Mobile Robots, Position, and Orientation

Robert Stengel
Robotics and Intelligent Systems MAE 345,
Princeton University, 2015

- **Math Review**
- **Ground Vehicles**
  - Legged creatures
  - Wheeled and tracked robots
  - Other
- **Frames of Reference and Pose**
- **Translation and Rotation**
- **Homogeneous Transformation**

Math Review

- *Matrix and Transpose*
- *Sums and Multiplication*
- *Matrix Products*
- *Identity Matrix*
- *Matrix Inverse*
- *Transformations*

Matrix and Transpose

- **Matrix**:
  - Usually bold capital or capital: **F** or F
  - Dimension = (m x n)
- **Transpose**:
  - Interchange rows and columns

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}$$

4 x 3

$$\mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}$$

3 x 4

Matrix Products

Matrix-vector product transforms one vector into another

$\mathbf{y} = A$

$$\mathbf{y} = \mathbf{Ax} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 + cx_3 \\ dx_1 + ex_2 + fx_3 \\ gx_1 + hx_2 + kx_3 \\ lx_1 + mx_2 + nx_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

Matrix-matrix product produces a new matrix

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}; \quad \mathbf{AB} = \begin{bmatrix} (a_1b_1 + a_2b_3) & (a_1b_2 + a_2b_4) \\ (a_3b_1 + a_4b_3) & (a_3b_2 + a_4b_4) \end{bmatrix}$$

$$(n \times m) = (n \times l)(l \times m)$$

Numerical Example 1

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

$$= \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Numerical Example 2

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad ; \quad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} (5+14) & (6+16) \\ (15+28) & (18+32) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\mathbf{x}_A = \mathbf{A}\mathbf{x}_B \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B$$

$$\mathbf{x}_B = \mathbf{B}\mathbf{x}_o \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o$$

$$\mathbf{x}_A = \mathbf{A}\mathbf{x}_B = \mathbf{AB}\mathbf{x}_o \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o$$

Square Matrix Identity and Inverse

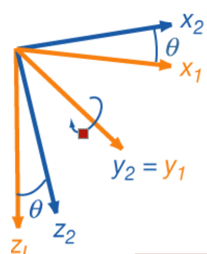
- **Identity matrix**: **no change** when it multiplies a conformable vector or matrix

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\mathbf{x} = \mathbf{I}\mathbf{x}}$$

- A **non-singular square matrix** multiplied by its inverse forms an identity matrix

$$\boxed{\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}}$$



Matrix Inverse Example

Transformation $\boxed{\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

Inverse Transformation $\boxed{\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

Consequently, ...

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1 = \mathbf{A}\mathbf{A}^{-1}\mathbf{x}_2 = \mathbf{x}_2$$

Computation of (n x n) Matrix Inverse

$$\mathbf{y} = \mathbf{A}\mathbf{x}; \quad \mathbf{x} = \mathbf{A}^{-1}\mathbf{y}$$

$$\dim(\mathbf{x}) = \dim(\mathbf{y}) = (n \times 1); \quad \dim(\mathbf{A}) = (n \times n)$$

$$[\mathbf{A}]^{-1} = \frac{\mathrm{Adj}(\mathbf{A})}{|\mathbf{A}|} = \frac{\mathrm{Adj}(\mathbf{A})}{\det \mathbf{A}} \quad \frac{(n \times n)}{(1 \times 1)}$$

$$= \frac{\mathbf{C}^T}{\det \mathbf{A}}; \quad \mathbf{C} = \textit{matrix of cofactors}$$

*Cofactors are <u>signed minors</u> of **A***

*ij^{th} minor of **A** is the <u>determinant</u> of **A** with the i^{th} row and j^{th} column removed*

MATLAB Code for Math Review

Use of Symbolic Variables

```
%   MAE 345 Lecture 2 Math Review
%   Rob Stengel

    clear
    disp('   ')
    disp('=====================================')
    disp('>>>MAE 345 Lecture 2 Math Review<<<')
    disp('=====================================')
    disp('   ')
    disp(['Date and Time are ', num2str(datestr(now))]);
    disp('   ')

%   Matrix
    syms A AT a b c d e f g h k l m n
    A   =   [a b c;d e f;g h k;l m n]   %   Matrix
    AT  =   A'                          %   Matrix Transpose

%   Matrix-Vector Product
    syms x x1 x2 x3 y1 y2 y3 y4
    x   =   [x1;x2;x3]
    y   =   [y1;y2;y3;y4]
    y   =   A * x
```


MATLAB Code for Math Review

```
%   Matrix-Matrix Product
    syms A a1 a2 a3 a4 B b1 b2 b3 b4 AB
    A   =   [a1 a2;a3 a4]
    B   =   [b1 b2;b3 b4]
    AB  =   A * B

%   Example 1
    syms A
    A   =   [2 4 6;3 -5 7;4 1 8;-9 -6 -3]
    y   =   A * x

%   Example 2
    A   =   [1 2;3 4]
    B   =   [5 6;7 8]
    AB  =   A * B

    syms xA xB x0
    x0  =   [x1;x2]
    xA  =   A * xB
    xB  =   B * x0
    xA  =   A * B * x0
```

MATLAB Code for Math Review

```
%   Matrix Identity and Inverse
    I3      =   eye(3)
    x       =   I3 * x
    syms A Ainv
    A       =   [a b c;d e f;g h k]
    Ainv    =   inv(A)
    I3      =   simplify(A * Ainv)
    I3      =   simplify(Ainv * A)

%   Matrix Inverse Example
    syms A Th cTh sTh Ainv
    A       =   [cTh 0 sTh;0 1 0;-sTh 0 cTh]
    Ainv    =   inv(A)
    detA    =   det(A)

    cTh     =   cos(Th)
    sTh     =   sin(Th)
    Th      =   pi / 4
    syms A Ainv
    A       =   [cos(Th) 0 sin(Th);0 1 0;-sin(Th) 0 cos(Th)]
    Ainv    =   inv(A)

%   Consequently, ...
    I3      =   A * Ainv

%   Computation of (n x n) Inverse
    detA    =   det(A)
    AdjA    =   Ainv * detA
```


MATLAB Command Window Output for Math Review

```
======================================
>>>MAE 345 Lecture 2 Math Review<<<
======================================

Date and Time are 03-Sep-2013 13:49:40

A =
[ a, b, c]
[ d, e, f]
[ g, h, k]
[ l, m, n]

AT =
[ conj(a), conj(d), conj(g), conj(l)]
[ conj(b), conj(e), conj(h), conj(m)]
[ conj(c), conj(f), conj(k), conj(n)]

x =
 x1
 x2
 x3

y =
 y1
 y2
 y3
 y4

y =
 a*x1 + b*x2 + c*x3
 d*x1 + e*x2 + f*x3
 g*x1 + h*x2 + k*x3
 l*x1 + m*x2 + n*x3
```

```
A =
[ a1, a2]
[ a3, a4]

B =
[ b1, b2]
[ b3, b4]

AB =
[ a1*b1 + a2*b3, a1*b2 + a2*b4]
[ a3*b1 + a4*b3, a3*b2 + a4*b4]

A =  2     4     6
     3    -5     7
     4     1     8
    -9    -6    -3

y = 2*x1 + 4*x2 + 6*x3
    3*x1 - 5*x2 + 7*x3
      4*x1 + x2 + 8*x3
  - 9*x1 - 6*x2 - 3*x3

A =  1     2
     3     4

B =  5     6
     7     8

AB = 19    22
     43    50
```

```
x0 =
 x1
 x2

xA =
[   xB, 2*xB]
[ 3*xB, 4*xB]

xB =
 5*x1 + 6*x2
 7*x1 + 8*x2

xA =
 19*x1 + 22*x2
 43*x1 + 50*x2

I3 =
     1     0     0
     0     1     0
     0     0     1

x =
 x1
 x2
 x3
```

MATLAB Command Window
Output for Math Review

```
A =
[ a, b, c]
[ d, e, f]
[ g, h, k]

Ainv =
[  (f*h - e*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k), -(c*h - b*k)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k), -(b*f - c*e)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]
[ -(f*g - d*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k),  (c*g - a*k)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k),  (a*f - c*d)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]
[ -(d*h - e*g)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k),  (a*h - b*g)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k), -(a*e - b*d)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]

I3 =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]

I3 =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]
```

```
A = [  cTh, 0, sTh]
     [    0, 1,   0]
   [ -sTh, 0, cTh]

Ainv =
[ cTh/(cTh^2 + sTh^2), 0, -sTh/(cTh^2 + sTh^2)]
[                   0, 1,                    0]
[ sTh/(cTh^2 + sTh^2), 0,  cTh/(cTh^2 + sTh^2)]

detA = cTh^2 + sTh^2

cTh = cos(Th)

sTh = sin(Th)

Th = 0.7854

A = 0.7071         0    0.7071
         0    1.0000         0
   -0.7071         0    0.7071

Ainv = 0.7071         0   -0.7071
            0    1.0000         0
       0.7071         0    0.7071

I3 = 1     0     0
     0     1     0
     0     0     1

detA = 1

AdjA = 0.7071         0   -0.7071
            0    1.0000         0
       0.7071         0    0.7071
```

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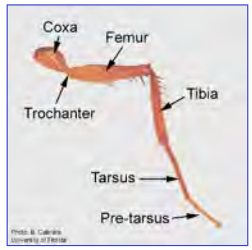
Legged Creatures

Walking, Running, and Jumping



Human Walking
https://www.youtube.com/watch?v=Fws-HYAQvq8

Spider Walking
http://www.youtube.com/watch?v=dE2QPYKju04

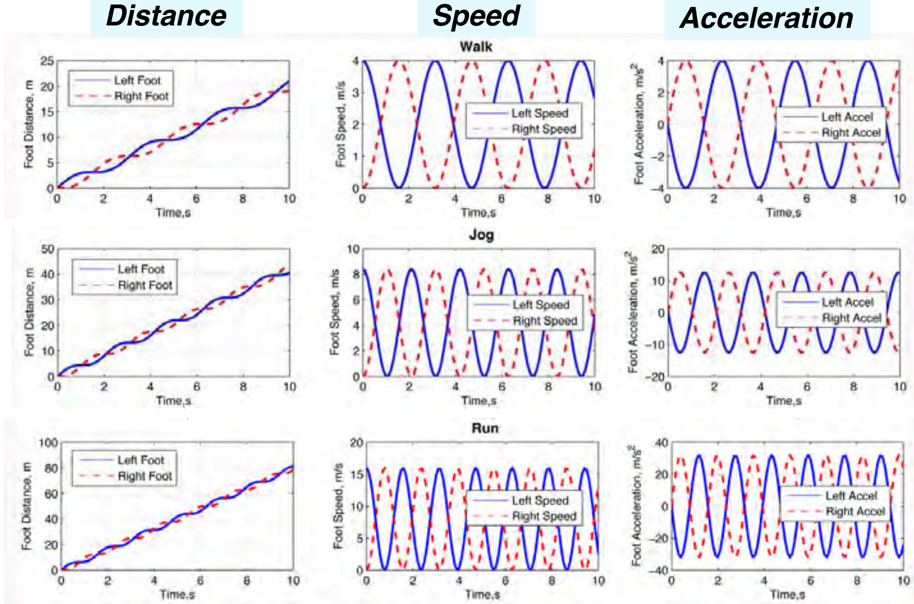
Spider Walk Animation
http://www.youtube.com/watch?v=MFx36uEPxV8&NR=1

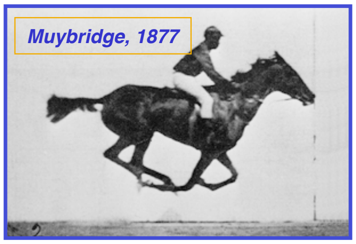
FreeRunning
http://www.youtube.com/watch?v=WEeqHj3Nj2c

Dynamic Effects Increase with Speed

- Horizontal foot motion ~ sinusoidal oscillation
- Increasing acceleration from walk to jog to run
- Increasing importance of forces and dynamics

Distance ***Speed*** ***Acceleration***

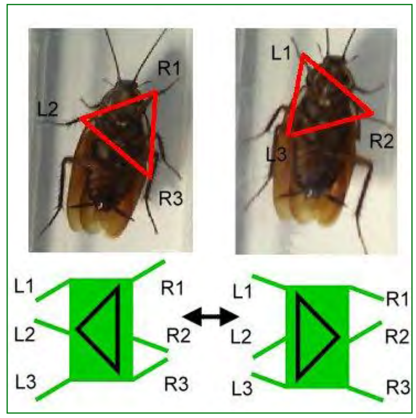




Gaits

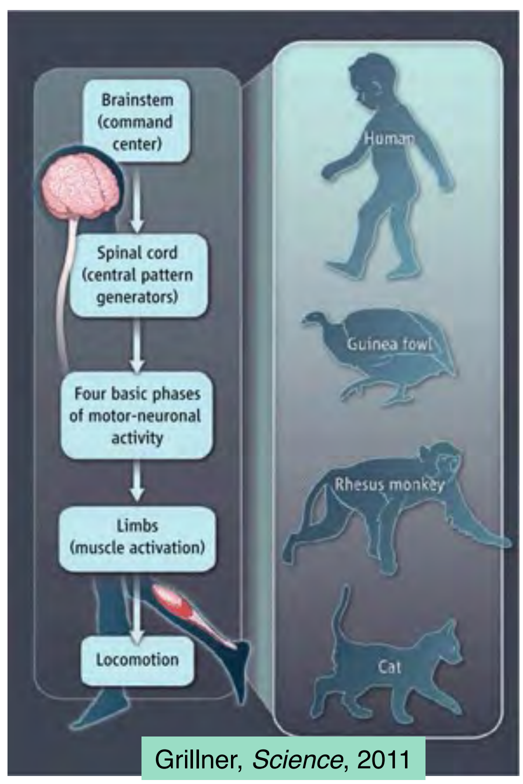
- Biped
- Quadraped
- Hexaped

- Walking
  - Statically stable
  - Statically unstable
- Running



Locomotor Primitives

- Common across legged vertebrate species
- Brain command
- Spinal column central pattern generator
- Phases of motor-neuronal activity
  - "Toe-off"
  - Flexion
  - Extension
  - Limb alternation
- Muscle activation



Grillner, *Science*, 2011

Biped Robots

Passive Walking
TU Delft

Cytron Kit Robot

Passive Walking Robots
http://www.youtube.com/watch?v=Njos0_r6TE4

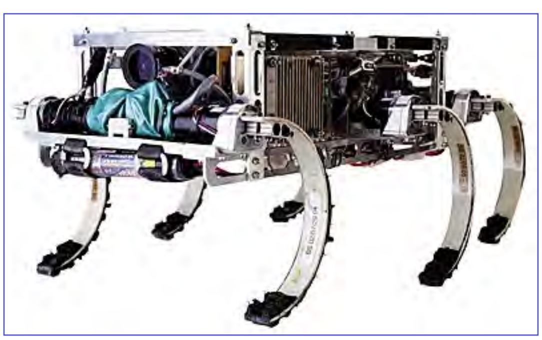
MIT Leglab Walking Robots
http://www.youtube.com/watch?v=vHjVV7AWaGM

Hexapod Robots

Combined walking and rolling motion
Alternating tripod gate

RHex (Boston Dynamics)
http://www.youtube.com/watch?v=a0NFrA-Nx4Y

iJus (Princeton '13 IW)
https://www.youtube.com/watch?v=35owx65Ei6g&hd=1

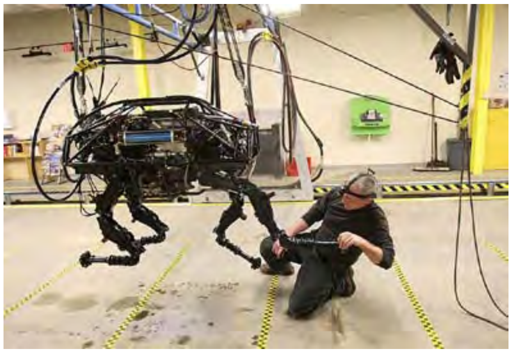


Rigid body

Flexible spine
(3 segments)

Big Dog and PETMAN

Boston Dynamics



http://www.youtube.com/watch?v=xqMVg5ixhd0

http://www.youtube.com/watch?feature=player_embedded&v=tFrjrgBV8K0

Robotic Exoskeleton (UC Berkeley)

BLEEX
http://www.youtube.com/watch?v=fRkg6H0ZP8A

Paraplegic student walks at 2011 UC Berkeley graduation
http://newscenter.berkeley.edu/2011/05/12/paraplegic-student-exoskeleton-graduation-walk/

Smart Knee and Robot Ankle

Stairs (Traditional Prosthetic)

Stairs (MIT Smart Knee)

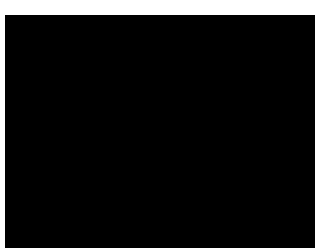
Robotic Ankle
http://www.youtube.com/watch?v=HhSVqsHzRI4

Hopping Robots

(Raibert, ~1990)

High inertia of "sprung" mass

2-D (Planar)



3-D

Kangeroo hopping
http://www.youtube.com/watch?v=OpYRIW314sE

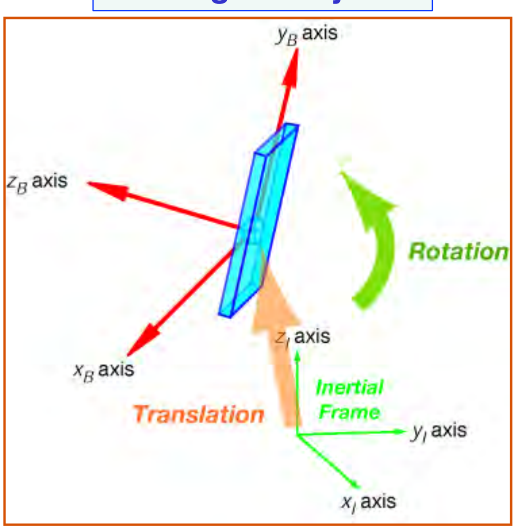
Sandia Robot
http://www.youtube.com/watch?v=SDSkqt2xpcc

Frames of Reference

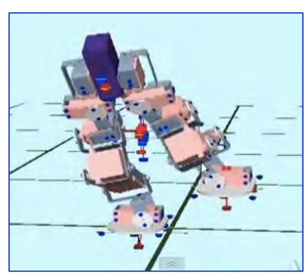
Pose of an Object

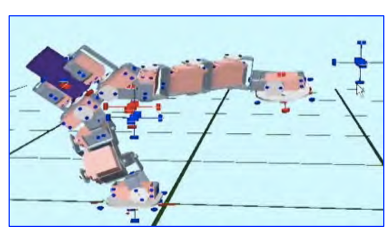
Expression of an object's frame(s) of reference with respect to the original frame

Rigid Body

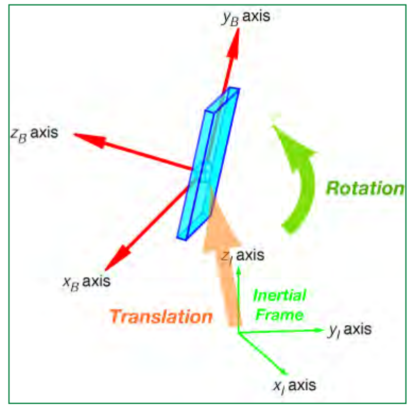


Multi-Link Body

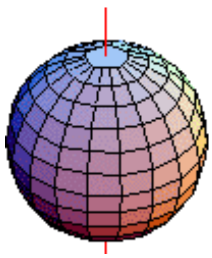




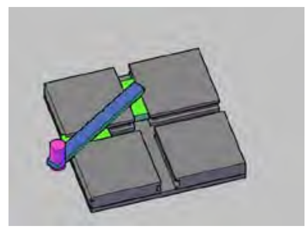
Transformations Between Reference Frames



Rotation



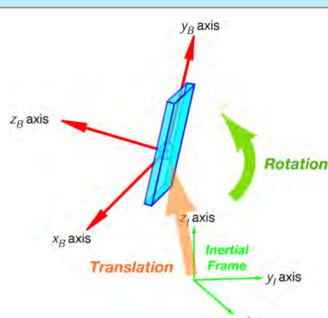
Translation



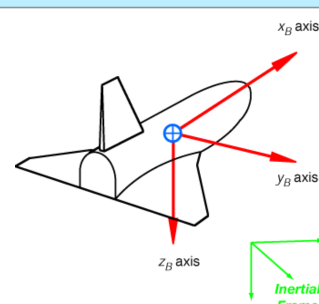
Cartesian Frames of Reference

- **Reference frames of interest**
  - ***I*: Inertial frame (fixed to inertial space, unmoving)**
  - ***B*: Body frame (fixed to body, moving, non-inertial)**
- **Translation**
  - **Linear position of the body frame origin with respect to the inertial frame origin**
- **Rotation**
  - **Orientation of the body frame axes with respect to the inertial frame axes**

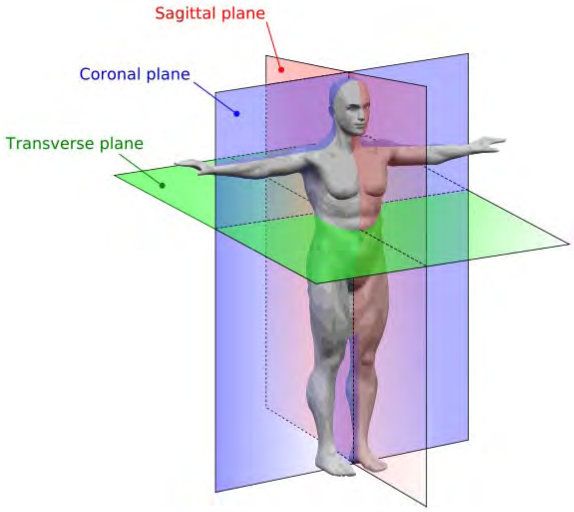
Common convention (*z* up)

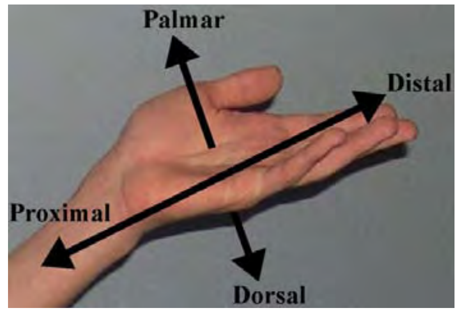


Aircraft convention (*z* down)

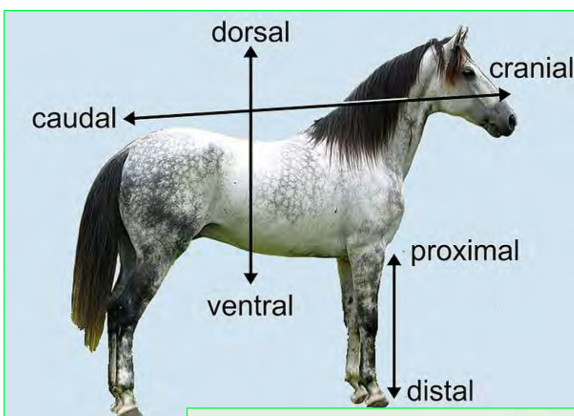


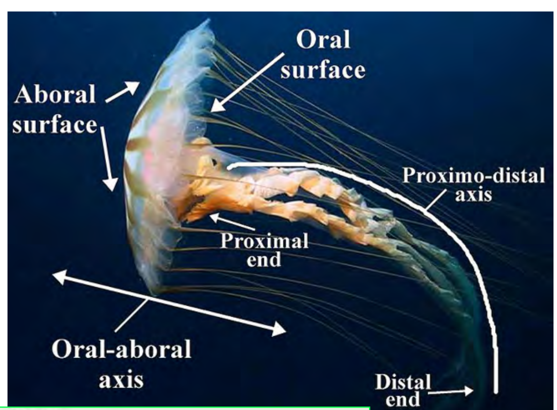
Human Anatomical Coordinates

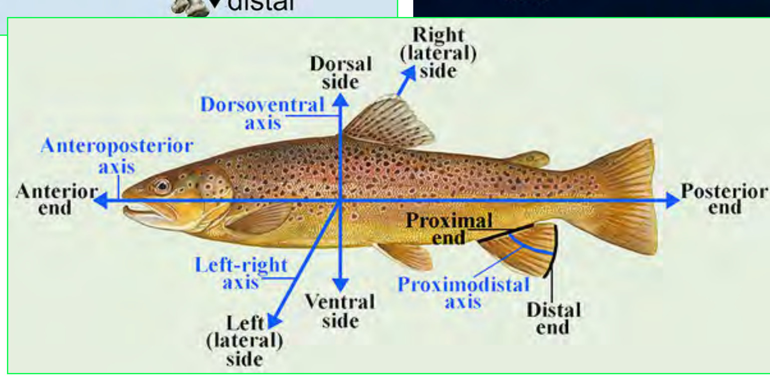




Animal Anatomical Coordinates

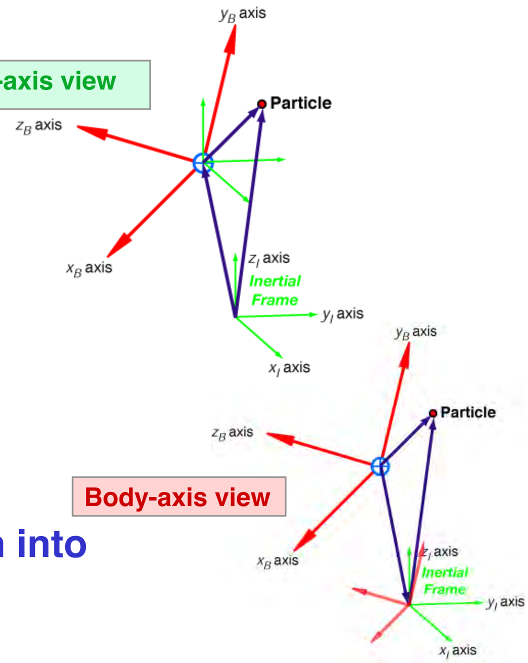






Measurement of Position in Alternative Frames - 1

Inertial-axis view



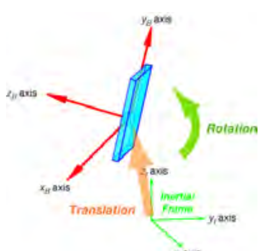
Body-axis view

Position vector

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\mathbf{r}_{particle} = \mathbf{r}_{origin} + \Delta\mathbf{r}_{w.r.t.origin}$$

Differences in frame orientations must be taken into account in adding vector components



Measurement of Position in Alternative Frames - 2

Inertial-axis view

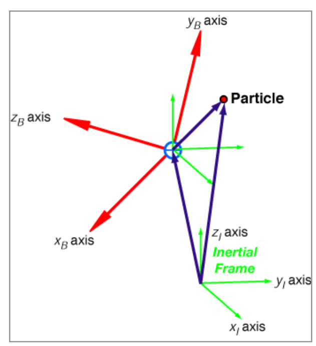
$$\mathbf{r}_{particle_I} = H_B^I \mathbf{r}_B + \mathbf{r}_{body\ origin_I}$$

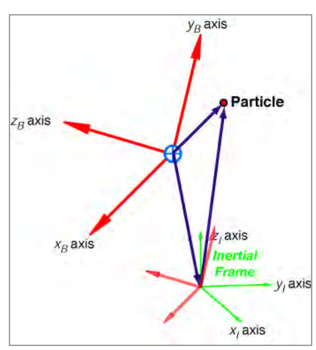
H_B^I : from Body-Axis Vector to Inertial-Axis Vector

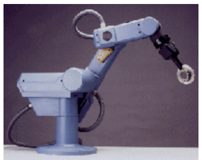
Body-axis view

$$\mathbf{r}_{particle_B} = H_I^B \mathbf{r}_I + \mathbf{r}_{inertial\ origin_B}$$

H_I^B : from Inertial-Axis Vector to Body-Axis Vector







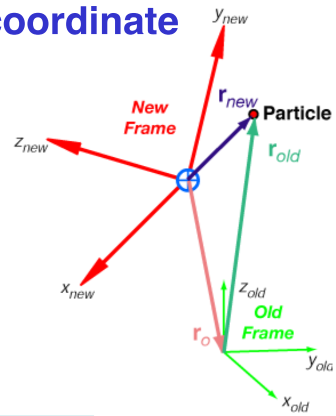
Rotation + Translation
(“Forward Kinematics”)

- **Expression of a vector in a new coordinate frame**
  - Displaced from old frame
  - Rotated w.r.t. old frame

$$\mathbf{r}_{new} = H_{old}^{new}\mathbf{r}_{old} + \mathbf{r}_{old_{new}}$$

Rotation matrix

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



- **Augmented vector**
  - Concatenate a “1” to r

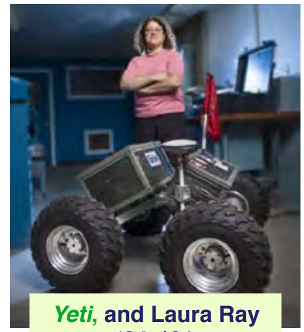
$$\mathbf{s} = \begin{bmatrix} \mathbf{r} \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \equiv$$

Homogeneous coordinate

Rolling Vehicles

Wheeled and Tracked Ground Vehicles

- Vacuum cleaners (*Roomba*)
- Military/Emergency robots (*PackBot*)
- Exploration robots (*Yeti*)



Yeti, and Laura Ray '84, *91

PackBot in Action
http://www.youtube.com/watch?v=eaP0waiz43w

Yeti in Greenland
https://www.youtube.com/watch?v=9DhX02R3QSo

http://www.youtube.com/watch?v=CLIPLiQDIk0

Figure 1: Blue quadrilateral defines the exclusion zone

DARPA Grand Challenge First Round

http://www.youtube.com/watch?v=uWLjqs2CEyE

Autonomous Automobiles

Stanford Stanley (DARPA Winner, 2005)

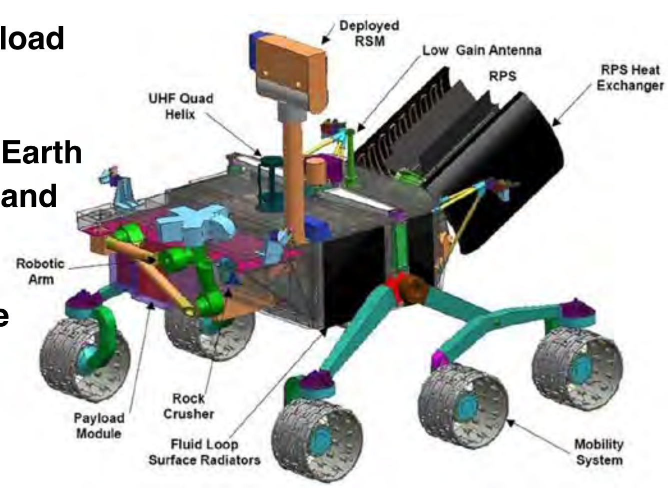
CMU-GM (DARPA 2007 Winner)

Princeton Autonomous Vehicle Engineering (PAVE) Prospect 11, Prospect 12, plus visitor

Google Self-Driving Car

Mars Science Laboratory
(Curiosity)

- **Transport science payload over Martian surface**
  - **Rocker-bogie design**
- **Communications with Earth**
- **Guidance, navigation, and control**
- **Power supply**
- **Support for deployable devices**
- **Size ~ Mini-Cooper**
- **Landed, 8/6/12, and operational**



Curiosity Trailer
http://www.jpl.nasa.gov/video/details.php?id=1014

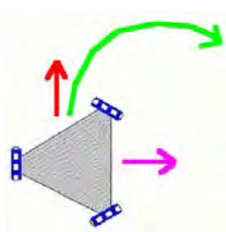
Sphero Ball and BB-8

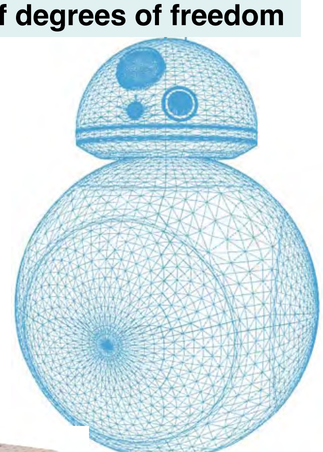
https://www.youtube.com/watch?v=A_K10fX9DSY

Holonomic Robots

Controllable # of degrees of freedom = Total # of degrees of freedom

Mecanum Wheel





NonHolonomic Robots

Controllable # of degrees of freedom ≠ Total # of degrees of freedom

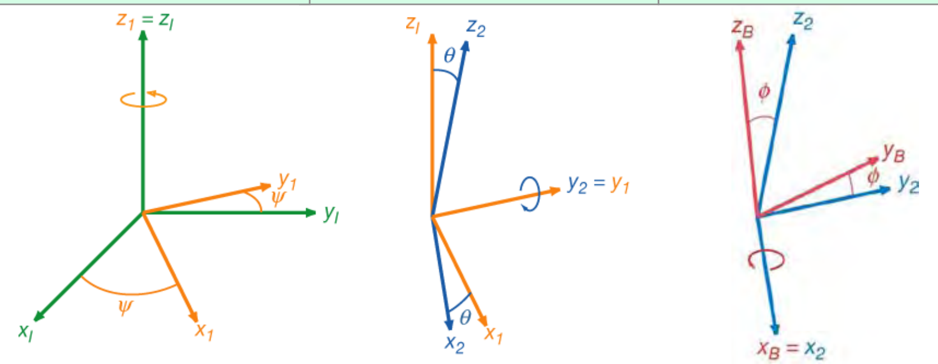
Rotational Orientation of a Rigid Body

Orientation of One Frame with Respect to Another

Euler Angles

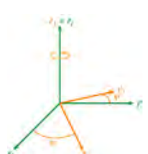
- **Conventional sequence of rotations from inertial to body frame**
  - **Each rotation occurs about a single axis**
  - **Right-hand rule**
  - **Yaw, then pitch, then roll**

Yaw rotation (ψ) about z_I	Pitch rotation (θ) about y_1	Roll rotation (ϕ) about x_2



Effects of Orientation on Vector Transformation

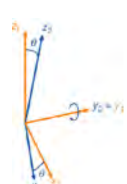
Yaw rotation (ψ) about z_I



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_I = \begin{bmatrix} (x_I \cos\psi + y_I \sin\psi) \\ (-x_I \sin\psi + y_I \cos\psi) \\ z_I \end{bmatrix}$$

$$\mathbf{r}_1 = \mathbf{H}_I^1 \mathbf{r}_I$$

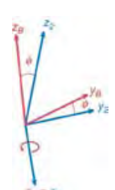
Pitch rotation (θ) about y_1



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

$$\mathbf{r}_2 = \mathbf{H}_1^2 \mathbf{r}_1 = \mathbf{H}_1^2 \mathbf{H}_I^1 \mathbf{r}_I = \mathbf{H}_I^2 \mathbf{r}_I$$

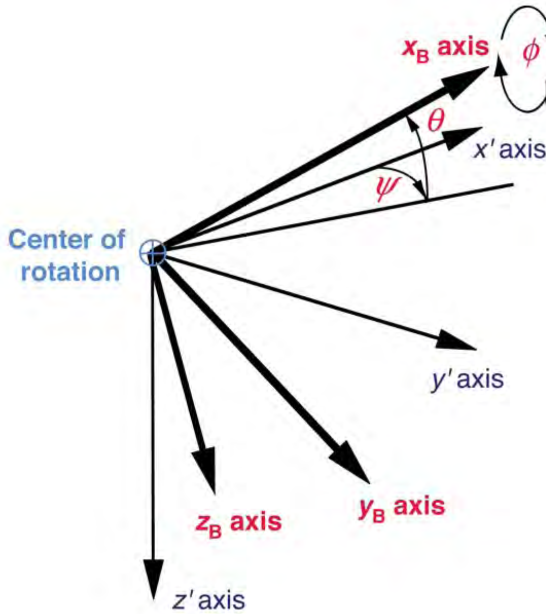
Roll rotation (ϕ) about x_2



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

$$\mathbf{r}_B = \mathbf{H}_2^B \mathbf{r}_2 = \mathbf{H}_2^B \mathbf{H}_1^2 \mathbf{r}_1$$
$$= \mathbf{H}_2^B \mathbf{H}_I^2 \mathbf{H}_I^1 \mathbf{r}_I = \mathbf{H}_I^B \mathbf{r}_I$$

Euler Angles (with *z* Axis down)



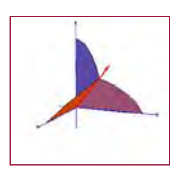
The Rotation Matrix*

$$\boxed{\mathbf{H}_I^B(\phi,\theta,\psi) = \mathbf{H}_2^B(\phi)\mathbf{H}_1^2(\theta)\mathbf{H}_I^1(\psi)}$$

$$\mathbf{H}_I^B(\phi,\theta,\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{c|c|c} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \hline -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \hline \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{array}\right]$$

*** also called *Direction Cosine Matrix (see supplement)***



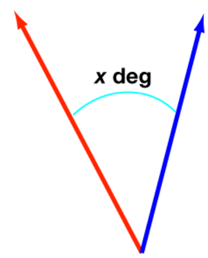
Properties of the Rotation Matrix

- The three-Euler-angle rotation matrix from *I* to *B* is the product of 3 single-angle rotation matrices

$$\boxed{\mathbf{H}_I^B(\phi,\theta,\psi) = \mathbf{H}_2^B(\phi)\mathbf{H}_1^2(\theta)\mathbf{H}_I^1(\psi)}$$

- The rotation matrix produces an orthonormal transformation
  - Angles are preserved
  - Lengths are preserved

$$\boxed{\begin{array}{l} |\mathbf{r}_I| = |\mathbf{r}_B| \quad ; \quad |\mathbf{s}_I| = |\mathbf{s}_B| \\ \angle(\mathbf{r}_I,\mathbf{s}_I) = \angle(\mathbf{r}_B,\mathbf{s}_B) \end{array}}$$



- With same origins, $\mathbf{r}_o = 0$

$$\boxed{\mathbf{r}_B = \mathbf{H}_I^B \mathbf{r}_I}$$

Orthonormal Transformation of Vector Coordinates

Same vector, different points of view

From inertial frame to body frame

$$\begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix}$$

From body frame to inertial frame

$$\begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix}$$

Orthonormal Rotation

- Inverse relationship: Transformation from *B* to *I*

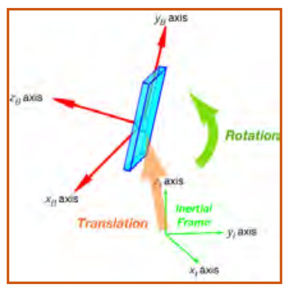
$$\boxed{\mathbf{r}_B = \mathbf{H}_I^B \mathbf{r}_I \quad ; \quad \mathbf{r}_I = \left(\mathbf{H}_I^B\right)^{-1} \mathbf{r}_B = \mathbf{H}_B^I \mathbf{r}_B}$$

- Because rotation transformation is orthonormal,
  - Inverse = transpose
  - Rotation matrix is always non-singular

$$\boxed{\mathbf{H}_B^I = \left(\mathbf{H}_I^B\right)^{-1} = \left(\mathbf{H}_I^B\right)^T} \quad \boxed{= \mathbf{H}_1^I \mathbf{H}_2^1 \mathbf{H}_B^2}$$

$$\boxed{\mathbf{H}_B^I \mathbf{H}_I^B = \mathbf{H}_I^B \mathbf{H}_B^I = \mathbf{I}}$$

Homogeneous Transformation Matrix



Express rotation and translation in a single transformation

$$\mathbf{s}_{new} = \left[\begin{array}{c|c} \begin{pmatrix}\text{Rotation} \\ \text{Matrix}\end{pmatrix}_{old}^{new} & \begin{pmatrix}\text{Location} \\ \text{of Old} \\ \text{Origin}\end{pmatrix}_{new} \\ \hline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array}\right] \mathbf{s}_{old} = \mathbf{A}_{old}^{new}\, \mathbf{s}_{old}$$

$$(4\times 1)_{new} = \left[\begin{array}{c|c} (3\times 3) & (3\times 1) \\ \hline (1\times 3) & (1\times 1) \end{array}\right] (4\times 1)_{old} = \left[(4\times 4)\right](4\times 1)_{old}$$

Homogeneous Transformation

- **Rotation <u>and</u> translation can be expressed in terms of homogeneous coordinates**
  - **Single matrix-vector product produces rotation and transformation**

$$\mathbf{s}_{new} = \begin{bmatrix} & H_{old}^{new} & & \mathbf{r}_{old_{new}} \\ \big(0 & 0 & 0 \big) & 1 \end{bmatrix} \mathbf{s}_{old} = \mathbf{A}\ \mathbf{s}_{old}$$

- or

$$\begin{bmatrix} x \\ y \\ x \\ 1 \end{bmatrix}_{new} = \left[\begin{array}{ccc|c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{old}$$

Equivalent Scalar Equations for Homogeneous Transformation

$$\mathbf{s}_{new} = \mathbf{A}_{old}^{new}\ \mathbf{s}_{old}$$

Matrix-Vector Multiplication

$$\begin{bmatrix} x \\ y \\ x \\ 1 \end{bmatrix}_{new} = \left[\begin{array}{ccc|c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{old}$$

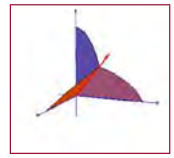
Individual Operations

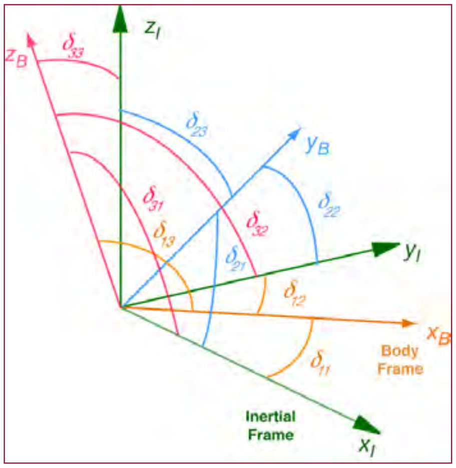
$$\begin{aligned} x_{new} &= h_{11}x_{old} + h_{12}y_{old} + h_{13}z_{old} + x_o \\ y_{new} &= h_{21}x_{old} + h_{22}y_{old} + h_{23}z_{old} + y_o \\ z_{new} &= h_{31}x_{old} + h_{32}y_{old} + h_{33}z_{old} + z_o \\ &----\\ 1 &= 1 \end{aligned}$$

Next Time:
Flying Robots, Motion, and Dynamics

Supplemental Material

Direction Cosine Matrix





Angles between each *I* axis and each *B* axis

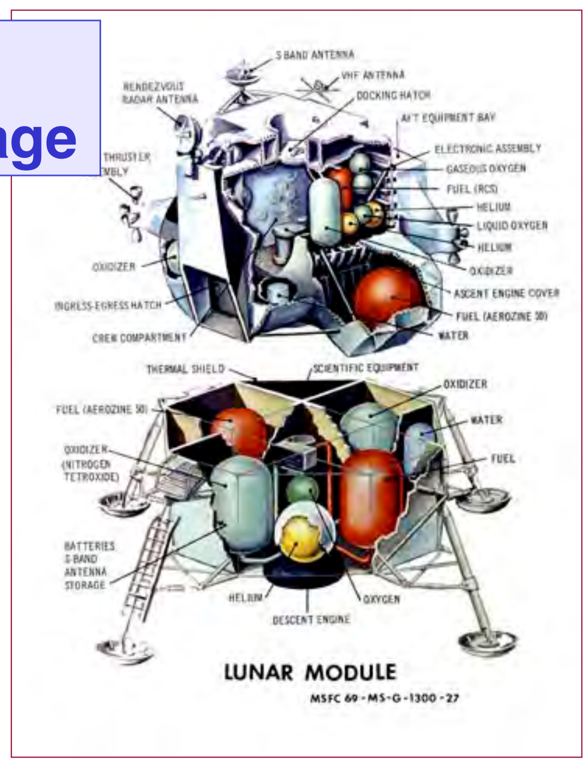
$$\mathbf{H}_I^B = \begin{bmatrix} \cos\delta_{11} & \cos\delta_{21} & \cos\delta_{31} \\ \cos\delta_{12} & \cos\delta_{22} & \cos\delta_{32} \\ \cos\delta_{13} & \cos\delta_{23} & \cos\delta_{33} \end{bmatrix}$$

Projection of inertial components of a vector onto body axes

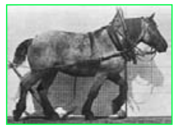
$$\begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = \begin{bmatrix} \cos\delta_{11} & \cos\delta_{21} & \cos\delta_{31} \\ \cos\delta_{12} & \cos\delta_{22} & \cos\delta_{32} \\ \cos\delta_{13} & \cos\delta_{23} & \cos\delta_{33} \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix}$$

Rotation of Lunar Module Ascent Stage

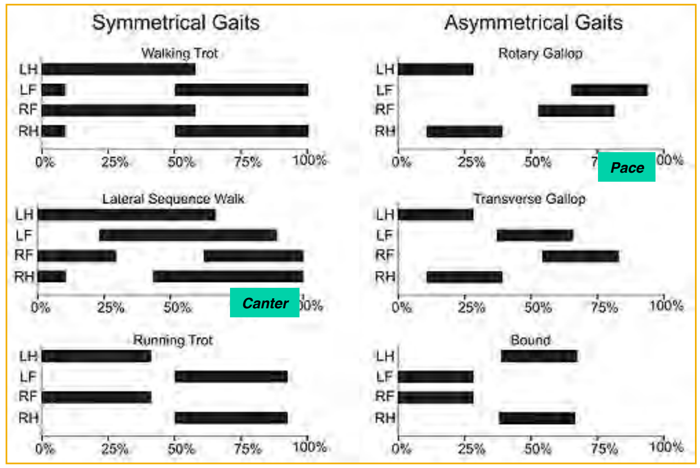
LM Ascent Stage from CSM



Quadraped Gaits

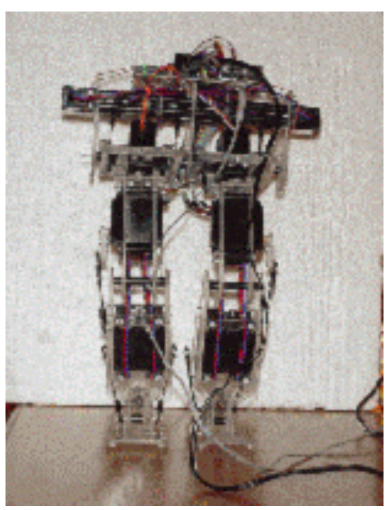


Feet on the Ground



American Android All-Terrain Biped

(David Handelman, *89)

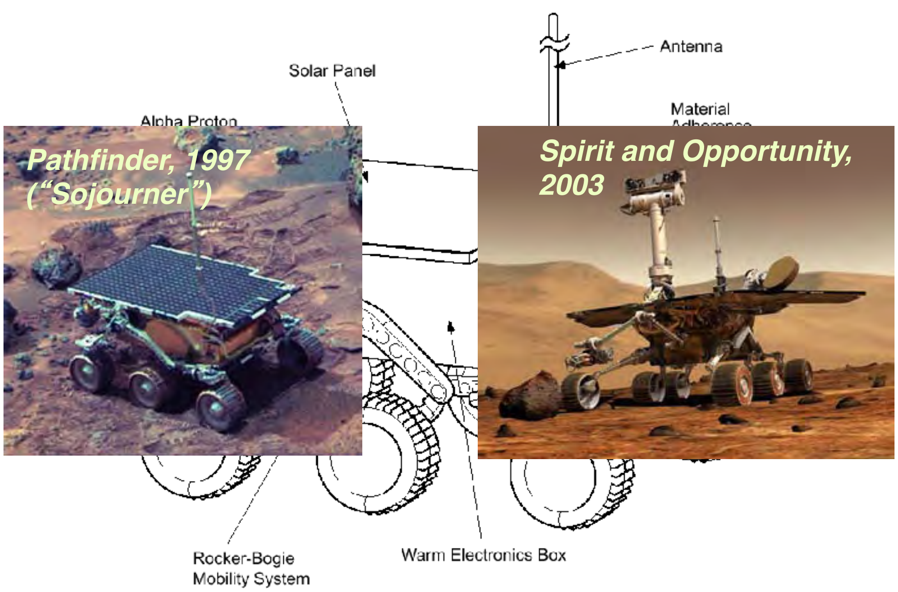


http://www.youtube.com/watch?v=UX0P11wNkcM

Mantis Hexapod Vehicle

http://www.youtube.com/watch?v=1sRIFQLwg3w

Mars Exploration Rovers



Personal Assistance

Surveillance Robots

SECOM Robot X
http://www.youtube.com/watch?v=0b6izpxj61o

Oculus Robot
http://www.youtube.com/watch?v=Q4L3Ujsclnk

Telepresence Robots

iRobot Ava 500 Video Collaboration Robot
http://www.youtube.com/watch?v=hVviDvsBQ78

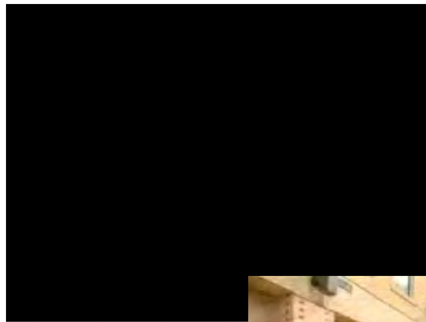
VGo Telepresence Robot
http://www.youtube.com/watch?v=8fdXStgdhEg

Personal Assistance Robots

iBot and Segway *(DEKA)*





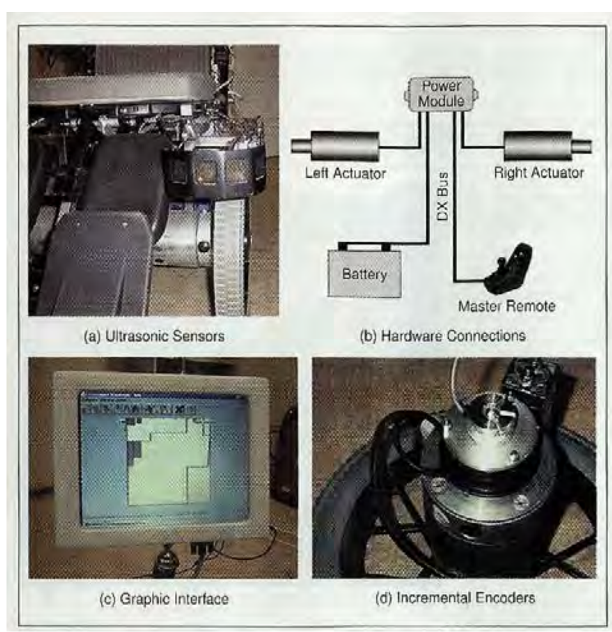


System Redundancy

Failure-tolerant Stability

Autonomous Wheelchairs

IEEE Robotics & Automation Magazine, March 2001



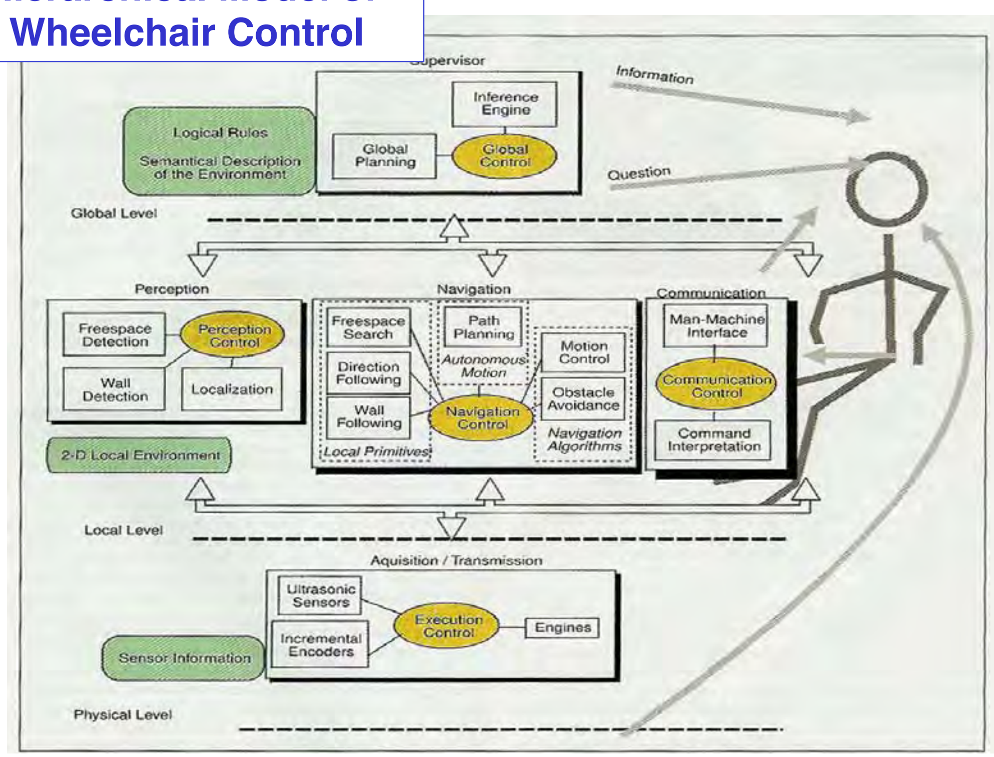
Robotic Friends for Young and Old

Paro Robotic Seal
http://www.youtube.com/watch?v=Vx8mv87e6wE

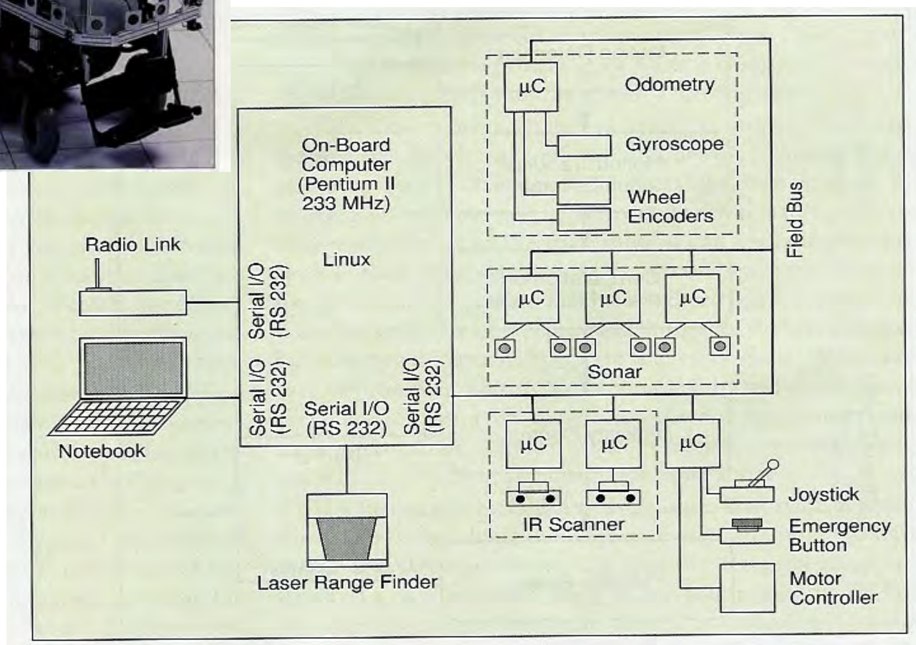
PaPeRo
http://www.youtube.com/watch?v=Z_QKHS3lydA

Furby
http://www.youtube.com/watch?v=lAZ8QG8uz9I

Hierarchical Model of Wheelchair Control



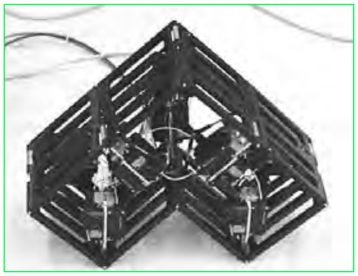
Wheelchair and Control Hardware

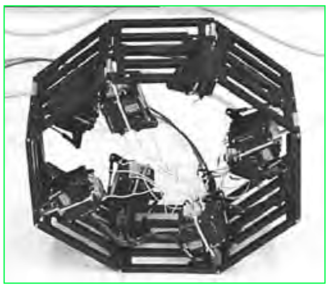


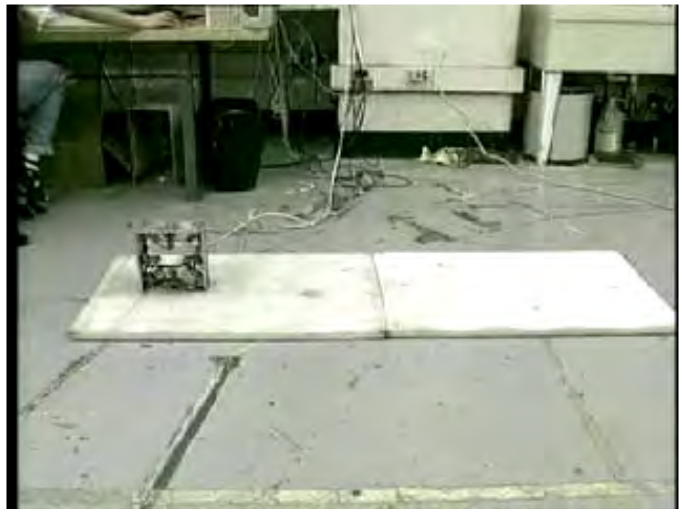
Other

The Blob

(MIT Leg Laboratory, 1995-97)







Meshworm Robot

(Seoul, MIT, Harvard)

Mesh of shape-memory alloy activated by differential heating

http://www.youtube.com/watch?v=EXkf62qGFII

Snake Robots

Games and Toys

Games

Toys

Toys and A.I.

The Uncanny Valley