Space Robotics

Space System Design, MAE 342, Princeton University
Robert Stengel

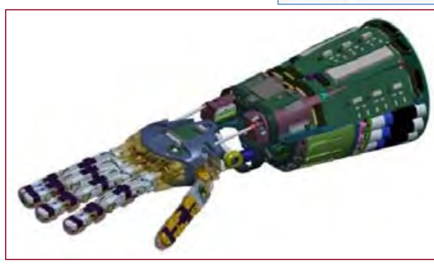
- Robots and Robotics
- Autonomous Spacecraft
- Planetary and Lunar Rovers
- Path Planning
- Robotic Arms
- Robonauts
- Deep Impact 1

Robots and Robotics

- Design, manufacture, control, and programming of robots
- Use of robots to solve problems
- Study of control processes, sensors, and algorithms used in humans, animals, and machines
- Application of control processes and algorithms to designing robots

Biomimetics (Bionics)

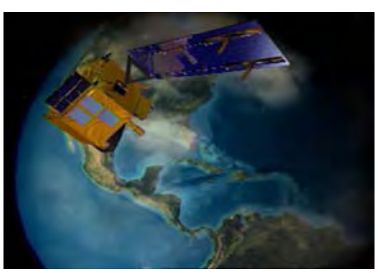
- Understanding biological principles and applying them to system design
  - Configuration
  - Structure
  - Behavior
  - Dynamics
  - Control

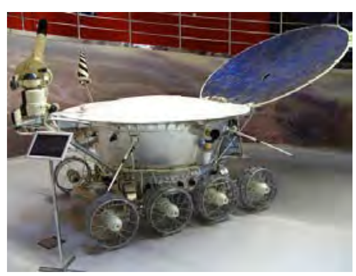


Autonomous Robots

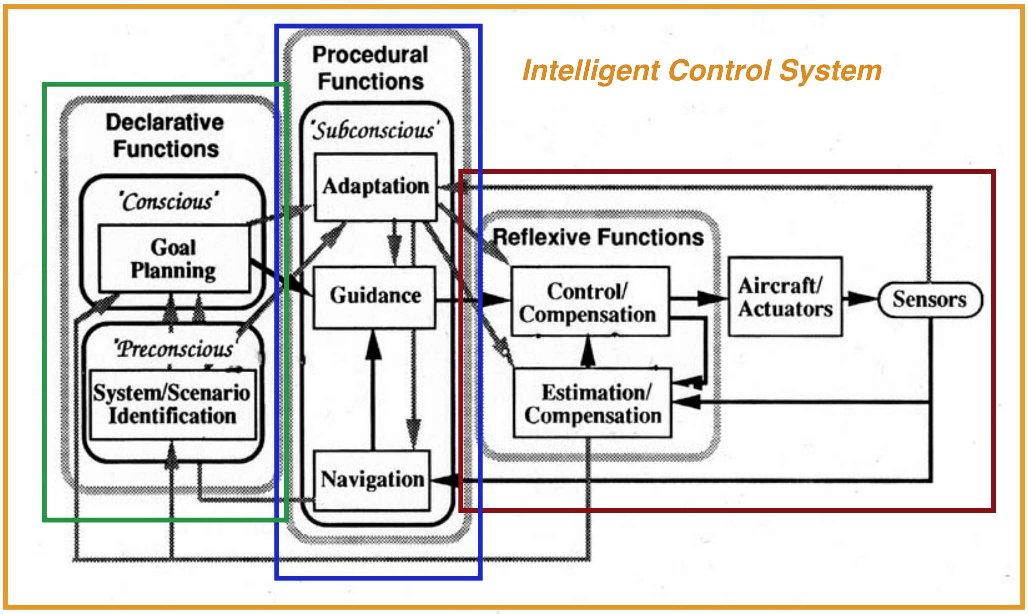
- Self control
- Self maintenance
- Awareness of environment
- Task orientation
- Mission specificity

- Power source
- Cooperation and collaboration
- = Intelligence?
- Self replication?
- Ethical issues





Elements of Intelligent Control



Declarative Functions	*Expert Systems, Decision Trees*
Procedural Functions	*Estimation and Control "Circuits"*
Reflexive Functions	*Control Laws, Neural Networks*

Robotic Vehicles

Expendable (Rocket) Launch Vehicles

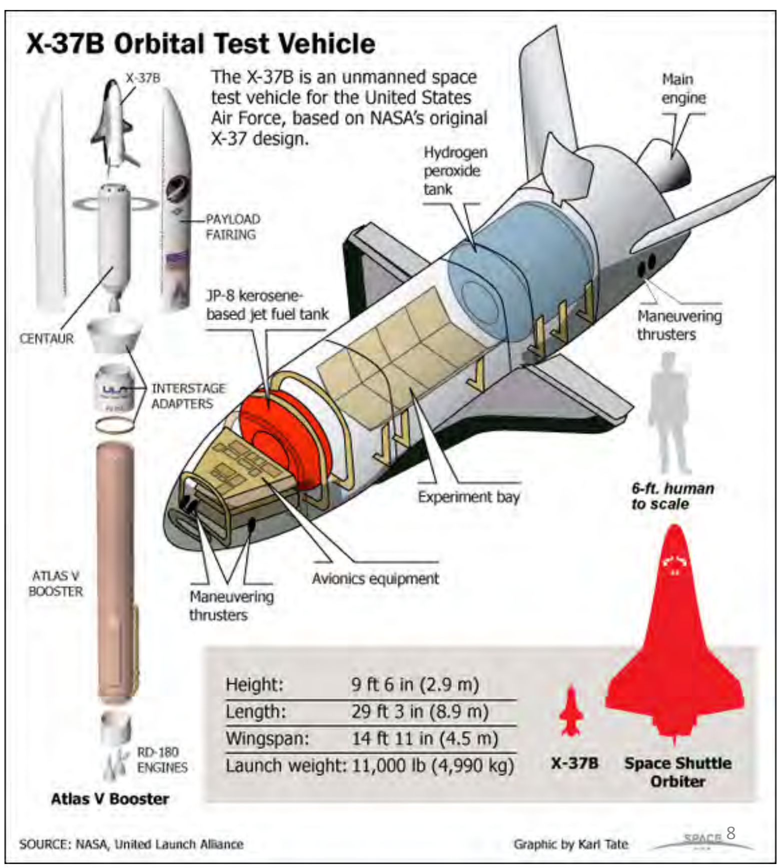
Current space launch vehicles are largely autonomous

Atlas V

[*http://www.youtube.com/watch?v=KxQbex7LJwg*](http://www.youtube.com/watch?v=KxQbex7LJwg)

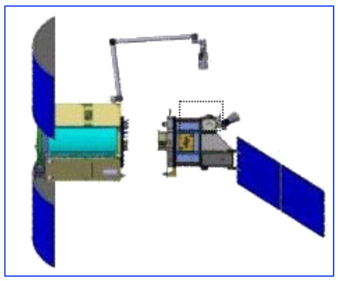
X-37B

- **Reusable experimental/ operational vehicle**
- **Unmanned "mini-Space Shuttle"**
- **Highly classified project**
- **1st 3 missions: 224, 469, & 675 days in orbit**
- **4th mission on-going**



Orbital Express: ASTRO and NEXTSat

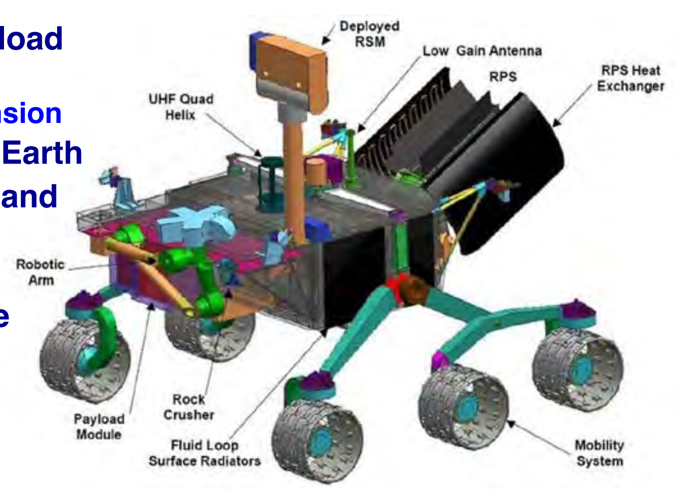
- DARPA, 2007
  - Automatic rendezvous, docking, and undocking
  - On-orbit transfer of replaceable units
  - 6DOF robot arm
  - Video guidance sensor
  - Atlas 5 launch



https://en.m.wikipedia.org/wiki/Robotic_spacecraft

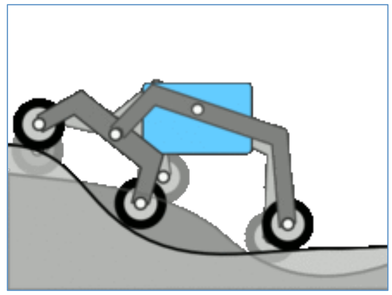
Mars Science Laboratory
(Curiosity)

- Transport science payload over Martian surface
  - Rocker-bogie suspension
- Communications with Earth
- Guidance, navigation, and control
- Power supply
- Support for deployable devices
- Size ~ Mini-Cooper
- Landed, 8/6/12, and operational



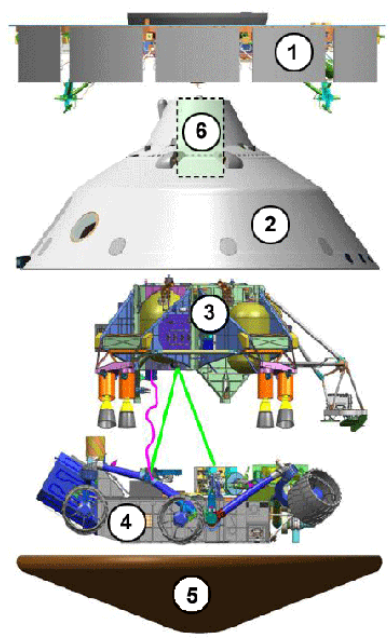
Curiosity Trailer
http://www.jpl.nasa.gov/video/details.php?id=1014

Curiosity Rocker-Bogie Suspension



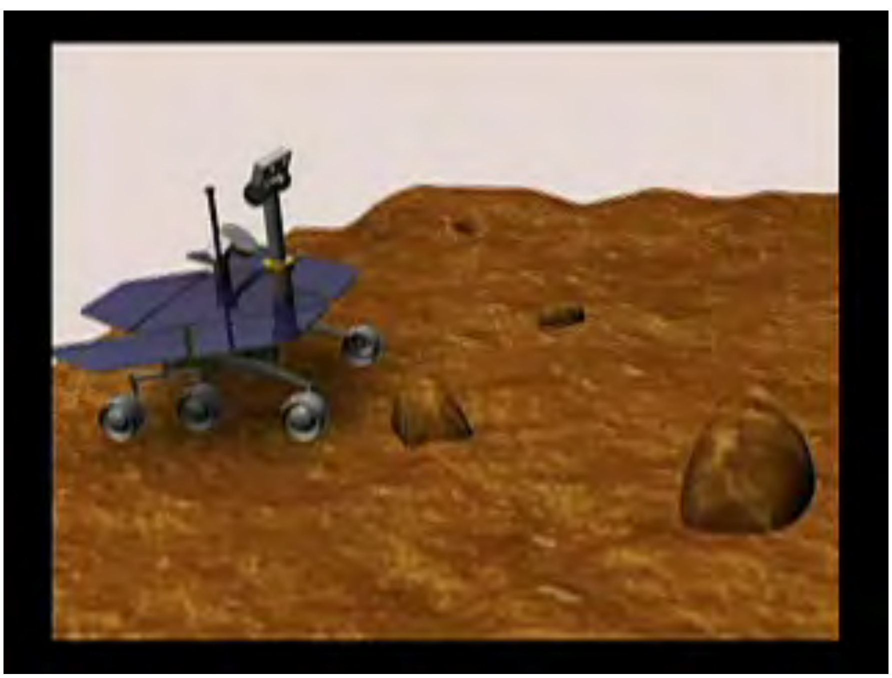
Curiosity Robotic Arms

Curiosity Preparation, Spacecraft, and Aeroshell



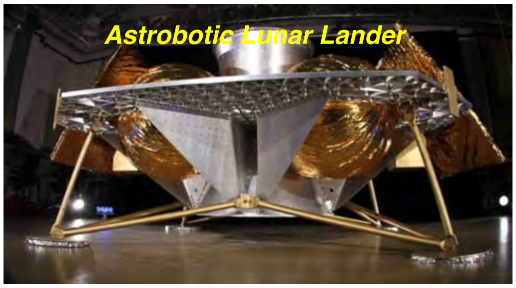
Curiosity Approach and Landing

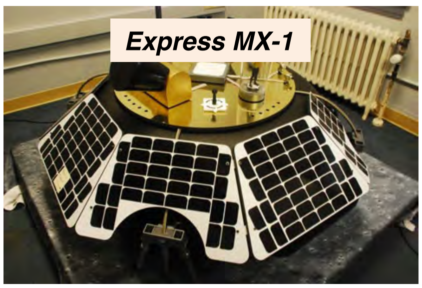
Mars Opportunity Rover Navigation



Google LunarX Prize

1st privately funded spacecraft to land on Moon, travel 500m, and stream live TV

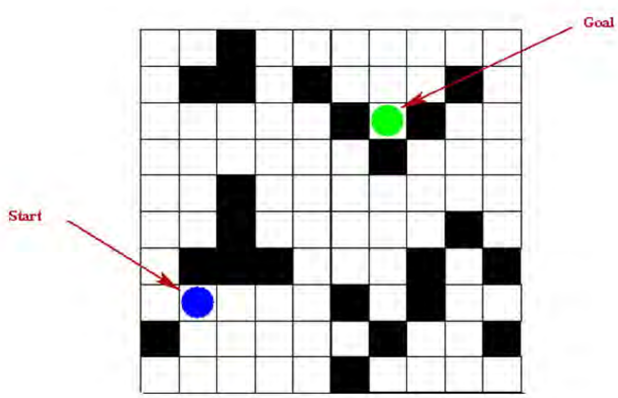




https://en.m.wikipedia.org/wiki/Google_Lunar_X_Prize

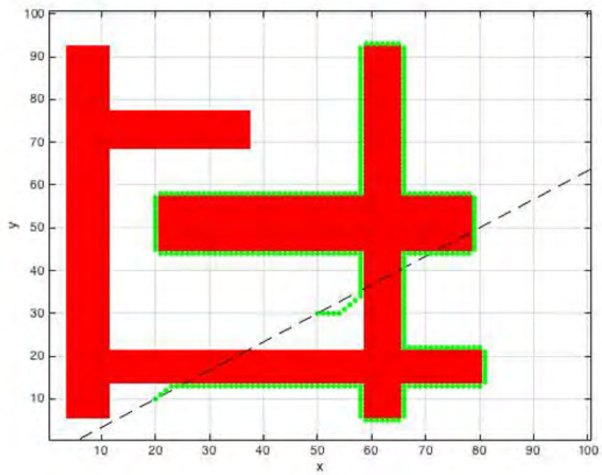
Path Planning on Occupancy Grid

Admissible and Inadmissible Blocks



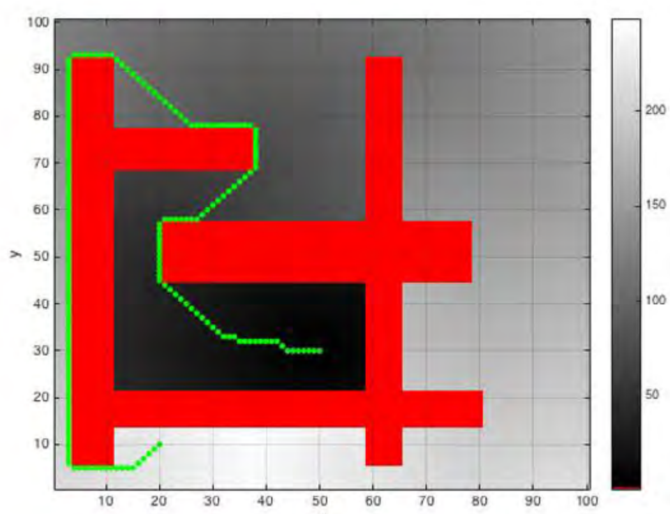
- **Identify feasible paths from Start to Goal**
- **Chose path that best satisfies criteria, e.g.,**
  - **Simplicity of calculation**
  - **Lowest cost**
  - **Highest performance**

Bug Path Planning



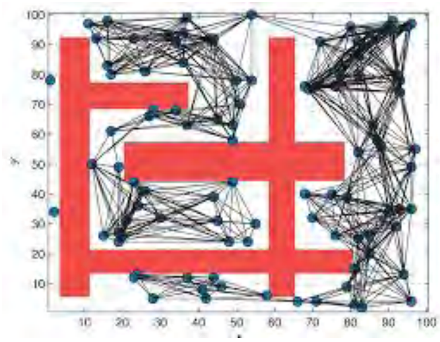
1) Identify shortest unconstrained path from Start to Goal
2) Chose path that navigates the boundary
    1) Stays as close as to possible to unconstrained path
    2) Satisfies constraint
    3) Follows simple rule, e.g., “stay to the left”

D* or A* Path Planning



- Determine ***occupancy cost*** of each block
- Chose path from Start to Goal that minimizes occupancy cost with each step

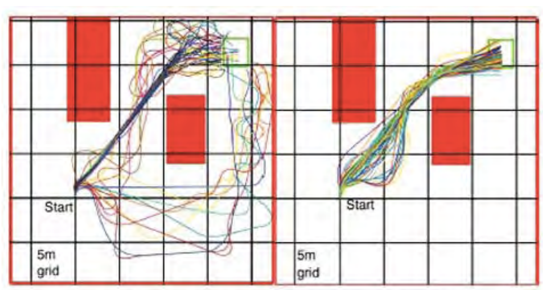
Probabilistic Road Map



- <u>Construction Phase</u>: Random configuration of admissible points
  - Connect admissible points to nearest neighbors
- <u>Assessment Phase</u>: Assess incremental cost of traveling along each "edge" between points
- <u>Query Phase</u>: Find all feasible paths from Start to Goal and <u>select lowest cost path</u>

Note that this approach would miss the lowest cost path found on the previous slide

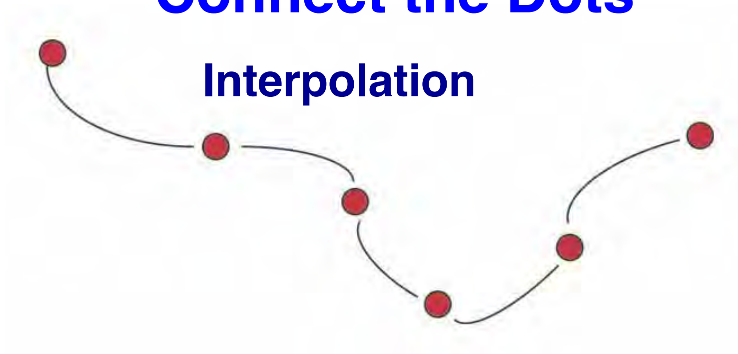
Rapidly Exploring Random Trees (RRT, RRT*)



(a) RRT (b) RRT*

- <u>RRT</u>
  - Space-filling tree evolves from Start
  - Open-loop trajectories with state constraints
  - Initially feasible solution converges to optimal solution through searching
- <u>RRT*</u>
  - "Committed trajectories"
  - Branch-and-bound tree adaptation
  - Provable convergence

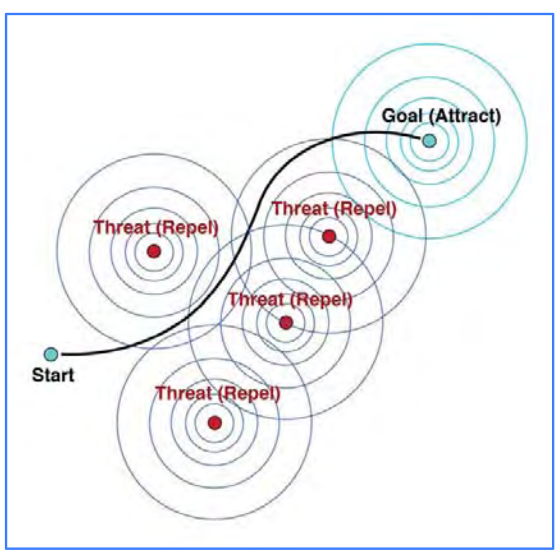
Connect the Dots



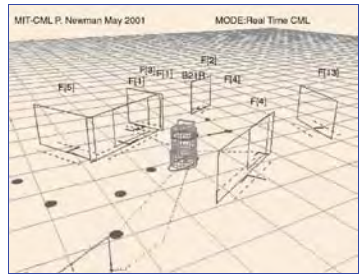
- **Piecewise polynomials (linear -> quintic)**
  - **End-point discontinuities**
  - **End-point constraints**
- **Single polynomial through all points**
  - **Polynomial degree = # of points – 1**
  - **Sensitivity to high-degree terms (e.g., ct^6)**
  - **Possibility of large excursions between points**
- **Polynomials through adjacent points**
  - **e.g., cubic B splines**

Path Planning with Potential Fields

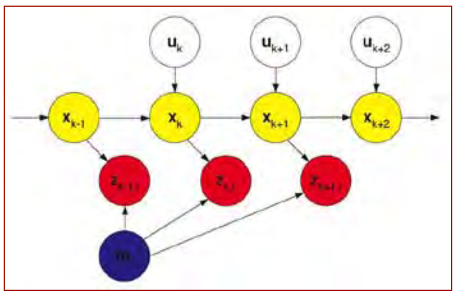
Features attract or repel path from Start to Goal, e.g., +/– gravity fields

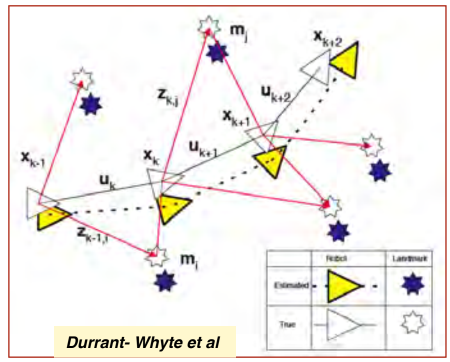


Simultaneous Location and Mapping (SLAM)

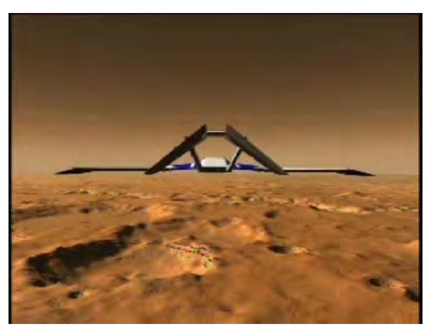


- Build or update a local map within an unknown environment
  - Stochastic map, defined by mean and covariance of many points
  - Landmark, terrain, and target tracking
  - Multi-sensor integration
  - SLAM Algorithm = Bank of state estimators





Mars Aerial Regional-Scale Environmental Survey (*ARES*) Research Airplane Concept, ~2008



https://www.youtube.com/watch?v=8YutbpJuFiI

https://www.youtube.com/watch?v=wAOTOmGFs5M

ARES System Layout

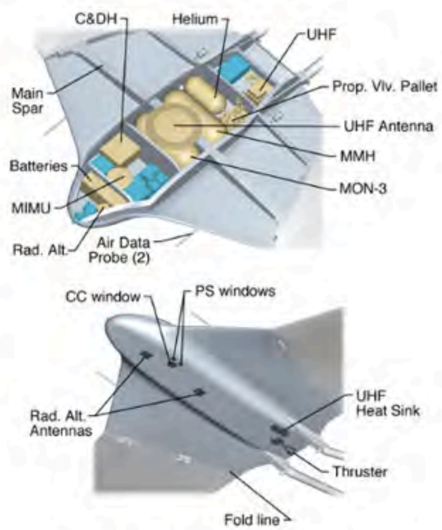


Figure 7: ARES Subsystem Layout.

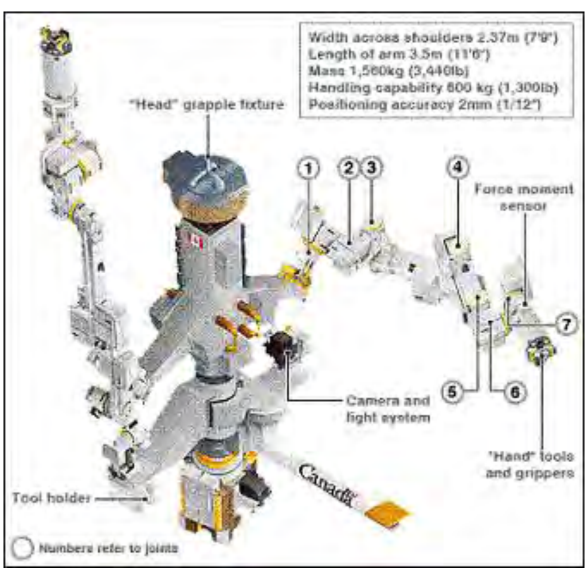
Table 3: Airplane Mass and Performance Summary

	Airplane Design		
	Current Best Estimate	Growth	Allocation
Airplane dry mass (kg)	82	101	127
Propellant mass (kg)	48	48	48
Airplane wet mass (kg)	130	149	175
Range, km	680	600	500
Endurance, min	81	71	60

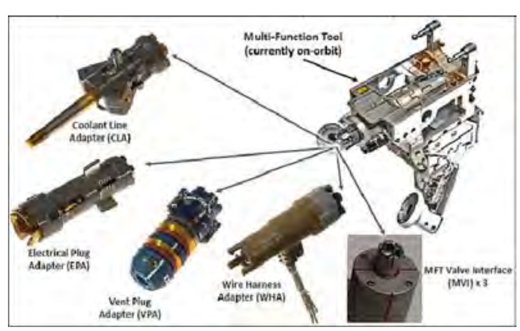
Articulated Robot Arms

Robot Arms for Space

DEXTRE

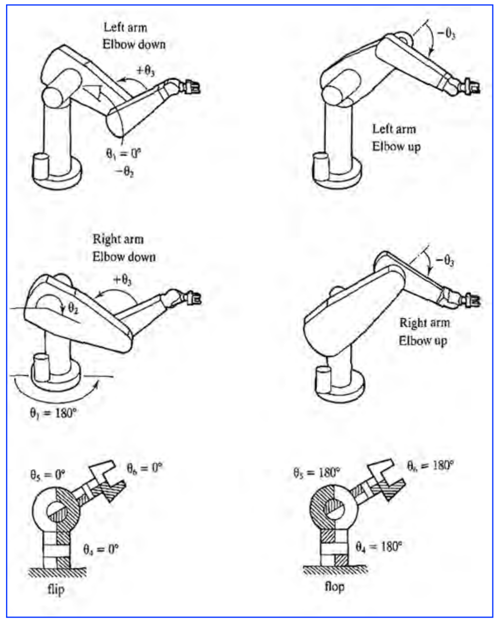


Dextre Specifications	
Height	12 feet
Width	7.7 feet (across shoulders)
Arm Length	11.48 feet linear stroke
Mass (approx.)	3,564 pounds
Mass Handling/Transportation Capacity	1,322.77 pounds
Degrees of Freedom	15
Peak Power (operational)	2,000 W
Avg. Power (keep alive)	600 W
Applied Tip Load Range	0-111 N
Stopping Distance (under max. load)	5.9 inches

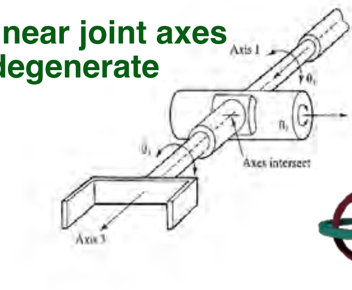


https://en.m.wikipedia.org/wiki/Dextre

Manipulator Redundancy and Degeneracy

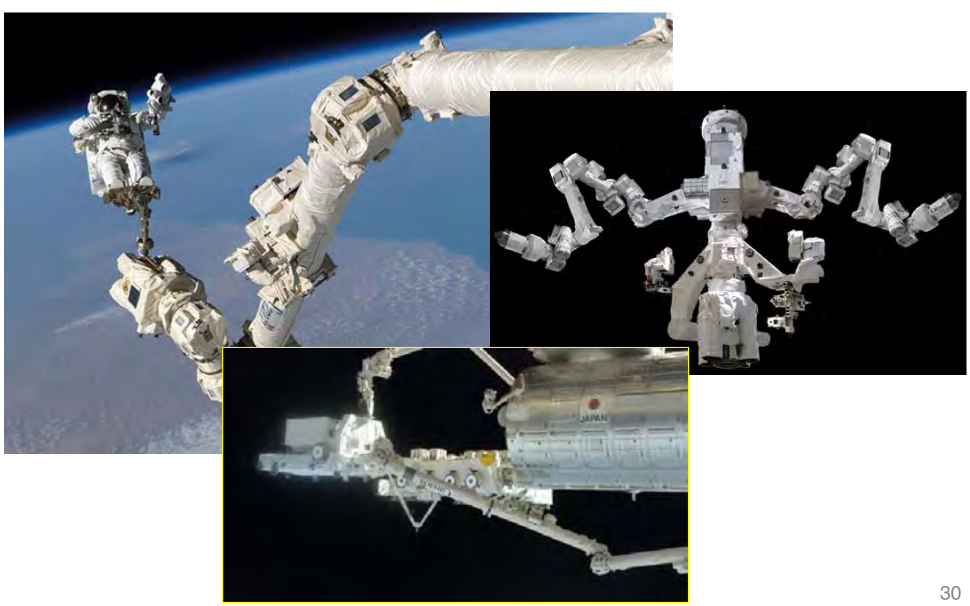


- More than one link configuration may provide a given end point if $\dim(x_J) \geq \dim(x_E) \geq \dim(x_T)$
- **Redundancy**: Finite number of joint vectors provide the same task-dependent vector
- **Degeneracy**: Infinite number of joint vectors provide the same task-dependent vector
- Co-linear joint axes are degenerate

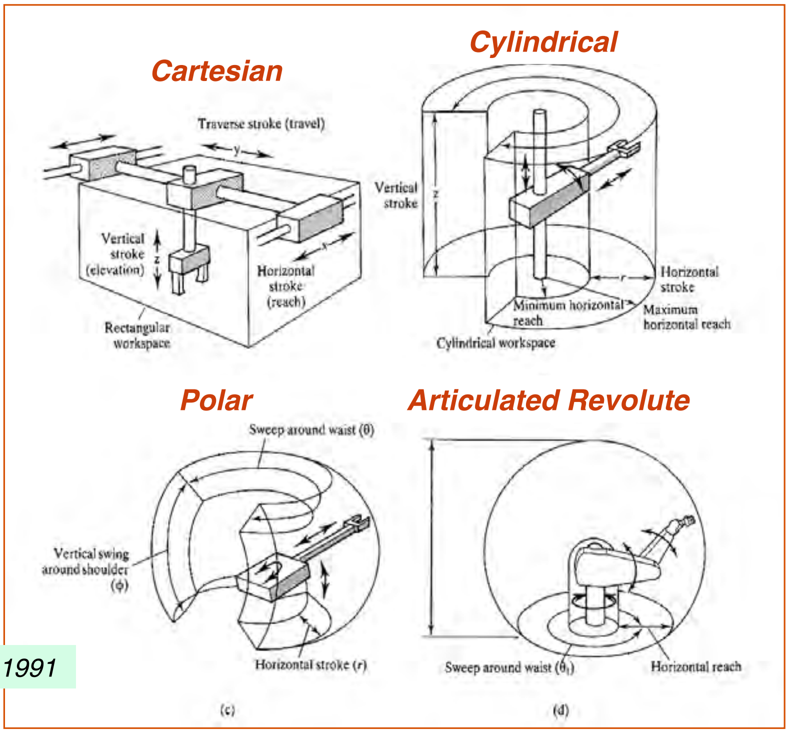


Space Robot Arms are Highly Redundant

Why?



Robot Arm Configurations

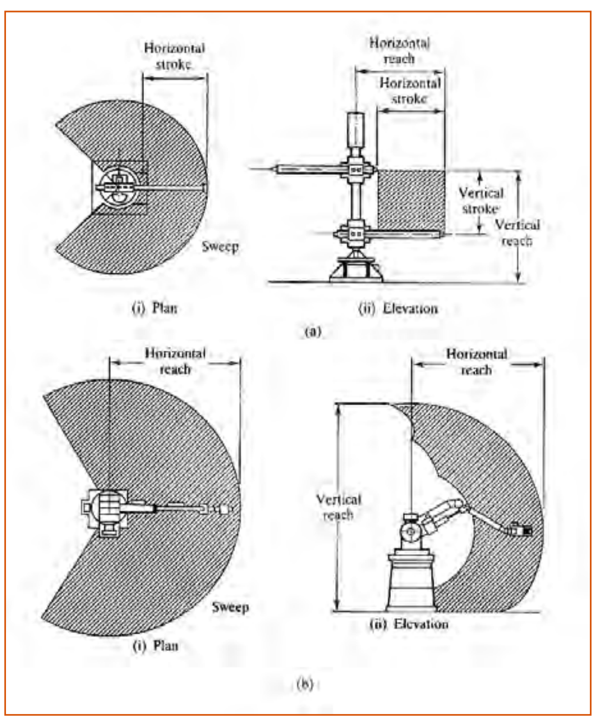


McKerrow, 1991

Robot Arm Workspaces

Cylindrical

Articulated Revolute

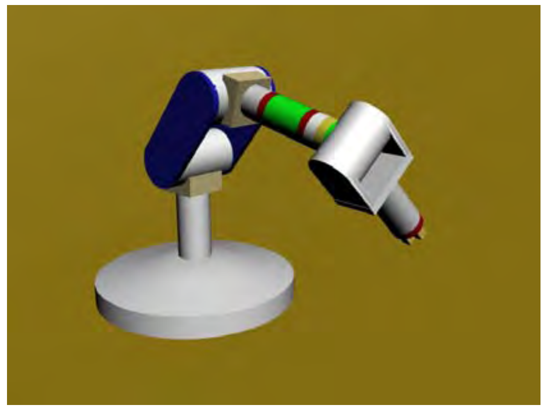


McKerrow, 1991

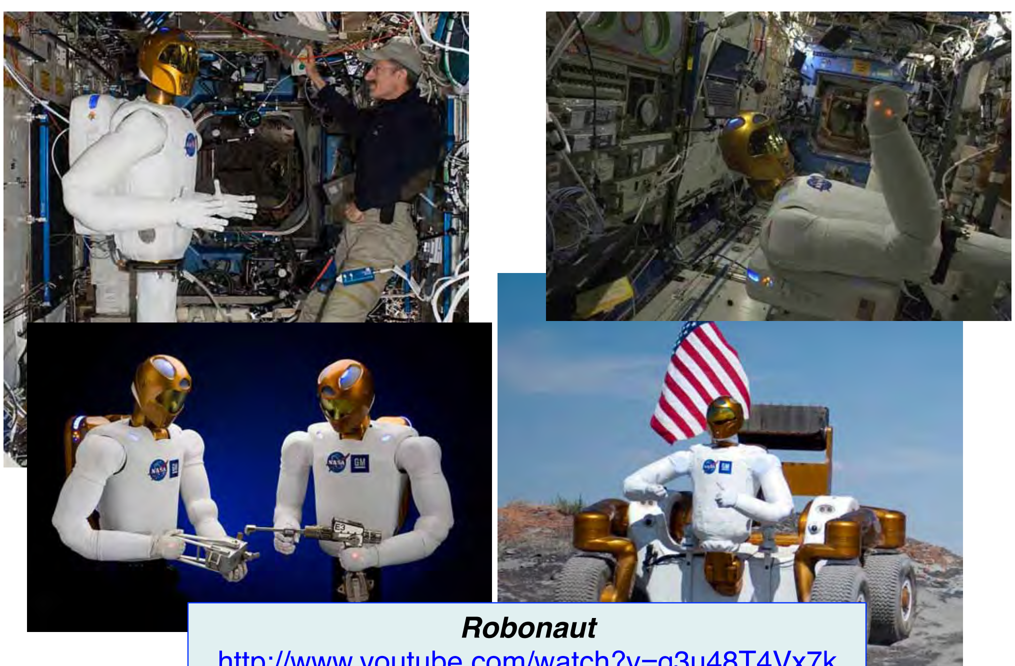
Serial Robotic Manipulators

***Proximal link*: closer to the base**
***Distal link*: farther from the base**

- **Serial chain of robotic links and joints**
  - **Large workspace**
  - **Low stiffness**
  - **Cumulative errors from link to link**
  - **Proximal links carry the weight and load of distal links**
  - **Actuation of proximal joints affects distal links**
  - **Limited load-carrying capability at end effecter**

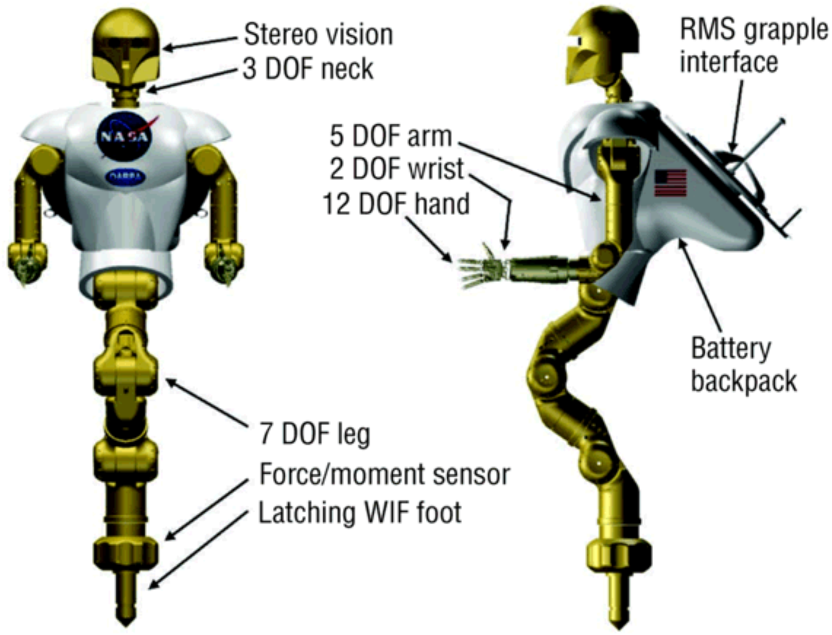


NASA/GM Robonaut R2

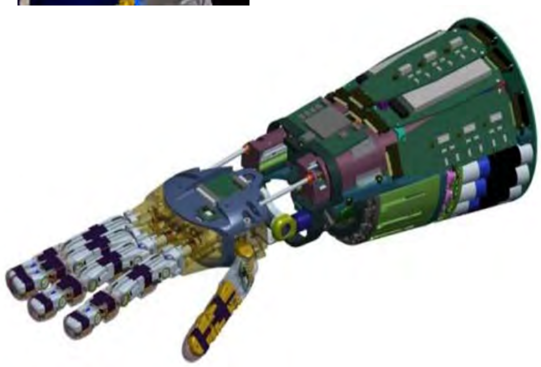


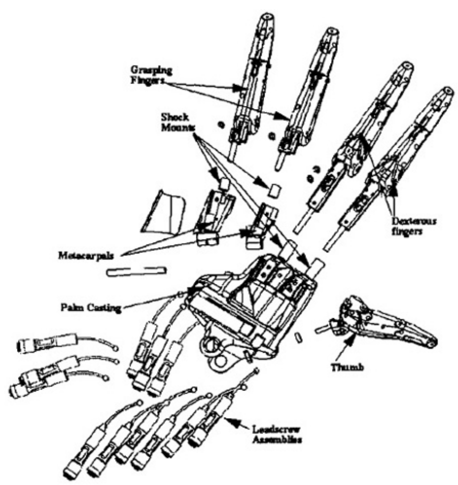
Robonaut
http://www.youtube.com/watch?v=g3u48T4Vx7k

Robonaut 2 Structure

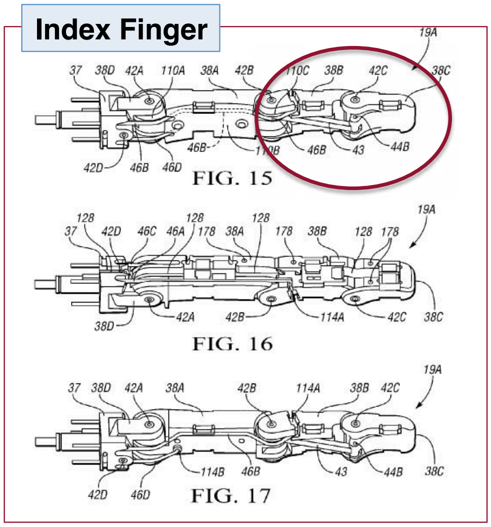


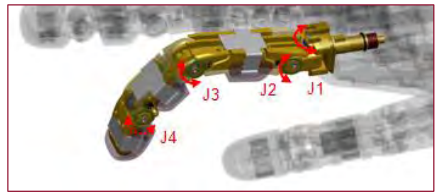
Robonaut 2 Hand

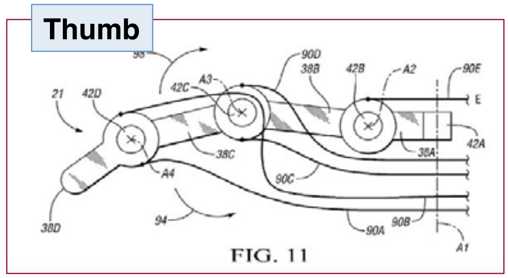


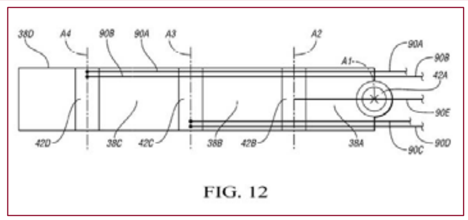


Robonaut 2 Fingers









- Control by tendons and four-bar linkages
- Linear actuators in forearm

Four-Bar Linkage

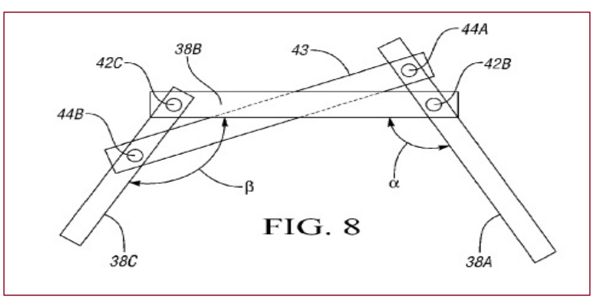
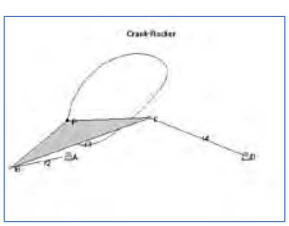
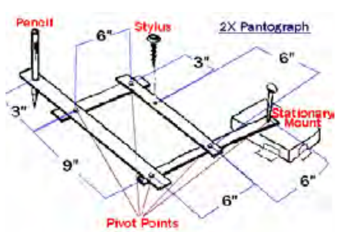
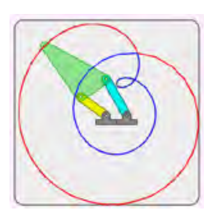


FIG. 8

- Closed-loop structure
- Rotational joints
- Planar motion
- Proportions of link lengths determine pattern of motion







Robonaut 2 Wrist and Forearm

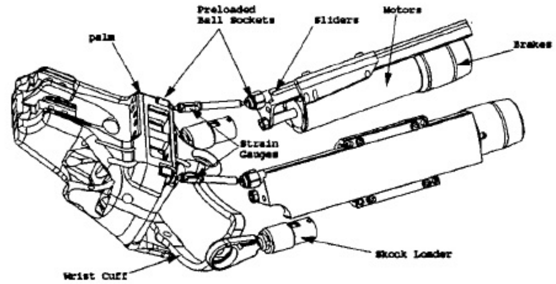


Figure 9 Wrist mechanism

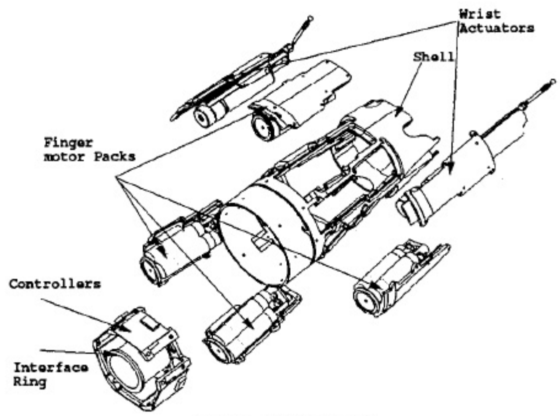
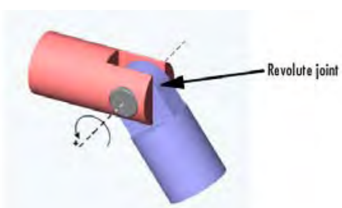


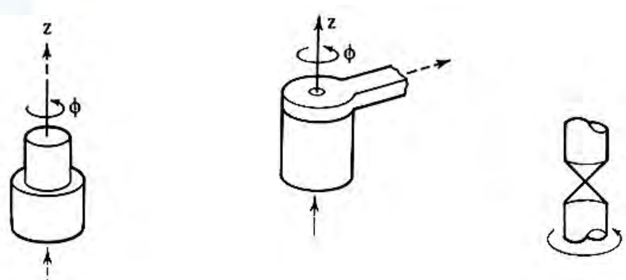
Figure 10: Forearm

Revolute Robotic Joints

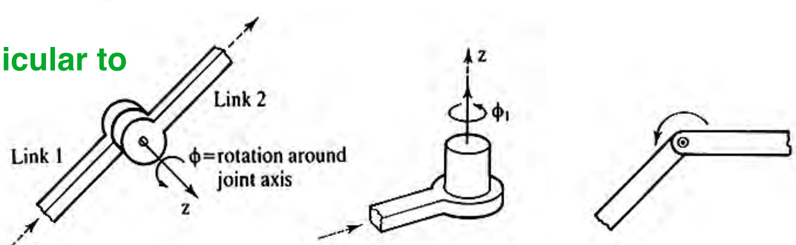


Rotation about a single axis

Parallel to Link

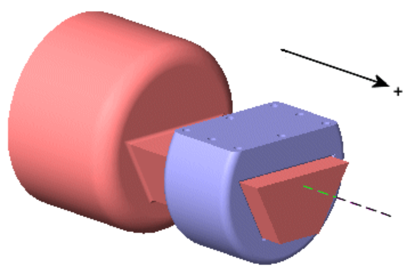


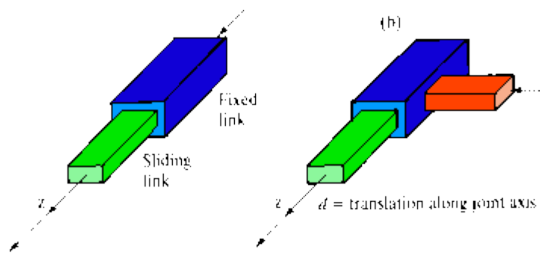
Perpendicular to Link



Prismatic Robotic Joints

Sliding along a single axis

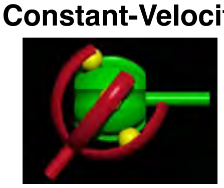




Universal

Other Robotic Joints

Constant-Velocity

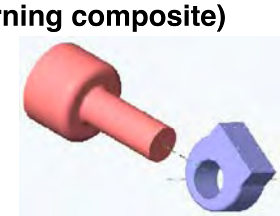


Flexible

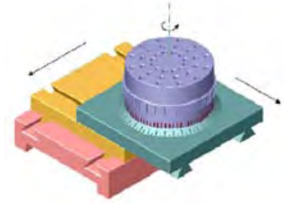
Spherical (or ball)

Roller Screw

Cylindrical (sliding and turning composite)



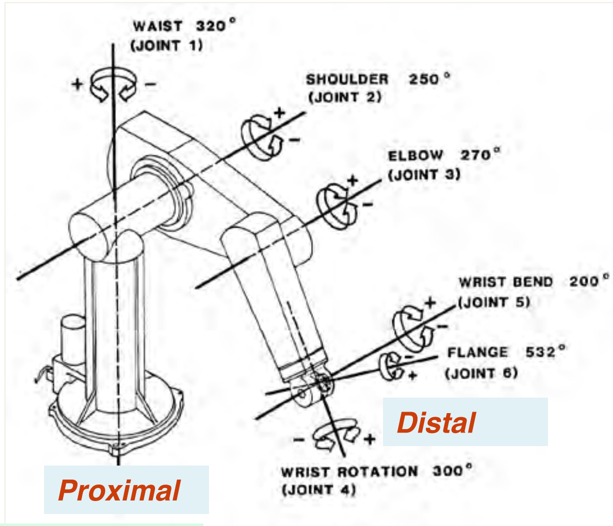
Planar (sliding and turning composite)



Characteristic Transformation of a Link

Link: solid structure between two joints

- **Each link type has a characteristic transformation matrix relating the proximal joint to the distal joint**
- **Link *n* has**
  - **Proximal end: Joint *n*, coordinate frame *n* – 1**
  - ***Distal end*: Joint *n* + 1, coordinate frame *n***

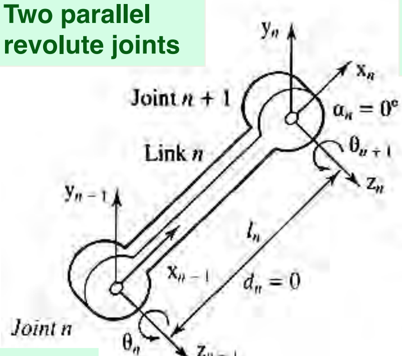


McKerrow, 1991

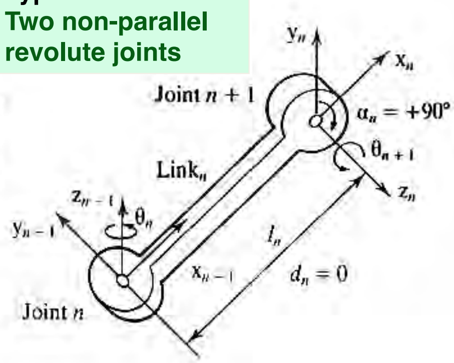
Links Between Revolute Joints

- **Link: solid structure between two joints**
  - **Proximal end: closer to the base**
  - **Distal end: farther from the base**
- **4 Link Parameters**
  - **Length of the link between rotational axes, *l*, along the common normal**
  - **Twist angle between axes, *α***
  - **Angle between 2 links, *θ* (revolute)**
  - **Offset between links, *d* (prismatic)**
- **Joint Variable: single link parameter that is free to vary**

Type 1 Link
Two parallel revolute joints

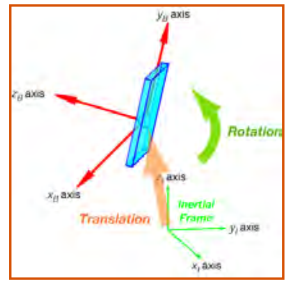


Type 2 Link
Two non-parallel revolute joints



McKerrow, 1991

Homogeneous Transformation Matrix



Express rotation and translation in a single transformation

$$\mathbf{s}_{new} = \left[\begin{array}{c|c} \begin{pmatrix}\text{Rotation}\\ \text{Matrix}\end{pmatrix}_{old}^{new} & \begin{pmatrix}\text{Location}\\ \text{of Old}\\ \text{Origin}\end{pmatrix}_{new} \\ \hline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array}\right] \mathbf{s}_{old} = \mathbf{A}_{old}^{new}\ \mathbf{s}_{old}$$

$$(4\times 1)_{new} = \left[\begin{array}{c|c} (3\times 3) & (3\times 1) \\ \hline (1\times 3) & (1\times 1) \end{array}\right] (4\times 1)_{old} = \left[(4\times 4)\right](4\times 1)_{old}$$

Homogeneous Transformation

- **Rotation <u>and</u> translation can be expressed in terms of homogeneous coordinates**
  - **Single matrix-vector product produces rotation and transformation**

$$\mathbf{s}_{new} = \begin{bmatrix} H_{old}^{new} & \mathbf{r}_{old_{new}} \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix} \mathbf{s}_{old} = \mathbf{A}\ \mathbf{s}_{old}$$

- **or**

$$\begin{bmatrix} x \\ y \\ x \\ 1 \end{bmatrix}_{new} = \left[\begin{array}{ccc|c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{old}$$

Equivalent Scalar Equations for Homogeneous Transformation

$$\mathbf{s}_{new} = \mathbf{A}_{old}^{new}\ \mathbf{s}_{old}$$

Matrix-Vector Multiplication

$$\begin{bmatrix} x \\ y \\ x \\ 1 \end{bmatrix}_{new} = \left[\begin{array}{ccc|c} h_{11} & h_{12} & h_{13} & x_o \\ h_{21} & h_{22} & h_{23} & y_o \\ h_{31} & h_{32} & h_{33} & z_o \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{old}$$

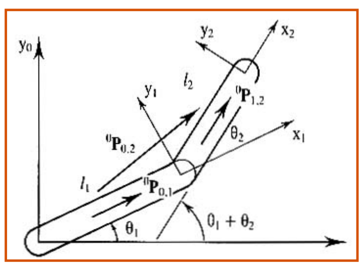
Individual Operations

$$x_{new} = h_{11}x_{old} + h_{12}y_{old} + h_{13}z_{old} + x_o$$
$$y_{new} = h_{21}x_{old} + h_{22}y_{old} + h_{23}z_{old} + y_o$$
$$z_{new} = h_{31}x_{old} + h_{32}y_{old} + h_{33}z_{old} + z_o$$
$$----$$
$$1 = 1$$

Series of Homogeneous Transformations

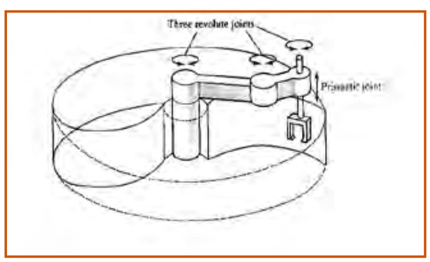
Two serial transformations can be combined in a single transformation

$$\mathbf{s}_2 = \mathbf{A}_1^2\mathbf{A}_0^1\ \mathbf{s}_0 = \mathbf{A}_0^2\ \mathbf{s}_0$$



Four transformations for SCARA robot

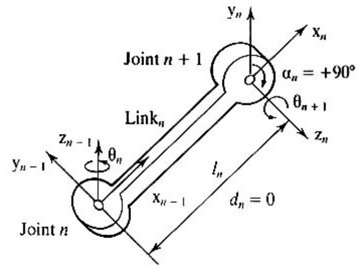
$$\mathbf{s}_4 = \mathbf{A}_3^4\mathbf{A}_2^3\mathbf{A}_1^2\mathbf{A}_0^1\ \mathbf{s}_0 = \mathbf{A}_0^4\ \mathbf{s}_0$$



Transformation for a Single Robotic Joint

- **Each joint requires four sequential transformations:**
  - **Rotation about α**
  - **Translation along d**
  - **Translation along l**
  - **Rotation about θ**

Type 2 Link

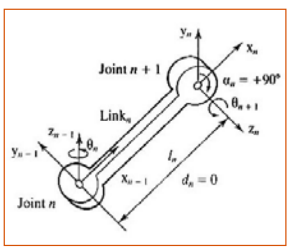


$$\mathbf{s}_{n+1} = \mathbf{A}_3^{n+1}\mathbf{A}_2^{3}\mathbf{A}_1^{2}\mathbf{A}_n^{1}\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$
$$= \mathbf{A}_\theta\mathbf{A}_d\mathbf{A}_l\mathbf{A}_\alpha\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$

- **... axes for each transformation (along or around) must be specified**

4th | 3rd | 2nd | 1st

$$\mathbf{s}_{n+1} = \mathbf{A}(z_{n-1},\theta_n)\mathbf{A}(z_{n-1},d_n)\mathbf{A}(x_{n-1},l_n)\mathbf{A}(x_{n-1},\alpha_n)\,\mathbf{s}_n = \mathbf{A}_n^{n+1}\,\mathbf{s}_n$$



Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

Then | **Then** | **Then** | **First**

$$\mathbf{A}_n = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A}_n = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward and Inverse Transformations

Forward transformation through links requires pre-multiplication of matrices

$$\mathbf{s}_1 = \mathbf{A}_0^1 \mathbf{s}_0 \quad ; \quad s_2 = \mathbf{A}_1^2 \mathbf{s}_1 = \mathbf{A}_1^2 \mathbf{A}_0^1 \mathbf{s}_0 = \mathbf{A}_0^2 \mathbf{s}_0$$

Reverse transformation uses the matrix inverse

$$\mathbf{s}_0 = \left(\mathbf{A}_0^2\right)^{-1} \mathbf{s}_2 = \mathbf{A}_2^0 \mathbf{s}_2 = \mathbf{A}_1^0 \mathbf{A}_2^1 \mathbf{s}_2$$

Homogeneous Transformation Matrix is not Orthonormal

$$\mathbf{A}_2^0 = \left(\mathbf{A}_0^2\right)^{-1} \neq \left(\mathbf{A}_0^2\right)^T$$

…but a useful identity makes inversion simple

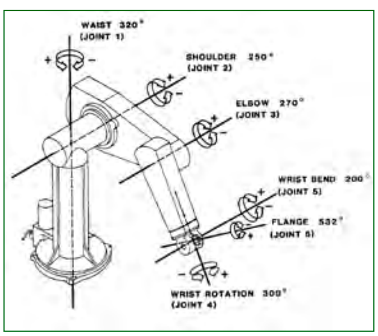
Matrix Inverse Identity

Forward transformation

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{old}^{new} & \mathbf{r}_o \\ \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{bmatrix}$$

Inverse

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix}^{-1} = \left[\begin{array}{c|c} \mathbf{H}_{new}^{old} & -\mathbf{H}_{new}^{old}\mathbf{r}_o \\ \hline \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} & 1 \end{array}\right]$$



Manipulator Maneuvering Spaces

Joint space: Vector of joint variables, e.g.,

$$\mathbf{r}_J = \begin{bmatrix} \theta_{waist} & \theta_{shoulder} & \theta_{elbow} & \theta_{wrist-bend} & \theta_{flange} & \theta_{wrist-twist} \end{bmatrix}^T$$

End-effecter space: Vector of end-effecter positions, e.g.,

$$\mathbf{r}_E = \begin{bmatrix} x_{tool} & y_{tool} & z_{tool} & \psi_{tool} & \theta_{tool} & \phi_{tool} \end{bmatrix}^T$$

Task space: Vector of task-dependent positions, e.g., locating a symmetric grinding tool above a horizontal surface:

$$\mathbf{r}_T = \begin{bmatrix} x_{tool} & y_{tool} & z_{tool} & \psi_{tool} & \theta_{tool} \end{bmatrix}^T$$

Forward and Inverse Transformations of a Robotic Assembly

Forward Transformation

Transforms homogeneous coordinates from tool frame to reference frame coordinates

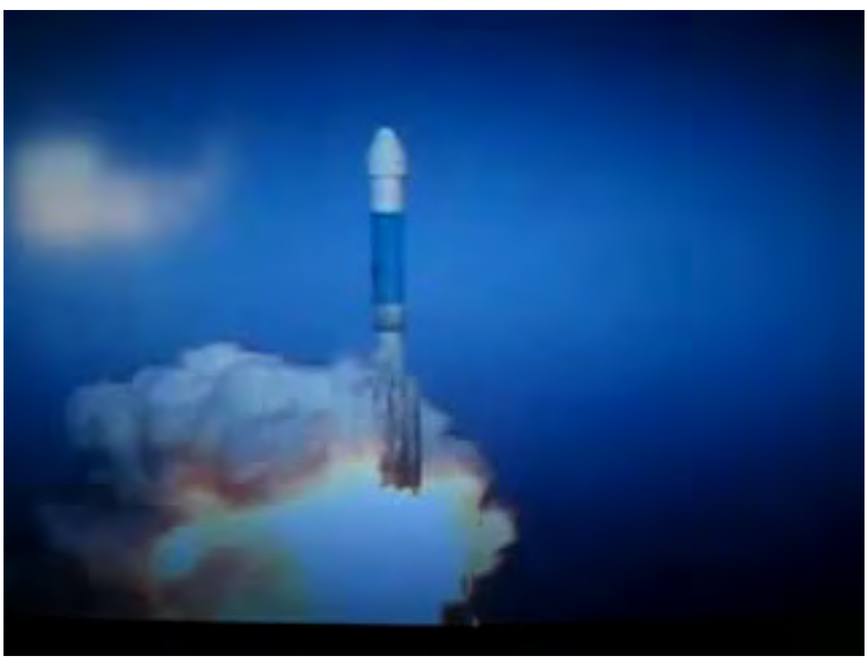
$$s_{base} = \mathbf{A}_{tool}^{base}\mathbf{s}_{tool}$$
$$= \mathbf{A}_{waist}\mathbf{A}_{shoulder}\mathbf{A}_{elbow}\mathbf{A}_{wrist-bend}\mathbf{A}_{flange}\mathbf{A}_{wrist-twist}\mathbf{s}_{tool}$$

Inverse Transformation

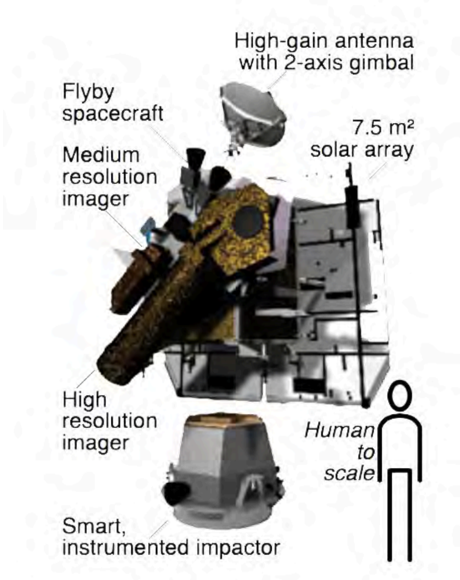
Transform homogeneous coordinate from reference frame to tool frame coordinates

$$s_{tool} = \mathbf{A}_{base}^{tool}\mathbf{s}_{base}$$
$$= \mathbf{A}^{-1}_{wrist-twist}\mathbf{A}^{-1}_{flange}\mathbf{A}^{-1}_{wrist-bend}\mathbf{A}^{-1}_{elbow}\mathbf{A}^{-1}_{shoulder}\mathbf{A}^{-1}_{waist}\mathbf{s}_{base}$$

Deep Impact 1



Deep Space 1 Flyby and Impactor Spacecraft



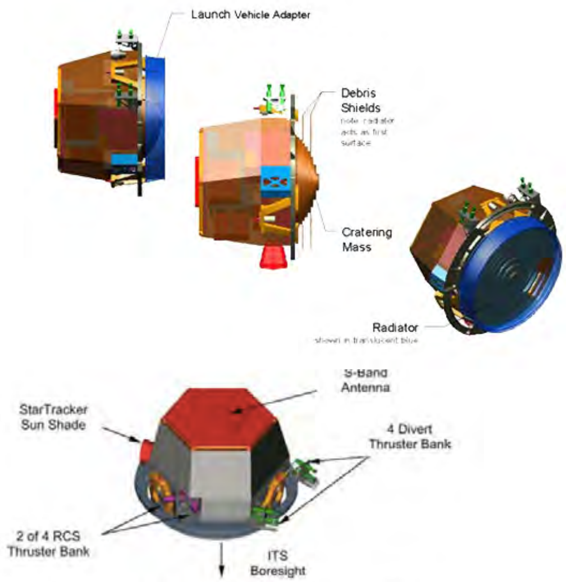
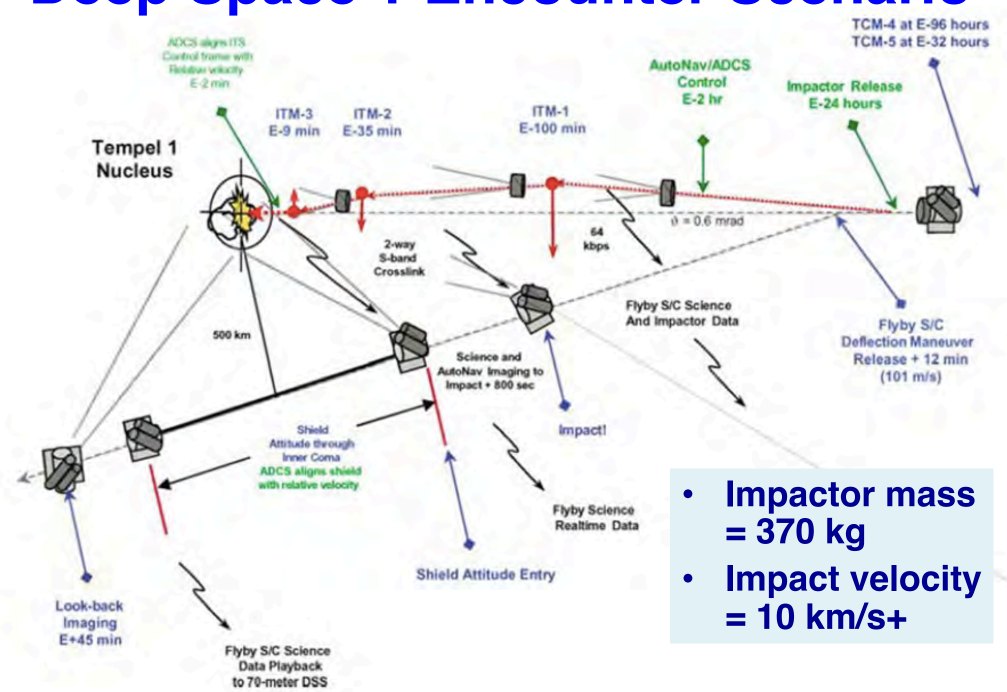


Figure 3. Impactor spacecraft flight system configuration.

Deep Space 1 Encounter Scenario



- **Impactor mass = 370 kg**
- **Impact velocity = 10 km/s+**

Impactor Characteristics

- Image data volume:
  - 17 MB (~35 images)
- Pointing accuracy:
  - $3\sigma = 2$ mrad
- Energy storage for 24-hr mission:
  - 28 KWh
- Propulsion/RCS:
  - 25 m/s divert, 1750 N-s impulse

Deep Space 1 Autonomous Navigation and Targeting

- Flyby spacecraft comes within 500 km of Tempel 1comet, observes impact event for 800 s
- Impactor guided toward comet nucleus by *AutoNav* software (s/w)
- Identical attitude determination and control *(ADCS)* s/w in Flyby and Impactor
- Impactor Targeting Sensor *(ITS)* commands images every 15 sec
- *"Scripted Autonomy"*

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Flyby Flight Control System

- Two RAD750 computers
- Medium Resolution Imager *(MRI)* for autonomous navigation during encounter
- High Resolution Imager *(HRI)* for approach phase optical navigation
- Power to both s/c while attached
- High-gain antenna for data return
- S-band antenna for communication with Impactor
- 3-axis momentum wheel attitude control
- 4 RCS thrusters for divert and momentum dumps
- Star Tracker/Inertial Reference Unit *(SSIRU)*

Mastrodemos, Kubitschek, Synnott, JPL, 2005

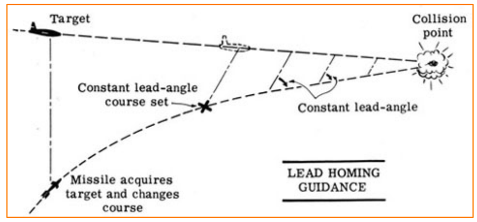
Impactor Flight Control System

- Battery power for 24-hr operation
- One RAD750 computer
- SSIRU
- Simple ITS
- Divert/RCS thrusters
- *AutoNav* components
  - Image processing
  - Orbit determination
  - Maneuver computation

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Impactor Targeting Strategy

- <u>Options</u>
  - Proportional Navigation
    - Measurement of closing velocity and line-of-sight angular rates
    - “Reduced dynamic approach”
  - Predictive Guidance
    - Equations of motion for target and interceptor
    - State estimation (“filtering”), like *SLAM*



- <u>Selected</u>: Predictive, Pulsed Guidance to desired location on target
- Best quality observations obtained during non-thrusting periods

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Impactor Targeting Strategy

- Large uncertainty in knowledge of prior position reduced via optical navigation
- Nucleus rotation rate and solar phase angle induce motion in center of brightness ***(CB)***
- Mitigated by batch filtering process and selection of 20-min data arc length

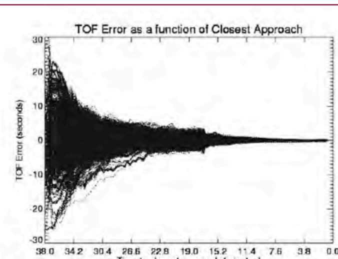


Figure 14. The decrease in the TOF error as a function of time to closest approach of the Flyby s/c from ~4,500 Monte Carlo simulation runs. The initial conditions simulated a 10 s (1σ) TOF error.

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Autonomous Navigation

- *AutoNav* modes
  - Star-relative mode
  - Star-less mode based on *ADCS* only and camera alignment
- Autonomous guidance process
  1) T_{impact} – 2 hr: Acquire comet nucleus images every 15 sec
  2) Compute pixel/line location of *CB*
  3) Compute measurement error
  4) Perform trajectory observation *(OD)* updates every min
  5) Perform 3 Impactor targeting maneuvers *(ITM)*, at T_{impact} – 100 min, T_{impact} – 35 min, T_{impact} – 7.5 min
  6) Compute scene-analysis-based offset prior to 3rd *ITM*
  7) T_{impact} – 4 min: Point *ITS* along estimated comet-relative velocity vector
  8) Flyby maintains tracking of impact point

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Trajectory Determination

- Updated once per minute for last 2 hr
- Sequential batch processing updates position, velocity, navigation line-of-sight attitude bias drift errors
- 80 observations in each 20-min arc
- Time series of Impactor position relative to the nucleus predicted with *Chebyshev polynomial* (i.e., smooth extrapolation of prior position estimates)

Mastrodemos, Kubitschek, Synnott, JPL, 2005

Impactor Targeting Maneuvers *(ITM)*

- Initiated via *AutoNav* sequence command
- Required impulsive maneuver ΔV magnitude and direction calculated
- Finite-burn start time computed, t_{start}
- Integration of accelerometer data indicates when ΔV has been reached, and burn is terminated, t_{finish}
- *ITM-1* removes Flyby pre-release delivery errors (~6 km), requiring $\Delta V \sim 1$ *m/s*
- *ITM-2* improves targeting, requiring $\Delta V \sim 11$ *cm/s*
- *ITM-3* refines fine targeting of illuminated nucleus based on *CB* observations, requiring $\Delta V \sim 7$*m/s*
  - B-plane correction of as much as 4 km
  - Accurate to ~ 54 m

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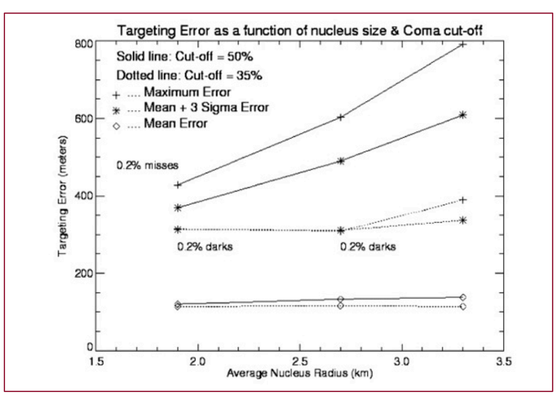
Expected Targeting Error



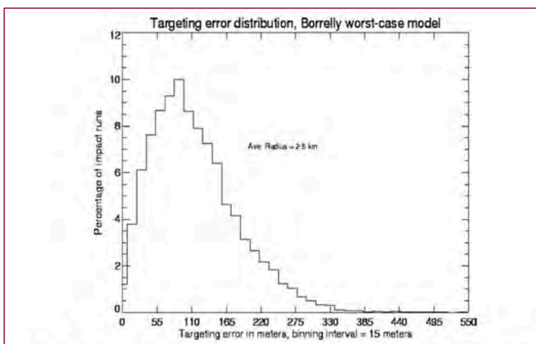


Figure 12. Distribution of targeting errors, which represent a typical example of the skewed distribution seen for all nucleus models examined. Impactor misses occur at the tail-end of the distribution.

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Expected Targeting Error

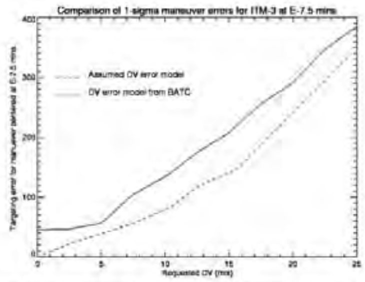


TABLE II
Impactor CB targeting results.

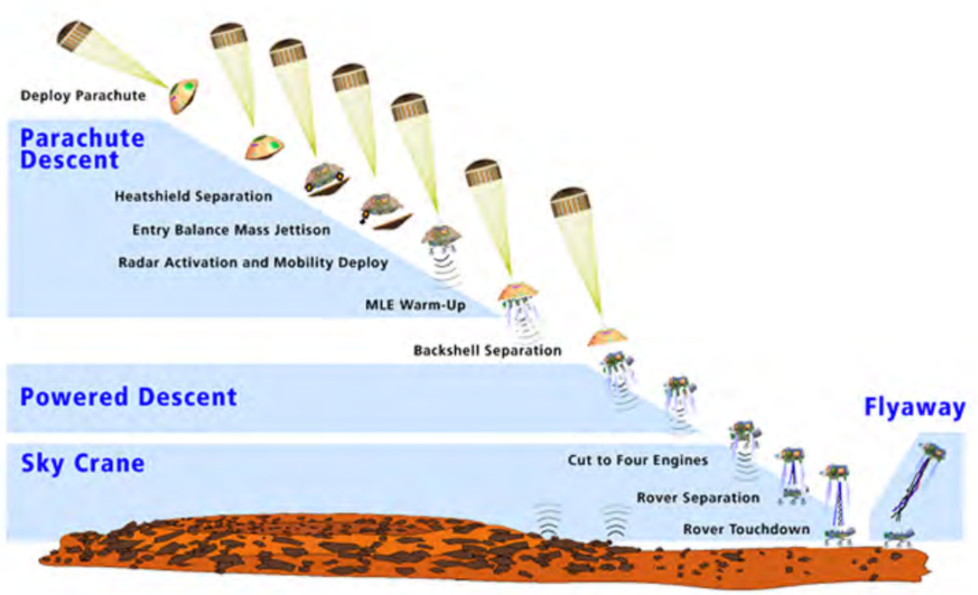
Nucleus model	Probability of illuminated impact (%)	Dark impact probability (%)	Probability of a miss (%)	Approximate number of simulations
Halley	99.8	0.2	0	1,500
Borrelly–worst case	90.75	6.75	2.75	9,000
Accretion	65	35	0.02	700

Mastrodemos, Kubitschek, Synnott, JPL, 2005

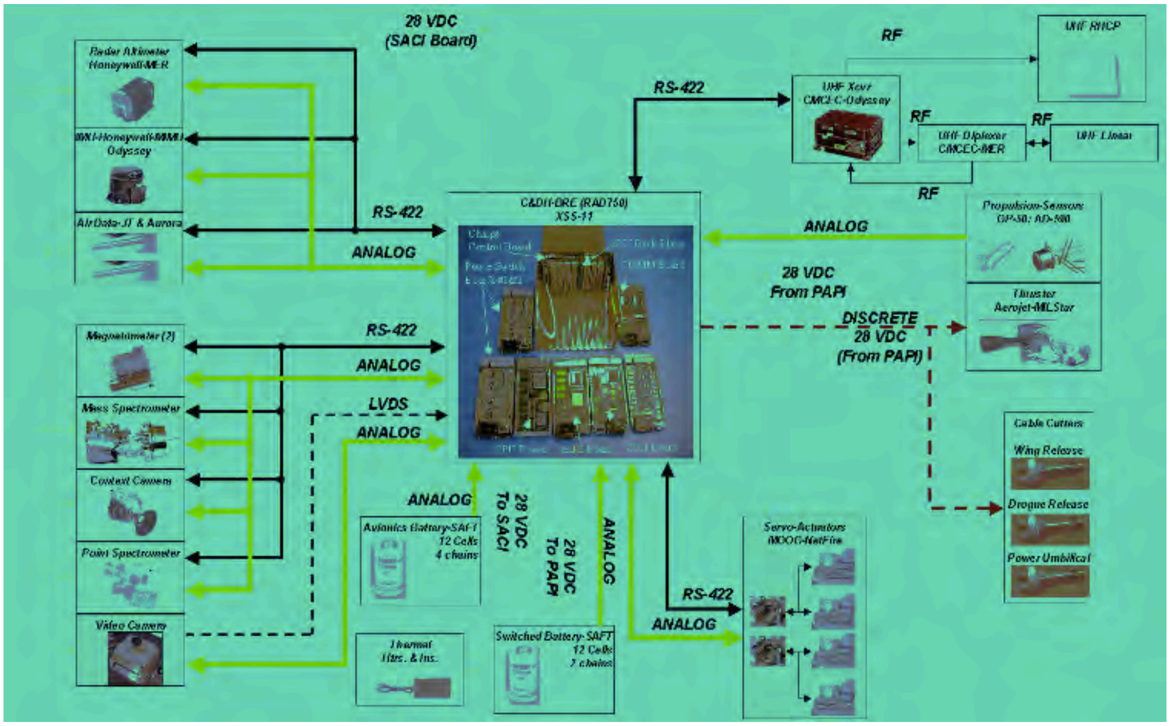
Next Time: Human Factors of Spaceflight

Supplemental Material

MSL Descent

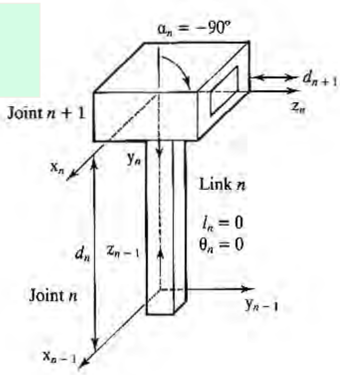


ARES Data Handling and Processing System

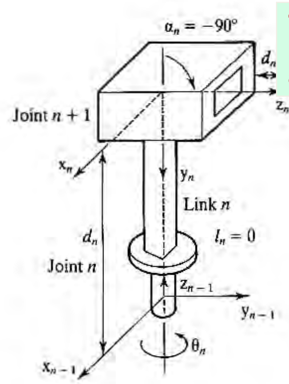


Links Involving Prismatic Joints

Type 5 Link
Intersecting prismatic joints



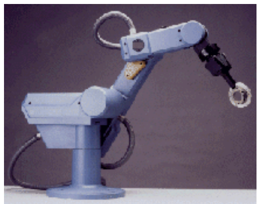
Type 6 Link
Intersecting revolute and prismatic joints



- **Link n extends along z_{n-1} axis**
  - **l_n = 0, along x_{n-1}**
  - **d_n = length, along z_{n-1} (variable)**
  - **θ_n = 0, about z_{n-1}**
  - **α_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

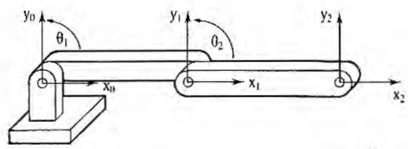
- **Link n extends along z_{n-1} axis**
  - **l_n = 0, along x_{n-1}**
  - **d_n = length, along z_{n-1} (fixed)**
  - **θ_n = variable joint angle n about z_{n-1}**
  - **α_n = fixed orientation of $n + 1$ prismatic axis about x_{n-1}**

McKerrow, 1991

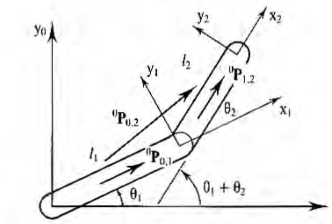


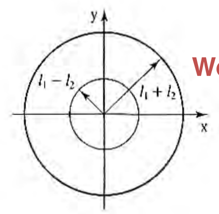
Two-Link/Three-Joint Manipulator

Parallel Rotation Axes



Manipulator in zero position





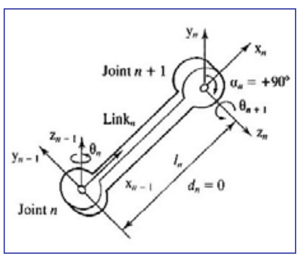
Workspace

Assignment of coordinate frames

Parameters and Variables for 2-link manipulator

- Link lengths (fixed)
- Joint angles (variable)

McKerrow, 1991



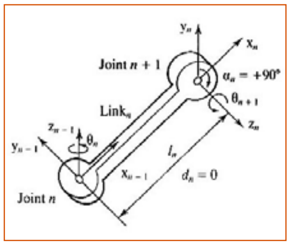
Denavit-Hartenberg Representation of Joint-Link-Joint Transformation

- **Like Euler angle rotation, transformational effects of the 4 link parameters are defined in a specific application sequence (right to left): {*θ, d, l, α*}**

- **4 link parameters**
  - **Angle between 2 links, *θ* (revolute)**
  - **Distance (offset) between links, *d* (prismatic)**
  - **Length of the link between rotational axes, *l*, along the common normal (prismatic)**
  - **Twist angle between axes, *α* (revolute)**

$$
\begin{aligned}
\mathbf{A}_n &= \mathbf{A}(z_{n-1},\theta_n)\mathbf{A}(z_{n-1},d_n)\mathbf{A}(x_{n-1},l_n)\mathbf{A}(x_{n-1},\alpha_n) \\
&= \mathrm{Rot}(z_{n-1},\theta_n)\ \mathrm{Trans}(z_{n-1},d_n)\ \mathrm{Trans}(x_{n-1},l_n)\ \mathrm{Rot}(x_{n-1},\alpha_n) \\
&\triangleq {}^n\mathbf{T}_{n+1} \quad \text{in some references (e.g., McKerrow, 1991)}
\end{aligned}
$$

Denavit-Hartenberg Demo
http://www.youtube.com/watch?v=10mUtjfGmzw



Four Transformations from One Joint to the Next (Single Link)

Rotation of θ_n about the z_{n-1} axis

$$\mathrm{Rot}(z_{n-1},\theta_n) = \begin{bmatrix} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of d_n along the z_{n-1} axis

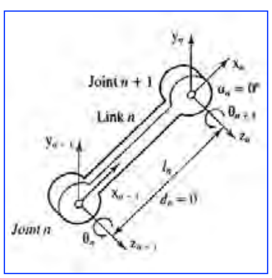
$$\mathrm{Trans}(z_{n-1},d_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation of l_n along the x_{n-1} axis

$$\mathrm{Trans}(x_{n-1},l_n) = \begin{bmatrix} 1 & 0 & 0 & l_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation of α_n about the x_{n-1} axis

$$\mathrm{Rot}(x_{n-1},\alpha_n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Example: Denavit-Hartenberg Representation of Joint-Link-Joint Transformation for Type 1 Link

Joint Variable = θ_n

$$\mathbf{A}_n = \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & l_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & l_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ = variable
$d = 0$ m
$l = 0.25$ m
$\alpha = 90$ deg

$$\mathbf{A}_n = \begin{bmatrix} \cos\theta_n & 0 & \sin\theta_n & 0.25\cos\theta_n \\ \sin\theta_n & 0 & -\cos\theta_n & 0.25\sin\theta_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta \triangleq 30$ deg
$d = 0$ m
$l = 0.25$ m
$\alpha = 90$ deg

$$\mathbf{A}_n = \begin{bmatrix} 0.866 & 0 & 0.5 & 0.217 \\ 0.5 & 0 & -0.866 & 0.125 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix Inverse Identity

Given: a square matrix, A, and its inverse, B

$$\mathbf{A} = \left[\begin{array}{c|c} \underset{m\times m}{\mathbf{A}_1} & \underset{m\times n}{\mathbf{A}_2} \\ \hline \underset{n\times m}{\mathbf{A}_3} & \underset{n\times n}{\mathbf{A}_4} \end{array}\right] \quad ; \quad \mathbf{B} \triangleq \mathbf{A}^{-1} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix}$$

Then

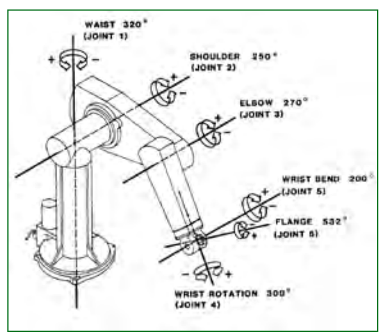
$$\mathbf{A}\,\mathbf{B} = \mathbf{A}\,\mathbf{A}^{-1} = \mathbf{I}_{m+n}$$

$$= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n \end{bmatrix} = \begin{bmatrix} (\mathbf{A}_1\mathbf{B}_1 + \mathbf{A}_2\mathbf{B}_3) & (\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_4) \\ (\mathbf{A}_3\mathbf{B}_1 + \mathbf{A}_4\mathbf{B}_3) & (\mathbf{A}_3\mathbf{B}_2 + \mathbf{A}_4\mathbf{B}_4) \end{bmatrix}$$

Equating like parts, and solving for B_i

$$\begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{B}_3 & \mathbf{B}_4 \end{bmatrix} = \left[\begin{array}{c|c} \left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & -\mathbf{A}_1^{-1}\mathbf{A}_2\left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \\ \hline -\mathbf{A}_4^{-1}\mathbf{A}_3\left(\mathbf{A}_1 - \mathbf{A}_2\mathbf{A}_4^{-1}\mathbf{A}_3\right)^{-1} & \left(\mathbf{A}_4 - \mathbf{A}_3\mathbf{A}_1^{-1}\mathbf{A}_2\right)^{-1} \end{array}\right]$$

Forward and Inverse Kinematics Between Joints, Tool Position, and Tool Orientation



Forward Kinematic Problem: Compute the position of the tool in the reference frame that corresponds to a given joint vector (i.e., vector of link variables)

$$s_{base} = \mathbf{A}_{waist}\mathbf{A}_{shoulder}\mathbf{A}_{elbow}\mathbf{A}_{wrist-bend}\mathbf{A}_{flange}\mathbf{A}_{wrist-twist}\mathbf{s}_{tool} = \mathbf{A}_{tool}^{base}\mathbf{s}_{tool}$$

$$\text{To Be Determined} \Leftarrow \text{Given}$$

Inverse Kinematic Problem: Find the vector of link variables that corresponds to a desired task-dependent position

$$\mathbf{A}_{waist}\mathbf{A}_{shoulder}\mathbf{A}_{elbow}\mathbf{A}_{wrist-bend}\mathbf{A}_{flange}\mathbf{A}_{wrist-twist}\mathbf{s}_{tool} = \mathbf{A}_{tool}^{base}\mathbf{s}_0 = s_{base}$$

$$\text{To Be Determined} \Leftarrow \text{Given}$$