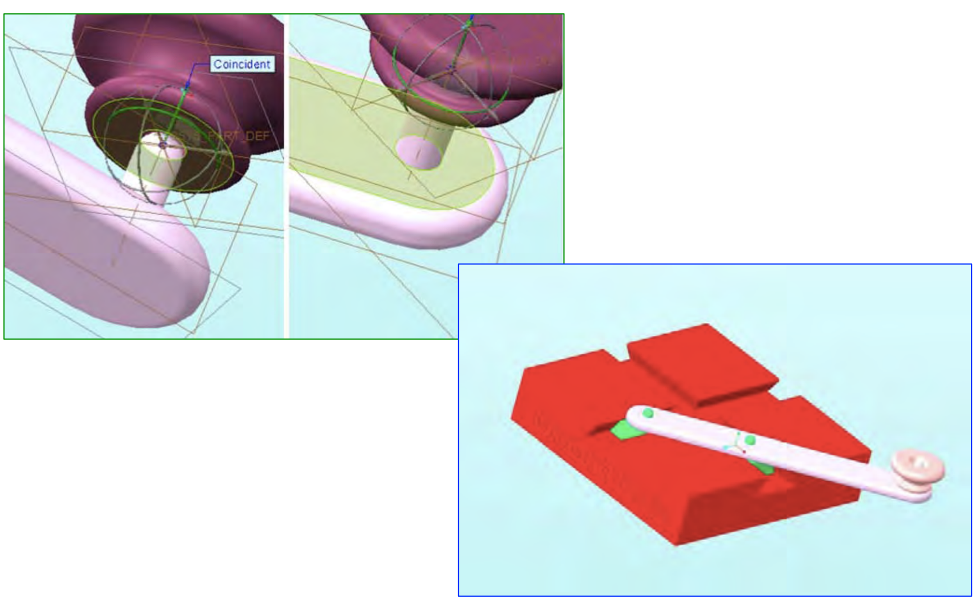


Fortescue, 2011

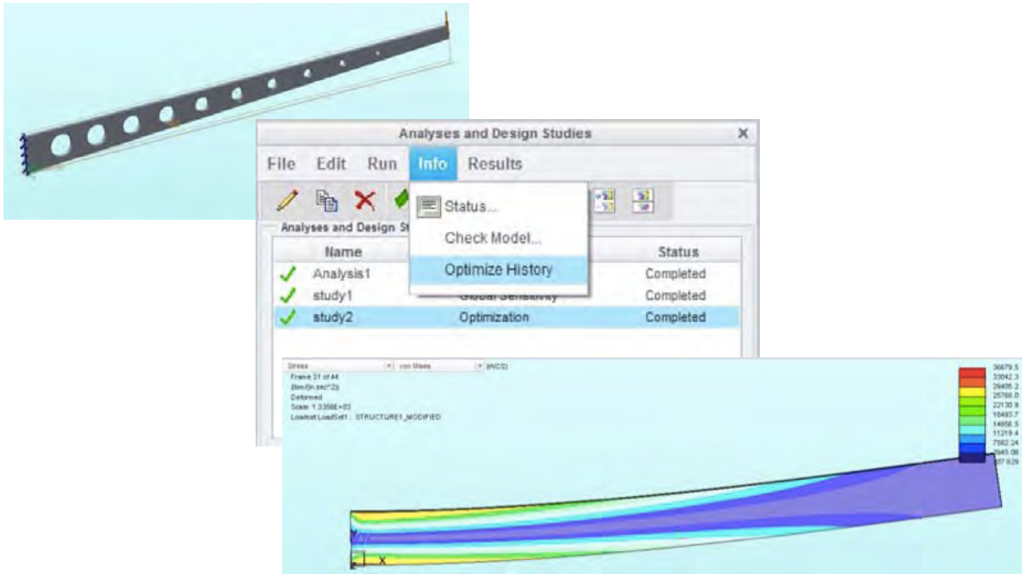
Structural Modeling Using PTC CREO



Assemble Parts Using PTC CREO



Analyze Loads Using PTC CREO



http://learningexchange.ptc.com/tutorial/799/creating-a-buckling-analysis

Next Time:
Spacecraft Configurations