Induced Drag and High-Speed Aerodynamics

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2014

Learning Objectives

- **Understand drag-due-to-lift and effects of wing planform**
- **Recognize effect of angle of attack on lift and drag coefficients**
- **How to estimate Mach number (i.e., air compressibility) effects on aerodynamics**
- **Be able to use Newtonian approximation to estimate lift and drag**

Reading:
Flight Dynamics
Aerodynamic Coefficients, 85-96
Airplane Stability and Control
Chapter 1

http://www.princeton.edu/~stengel/MAE331.html
http://www.princeton.edu/~stengel/FlightDynamics.html

Early Developments in Stability and Control

Chapter 1, *Airplane Stability and Control*, Abzug and Larrabee

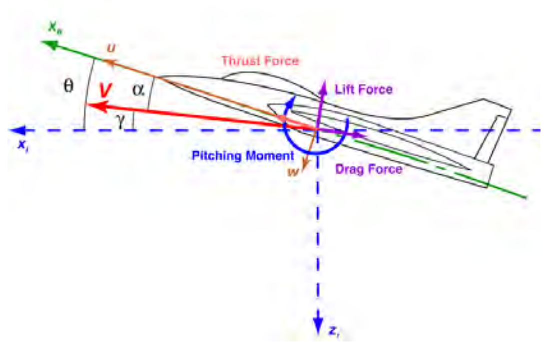
- What are the principal subject and scope of the chapter?
- What technical ideas are needed to understand the chapter?
- During what time period did the events covered in the chapter take place?
- What are the three main "takeaway" points or conclusions from the reading?
- What are the three most surprising or remarkable facts that you found in the reading?

Induced Drag

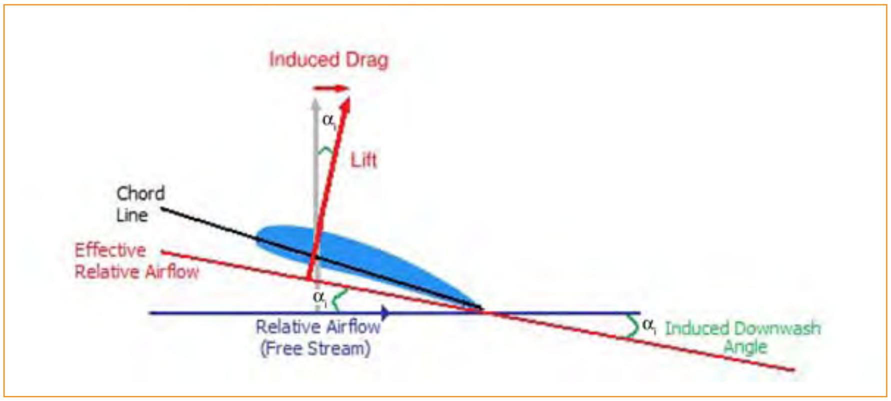
Aerodynamic Drag

$$Drag = C_D \frac{1}{2}\rho V^2 S \approx \left(C_{D_0} + \varepsilon C_L^2\right)\frac{1}{2}\rho V^2 S$$

$$\approx \left[C_{D_0} + \varepsilon\left(C_{L_o} + C_{L_\alpha}\alpha\right)^2\right]\frac{1}{2}\rho V^2 S$$



Induced Drag of a Wing, εC_L^2



- **Lift produces downwash (angle proportional to lift)**
  - **Downwash rotates local velocity vector CW in figure**
  - **Lift is perpendicular to velocity vector**
  - **Axial component of rotated lift induces drag**

Three Expressions for Induced Drag of a Wing

$$C_{D_i} = C_{L_i} \sin\alpha_i \approx \left(C_{L_0} + C_{L_\alpha}\alpha\right)\sin\alpha_i$$

$$\approx \left(C_{L_0} + C_{L_\alpha}\alpha\right)\alpha_i \equiv \varepsilon C_L^2$$

$$\equiv \frac{C_L^2}{\pi e AR} = \frac{C_L^2(1+\delta)}{\pi AR}$$

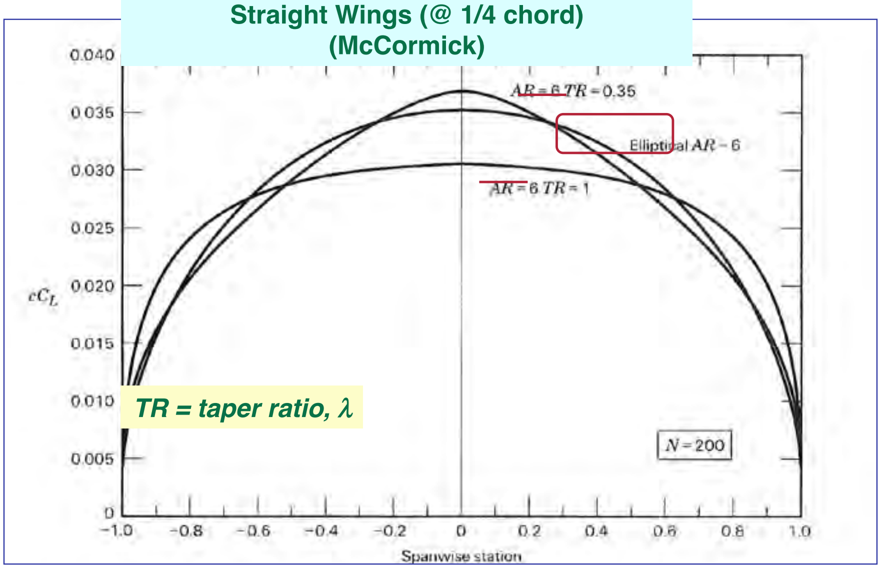
where

$e = Oswald\ efficiency\ factor = 1$ *for elliptical distribution*

$\delta = departure\ from\ ideal$ *elliptical lift distribution*

Spanwise Lift Distribution of 3-D (Trapezoidal) Wings

Straight Wings (@ 1/4 chord)
(McCormick)

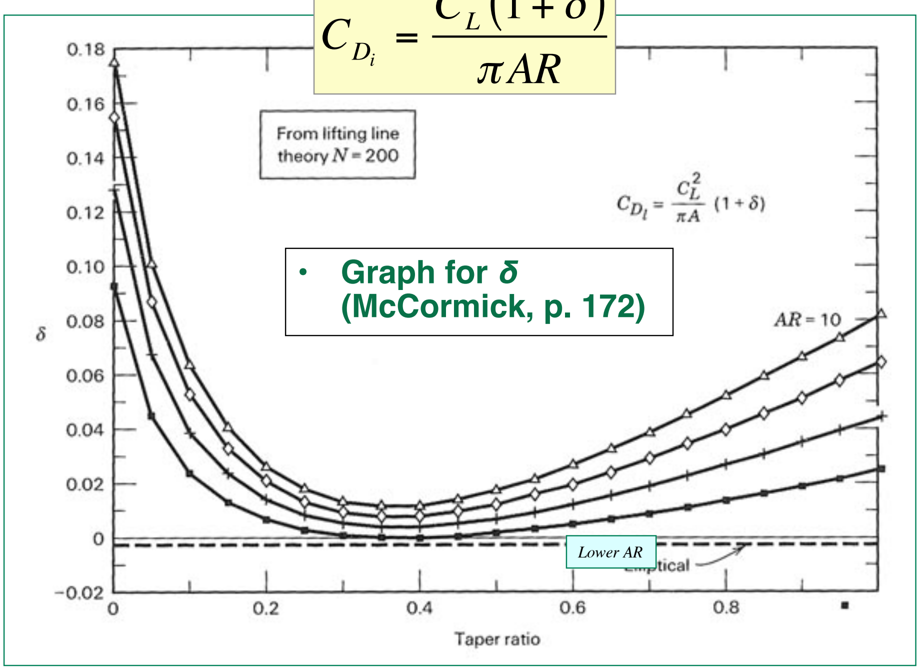


TR = taper ratio, λ

For some taper ratio between 0.35 and 1, lift distribution is nearly elliptical

Induced Drag Factor, *δ*

$$C_{D_i} = \frac{C_L^2 \left(1+\delta\right)}{\pi AR}$$



- **Graph for *δ* (McCormick, p. 172)**

Oswald Efficiency Factor, *e*

$$C_{D_i} = \frac{C_L^2}{\pi e AR}$$

Approximations for *e*

Pamadi

$$\kappa = \frac{AR\,\lambda}{\cos\Lambda_{LE}}$$

$$R = 0.0004\kappa^3 - 0.008\kappa^2 + 0.05\kappa + 0.86$$

$$e \approx \frac{1.1 C_{L_\alpha}}{R C_{L_\alpha} + (1-R)\pi AR}$$

Raymer

$$e \approx 1.78\left(1 - 0.045 AR^{0.68}\right) - 0.64 \quad \text{[Straight wing]}$$

$$e \approx 4.61\left(1 - 0.045 AR^{0.68}\right)\left(\cos\Lambda_{LE}\right)^{0.15} - 3.1 \quad \text{[Swept wing]}$$

Maximum Lift-to-Drag Ratio

- Maximize ***L/D*** by proper choice of C_L

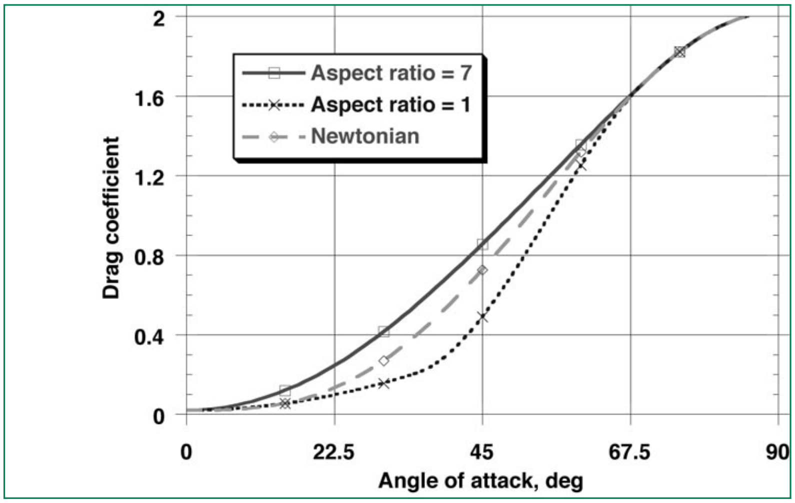
$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D_o} + \varepsilon C_L^2}$$

$$\frac{\partial(L/D)}{\partial C_L} = 0$$

$$\frac{\partial(L/D)}{\partial C_L} = 0 = \frac{\left(C_{D_o} + \varepsilon C_L^2\right) - C_L\left(2\varepsilon C_L\right)}{\left(C_{D_o} + \varepsilon C_L^2\right)^2} = \frac{\left(C_{D_o} - \varepsilon C_L^2\right)}{\left(C_{D_o} + \varepsilon C_L^2\right)^2}$$

$$\left(C_L\right)_{(L/D)_{\max}} = \sqrt{\frac{C_{D_o}}{\varepsilon}}$$

Large Angle Variations in Subsonic Drag Coefficient (0° < α < 90°)

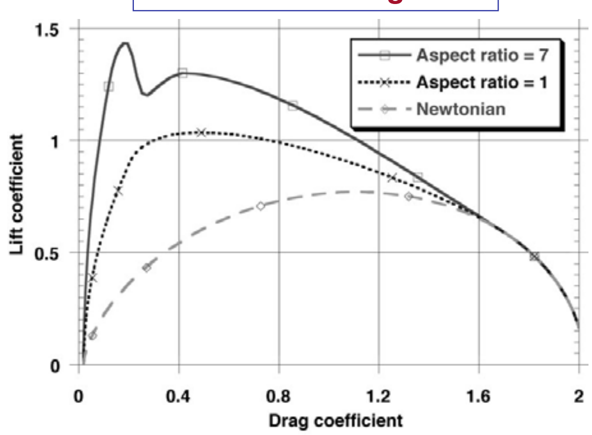


All wing drag coefficients converge to Newtonian-like values at high angle of attack

Low-*AR* wing has less drag than high-*AR* wing at given α

Lift vs. Drag for Large Variation in Angle-of-Attack (0° < α < 90°)

Subsonic Lift-Drag Polar

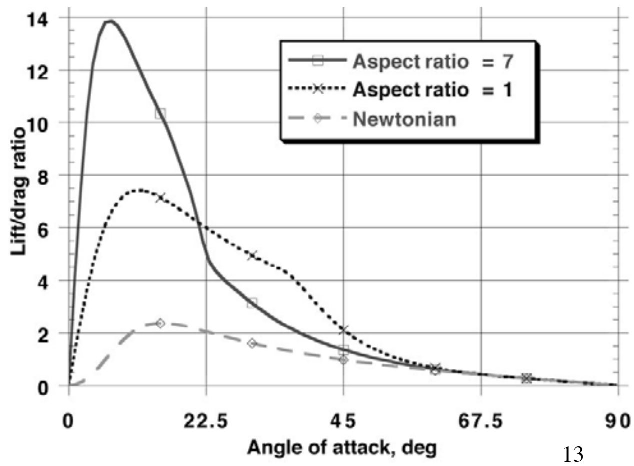


- Low-*AR* wing has less drag than high-*AR* wing, but less lift as well
- High-*AR* wing has the best overall *L/D*

Lift-to-Drag Ratio vs. Angle of Attack

- L/D is an important performance metric for aircraft
- High-*AR* wing has best overall L/D
- Low-*AR* wing has best L/D at intermediate angle of attack

$$\boxed{\frac{L}{D} = \frac{C_L \bar{q} S}{C_D \bar{q} S} = \frac{C_L}{C_D}}$$



Historical Factoid

Conversions from Propellers to Jets

Douglas XB-42 Mixmaster

Douglas XB-43

Northrop YB-35

Northrop XB-49

Convair B-36

Convair YB-60

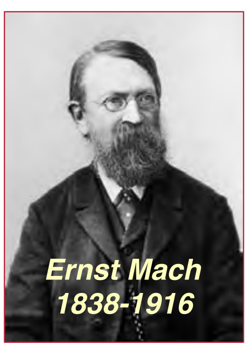
Historical Factoid

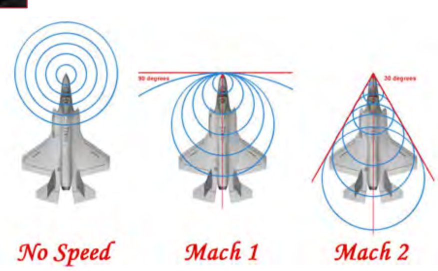
Jets at an Awkward Age

- Performance of the first jet aircraft outstripped stability and control technology
  - Lacked satisfactory actuators, sensors, and control electronics
  - Transistor: 1947, integrated circuit: 1958
- Dramatic dynamic variations over larger flight envelope
  - Control mechanisms designed to lighten pilot loads were subject to instability
- Reluctance of designers to embrace change, fearing decreased reliability, increased cost, and higher weight

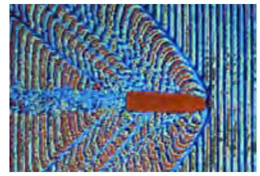
Mach Number Effects

$$\text{Mach Number} = \frac{\text{True Airspeed}}{\text{Speed of Sound}}$$





Drag Due to Pressure Differential



$$C_{D_{base}} = C_{pressure_{base}} \frac{S_{base}}{S} \approx \frac{0.029}{\sqrt{C_{friction} \frac{S_{wet}}{S_{base}}}} \frac{S_{base}}{S} \quad (M < 1) \quad [Hoerner]$$

Blunt base pressure drag

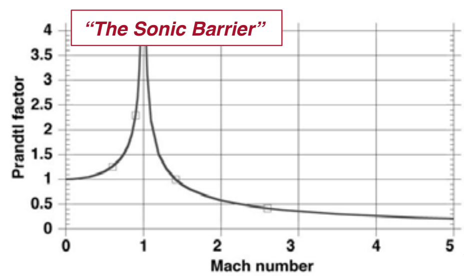
$$< \frac{2}{\gamma M^2}\left(\frac{S_{base}}{S}\right) \quad (M > 2, \quad \gamma = specific\ heat\ ratio)$$

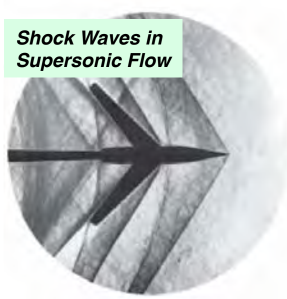
$$C_{D_{wave}} \approx \frac{C_{D_{incompressible}}}{\sqrt{1 - M^2}} \quad (M < 1)$$

$$\approx \frac{C_{D_{compressible}}}{\sqrt{M^2 - 1}} \quad (M > 1)$$

$$\approx \frac{C_{D_{M \approx \sqrt{2}}}}{\sqrt{M^2 - 1}} \quad (M > 1)$$

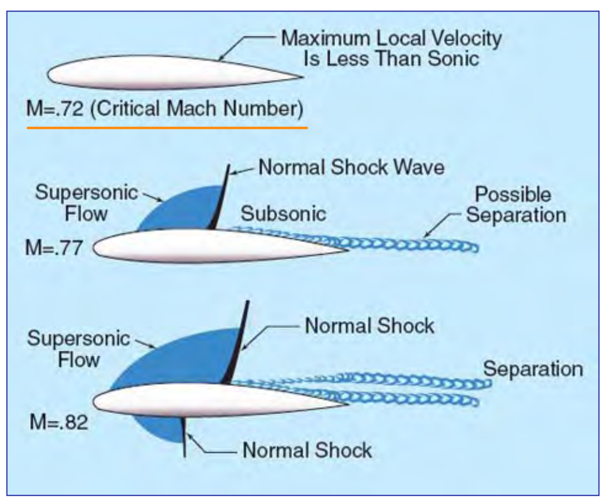
Prandtl factor





Air Compressibility Effect

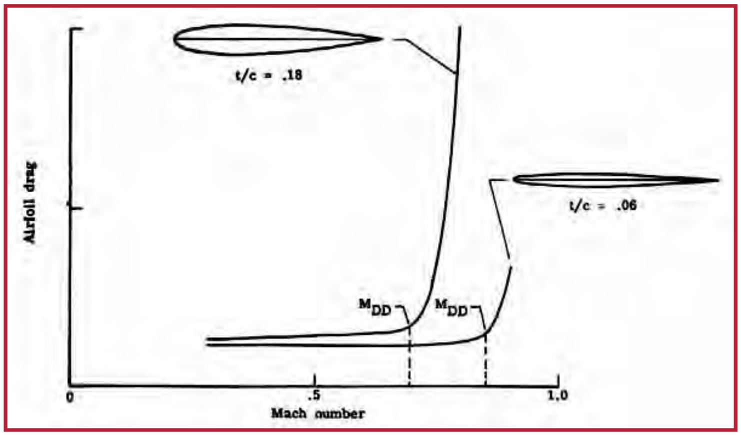
- **Drag rises due to pressure increase across a shock wave**
- **Subsonic flow**
  - **Local airspeed is less than sonic (i.e., speed of sound) everywhere**
- **Transonic flow**
  - **Airspeed is less than sonic at some points, greater than sonic elsewhere**
- **Supersonic flow**
  - **Local airspeed is greater than sonic virtually everywhere**



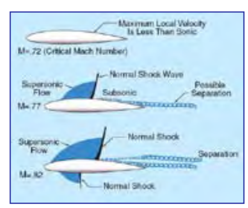
- **Critical Mach number**
  - **Mach number at which local flow first becomes sonic**
  - **Onset of drag-divergence**
  - **M_{crit} ~ 0.7 to 0.85**

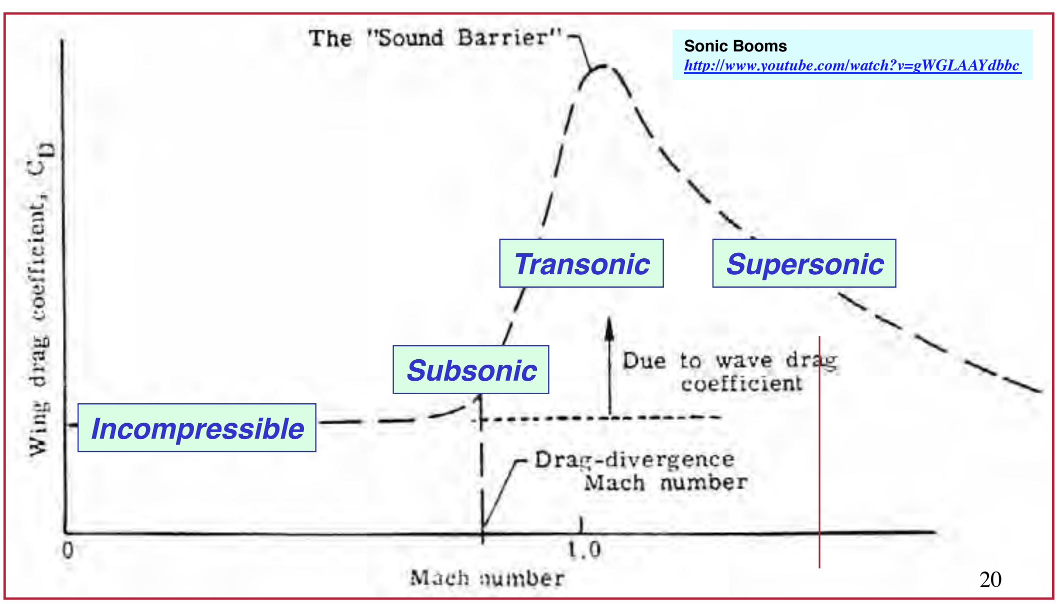
Effect of Chord Thickness on Wing Pressure Drag

- Thinner chord sections lead to higher M_{crit}, or drag-divergence Mach number

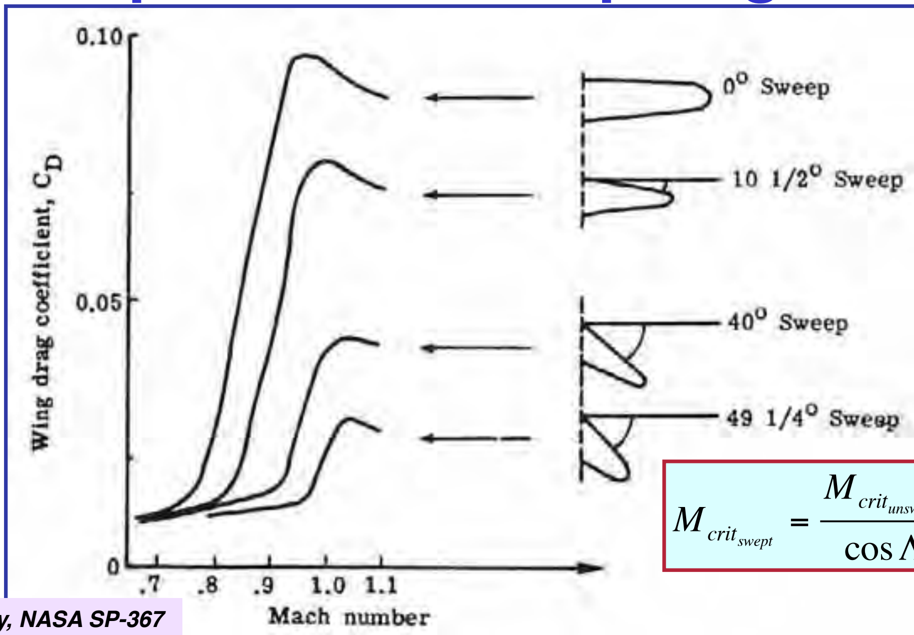


Air Compressibility Effect on Wing Drag





Pressure Drag on Wing Depends on Sweep Angle



$$M_{crit_{swept}} = \frac{M_{crit_{unswept}}}{\cos \Lambda}$$

Talay, NASA SP-367

Historical Factoid

From Straight to Swept Wings

- **Straight-wing models were redesigned with swept wings to reduce compressibility effects on drag and increase speed**
- **Dramatic change in stability, control, and flying qualities**

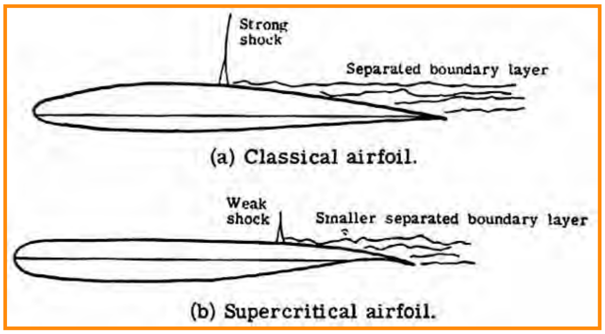
- **North American FJ-1 and FJ-4 Fury**
- **Republic F-84B Thunderbird and F-84F Thunderstreak**
- **Grumman F9F-2 Panther and F9F-6 Cougar**

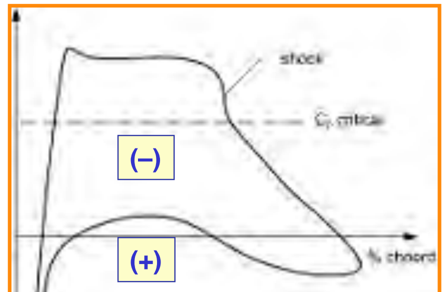





Supercritical Wing

- Richard Whitcomb's supercritical airfoil
  - Wing upper surface flattened to increase M_{crit}
  - Wing thickness can be restored
    - Important for structural efficiency, fuel storage, etc.



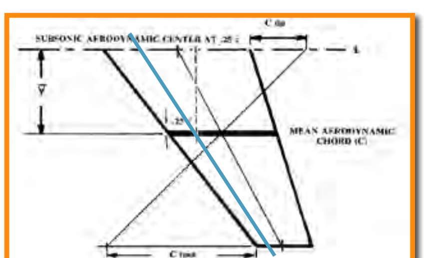


Pressure Distribution on Supercritical Airfoil ~ Section Lift

Subsonic Air Compressibility and Sweep Effects on 3-D Wing Lift Slope

- Subsonic 3-D wing, with sweep effect

$$C_{L_\alpha} = \frac{\pi AR}{\left[1 + \sqrt{1 + \left(\frac{AR}{2\cos\Lambda_{1/4}}\right)^2 \left(1 - M^2 \cos\Lambda_{1/4}\right)}\right]}$$

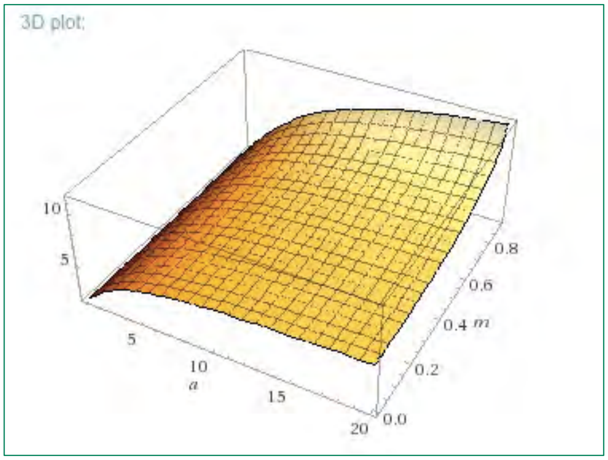


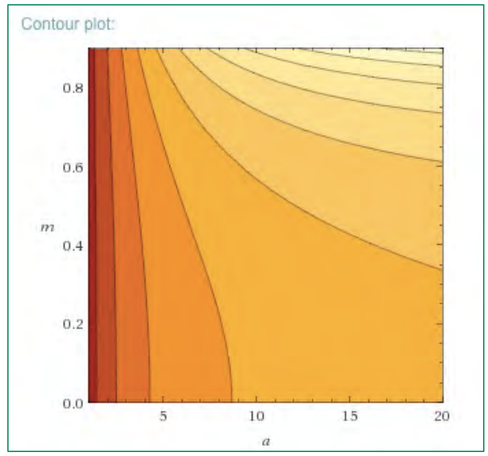
$$\Lambda_{1/4} = \textit{sweep angle of quarter chord}$$

Subsonic Air Compressibility Effects on 3-D Wing Lift Slope

- **Subsonic 3-D wing, sweep = 0**

plot(pi A / (1+sqrt(1 + ((A / 2)^2) (1 - M^2))), A=1 to 20, M = 0 to 0.9)

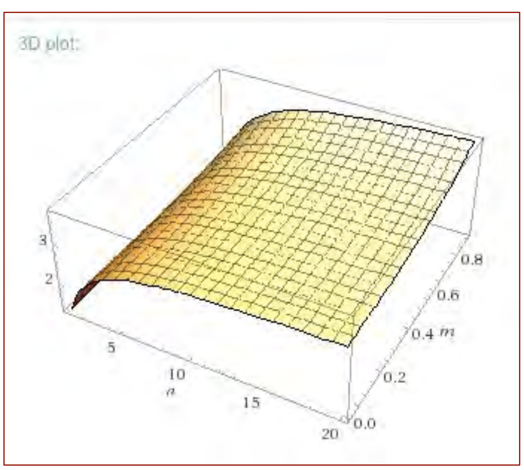


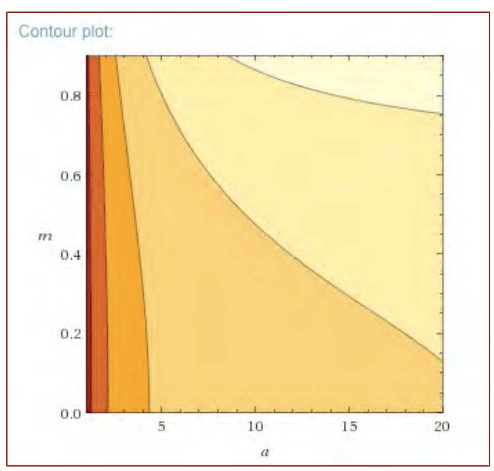


Subsonic Air Compressibility Effects on 3-D Wing Lift Slope

- **Subsonic 3-D wing, sweep = 60°**

plot(pi A / (1+sqrt(1 + (A ^2) (1 – 0.5 M^2))), A=1 to 20, M = 0 to 0.9)

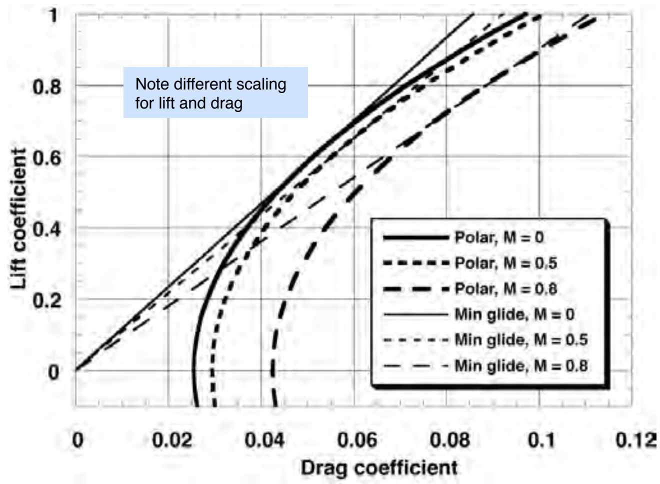




Lift-Drag Polar for a Typical Bizjet

- Lift-Drag Polar: Cross-plot of $C_L(\alpha)$ vs. $C_D(\alpha)$

- L/D equals slope of line drawn from the origin
  - Single maximum for a given polar
  - Two solutions for lower L/D (high and low airspeed)



Wing Lift Slope at M = 1

Approximation for <u>all</u> wing planforms

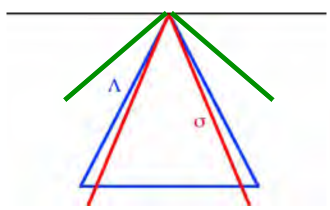
$$\boxed{C_{L_\alpha} = \frac{\pi AR}{2} = 2\pi\left(\frac{AR}{4}\right)}$$

Supersonic Compressibility Effects on Triangular Wing Lift Slope

- **Supersonic delta (triangular) wing**

Supersonic leading edge

$$C_{L_\alpha} = \frac{4}{\sqrt{M^2 - 1}}$$



Subsonic leading edge

$$C_{L_\alpha} = \frac{2\pi^2 \cot \Lambda}{(\pi + \lambda)}$$

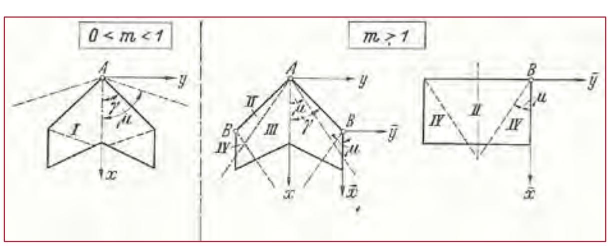
$$where \quad \lambda = m\left(0.38 + 2.26m - 0.86m^2\right)$$

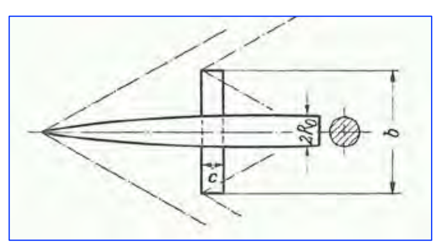
$$m = \cot \Lambda_{LE} / \cot \sigma$$

$$\Lambda_{LE} = sweep\ angle\ of\ leading\ edge$$

Supersonic Effects on Arbitrary Wing and Wing-Body Lift Slope

- **Impinging shock waves**
- **Discrete areas with differing *M* and local pressure coefficients, c_p**
- **Areas change with α**
- **No simple equations for lift slope**





Schlicting & Truckenbrodt, 1979

Historical Factoid

Fighter Jets of the 1950s: “Century Series”

- Emphasis on supersonic speed

Historical Factoid

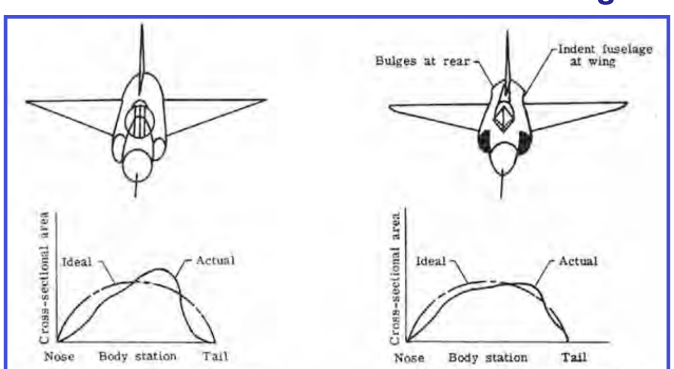
What Happened to the F-103?

Transonic Drag Rise and the Area Rule

- **Richard Whitcomb (NASA Langley) and Wallace Hayes (Princeton)**
- ***YF-102A* (left) could not break the speed of sound in level flight; *F-102A* (right) could**

Transonic Drag Rise and the Area Rule

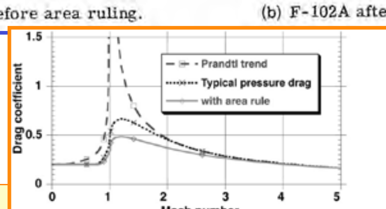
- **Cross-sectional area of the total configuration should gradually increase and decrease to minimize transonic drag**



(a) YF-102A before area ruling.

(b) F-102A after area ruling.

Talay, NASA SP-367

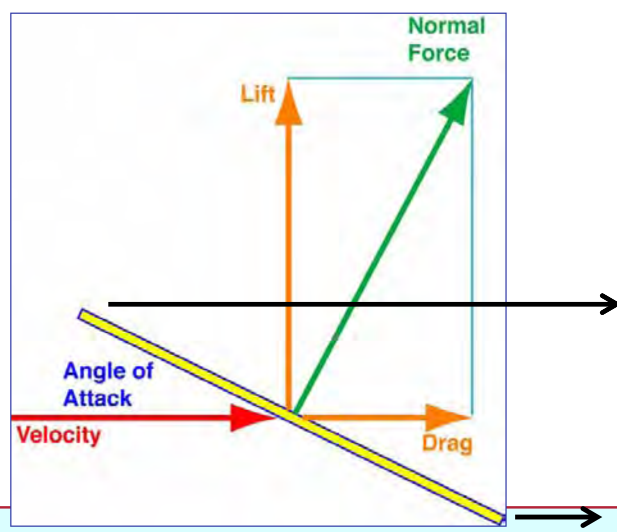


Sears-Haack Body
http://en.wikipedia.org/wiki/Sears-Haack_body

Newtonian Flow and High-Angle-of-Attack Lift and Drag

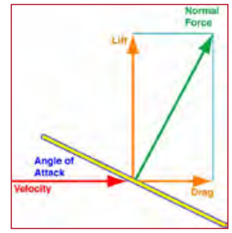
Newtonian Flow

- No circulation
- “Cookie-cutter” flow
- Equal pressure across bottom of the flat plate



$$\text{Normal Force} = \left(\frac{\text{Mass flow rate}}{\text{Unit area}}\right)(\text{Change in velocity})(\text{Projected Area})(\text{Angle between plate and velocity})$$

Newtonian Flow



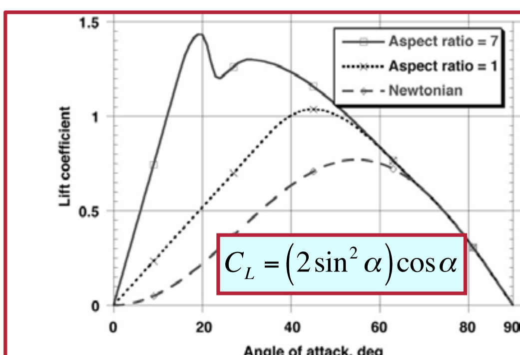
$$
\begin{aligned}
N &= (\rho V)(V)(S\sin\alpha)(\sin\alpha) \\
&= (\rho V^2)(S\sin^2\alpha) \\
&= (2\sin^2\alpha)\left(\frac{1}{2}\rho V^2\right)S \\
&\equiv C_N\left(\frac{1}{2}\rho V^2\right)S = C_N\bar{q}S
\end{aligned}
$$

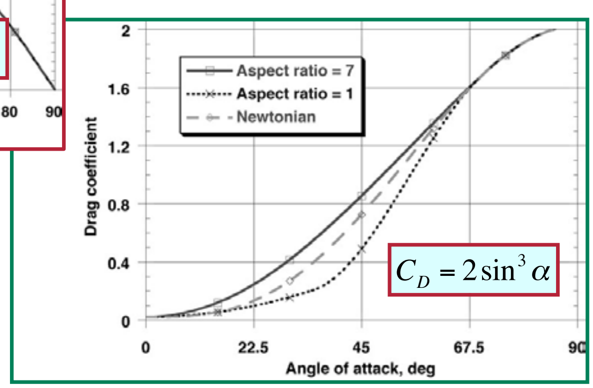
Lift and drag coefficients

$$
\mathbf{\mathit{Lift}} = N\cos\alpha \qquad C_L = (2\sin^2\alpha)\cos\alpha
$$

$$
\mathbf{\mathit{Drag}} = N\sin\alpha \qquad C_D = 2\sin^3\alpha
$$

Newtonian Lift and Drag Coefficients





Application of Newtonian Flow

- **Hypersonic flow (M ~> 5)**
  - Shock wave close to surface (thin shock layer), merging with the boundary layer
  - Flow is ~ parallel to the surface
  - Separated upper surface flow

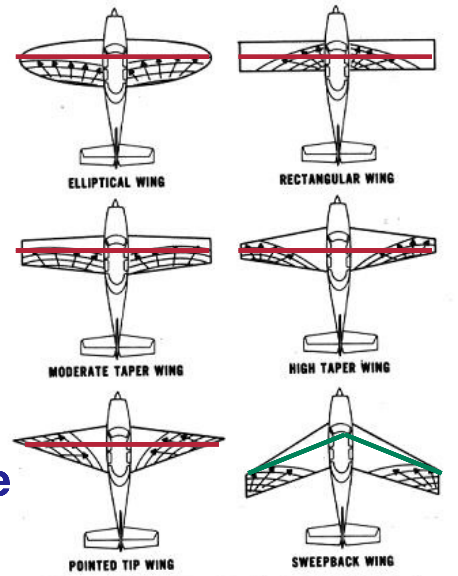
- **All Mach numbers at high angle of attack**
  - Separated flow on upper (leeward) surfaces

Next Time: Aerodynamic Moments (i.e., Torques)

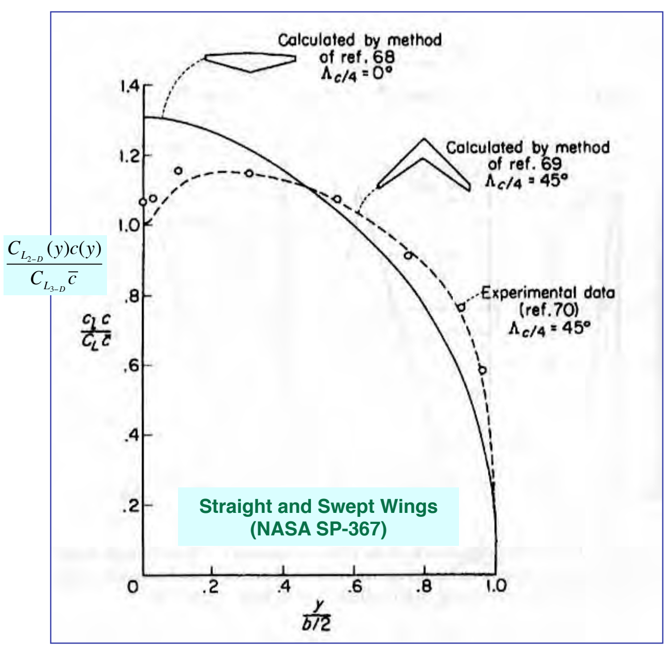
Reading:
Flight Dynamics
Aerodynamic Coefficients, 96-118
Airplane Dynamics and Control
Chapter 6

Straight, Swept, and Tapered Wings

- Straight at the quarter chord
- Swept at the quarter chord
- Progression of separated flow from trailing edge with increasing angle of attack



Spanwise Lift Distribution of 3-D Wings

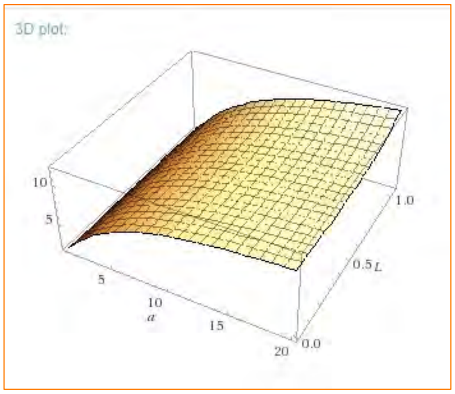


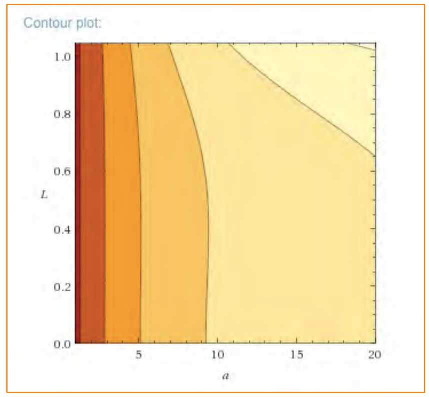
- Wing does not have to have a geometrically elliptical planform to have a nearly elliptical lift distribution
- Sweep moves lift distribution toward tips

Transonic Sweep Effects on 3-D Wing Lift Slope

- **Subsonic 3-D wing, M = 0.85**

plot(pi A / (1+sqrt(1 + ((A / 2 cos(L)) ^2) (1 – cos(L) 0.85^2))), A=1 to 20, L = 0 to (pi / 3))





Sweep Reduces Subsonic Lift Slope

Swept Wing

$$C_{L_\alpha} = \frac{\pi AR}{\left[1+\sqrt{1+\left(\dfrac{AR}{2\cos\Lambda_{1/4}}\right)^2\left(1-M^2\cos\Lambda_{1/4}\right)}\right]}$$

$$= \frac{\pi AR}{\left[1+\sqrt{1+\left(\dfrac{AR}{2\cos\Lambda_{1/4}}\right)^2}\right]}$$ [Incompressible flow]

Triangular Wing

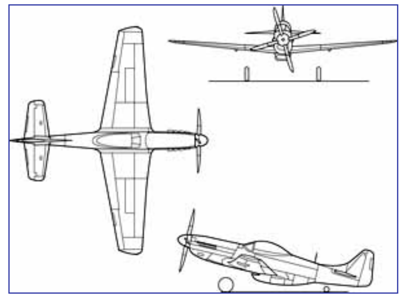
$$C_{L_\alpha} = \frac{2\pi^2 \cot\Lambda_{LE}}{(\pi+\lambda)}$$

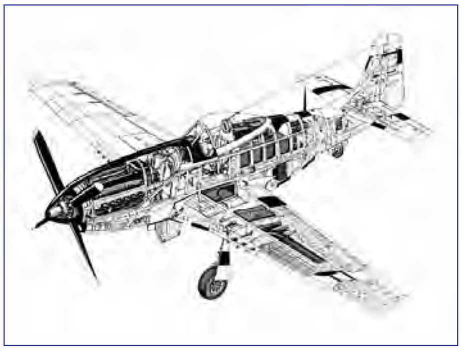
$$where \quad \lambda = m\left(0.38+2.26m-0.86m^2\right)$$

$$m = \cot\Lambda_{LE} / \cot\sigma$$

$\Lambda_{LE}, \sigma :$ measured from y axis

P-51 Mustang

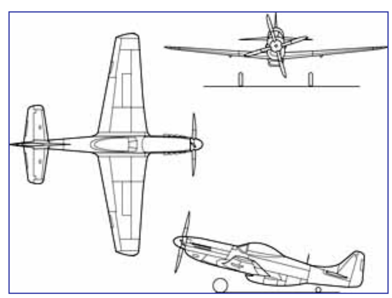




$$\begin{aligned}
&Wing\ Span = 37\ ft\ (9.83\ m)\\
&Wing\ Area = 235\ ft\ (21.83\ m^2)\\
&Loaded\ Weight = 9{,}200\ lb\ (3{,}465\ kg)\\
&Maximum\ Power = 1{,}720\ hp\ (1{,}282\ kW)\\
&C_{D_o} = 0.0163\\
&AR = 5.83\\
&\lambda = 0.5
\end{aligned}$$

http://en.wikipedia.org/wiki/P-51_Mustang

P-51 Mustang Example



$$C_{L_\alpha} = \frac{\pi AR}{\left[1+\sqrt{1+\left(\frac{AR}{2}\right)^2}\right]} = 4.49\ per\ rad\ (wing\ only)$$

$e = 0.947$

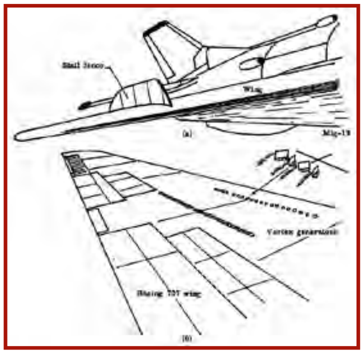
$\delta = 0.0557$

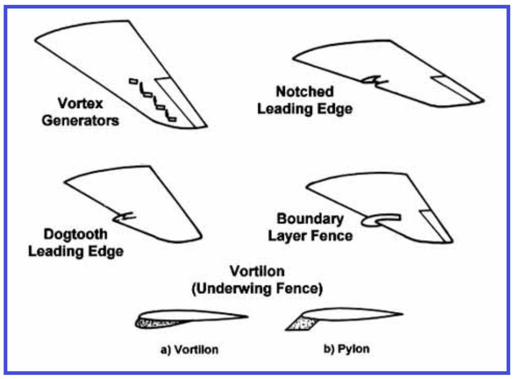
$\varepsilon = 0.0576$

$$C_{D_i} = \varepsilon C_L^2 = \frac{C_L^2}{\pi e AR} = \frac{C_L^2(1+\delta)}{\pi AR}$$

http://www.youtube.com/watch?v=WE0sr4vmZtU

Secondary Wing Structures

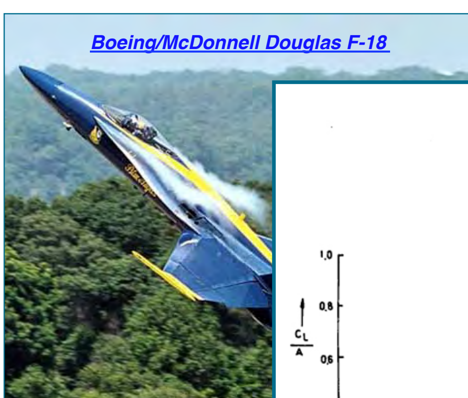


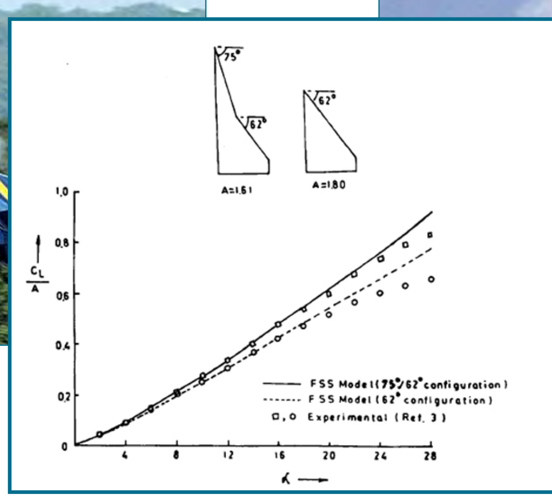


- **Vortex generators, fences, vortilons, notched or dog-toothed wing leading edges**
  - **Boundary layer control**
  - **Maintain attached flow with increasing α**
  - **Avoid tip stall**

Leading-Edge Extensions

- **Strakes or leading edge extensions**
  - **Maintain lift at high α**
  - **Reduce c.p. shift at high Mach number**



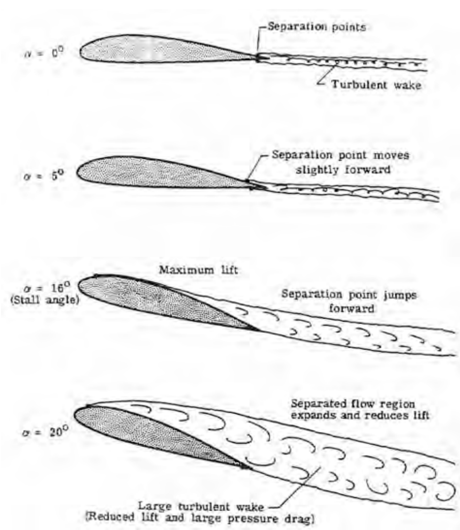


Wingtip Design

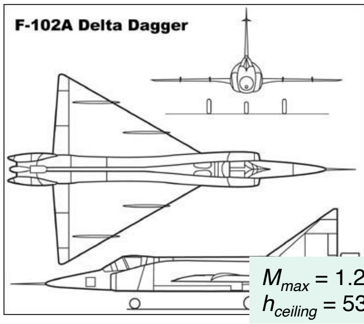
- **Winglets, rake, and Hoerner tip** reduce induced drag by controlling the tip vortices
- **End plate, wingtip fence** straightens flow, increasing apparent aspect ratio *(L/D)*
- **Chamfer** produces favorable roll w/ sideslip

Design for Satisfactory Stalls

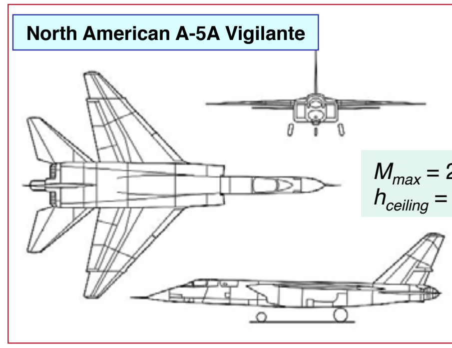
- Marked by noticeable, uncommanded changes in pitch, yaw, or roll and/or by a marked increase in buffet
- Stall must be detectable
- Aircraft must pitch down when it occurs
- Up to the stall break, ailerons and rudder should operate properly
- Inboard stall strips to prevent tip stall and loss of roll control before the stall
- Strakes for improved high-α flight



Low Aspect Ratio Configurations

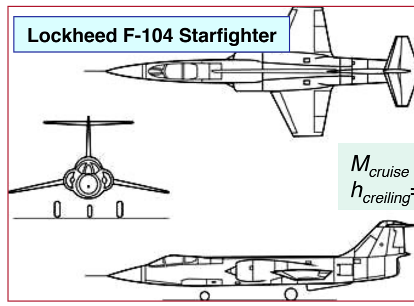


M_{max} = 1.25
$h_{ceiling}$ = 53 kft



M_{max} = 2
$h_{ceiling}$ = 52 kft

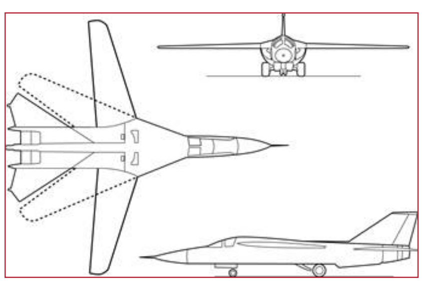
- Typical for supersonic aircraft



M_{cruise} = 1.4
$h_{creiling}$= 50 kft

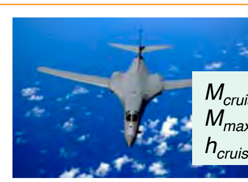
Variable Aspect Ratio Configurations

General Dynamics F-111



North American B-1

M_{max} = 2.5
$h_{ceiling}$ = 65 kft



M_{cruise} = 0.9
M_{max} = 1.25
h_{cruise} = 50 kft

- Aerodynamic efficiency at sub- and supersonic speeds

Sweep Effect on Thickness Ratio

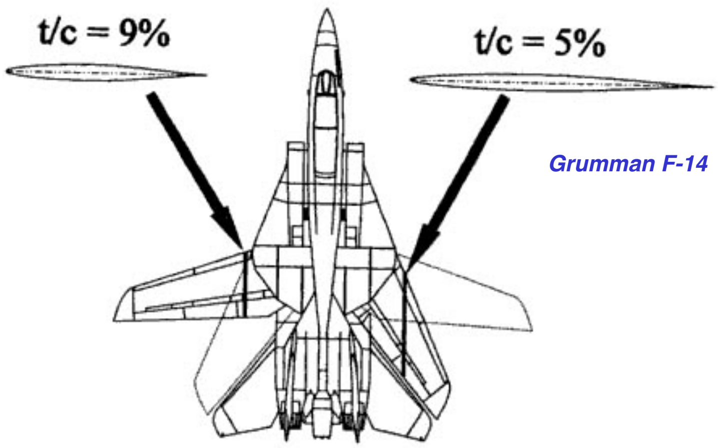


Fig. 1.18 Effect of sweep on relative thickness of wing sections.

from Asselin

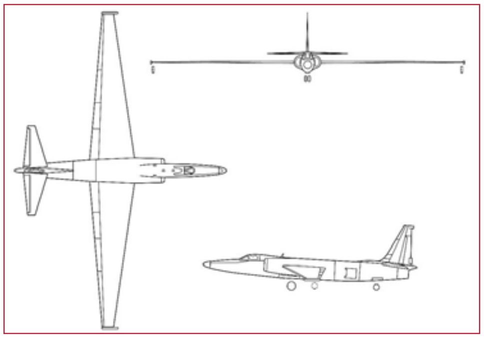
Lifting Body Re-Entry Vehicles

http://www.youtube.com/watch?v=K13G1uxNYks

http://www.youtube.com/watch?v=YCZNW4NrLVY

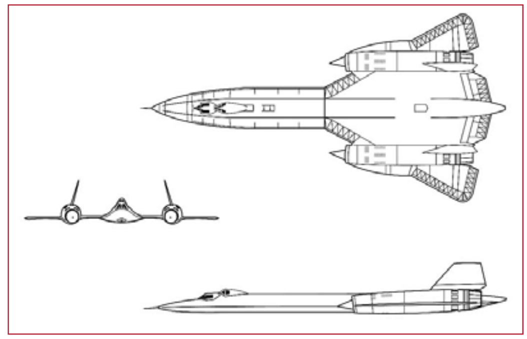
Reconnaissance Aircraft

Lockheed U-2 (ER-2)



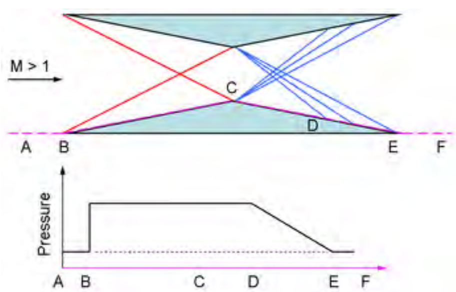
- **Subsonic, high-altitude flight**

Lockheed SR-71 Trainer



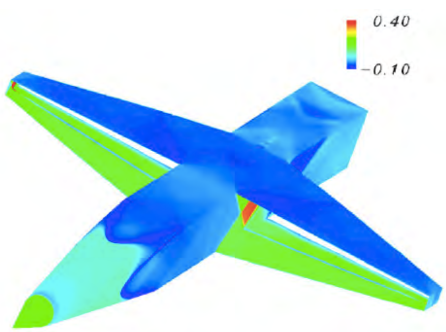
- **Supersonic, high-altitude flight**

Supersonic Biplane



- **Concept of Adolf Busemann (1935)**
  - **Shock wave cancellation at one specific Mach number**
  - **2-D wing**

http://en.wikipedia.org/wiki/Adolf_Busemann



- **Kazuhiro Kusunose *et al* , Tohoku U (PAS, 47, 2011, 53-87)**
  - **Adjustable flaps**
  - **Tapered, variably spaced 3-D wings**
  - **Fuselage added**

Supersonic Transport Concept

- Rui Hu, Qiqi Wang (MIT), Antony Jameson (Stanford), AIAA-2011-1248
  - Optimization of biplane aerodynamics
  - Sketch of possible configuration