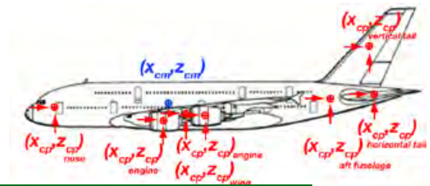
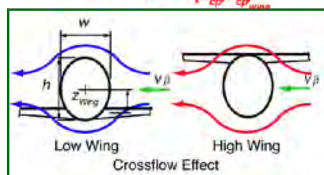
Aerodynamic Moments (i.e., Torques)

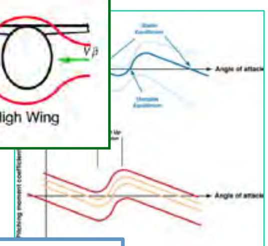
Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2014

Learning Objectives

- Expressions for aerodynamic balance and moment
- Concepts of aerodynamic center, center of pressure, and static margin
- Configuration and angle-of-attack effects on pitching moment and stability
- Calculate configuration and sideslip-angle effects on lateral-directional (i.e., rolling and yawing) aerodynamic moments
- Tail design effects on airplane aerodynamics







Reading:
Flight Dynamics
Aerodynamic Coefficients, 96-118

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

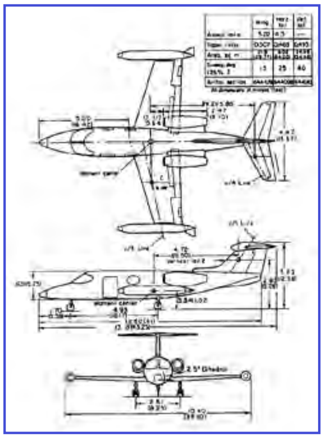
Assignment #3
due: End of day, Oct 3, 2014

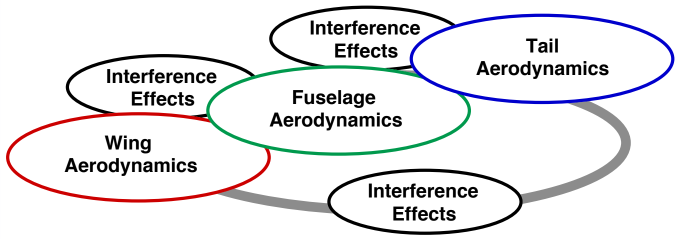
- ***Estimate and plot the non-dimensional force and moment aerodynamic coefficients and control effects for the Cessna Citation Mustang 510 in the Mach number range from 0 to 0.65***
- ***3- and 4-member teams***

https://www.youtube.com/watch?v=wOSJ3mdwDa0

Handbook Approach to Aerodynamic Estimation

- **Build estimates from component effects**
  - **USAF Stability and Control DATCOM (download at http://www.pdas.com/datcomb.html)**
  - **USAF Digital DATCOM (see *Wikipedia* page)**
  - **ESDU Data Sheets (see *Wikipedia* page)**

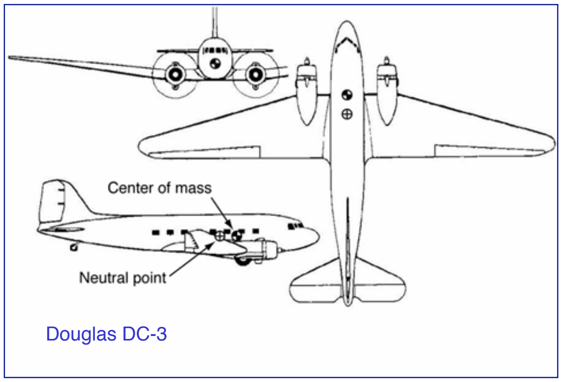




Moments of the Airplane

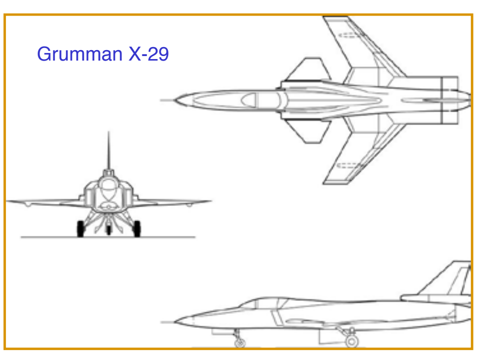
Airplane Balance

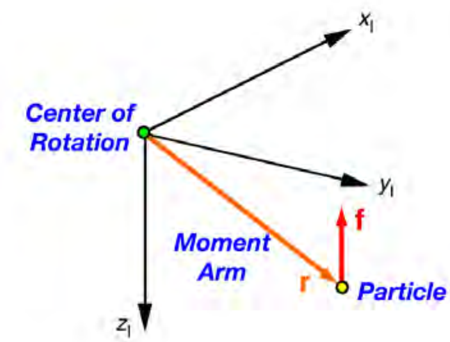
- <u>Conventional aft-tail configuration</u>
  - c.m. near wing's aerodynamic center (point at which wing's pitching moment coefficient is invariant with angle of attack ~25% mac)
- <u>Tailless airplane</u>: c.m. ahead of the neutral point



Airplane Balance

- <u>Canard configuration</u>:
  - Neutral point moved forward by canard surfaces
  - Center of mass may be behind the neutral point, requiring closed-loop stabilization
- Fly-by-wire feedback control can expand envelope of allowable center-of-mass locations





Moment Produced By Force on a Particle

$\tilde{\mathbf{r}}$: Cross-product-equivalent matrix

Cross Product of Vectors

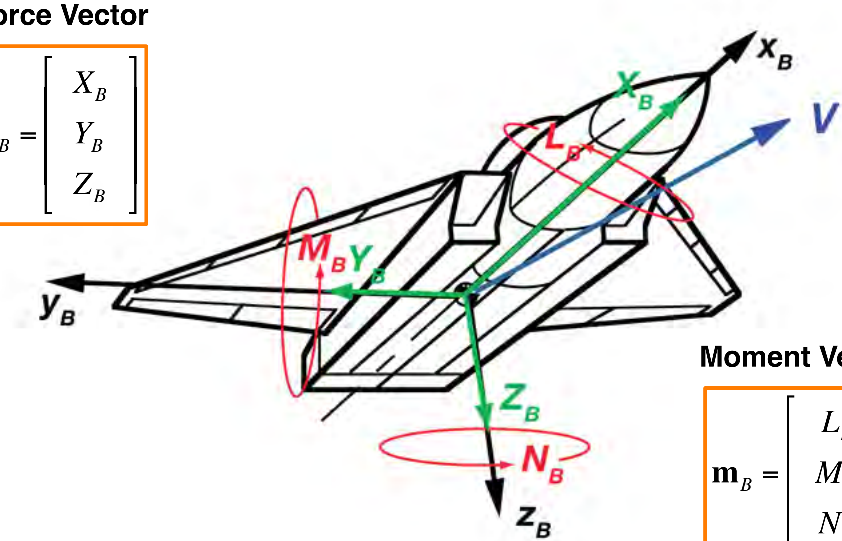
$$\mathbf{r}\times\mathbf{f} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ f_x & f_y & f_z \end{vmatrix} = \left(yf_z - zf_y\right)\boldsymbol{i} + \left(zf_x - xf_z\right)\boldsymbol{j} + \left(xf_y - yf_x\right)\boldsymbol{k}$$

$$\mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} = \begin{bmatrix} \left(yf_z - zf_y\right) \\ \left(zf_x - xf_z\right) \\ \left(xf_y - yf_x\right) \end{bmatrix} = \mathbf{r}\times\mathbf{f} \triangleq \tilde{\mathbf{r}}\mathbf{f} = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

Forces and Moments Acting on Entire Airplane

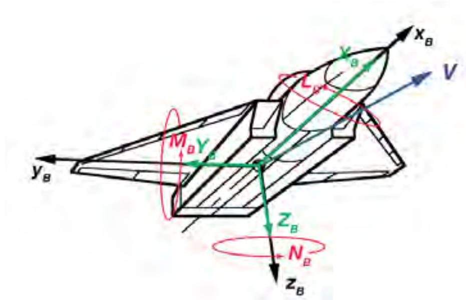
Force Vector

$$\mathbf{f}_B = \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$



Moment Vector

$$\mathbf{m}_B = \begin{bmatrix} L_B \\ M_B \\ N_B \end{bmatrix}$$



Aerodynamic Force and Moment Vectors of the Airplane

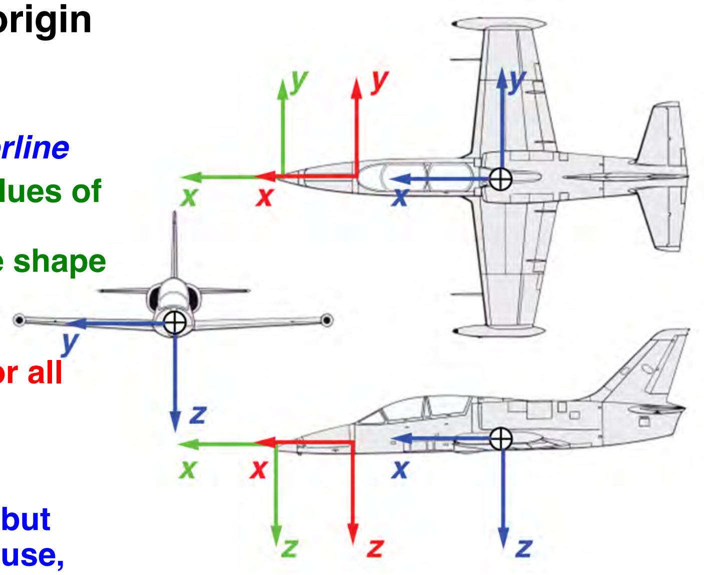
$$\mathbf{f}_B = \int_{Surface} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} dx\,dy\,dz = \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix}$$

$$\mathbf{m}_B = \int_{Surface} \begin{bmatrix} (yf_z - zf_y) \\ (zf_x - xf_z) \\ (xf_y - yf_x) \end{bmatrix} dx\,dy\,dz = \begin{bmatrix} L_B \\ M_B \\ N_B \end{bmatrix}$$

Pitching Moment of the Airplane

Body-Axis Reference Frames

- Reference frame origin is arbitrary; it is a *fiducial point*
  - *x* axis along *centerline*
  - Tip of nose: All values of *x* on airframe are negative, but nose shape could change
  - Foreward-most bulkhead: Fixed for all manufacturing measurements
  - Center of mass: Rotational center, but changes with fuel use, payload, etc.



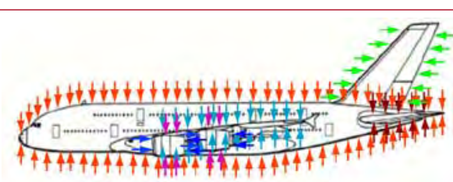
Pitching Moment
(moment about the *y* axis)

- Pressure and shear stress differentials times moment arms integrate over the airplane surface to produce a net pitching moment

$$\text{Body - Axis Pitching Moment} = M_B$$

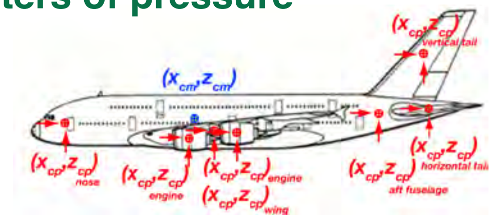
$$= -\iint_{surface} \left[\Delta p_z(x,y) + \Delta s_z(x,y)\right](x - x_{cm})\,dx\,dy$$

$$+ \iint_{surface} \left[\Delta p_x(y,z) + \Delta s_x(y,z)\right]\Delta p_x(z - z_{cm})\,dy\,dz$$



Pitching Moment
(moment about the y axis)

- Distributed effects can be aggregated to local centers of pressure



$$\boxed{\begin{aligned} M_B \approx &-\sum_{i=1}^{I} Z_i\left(x_i - x_{cm}\right) + \sum_{i=1}^{I} X_i\left(z_i - z_{cm}\right) \\ &+ Interference\ Effects + Pure\ Couples \end{aligned}}$$

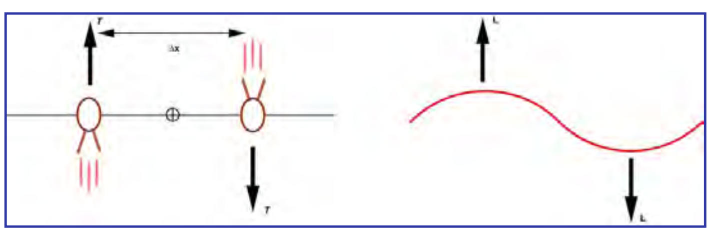
- Net effect expressed as

$$\boxed{M_B = C_m \bar{q} S \bar{c}}$$

Pure Couple

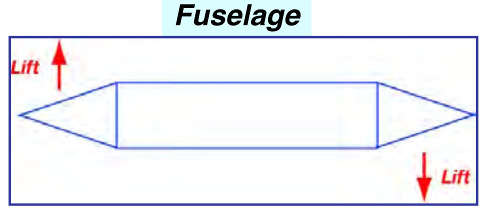
- Net force = 0
- Net moment ≠ 0

Rockets ***Cambered Lifting Surface***



- Cross-sectional area, A
- x positive to the right
- At small α
  - Positive lift slope with $dA/dx > 0$
  - Negative lift slope with $dA/dx < 0$
- Fuselage typically produces a destabilizing (positive) pitching moment

Fuselage



Net Center of Pressure

Local centers of pressure can be aggregated at a net center of pressure (or neutral point) along the body *x* axis

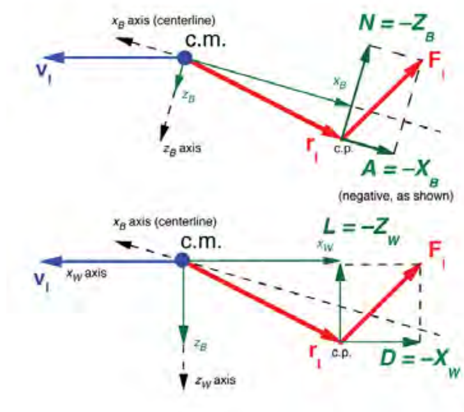
$$x_{cp_{net}} = \frac{\left[\left(x_{cp}C_N\right)_{wing} + \left(x_{cp}C_N\right)_{fuselage} + \left(x_{cp}C_N\right)_{tail} + \ldots\right]}{C_{N_{total}}}$$

***S* = reference area**

$$C_N = -C_Z$$
$$C_A = -C_X$$

Body Axes

Wind Axes

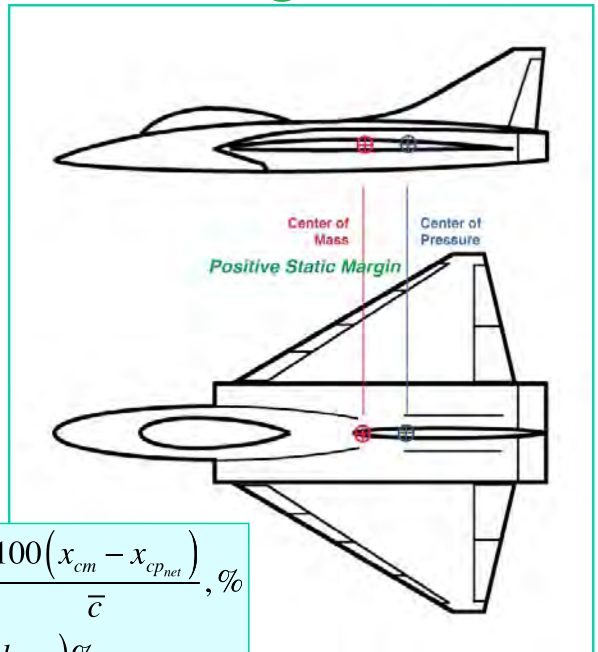


Static Margin

- **Static margin *(SM)* reflects the distance between the center of mass *(cm)* and the net center of pressure *(cp)***
  - **Body axes**
  - **Normalized by mean aerodynamic chord**
  - **Does not reflect *z* position of *center of pressure***
- **Positive *SM* if *cp* is behind *cm***

$$\text{Static Margin} \triangleq SM = \frac{100\left(x_{cm} - x_{cp_{net}}\right)_B}{\bar{c}}, \%$$
$$\equiv 100\left(h_{cm} - h_{cp_{net}}\right)\%$$

Static Margin

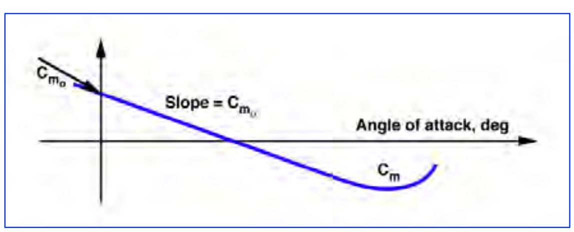


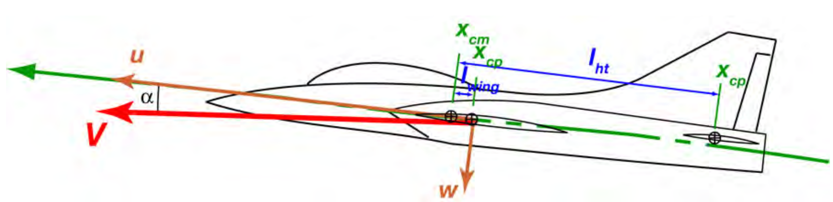
$$\textit{Static Margin} = SM = \frac{100\left(x_{cm} - x_{cp_{net}}\right)}{\bar{c}}, \%$$
$$\equiv 100\left(h_{cm} - h_{cp_{net}}\right)\%$$

Pitch-Moment Coefficient Sensitivity to Angle of Attack

For small angle of attack and no control deflection

$$M_B = C_m \bar{q} S \bar{c} \approx \left(C_{m_o} + C_{m_\alpha} \alpha\right) \bar{q} S \bar{c}$$





Pitch-Moment Coefficient Sensitivity to Angle of Attack

- For small angle of attack and no control deflection

$$C_{m_\alpha} \approx -C_{N_{\alpha_{net}}}\left(h_{cm} - h_{cp_{net}}\right) \approx -C_{L_{\alpha_{net}}}\left(h_{cm} - h_{cp_{net}}\right)$$

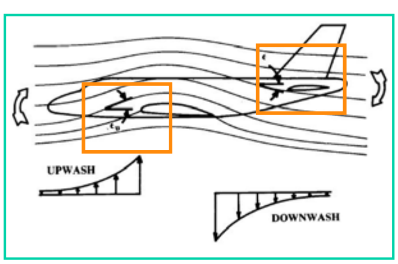
$$\approx -C_{L_{\alpha_{wing}}}\left(\frac{x_{cm} - x_{cp_{wing}}}{\bar{c}}\right) - C_{L_{\alpha_{ht}}}\left(\frac{x_{cm} - x_{cp_{ht}}}{\bar{c}}\right)$$

referenced to wing area, S

Horizontal Tail Lift Sensitivity to Angle of Attack

$$\left[\left(C_{L_{\alpha_{ht}}}\right)_{\substack{horizontal\\tail}}\right]_{ref=S} = \left(C_{L_{\alpha_{ht}}}\right)_{ref=S_{ht}}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)\left(\frac{V_{ht}}{V_N}\right)^2$$

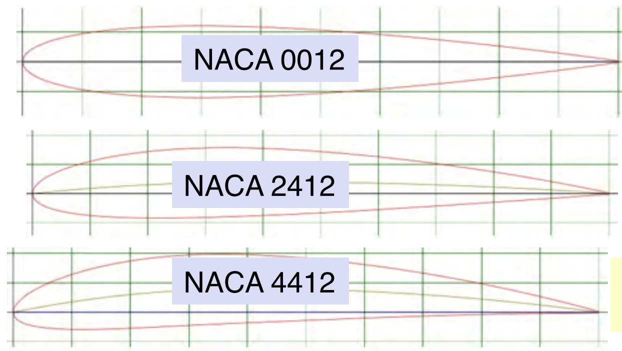
V_{ht} :	Airspeed at horizontal tail
ε :	**Downwash angle** due to wing at tail
$\partial\varepsilon/\partial\alpha$:	**Sensitivity** of downwash to angle of attack
η_{elas} :	**Aeroelastic effect**



- Downwash effect on aft horizontal tail
- Upwash effect on a canard (i.e., forward) surface

Aerodynamic Center of a Wing

$$x_{ac} = x \text{ for which } \frac{\partial C_m}{\partial \alpha} \equiv 0$$
$$= x_{cp} \text{ for a symmetric airfoil}$$
$$\neq x_{cp} \text{ for an asymmetric airfoil}$$



Airfoil Tools
http://airfoiltools.com

Effect of Static Margin on Pitching Moment

- **For small angle of attack and no control deflection**

$$M_B = C_m \bar{q} S\bar{c} \approx \left[C_{m_o} - C_{N_\alpha} \left(h_{cm} - h_{cp_{net}} \right) \alpha \right] \bar{q} S\bar{c}$$
$$\approx \left[C_{m_o} - C_{L_\alpha} \left(h_{cm} - h_{cp_{net}} \right) \alpha \right] \bar{q} S\bar{c}$$

Effect of Static Margin on Pitching Moment

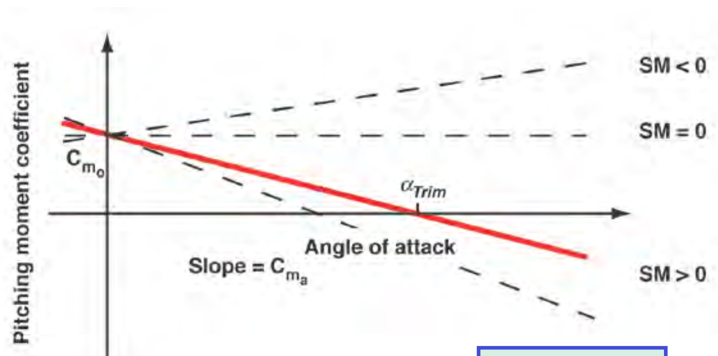
- **Sum of moments is zero in *trimmed condition***

$$M_B = \left(C_{m_o} + C_{m_\alpha}\alpha\right)\bar{q}\,S\bar{c}$$
$$= 0 \quad \textit{in trimmed (equilibrium) flight}$$

- **Typically, static margin is positive and $\partial C_m/\partial \alpha$ is negative for static pitch stability**

Effect of Static Margin on Pitching Coefficient

- **Zero crossing determines trim angle of attack, i.e., sum of moments = 0**
- **Negative slope required for static stability**
- **Slope, $\partial C_m/\partial \alpha$, varies with static margin**



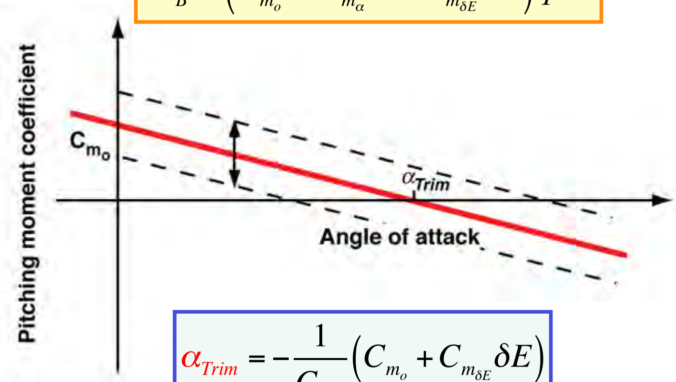
$$M_B = \left(C_{m_o} + C_{m_\alpha}\alpha\right)\bar{q}\,S\bar{c}$$

$$\alpha_{Trim} = -\frac{C_{m_o}}{C_{m_\alpha}}$$

Effect of Elevator Deflection on Pitching Coefficient

- **Control deflection** shifts curve up and down, affecting trim angle of attack

$$M_B = \left(C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\delta E}}\delta E\right)\bar{q}S\bar{c}$$



$$\alpha_{Trim} = -\frac{1}{C_{m_\alpha}}\left(C_{m_o} + C_{m_{\delta E}}\delta E\right)$$

Lateral-Directional Effects of Sideslip Angle

Rolling and Yawing Moments of the Airplane

Distributed effects can be aggregated to local centers of pressure

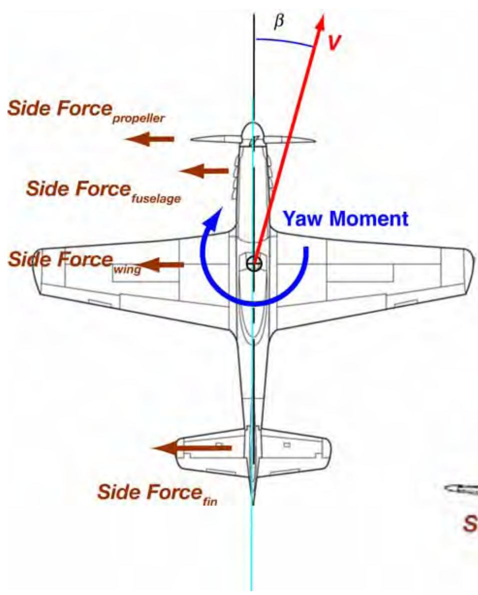
Rolling Moment

$$L_B \approx \sum_{i=1}^{I} Z_i \left(y_i - y_{cm} \right) - \sum_{i=1}^{I} Y_i \left(z_i - z_{cm} \right)$$
$$+ Interference\ Effects + Pure\ Couples$$

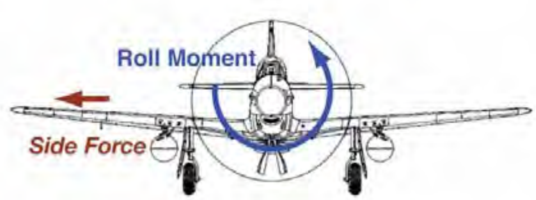
Yawing Moment

$$N_B \approx \sum_{i=1}^{I} Y_i \left(x_i - x_{cm} \right) - \sum_{i=1}^{I} X_i \left(y_i - y_{cm} \right)$$
$$+ Interference\ Effects + Pure\ Couples$$

Sideslip Angle Produces Side Force, Yawing Moment, and Rolling Moment



- **Sideslip usually a small angle (±5 deg)**
- **Side force generally not a significant effect**
- **Yawing and rolling moments are principal effects**



Side Force due to Sideslip Angle

$$Y \approx \frac{\partial C_Y}{\partial \beta}\bar{q}S \bullet \beta = C_{Y_\beta}\bar{q}S \bullet \beta$$

- **Fuselage, vertical tail, and wing are main contributors**

$$C_{Y_\beta} \approx \left(C_{Y_\beta}\right)_{Fuselage} + \left(C_{Y_\beta}\right)_{Vertical\ Tail} + \left(C_{Y_\beta}\right)_{Wing}$$

***S* = reference area**

Side Force due to Sideslip Angle

$$\left(C_{Y_\beta}\right)_{Vertical\ Tail} \approx \left(\frac{\partial C_Y}{\partial \beta}\right)_{ref=S_{vt}} \eta_{vt}\left(\frac{S_{vt}}{S}\right)$$

$$\left(C_{Y_\beta}\right)_{Fuselage} \approx -2\frac{S_{Base}}{S}; \quad S_B = \pi d_{Base}^2 / 4$$

$$\left(C_{Y_\beta}\right)_{Wing} \approx -C_{D_{Parasite,Wing}} - k\Gamma^2$$

η_{vt} = Vertical tail efficiency

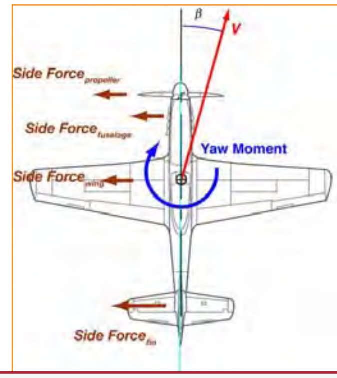
$$k = \frac{\pi AR}{1+\sqrt{1+AR^2}}$$

Γ = Wing dihedral angle, rad

Yawing Moment due to Sideslip Angle

$$N \approx \frac{\partial C_n}{\partial \beta}\left(\frac{\rho V^2}{2}\right)Sb \bullet \beta = C_{n_\beta}\left(\frac{\rho V^2}{2}\right)Sb \bullet \beta$$

- **Side force contributions times respective moment arms**
  - **Non-dimensional stability derivative**



$$C_{n_\beta} \approx \left(C_{n_\beta}\right)_{Vertical\ Tail} + \left(C_{n_\beta}\right)_{Fuselage} + \left(C_{n_\beta}\right)_{Wing} + \left(C_{n_\beta}\right)_{Propeller}$$

S = reference area

Rolling Moment due to Sideslip Angle

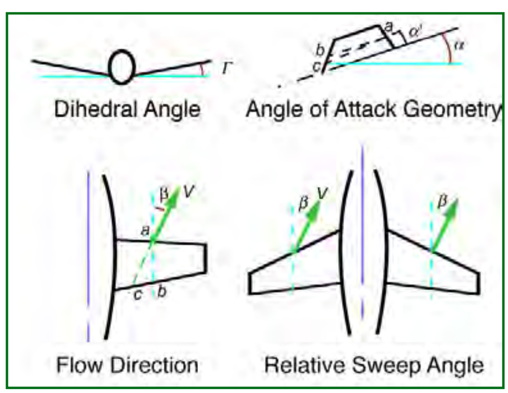
Dihedral effect

$$L \approx C_{l_\beta}\overline{q}Sb \bullet \beta$$

$$C_{l_\beta} \approx \left(C_{l_\beta}\right)_{Wing} + \left(C_{l_\beta}\right)_{Wing-Fuselage} + \left(C_{l_\beta}\right)_{Vertical\ Tail}$$

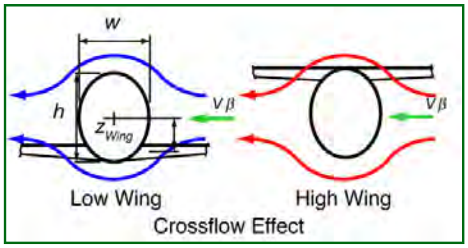
S = reference area

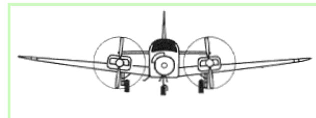
Unequal lift on left and right wings induces rolling motion

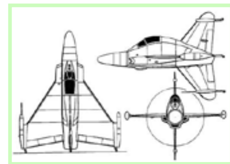


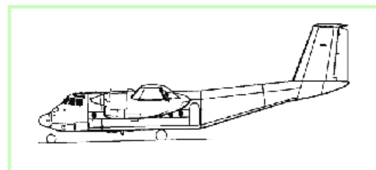
Rolling Moment due to Sideslip Angle

- **Wing vertical location effect: Crossflow produces up- and down-wash**
  - **Rolling effect depends on vertical location of the wing**







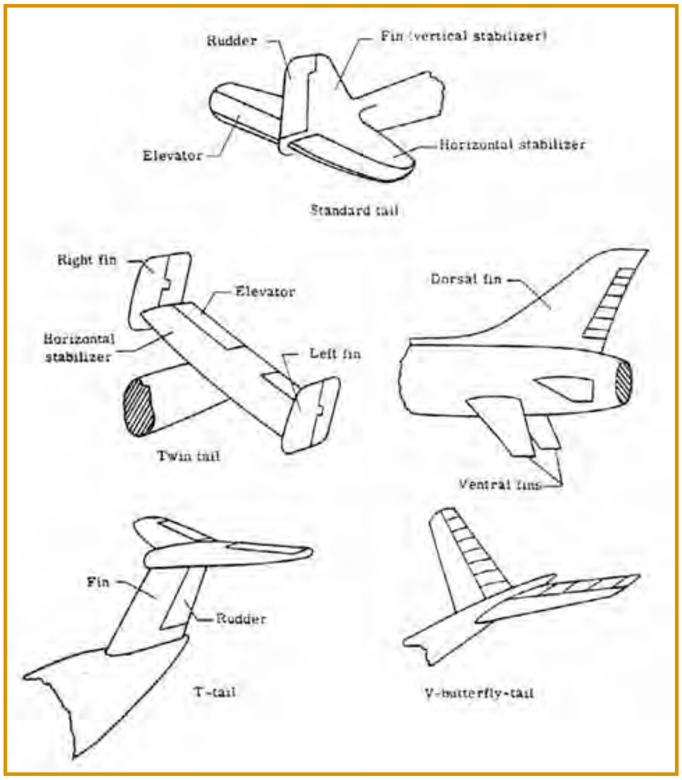


- **Vertical tail effect**

Tail Design Effects

Tail Design Effects

- Aerodynamics analogous to those of the wing
- Longitudinal stability
  - Horizontal stabilizer
  - Short period natural frequency and damping
- Directional stability
  - Vertical stabilizer (fin)
    - Ventral fins
    - Strakes
    - Leading-edge extensions
    - Multiple surfaces
    - Butterfly (V) tail
  - Dutch roll natural frequency and damping
- Stall or spin prevention/ recovery
- Avoid *rudder lock (TBD)*



Horizontal Tail Location and Size

- 15-30% of wing area
- ~ wing semi-span behind the c.m.
- Must trim neutrally stable airplane at maximum lift in ground effect
- Effect on short period mode
- Horizontal Tail Volume: Typical value = 0.48

$$\boxed{\mathbb{V}_{HT} = \frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}}$$

Pitching Moment due to Elevator Deflection

- **Normal force coefficient variation due to elevator deflection**

$$C_{L_{\delta E}} \triangleq \frac{\partial C_L}{\partial \delta E} = \tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\frac{S_{ht}}{S} \approx C_{N_{\delta E}}$$

$$\Delta C_N = C_{N_{\delta E}}\delta E$$

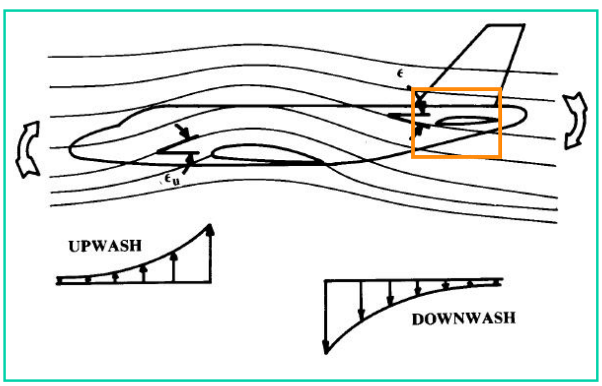
- **Pitching moment coefficient variation due to elevator deflection**

$$C_{m_{\delta E}} = C_{N_{\delta E}}\frac{l_{ht}}{\bar{c}} \approx -\tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\frac{S_{ht}}{S}\frac{l_{ht}}{\bar{c}}$$

$$= -\tau_{ht}\eta_{ht}\left(C_{L_\alpha}\right)_{ht}\mathbb{V}_{HT}$$

Downwash and Elasticity Also Effect Elevator Sensitivity

$$\left[\left(\frac{\partial C_L}{\partial \delta E}\right)_{ht}\right]_{ref=S} = \left(C_{L_{\delta E}}\right)_{ref=S} = \left(C_{L_{\delta E}}\right)_{ref=S_{ht}}\left(\frac{V_{tail}}{V_N}\right)^2\left(1-\frac{\partial \varepsilon}{\partial \alpha}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)$$



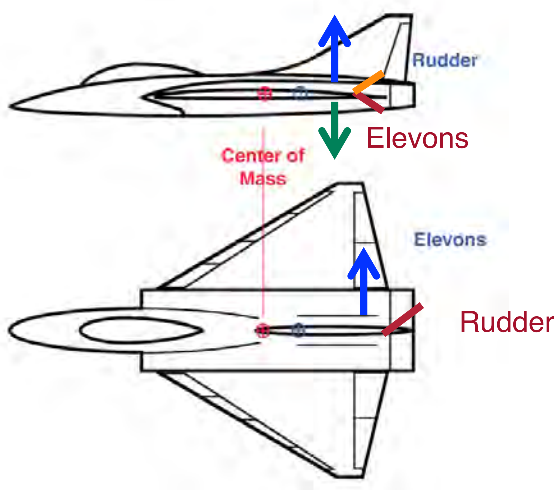
Vertical Tail Location and Size

- Analogous to horizontal tail volume
- Effect on Dutch roll mode
- Powerful rudder for spin recovery
  - Full-length rudder located behind the elevator
  - High horizontal tail so as not to block the flow over the rudder
- Vertical Tail Volume: Typical value = 0.18

Otto Koppen
http://en.wikipedia.org/wiki/Otto_C._Koppen

$$\boldsymbol{V_{VT}} = \frac{S_{vt}}{S}\frac{l_{vt}}{b}$$

Lateral-Directional Control Surfaces



Yawing Moment due to Rudder Deflection

- **Side force coefficient variation due to rudder deflection**

$$\left(C_{Y_{\delta R}}\right)_{ref=S} \triangleq \left(\frac{\partial C_Y}{\partial \delta R}\right)_{ref=S} = \left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \tau_{vt}\eta_{vt}\frac{S_{vt}}{S}$$

$$\Delta C_Y = C_{Y_{\delta R}}\delta R$$

- **Yawing moment coefficient variation due to rudder deflection**

$$\left(C_{n_{\delta R}}\right)_{ref=S} = -\left(C_{Y_{\delta R}}\right)_{ref=S}\frac{l_{vt}}{b} \approx -\left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \tau_{vt}\eta_{vt}\frac{S_{vt}}{S}\frac{l_{vt}}{\bar{c}}$$

$$= -\tau_{vt}\eta_{vt}\left[\left(C_{L_\alpha}\right)_{vt}\right]_{ref=S_{vt}} \mathbf{V}_{VT}$$

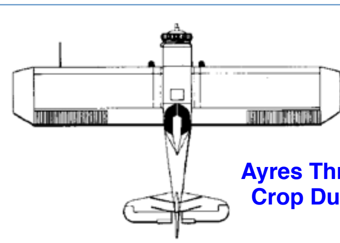
Rolling Moment due to Aileron Deflection

$$\boxed{L \approx C_{l_{L_{\delta A}}}\bar{q}Sb \bullet \delta A}$$

- **For a trapezoidal planform, subsonic flow**

$$\left(C_{l_{\delta A}}\right)_{3D} \simeq \left(\frac{C_{L_\delta}}{C_{L_\alpha}}\right)_{2D}\frac{\left(C_{L_\alpha}\right)_{3D}}{1+\lambda}\left[\frac{1-k^2}{3} - \frac{1-k^3}{3}(1-\lambda)\right]$$

$$k \triangleq \frac{y}{b/2}, \quad y = \text{ Inner edge of aileron}, \lambda = \text{ Taper ratio}$$



Ayres Thrush Crop Duster

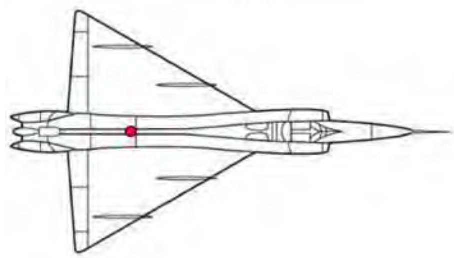
Next Time: Aircraft Performance

Reading:
Flight Dynamics
Aerodynamic Coefficients, 118-130
Airplane Stability and Control
Chapter 6

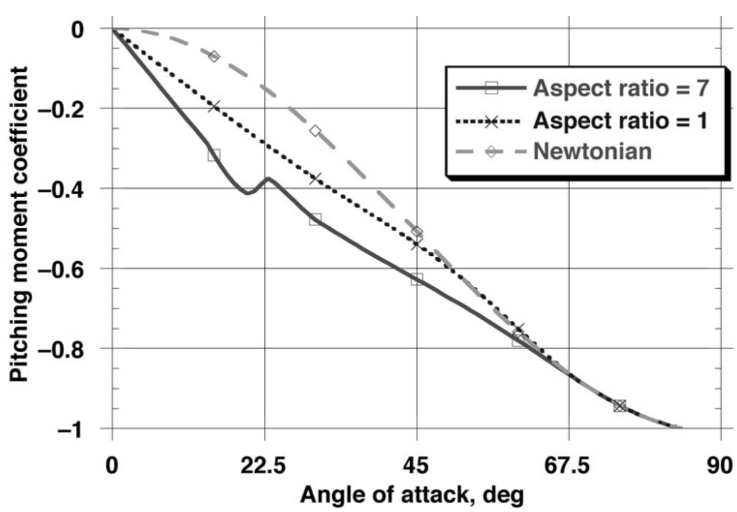
Supplemental Material

Planform Effect on Center of Pressure Variation with Mach Number

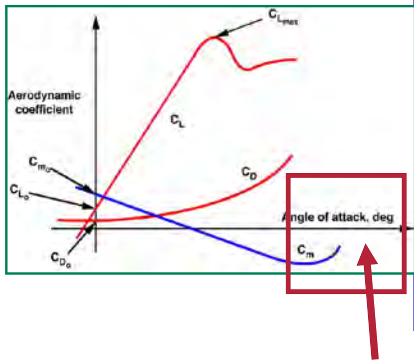
- **Straight Wing**
  - Subsonic center of pressure (c.p.) at ~1/4 mean aerodynamic chord (m.a.c.)
  - Transonic-supersonic c.p. at ~1/2 m.a.c.
- **Delta Wing**
  - Subsonic-supersonic c.p. at ~2/3 m.a.c.



Subsonic Pitching Coefficient vs. Angle of Attack $(0° < \alpha < 90°)$



“Pitch Up” and Deep Stall



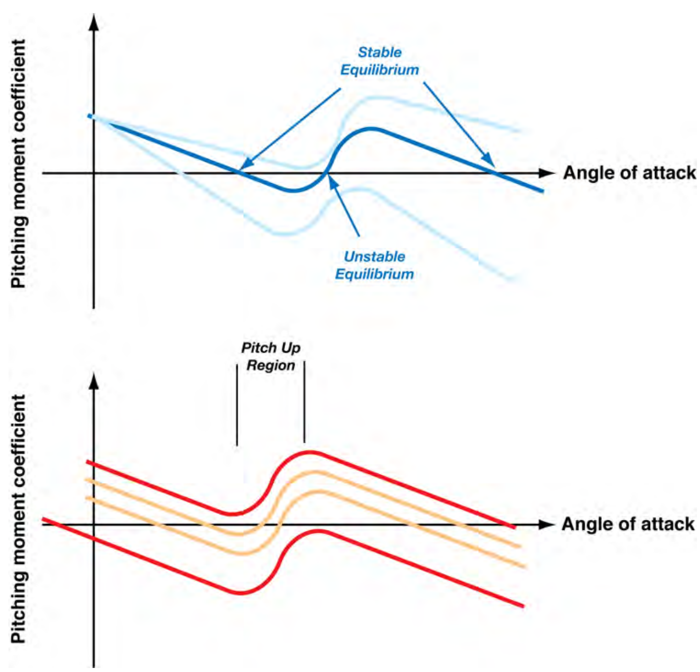
- **Possibility of 2 stable equilibrium (trim) points with same control setting**
  - **Low α**
  - **High α**
- **High-angle trim is called deep stall**
  - **Low lift**
  - **High drag**
- **Large control moment required to regain low-angle trim**

TU-154 Pitch Up Accident
http://www.youtube.com/watch?v=IpZ8YukAwwI&feature=related

BAC 1-11 Deep Stall Flight Testing Accident
http://en.wikipedia.org/wiki/BAC_One-Eleven

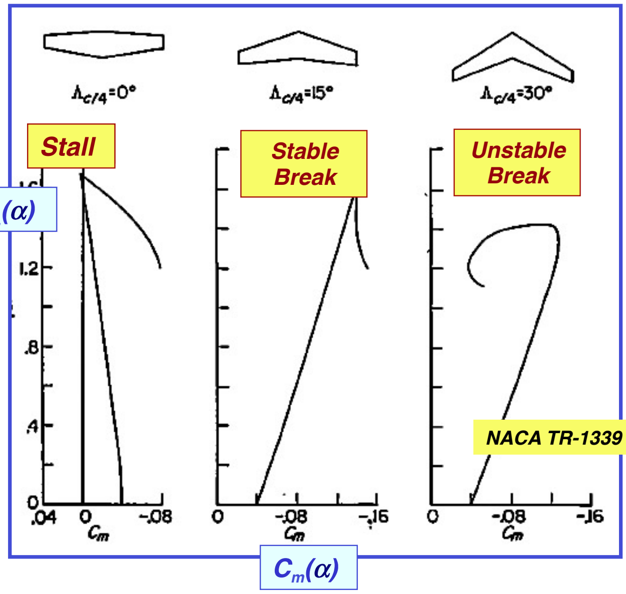
Pitch Up and Deep Stall, C_m vs. a

- **Possibility of 2 stable equilibrium (trim) points with same control setting**
  - **Low α**
  - **High α**
- **High-angle trim is called deep stall**
  - **Low lift**
  - **High drag**
- **Large control moment required to regain low-angle trim**



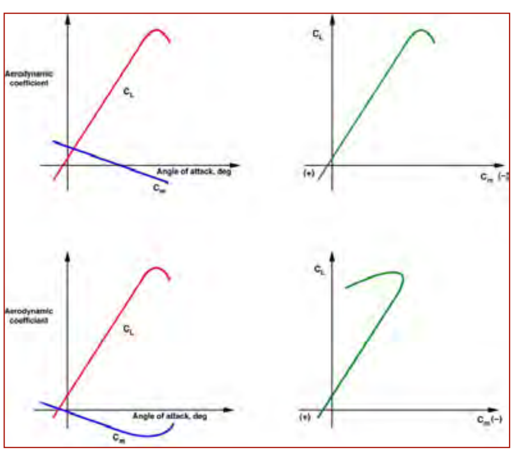
Sweep Effect on Pitch Moment Coefficient, C_L vs. C_m

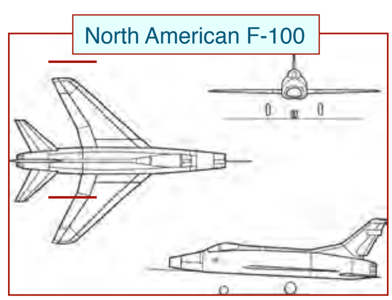
- $\Lambda_{c/4} = 0$
  - Low α center of pressure (c.p.) in front of the quarter chord
  - Stable break at stall (c.p. moves aft)
- $\Lambda_{c/4} = 15°$
  - Low α c.p. aft of the quarter-chord
  - Stable break at stall (c.p. moves aft)
- $\Lambda_{c/4} = 30°$
  - Low α c.p. aft of the quarter-chord
  - Unstable break at stall (c.p. moves forward)
    - Outboard wing stalls before inboard wing ("tip stall")



Pitch Up: Explanation of C_L vs. C_m Cross-plot

- Crossplot C_L vs. C_m to obtain plots such as those shown on previous slide
- Positive break in C_m is due to forward movement of net center of pressure, decreasing static margin

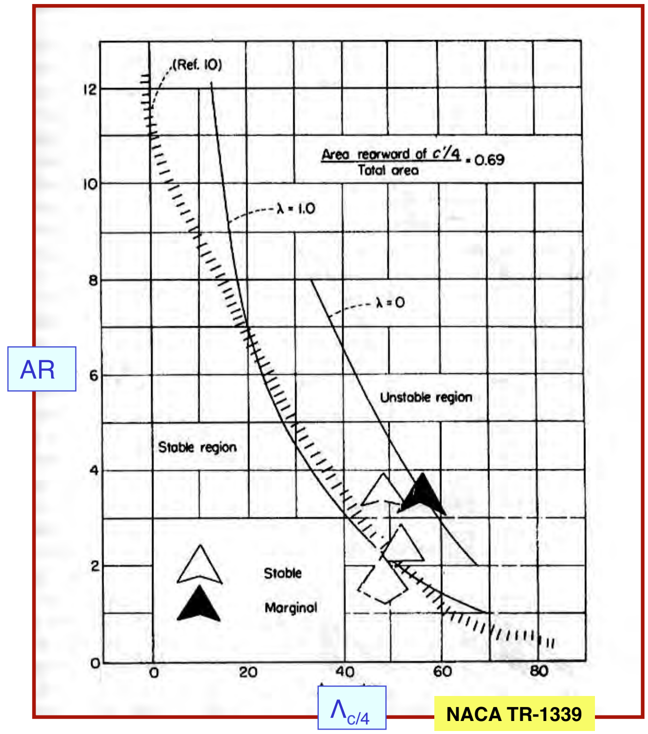




http://www.youtube.com/watch?v=mZL0x-gEDM8

Shortal-Maggin Longitudinal Stability Boundary for Swept Wings

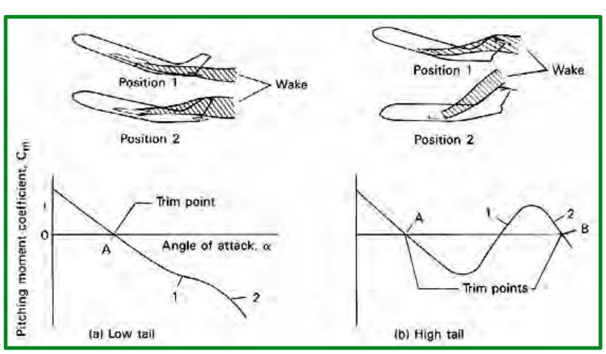
- Stable or unstable pitch break at the stall
- Stability boundary is expressed as a function of
  - Aspect ratio
  - Sweep angle of the quarter chord
  - Taper ratio



NACA TR-1339

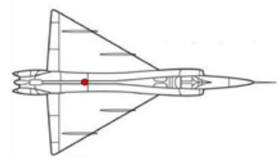
Horizontal Tail Location

- Horizontal tail and elevator in wing wake at selected angles of attack
- Effectiveness of high-mounted elevator is unaffected by wing wake at low to moderate angle of attack
- Effectiveness of low tail is unaffected by wing wake at high angle of attack



Effects of Wing Aspect Ratio and Sweep Angle

- Lift slope
- Pitching moment slope
- Lift-to-drag ratio
- All contribute to
  – Phugoid damping
  – Short period natural frequency and damping
  – Roll damping



Effects of Wing Aspect Ratio

- Neglecting air compressibility
- Angles of attack below stall

- Lift slope

$$C_{L_{\alpha_{wing}}} = \frac{\pi AR}{\left[1 + \sqrt{1 + \left(\frac{AR}{2}\right)^2}\right]}$$

- Pitching moment slope

$$C_{m_\alpha} \approx -C_{L_{\alpha_{total}}} \left(\frac{\text{Static Margin (\%)}}{100}\right)$$

- Lift-to-drag ratio

$$L/D = \frac{C_{L_{total}}}{\left(C_{D_o} + \varepsilon C_L^2\right)_{total}} = \frac{\left(C_{L_o} + C_{L_\alpha}\alpha\right)_{total}}{\left[C_{D_o} + \varepsilon C_L^2\right]_{total}}$$

- Roll damping

Wing with taper

$$\left(C_{l_{\hat{p}}}\right)_{wing} = \frac{\partial\left(\Delta C_l\right)_{wing}}{\partial \hat{p}} = -\frac{C_{L_{\alpha_{wing}}}}{12}\left(\frac{1+3\lambda}{1+\lambda}\right)$$

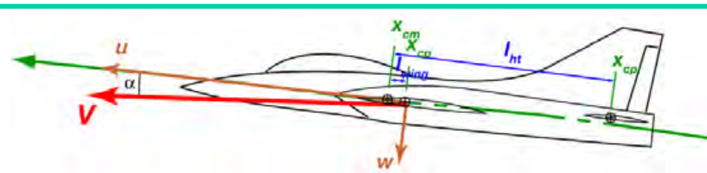
Thin triangular wing

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = -\frac{\pi AR}{32}$$

Tail Moment Sensitivity to Angle of Attack

$$C_{m_{\alpha_{ht}}} = -\left(C_{L_{\alpha_{ht}}}\right)_{ht}\left(\frac{V_{ht}}{V_N}\right)^2\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)\eta_{elas}\left(\frac{S_{ht}}{S}\right)\left(\frac{l_{ht}}{\bar{c}}\right)$$
$$= -\left(C_{L_{\alpha_{ht}}}\right)_{ht}\left(\frac{V_{ht}}{V_N}\right)^2\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)\eta_{elas}\mathbb{V}_{HT}$$

$$\mathbb{V}_{HT} = \frac{S_{ht}l_{ht}}{S\bar{c}} = \textbf{Horizontal Tail Volume Ratio}$$



Yawing Moment due to Sideslip Angle

Vertical tail contribution

$$\left(C_{n_\beta}\right)_{Vertical\ Tail} \approx -C_{Y_{\beta_{vt}}}\eta_{vt}\frac{S_{vt}l_{vt}}{Sb} \triangleq -C_{Y_{\beta_{vt}}}\eta_{vt}\mathbb{V}_{VT}$$

$l_{vt} \triangleq$ **Vertical tail length** $(+)$

= distance from center of mass to tail center of pressure

$= x_{cm} - x_{cp_{vt}}$ [x is positive forward; both are negative numbers]

$$\eta_{vt} = \eta_{elas}\left(1+{\partial\sigma}/{\partial\beta}\right)\left(\frac{V_{vt}^2}{V_N^2}\right)$$

$$\mathbb{V}_{VT} = \frac{S_{vt}l_{vt}}{Sb} = \textbf{Vertical Tail Volume Ratio}$$

Yawing Moment due to Sideslip Angle

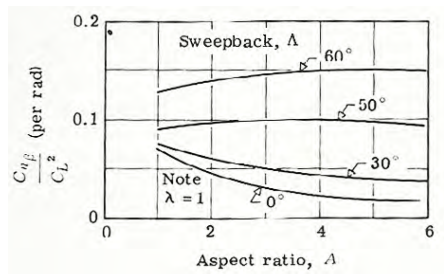
Fuselage contribution

$$\left(C_{n_\beta}\right)_{Fuselage} = \frac{-2K\ Volume_{Fuselage}}{Sb}$$

$$K = \left(1 - {d_{max}}\Big/{Length_{fuselage}}\right)^{1.3}$$

Wing (differential lift and induced drag) contribution

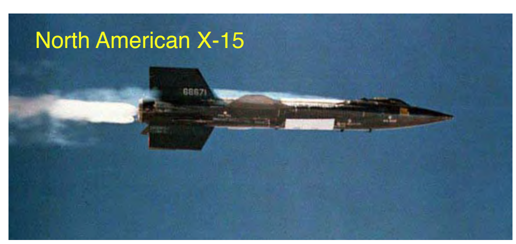
$$\left(C_{n_\beta}\right)_{Wing} = 0.75 C_{L_N}\Gamma + fcn\left(\Lambda, AR, \lambda\right) C_{L_N}^2$$



Seckel, from NACA TR-1098, 1950

Ventral Fin Effects

- Increase directional stability
- Counter roll due to sideslip of the dorsal fin



V (Butterfly) Tails

- Analogous to conventional tail at low angles of attack and sideslip
- Control surface deflection
  - Sum: Pitch control
  - Difference: Yaw control
- Nonlinear effects at high angle of attack are quite different from conventional tail

Twin and Triple Vertical Tails

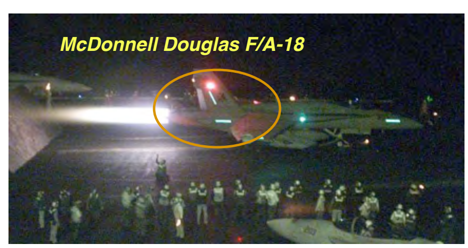
- Increased tail area with no increase in vertical height
- End-plate effect for horizontal tail improves effectiveness
- Proximity to propeller slipstream

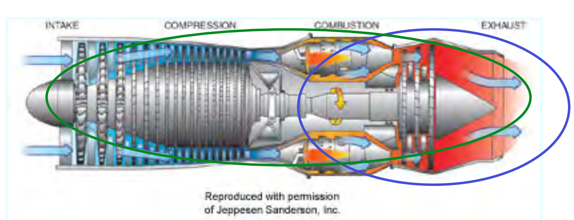
Propeller Effects

- ***Slipstream*** **over wing, tail, and fuselage**
  - **Increased dynamic pressure**
  - **Swirl of flow**
  - **Downwash and sidewash at the tail**
- ***DH-2*** **unstable with engine out**
- **Single- and multi-engine effects**
- **Design factors: fin offset (correct at one airspeed only), *c.m.* offset**
- ***Propeller fin effect*****: Visualize lateral/ horizontal projections of the propeller as forward surfaces**
- **Counter-rotating propellers minimize torque and swirl**

Jet Effects on Rigid-Body Motion

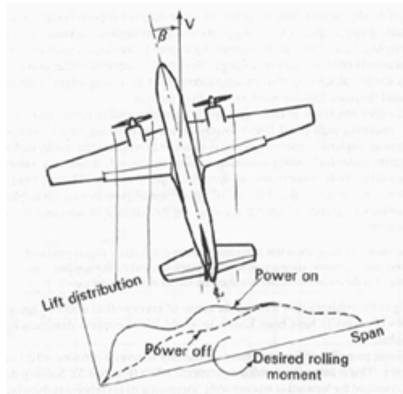
- **Normal force at intake (analogous to propeller fin effect) (*F-86*)**
- **Deflection of airflow past tail due to entrainment in exhaust (*F/A-18*)**
- **Pitch and yaw damping due to internal exhaust flow**
- **Angular momentum of rotating machinery**





Loss of Engine

- Loss of engine produces large yawing (and sometimes rolling) moment(s), requiring major application of controls
- Engine-out training can be as hazardous, especially during takeoff, for both propeller and jet aircraft
- Acute problem for general-aviation pilots graduating from single-engine aircraft



Configurational Solutions to the Engine-Out Problem

- Engines on the centerline (*Cessna 337 Skymaster*)
- More engines (*B-36*)
- Cross-shafting of engines (*V-22*)
- Large vertical tail (*Boeing 737*)

Anatomy of a Cirrus Stall Accident

http://www.youtube.com/watch?v=7nm_hoHhbFo

Some Videos

XF-92A, 1948

http://www.youtube.com/watch?v=hVjaiMXvCTQ

First flight of B-58 Hustler, 1956

http://www.youtube.com/watch?v=saeejPWQTHw

Century series fighters, bombers, 1959

http://www.youtube.com/watch?v=WmseXJ7DV4c&feature=related

Bird of Prey, 1990s, and X-45, 2000s

http://www.youtube.com/watch?v=BMcuVhzCrX8&feature=related

YF-12A supersonic flight past the sun

http://www.youtube.com/watch?v=atItRcfFwgw&feature=related

Supersonic flight, sonic booms

http://www.youtube.com/watch?v=gWGLAAYdbbc&list=LP93BKTqpxbQU&index=1&feature=plcp