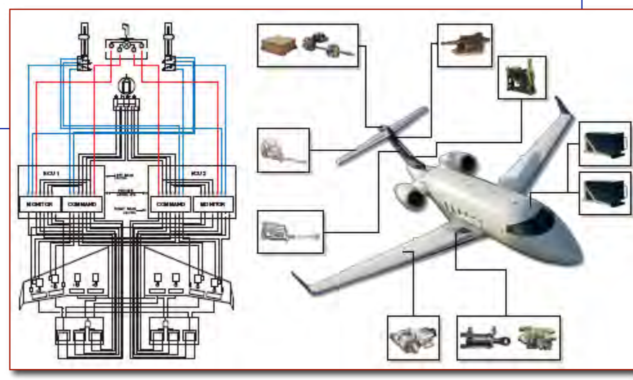
Sensors and Actuators

Robert Stengel
Robotics and Intelligent Systems, MAE 345,
Princeton University, 2015

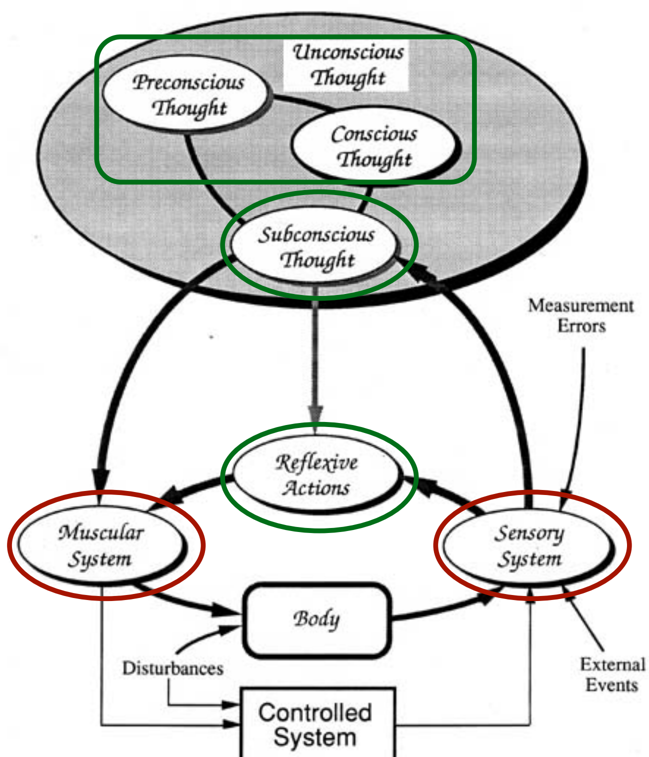
- Biological Antecedents
- Critical Elements for System Observation and Control
- Control Effecters
- Output Sensors
- Navigation



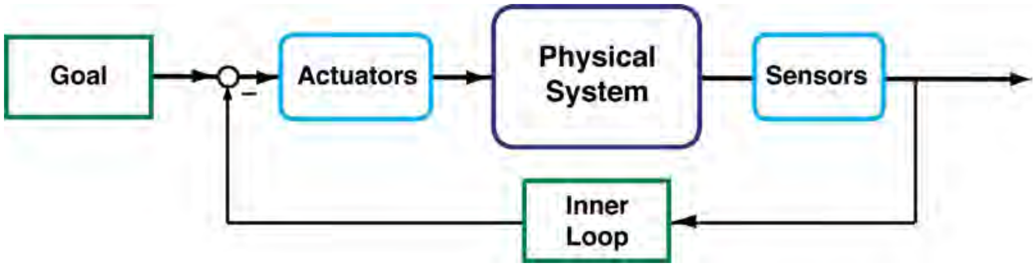
http://www.princeton.edu/~stengel/MAE345.html

Biologically Inspired Control

- Declarative Planning
- Procedural Formatting
- Reflexive Control
- Sensory input
- Motor output



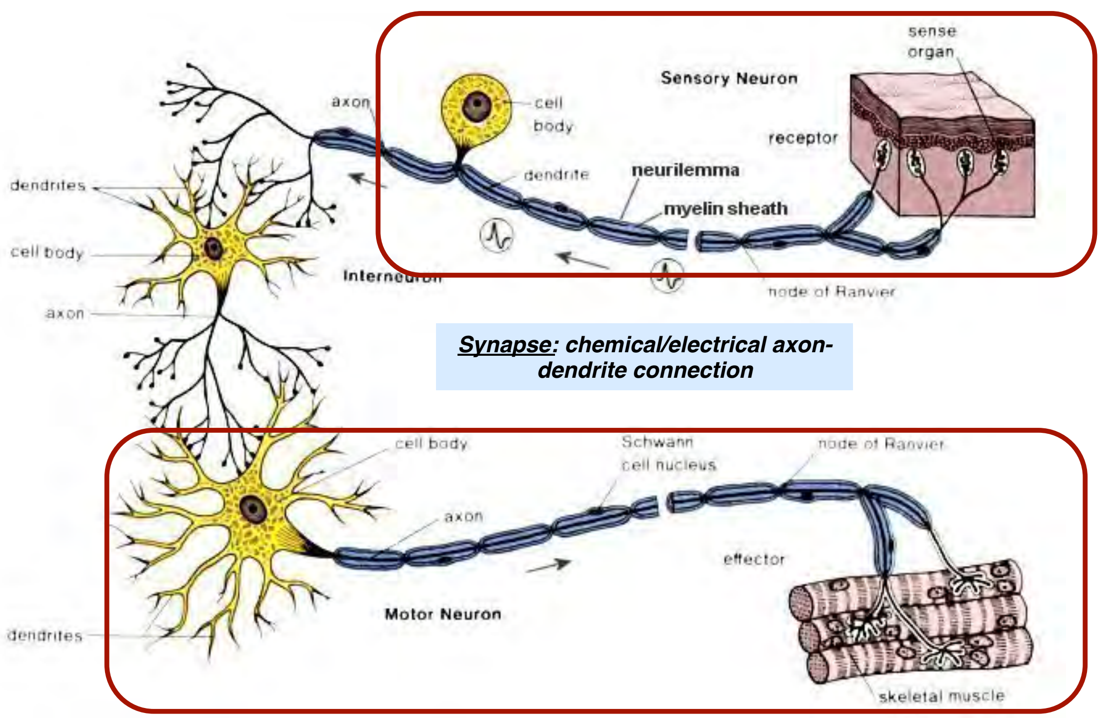
Feedback Control Requires Sensors and Actuators



- *Sensors* and *actuators* have their own dynamic characteristics

- Desirable properties
  - High bandwidth ("faster" than system to be controlled)
  - Accuracy
  - Precision
  - Large dynamic range
  - Sufficient power for control
  - Reliability
  - Low cost

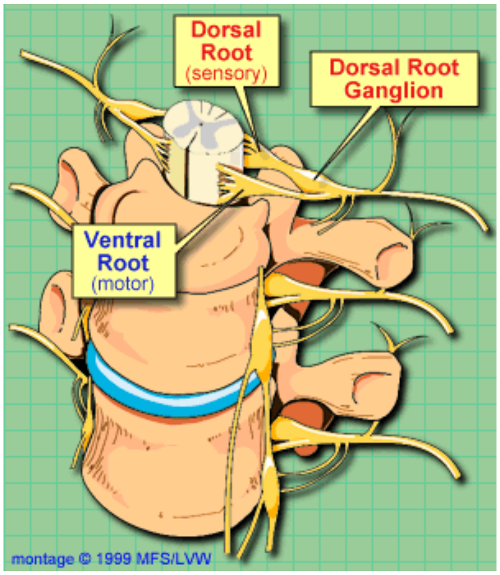
Peripheral Sensory and Motor Neurons

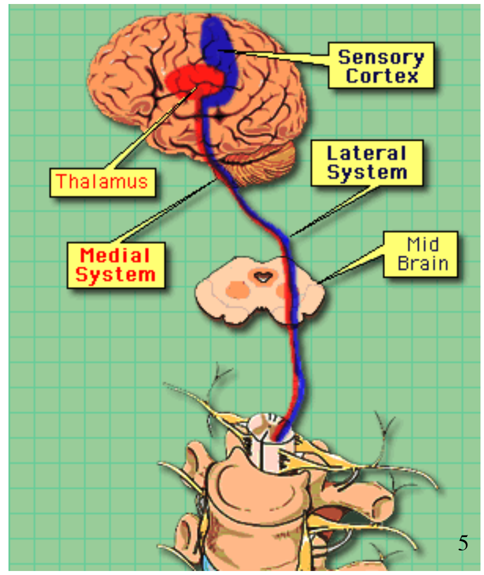


***Synapse**: chemical/electrical axon-dendrite connection*

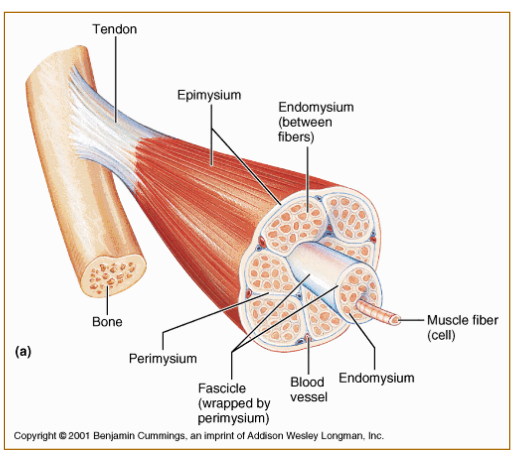
Sensory and Motor Signal Paths to the Brain

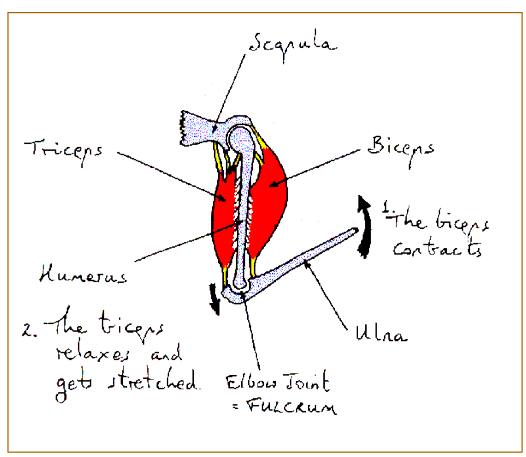
Reflexive response is processed in the spinal roots
Declarative and procedural response is processed in the brain





Skeletal Muscle



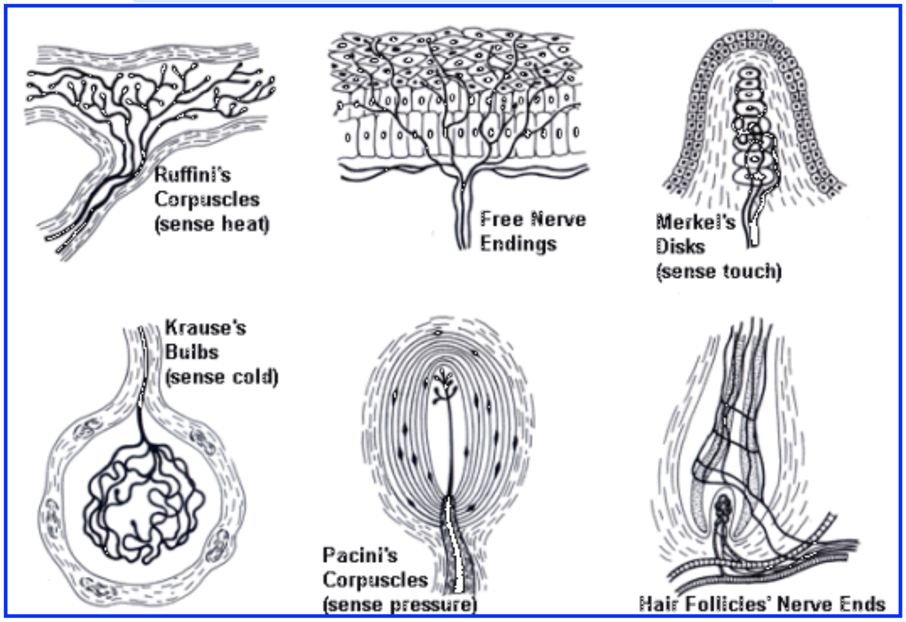


- **Attached to the skeleton to produce motion of limbs, torso, neck, and head**
- **Agonist-antagonist muscle pairs produce opposing motion (flexion and extension)**
- **End-effecter strength depends on lever arm and varies with joint angle**
- **Voluntary (declarative) commands from somatic central nervous system**

Sensory Neuron Receptors

Neuron Receptors (corpuscles, disks, cells, muscle spindles) generate action potentials that are transmitted to the spinal cord

Cutaneous and Sub-Cutaneous Receptors



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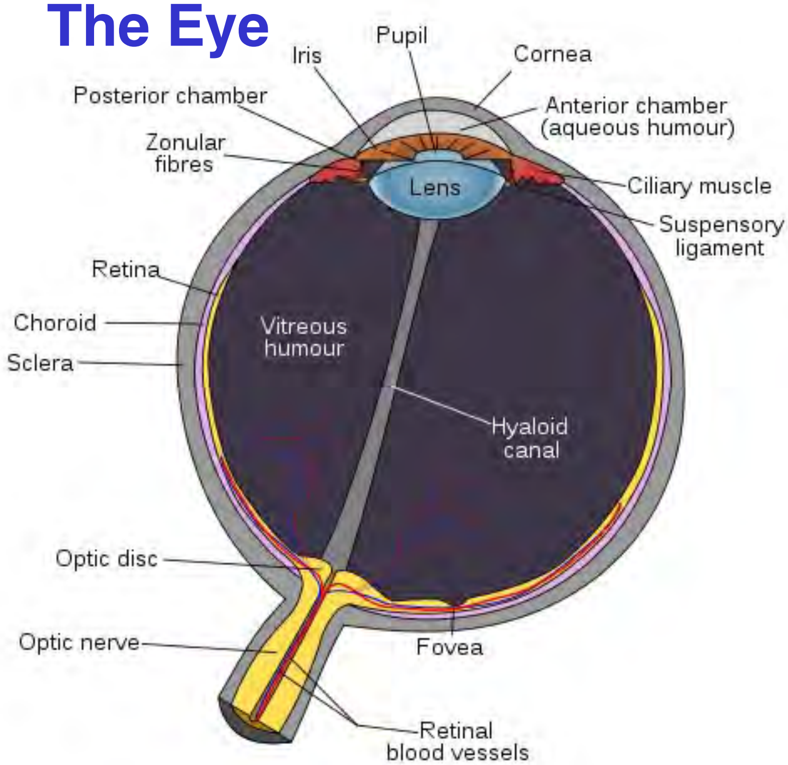
The Eye



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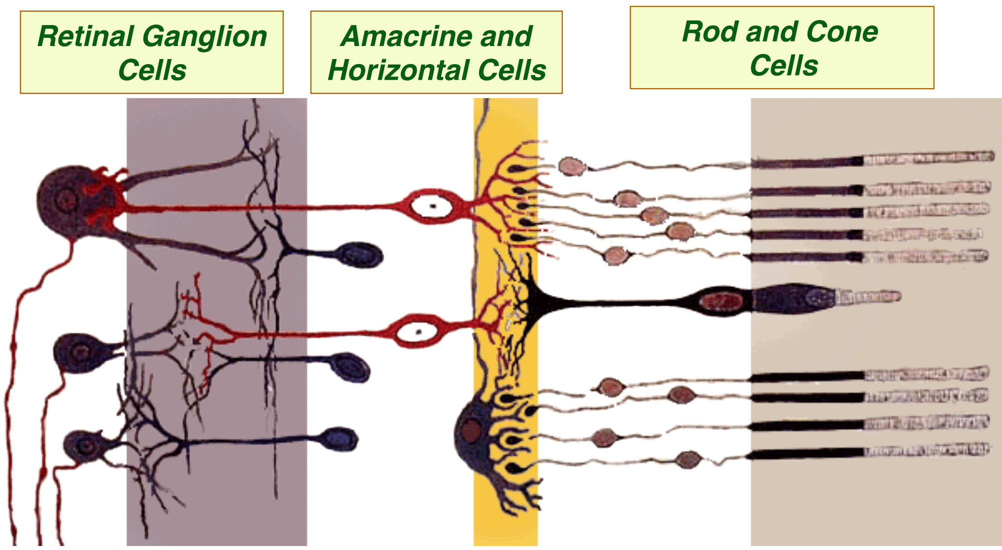
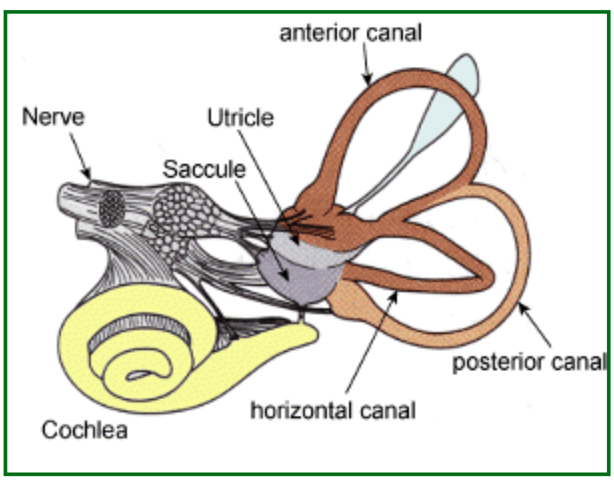
Retinal Cross Section

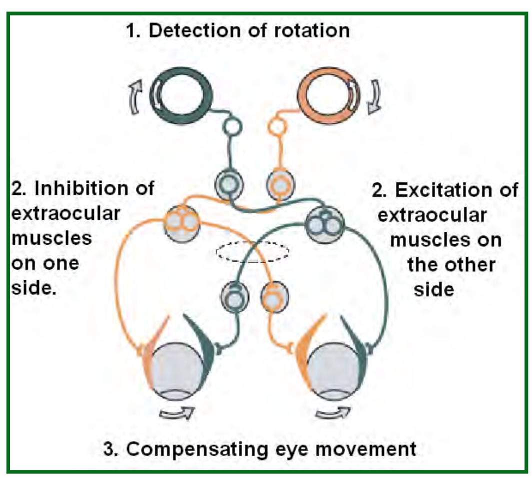


Biological Inertial Measurement: The Inner Ear

Vestibular system measures linear and angular acceleration



Integration with eye motion

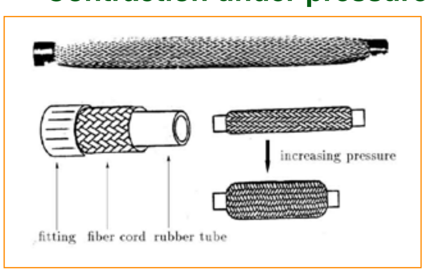


Actuators

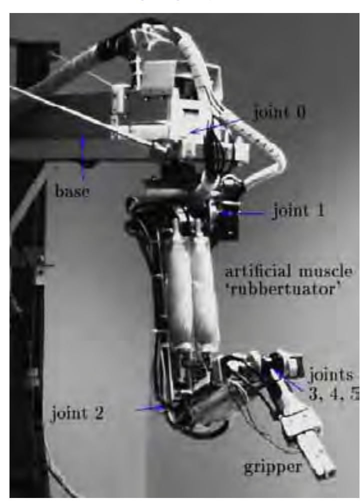
Rubbertuator

Pneumatic analog of muscle

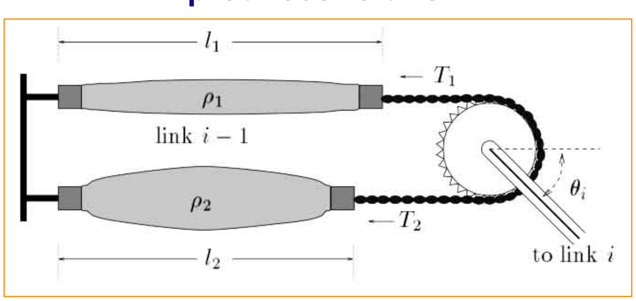
Contraction under pressure



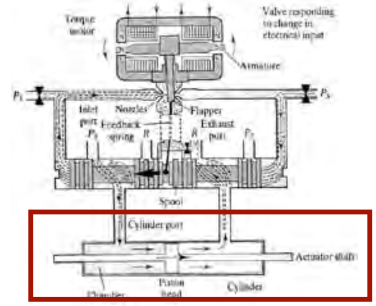
Robot arm

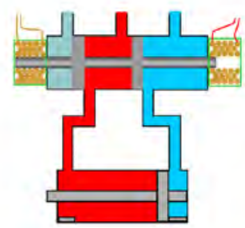


Agonist-antagonist action produces rotation

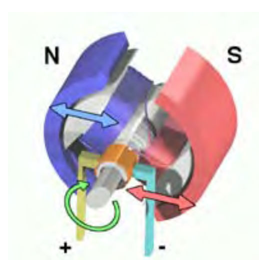


Linear Hydraulic Actuator

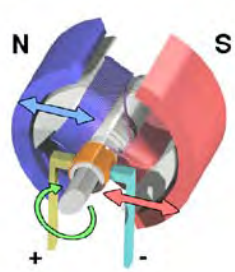


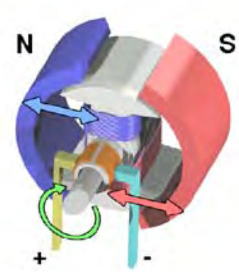


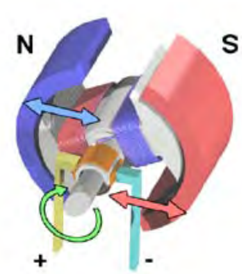
Electric Actuator
Brushed DC Motor



Two-pole DC Motor



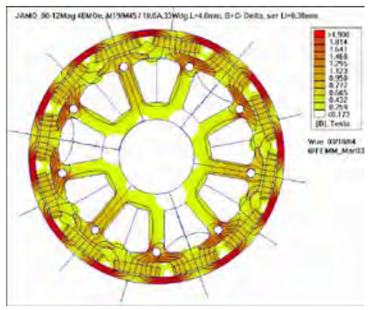


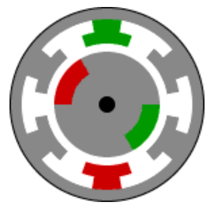


- **Current flowing through armature generates a magnetic field**
- **Permanent magnets torque the armature**
- **When armature is aligned with magnets, commutator reverses current and magnetic field**
- **Multiple poles added to allow motor to smooth output torque and to start from any position**

Electric Actuator
Brushless DC Motor

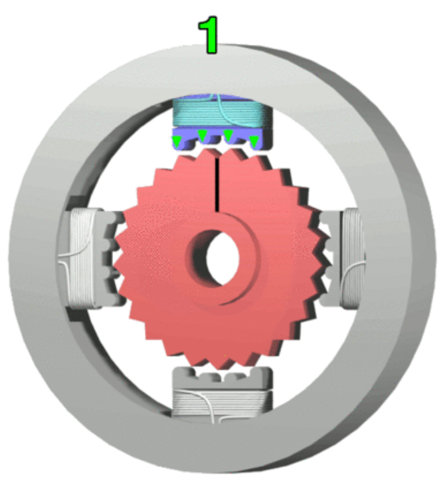
- Armature is fixed, and permanent magnets rotate
- Electronic controller commutates the electromagnetic force, providing a rotating field
- Advantages
  - Efficiency
  - Noise
  - Lifetime
  - Reduced EMI
  - Cooling
  - Water-resistant

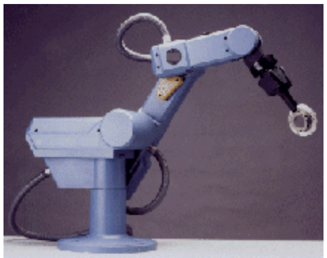




Electric Actuator
Stepper Motor

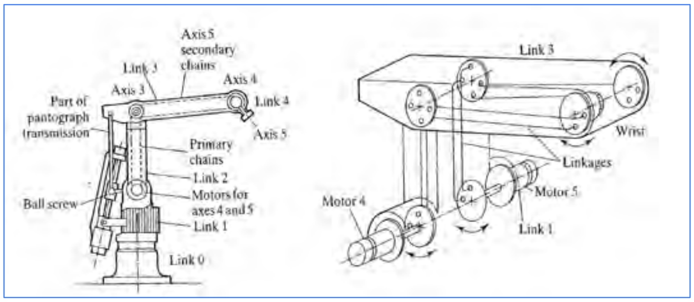
- Brushless, synchronous motor that moves in discrete steps
- Precise, quantized control without feedback
- Armature teeth offset to induce rotary motion





Actuation Linkages

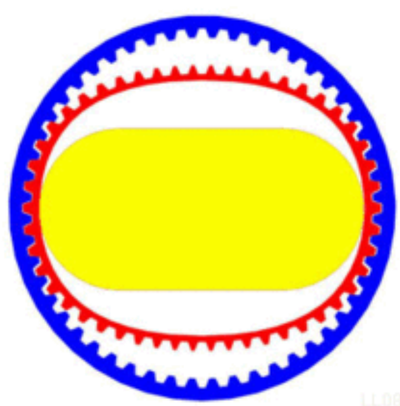
- Gearing, leverage
- Gears
- Belts, Chains, Cables
- Bellcranks



Belt Linkage
http://www.youtube.com/watch?v=FV_P7GBAAgo

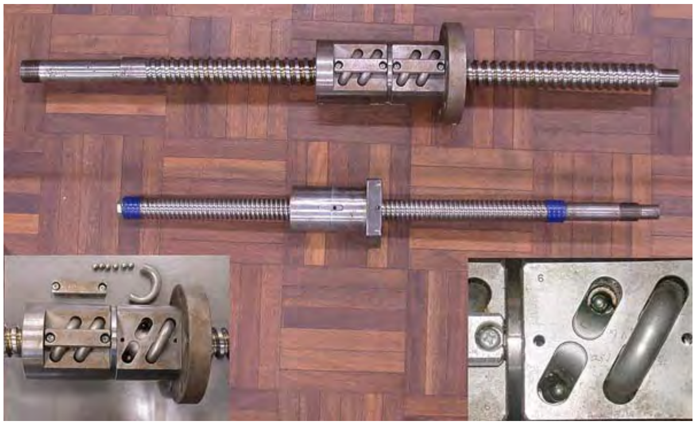
Harmonic Drive

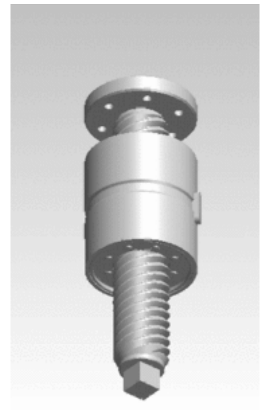
- Strain wave gearing on motor output
- No backlash
- High gear ratios
- Good resolution and repeatability
- High torque



Ball/Roller Screw

Transforms rotary to linear motion



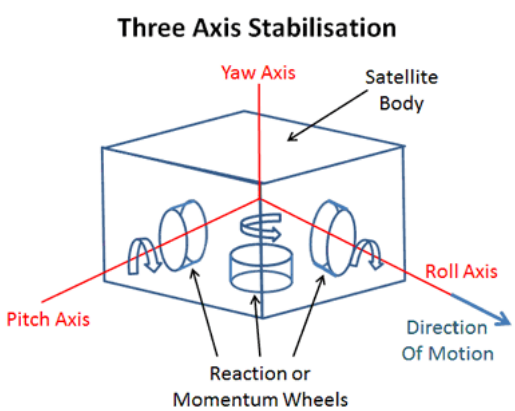


Reaction Wheel

Flywheel on a motor shaft

Reaction wheel rpm is varied to trade angular momentum with a spacecraft for control

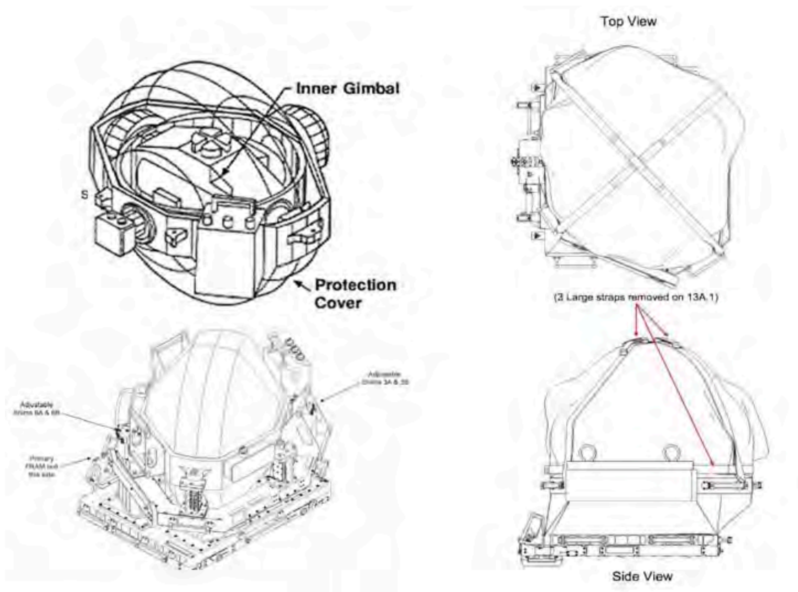
- **Three orthogonal wheels vary all components of angular momentum**
- **Fourth wheel at oblique angle would provide redundancy**



Control-Moment Gyro

Flywheel on a motor shaft

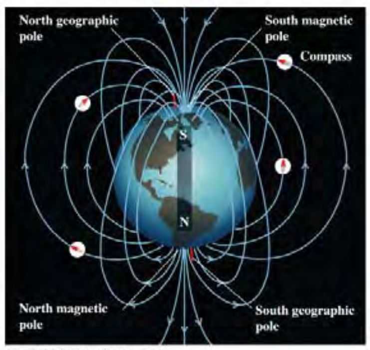
RPM is fixed, axis is rotated to impart torque

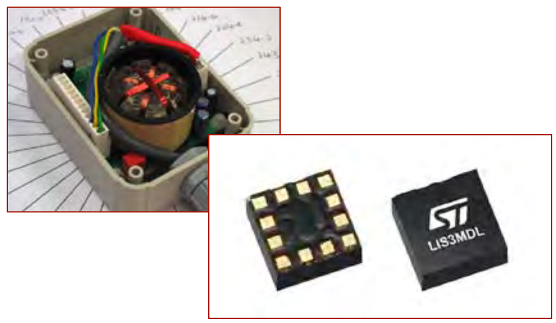


Sensors

Magnetometer

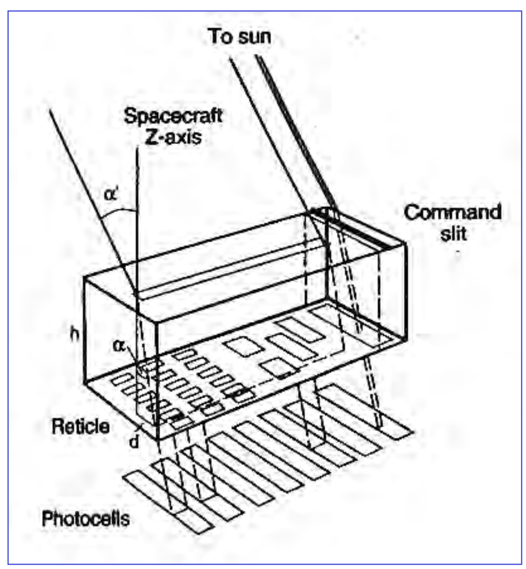
- **Flux gate "compass"**
  - **Alternating current passed through one coil**
  - **Permalloy core alternately magnetized by electromagnetic field**
  - **Corresponding magnetic field sensed by second coil**
  - **Distortion of oscillating field is a measure of one component of the Earth's magnetic field**
- **Three magnetometers required to determine Earth's magnetic field vector**





Sun Angle Sensor

- **Distance from centerline measured by sensed pattern, which determines angle, α**
- **With index of refraction, *n*, angle to sun, α', is determined**
- **Photodetectors may provide digital (coarse) or analog (fine) outputs**

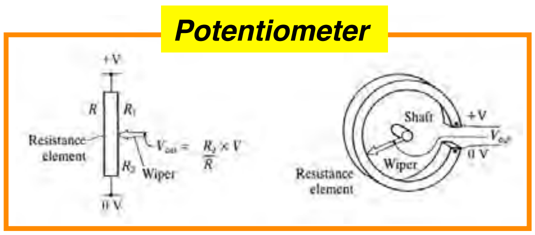


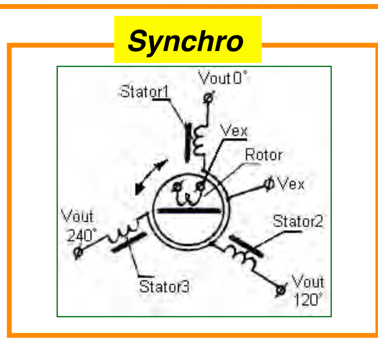
$$\tan\alpha = d / h$$

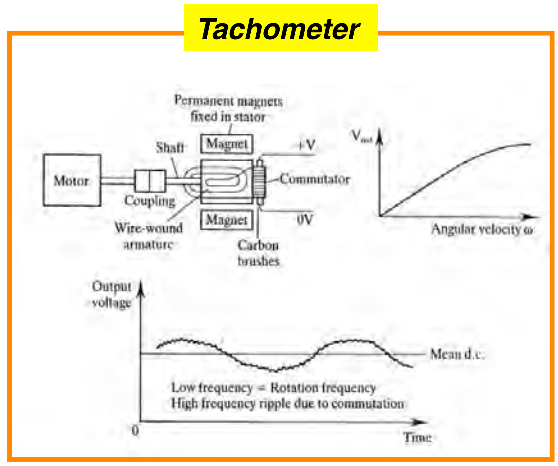
$$\sin\alpha' = n\sin\alpha \quad (Snell's\ law)$$

$$n = index\ of\ refraction$$

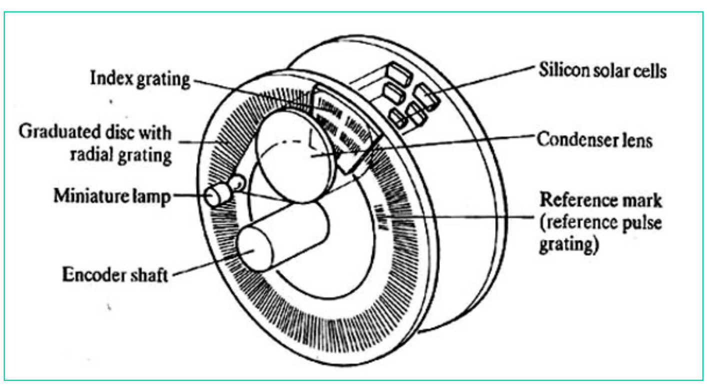
Potentiometer, Synchro, and Tachometer

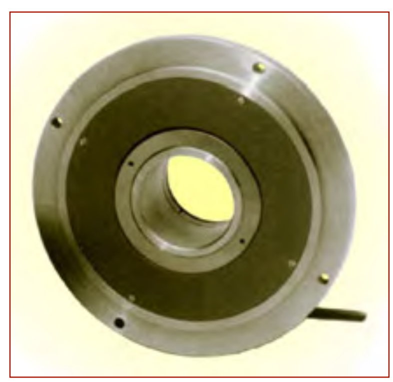


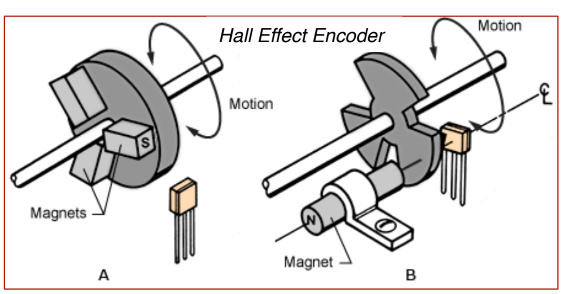


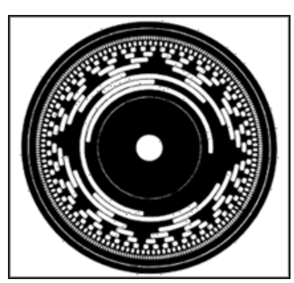


Angular Encoder

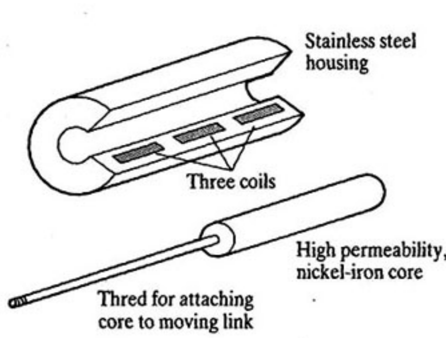


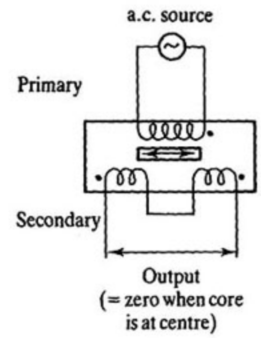


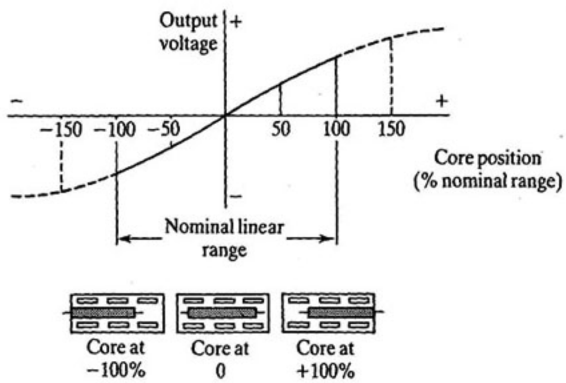




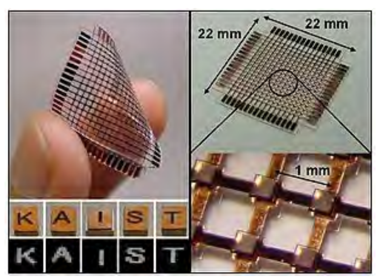
Linear Variable Differential Transformer



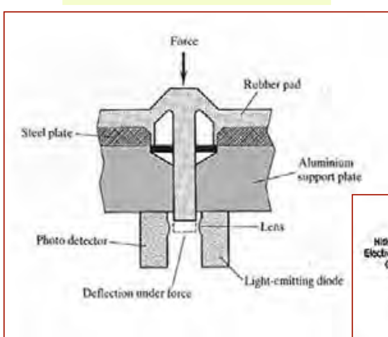




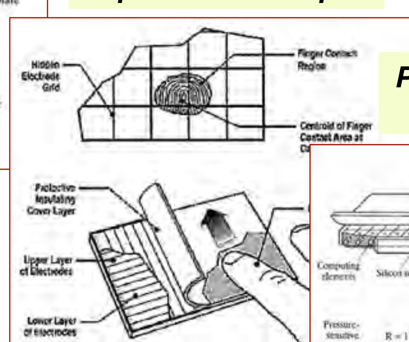
Tactile Sensors



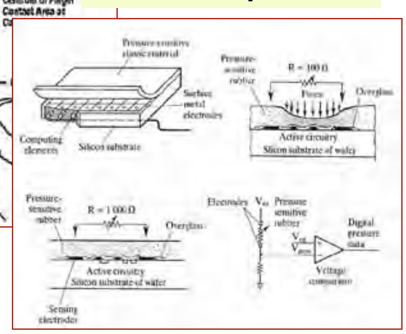
Photoelectric Key



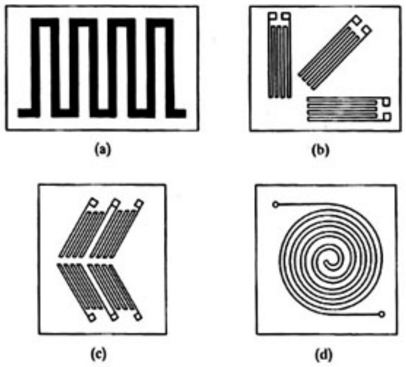
Capacitive Touchpad



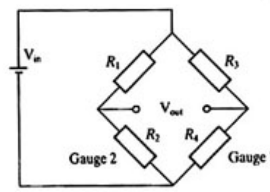
Pressure-Sensitive Touchpad



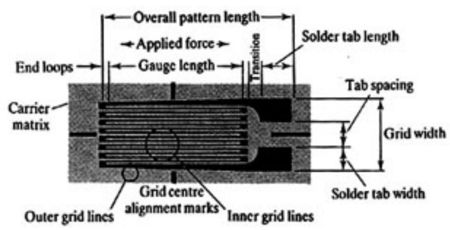
Strain Gauge



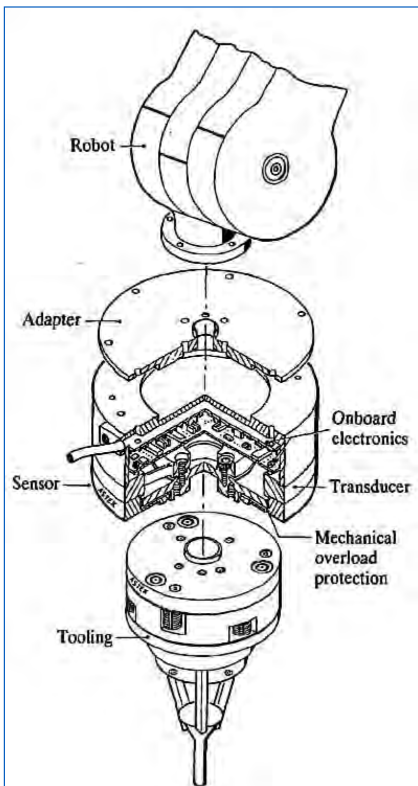
Wheatstone Bridge



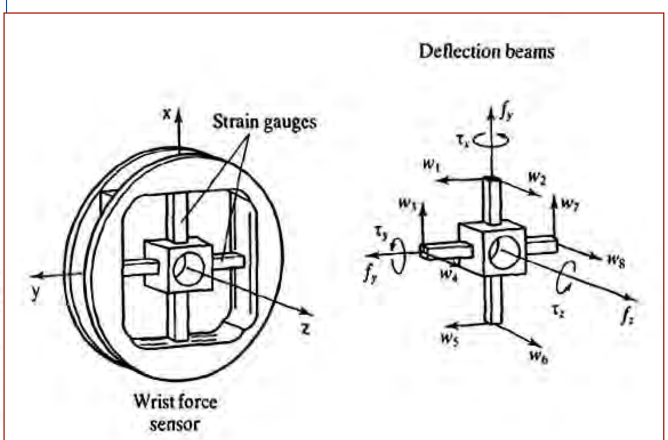
$$V_{out} = \left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right)V_{in}$$



$$\varepsilon = \left(\frac{\Delta R}{R_o}\right) \Big/ Gauge\ Factor$$



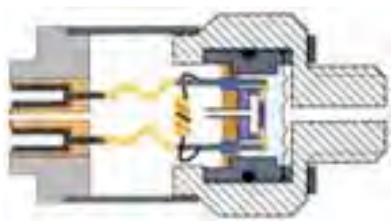
Force Sensors



Force $\propto$
Stiffness x Displacement(Strain)

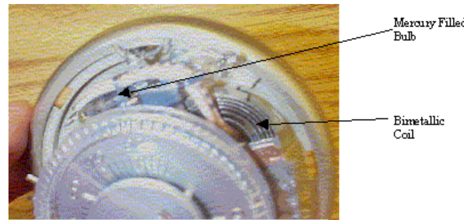
Pressure and Temperature Sensors

Deflection of Diaphragm Between Chambers at Different Pressure



- Variation in
  - Capacitance or
  - Resistance

Deflection of Bi-Metallic Element

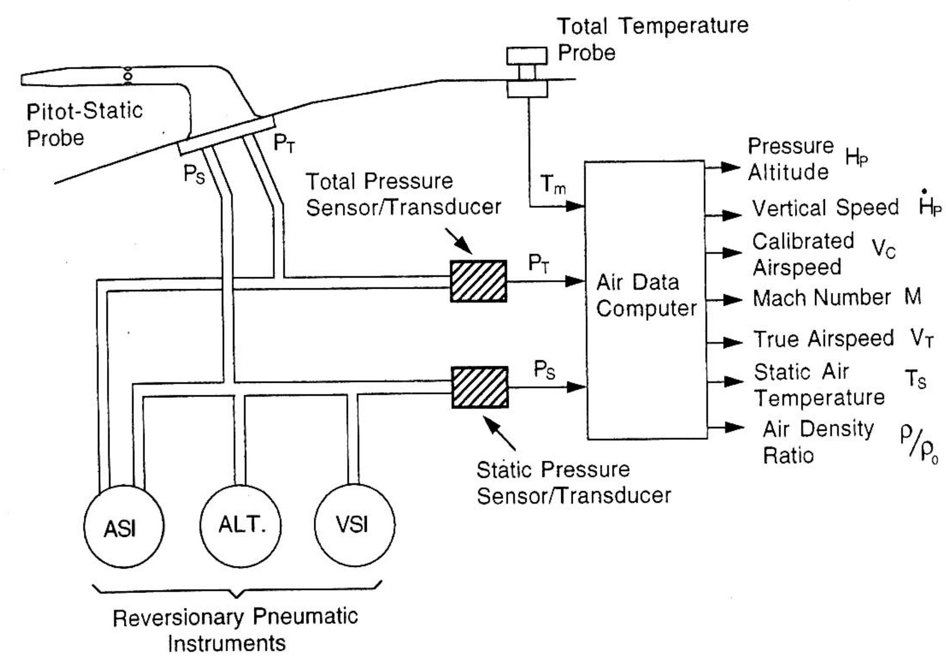


- Mercury switch - on/off

Thermistors

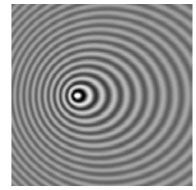
- Variation in Resistance

Air Data Sensors

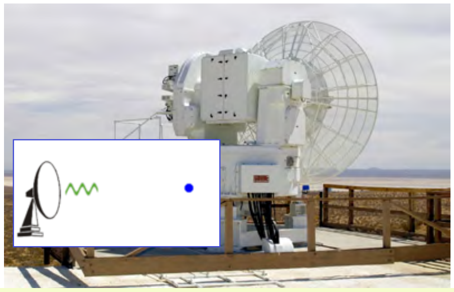


Radar and Sonar

Doppler Effect (wave source moving to the left)



Tracking (Pulse) Radar



http://www.youtube.com/watch?v=LOgRBtbEuig

Adaptive Cruise Control Radar

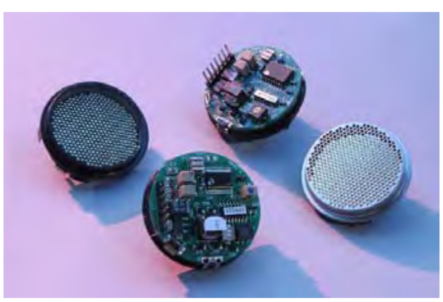
Active Electronically Steered Array Tracking Radar

Handheld Sonar

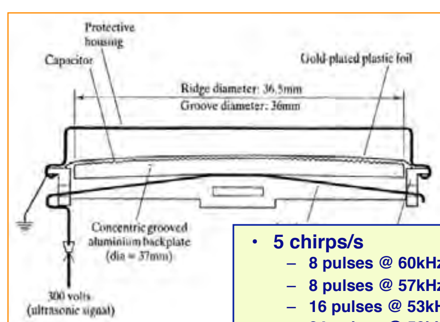
(Doppler) Radar Gun

Ultrasonic Rangefinder

SensComp ("Polaroid") Devices

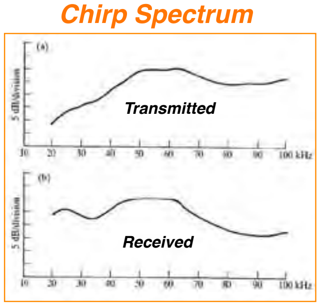


Transmit/Receive Unit

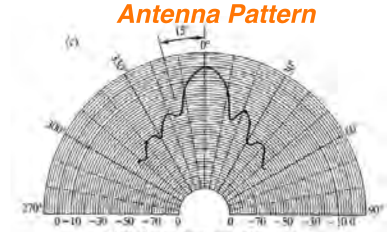


- 5 chirps/s
  - 8 pulses @ 60kHz
  - 8 pulses @ 57kHz
  - 16 pulses @ 53kHz
  - 24 pulses @ 50kHz

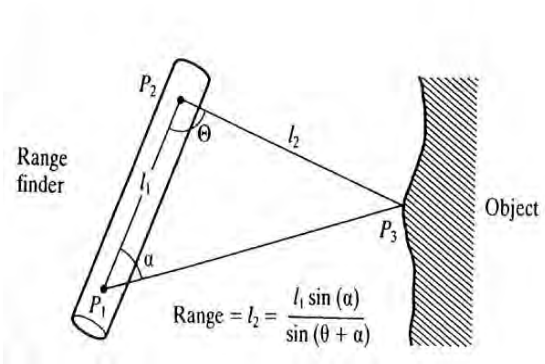
Chirp Spectrum

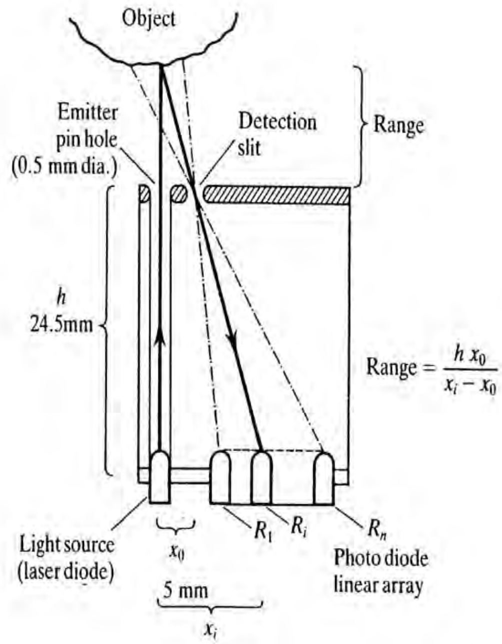


Antenna Pattern



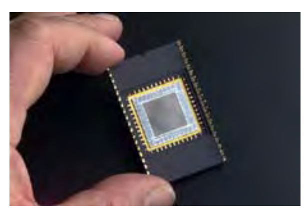
Triangulation Rangefinders



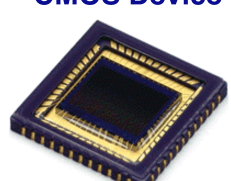


Video and Computer Vision

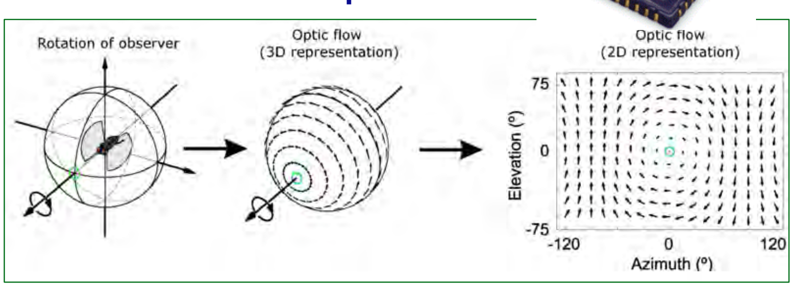
CCD Sensor



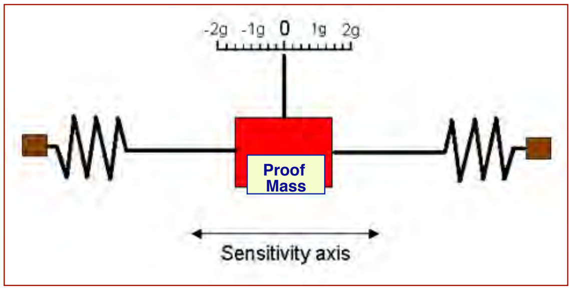
CMOS Device



Optic Flow



Spring Deflection Accelerometer

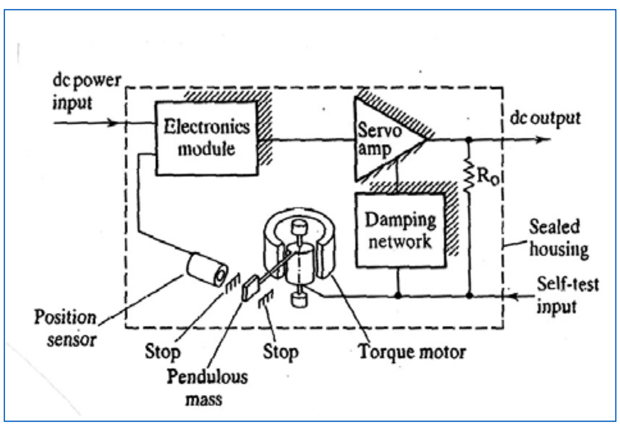


$$\Delta\ddot{x} = -k_s \Delta x / m$$

$$\Delta x = \frac{m}{k_s} \Delta\ddot{x}$$

- Deflection is proportional to acceleration
- Damping required to reduce oscillation

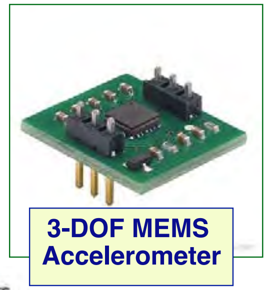
Force Rebalance Accelerometer

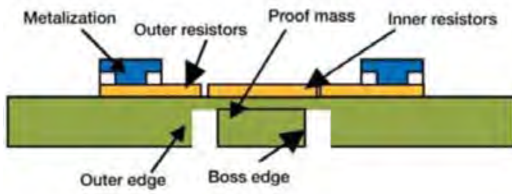


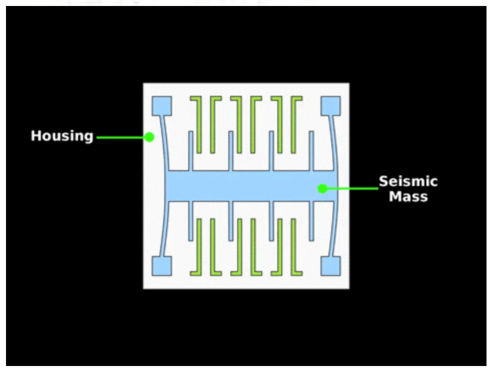
$$\Delta\ddot{x} = f_x / m = \frac{torque / moment\ arm}{m} \Rightarrow \Delta x \simeq 0$$

- Torquer voltage required to re-center the proof mass becomes the measure of acceleration
- Example of closed-loop control

MicroElectroMechanical System (MEMS) Accelerometer

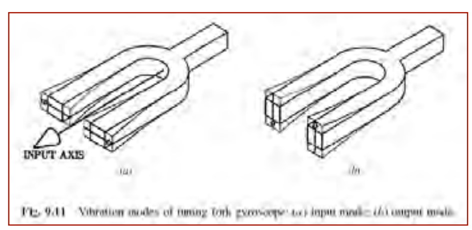


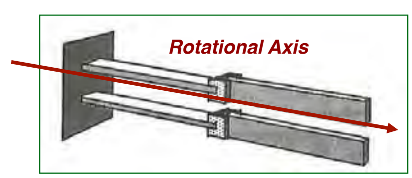




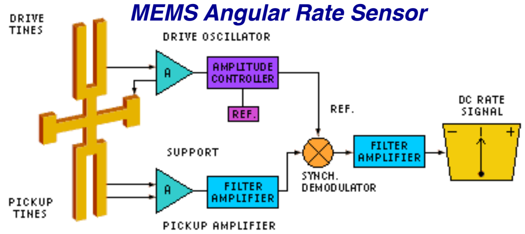
Vibrating Piezoelectric Crystal Angular Rate Sensor

- “Tuning fork” principle
- 4 piezoelectric crystals
  - 2 active, oscillating out of phase with each other
  - 2 sensors, mounted perpendicular to the active crystals
- With zero rate along the long axis, sensors do not detect vibration
- Differential output of the sensors is proportional to angular rate



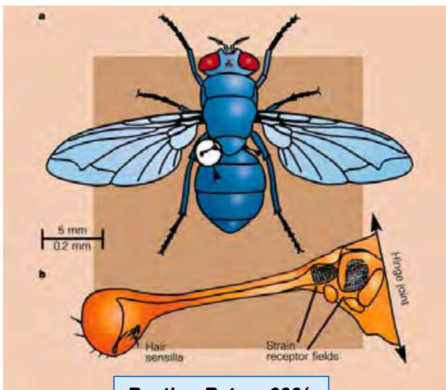


MEMS Angular Rate Sensor

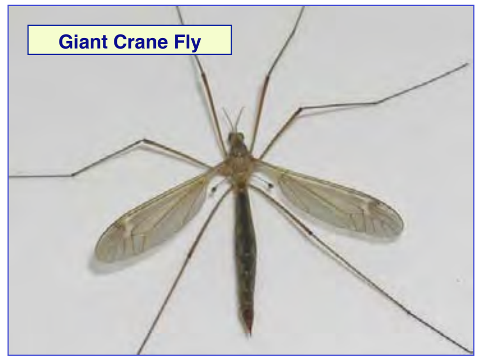


Halteres:
Biological Angular Rate Sensors

Vestigal second pair of wings



Beating Rate: ~600/s



All in Your Pocket

iPhone 6s

- 3-axis accelerometer
- 3-axis angular rate
- 2-axis magnetometer compass
- GPS position measurement
- 64-bit, 1.8 GHz processor
- 2 GB RAM
- 128 GB flash memory
- 2 cameras, mic, speakers

$$\mathbf{z} = \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ p \\ q \\ r \\ \varepsilon_{horizontal} \\ \varepsilon_{vertical} \\ L \\ \lambda \\ h \end{bmatrix}$$

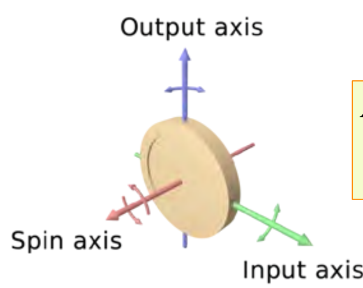
Parrot AR.Drone 2.0

- 1GHz 32 bit ARM Cortex A8 processor with 800MHz video DSP TMS320DMC64x
- Linux 2.6.32
- 1Gbit DDR2 RAM at 200MHz
- USB 2.0 high speed for extensions
- Wi-Fi b,g,n
- 3 axis gyroscope 2000°/second precision
- 3 axis accelerometer +-50mg precision
- 3 axis magnetometer 6° precision
- Pressure sensor +/- 10 Pa precision
- Ultrasound sensors for ground altitude measur
- 60 fps vertical ground speed m

- 4 brushless inrunner motors. 14.5W 28,500 RMP
- Micro ball bearing
- Low noise Nylatron gears for 1/8.75 propeller reductor
- Tempered steel propeller shaft
- Self-lubrificating bronze bearing
- Specific high propelled drag for great maneuverability
- 8 MIPS AVR CPU per motor controller
- 3 elements 1000 mA/H LiPo rechargeable battery (Autonomy: 12 minutes)
- Emergency stop controlled by software
- Fully reprogrammable motor controller
- Water resistant motor's electronic controller

- HD Camera. 720p 30fps
- Wide angle lens : 92° diagonal
- H264 encoding base profile
- Low latency streaming
- Video storage on the fly with the remote device
- JPEG photo
- Video storage on the fly with Wi-Fi directly on your remote device or on a USB key

Mechanical Gyroscope



Angular momentum

$$\mathbf{h}_B = \boldsymbol{I}_B \boldsymbol{\omega}_B$$

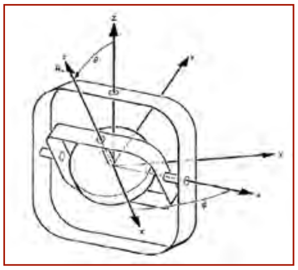
- **Body-axis moment equation**

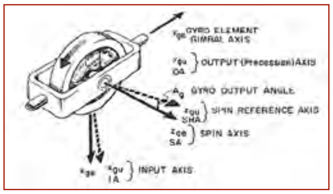
$$\boxed{\mathbf{M}_B = \dot{\mathbf{h}}_B + \tilde{\boldsymbol{\omega}}_B \mathbf{h}_B = \mathbf{I}_B \dot{\boldsymbol{\omega}}_B + \tilde{\boldsymbol{\omega}}_B \mathbf{h}_B}$$

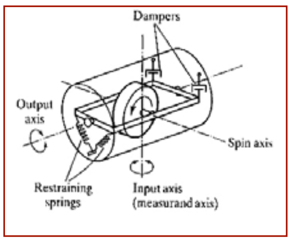
$$\boxed{\dot{\boldsymbol{\omega}}_B = \boldsymbol{I}_B^{-1}\left(M_B - \tilde{\boldsymbol{\omega}}_B \boldsymbol{I}_B \boldsymbol{\omega}_B\right)}$$

Constant nominal spin rate, n, about z axis
$I_{xx} = I_{yy} << I_{zz}$
Small perturbations in ω_x and ω_y

Types of Mechanical Gyroscope



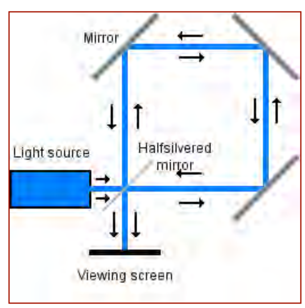




- **Two-degree-of-freedom gyro**
  - Free gyro mounted on a gimbaled platform
  - Gyro "stores" reference direction in space
  - Angle" pickoffs" (encoders) on gimbal axes measure pitch and yaw angles
- **Single-degree-of-freedom gyro**
  - Gyro axis constrained to rotate in its case with respect to the output axis, ***y***, only
  - "Synchro" measures axis rotation, and "torquer" keeps θ small
  - Torque applied is a measure of the input about the ***x*** axis
- **Rate and integrating gyros**
  - Large angle feedback produces a **rate gyro**
  - Large rate feedback produces an **integrating gyro**

Optical "Gyroscope"

- **Sagnac interferometer measures rotational rate, ω**
  - $\omega = 0$, photons traveling in opposite directions complete the circuit in the same time
  - $\omega \neq 0$, travel length and time are different
- **On a circular path of radius *R*:**



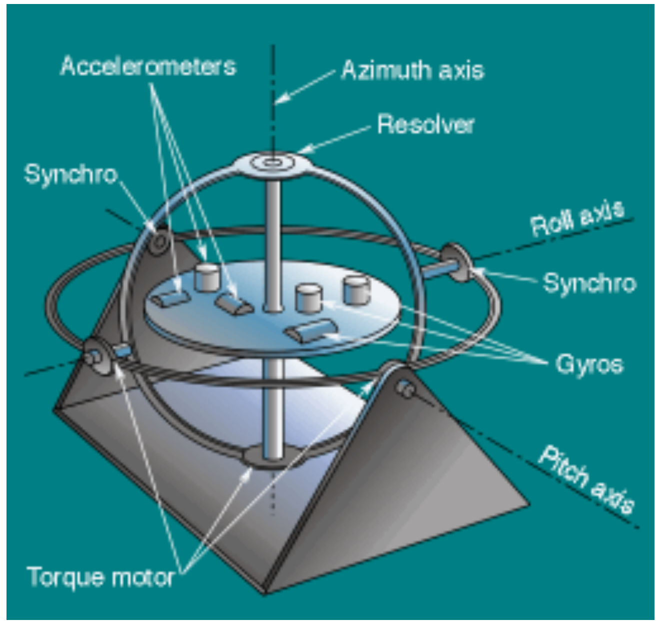
$$t_{CCW} = \frac{2\pi R}{c}\left(1 - \frac{R\omega}{c}\right); \quad t_{CW} = \frac{2\pi R}{c}\left(1 + \frac{R\omega}{c}\right)$$

$$\Delta t = t_{CW} - t_{CCW} = \frac{4\pi R^2}{c^2}\omega = \frac{4A}{c^2}\omega$$

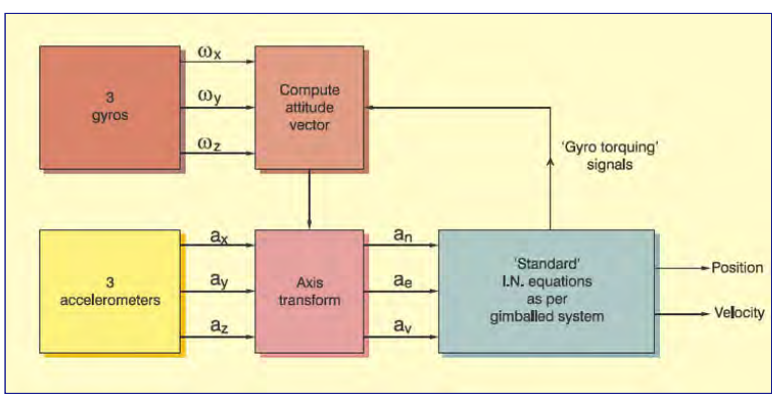
$c:$ speed of light
$R:$ radius
$A:$ area

Physical Platform Inertial Reference Unit

- Physical platform is servo-driven to maintain reference orientation
  - Instrument feedback
  - Schuler pendulum
  - Gyro-compassing
  - Star trackers
  - GPS
- 3 Accelerometers
- 3 Angle or Angular Rate Gyros



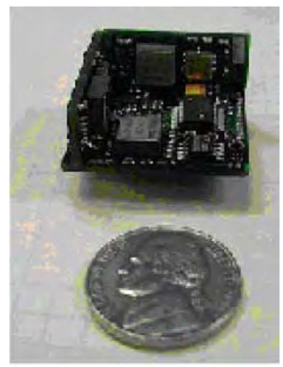
Strapdown Inertial Measurement Unit



- Rate gyros and accelerometers rotate with the vehicle
  - High dynamic range of instruments is required
  - Inertial reference frame is computed rather than physical
  - Use of direction cosine matrix and quaternions for attitude reference

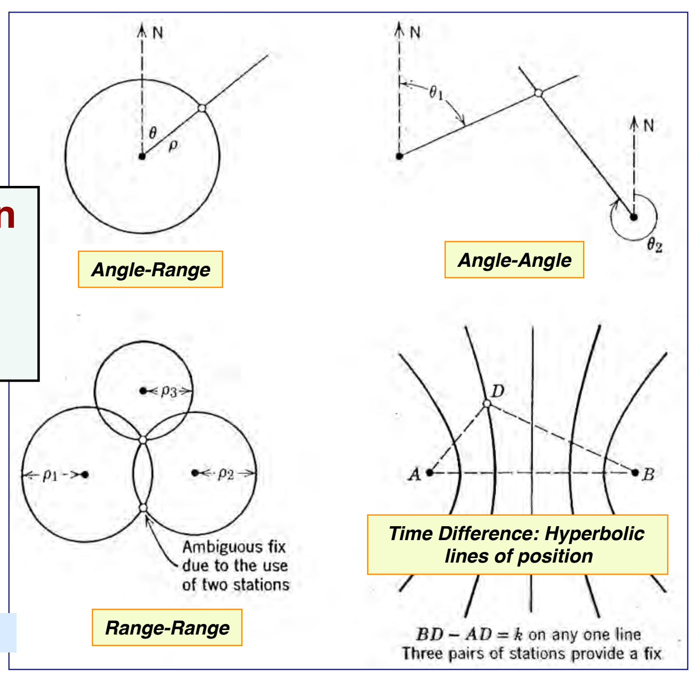
MicroElectroMechanical (MEMS) Strapdown Inertial Measurement Unit

- 3 linear accelerometers, 3 angular rate sensors
  - High drift rates produce worsening navigation accuracy
  - Short-term accuracy sufficient for many applications
  - Inexpensive
  - GPS position updating counters the drift rate



Position Fixing for Navigation (2-D Examples)

- Lines of position
  - Straight line
  - Circle
  - Hyperbola

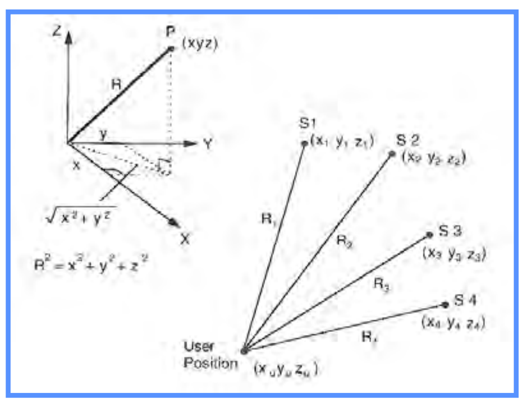


Kayton & Fried

Global Positioning System (GPS)

- Six orbital planes with four satellites each
  - Altitude: 20,200 km (10,900 nm)
  - Inclination : 55 deg
  - Constellation planes separated by 60 deg
- Each satellite contains an atomic clock and broadcasts a 30-sec message at 50 bps
  - Ephemeris
  - ID
  - Clock data
- Details of satellite signal at http://en.wikipedia.org/wiki/Gps
- http://www.youtube.com/watch?v=v_6yeGcpoyE

Position Fixing from Four GPS Satellites



- Pseudorange estimated from *speed of light* and *time required* to receive signal

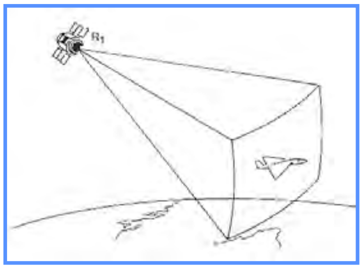
$$\boxed{\Delta t_i = \left(t_{received} - t_{sent}\right)_{Satellite\,\#\,i}}$$

$$Satellite\ \#1 : R_{1_p} = c\Delta t_1$$
$$Satellite\ \#2 : R_{2_p} = c\Delta t_2$$

$$Satellite\ \#3 : R_{3_p} = c\Delta t_3$$
$$Satellite\ \#4 : R_{4_p} = c\Delta t_4$$

User clock inaccuracy **produces error, C_u**

$$\boxed{C_u = c\Delta t_{user\ clock\ error}}$$



Position Fixing from Four GPS Satellites

Satellite position: (x_i, y_i, z_i)

User position: (x_u, y_u, z_u)

- **Satellite transmits transmit time and position via *ephemeris***

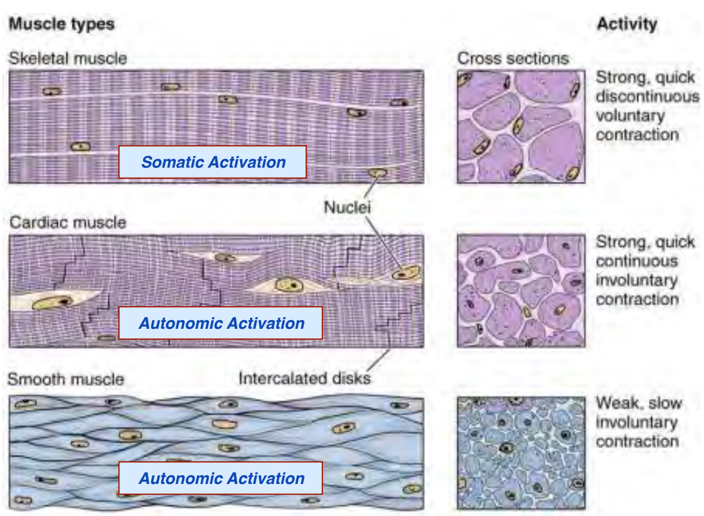
$$
\begin{aligned}
R_1 &= \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} = R_{1_p} + C_u \\
R_2 &= \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} = R_{2_p} + C_u \\
R_3 &= \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} = R_{3_p} + C_u \\
R_4 &= \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} = R_{4_p} + C_u
\end{aligned}
$$

- **Four equations and four unknowns (x_u, y_u, z_u, C_u)**
- **Accuracy improved using data from more than 4 satellites**

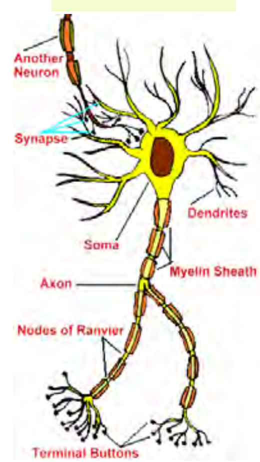
Next Time: Introduction to Optimization

Supplementary Material

Muscle and Motor (*Efferent*) Neurons

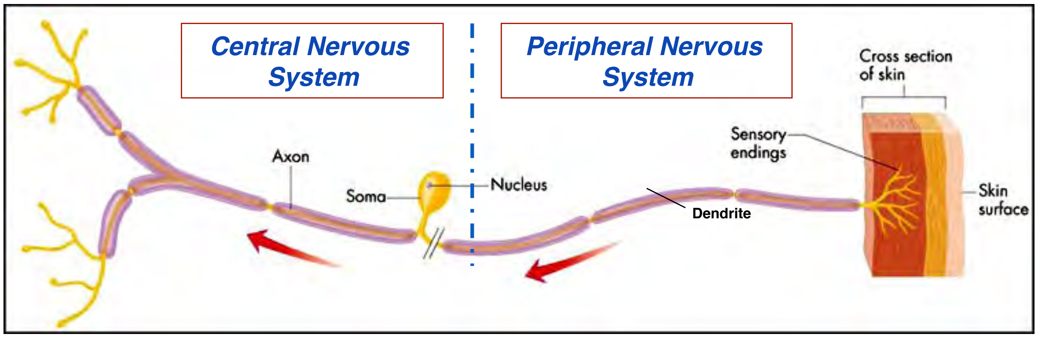


Multipolar neuron



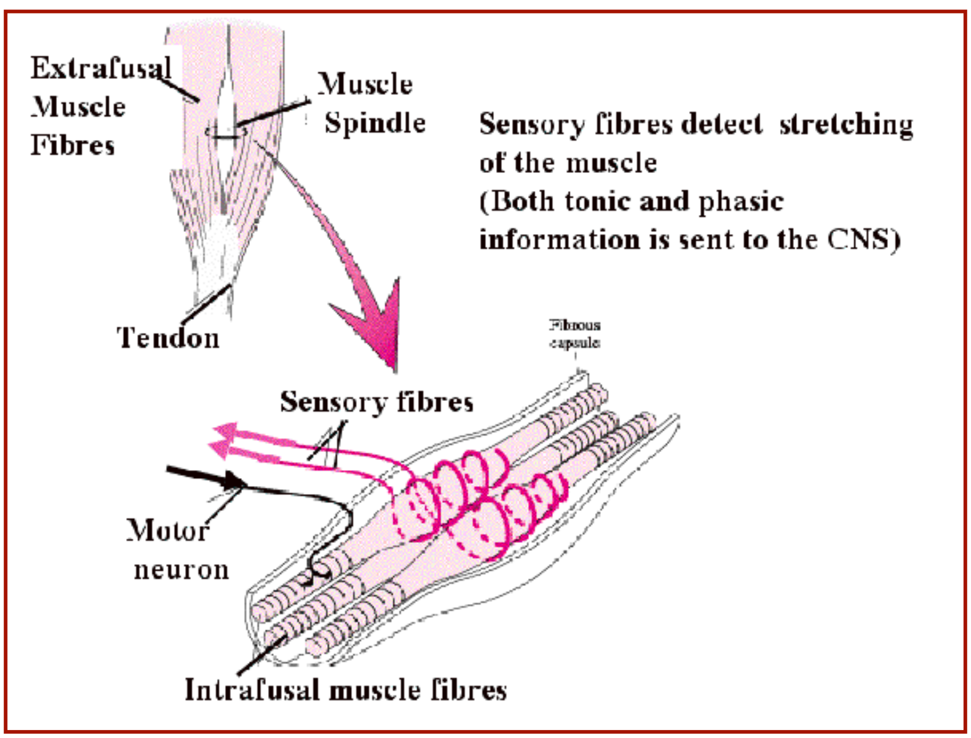
- Force is produced by contraction of individual muscle cells
- Motor neurons command muscles
- Each muscle cell is innervated by many overlapping neurons
- Motor neuron soma are in ventral root ganglia of the spine

Sensory (*Afferent*) Neurons

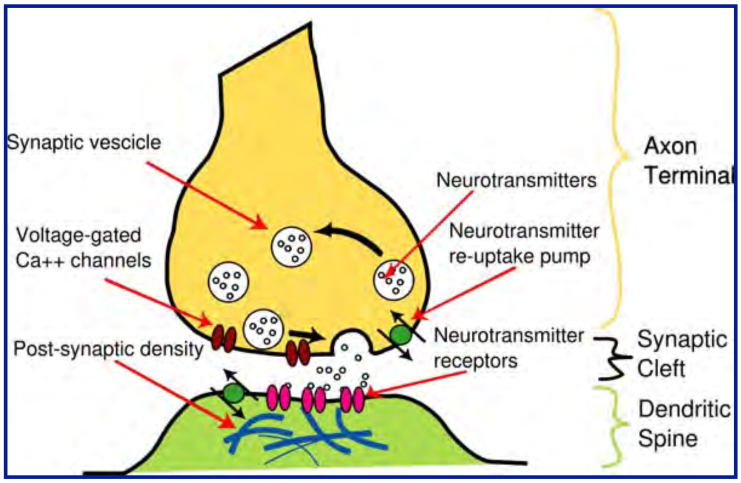


- Components of the peripheral nervous system that measure *pressure*, *temperature*, *vibration*, etc.
- Neuron Soma located in the dorsal root at the base of the spine
- The sensory neuron is pseudo-unipolar
  - Input from a single receptor's axon
  - Output to a single axon to synapses in the spinal column

Motor Neuron Receptors

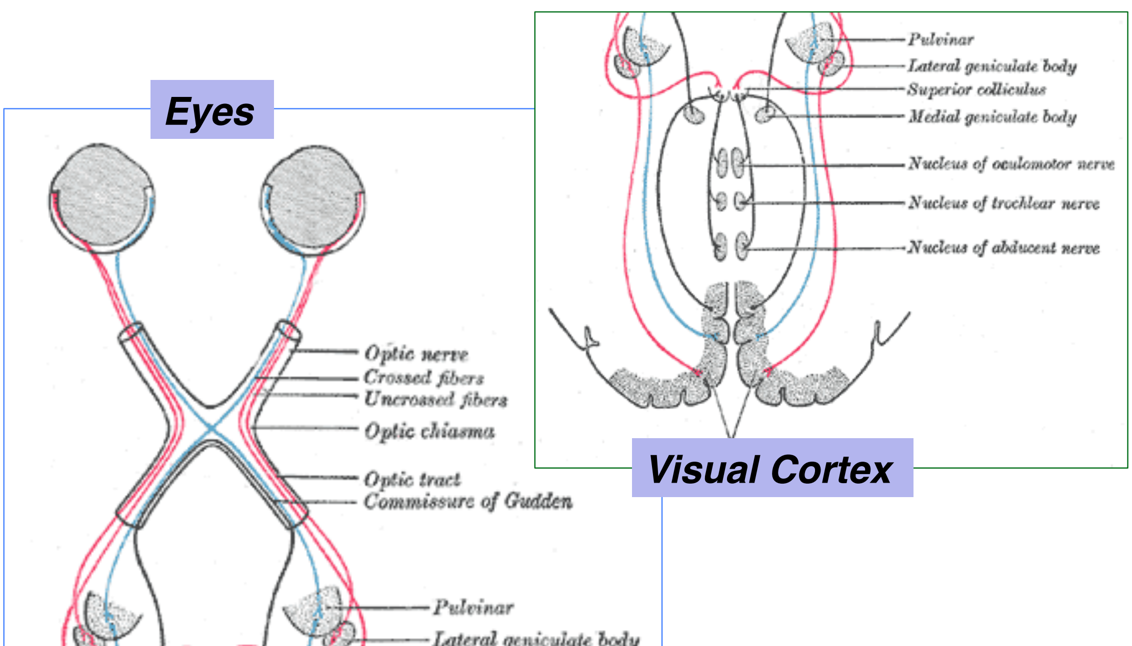


Synapses Excite or Inhibit Downstream Cellular Activity



- **Post-synaptic cell can be a neuron, a muscle, or a gland**

Optic Schema



ADVANTAGES AND DISADVANTAGES OF HYDRAULIC ACTUATORS (from McKerrow)

Hydraulic actuators

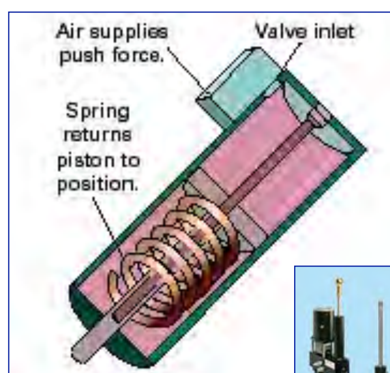
Advantages

- Large lift capacity
- High power to weight ratio
- Moderate speeds
- Oil is incompressible, hence once positioned joints can be locked to a stiff structure
- Very good servo control can be achieved
- Self lubricating and self cooling
- Operate in stalled condition with no damage
- Fast response
- Intrinsically safe in flammable and explosive atmospheres
- Smooth operation at low speeds

Disadvantages

- Hydraulic systems are expensive
- Maintenance problems with seals causing leakage
- Not suitable for high speed cycling
- Need for a return line
- Hard to miniaturize because high pressures and flow rates
- Need for remote power source which uses floor space
- Cannot back drive links against valves

ADVANTAGES AND DISADVANTAGES OF PNEUMATIC ACTUATORS (from McKerrow)



Pneumatic actuators

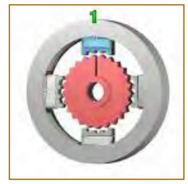
Advantages

- Relatively inexpensive
- High speed
- Do not pollute work area with fluids
- Can be used in laboratory work
- No return line required
- Common energy source in industry
- Suits modular robot designs
- Actuator can stall without damage

Disadvantages

- Compressibility of air limits control and accuracy aspects
- Noise pollution from exhausts
- Leakage of air can be of concern
- Additional drying/filtering may be required
- Difficulties with control of speeds, take up of loads, and exhausting of lines

Advantages and Disadvantages of Electric Actuators (DC motor and stepper motor) (from McKerrow)



Electric actuators (DC motors and stepper motors)

Advantages

- Actuators are fast and accurate
- Possible to apply sophisticated control techniques to motion
- Relatively inexpensive
- Very fast development times for new models
- New rare earth motors have high torques, reduced weight, and fast response times

Disadvantages

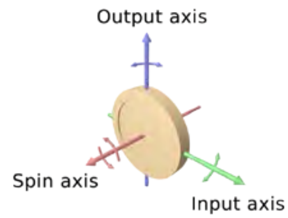
- Inherently high speed with low torque, hence gear trains or other power transmission units are needed
- Gear backlash limits precision
- Electrical arcing may be a consideration in flammable atmospheres
- Problems of overheating in stalled condition
- Brakes are needed to lock them in position

Autonomous Control of Miniature Aircraft Using Optical Flow

http://www.youtube.com/watch?v=F7QxDJiZHwl&feature=related

Swinglet, Ecole Polytechnique, Lausanne

Gyroscope Equations of Motion

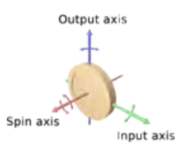


Linearized equations of angular rate change

$$\begin{bmatrix}\Delta\dot{\omega}_x \\ \Delta\dot{\omega}_y \\ 0\end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz}\end{bmatrix}^{-1} \left[\begin{bmatrix} M_x \\ M_y \\ 0\end{bmatrix} - \begin{pmatrix} 0 & -n & \Delta\omega_y \\ n & 0 & -\Delta\omega_x \\ -\Delta\omega_y & \Delta\omega_x & 0\end{pmatrix}\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz}\end{pmatrix}\begin{pmatrix}\Delta\omega_x \\ \Delta\omega_y \\ n\end{pmatrix}\right]$$

$$\begin{bmatrix}\Delta\dot{\omega}_x \\ \Delta\dot{\omega}_y \\ 0\end{bmatrix} = \begin{bmatrix} \left[M_x - n\left(I_{zz} - I_{yy}\right)\Delta\omega_y\right]/I_{xx} \\ \left[M_y - n\left(I_{xx} - I_{zz}\right)\Delta\omega_x\right]/I_{yy} \\ 0\end{bmatrix}$$

$$\begin{bmatrix}\Delta\dot{\omega}_x \\ \Delta\dot{\omega}_y\end{bmatrix} = \begin{bmatrix} 0 & n\left(I_{yy} - I_{zz}\right)/I_{xx} \\ n\left(I_{zz} - I_{xx}\right)/I_{yy} & 0\end{bmatrix}\begin{bmatrix}\Delta\omega_x \\ \Delta\omega_y\end{bmatrix} + \begin{bmatrix} M_x / I_{xx} \\ M_y / I_{yy}\end{bmatrix}$$



Gyroscope Natural Frequency

Laplace transform of dynamic equation

$$\begin{bmatrix} s & -n\left(I_{yy} - I_{zz}\right)/I_{xx} \\ -n\left(I_{zz} - I_{xx}\right)/I_{yy} & s\end{bmatrix}\begin{bmatrix}\Delta\omega_y(s) \\ \Delta\omega_y(s)\end{bmatrix} = \begin{bmatrix} M_x(s)/I_{xx} \\ M_y(s)/I_{yy}\end{bmatrix}$$

- **Characteristic equation**

$$\Delta(s) = s^2 + n^2\left(\frac{I_{zz}}{I_{xx}} - 1\right)^2 = 0$$

- **Natural frequency, ω_n, of small perturbations**

$$\omega_n = n\left(\frac{I_{zz}}{I_{xx}} - 1\right) \quad rad/\sec$$

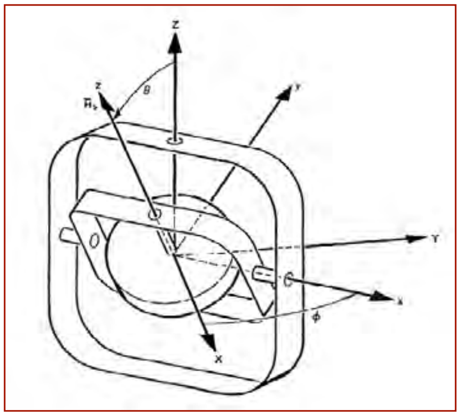
Example

$$n = 36{,}000\ rpm = 3{,}770\ rad/sec$$

$$\text{Thin disk: } \frac{I_{zz}}{I_{xx}} = 2$$

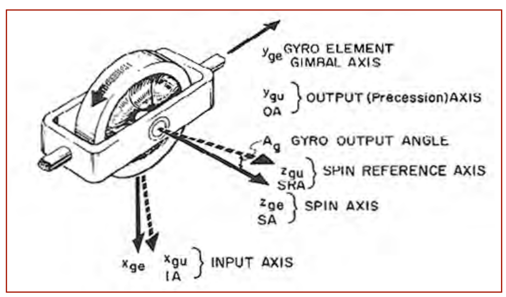
$$\omega_n = 3{,}770\ rad/sec = 600\ Hz$$

Two-Degree of Freedom Gyroscope



- Free gyro mounted on a gimbaled platform
- Gyro "stores" reference direction in space
- Angle "pickoffs" (encoders) on gimbal axes measure pitch and yaw angles
- Direction can be precessed by applying a torque

Single-Degree-of-Freedom Gyroscope



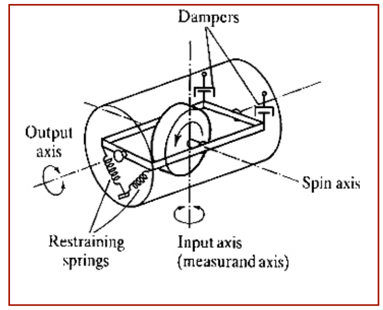
Gyro axis constrained to rotate in its case with respect to the output axis, *y*, only

$$\begin{bmatrix} \Delta\dot{\theta} \\ \Delta\dot{\omega}_y \end{bmatrix} = \begin{bmatrix} \Delta\omega_y \\ \left(h_{rotor}\Delta\omega_x + M_{y_{control}}\right)/I_{yy} \end{bmatrix}$$

"Synchro" measures axis rotation, and "torquer" keeps θ small
Torque applied is a measure of the input about the *x* axis

$$M_{y_{control}} = k_\theta \Delta\theta + k_\omega \Delta\omega_y + k_c \Delta u_c$$

Rate and Integrating Gyroscopes



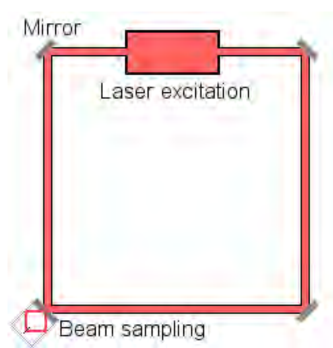
- **Large angle feedback produces a rate gyro**
  - Analogous to a mechanical spring restraint

$$\Delta\dot{\omega}_{y_{SS}} = 0 = \left(h_{rotor}\Delta\omega_{x_{SS}} + k_\theta \Delta\theta_{SS}\right)/I_{yy}$$
$$\Delta\theta_{SS} = -\frac{h_{rotor}}{k_\theta}\Delta\omega_{x_{SS}}$$

- **Large rate feedback produces an integrating gyro**
  - Analogous to a mechanical damper restraint

$$\Delta\dot{\omega}_{y_{SS}} = 0 = \left(h_{rotor}\Delta\omega_{x_{SS}} + k_\omega \Delta\omega_{y_{SS}}\right)/I_{yy}$$
$$\Delta\omega_{y_{SS}} = -\frac{h_{rotor}}{k_\omega}\Delta\omega_{x_{SS}}$$
$$\Delta\theta_{SS} = \Delta\phi_{SS}$$

Ring Laser Gyro



- **Laser in optical path creates photon resonance at wavelength λ**
- **Frequency change in cavity is proportional to angular rate**
- **Three RLGs needed to measure three angular rates**

$$\Delta f = \frac{4A}{\lambda P}\omega$$
$$P: \quad perimeter\ length$$

Fiber Optic Gyro

- **Long length of fiber cable wrapped in a circle**
- **Photon source and sensor are external to the fiber optics**
- **Length difference for opposite beams is**

$$\boxed{\Delta L = \frac{4AN}{c}\omega}$$

$A:\ \ included\ area$

$N:\ \ number\ of\ turns$

- **Phase difference is proportional to angular rate**

$$\boxed{\Delta\varphi = \frac{8\pi AN}{\lambda c}\omega}$$