Point-Mass Dynamics and Aerodynamic/Thrust Forces

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2014

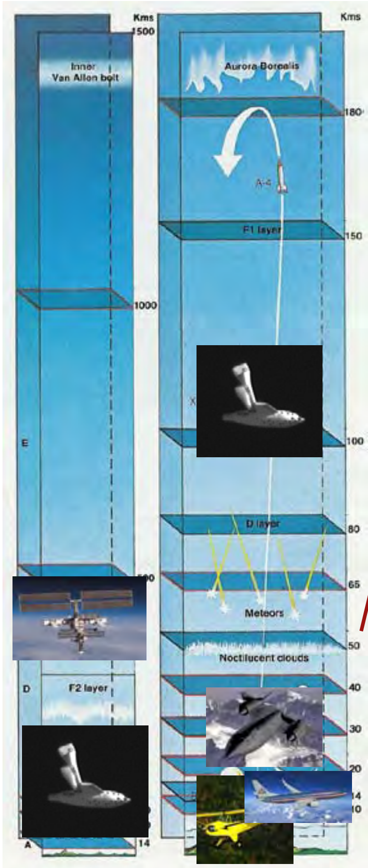
Learning Objectives

- Properties of atmosphere and dynamic pressure
- Frames of reference for position and motion
- Velocity and momentum in inertial frame
- Newton's three laws of motion and "flat-earth" gravitation
- Longitudinal and lateral-directional axes of airplane
- Lift and drag expressed using non-dimensional coefficients
- Simplified equations for longitudinal motion
- Aircraft powerplants and thrust

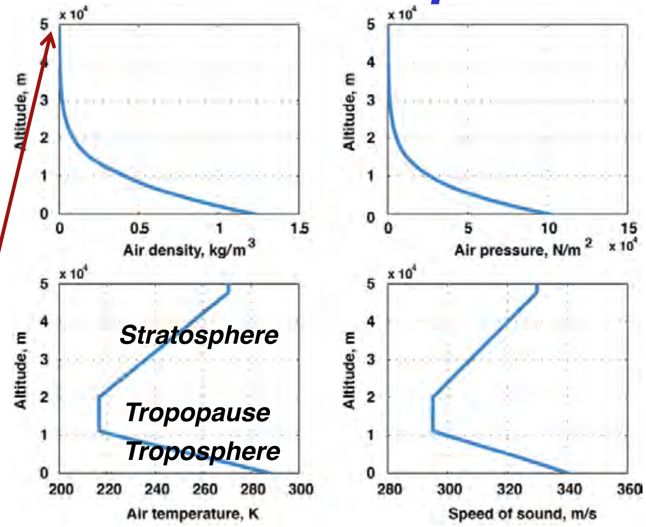
Reading:
Flight Dynamics
Introduction, 1-27
The Earth's Atmosphere, 29-34
Kinematic Equations, 38-53
Forces and Moments, 59-65
Introduction to Thrust, 103-107

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

The Atmosphere



Properties of the Lower Atmosphere*



- Air density and pressure decay exponentially with altitude
- Air temperature and speed of sound are piecewise-linear functions of altitude

* 1976 US Standard Atmosphere

Air Density, Dynamic Pressure, and Mach Number

$$\rho = Air\, density,\ function\, of\, height$$
$$= \rho_{sea\,level} e^{-\beta h} = \rho_{sea\,level} e^{\beta z}$$
$$\rho_{sea\,level} = 1.225\, kg/m^3; \quad \beta = 1/9{,}042\, m$$

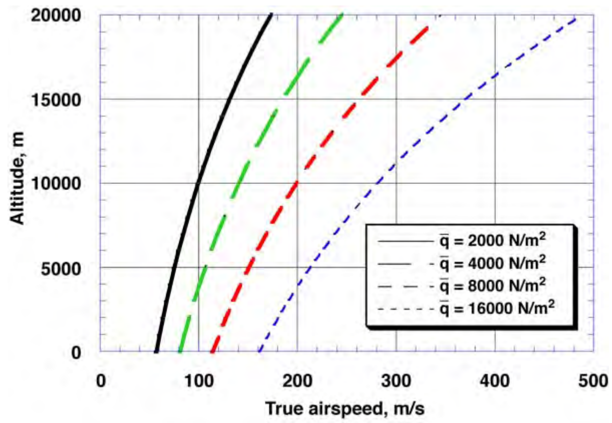
$$V_{air} = \left[v_x^2 + v_y^2 + v_z^2\right]_{air}^{1/2} = \left[\mathbf{v}^T\mathbf{v}\right]_{air}^{1/2} = Airspeed$$

$$Dynamic\ pressure = \bar{q} = \frac{1}{2}\rho(h)V_{air}^2$$

$$Mach\ number = \frac{V_{air}}{a(h)}; \quad a = speed\ of\ sound, m/s$$

Contours of Constant Dynamic Pressure, $\bar{q}$

- In steady, cruising flight, $Weight = Lift = C_L \frac{1}{2}\rho V_{air}^2 S = C_L \bar{q} S$

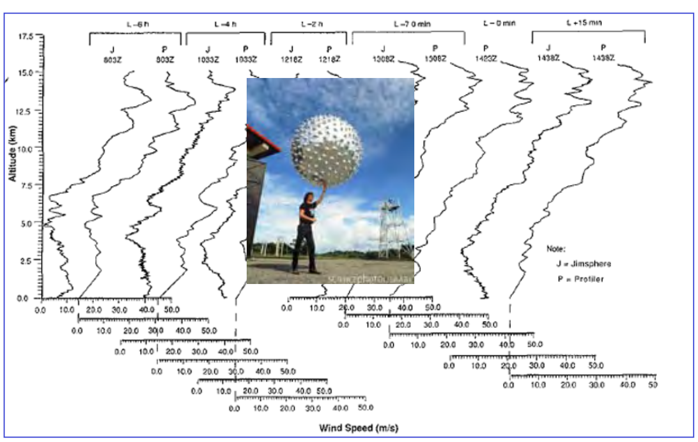


Airspeed must increase as altitude increases to maintain constant dynamic pressure

Wind: Motion of the Atmosphere

- Zero wind at Earth's surface = Rotating air mass
- Wind measured with respect to Earth's rotating surface

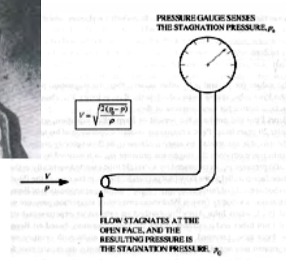
Wind Velocity Profiles vary over Time



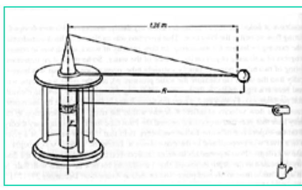
- Airspeed = Airplane's speed with respect to air mass
- Earth-relative velocity = Wind velocity ± True airspeed

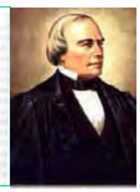
Historical Factoids

- **Henri Pitot:** Pitot tube (1732)



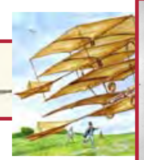
- **Benjamin Robins:** Whirling arm "wind tunnel" (1742)

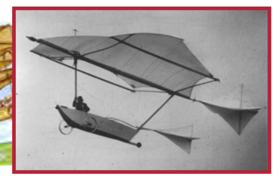




Sir George Cayley

- Sketches "modern" airplane configuration (1799)
- Hand-launched glider (1804)
- Papers on applied aerodynamics (1809-1810)
- Triplane glider carrying 10-yr-old boy (1849)
- Monoplane glider carrying coachman (1853)
  - Cayley's coachman had a steering oar with cruciform blades
  - Modern reconstruction (*right*)

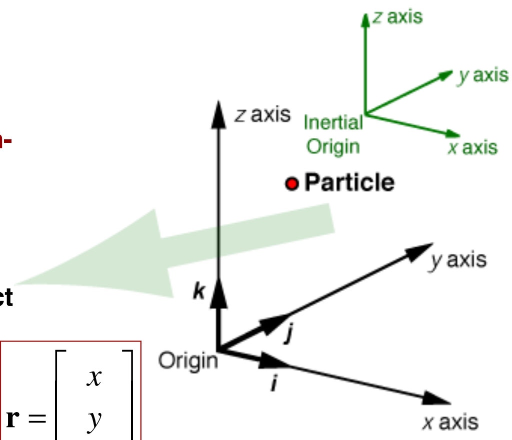




Equations of Motion for a Particle (Point Mass)

Newtonian Frame of Reference

- Newtonian (Inertial) Frame of Reference
  - Unaccelerated Cartesian frame: origin referenced to inertial (non-moving) frame
  - Right-hand rule
  - Origin can translate at constant linear velocity
  - Frame cannot rotate with respect to inertial origin



- Position: 3 dimensions
- What is a non-moving frame?

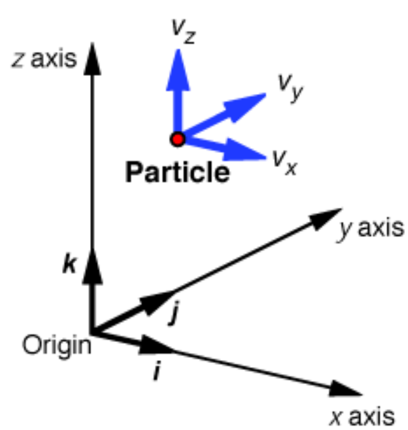
$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Translation changes the position of an object

Velocity and Momentum

- Velocity of a particle

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$



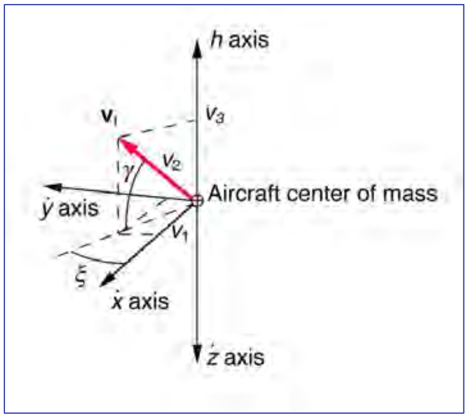
- Linear momentum of a particle

$$\mathbf{p} = m\mathbf{v} = m\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

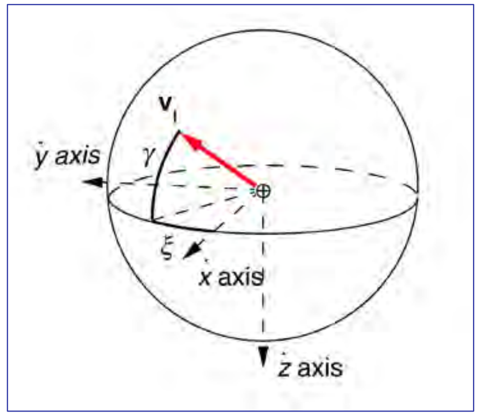
$$\textit{where} \quad m = \textit{mass of particle}$$

Inertial Velocity Expressed in Polar Coordinates

Polar Coordinates



Projected on a Sphere



γ : Vertical Flight Path Angle, rad or deg

ξ : Horizontal Flight Path Angle (Heading Angle), rad or deg

Newton's Laws of Motion: Dynamics of a Particle

- **First Law: If no force acts on a particle,**
  - **it remains at rest or**
  - **continues to move in a straight line at constant velocity, as observed in an inertial reference frame**
  - **Momentum is conserved**

$$\frac{d}{dt}(m\mathbf{v}) = 0 \quad ; \quad m\mathbf{v}\big|_{t_1} = m\mathbf{v}\big|_{t_2}$$

Newton's Laws of Motion:
Dynamics of a Particle

- **Second Law**: A particle of fixed mass acted upon by a force
  - changes velocity with acceleration proportional to and in the direction of the force, as observed in an inertial frame;
  - The ratio of force to acceleration is the mass of the particle:

$$\boxed{\mathbf{F} = m\mathbf{a}}$$

$$\frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = \mathbf{F} \quad ; \quad \mathbf{F} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$\therefore \quad \frac{d\mathbf{v}}{dt} = \frac{1}{m}\mathbf{F} = \frac{1}{m}\mathbf{I}_3\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

Newton's Laws of Motion:
Dynamics of a Particle

- **Third Law**
  - For every action, there is an equal and opposite reaction

Force on Rocket = Force on Exhaust Gasses

Equations of Motion for a Particle: Position and Velocity

Force vector

$$\mathbf{F}_I = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I = \left[\mathbf{F}_{gravity} + \mathbf{F}_{aerodynamics} + \mathbf{F}_{thrust}\right]_I$$

Rate of change of velocity

$$\frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix}_I = \frac{1}{m}\mathbf{F} = \begin{bmatrix} 1/m & 0 & 0 \\ 0 & 1/m & 0 \\ 0 & 0 & 1/m \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}_I$$

Rate of change of position

$$\frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_I = \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_I$$

Integration for Velocity with Constant Force

$$\frac{d\mathbf{v}(t)}{dt} = \dot{\mathbf{v}}(t) = \frac{1}{m}\mathbf{F} = \begin{bmatrix} f_x/m \\ f_y/m \\ f_z/m \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\mathbf{v}(T) = \int_0^T \frac{d\mathbf{v}(t)}{dt}dt + \mathbf{v}(0) = \int_0^T \frac{1}{m}\mathbf{F}(t)dt + \mathbf{v}(0) = \int_0^T \mathbf{a}(t)dt + \mathbf{v}(0)$$

$$\begin{bmatrix} v_x(T) \\ v_y(T) \\ v_z(T) \end{bmatrix} = \int_0^T \begin{bmatrix} f_x(t)/m \\ f_y(t)/m \\ f_z(t)/m \end{bmatrix} dt + \begin{bmatrix} v_x(0) \\ v_y(0) \\ v_z(0) \end{bmatrix} = \int_0^T \begin{bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{bmatrix} dt + \begin{bmatrix} v_x(0) \\ v_y(0) \\ v_z(0) \end{bmatrix}$$

Integration for Position with Varying Velocity

$$\frac{d\mathbf{r}(t)}{dt} = \dot{\mathbf{r}}(t) = \mathbf{v}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix}$$

$$\mathbf{r}(T) = \int_0^T \frac{d\mathbf{r}(t)}{dt} dt + \mathbf{r}(0) = \int_0^T \mathbf{v}(t) dt + \mathbf{r}(0)$$

$$\begin{bmatrix} x(T) \\ y(T) \\ z(T) \end{bmatrix} = \int_0^T \begin{bmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{bmatrix} dt + \begin{bmatrix} x(0) \\ y(0) \\ z(0) \end{bmatrix}$$

Gravitational Force: Flat-Earth Approximation

- **Approximation**
  - **Flat earth reference is an inertial frame, e.g.,**
    - **North, East, Down**
    - **Range, Crossrange, Altitude (–)**
- **g** is gravitational acceleration
- *m***g** is gravitational force
- Independent of position
- *z* measured down

$$\left(\mathbf{F}_{gravity}\right)_I = \left(\mathbf{F}_{gravity}\right)_E = m\mathbf{g}_E = m\begin{bmatrix} 0 \\ 0 \\ g_o \end{bmatrix}_E$$

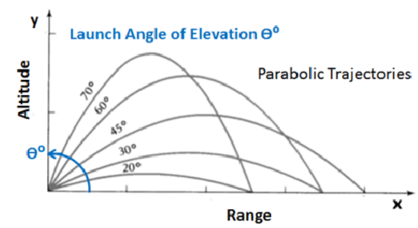
$$g_o \simeq 9.807\ m/s^2 \text{ at earth's surface}$$

Flight Path Dynamics, Constant Gravity and No Aerodynamics

$$
\begin{aligned}
v_x(0) &= v_{x_0} \\
v_z(0) &= v_{z_0} \\
x(0) &= x_0 \\
z(0) &= z_0
\end{aligned}
$$

$$
\begin{aligned}
\dot{v}_x(t) &= 0 \\
\dot{v}_z(t) &= -g \quad (z \text{ positive up}) \\
\dot{x}(t) &= v_x(t) \\
\dot{z}(t) &= v_z(t)
\end{aligned}
$$

$$
\begin{aligned}
v_x(T) &= v_{x_0} \\
v_z(T) &= v_{z_0} - \int_0^T g\,dt = v_{z_0} - gT \\
x(T) &= x_0 + v_{x_0}T \\
z(T) &= z_0 + v_{z_0}T - \int_0^T gt\,dt = z_0 + v_{z_0}T - gT^2/2
\end{aligned}
$$



MATLAB Scripts for Flat-Earth Trajectory, No Aerodynamics

Analytical Solution

```
g   =   9.8;
t   =   0:0.1:40;

vx0 =   10;
vz0 =   100;
x0  =   0;
z0  =   0;

vx1 =   vx0;
vz1 =   vz0 - g*t;
x1  =   x0 + vx0*t;
z1  =   z0 + vz0*t - 0.5*g*t.* t;
```

Numerical Solution

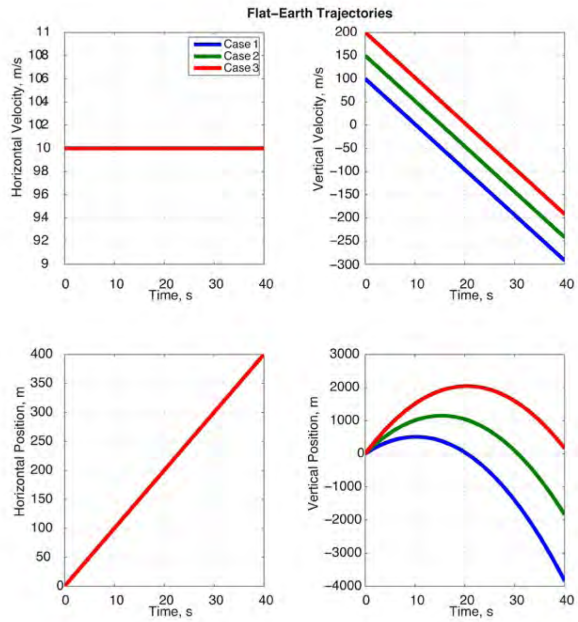
Calling Routine

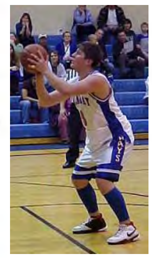
```
tspan   = 40;
xo      = [10;100;0;0];
[t1,x1] = ode45('FlatEarth',tspan,xo);
```

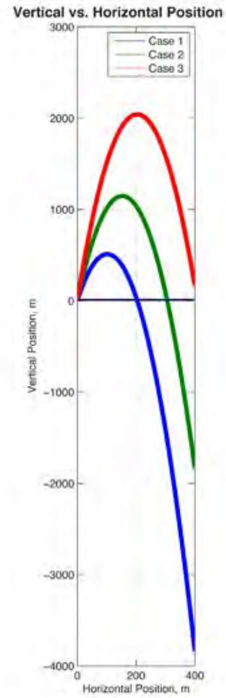
Equations of Motion

```
function xdot = FlatEarth(t,x)
%   x(1)    =   vx
%   x(2)    =   vz
%   x(3)    =   x
%   x(4)    =   z
    g       =   9.8;
    xdot(1) =   0;
    xdot(2) =   -g;
    xdot(3) =   x(1);
    xdot(4) =   x(2);
    xdot    =   xdot';
end
```

Flight Path with Constant Gravity and No Aerodynamics







Aerodynamic Force on an Airplane

Earth-Reference Frame	Body-Axis Frame	Wind-Axis Frame
$\mathbf{F}_I = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_E = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_E \frac{1}{2}\rho V_{air}^2 S = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_E \bar{q} S$	$\mathbf{F}_B = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B \bar{q} S$	$\mathbf{F}_V = \begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} \bar{q} S$
Referenced to the Earth, not the aircraft	Aligned with the aircraft axes	Aligned with and perpendicular to the direction of motion

Non-Dimensional Aerodynamic Coefficients

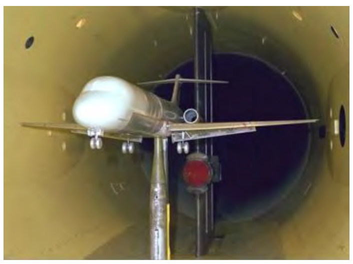
Body-Axis Frame

$$\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B = \begin{bmatrix} \textit{axial force coefficient} \\ \textit{side force coefficient} \\ \textit{normal force coefficient} \end{bmatrix}$$

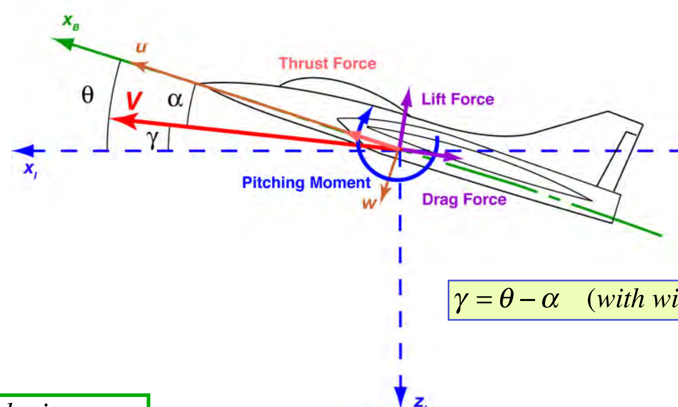
Wind-Axis Frame

$$\begin{bmatrix} C_D \\ C_Y \\ C_L \end{bmatrix} = \begin{bmatrix} \textit{drag coefficient} \\ \textit{side force coefficient} \\ \textit{lift coefficient} \end{bmatrix}$$

- **Functions of flight condition, control settings, and disturbances, e.g., $C_L = C_L(\delta, M, \delta E)$**
- **Non-dimensional coefficients allow application of sub-scale model wind tunnel data to full-scale airplane**



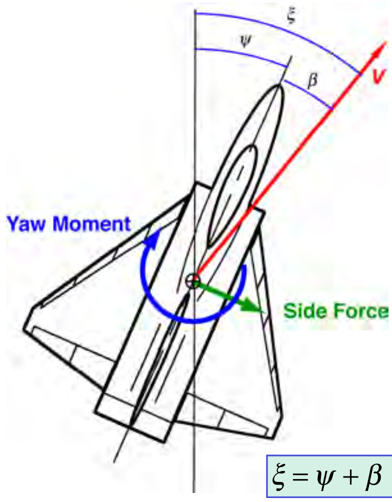
Longitudinal Variables

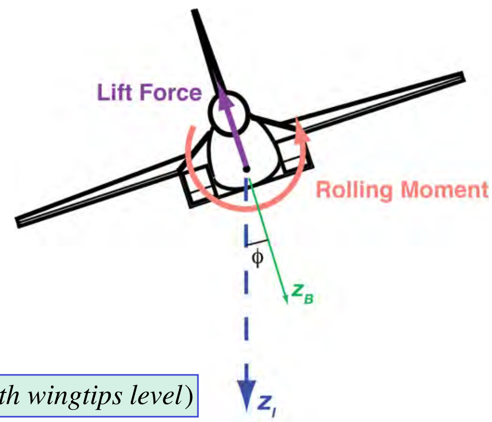


$$\gamma = \theta - \alpha \quad (\textit{with wingtips level})$$

- $u(t)$: *axial velocity* — along vehicle centerline
- $w(t)$: *normal velocity* — perpendicular to centerline
- $V(t)$: *velocity magnitude* — along net direction of flight
- $\alpha(t)$: *angle of attack* — angle between centerline and direction of flight
- $\gamma(t)$: *flight path angle* — angle between direction of flight and local horizontal
- $\theta(t)$: *pitch angle* — angle between centerline and local horizontal

Lateral-Directional Variables





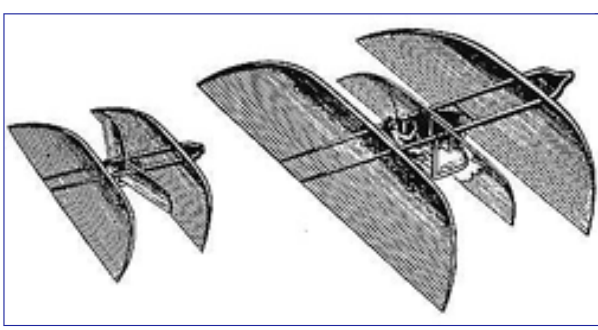
$$\xi = \psi + \beta \quad (with\ wingtips\ level)$$

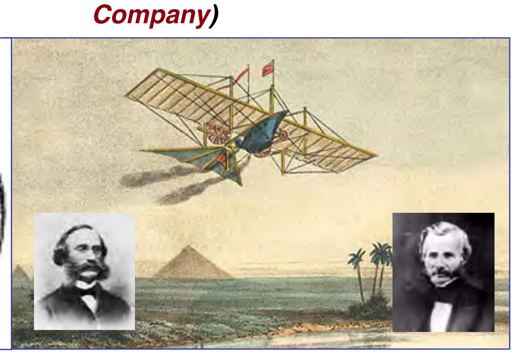
- $\beta(t)$: *sideslip angle* — angle between centerline and direction of flight
- $\psi(t)$: *yaw angle* — angle between centerline and local horizontal
- $\xi(t)$: *heading angle* — angle between direction of flight and compass reference (e.g., north)
- $\phi(t)$: *roll angle* — angle between true vertical and body *z* axis

Historical Factoids

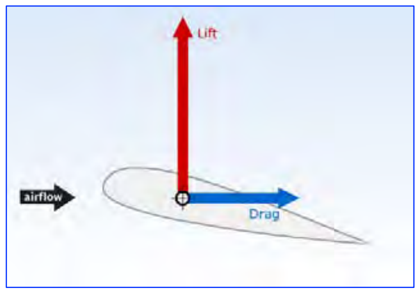
Visionaries and Theorists

- **1831: Thomas Walker**
  - **Various glider concepts**
  - **Tandem-wing design influenced Langley**
- ***1843: William Henson & John Stringfellow***
  - ***Aerial steam carriage concept***
  - ***Vision of commercial air transportation (with Marriott and Columbine, The Aerial Transit Company)***

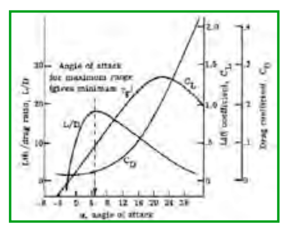




Introduction to Lift and Drag



Lift and Drag are Oriented w.r.t. the Velocity Vector



$$Lift = C_L \frac{1}{2}\rho V_{air}^2 S \approx \left[C_{L_0} + \frac{\partial C_L}{\partial \alpha}\alpha\right]\frac{1}{2}\rho V_{air}^2 S$$

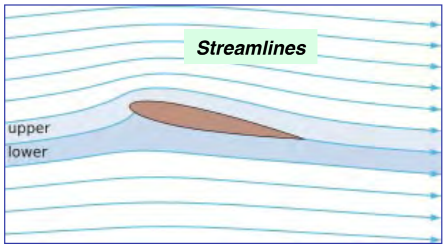
- **Lift components sum to produce total lift**
  - **Pressure differential between upper and lower surfaces**
    - **Wing**
    - **Fuselage**
    - **Horizontal tail**

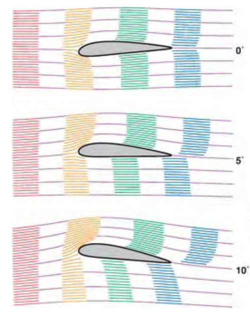
$$Drag = C_D \frac{1}{2}\rho V_{air}^2 S \approx \left[C_{D_0} + \varepsilon C_L^2\right]\frac{1}{2}\rho V_{air}^2 S$$

- **Drag components sum to produce total drag**
  - **Skin friction**
  - **Base pressure differential**
  - **Shock-induced pressure differential (M > 1)**

Aerodynamic Lift

$$Lift = C_L \frac{1}{2}\rho V_{air}^2 S \approx \left(C_{L_{wing}} + C_{L_{fuselage}} + C_{L_{tail}}\right)\frac{1}{2}\rho V_{air}^2 S \approx \left[C_{L_0} + \boxed{\frac{\partial C_L}{\partial \alpha}}\alpha\right]\bar{q}S$$





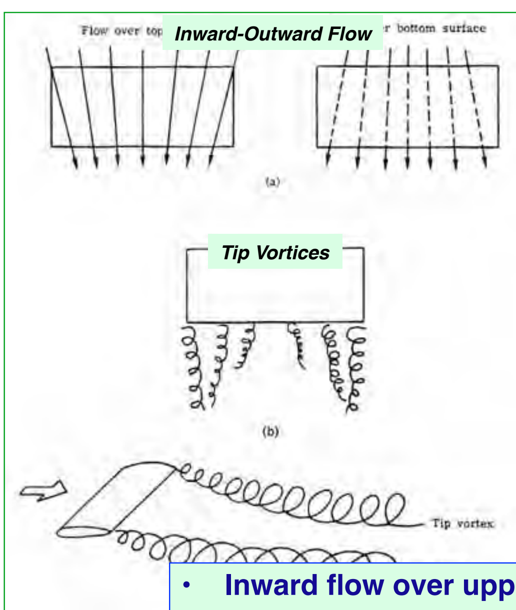
$\alpha = 0°$

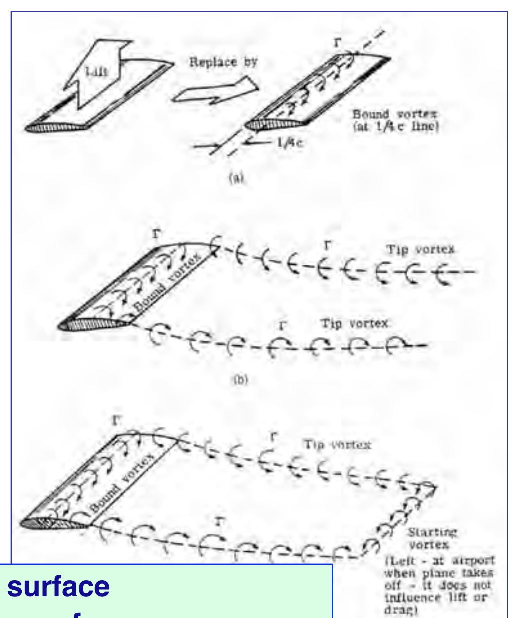
$\alpha = 5°$

$\alpha = 10°$

- Fast flow over top + slow flow over bottom = Mean flow + Circulation
- Speed difference proportional to angle of attack
- Kutta condition (stagnation points at leading and trailing edges)

2D vs. 3D Lift

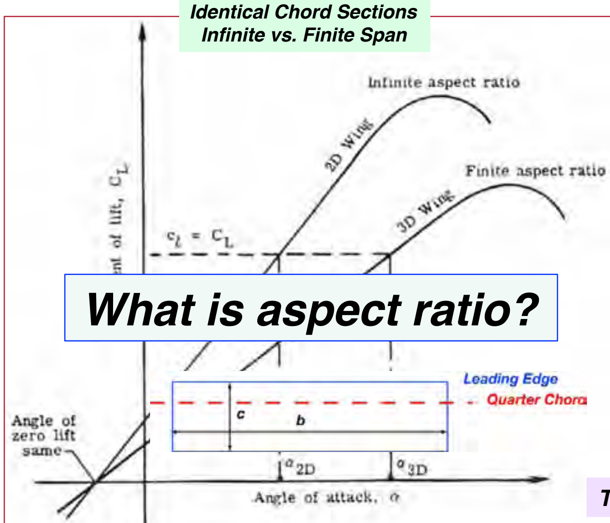




- Inward flow over upper surface
- Outward flow over lower surface
- Bound vorticity of wing produces tip vortices

2D vs. 3D Lift

Identical Chord Sections
Infinite vs. Finite Span



What is aspect ratio?

Talay, NASA-SP-367

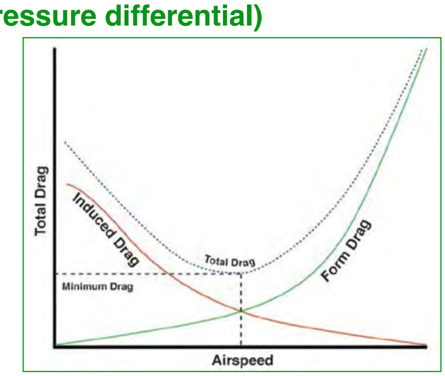
- **Finite aspect ratio reduces lift slope**

Aerodynamic Drag

$$Drag = C_D \frac{1}{2}\rho V_{air}^2 S \approx \left(C_{D_p} + C_{D_i} + C_{D_w}\right)\frac{1}{2}\rho V_{air}^2 S \approx \left[C_{D_0} + \boxed{\varepsilon C_L^2}\right]\bar{q}S$$

- **Drag components**
  - **Parasite drag (friction, interference, base pressure differential)**
  - **Induced drag (drag due to lift generation)**
  - **Wave drag (shock-induced pressure differential)**

- **In steady, subsonic flight**
  - **Parasite (form) drag increases as V^2**
  - **Induced drag proportional to $1/V^2$**
  - **Total drag minimized at one particular airspeed**



Historical Factoids

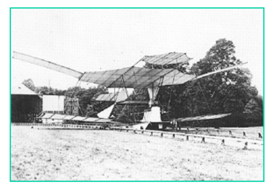
- **1868: Jean Marie Le Bris**
  - ***Artificial Albatross*** **glides a short distance**

- **1874: Felix du Temple's hot-air engine manned monoplane**
  - **Flies down a ramp**
- **1884: Alexander Mozhaisky's steam-powered manned airplane**
  - **brief hop off the ground**
  - **flat-plate wings**

- **1891-96: Hang-glider flights**
  - **Otto Lilienthal**
  - **Chanute, Pilcher, ...**

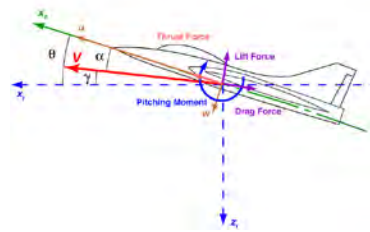
- **1890: Clement Ader**
  - **Steam-powered *Eole* hops**

- **1894: Sir Hiram Maxim**
  - **Steam-powered biplane hops**
  - **Vertical gyro/servo control of the elevator**



2-D Equations of Motion with Aerodynamics and Thrust

2-D Equations of Motion for a Point Mass



- Restrict motions to a vertical plane (i.e., motions in *y* direction = 0)
- Inertial frame, wind = 0
- *z* positive <u>down</u>, flat-earth assumption

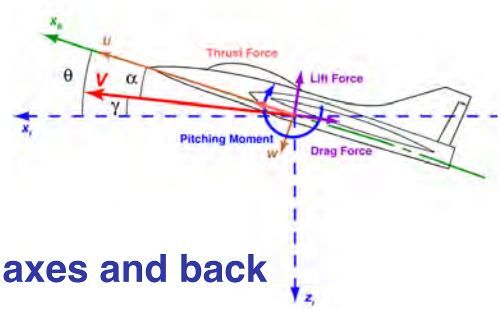
$$\bar{q} = \frac{1}{2}\rho(z)\left(v_x^2 + v_z^2\right)$$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \\ \dot{v}_x \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \\ f_x / m \\ f_z / m \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ z \\ v_x \\ v_z \end{bmatrix} + \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ \hline 1/m & 0 \\ 0 & 1/m \end{array}\right] \begin{bmatrix} \left(C_T \cos\theta + C_{X_I}\right)\bar{q}S \\ \left(C_T \sin\theta + C_{Z_I}\right)\bar{q}S + mg_o \end{bmatrix}$$

- Assume point-mass location coincides with aircraft's center of mass

$$C_T = \text{Thrust coefficient}$$
$$\theta = \text{Pitch angle}$$

Transform Velocity from Cartesian to Polar Coordinates



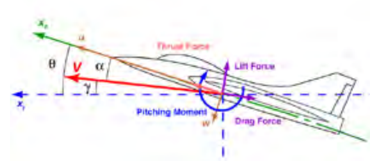
- Inertial axes -> wind axes and back

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \begin{bmatrix} V\cos\gamma \\ -V\sin\gamma \end{bmatrix} \Rightarrow \begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{\dot{x}^2 + \dot{z}^2} \\ -\sin^{-1}\left(\dfrac{\dot{z}}{V}\right) \end{bmatrix} = \begin{bmatrix} \sqrt{v_x^2 + v_z^2} \\ -\sin^{-1}\left(\dfrac{v_z}{V}\right) \end{bmatrix}$$

- Rates of change of velocity and flight path angle

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} \dfrac{d}{dt}\sqrt{v_x{}^2 + v_z{}^2} \\ -\dfrac{d}{dt}\sin^{-1}\left(\dfrac{v_z}{V}\right) \end{bmatrix}$$

Longitudinal Point-Mass Equations of Motion



$$x = range$$
$$z = -height\ (altitude)$$
$$V = velocity$$
$$\gamma = flight\ \ path\ angle$$

$$\dot{x}(t) = v_x = V(t)\cos\gamma(t)$$
$$\dot{z}(t) = v_z = -V(t)\sin\gamma(t)$$
$$\dot{V}(t) = \frac{\left(C_T\cos\alpha - C_D\right)\frac{1}{2}\rho(z)V^2(t)S - mg_o\sin\gamma(t)}{m}$$
$$\dot{\gamma}(t) = \frac{\left(C_T\sin\alpha + C_L\right)\frac{1}{2}\rho(z)V^2(t)S - mg_o\cos\gamma(t)}{mV(t)}$$

Steady, Level (i.e., Cruising) Flight

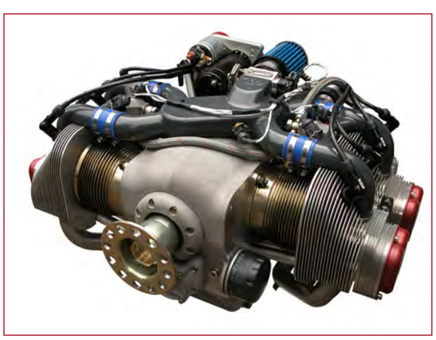
- In steady, level flight with cos α ~ 1, sin α ~ 0
  - Thrust = Drag
  - Lift = Weight

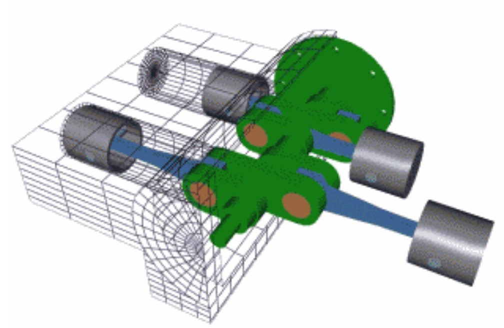
$$\dot{x}(t) = v_x = V_{cruise}$$
$$\dot{z}(t) = v_z = 0$$
$$0 = \frac{\left(C_T - C_D\right)\frac{1}{2}\rho(z)V_{cruise}^2 S}{m}$$
$$0 = \frac{C_L\frac{1}{2}\rho(z)V_{cruise}^2 S - mg(z)}{mV_{cruise}}$$

Introduction to Aeronautical Propulsion

Internal Combustion Reciprocating Engine

Linear motion of pistons converted to rotary motion to drive propeller





Early Reciprocating Engines

- **Rotary Engine:**
  - Air-cooled
  - Crankshaft fixed
  - Cylinders turn with propeller
  - On/off control: No throttle

- **V-8 Engine:**
  - Water-cooled
  - Crankshaft turns with propeller

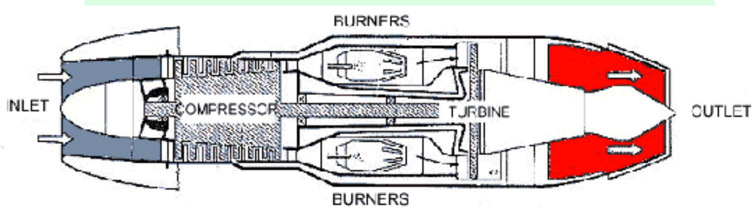
Sopwith Triplane

SPAD S.VII

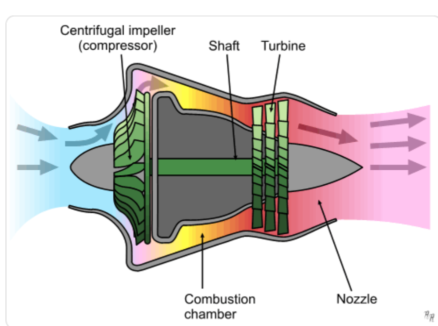
Turbojet Engines (1930s)

Thrust produced directly by exhaust gas

Axial-flow Turbojet (von Ohain, Germany)

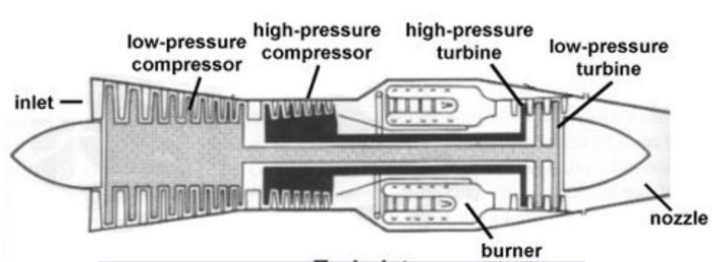


Centrifugal-flow Turbojet (Whittle, UK)

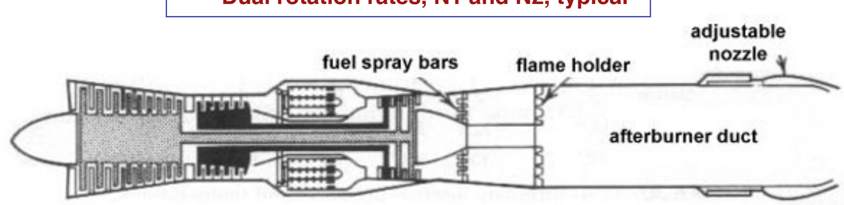


Birth of the Jet Airplane

Turbojet + Afterburner (1950s)



- Dual rotation rates, N1 and N2, typical

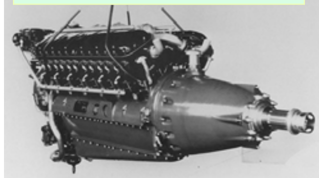


Turbojet with Afterburner

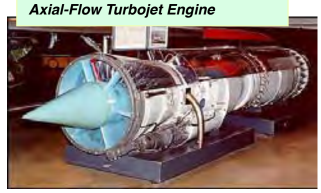
- Fuel added to exhaust
- Additional air may be introduced

Fighter Aircraft and Engines

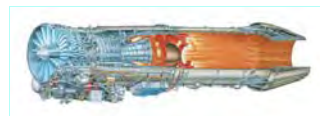
Allison V-1710
Turbocharged Reciprocating Engine



P&W J57
Axial-Flow Turbojet Engine

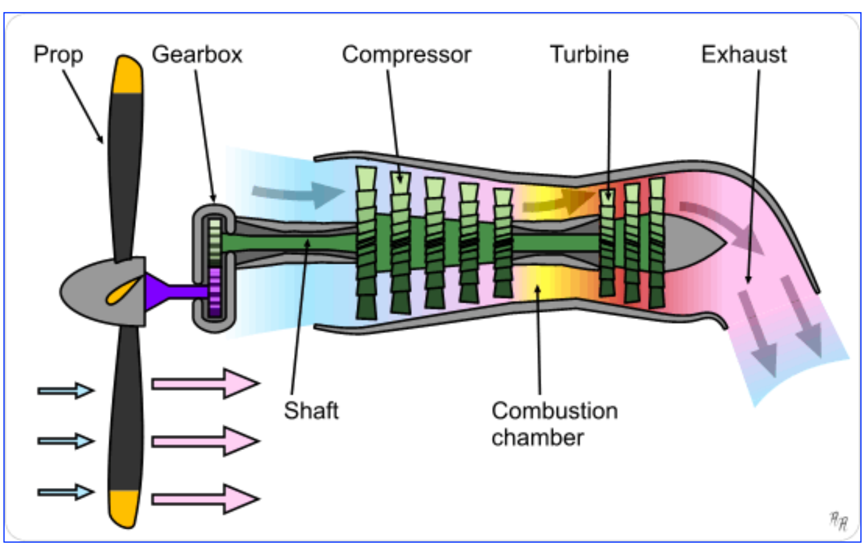


GE F404
Afterburning Turbofan Engine



Turboprop Engines (1940s)

Exhaust gas drives a propeller to produce thrust

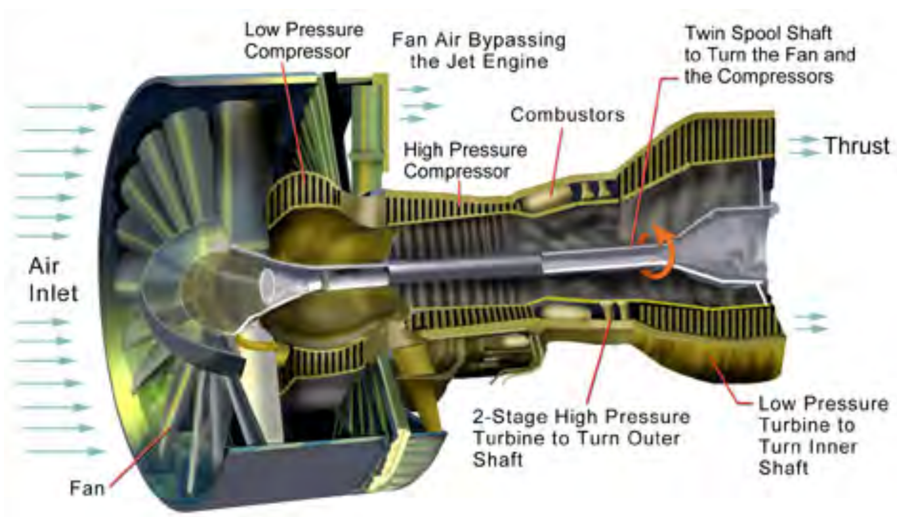


Propeller-Driven Aircraft of the 1950s

Reciprocating Engines

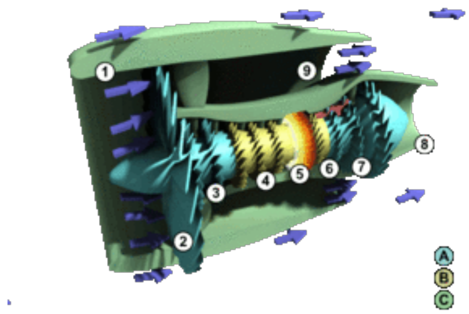
Turboprop Engines

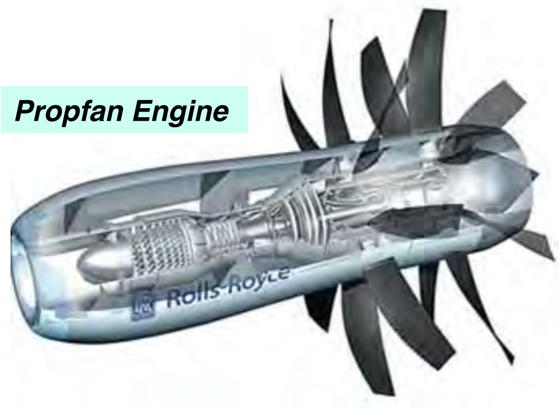
Turbofan Engine (1960s)



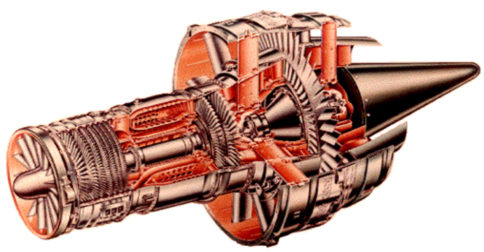
- **Dual or triple rotation rates**

High Bypass Ratio Turbofan



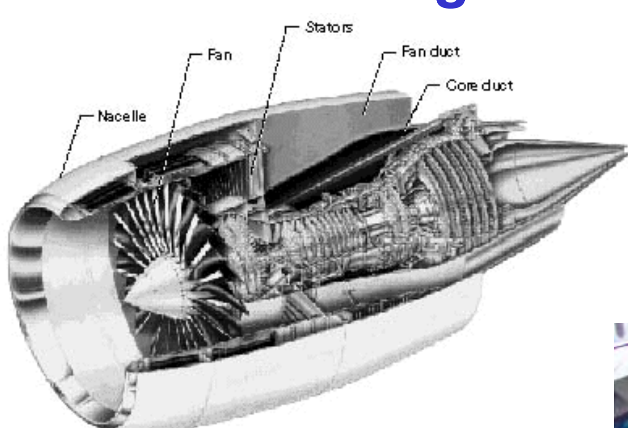


Propfan Engine



Aft-fan Engine

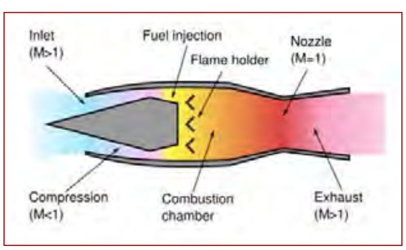
Jet Engine Nacelles



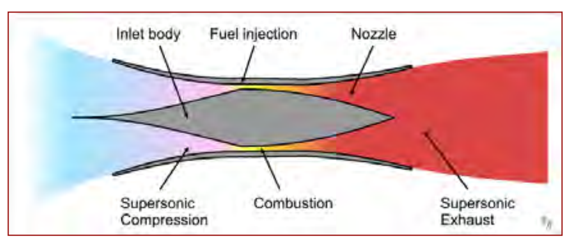
Jet Transports of the 2000s

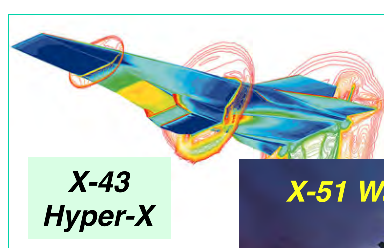
Ramjet and Scramjet

Ramjet (1940s)



Scramjet (1950s)



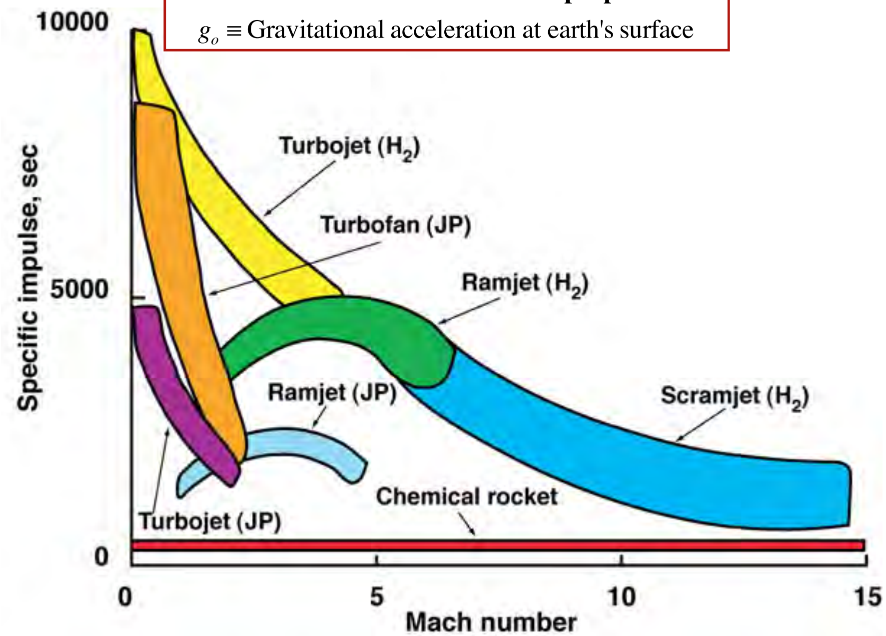


Thrust and Specific Impulse

$$I_{sp} = \frac{Thrust}{\dot{m}\, g_o} \triangleq \text{Specific Impulse}, \quad \text{Units} = \frac{\text{m/s}}{\text{m/s}^2} = \text{sec}$$

$\dot{m} \equiv$ Mass flow rate of **on-board propellant**

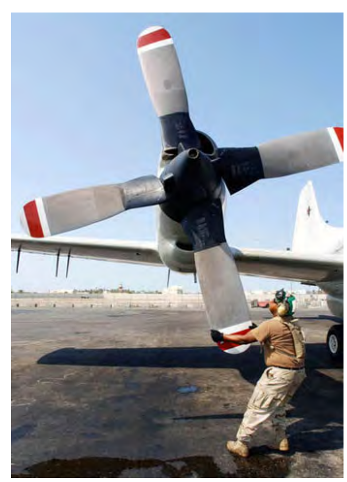
$g_o \equiv$ Gravitational acceleration at earth's surface



Thrust and Thrust Coefficient

$$Thrust \equiv C_T \frac{1}{2} \rho V^2 S$$

- Non-dimensional thrust coefficient, C_T
  - C_T is a function of power/throttle setting, fuel flow rate, blade angle, Mach number, ...
- Reference area, S, may be aircraft wing area, propeller disk area, or jet exhaust area

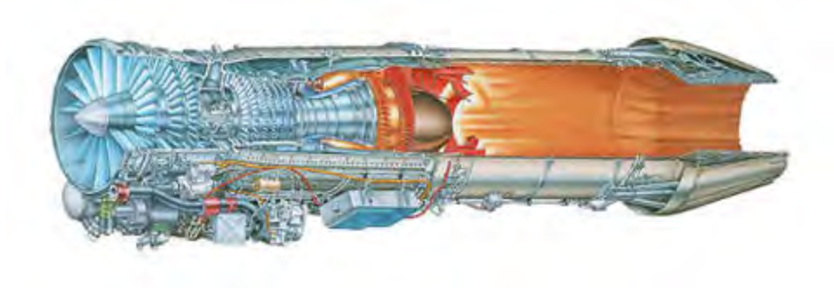


Sensitivity of Thrust to Airspeed

$$Nominal\ Thrust = T_N \equiv C_{T_N} \frac{1}{2} \rho V_N^2 S$$

$$(.)_N = Nominal\ (or\ reference)\ value$$

Turbojet thrust is <u>independent</u> of airspeed over a wide range



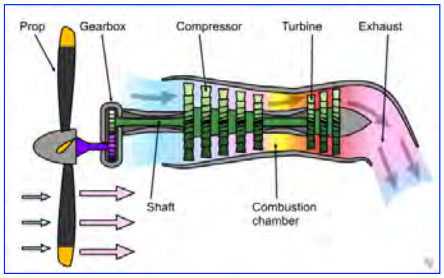
Power

Assuming thrust is aligned with airspeed vector

$$Power = P = Thrust \times Velocity \equiv C_T \frac{1}{2} \rho V^3 S$$

Propeller–driven power is <u>independent</u> of airspeed over a wide range

(reciprocating or turbine engine, with constant RPM or variable-pitch prop)

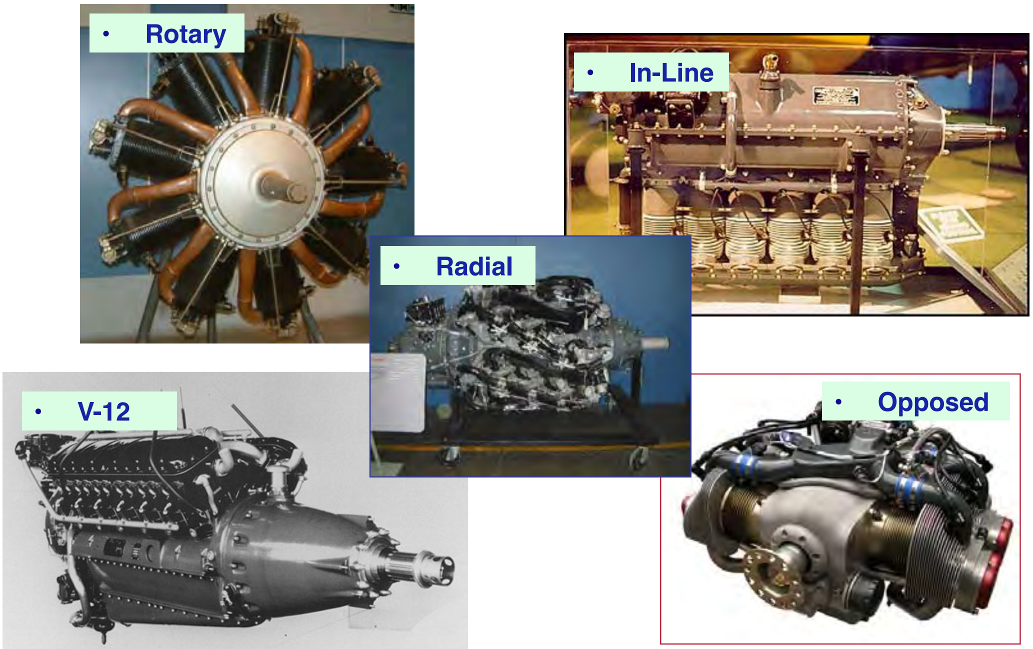


Next Time:
Low-Speed Aerodynamics

Reading:
Flight Dynamics
Aerodynamic Coefficients, 65-84

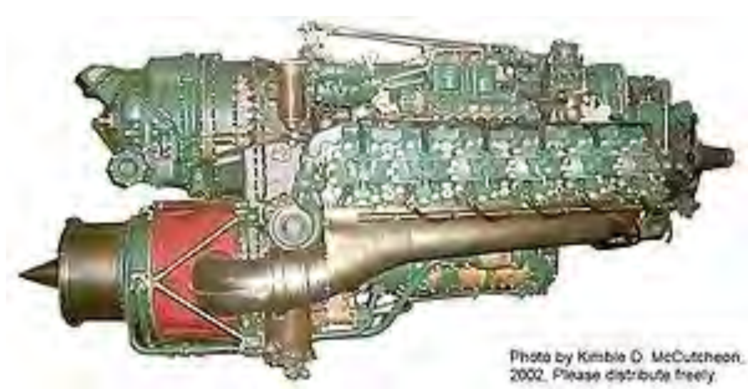
Supplementary Material

Reciprocating Engines



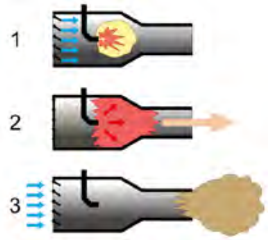
Turbo-compound Reciprocating Engine

- Exhaust gas drives the turbo-compressor
- Napier Nomad II shown (1949)

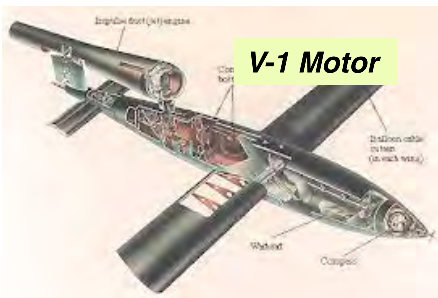


Pulsejet

Flapper-valved motor (1940s)



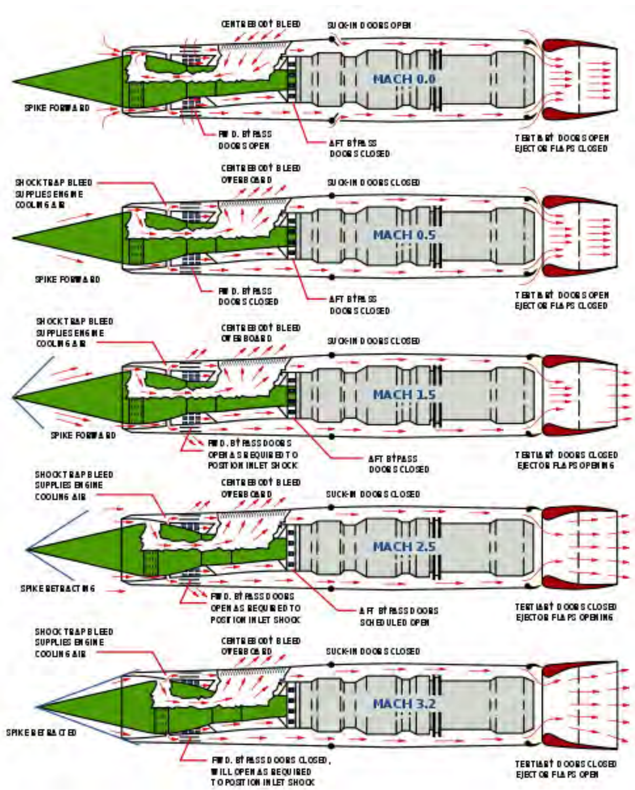
Dynajet Red Head (1950s)



V-1 Motor

Pulse Detonation Engine on Long EZ (1981)

http://airplanesandrockets.com/motors/dynajet-engine.htm



SR-71: P&W J58 Variable-Cycle Engine (Late 1950s)

Hybrid Turbojet/ Ramjet