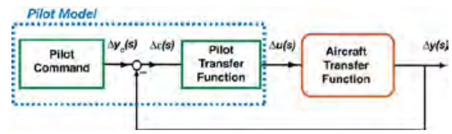
Advanced Problems of Longitudinal Dynamics

Robert Stengel, Aircraft Flight Dynamics
MAE 331, 2014

- Angle-of-attack-rate aero effects
- Fourth-order dynamics
  - Steady-state response to control
  - Transfer functions
  - Frequency response
  - Root locus analysis of parameter variations
- Nichols chart
- Pilot-aircraft interactions



Flight Dynamics
204-206, 503-525
Airplane Stability and Control
Chapter 10

Primary and Coupling Blocks of the Fourth-Order Longitudinal Model

$$\mathbf{F}_{Lon} = \left[\begin{array}{cc|cc} -D_V & -g & 0 & -D_\alpha \\ L_V/V_N & 0 & 0 & L_\alpha/V_N \\ \hline M_V & 0 & M_q & M_\alpha \\ -L_V/V_N & 0 & 1 & -L_\alpha/V_N \end{array}\right] = \begin{bmatrix} \mathbf{F}_{Ph} & \mathbf{F}_{SP}^{Ph} \\ \mathbf{F}_{Ph}^{SP} & \mathbf{F}_{SP} \end{bmatrix}$$

- Some stability derivatives appear only in primary blocks (D_V, M_q, M_α)
  - Effects are well-described by 2nd-order models
- Some stability derivatives appear only in coupling blocks (M_V, D_α)
  - Effects are ignored by 2nd-order models
- Some stability derivatives appear in both (L_V, L_α)
  - May require 4th-order modeling

How do the 4th-order roots vary when we change pitch-rate damping, M_q?

Identify M_q terms in the characteristic polynomial

$$
\begin{aligned}
\Delta_{Lon}(s) = {} & s^4 + \left(D_V + L_\alpha / V_N\right)s^3 + \left[\left(g - D_\alpha\right) L_V / V_N + D_V\left(L_\alpha / V_N\right) - M_\alpha\right]s^2 \\
& + \left\{D_\alpha M_V - D_V M_\alpha\right\}s + g\left(M_V L_\alpha / V_N - M_\alpha L_V / V_N\right) \\
& - M_q s^3 - \left[D_V\left(M_q\right) + M_q L_\alpha / V_N\right]s^2 + M_q\left[\left(D_\alpha - g\right) L_V / V_N - D_V L_\alpha / V_N\right]s \\
& = 0
\end{aligned}
$$

How do the 4th-order roots vary when we change pitch-rate damping, M_q?

Group M_q terms in the characteristic polynomial

$$
\begin{aligned}
\Delta_{Lon}(s) &= d(s) - M_q\left\{s^3 + \left[D_V + L_\alpha / V_N\right]s^2 - \left[\left(D_\alpha - g\right) L_V / V_N - D_V L_\alpha / V_N\right]s\right\} \\
&= d(s) - M_q s\left\{s^2 + \left[D_V + L_\alpha / V_N\right]s - \left[\left(D_\alpha - g\right) L_V / V_N - D_V L_\alpha / V_N\right]\right\} \\
&= d(s) + kn(s) = 0
\end{aligned}
$$

$$
\boxed{k\frac{n(s)}{d(s)} = -1}
$$

How do the 4th-order roots vary when we change pitch-rate damping, M_q?

- Factor terms that are multiplied by M_q to find the 3 zeros
  - 2 zeros near origin similar to approximate phugoid roots, effectively canceling M_q effect on them

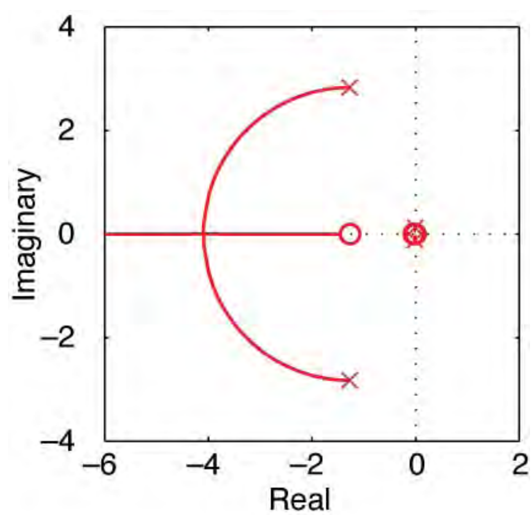
$$-M_q \frac{s\left\{s^2+\left(D_V+L_\alpha/V_N\right)s-\left[\left(D_\alpha-g\right)L_V/V_N - D_V L_\alpha/V_N\right]\right\}}{\left\{\begin{array}{c} s^4+\left(D_V+L_\alpha/V_N\right)s^3+\left[\left(g-D_\alpha\right)L_V/V_N + D_V\left(L_\alpha/V_N\right)-M_\alpha\right]s^2 \\ +\left\{D_\alpha M_V - D_V M_\alpha\right\}s + g\left(M_V L_\alpha/V_N - M_\alpha L_V/V_N\right)\end{array}\right\}} = -1$$

$$-M_q \frac{s\left(s-z_1\right)\left(s-z_2\right)}{\left(s^2+2\zeta_P\omega_{n_P}s+\omega_{n_P}^2\right)\left(s^2+2\zeta_{SP}\omega_{n_{SP}}s+\omega_{n_{SP}}^2\right)} = -1$$

$$s\left(s-z_1\right) = \left(s^2 - z_1 s + 0\right) \approx \left(s^2+2\zeta_P\omega_{n_P}s+\omega_{n_P}^2\right)$$

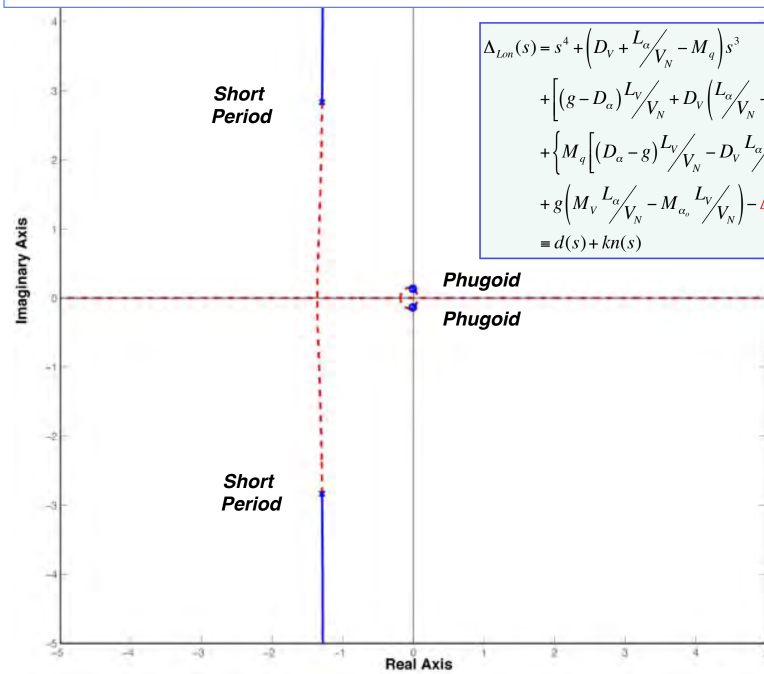
M_q variation has virtually no effect on phugoid roots
Effect on short-period roots is predicted by 2nd-order model

$$-M_q \frac{\left(\cancel{s^2 - z_1 s}\right)\left(s-z_2\right)}{\left(\cancel{s^2+2\zeta_P\omega_{n_P}s+\omega_{n_P}^2}\right)\left(s^2+2\zeta_{SP}\omega_{n_{SP}}s+\omega_{n_{SP}}^2\right)} \simeq \frac{-M_q\left(s-z_2\right)}{\left(s^2+2\zeta_{SP}\omega_{n_{SP}}s+\omega_{n_{SP}}^2\right)} = -1$$



M_α Effect on 4th-Order Roots

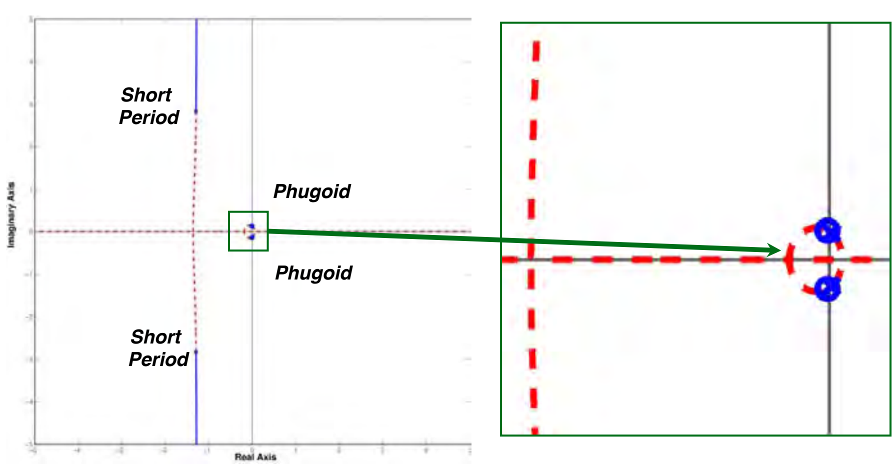
Group all terms multiplied by M_α to form numerator for M_α



$$\begin{aligned}\Delta_{Lon}(s) &= s^4 + \left(D_V + L_\alpha/V_N - M_q\right)s^3 \\ &+ \left[(g - D_\alpha)L_V/V_N + D_V\left(L_\alpha/V_N - M_q\right) - M_q L_\alpha/V_N - M_{\alpha_o}\right]s^2 \\ &+ \left\{M_q\left[(D_\alpha - g)L_V/V_N - D_V L_\alpha/V_N\right] + D_\alpha M_V - D_V M_{\alpha_o}\right\}s \\ &+ g\left(M_V L_\alpha/V_N - M_{\alpha_o} L_V/V_N\right) - \Delta M_\alpha\left(s^2 + D_V s + g L_V/V_N\right) \\ &\equiv d(s) + kn(s)\end{aligned}$$

M_α Effect on 4th-Order Roots

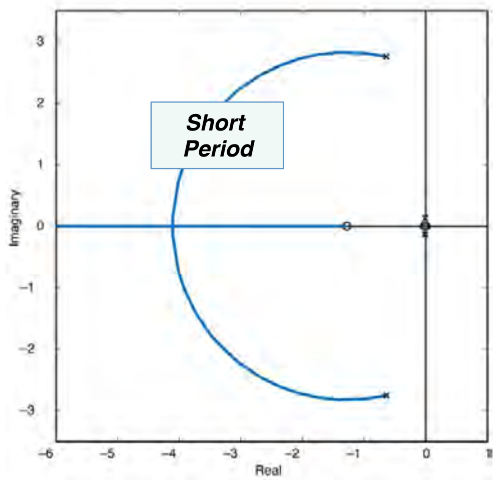
- **Primary effect: Same as the approximate short-period model**
- **Numerator zeros**
  - **Same as the approximate phugoid mode characteristic polynomial**
  - **Effect of M_α variation on phugoid mode is small**

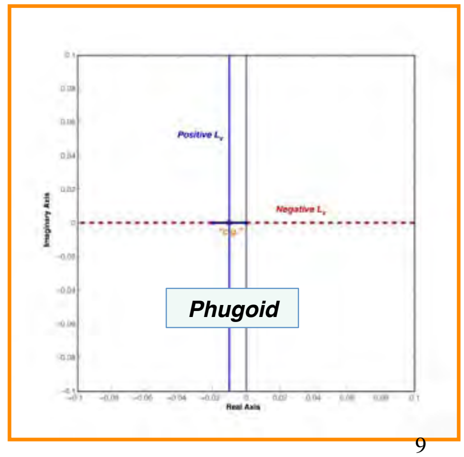


L_α / V_N and L_V / V_N Effects on Fourth-Order Roots

- L_α / V_N: Increased damping of the short-period
- Small effect on the phugoid mode

- L_V / V_N: Damped natural frequency of the phugoid
- Negligible effect on the short-period





Longitudinal Model Transfer Function Matrix ($H_x = I$, $H_u = 0$)

$$
\mathscr{H}_{Lon}(s) = \frac{\begin{bmatrix} n^V_{\delta E}(s) & n^V_{\delta T}(s) & n^V_{\delta F}(s) \\ n^\gamma_{\delta E}(s) & n^\gamma_{\delta T}(s) & n^\gamma_{\delta F}(s) \\ n^q_{\delta E}(s) & n^q_{\delta T}(s) & n^q_{\delta F}(s) \\ n^\alpha_{\delta E}(s) & n^\alpha_{\delta T}(s) & n^\alpha_{\delta F}(s) \end{bmatrix}}{\left(s^2 + 2\zeta_P \omega_{n_P} s + \omega_{n_P}^2\right)\left(s^2 + 2\zeta_{SP} \omega_{n_{SP}} s + \omega_{n_{SP}}^2\right)}
$$

$$
\begin{bmatrix} \Delta V(s) \\ \Delta\gamma(s) \\ \Delta q(s) \\ \Delta\alpha(s) \end{bmatrix} = \mathbf{H_x A}(s)\mathbf{G} \begin{bmatrix} \Delta\delta E(s) \\ \Delta\delta T(s) \\ \Delta\delta F(s) \end{bmatrix} = \mathscr{H}_{Lon}(s) \begin{bmatrix} \Delta\delta E(s) \\ \Delta\delta T(s) \\ \Delta\delta F(s) \end{bmatrix}
$$

Elevator-to-Normal-Velocity Transfer Function

$$\frac{\Delta w(s)}{\Delta\delta E(s)} = \frac{n_{\delta E}^{w}(s)}{\Delta_{Lon}(s)} = \frac{M_{\delta E}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Approx\ Ph}\left(s - z_3\right)}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Ph}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{SP}}$$

- **Normal velocity transfer function is analogous to angle of attack transfer function ($\Delta\alpha \approx \Delta w/V_N$)**
- **z_3 often neglected due to high frequency**

Transfer Functions of Elevator Input to Angle Output*

- **Elevator-to-Flight Path Angle transfer function**

$$\frac{\Delta\gamma(s)}{\Delta\delta E(s)} = \frac{n_{\delta E}^{\gamma}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta E}^{\gamma}(s) = M_{\delta E}\frac{L_\alpha}{V_N}\left(s + 1/T_{\gamma_1}\right)$$

- **Elevator-to-Angle of Attack transfer function**

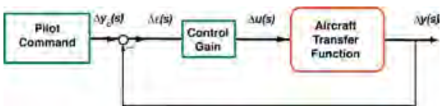
$$\frac{\Delta\alpha(s)}{\Delta\delta E(s)} = \frac{n_{\delta E}^{\alpha}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta E}^{\alpha}(s) = M_{\delta E}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Approx\ Ph}$$

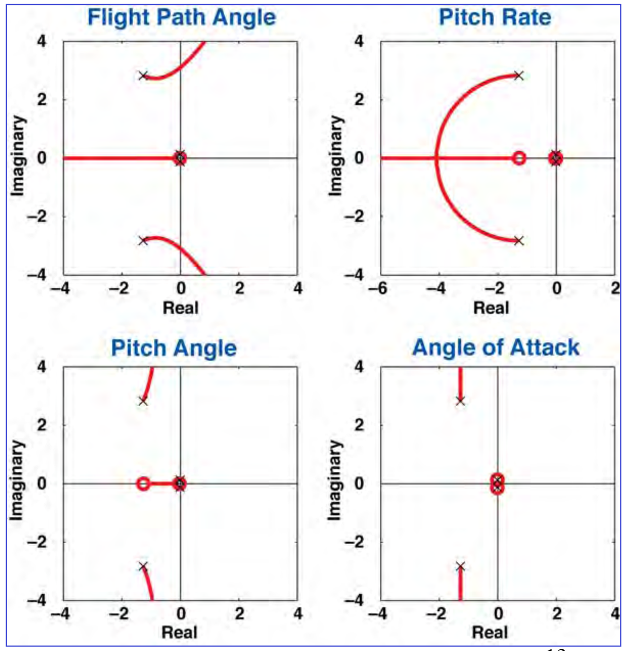
- **Elevator-to-Pitch Angle transfer function**

$$\frac{\Delta\theta(s)}{\Delta\delta E(s)} = \frac{n_{\delta E}^{\theta}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta E}^{\theta}(s) = M_{\delta E}\left(s + 1/T_{\theta_1}\right)\left(s + 1/T_{\theta_2}\right)$$

* Flying qualities notation for zero time constants

Feedback Control: Angles to Elevator



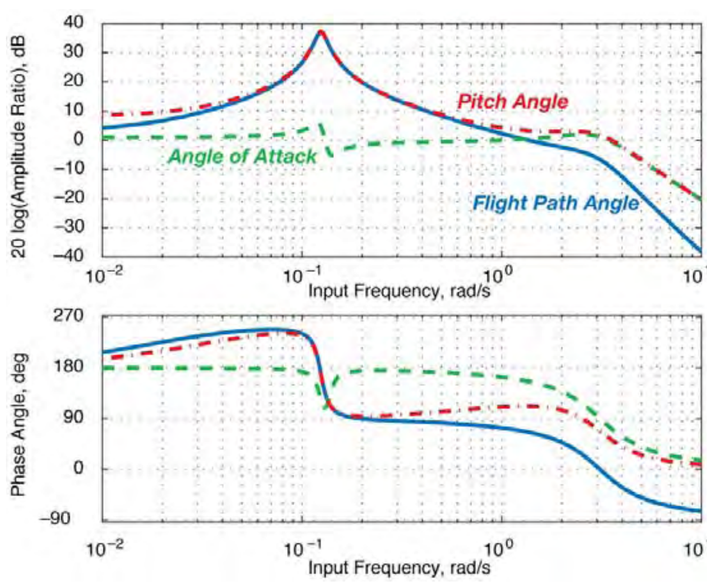


- Variations in control gain
- Principal effect is on short-period roots

Frequency Response of Angles to Elevator Input

- Pitch angle frequency response ($\Delta\theta = \Delta\gamma + \Delta\alpha$)
  - Similar to flight path angle near phugoid natural frequency
  - Similar to angle of attack near short-period natural frequency

$$\Delta\gamma_{SS} = c\Delta\delta E_{SS}$$
$$\Delta\alpha_{SS} = f\Delta\delta E_{SS}$$
$$\Delta\theta_{SS} = (c - f)\Delta\delta E_{SS}$$

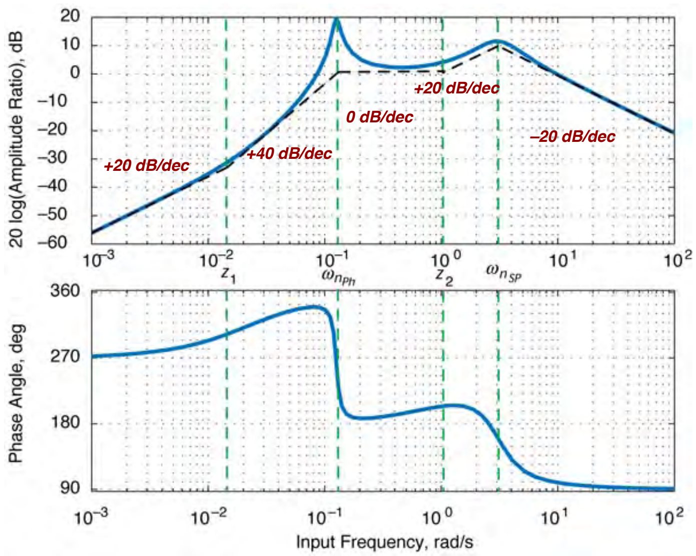


Elevator-to-Pitch-Rate Frequency Response

$$\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{n_{\delta E}^{q}(s)}{\Delta_{Lon}(s)} \approx \frac{M_{\delta E}\, s\left(s - z_1\right)\left(s - z_2\right)}{\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)_{Ph}\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)_{SP}}$$

$$\triangleq \frac{M_{\delta E}\, s\left(s + 1/T_{\theta_1}\right)\left(s + 1/T_{\theta_2}\right)}{\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)_{Ph}\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)_{SP}}$$

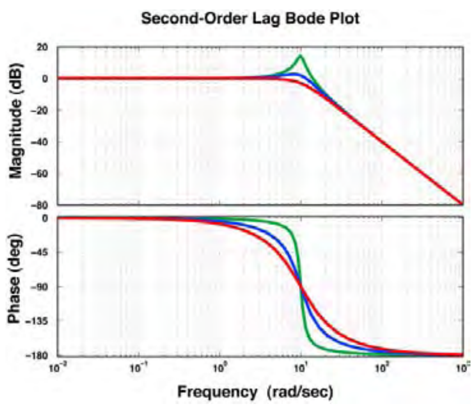
- **$(n - q) = 1$**
- **Negligible low-frequency response, except at phugoid natural frequency**
- **High-frequency response well predicted by 2nd-order model**

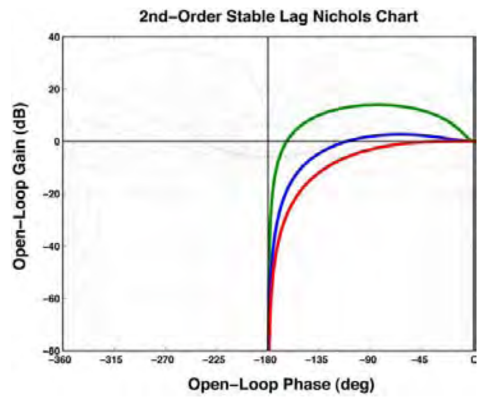


Gain and Phase Margins: The Nichols Chart

Nichols Chart: Gain vs. Phase Angle

- **Bode Plot**
  - Two plots
  - Open-Loop Gain (dB) vs. $\log_{10}\omega$
  - Open-Loop Phase Angle vs. $\log_{10}\omega$
- **Nichols Chart**
  - Single crossplot; input frequency not shown
  - Open-Loop Gain (dB) vs. Open-Loop Phase Angle





Gain and Phase Margins

- **Gain Margin**
  - Measured at the input frequency, ω, for which $\varphi(j\omega) = -180°$
  - Difference between 0 dB and transfer function magnitude, $20 \log_{10} AR(j\omega)$
- **Phase Margin**
  - Measured at the input frequency, ω, for which $20 \log_{10} AR(j\omega) = 0$ dB
  - Difference between the phase angle $\varphi(j\omega)$, and $-180°$

- **Axis intercepts on the Nichols Chart identify GM and PM**

Examples of Gain and Phase Margins: 2nd-Order System with Low-Pass Filter

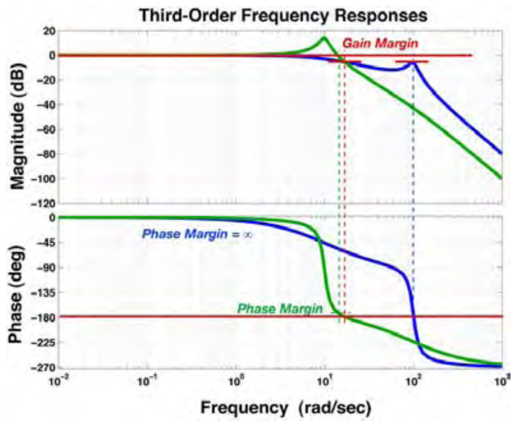
Low-Bandwidth Filter

$$H_{blue}(j\omega) = \left[\frac{10}{(j\omega+10)}\right]\left[\frac{100^2}{(j\omega)^2 + 2(0.1)(100)(j\omega) + 100^2}\right]$$

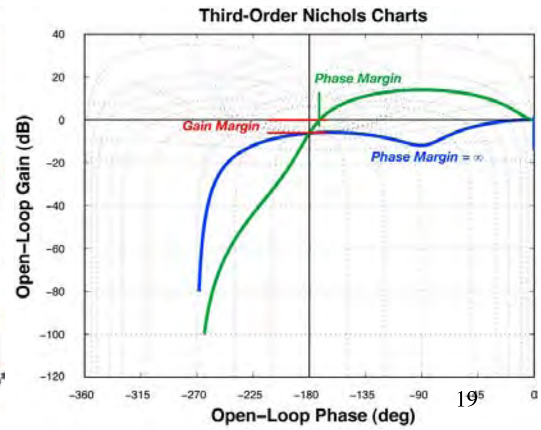
High-Bandwidth Filter

$$H_{green}(j\omega) = \left[\frac{10^2}{(j\omega)^2 + 2(0.1)(10)(j\omega) + 10^2}\right]\left[\frac{100}{(j\omega+100)}\right]$$

Bode Plot

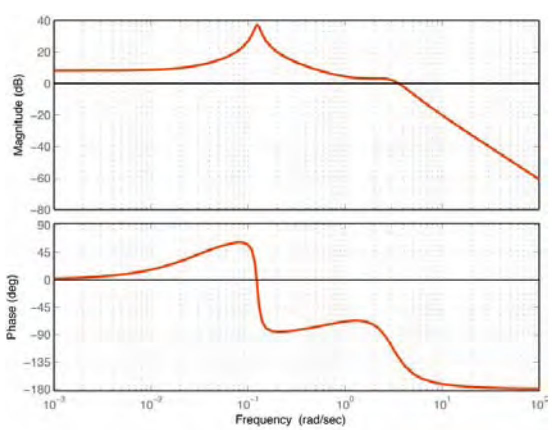


Nichols Chart

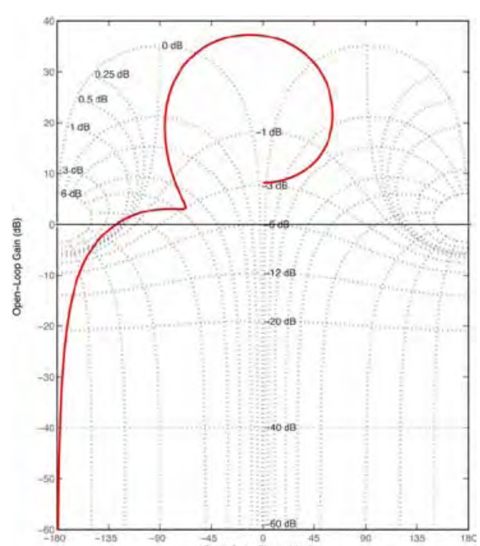


Gain and Phase Margins in Pitch-Tracking Task

Elevator-to-Pitch-Angle Bode Plot



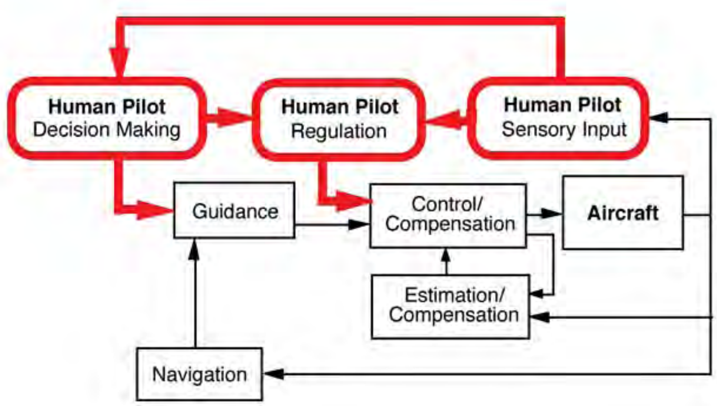
Elevator-to-Pitch-Angle Nichols Chart



- **Gain Margin:** Amplitude ratio below 0 dB when phase angle = 180°
- **Phase Margin:** Phase angle above –180° when amplitude ratio = 0 dB

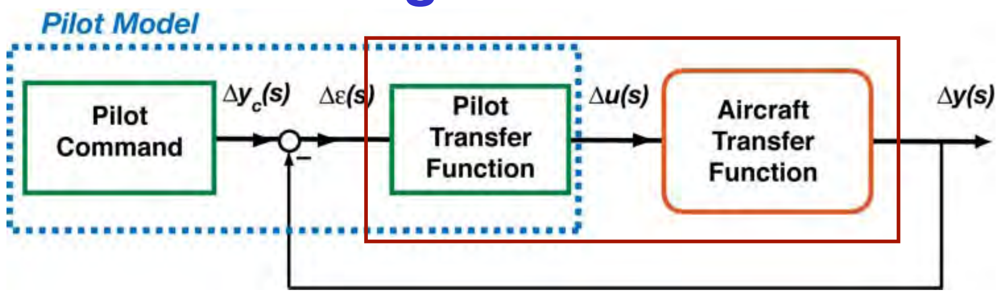
Pilot-Vehicle Interactions

Pilot Inputs to Control



* p. 421-425, *Flight Dynamics*

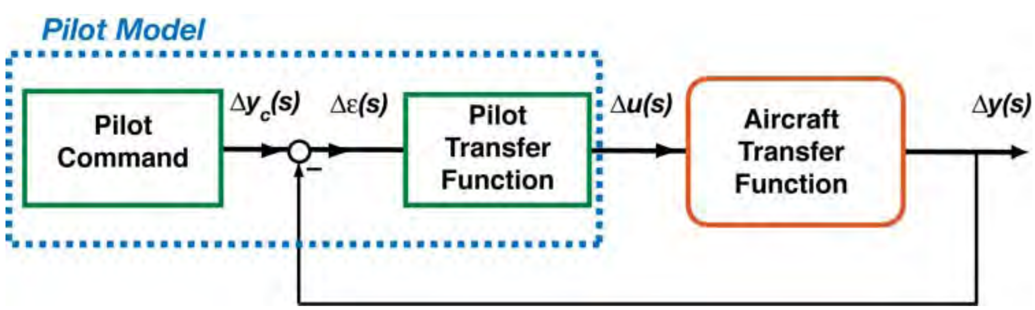
Effect of Pilot Dynamics on Pitch-Angle Control Task



Open-Loop 1st-Order-Pilot/Aircraft Transfer Function

$$H(s) = K_P\left[\frac{1/T_P}{(s+1/T_P)}\right]\left[\frac{M_{\delta E}\left(s+{}^{1}\!/\!{}_{T_{\theta_1}}\right)\left(s+{}^{1}\!/\!{}_{T_{\theta_2}}\right)}{\left(s^2+2\zeta\omega_n s+\omega_n^{\,2}\right)_{Ph}\left(s^2+2\zeta\omega_n s+\omega_n^{\,2}\right)_{SP}}\right]$$

Effect of Pilot Dynamics on Pitch-Angle Control Task

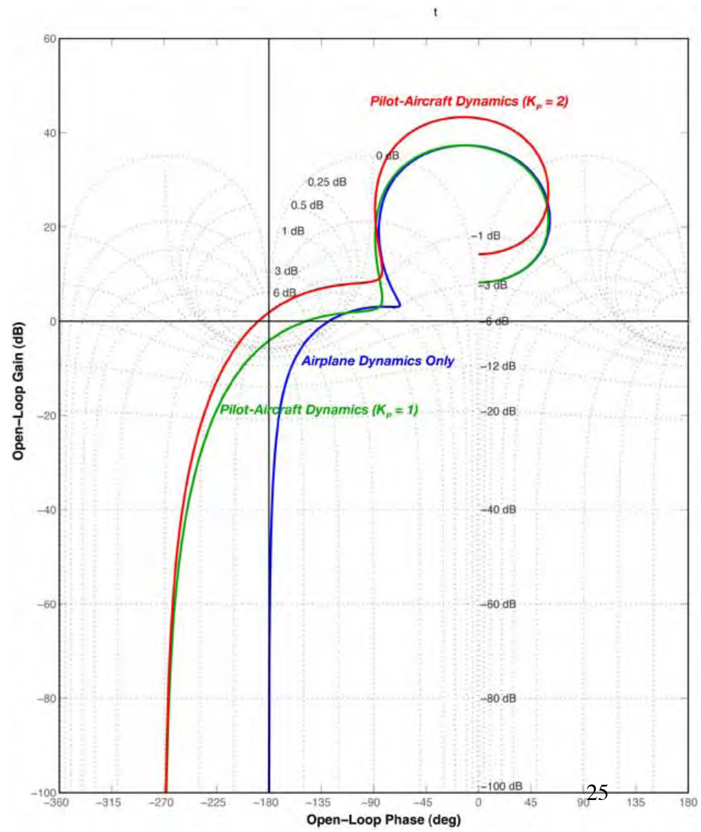


- **Pilot introduces neuromuscular lag while closing the control loop**
- **Example**
  - **Model the lag by a 1st-order time constant, T_P, of 0.25 s**
  - **Pilot's gain, K_P, is either 1 or 2**

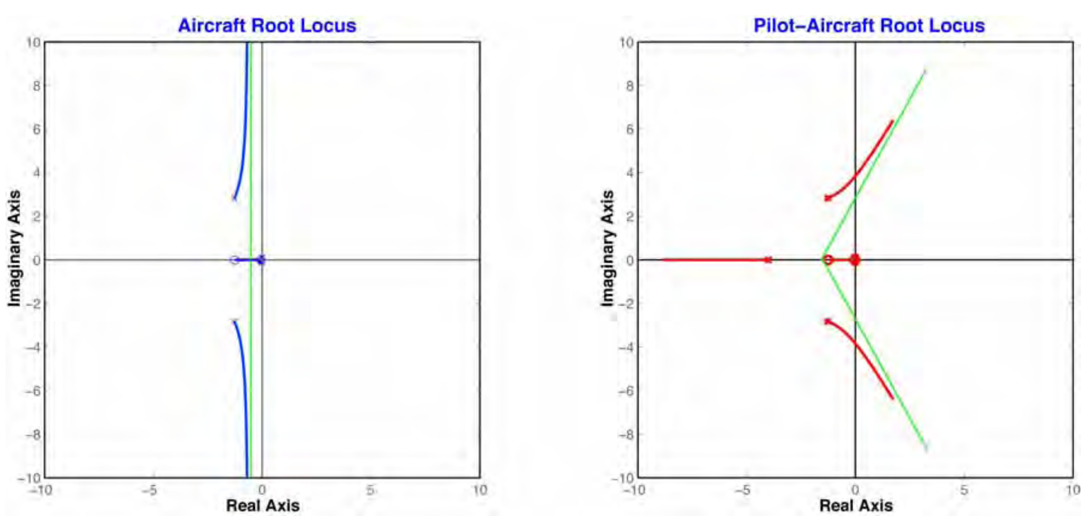
$$Pilot\ Transfer\ Function = \frac{\Delta u(s)}{\Delta\varepsilon(s)} = K_P\frac{1/T_P}{s+1/T_P} = K_P\frac{1/0.25}{s+1/0.25}$$

Effect of Pilot Dynamics on Pitch-Angle Control Task

- **Gain and phase margins become negative for pilot gain between 1 and 2**
- **Then, pilot destabilizes the system (PIO)**



Effect of 1st-Order Pilot Dynamics on Elevator/Pitch-Angle Control Root Locus



Pilot transfer function changes asymptotes of the root locus

Pilot-Induced Oscillations

Uncommanded aircraft is stable but piloting actions couple with aircraft dynamics to produce instability

Pilot-Induced Oscillations

NASA Digital-Fly-By-Wire F-8
Simulation of Space Shuttle

http://www.dfrc.nasa.gov/Gallery/Movie/F-8DFBW/Medium/EM-0044-01.mov

http://www.dfrc.nasa.gov/Gallery/Movie/F-8DFBW/Medium/EM-0044-02.mov

Power Effects on 4th-Order Longitudinal Modes

Power Effects on Stability and Control

- ***Gee Bee R1 Racer***: **an engine with wings and almost no tail**
- **During W.W.II, the size of fighters remained about the same, but installed horsepower doubled (*F4F* vs. *F8F*)**
- **Use of flaps means high power at low speed, increasing relative significance of thrust effects**
- **Short-Takeoff-and-Landing (STOL) aircraft augment takeoff/landing lift in many ways, e.g.,**
  - **Full-span flaps**
  - **Deflected thrust**

Direct Thrust Effect on Speed Stability, T_V

- In steady, level flight, nominal thrust balances nominal drag

$$T_N - D_N = C_{T_N} \frac{1}{2} \rho V_N^2 S - C_{D_N} \frac{1}{2} \rho V_N^2 S = 0$$

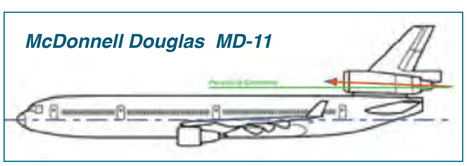
- **Effect of velocity change**

$$\frac{\partial T}{\partial V} = \begin{cases} < 0, & \textit{for propeller aircraft} \\ \approx 0, & \textit{for turbojet aircraft} \\ > 0, & \textit{for ramjet aircraft} \end{cases}$$

- **Small velocity perturbation grows if**
- **Therefore**
  - propeller is stabilizing for velocity change
  - turbojet has neutral effect
  - ramjet is destabilizing

$$\frac{\partial T}{\partial V} - \frac{\partial D}{\partial V} > 0$$

Pitching Moment Due to Thrust, M_V

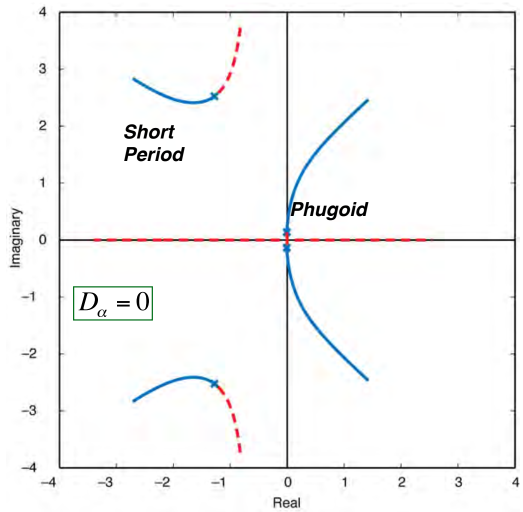


- Negative $\partial M/\partial V$ (Pitch-down effect) tends to increase velocity
- Positive $\partial M/\partial V$ (Pitch-up effect) tends to decrease velocity
- With propeller thrust line above the *c.m.*, increased velocity decreases thrust, producing a pitch-up moment
- Tilting the thrust line can have benefits
  - Up: Lake Amphibian, MD-11
  - Down: F6F, F8F, AD-1

M_V Effect on 4th-Order Roots

$$
\begin{aligned}
\Delta_{Lon}(s) = s^4 &+ \left(D_V + \frac{L_\alpha}{V_N} - M_q\right)s^3 \\
&+ \left[(g - D_\alpha)\frac{L_V}{V_N} + D_V\left(\frac{L_\alpha}{V_N} - M_q\right) - M_q\frac{L_\alpha}{V_N} - M_\alpha\right]s^2 \\
&+ \left\{M_q\left[(D_\alpha - g)\frac{L_V}{V_N} - D_V\frac{L_\alpha}{V_N}\right] + D_\alpha M_V - D_V M_\alpha\right\}s \\
&gM_\alpha\frac{L_V}{V_N} + M_V\left(D_\alpha s + g\frac{L_\alpha}{V_N}\right) = 0
\end{aligned}
$$

- Large positive value produces oscillatory phugoid instability
- Large negative value produces real phugoid divergence



Steady-State Response of the 4th-Order LTI Longitudinal Model

$$\Delta\dot{\mathbf{x}}(t) = \mathbf{F}\Delta\mathbf{x}(t) + \mathbf{G}\Delta\mathbf{u}(t)$$

- How do we calculate the equilibrium response to control?

$$\boxed{\Delta\mathbf{x}_{SS} = -\mathbf{F}^{-1}\,\mathbf{G}\,\Delta\mathbf{u}_{SS}}$$

- For the longitudinal model

$$
\begin{bmatrix} \Delta V_{SS} \\ \Delta\gamma_{SS} \\ \Delta q_{SS} \\ \Delta\alpha_{SS} \end{bmatrix}
= -\left[\begin{array}{cc|cc}
-D_V & -g & 0 & -D_\alpha \\
L_V/V_N & 0 & 0 & L_\alpha/V_N \\
\hline
M_V & 0 & M_q & M_\alpha \\
-L_V/V_N & 0 & 1 & -L_\alpha/V_N
\end{array}\right]^{-1}
\begin{bmatrix}
0 & T_{\delta T} & 0 \\
0 & 0 & L_{\delta F}/V_N \\
M_{\delta E} & 0 & 0 \\
0 & 0 & -L_{\delta F}/V_N
\end{bmatrix}
\begin{bmatrix} \Delta\delta E_{SS} \\ \Delta\delta T_{SS} \\ \Delta\delta F_{SS} \end{bmatrix}
$$

Algebraic Equation for Equilibrium Response

$$
\begin{bmatrix}\Delta V_{SS}\\ \Delta\gamma_{SS}\\ \Delta q_{SS}\\ \Delta\alpha_{SS}\end{bmatrix}
=\frac{\begin{bmatrix}
\left[-gM_{\delta E}\,{}^{L_\alpha}\!/\!{}_{V_N}\right] & 0 & \left[gM_\alpha L_{\delta F}/V_N\right]\\
\left[\left(D_V\,{}^{L_\alpha}\!/\!{}_{V_N}-D_\alpha\,{}^{L_V}\!/\!{}_{V_N}\right)M_{\delta E}\right] & \left[\left(M_V\,{}^{L_\alpha}\!/\!{}_{V_N}-M_\alpha\,{}^{L_V}\!/\!{}_{V_N}\right)T_{\delta T}\right] & \left[\left(D_\alpha M_V-D_V M_\alpha\right)L_{\delta F}/V_N\right]\\
0 & 0 & 0\\
\left[-gM_{\delta E}\,{}^{L_V}\!/\!{}_{V_N}\right] & 0 & \left[L_{\delta F}/V_N\right]
\end{bmatrix}}{g\left(M_V\,{}^{L_\alpha}\!/\!{}_{V_N}-M_\alpha\,{}^{L_V}\!/\!{}_{V_N}\right)}
\begin{bmatrix}\Delta\delta E_{SS}\\ \Delta\delta T_{SS}\\ \Delta\delta F_{SS}\end{bmatrix}
$$

- Roles of stability and control derivatives identified
- Result is a simple equation relating input and output

$$
\begin{bmatrix}\Delta V_{SS}\\ \Delta\gamma_{SS}\\ \Delta q_{SS}\\ \Delta\alpha_{SS}\end{bmatrix}
=\begin{bmatrix} a & 0 & b\\ c & d & e\\ 0 & 0 & 0\\ f & 0 & g\end{bmatrix}
\begin{bmatrix}\Delta\delta E_{SS}\\ \Delta\delta T_{SS}\\ \Delta\delta F_{SS}\end{bmatrix}
$$

4th-Order Steady-State Response May Be Counterintuitive

$$
\begin{aligned}
\Delta V_{SS} &= a\Delta\delta E_{SS} + (0)\Delta\delta T_{SS} + b\Delta\delta F_{SS}\\
\Delta\gamma_{SS} &= c\Delta\delta E_{SS} + d\Delta\delta T_{SS} + e\Delta\delta F_{SS}\\
\Delta q_{SS} &= (0)\Delta E_{SS} + (0)\Delta\delta T_{SS} + (0)\Delta\delta F_{SS}\\
\Delta\alpha_{SS} &= f\Delta\delta E_{SS} + (0)\Delta\delta T_{SS} + g\Delta\delta F_{SS}
\end{aligned}
$$

- **Observations**
  - Thrust command
  - Elevator and flap commands
  - Steady-state pitch rate is zero
  - *4th-order model neglects air density gradient effects*

Steady-state pitch angle

$$
\Delta\theta_{SS} = \Delta\gamma_{SS} + \Delta\alpha_{SS} = (c+f)\Delta\delta E_{SS} + d\Delta\delta T_{SS} + (e+g)\Delta\delta F_{SS}
$$

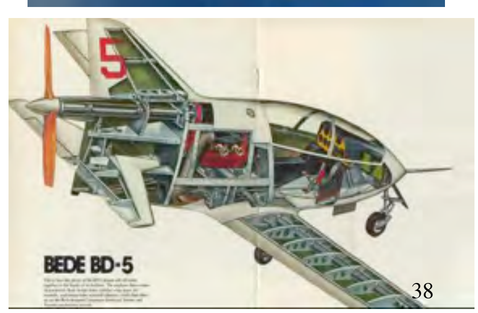
Personal Aircraft Factoids

Ercoupe Approach to Safe, Affordable Flying

- Limited control authority; 2-control cockpits (rudder interconnected to aileron)
- Limited center-of-mass travel
- Limited speed range
- Wing leveling and lateral stability (stable spiral mode)
- Fixed, tricycle landing gear

Less-Safe Airplanes

- However -- many designers of personal aircraft had other ideas
  - *Mignet Flying Flea* (Homebuilt, pivoting main wing, no ailerons, unrecoverable dive)
  - V-tail *Beechcraft Bonanza Model 35* (10,000 built, 250 in-flight structural failures)
  - *American Yankee AA-1* (BD-1, "hot", stalls and spins)
  - *Bede BD-5* (Home-built, unforgiving flying qualities)



Do "Safe" Airplanes Have Fewer Accidents?

- The *Ercoupe's* safety record is about average
- Many *J-3 Cubs* involved in fatal stall/spin accidents
- *Cirrus SR-20/22* has had 26 fatal crashes, killing 52 to date (1999-2006)

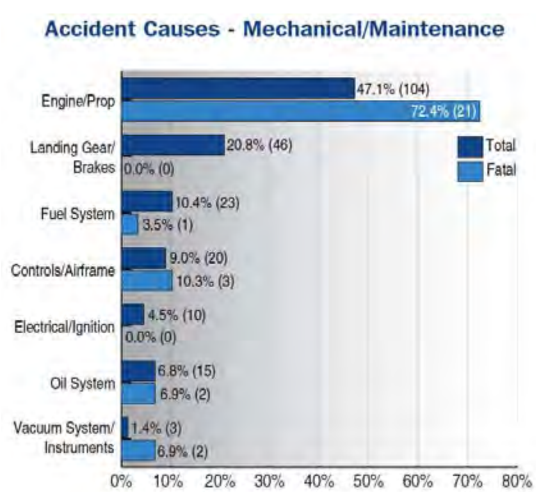
- Top causes of general aviation accidents
  - Maneuvering
  - Weather
  - Takeoff/Climb
  - Descent/Approach
  - Engine/Prop
  - Fuel Management

- Probable cause of Ercoupe fatal accidents: 1994-2004
  - VFR flight in IFR conditions
  - Pilot's lack of night-flying experience
  - Pilot had <u>no</u> flight training
  - Inadvertent stall
  - Loss of engine power
  - In-flight breakup (corrosion)

In comparison, there were 69 fatalities during 36 years of flying the F-14 fighter

2004 General Aviation Accidents

(AOPA)



Nall Report, 2012-13, http://www.aopa.org/-/media/Files/AOPA/Home/Pilot%20Resources/Safety%20&%20Proficiency/Accident%20Analysis/Nall%20Report/Scorecard_1213.pdf

How Safe is “Safe”?

(NTSB)

Accidents, Fatalities, and Rates, 1986 through 2005,
U.S. General Aviation

	Accidents		Fatalities			Accidents per 100,000 Flight Hours	
Year	All	Fatal	Total	Aboard	Flight Hours	All	Fatal
1986	2,581	474	967	879	27,073,000	9.49	1.73
1987	2,495	446	837	822	26,972,000	9.18	1.63
1988	2,388	460	797	792	27,446,000	8.65	1.66
1989	2,242	432	769	766	27,920,000	7.97	1.52
1990	2,242	444	770	765	28,510,000	7.85	1.55
1991	2,197	439	800	786	27,678,000	7.91	1.57
1992	2,111	451	867	865	24,780,000	8.51	1.82
1993	2,064	401	744	740	22,796,000	9.03	1.74
1994	2,021	404	730	723	22,235,000	9.08	1.81
1995	2,056	413	735	728	24,906,000	8.21	1.63
1996	1,908	361	636	619	24,881,000	7.65	1.45
1997	1,844	350	631	625	25,591,000	7.19	1.36
1998	1,905	365	625	619	25,518,000	7.44	1.41
1999	1,905	340	619	615	29,246,000	6.5	1.16
2000	1,837	345	596	585	27,838,000	6.57	1.21
2001	1,727	325	562	558	25,431,000	6.78	1.27
2002	1,715	345	581	575	25,545,000	6.69	1.33
2003	1,739	352	632	629	25,706,000	6.75	1.36
2004	1,617	314	558	558	24,888,000	6.49	1.26
2005	1,669	321	562	557	24,401,000	6.83	1.31

Accidents, Fatalities, and Rates, 1986 through 2005,
for U.S. Air Carriers Operating Under 14 CFR 121

	Accidents		Fatalities			Accidents per 100,000 Flight Hours	
Year	All	Fatal	Total	Aboard	Flight Hours	All	Fatal
1986	21	2	5	4	9,495,158	0.211	0.011
1987	32	4	231	229	10,115,407	0.306	0.03
1988	29	3	285	274	10,521,052	0.266	0.019
1989	23	8	131	130	10,597,922	0.217	0.075
1990	21	6	39	12	11,524,726	0.182	0.052
1991	22	4	62	49	11,139,166	0.198	0.036
1992	16	4	33	31	11,732,026	0.136	0.034
1993	22	1	1	0	11,981,347	0.184	0.008
1994	18	4	239	237	12,292,356	0.138	0.033
1995	30	1	160	160	12,776,679	0.235	0.008
1996	31	3	342	342	12,971,676	0.239	0.023
1997	43	3	3	2	15,061,662	0.285	0.02
1998	41	1	1	0	15,921,447	0.258	0.006
1999	40	2	12	11	16,693,365	0.24	0.012
2000	49	2	89	89	17,478,519	0.28	0.011
2001	41	6	531	525	17,157,858	0.216	0.012
2002	35	0	0	0	16,718,781	0.209	-
2003	51	2	22	21	16,887,756	0.302	0.012
2004	23	1	13	13	18,184,016	0.126	0.005
2005	32	3	22	20	18,728,000	0.171	0.016

Next Time:
Flying Qualities Criteria

Flight Dynamics
419-428, 525-533
Airplane Stability and Control
Chapter 21

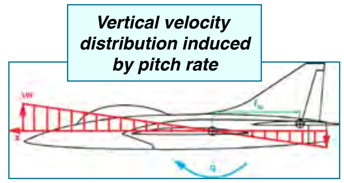
Supplemental Material

Apparent Mass and Unsteady Aerodynamics

Distinction Between Angle-of-Attack Rate and Pitch Rate

- With no vertical motion of the *c.m.*, pitch rate and angle-of-attack rate are the same

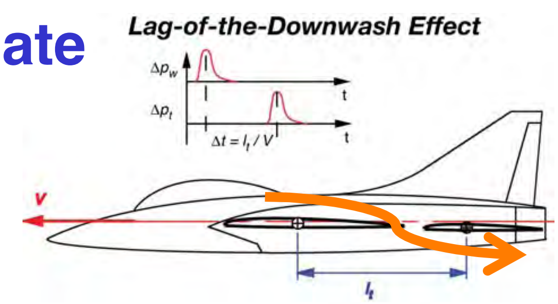
$$\dot{\alpha} = q$$



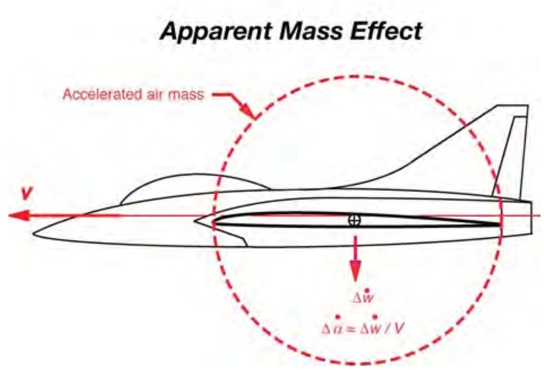
- With no pitching, vertical heaving (or plunging) motion of the *c.m.*, produces angle-of-attack rate but no pitch rate

$$\dot{\alpha} \neq 0 \neq q; \quad q = 0$$

Angle-of-Attack Rate Has Two Effects



- Pressure variations at wing convect downstream, arriving at tail Δt sec later
  - Lag of the downwash
  - Delayed tail-lift/pitch-moment effect



- Vertical force opposed by a mass of air ("apparent mass") as well as airplane mass
  - Vertical acceleration produces added lift and moment

Flight Dynamics, pp. 204-206, 284-285

Angle-of-Attack-Rate Effects Principally Affect the Short-Period Mode

Lift and pitching moment proportional to angle-of-attack rate

$$\Delta\dot{q} = M_q\Delta q + M_\alpha\Delta\alpha + M_{\delta E}\Delta\delta E + M_{\dot{\alpha}}\Delta\dot{\alpha}$$

$$\Delta\dot{\alpha} = \left(1 - L_q/V_N\right)\Delta q - \left(L_\alpha/V_N\right)\Delta\alpha - \left(L_{\delta E}/V_N\right)\Delta\delta E - \left(L_{\dot{\alpha}}/V_N\right)\Delta\dot{\alpha}$$

Bring effects to left side

$$\Delta\dot{q} - M_{\dot{\alpha}}\Delta\dot{\alpha} = M_q\Delta q + M_\alpha\Delta\alpha + M_{\delta E}\Delta\delta E$$

$$\Delta\dot{\alpha} + \left(L_{\dot{\alpha}}/V_N\right)\Delta\dot{\alpha} = \left(1 - L_q/V_N\right)\Delta q - \left(L_\alpha/V_N\right)\Delta\alpha - \left(L_{\delta E}/V_N\right)\Delta\delta E$$

Vector-matrix form

$$\begin{bmatrix} 1 & -M_{\dot{\alpha}} \\ 0 & \left[1 + \left(L_{\dot{\alpha}}/V_N\right)\right] \end{bmatrix} \begin{bmatrix} \Delta\dot{q} \\ \Delta\dot{\alpha} \end{bmatrix} = \begin{bmatrix} M_q & M_\alpha \\ \left(1 - L_q/V_N\right) & -\left(L_\alpha/V_N\right) \end{bmatrix} \begin{bmatrix} \Delta q \\ \Delta\alpha \end{bmatrix} + \begin{bmatrix} M_{\delta E} \\ -\left(L_{\delta E}/V_N\right) \end{bmatrix} \Delta\delta E$$

Angle-of-Attack-Rate Effects

Pre-multiply both sides by inverse

$$\begin{bmatrix} \Delta\dot{q} \\ \Delta\dot{\alpha} \end{bmatrix} = \begin{bmatrix} 1 & -M_{\dot{\alpha}} \\ 0 & \left[1 + \left(L_{\dot{\alpha}}/V_N\right)\right] \end{bmatrix}^{-1} \left\{ \begin{bmatrix} M_q & M_\alpha \\ \left(1 - L_q/V_N\right) & -\left(L_\alpha/V_N\right) \end{bmatrix} \begin{bmatrix} \Delta q \\ \Delta\alpha \end{bmatrix} + \begin{bmatrix} M_{\delta E} \\ -\left(L_{\delta E}/V_N\right) \end{bmatrix} \Delta\delta E \right\}$$

Inverse of the apparent mass matrix

$$\begin{bmatrix} 1 & -M_{\dot{\alpha}} \\ 0 & \left[1 + \left(L_{\dot{\alpha}}/V_N\right)\right] \end{bmatrix}^{-1} = \frac{\begin{bmatrix} \left[1 + \left(L_{\dot{\alpha}}/V_N\right)\right] & M_{\dot{\alpha}} \\ 0 & 1 \end{bmatrix}}{\left[1 + \left(L_{\dot{\alpha}}/V_N\right)\right]}$$

Angle-of-Attack-Rate Effects

Substitute

$$\begin{bmatrix}\Delta\dot q\\ \Delta\dot\alpha\end{bmatrix} = \frac{\begin{bmatrix}\left[1+\left(L_{\dot\alpha}/V_N\right)\right] & M_{\dot\alpha}\\ 0 & 1\end{bmatrix}}{\left[1+\left(L_{\dot\alpha}/V_N\right)\right]}\left\{\begin{bmatrix}M_q & M_\alpha\\ \left(1-L_q/V_N\right) & -L_\alpha/V_N\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\alpha\end{bmatrix}+\cdots\right\}$$

Multiply matrices

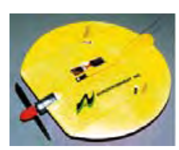
$$\begin{bmatrix}\Delta\dot q\\ \Delta\dot\alpha\end{bmatrix} = \frac{1}{\left[1+\left(L_{\dot\alpha}/V_N\right)\right]}\left\{\begin{bmatrix}\left\{\left[1+\left(L_{\dot\alpha}/V_N\right)\right]M_q + M_{\dot\alpha}\left(1-L_q/V_N\right)\right\} & \left\{\left[1+\left(L_{\dot\alpha}/V_N\right)\right]M_\alpha - M_{\dot\alpha}\left(L_\alpha/V_N\right)\right\}\\ \left(1-L_q/V_N\right) & -L_\alpha/V_N\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\alpha\end{bmatrix} + \begin{bmatrix}\left[1+\left(L_{\dot\alpha}/V_N\right)\right]M_{\delta E} - M_{\dot\alpha}\left(L_{\delta E}/V_N\right)\\ -L_{\delta E}/V_N\end{bmatrix}\Delta\delta E\right\}$$

Simplification of Angle-of-Attack-Rate Effects

- **Typically**

L_q and $L_{\dot\alpha}$ have small effects for large aircraft*
M_q and $M_{\dot\alpha}$ are same order of magnitude
and have more significant effects

* but not for small aircraft, e.g., R/C models and micro-UAVs



- **Neglecting** L_q and $L_{\dot\alpha}$

$$\begin{bmatrix}\Delta\dot q\\ \Delta\dot\alpha\end{bmatrix} \simeq \begin{bmatrix}\left\{M_q + M_{\dot\alpha}\right\} & \left\{M_\alpha - M_{\dot\alpha}\left(L_\alpha/V_N\right)\right\}\\ 1 & -\left(L_\alpha/V_N\right)\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\alpha\end{bmatrix} + \begin{bmatrix}M_{\delta E} - M_{\dot\alpha}\left(L_{\delta E}/V_N\right)\\ -\left(L_{\delta E}/V_N\right)\end{bmatrix}\Delta\delta E$$

2^{nd}-Degree Characteristic Polynomial with

$$\boxed{L_q \text{ and } L_{\dot{\alpha}} \simeq 0}$$

Short-period characteristic polynomial

$$\Delta(s) = \begin{vmatrix} \left[s-\left(M_q+M_{\dot{\alpha}}\right)\right] & -\left[M_\alpha - M_{\dot{\alpha}}\left(L_\alpha/V_N\right)\right] \\ -1 & \left[s+\left(L_\alpha/V_N\right)\right] \end{vmatrix} = \left[s-\left(M_q+M_{\dot{\alpha}}\right)\right]\left[s+\left(L_\alpha/V_N\right)\right]-\left[M_\alpha - M_{\dot{\alpha}}\left(L_\alpha/V_N\right)\right]$$

$$= s^2 + \left[\left(L_\alpha/V_N\right)-\left(M_q+M_{\dot{\alpha}}\right)\right]s + \left\{\left[M_\alpha - \left(M_q+M_{\dot{\alpha}}\right)\left(L_\alpha/V_N\right)\right]+M_{\dot{\alpha}}\left(L_\alpha/V_N\right)\right\}$$

Damping is increased
Natural frequency is unaffected

$$\Delta(s) = s^2 + \left[\left(L_\alpha/V_N\right)-\left(M_q+M_{\dot{\alpha}}\right)\right]s + \left\{\left[M_\alpha - M_q\left(L_\alpha/V_N\right)\right]\right\}$$
$$= s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Comparison of Bizjet Fourth- and Second-Order Models and Eigenvalues

Fourth-Order Model

F =

-0.0185	-9.8067	0	0
0.0019	0	0	1.2709
0	0	-1.2794	-7.9856
-0.0019	0	1	-1.2709

G =

0	4.6645
0	0
-9.069	0
0	0

Eigenvalue	Damping	Freq. (rad/s)
-8.43e-03 + 1.24e-01j	6.78E-02	1.24E-01
-8.43e-03 - 1.24e-01j	6.78E-02	1.24E-01
-1.28e+00 + 2.83e+00j	4.11E-01	3.10E+00
-1.28e+00 - 2.83e+00j	4.11E-01	3.10E+00

Phugoid Approximation

F =

-0.0185	-9.8067
0.0019	0

G =

4.6645
0

Eigenvalue	Damping	Freq. (rad/s)
-9.25e-03 + 1.36e-01j	6.78E-02	1.37E-01
-9.25e-03 - 1.36e-01j	6.78E-02	1.37E-01

Short-Period Approximation

F =

-1.2794	-7.9856
1	-1.2709

G =

-9.069
0

Eigenvalue	Damping	Freq. (rad/s)
-1.28e+00 + 2.83e+00j	4.11E-01	3.10E+00
-1.28e+00 - 2.83e+00j	4.11E-01	3.10E+00

- **Approximations are very close to 4^{th}-order values because natural frequencies are widely separated**

A Little More About Output Matrices

With $H_x = I$ and $H_u = 0$

$$\Delta\mathbf{y} = \Delta\mathbf{x} = \mathbf{H}_x\Delta\mathbf{x};\quad \text{then}\quad \mathbf{H}_x = \mathbf{I}_4$$

and

$$\begin{bmatrix}\Delta y_1\\ \Delta y_2\\ \Delta y_3\\ \Delta y_4\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\Delta x_1\\ \Delta x_2\\ \Delta x_3\\ \Delta x_4\end{bmatrix} \triangleq \begin{bmatrix}\Delta V\\ \Delta\gamma\\ \Delta q\\ \Delta\alpha\end{bmatrix}$$

Only output is ΔV

$$\Delta y = \Delta V = \begin{bmatrix}1&0&0&0\end{bmatrix}\begin{bmatrix}\Delta V\\ \Delta\gamma\\ \Delta q\\ \Delta\alpha\end{bmatrix}$$

ΔV and $\Delta\alpha$ are measured

$$\Delta\mathbf{y} = \begin{bmatrix}\Delta y_1\\ \Delta y_2\end{bmatrix} = \begin{bmatrix}\Delta V\\ \Delta\alpha\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}\Delta V\\ \Delta\gamma\\ \Delta q\\ \Delta\alpha\end{bmatrix}$$

A Little More About Output Matrices

- **Output (measurement) of body-axis velocity and pitch rate and angle**
- **Transformation from [$\Delta V, \Delta\gamma, \Delta q, \Delta\theta$] to [$\Delta u, \Delta w, \Delta q, \Delta\alpha$]**

$$\begin{bmatrix}\Delta u\\ \Delta w\\ \Delta q\\ \Delta\theta\end{bmatrix} = \begin{bmatrix}\cos\alpha_N & 0 & 0 & -V_N\sin\alpha_N\\ \sin\alpha_N & 0 & 0 & V_N\cos\alpha_N\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}\begin{bmatrix}\Delta V\\ \Delta\gamma\\ \Delta q\\ \Delta\alpha\end{bmatrix}$$

- **Separate measurement of state and control perturbations**

$$\Delta\mathbf{y} = \begin{bmatrix}\Delta\mathbf{x}\\ \Delta\mathbf{u}\end{bmatrix} = \mathbf{H}_x\Delta\mathbf{x} + \mathbf{H}_u\Delta\mathbf{u}$$

$$\begin{bmatrix}\Delta y_1\\ \Delta y_2\\ \Delta y_3\\ \Delta y_4\\ \Delta y_5\\ \Delta y_6\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}\Delta V\\ \Delta\gamma\\ \Delta q\\ \Delta\alpha\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}\Delta\delta E\\ \Delta\delta T\end{bmatrix}$$

Elevator-to-Normal-Velocity Numerator

($L_{\delta E} = 0$)

Transform though α_N back to body axes

$$\mathbf{H_x} Adj(s\mathbf{I} - \mathbf{F}_{Lon})\mathbf{G} = \begin{bmatrix} \sin\alpha_N & 0 & 0 & V_N \cos\alpha_N \end{bmatrix} \begin{bmatrix} n_V^V(s) & n_\gamma^V(s) & n_q^V(s) & n_\alpha^V(s) \\ n_V^\gamma(s) & n_\gamma^\gamma(s) & n_q^\gamma(s) & n_\alpha^\gamma(s) \\ n_V^q(s) & n_\gamma^q(s) & n_q^q(s) & n_\alpha^q(s) \\ n_V^\alpha(s) & n_\gamma^\alpha(s) & n_q^\alpha(s) & n_\alpha^\alpha(s) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_{\delta E} \\ 0 \end{bmatrix} = n_{\delta E}^w(s)$$

Scalar transfer function numerator

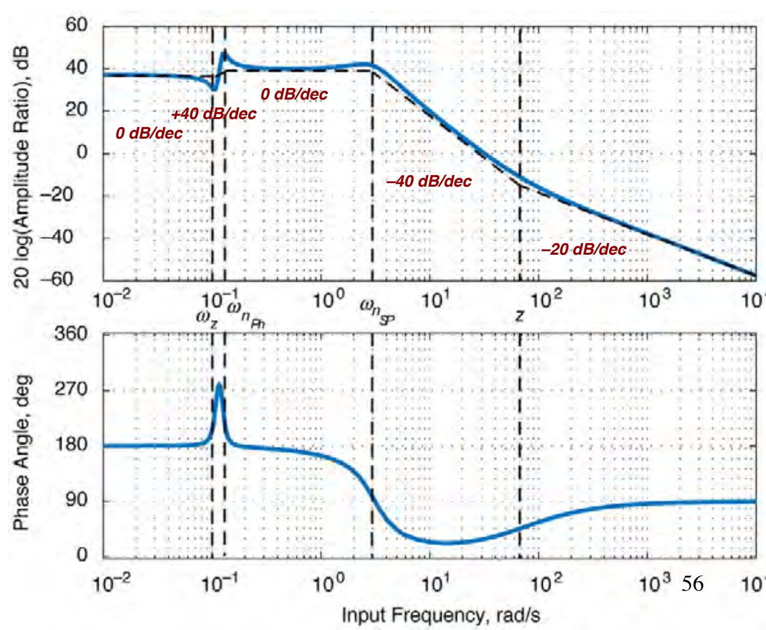
$$n_{\delta E}^w(s) = \begin{bmatrix} \sin\alpha_N & 0 & 0 & V_N \cos\alpha_N \end{bmatrix} \begin{bmatrix} n_q^V(s) \\ n_q^\gamma(s) \\ n_q^q(s) \\ n_q^\alpha(s) \end{bmatrix} M_{\delta E}$$

$$= M_{\delta E}\left[(\sin\alpha_N) n_q^V(s) + (V_N \cos\alpha_N) n_q^\alpha(s)\right]$$

Elevator-to-Normal-Velocity Frequency Response

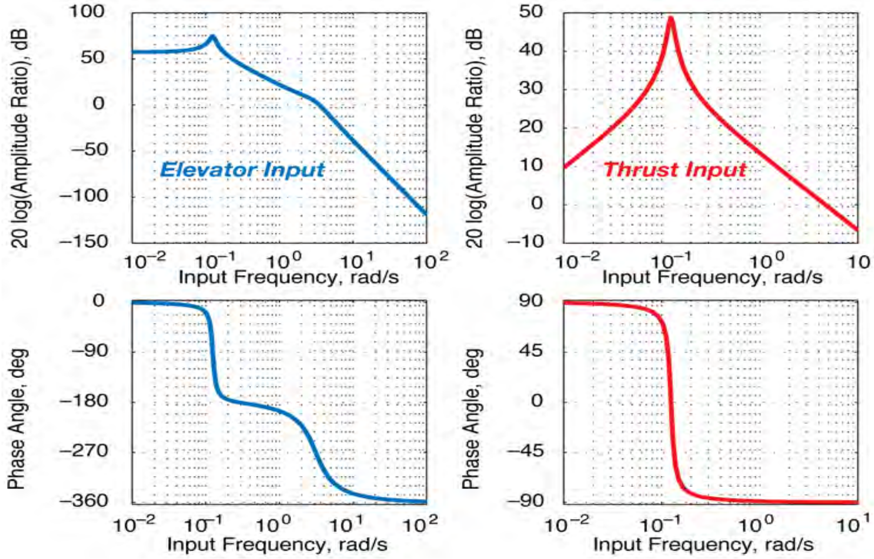
$$\frac{\Delta w(s)}{\Delta \delta E(s)} = \frac{n_{\delta E}^w(s)}{\Delta_{Lon}(s)} \approx \frac{M_{\delta E}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Approx\ Ph}(s - z_3)}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Ph}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{SP}}$$

- $(n - q) = 1$
- Complex zero almost (but not quite) cancels phugoid response



Airspeed Frequency Response to Elevator and Thrust Inputs

Response is primarily through the lightly damped phugoid mode

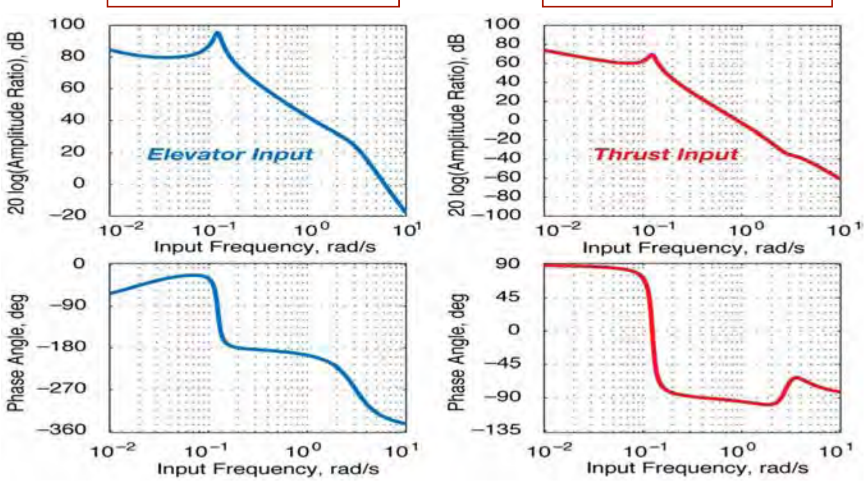


Altitude Frequency Response to Elevator and Thrust Inputs

Altitude perturbation: Integral of the flight path angle perturbation

$$\boxed{\Delta z(t) = -V_N \int_0^t \Delta\gamma(\tau)\, d\tau}$$

$$\boxed{\frac{\Delta z(s)}{\Delta\delta E(s)} = -\left(\frac{V_N}{s}\right)\frac{\Delta\gamma(s)}{\Delta\delta E(s)}} \qquad \boxed{\frac{\Delta z(s)}{\Delta\delta T(s)} = -\left(\frac{V_N}{s}\right)\frac{\Delta\gamma(s)}{\Delta\delta T(s)}}$$



High- and Low-Frequency Limits of Frequency Response Function

$$\boxed{H_{ij}(j\omega) = AR(\omega)\, e^{j\phi(\omega)}}$$

$$H_{ij}(j\omega \to \infty) \to \frac{k_{ij}\left[(j\omega)^q + b_{q-1}(j\omega)^{q-1} + \ldots + b_1(j\omega) + b_0\right]}{\left[(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \ldots + a_1(j\omega) + a_0\right]} \to \frac{k_{ij}}{(j\omega)^{n-q}}$$

$$H_{ij}(j\omega \to 0) \to \frac{k_{ij}\left[(j\omega)^q + b_{q-1}(j\omega)^{q-1} + \ldots + b_1(j\omega) + b_0\right]}{\left[(j\omega)^n + a_{n-1}(j\omega)^{n-1} + \ldots + a_1(j\omega) + a_0\right]} \to \begin{cases} \dfrac{k_{ij}b_0}{a_0}, & b_0 \neq 0 \\ \dfrac{k_{ij}j(0)b_1}{a_0}, & b_0 = 0, b_1 \neq 0, etc. \end{cases}$$

Elevator-to-Pitch-Rate Numerator and Transfer Function

$$\mathbf{H_x}Adj(s\mathbf{I} - \mathbf{F}_{Lon})\mathbf{G} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} n_V^V(s) & n_\gamma^V(s) & n_q^V(s) & n_\alpha^V(s) \\ n_V^\gamma(s) & n_\gamma^\gamma(s) & n_q^\gamma(s) & n_\alpha^\gamma(s) \\ n_V^q(s) & n_\gamma^q(s) & n_q^q(s) & n_\alpha^q(s) \\ n_V^\alpha(s) & n_V^\alpha(s) & n_V^\alpha(s) & n_V^\alpha(s) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ M_{\delta E} \\ 0 \end{bmatrix} = n_{\delta E}^q(s)$$

$$\frac{\Delta q(s)}{\Delta \delta E(s)} = \frac{n_{\delta E}^q(s)}{\Delta_{Lon}(s)} \approx \frac{M_{\delta E}\, s\,(s - z_1)(s - z_2)}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Ph}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{SP}}$$

"Free *s*" in numerator differentiates pitch angle transfer function

Transfer Functions of Thrust Input to Angle Output

Thrust-to-Flight Path Angle transfer function

$$\boxed{\frac{\Delta\gamma(s)}{\Delta\delta T(s)} = \frac{n_{\delta T}^{\gamma}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta T}^{\gamma}(s) = T_{\delta T}\frac{L_V}{V_N}\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)_{Approx\ SP}}$$

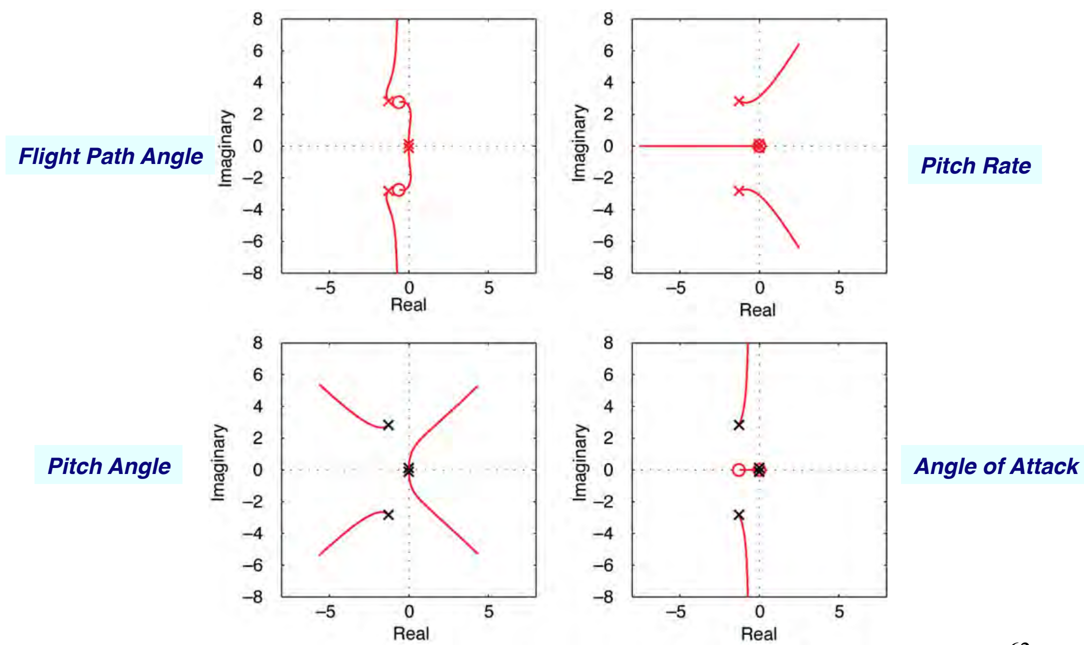
Thrust-to-Angle of Attack transfer function

$$\boxed{\frac{\Delta\alpha(s)}{\Delta\delta T(s)} = \frac{n_{\delta T}^{\alpha}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta T}^{\alpha}(s) = T_{\delta T}\, s\left(s + {}^{1}\!/_{T_{\alpha_T}}\right)}$$

Thrust-to-Pitch Angle transfer function

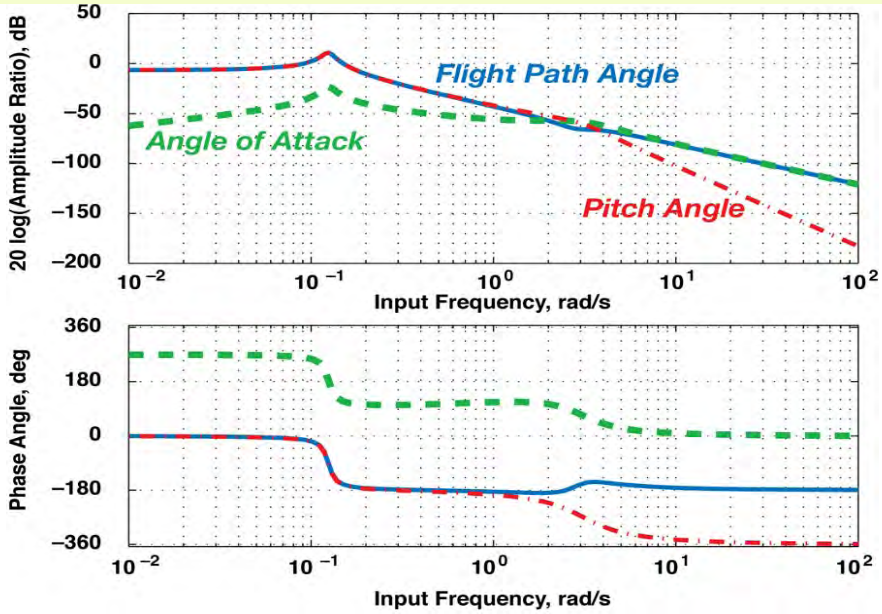
$$\boxed{\frac{\Delta\theta(s)}{\Delta\delta T(s)} = \frac{n_{\delta T}^{\theta}(s)}{\Delta_{Lon}(s)}; \quad n_{\delta T}^{\theta}(s) = T_{\delta T}\left(s + {}^{1}\!/_{T_{\theta_T}}\right)}$$

Root Locus Analysis of Angular Feedback to Thrust (4th-Order Model)



Frequency Response of Angles to Thrust Input

- **Primarily effects flight path angle and low-frequency pitch angle**



Control Surface Dynamic Coupling

Dynamic Model of a Control Surface Mechanism

Approximate control dynamics by a 2nd-order LTI system

$$\ddot{\delta} - H_{\dot{\delta}}\dot{\delta} - H_{\delta}\delta = H_{\alpha}\alpha + H_{command} + \ldots$$

mechanism dynamics = external forcing

Bring all torques and inertias to right side

$$\ddot{\delta}E = \frac{H_{elevator}}{I_{elevator}} = \frac{C_{H_{elevator}}\frac{1}{2}\rho V^2 S\overline{c}}{I_{elevator}}$$

$$= \left[C_{H_{\dot{\delta}E}}\dot{\delta}E + C_{H_{\delta E}}\delta E + C_{H_{\alpha}}\alpha + C_{H_{command}} + \ldots\right]\frac{\frac{1}{2}\rho V^2 S\overline{c}}{I_{elevator}}$$

$$\equiv H_{\dot{\delta}E}\dot{\delta}E + H_{\delta E}\delta E + H_{\alpha}\alpha + H_{command} + \ldots$$

Coupling of System Model and Control Mechanism Dynamics

- **2nd-order model of control-deflection dynamics**
  - Command input from cockpit
  - Forcing by aerodynamic effects
    - Control surface deflection
    - Aircraft angle of attack and angular rates

$$\Delta\dot{\mathbf{x}}_{\delta E} = \mathbf{F}_{\delta E}\Delta\mathbf{x}_{\delta E} + \mathbf{G}_{\delta E}\Delta\mathbf{u}_{\delta E} + \mathbf{F}_{SP}^{\delta E}\Delta\mathbf{x}_{SP}$$

$$\begin{bmatrix}\Delta\dot{\delta}E\\ \Delta\ddot{\delta}E\end{bmatrix} \approx \begin{bmatrix}0 & 1\\ H_{\delta E} & H_{\dot{\delta}E}\end{bmatrix}\begin{bmatrix}\Delta\delta E\\ \Delta\dot{\delta}E\end{bmatrix} + \begin{bmatrix}0\\ -H_{\delta E}\end{bmatrix}\Delta\delta E_{command} + \begin{bmatrix}0 & 0\\ H_q & H_{\alpha}\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\alpha\end{bmatrix}$$

- **Short period approximation**
- **Coupling with mechanism dynamics**

$$\Delta\dot{\mathbf{x}}_{SP} = \mathbf{F}_{SP}\Delta\mathbf{x}_{SP} + \mathbf{G}_{SP}\Delta\mathbf{u}_{SP} = \mathbf{F}_{SP}\Delta\mathbf{x}_{SP} + \mathbf{F}_{\delta E}^{SP}\Delta\mathbf{x}_{\delta E}$$

$$\begin{bmatrix}\Delta\dot{q}\\ \Delta\dot{\alpha}\end{bmatrix} \approx \begin{bmatrix}M_q & M_{\alpha}\\ 1 & -L_{\alpha}/V_N\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta\alpha\end{bmatrix} + \begin{bmatrix}M_{\delta E} & 0\\ -L_{\delta E}/V_N & 0\end{bmatrix}\begin{bmatrix}\Delta\delta E\\ \Delta\dot{\delta}E\end{bmatrix}$$

Short Period Model Augmented by Control Mechanism Dynamics

Augmented dynamic equation

$$\Delta\dot{\mathbf{x}}_{SP/\delta E} = \mathbf{F}_{SP/\delta E}\Delta\mathbf{x}_{SP/\delta E} + \mathbf{G}_{SP/\delta E}\Delta\delta E_{command}$$

State Vector

$$\Delta\mathbf{x}_{SP'} = \begin{bmatrix} \Delta q \\ \Delta\alpha \\ \Delta\delta E \\ \Delta\dot{\delta} E \end{bmatrix}$$

Augmented stability and control matrices

$$\mathbf{F}_{SP/\delta E} = \begin{bmatrix} \mathbf{F}_{SP} & \mathbf{F}_{\delta E}^{SP} \\ \mathbf{F}_{SP}^{\delta E} & \mathbf{F}_{\delta E} \end{bmatrix} = \left[\begin{array}{cc|cc} M_q & M_\alpha & M_{\delta E} & 0 \\ 1 & -L_\alpha/V_N & -L_{\delta E}/V_N & 0 \\ \hline 0 & 0 & 0 & 1 \\ H_q & H_\alpha & H_{\delta E} & H_{\dot{\delta} E} \end{array}\right]$$

$$\mathbf{G}_{SP/\delta E} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ H_{\delta E} \end{bmatrix}$$

Roots of the Augmented Short Period Model

Characteristic equation for short-period/elevator dynamics

$$\Delta_{SP/\delta E}(s) = \left|s\mathbf{I}_n - \mathbf{F}_{SP/\delta E}\right| = \left|\begin{array}{cc|cc} (s - M_q) & -M_\alpha & -M_{\delta E} & 0 \\ -1 & \left(s + L_\alpha/V_N\right) & L_{\delta E}/V_N & 0 \\ \hline 0 & 0 & s & -1 \\ -H_q & -H_\alpha & -H_{\delta E} & (s - H_{\dot{\delta} E}) \end{array}\right| = 0$$

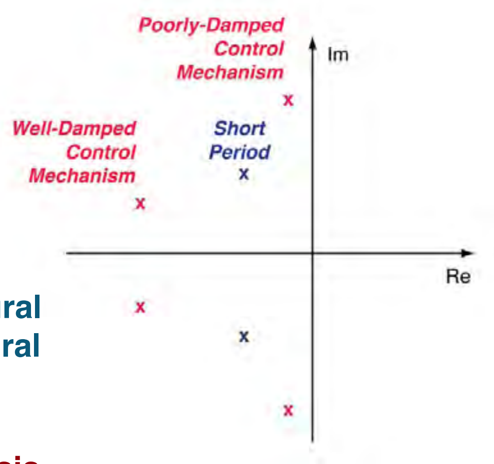
Short Period   Control Mechanism

$$\Delta_{SP/\delta E}(s) = \left(s^2 + 2\zeta_{SP}\omega_{n_{SP}}s + \omega_{n_{SP}}^2\right)\left(s^2 + 2\zeta_{\delta E}\omega_{n_{\delta E}}s + \omega_{n_{\delta E}}^2\right)$$

Roots of the Augmented Short Period Model

- Coupling of the modes depends on design parameters

$$\boxed{M_{\delta E}, \frac{L_{\delta E}}{V_N}, H_q, \text{ and } H_\alpha}$$



- Desirable for mechanical natural frequency > short-period natural frequency
- Coupling dynamics can be evaluated by root locus analysis