Maneuvering at High Angles and Rates

Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2014

- High angle of attack and angular rates
- Asymmetric flight
- Nonlinear aerodynamics
- Inertial coupling
- Spins and tumbling

Flight Dynamics
681-785
Airplane Stability and Control
Chapter 8

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

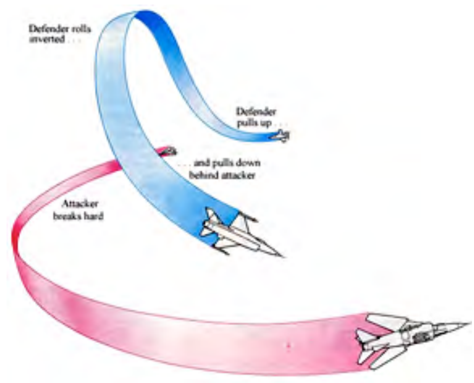
The Discovery of Inertial Coupling

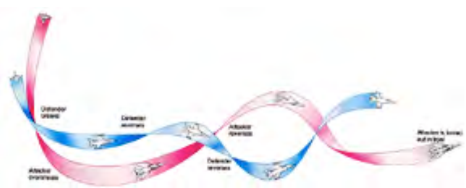
Chapter 8, *Airplane Stability and Control*, Abzug and Larrabee

- What are the principal subject and scope of the chapter?
- What technical ideas are needed to understand the chapter?
- During what time period did the events covered in the chapter take place?
- What are the three main "takeaway" points or conclusions from the reading?
- What are the three most surprising or remarkable facts that you found in the reading?

Tactical Airplane Maneuverability

- **Maneuverability parameters**
  - Stability
  - Roll rate and acceleration
  - Normal load factor
  - Thrust/weight ratio
  - Pitch rate
  - Transient response
  - Control forces
- **Dogfights**
  - Preferable to launch missiles at *long range*
  - Dogfight is a *backup tactic*
  - Preferable to have an *unfair advantage*
- **Air-combat sequence**
  - Detection
  - Closing
  - Attack
  - Maneuvers, e.g.,
    - *Scissors*
    - *High yo-yo*
  - Disengagement





Coupling of Longitudinal and Lateral-Directional Motions

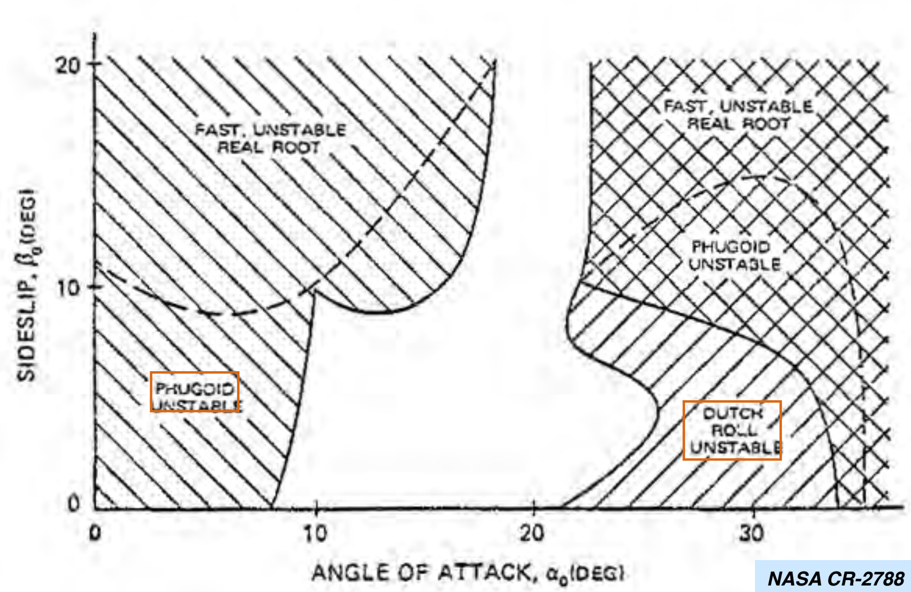
Longitudinal Motions can Couple to Lateral-Directional Motions

- **Linearized equations have limited application to high-angle/high-rate maneuvers**
  - **Steady, non-zero sideslip angle (Sec. 7.1, *FD*)**
  - **Steady turn (Sec. 7.1, *FD*)**
  - **Steady roll rate**

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{Lon} & \mathbf{F}_{Lat-Dir}^{Lon} \\ \mathbf{F}_{Lon}^{Lat-Dir} & \mathbf{F}_{Lat-Dir} \end{bmatrix}$$

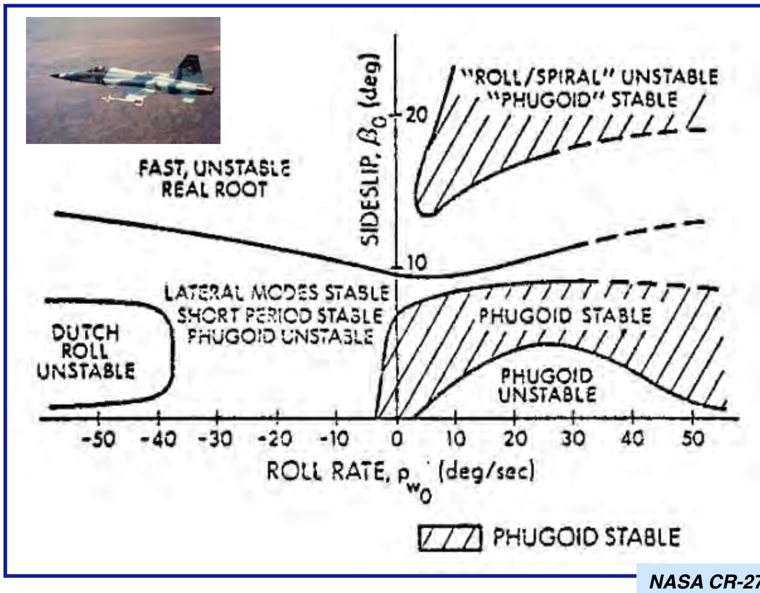
$$\boxed{\mathbf{F}_{Lat-Dir}^{Lon}, \mathbf{F}_{Lon}^{Lat-Dir} \neq 0}$$

Stability Boundaries Arising From Asymmetric Flight



NASA CR-2788

Stability Boundaries with Nominal Sideslip, β_o, and Roll Rate, p_o



NASA CR-2788

Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

- **Combine 2nd-order short period and Dutch roll modes**
  - **Body axes**
  - **Constant roll rate = p_o, rad/s**

- **State vector**

$$\Delta\mathbf{x}(t) = \begin{bmatrix} \Delta\mathbf{x}_{Lon} \\ \Delta\mathbf{x}_{LD} \end{bmatrix} = \begin{bmatrix} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{bmatrix} \begin{matrix} \textit{Normal velocity, m / s} \\ \textit{Pitch rate, rad / s} \\ \textit{Side velocity, m / s} \\ \textit{Yaw rate, rad / s} \end{matrix}$$

- **Control input vector**

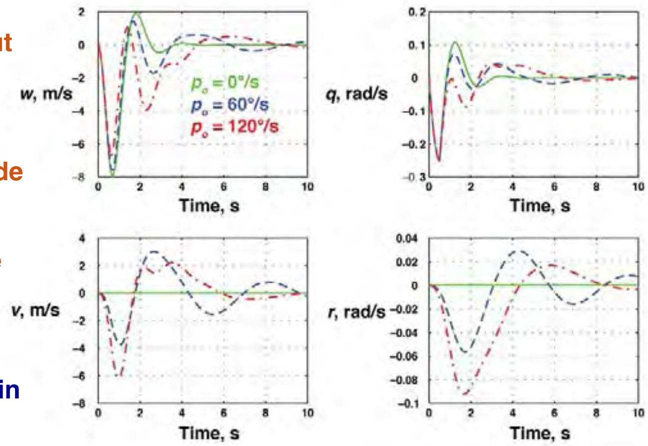
$$\Delta\mathbf{u}(t) = \begin{bmatrix} \Delta\delta E \\ \Delta\delta A \\ \Delta\delta R \end{bmatrix} \begin{matrix} \textit{Elevator, deg or rad} \\ \textit{Ailerons, deg or rad} \\ \textit{Rudder, deg or rad} \end{matrix}$$

- **4th-order dynamic model**

$$\begin{bmatrix} \Delta\dot{\mathbf{x}}_{Lon} \\ \Delta\dot{\mathbf{x}}_{LD} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{Lon} & \mathbf{F}_{LD}^{Lon} \\ \mathbf{F}_{Lon}^{LD} & \mathbf{F}_{LD} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x}_{Lon} \\ \Delta\mathbf{x}_{LD} \end{bmatrix} + \begin{bmatrix} \mathbf{G}_{Lon} \\ \mathbf{G}_{LD} \end{bmatrix} \Delta u$$

Time Response to Elevator Step Input

- When $p_o = 0°/s$
  - Elevator input produces longitudinal response but <u>no lateral-directional response</u>
- At $p_o = 60°/s$
  - Short-period (faster) mode dominates longitudinal response
  - Dutch-roll (slower) mode dominates lateral directional response
- At $p_o = 120°/s$
  - Both modes are evident in both responses
  - Fast mode is even faster
  - Slow mode is even slower



Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

- 4^{th}-order stability matrix
  - Body axes
  - Neglible v_o, $u_o \sim V_N$
  - Negligible coupling aerodynamic effects
- Constant roll rate is only source of coupling

$$\begin{bmatrix} \mathbf{F}_{Lon} & \mathbf{F}_{LD}^{Lon} \\ \mathbf{F}_{Lon}^{LD} & \mathbf{F}_{LD} \end{bmatrix} = \begin{bmatrix} \overbrace{\begin{bmatrix} Z_w & u_o \\ M_w & M_q \end{bmatrix}}^{\text{Short Period}} & \overbrace{\begin{bmatrix} -p_o & 0 \\ 0 & \dfrac{(I_{zz} - I_{xx})}{I_{yy}} p_o \end{bmatrix}}^{\text{Yaw-to-Pitch Coupling}} \\ \underbrace{\begin{bmatrix} p_o & 0 \\ 0 & \dfrac{(I_{xx} - I_{yy})}{I_{zz}} p_o \end{bmatrix}}_{\text{Pitch-to-Yaw Coupling}} & \underbrace{\begin{bmatrix} Y_v & -u_o \\ N_v & N_r \end{bmatrix}}_{\text{Dutch Roll}} \end{bmatrix}$$

$$\Delta\mathbf{x}(t) = \begin{bmatrix} \Delta w \\ \Delta q \\ \Delta v \\ \Delta r \end{bmatrix}$$

Pitch-Yaw Coupling Due To Steady Roll Rate, p_o

Characteristic Polynomial

$$\Delta_{rolling}(s) = \left\{\left[(s-Z_w)(s-M_q)-u_oM_w\right]\left[(s-Y_v)(s-N_r)+u_oN_v\right]\right\}$$
$$+p_o^2\left\{(s-M_q)(s-N_r)-(s-Z_w)(s-Y_v)\frac{(I_{zz}-I_{xx})}{I_{yy}}\frac{(I_{xx}-I_{yy})}{I_{zz}}-u_oM_w\frac{(I_{xx}-I_{yy})}{I_{zz}}-u_oN_v\frac{(I_{zz}-I_{xx})}{I_{yy}}\right\}$$
$$-p_o^4\frac{(I_{zz}-I_{xx})}{I_{yy}}\frac{(I_{xx}-I_{yy})}{I_{zz}}$$

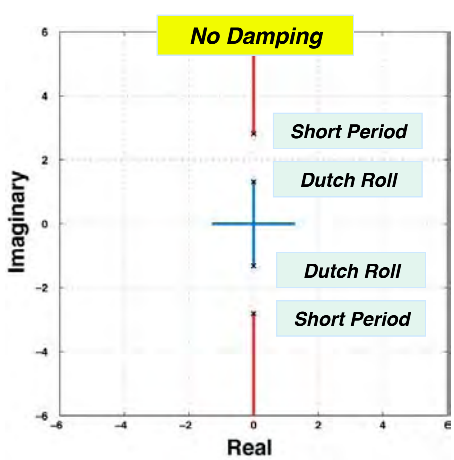
- Coupling effect is proportional to p_o^2 and p_o^4
- Effect on roots is independent of the sign of p_o
- Cannot use Evans' s root-locus rules with $k = p_o^2$, as k^2 also appears
- Can compute effect of p_o^2 on roots using MATLAB's *eig*

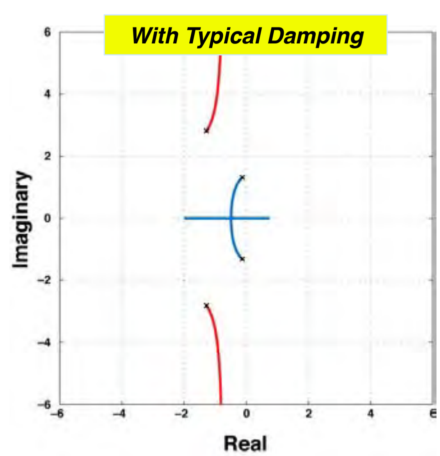
$$\Delta_{rolling}(s) = \left[\Delta_{SP}(s)\Delta_{DR}(s)\right]+p_o^2\left[fcn\left(s,M_q,N_r,Z_w,Y_v,I_{xx},I_{yy},I_{zz},u_o,M_w,N_v\right)\right]-p_o^4\frac{(I_{zz}-I_{xx})}{I_{yy}}\frac{(I_{xx}-I_{yy})}{I_{zz}}$$

Thunderbird F-16 Barrel Roll
http://www.youtube.com/watch?v=ovSOStIncbU

Effect of Steady Roll Rate, p_o, on Pitching and Yawing Roots

- Factor $\Delta_{rolling}(s)$ for various values of p_o^2
- p_o^2 = root locus gain, k
- Faster mode gets faster
- Slower mode gets slower and may become unstable

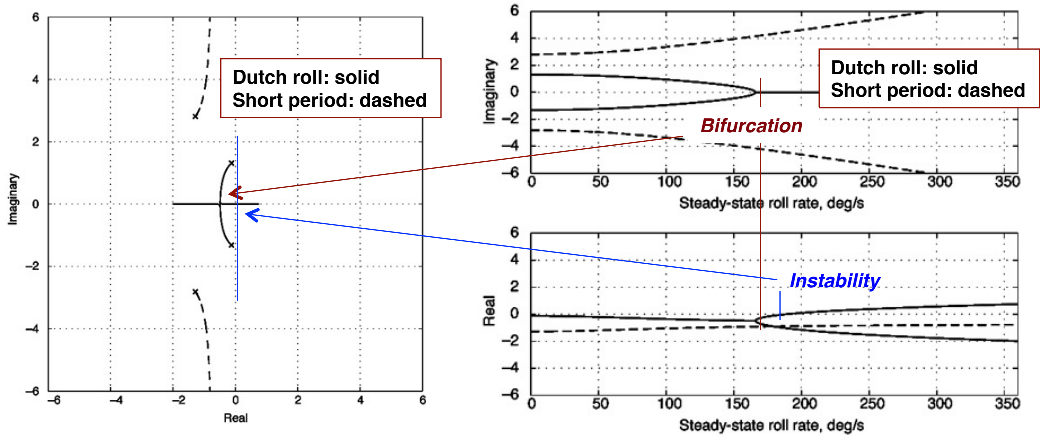




Steady Roll Rate, p_o, Effect Expressed by Root Locus or Parameter Plot

Divergent root at high roll rate

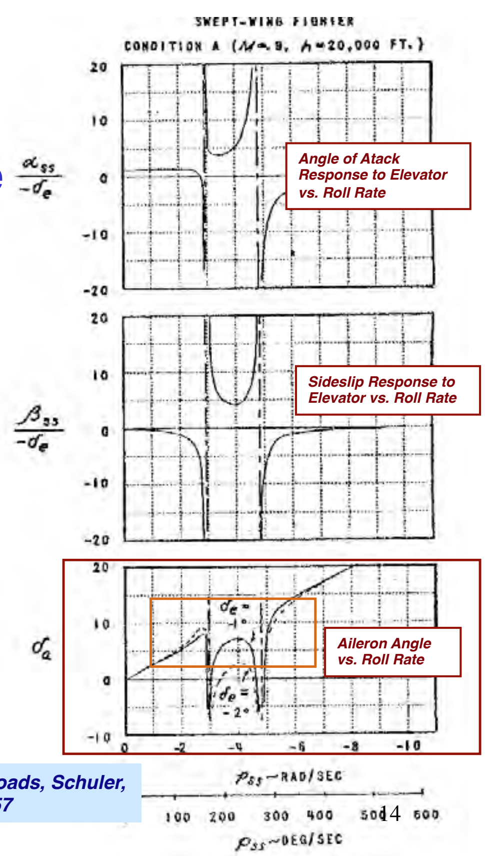
Parameter plot: variation of real and imaginary parts of roots vs. roll rate,



Steady-State Response as Well as Stability is Affected by High Roll Rate

$$\boxed{\mathbf{f}\left(v, w, p, q, r, \Delta\delta A, \Delta\delta E, \Delta\delta R\right)_{SS} = 0}$$

- **Effects of steady roll rate on nonlinear equilibrium control response**
  - **Pitch-yaw coupling**
  - **"p jump" or "p acceleration"**
- **Multiple equilibria for same control settings**
  - **Up to 9 possible roll rates for one aileron setting**
  - **Sensitivity to elevator setting**
    - *Flight Dynamics*, 7.3



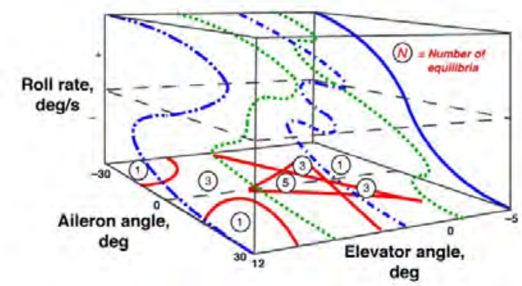
Rhoads, Schuler, 1957

The Butterfly Catastrophe*

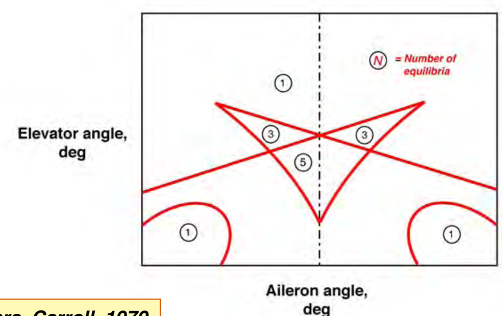
$$\begin{array}{|l|}\hline \mathbf{f}_1\left(v,w,p,q,r,\Delta\delta A,\Delta\delta E,\Delta\delta R\right)_{SS} = 0 \\ p_{SS} = \mathbf{f}_2\left(v,w,q,r,\Delta\delta A,\Delta\delta E,\Delta\delta R\right)_{SS} \\ \hline \end{array}$$

- **Surface of equilibrium solutions for roll rate**
- **Possibility of an unrecoverable spin**

Roll Rate Equilibrium Surface



The Butterfly Catastrophe



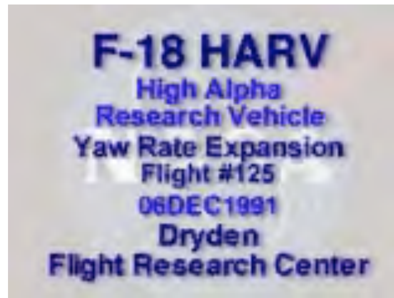
*** René Thom, 1974**

after Mehra, Carroll, 1979

Tumbling and Spins

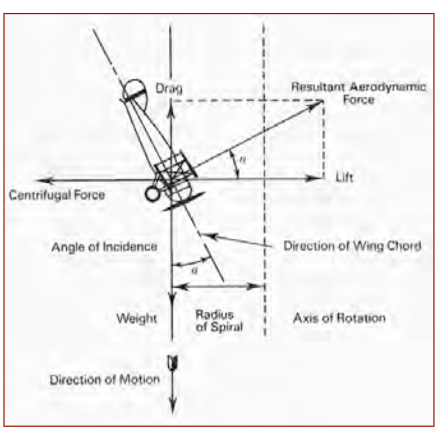
Tumbling, Spins, and Recovery

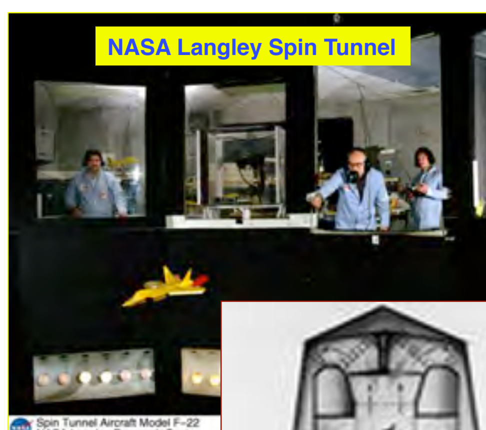
- Strong nonlinear effects
- Aircraft-specific control strategy for recovery



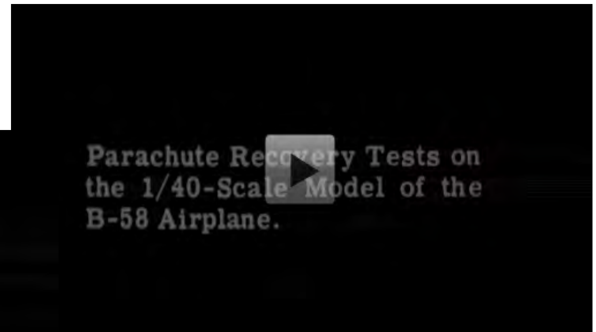
Wind Tunnel Spin Testing

- Sidney B. Gates, RAE: "The Spinning of Aeroplanes" (with L.W. Bryant, 1926), neutral and maneuver points, stick force per *g*
- Continued research on stalls and spins at NASA, USAF, and in many other countries





NASA Langley Spin Tunnel Testing



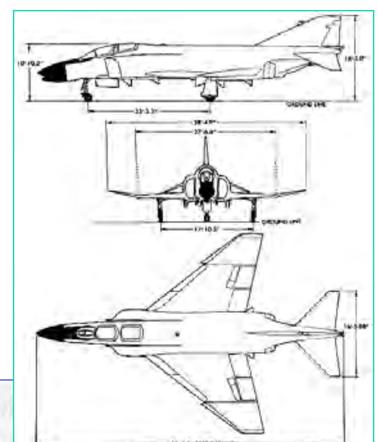
http://www.youtube.com/watch?v=u7FCqLpTgkk

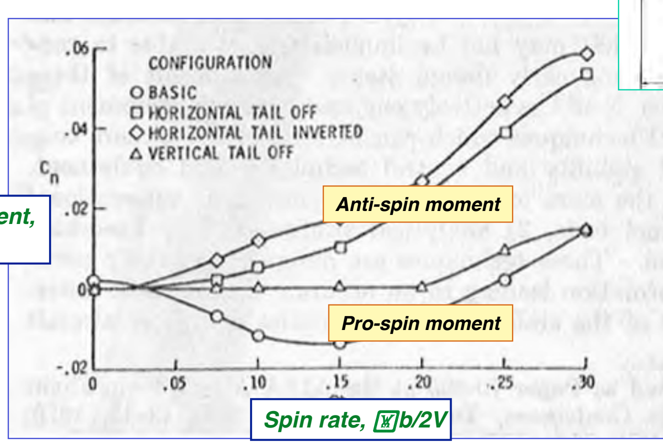
http://www.youtube.com/watch?v=tQwMCml55Q0

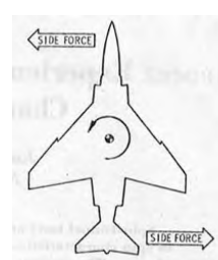
http://www.youtube.com/watch?v=VUKTBUY1RII

Tails with Negative Dihedral (Anhedral)

- Horizontal tail below wing's wake
- May have adverse effect on spin characteristics
- F-4 model test

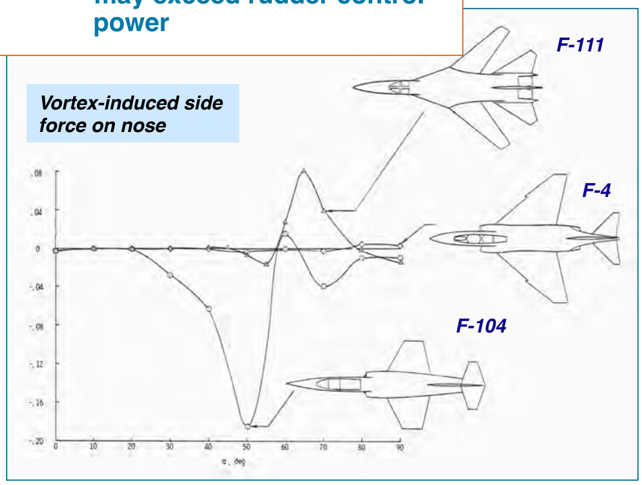


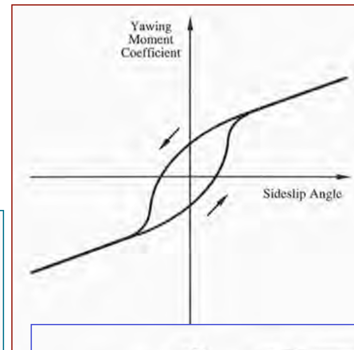


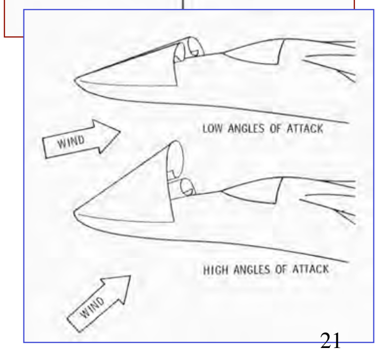


Yawing Moment at High Angle of Attack

- **Dynamic as well as static effects,e.g., *hysteresis***
- **Random asymmetric yawing moments *(left or right)***
  - **generated by slender nose at zero sideslip angle**
  - **may exceed rudder control power**







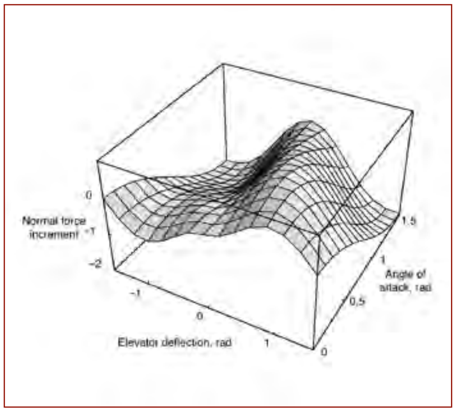
Controlling Yawing Moment at High Angle of Attack

- ***Sucking*, *blowing*, or *movable strakes* to control nose vortices**
- ***X-29, F/A-18 HARV***
- **Vortex bursting effect on tail**

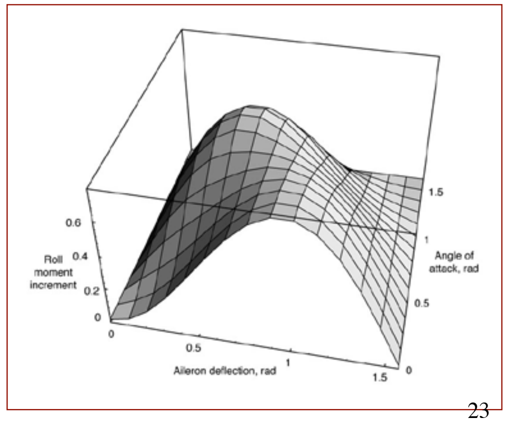
Control Effectiveness at High Angle of Attack and Deflection Angle

- **Assumption of Newtonian flow**

Elevator Effect



Aileron Effect



Control at High Aerodynamic Angles

Supermaneuverability

- Means of forcing opponent to *overshoot*
- *Pugachev's Cobra maneuver*, first done in *Sukhoi Su-27*
- Beneficial effect of thrust-vector control (*X-31*)
- *Mongoose maneuver* (*X-31*)
- Essentially low-speed maneuvers, not where you want to be in air combat (i.e., high energy-state)

Dryden Flight Research Center

Thrust Vector Control

Pitch and Yaw Control (X-31)

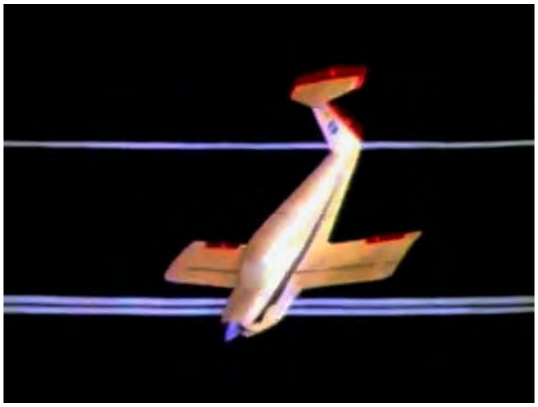
Pitch Control (F-22)

Next Time: Aeroelasticity and Fuel Slosh

Flight Dynamics
418-419, 549-569, 665-678
Airplane Stability and Control
Chapter 19

Supplemental Material

Stall-Spin Studies of General Aviation Aircraft



http://www.youtube.com/watch?v=TmWB6oyJ9lE