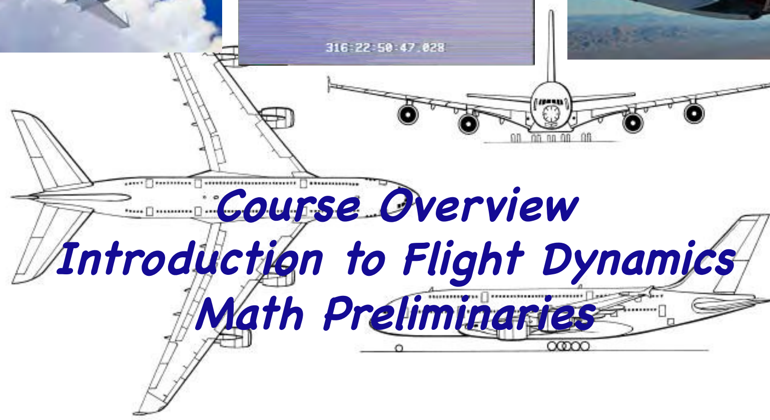
Aircraft Flight Dynamics

Robert Stengel, Princeton University, 2014



Course Overview
Introduction to Flight Dynamics
Math Preliminaries

At the End of the Course, you should be able to:

- Understand aircraft configuration aerodynamics, performance, stability, and control
- Estimate an aircraft's aerodynamic characteristics from geometric and inertial properties
- Analyze linear and nonlinear dynamic systems
- Recognize airplane modes of motion and their significance
- Compute aircraft motions
- Appreciate historical development of aviation

Syllabus, First Half

- Introduction, Math Preliminaries
- Point Mass Dynamics
- Aerodynamics of Airplane Configurations
- Cruising Flight Performance
- Gliding, Climbing, and Turning Performance
- Wind Tunnel and Flight Testing
- Nonlinear, 6-DOF Equations of Motion
- Aircraft Control Devices and Systems
- Linearized Equations of Motion
- Longitudinal Dynamics
- Lateral-Directional Dynamics

Details, reading, homework assignments, and references at
http://blackboard.princeton.edu/

Syllabus, Second Half

- Analysis of Linear Systems
  - Time Response
  - Transfer Functions and Frequency Response
  - Root Locus Analysis
- Advanced Problems in Longitudinal Dynamics
- Advanced Problems in Lateral-Directional Dynamics
- Flying Qualities Criteria
- Maneuvering at High Angles and Rates
- Aeroelasticity and Fuel Slosh
- Problems of High Speed and Altitude
- Atmospheric Hazards to Flight

Your interested in MAE 331 because …?

Text and References

- **Science, Engineering, and Math:**
  - *Flight Dynamics*, RFS, Princeton University Press, 2004
  - http://pup.princeton.edu/titles/7909.html
- **Case Studies, Historical Context**
  - *Airplane Stability and Control*, Abzug and Larrabee, Cambridge University Press, 2002
- **Supplemental reference**
  - Virtual reference book, 2013

Quick Quizzes
First 5 Minutes of 10 Classes

- One question about the lectures and reading assignments from the previous week
- Largely qualitative but may require simple calculations
- Be sure to bring a pencil, paper, and calculator to class

Homework Assignments

- All assignments will be done in groups of 2 or 3 students
- Team members for each assignment will be
  - different
  - chosen using a spreadsheet and random number generator (TBD)
- Each member of each team will receive the same grade as the others

Flight Tests Using Balsa Glider and Cockpit Flight Simulator

- *Compare actual flight of the glider with trajectory simulation*

- *Flight envelope of full-scale aircraft simulation*
  - *Maximum speed, altitude ceiling, stall speed, …*
- *Performance*
  - *Time to climb, minimum sink rate, …*
- *Turning Characteristics*
  - *Maximum turn rate, …*

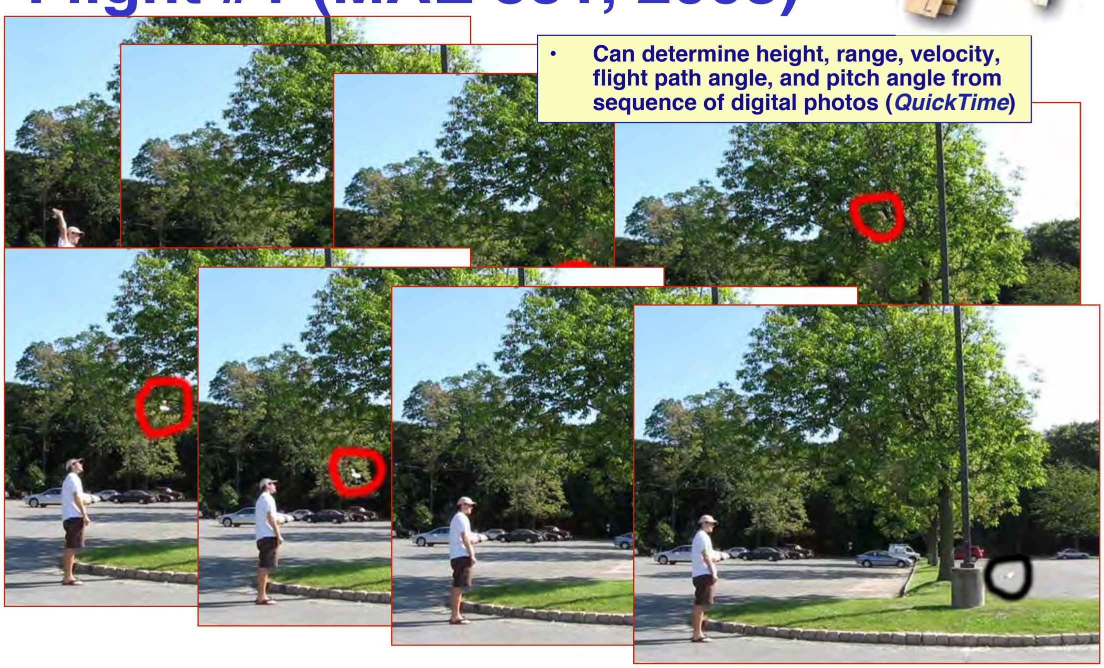
Assignment #1
due: September 19, 2014

- ***Document the physical characteristics and flight behavior of a balsa glider***
  - *Everything that you know about the <u>physical characteristics</u> of the glider*
  - *Everything that you know about the <u>flight characteristics</u> of the glider*
- ***2-person team, joint write-up***

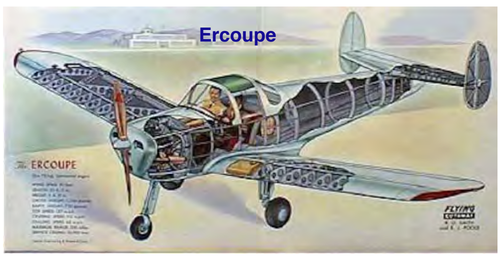
Luke Nash's Biplane Glider Flight #1 (MAE 331, 2008)

Luke Nash's Biplane Glider Flight #1 (MAE 331, 2008)

- Can determine height, range, velocity, flight path angle, and pitch angle from sequence of digital photos (*QuickTime*)



Stability and Control Case Studies



Reading Assignments

- ***All students are expected to do all reading assignments before class***
- *Reading for Case Studies and Historical Context, Airplane Stability and Control*
  - *10-minute synopses by groups of 3 students*
    - *Principal subject and scope of the chapter*
    - *Technical ideas needed to understand the chapter*
    - *When did the events covered in the chapter occur?*
    - *Three main "takeaway points" or conclusions*
    - *Three most surprising or remarkable facts*
  - *1st synopsis: Sept 23rd, team members TBD*

Goals for Airplane Design

- Shape of the airplane determined by its purpose
- Safety, handling, performance, functioning, and comfort
- Agility vs. sedateness
- Control surfaces adequate to produce needed moments (i.e., torques)
- Center of mass location
  - too far forward increases unpowered control-stick forces
  - too far aft degrades static stability

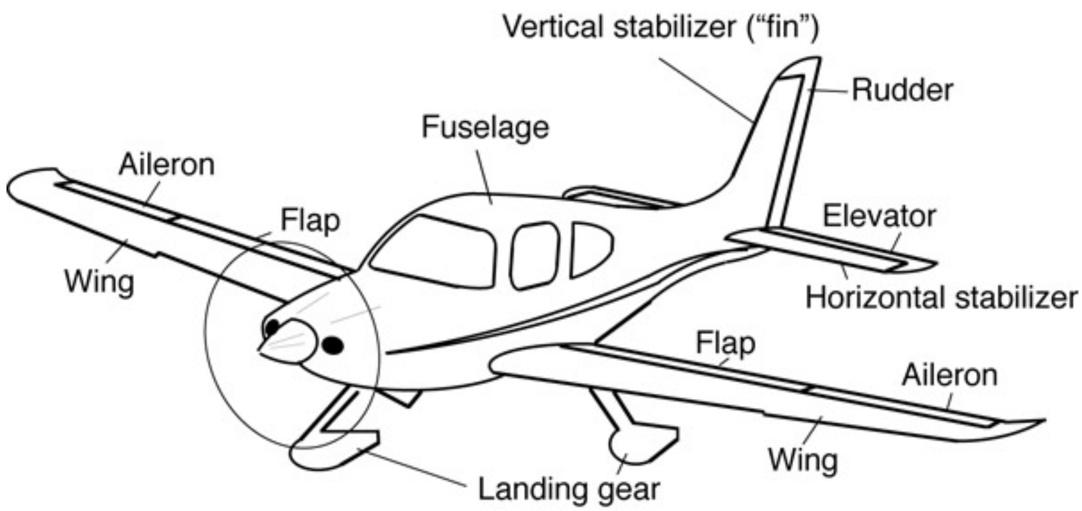
Configuration Driven By The Mission and Flight Envelope

Inhabited Air Vehicles

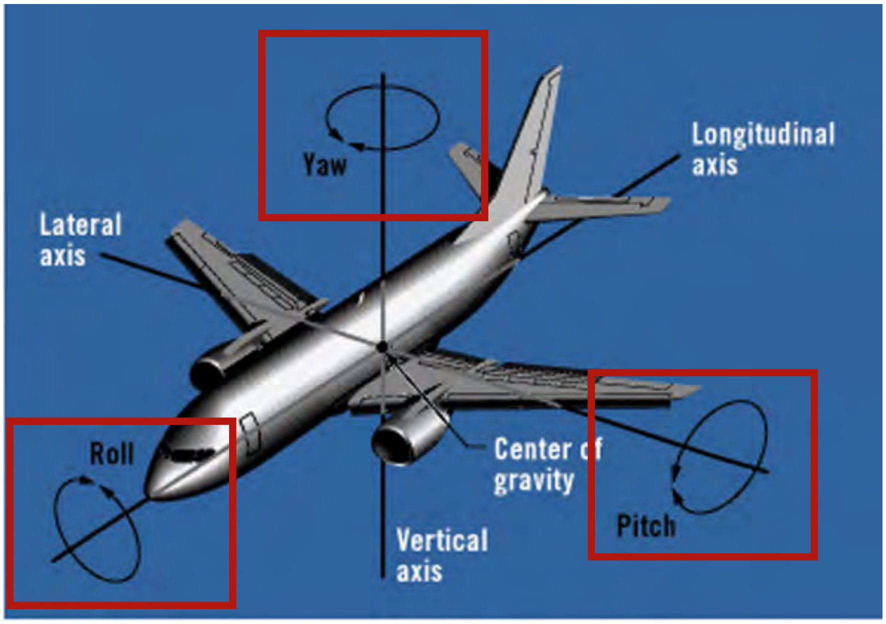
Uninhabited Air Vehicles (UAV)

Introduction to Flight Dynamics

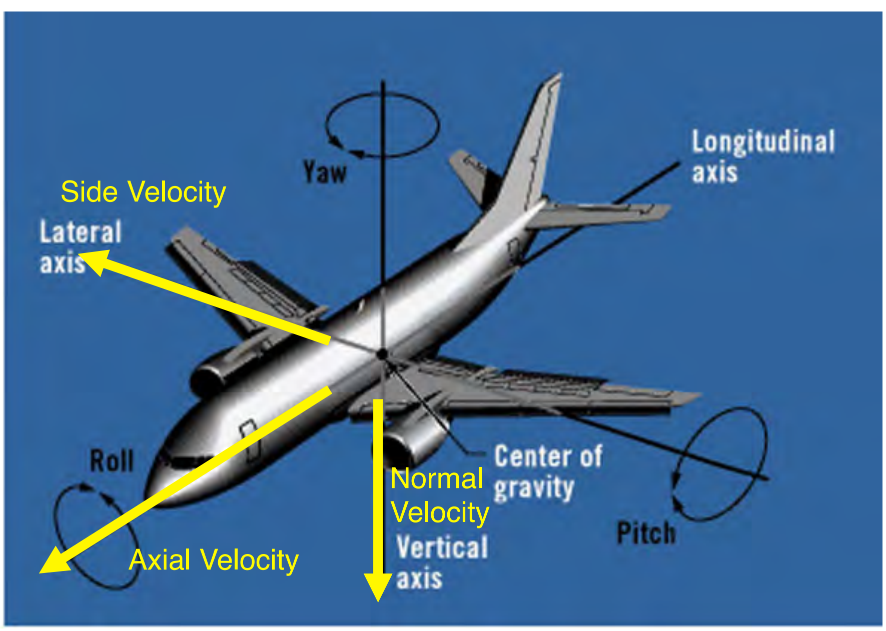
Airplane Components



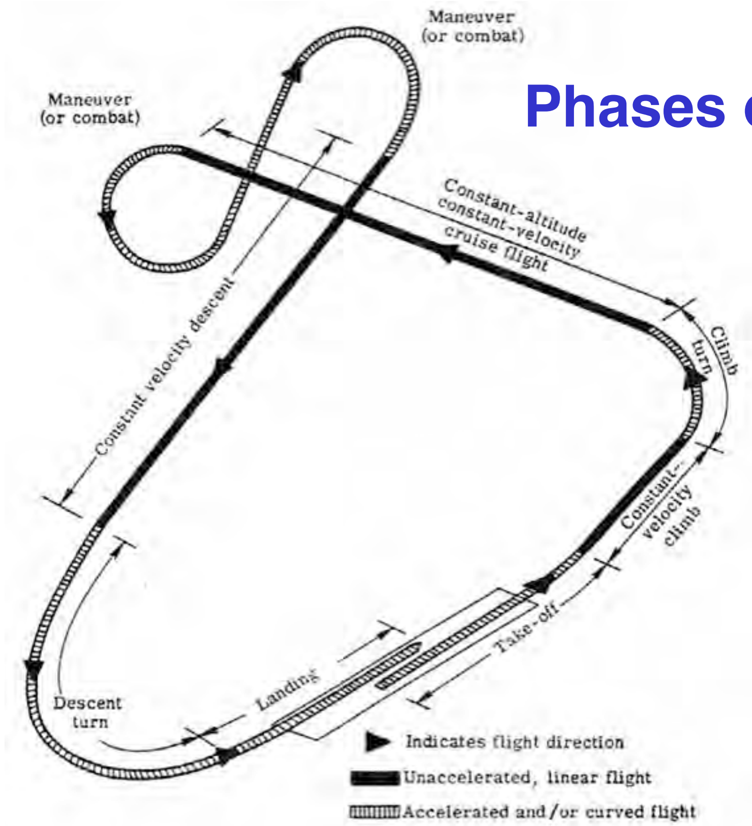
Airplane Rotational Degrees of Freedom



Airplane Translational Degrees of Freedom



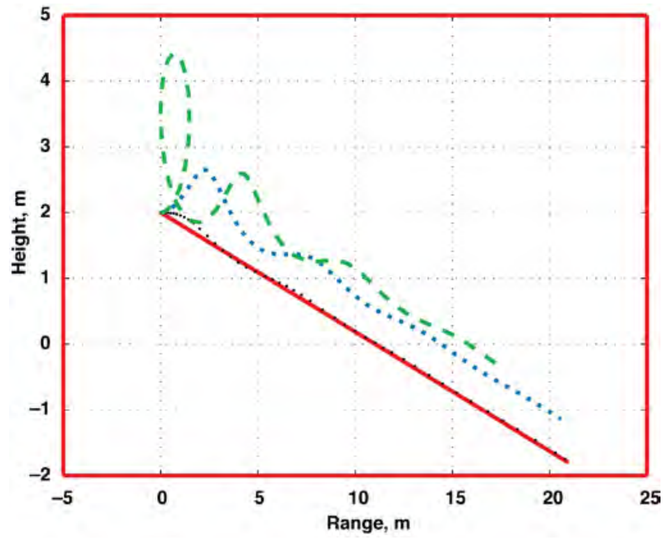
Phases of Flight



Flight of a Paper Airplane

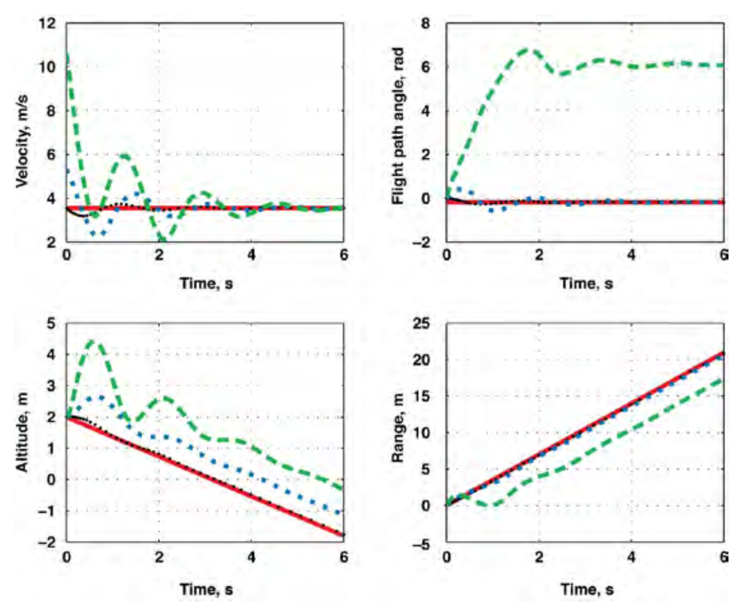
Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

- Equations of motion integrated numerically to estimate the flight path
- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop



Flight of a Paper Airplane
Example 1.3-1, *Flight Dynamics*

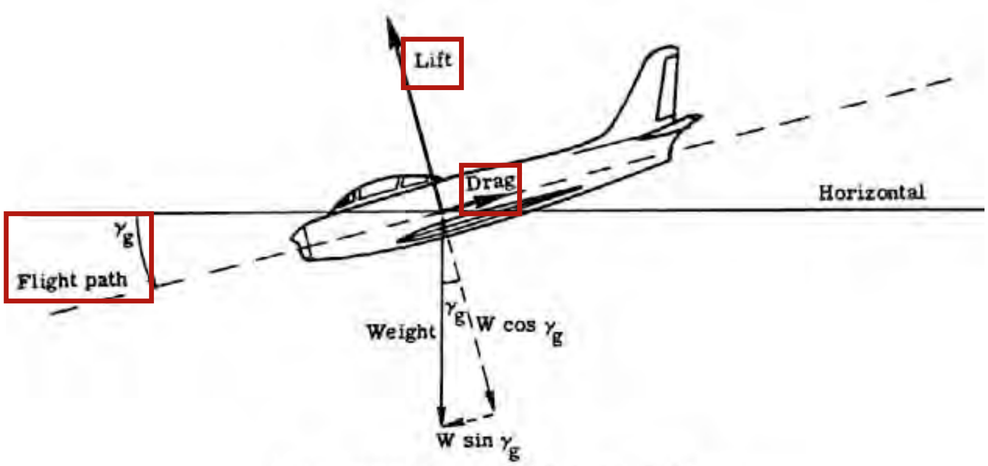
- Red: Equilibrium flight path
- Black: Initial flight path angle = 0
- Blue: plus increased initial airspeed
- Green: loop



Assignment #2

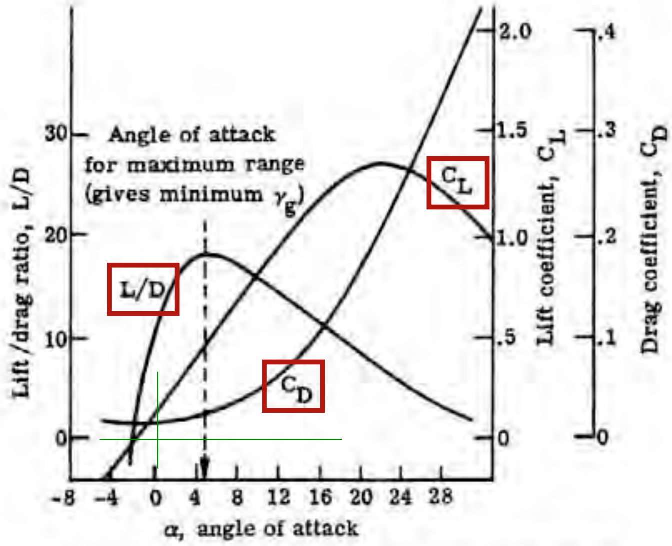
- Compute the trajectory of a balsa glider
- Computer simulation of the equations of motion
- Compare to the actual flight of the glider (Assignment #1)
- Similar to the flight of a paper airplane
- 2-person team assignment

Gliding Flight



(a) Unaccelerated glide conditions.

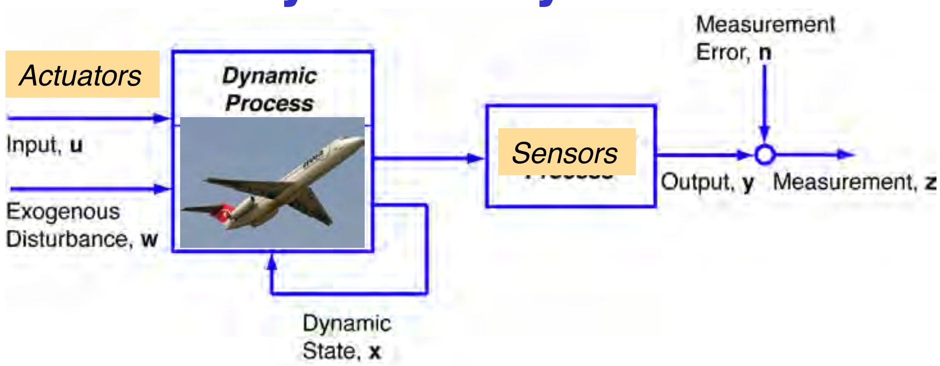
Configuration Aerodynamics



(b) Glide aerodynamic characteristics.

Math Preliminaries

Dynamic Systems



Dynamic Process: Current state depends on prior state

x	= dynamic state
u	= input
w	= exogenous disturbance
p	= parameter
t or *k*	= time or event index

Observation Process: Measurement may contain error or be incomplete

y	= output (error-free)
z	= measurement
n	= measurement error

- All of these quantities are multidimensional
- They can be expressed as **vectors**

Notation for Scalars and Vectors

- **Scalar**: usually lower case: *a, b, c, …, x, y, z*

$$a = 12; \quad b = 7; \quad c = a + b = 19; \quad x = a + b^2 = 12 + 49 = 61$$

- **Vector**: usually bold or with underbar: $\mathbf{x}$ or $\underline{x}$
  - Ordered set
  - Column of scalars
  - Dimension = *n* x 1

$$\mathbf{a} = \begin{bmatrix} 2 \\ -7 \\ 16 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Matrices and Transpose

- **Matrix**: usually bold capital or capital: **F** or F
  - Dimension = (m x n)

$$\mathbf{x} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}; \quad \mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}$$

(3×1) (4×3)

- **Transpose**: interchange rows and columns

$$\mathbf{x}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}$$

Multiplication

- **Operands must be conformable**
- **Multiplication of vector by scalar is associative, commutative, and distributive**

$$a\mathbf{x} = \mathbf{x}a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}$$

$$a(\mathbf{x}+\mathbf{y}) = (\mathbf{x}+\mathbf{y})a = (a\mathbf{x}+a\mathbf{y})$$

$\dim(\mathbf{x}) = \dim(\mathbf{y})$

$$a\mathbf{x}^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}$$

- **Could we add** $(\mathbf{x}+a)$ **?**
- **Only if** $\dim(\mathbf{x}) = (1 \times 1)$

Addition

- **Conformable vectors and matrices are added term by term**

$$\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix} \quad ; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\mathbf{x} + \mathbf{z} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$$

Inner Product

- **Inner (dot) product of vectors produces a scalar result**

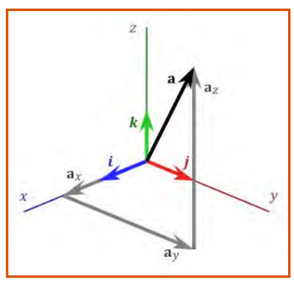
$$\mathbf{x}^T\mathbf{x} = \mathbf{x} \bullet \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$(1 \times m)(m \times 1) = (1 \times 1)$

$$= (x_1^2 + x_2^2 + x_3^2)$$

- **Length (or magnitude) of vector is square root of dot product**

$$= (x_1^2 + x_2^2 + x_3^2)^{1/2}$$



Vector Transformation

- **Matrix-vector product transforms one vector into another**
- **Matrix-matrix product produces a new matrix**

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

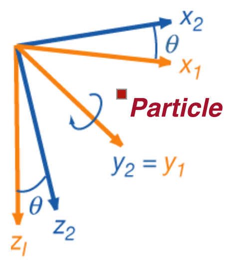
$$= \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Derivatives and Integrals of Vectors

- **Derivatives and integrals of vectors are vectors of derivatives and integrals**

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{bmatrix}$$

$$\int \mathbf{x}\,dt = \begin{bmatrix} \int x_1\,dt \\ \int x_2\,dt \\ \int x_3\,dt \end{bmatrix}$$



Matrix Inverse

Transformation

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

Inverse Transformation

$$\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

Matrix Identity and Inverse

- **Identity matrix: no change when it multiplies a conformable vector or matrix**

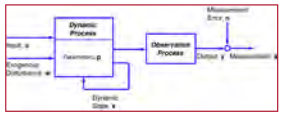
$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{y} = \mathbf{I}\mathbf{y}$$

- **A non-singular square matrix multiplied by its inverse forms an identity matrix**

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\begin{aligned} \mathbf{A}\mathbf{A}^{-1} &= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^{-1} \\ &= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$



Mathematical Models of Dynamic Systems are Differential Equations

Continuous-time dynamic process: Vector Ordinary Differential Equation

$$\dot{\mathbf{x}}(t) \triangleq \frac{d\mathbf{x}(t)}{dt} = \mathbf{f}[\mathbf{x}(t),\mathbf{u}(t),\mathbf{w}(t),\mathbf{p}(t),t]$$

$$\dim(\mathbf{x}) = (n \times 1)$$
$$\dim(\mathbf{f}) = (n \times 1)$$
$$\dim(\mathbf{u}) = (m \times 1)$$
$$\dim(\mathbf{w}) = (s \times 1)$$
$$\dim(\mathbf{p}) = (l \times 1)$$

Output Transformation

$$\mathbf{y}(t) = \mathbf{h}[\mathbf{x}(t),\mathbf{u}(t)]$$

$$\dim(\mathbf{y}) = (r \times 1)$$
$$\dim(\mathbf{h}) = (r \times 1)$$

Measurement with Error

$$\mathbf{z}(t) = \mathbf{y}(t) + \mathbf{n}(t)$$

$$\dim(\mathbf{z}) = (r \times 1)$$
$$\dim(\mathbf{n}) = (r \times 1)$$

Next Time:

Learning Objectives
Point-Mass Dynamics
Aerodynamic/Thrust Forces

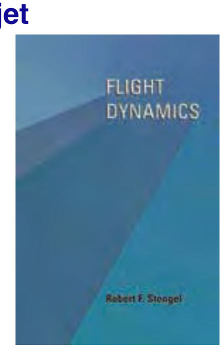
Reading:

Flight Dynamics
Introduction, 1-27
The Earth's Atmosphere, 29-34
Kinematic Equations, 38-53
Forces and Moments, 59-65
Introduction to Thrust, 103-107

Supplemental Material

Flight Dynamics Book and Computer Code

- All programs are accessible from the *Flight Dynamics* web page
  - http://www.princeton.edu/~stengel/FlightDynamics.html
- ... or directly
- *Errata* for the book are listed there
- 6-degree-of-freedom nonlinear simulation of a business jet aircraft (MATLAB)
  - http://www.princeton.edu/~stengel/FDcodeB.html
- Linear system analysis (MATLAB)
  - http://www.princeton.edu/~stengel/FDcodeC.html
- Paper airplane simulation (MATLAB)
  - http://www.princeton.edu/~stengel/PaperPlane.html
- Performance analysis of a business jet aircraft (Excel)
  - http://www.princeton.edu/~stengel/Example261.xls



Helpful Resources

- Web pages
  - http://blackboard.princeton.edu/
  - http://www.princeton.edu/~stengel/MAE331.html
  - http://www.princeton.edu/~stengel/FlightDynamics.html
- Princeton University Engineering Library (paper and on-line)
  - http://lib-terminal.princeton.edu/ejournals/by_title_zd.asp
  - http://sfx.princeton.edu:9003/sfx_pul/az
- NACA/NASA pubs
  - http://ntrs.nasa.gov/search.jsp

Course Learning Objectives

(Accreditation Board for Engineering and Technology)

Course Learning Objectives	ABET Criterion 3
Understanding of the dynamics and control of aircraft.	a
Ability to estimate aerodynamic coefficients and stability derivatives from aircraft geometry and flight envelope.	a, c
Facility in analyzing mathematical descriptions of the rigid-body motions of flying vehicles.	a
Ability to estimate the performance, stability, and control characteristics of aircraft.	b
Development of appreciation for flight-testing methods and results.	b, k
Ability to apply systems-engineering approach to the analysis, design, and testing of aircraft.	b, c
Demonstration of ability to work in multidisciplinary teams.	d
Demonstration of computational problem-solving, through thorough knowledge, application, and development of analytical software.	e, k
Appreciation of the historical context within which airplanes have evolved to present-day configurations.	f, h, i, j
Competence in presenting ideas.	g