STAT 443: Lab 3

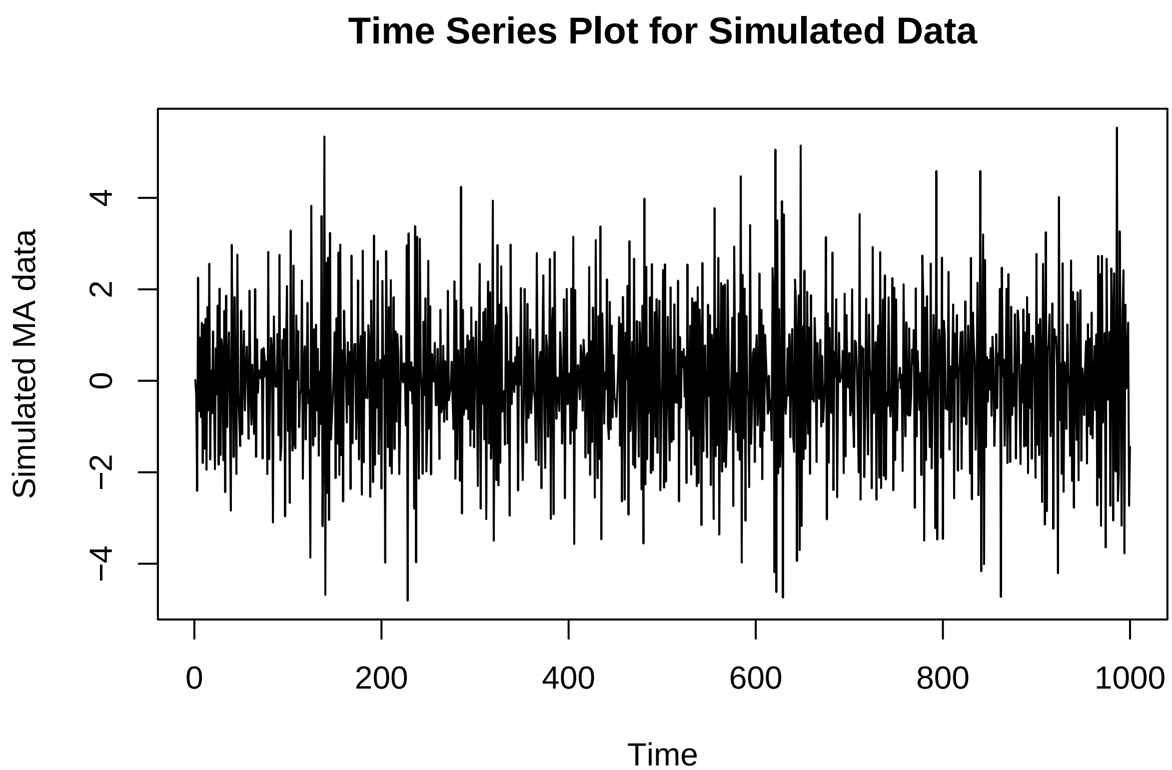
Wenxuan Zan (61336194)

27 January, 2023

Question 1

a)

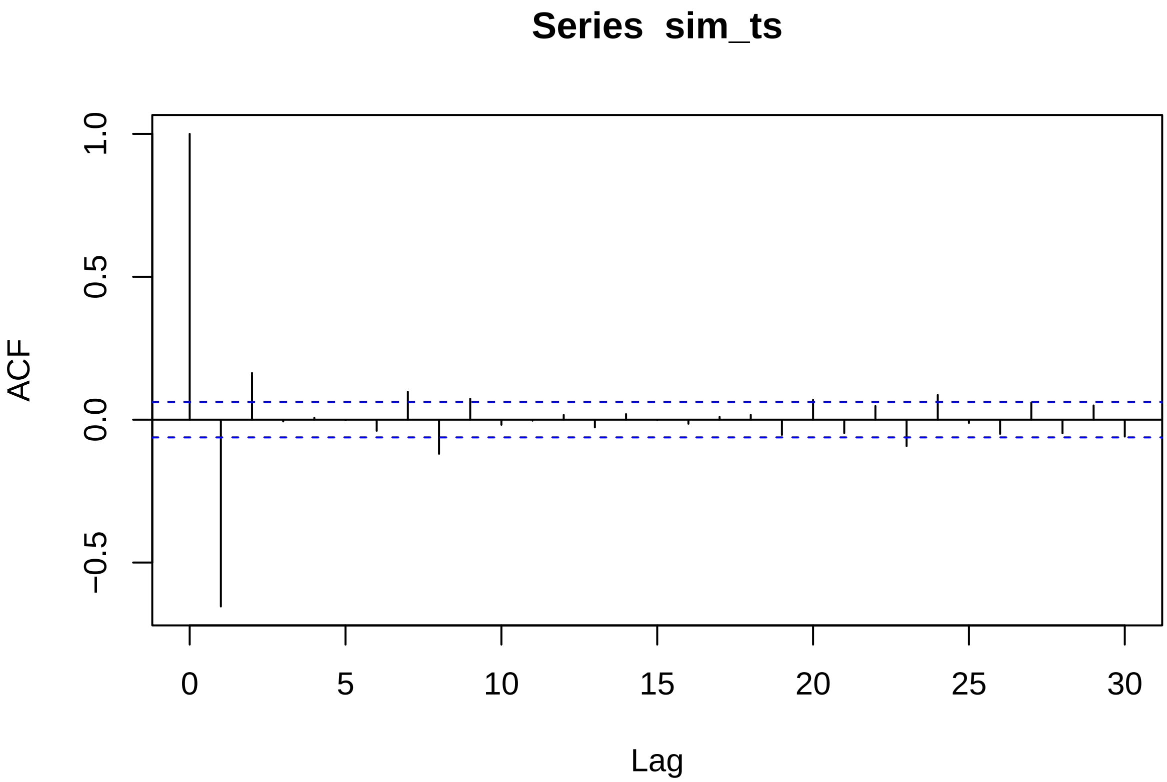
```
sim_data <- arima.sim(n = 1000, list(ma = c(-1.3,0.4), sd = sqrt(0.1)))
sim_ts <- ts(sim_data)
plot(sim_ts,
     main = "Time Series Plot for Simulated Data",
     ylab = "Simulated MA data")
```



Looking at the above time series plot, there is the presence of negative serial correlation, therefore the acf should have an alternating pattern that a positive value tends to be followed by a negative value.

b)

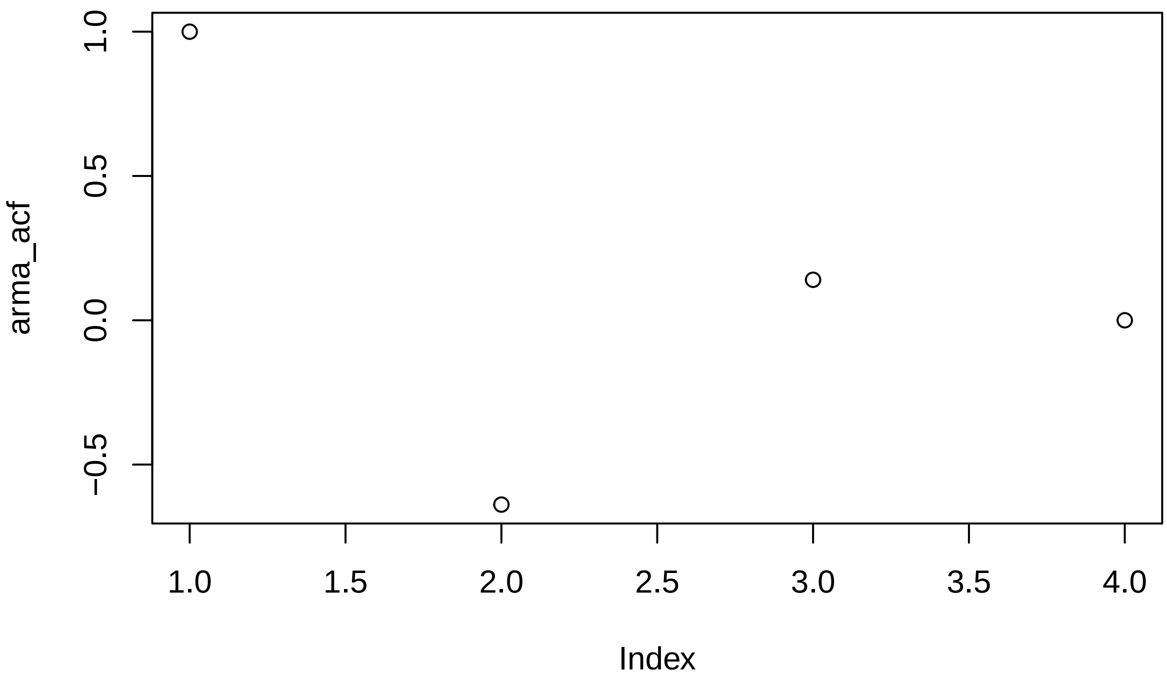
```r
acf(sim_ts)
```



Yes, the acf plot looks as what we would expect in part a).After repeating for a few times, we can see that the acf plot is composed of alternating positive and negative autocorrelation values.

c)

```r
arma_acf <- ARMAacf(ma = c(-1.3,0.4))
plot(arma_acf)
```



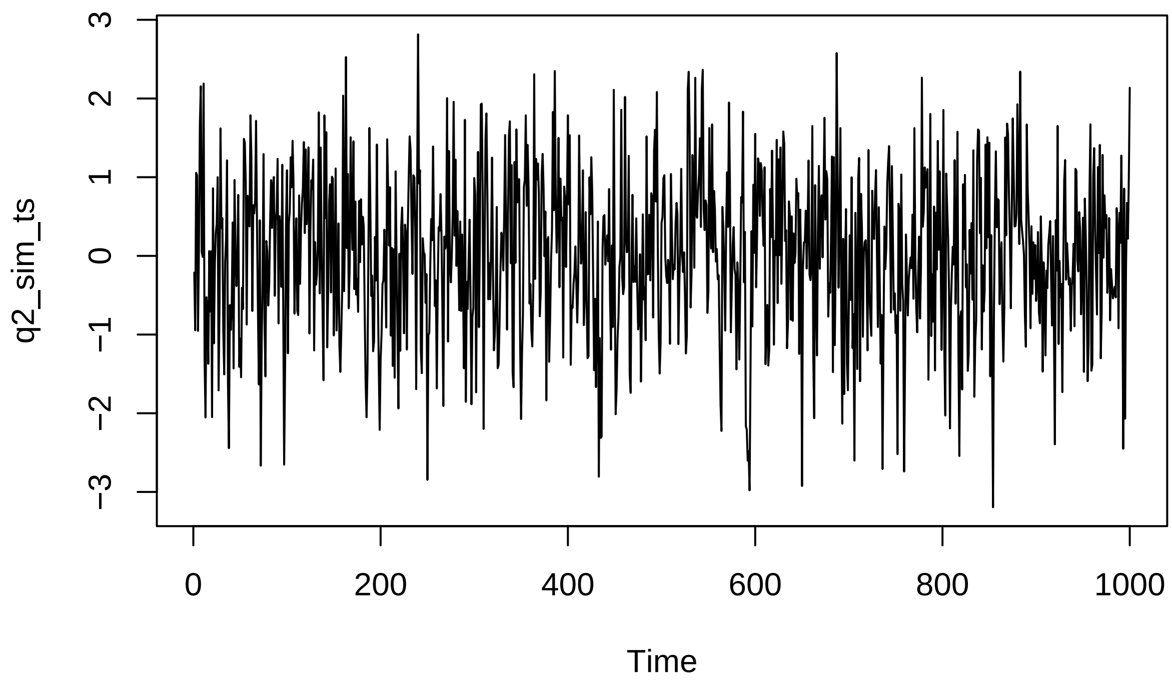
d)

The sample acf also shows a pattern of alternating values where one positive autocorrelation is followed by a negative autocorrelation which matches to the simulated data in b). We also see that sample acf decays with lag and at lag 4, the acf equals 0, this implies the time serie is stationary.

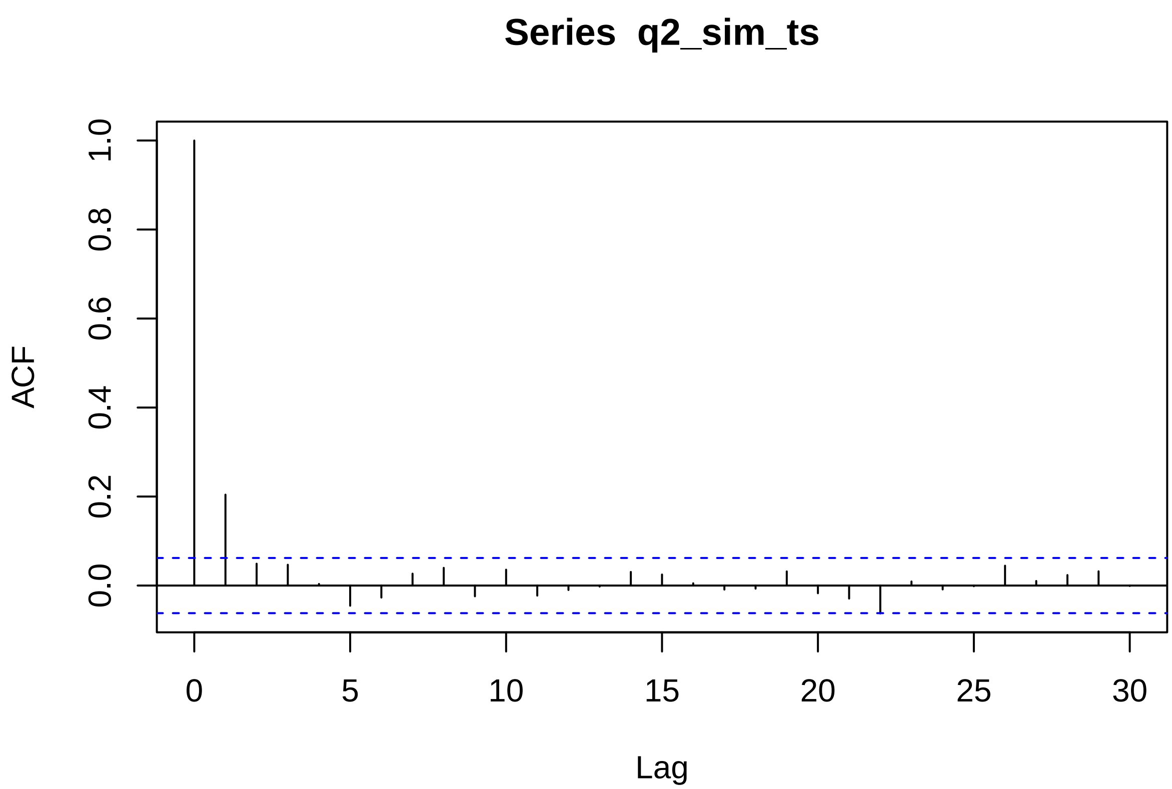
Question 2

a)

```
q2_sim <- arima.sim(n = 1000, list(ma = c(0.25), sd = sqrt(0.4)))
q2_sim_ts <- ts(q2_sim)
plot(q2_sim_ts)
```



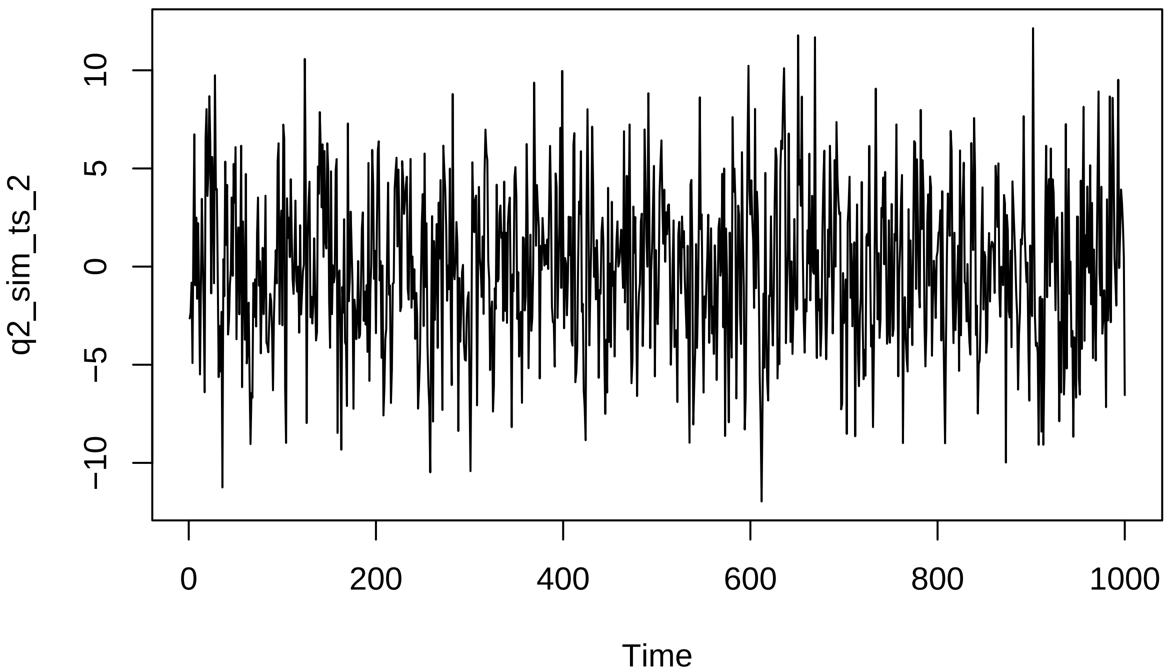

```
acf(q2_sim_ts)
```



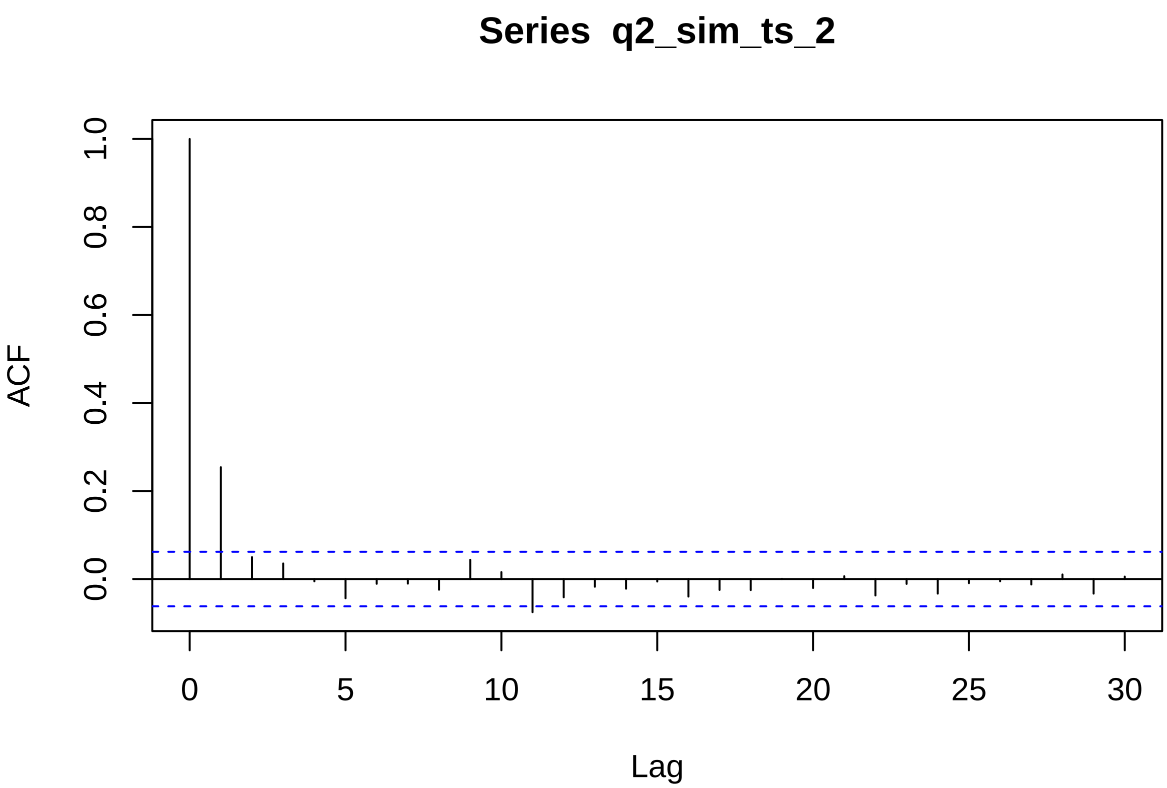
First of all, the sample acf decays with lag. Then we observe runs of positive and negative acfs, therefore it is likely that there is a positive temporal correlation in the simulated data.

b)

```
q2_sim_2 <- arima.sim(n = 1000, list(ma = c(4), sd = sqrt(0.4)))
q2_sim_ts_2 <- ts(q2_sim_2)
plot(q2_sim_ts_2)
```




```
acf(q2_sim_ts_2)
```



We observe the acf is decaying with lag and is approaching 0, therefore the time serie data is likely stationary. We also observe runs of positive and negative acfs, therefore it is likely that there is a positive temporal correlation in the simulated data.

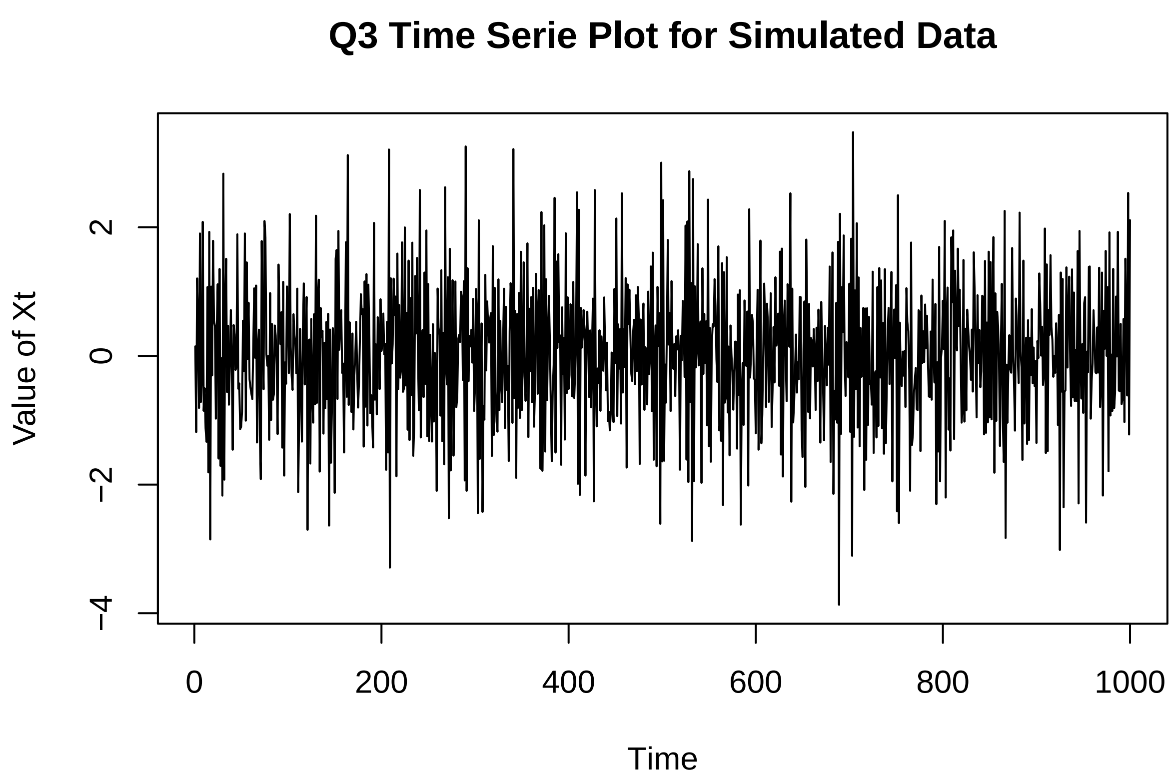
c)

part a) and b) look similar, this is because the coefficient in both models are the inverse of each other.

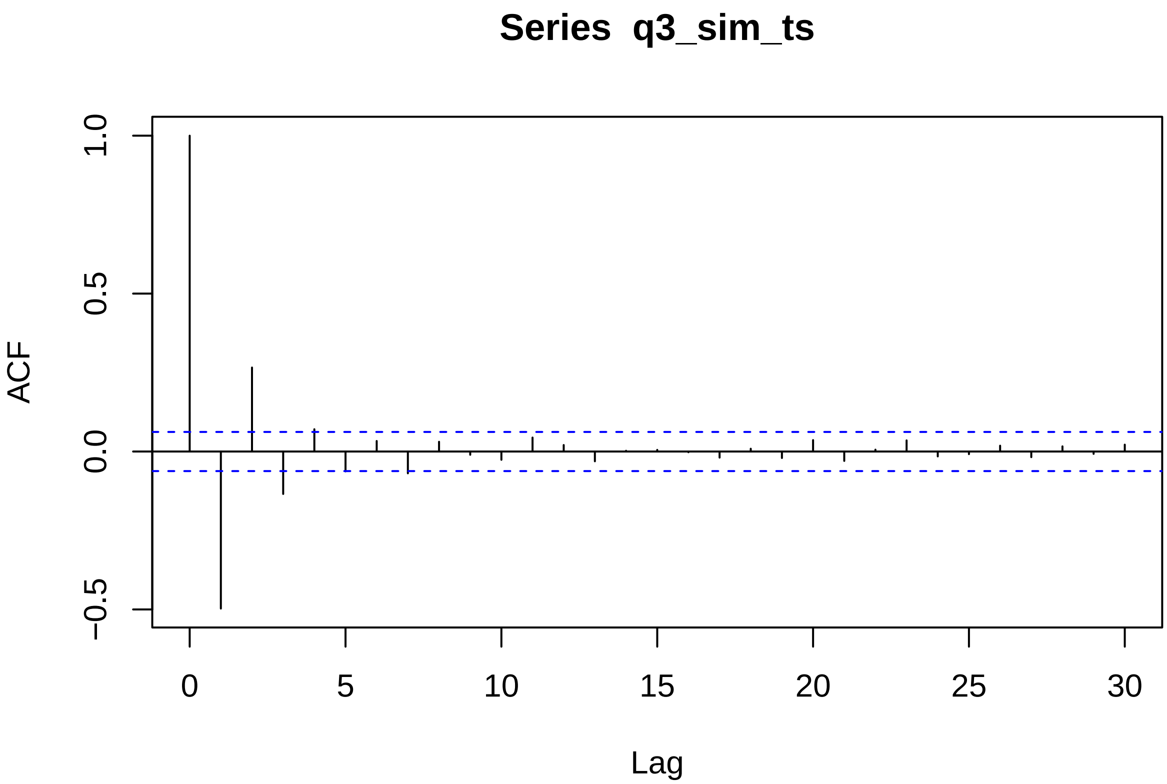
Question 3

a)

```
alpha = -0.5
q3_sim <- arima.sim(n = 1000, list(ar = c(alpha), sd = 0.1))
q3_sim_ts <- ts(q3_sim)
plot(q3_sim_ts,
     ylab = "Value of Xt",
     main = "Q3 Time Serie Plot for Simulated Data")
```



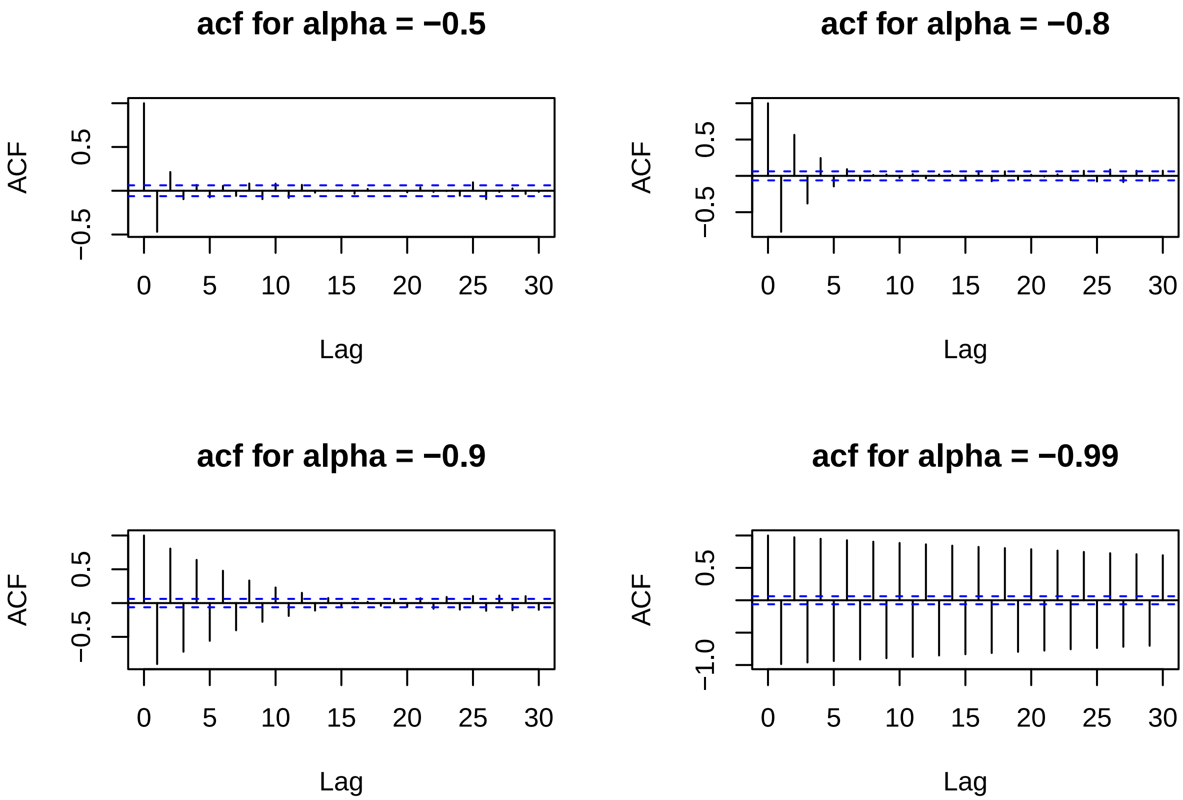

```
acf(q3_sim_ts)
```



The acf values are alternating where a positive autocorrelation coefficient is followed by a negative autocorrelation coefficient, this indicates that this time series data exhibits negative temporal correlation. We also observe that the acf values decay with lag and approaches zero as lag increases.

b)

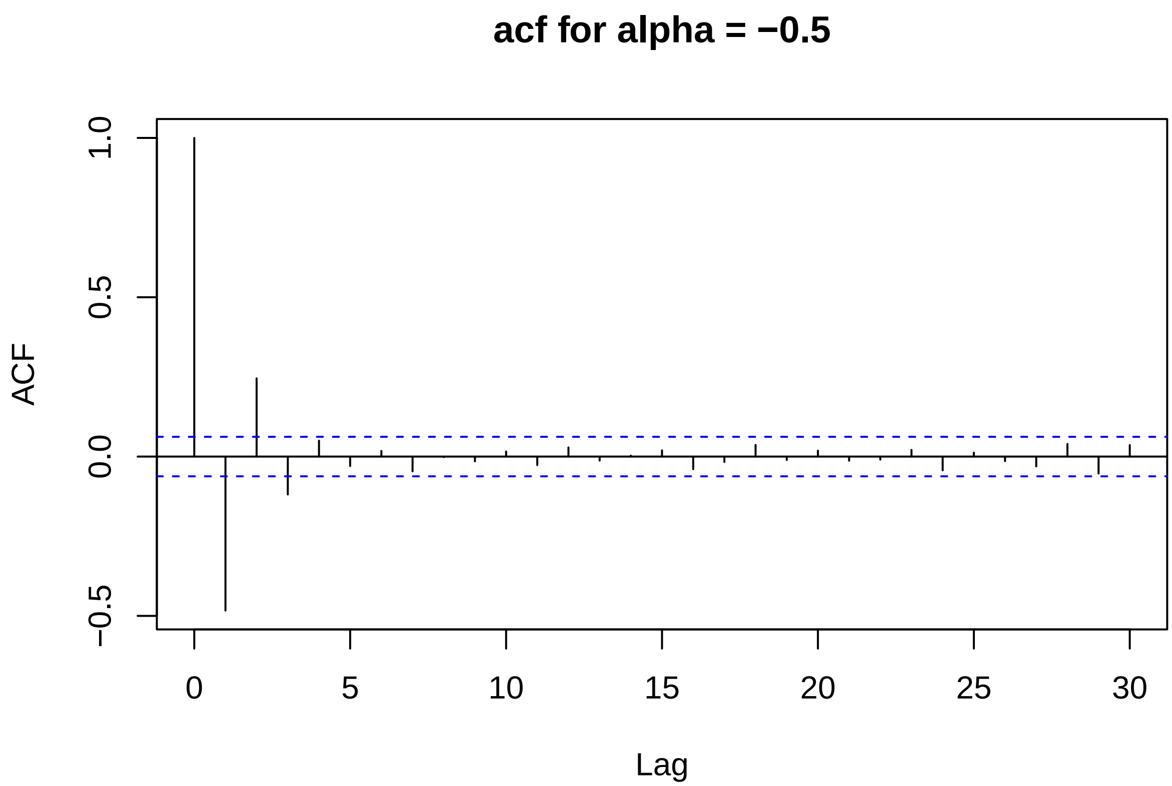
```
par(mfrow = c(2,2))
alpha = c(seq(from = -0.5, to = -1, by = -0.1))
q3b_sim_ts_1 = ts(arima.sim(n = 1000, list(ar = c(alpha[1]), sd = 0.1)))
q3b_sim_ts_4 = ts(arima.sim(n = 1000, list(ar = c(alpha[4]), sd = 0.1)))
q3b_sim_ts_5 = ts(arima.sim(n = 1000, list(ar = c(alpha[5]), sd = 0.1)))
q3b_sim_ts_6 = ts(arima.sim(n = 1000, list(ar = c(-0.99), sd = 0.1)))

acf(q3b_sim_ts_1, main = "acf for alpha = -0.5")
acf(q3b_sim_ts_4, main = "acf for alpha = -0.8")
acf(q3b_sim_ts_5, main = "acf for alpha = -0.9")
acf(q3b_sim_ts_6, main = "acf for alpha = -0.99")
```



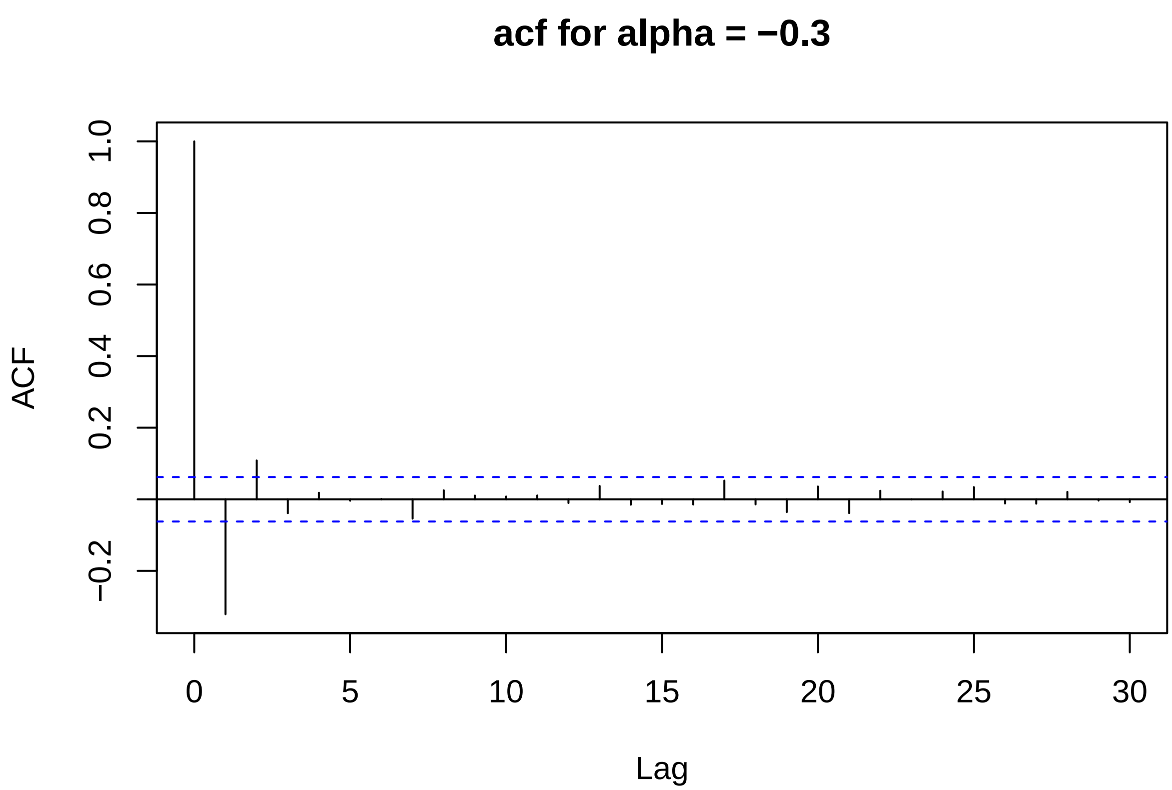
As *alpha* $\to -1$, the decay of autocorrelation coefficient becomes more and more slowly. We see a stronger negative temporal correlation in the time series data. Also we can see that when *alpha* $\to -1$, the time series becomes non stationary.

c)

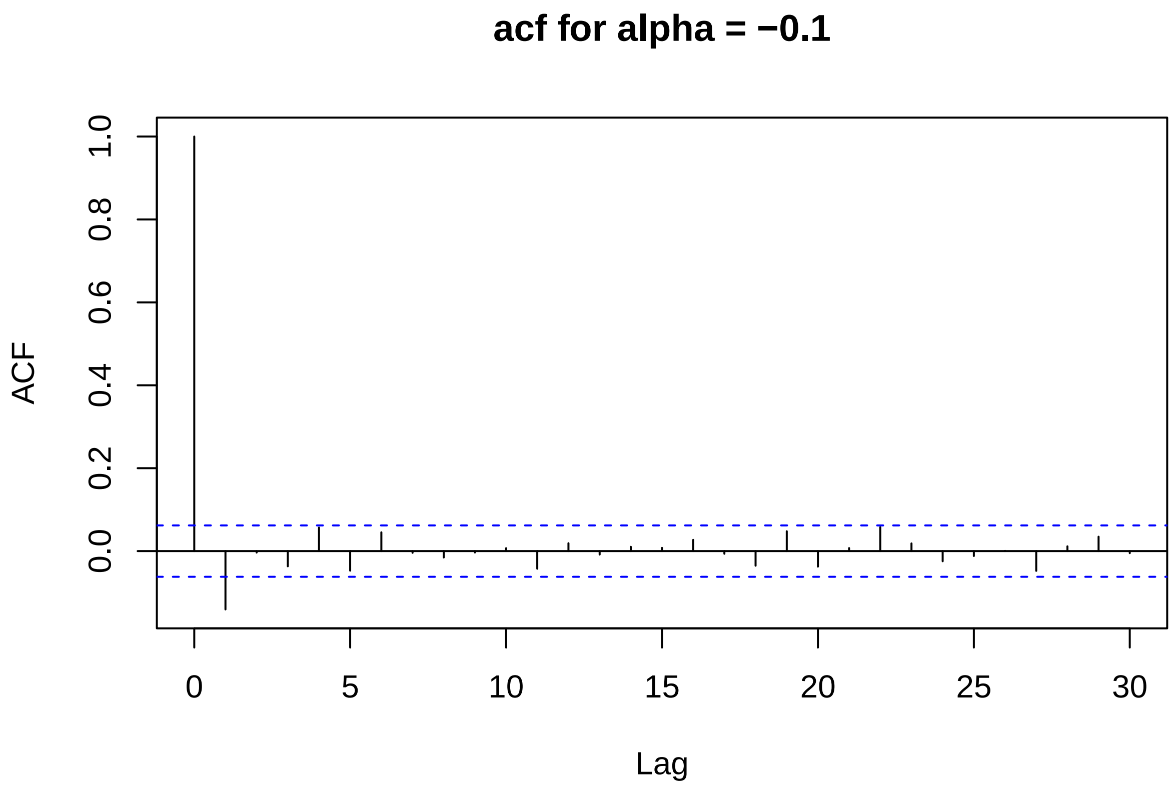
```
q3c_sim_ts_1 = ts(arima.sim(n = 1000, list(ar = c(-0.5), sd = 0.1)))
q3c_sim_ts_4 = ts(arima.sim(n = 1000, list(ar = c(-0.3), sd = 0.1)))
q3c_sim_ts_5 = ts(arima.sim(n = 1000, list(ar = c(-0.1), sd = 0.1)))
q3c_sim_ts_6 = ts(arima.sim(n = 1000, list(ar = c(-0.01), sd = 0.1)))

acf(q3c_sim_ts_1, main = "acf for alpha = -0.5")
```



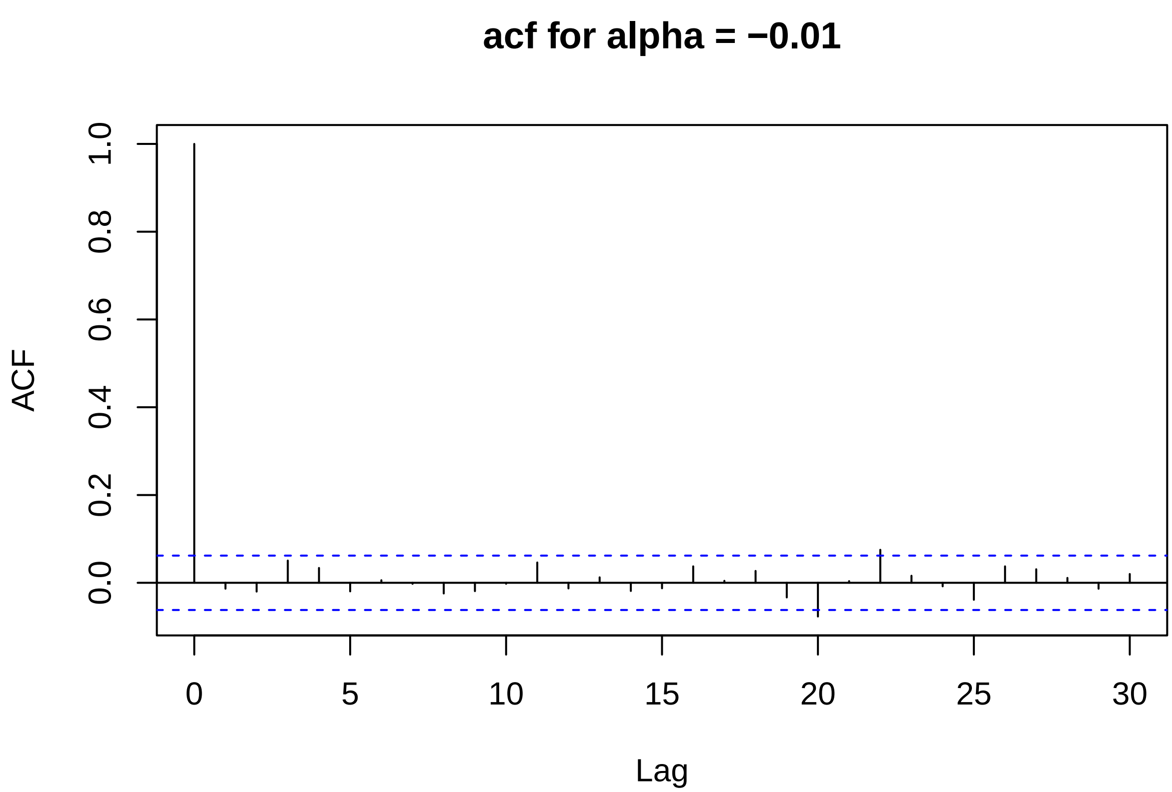

```r
acf(q3c_sim_ts_4, main = "acf for alpha = -0.3")
```



```r
acf(q3c_sim_ts_5, main = "acf for alpha = -0.1")
```



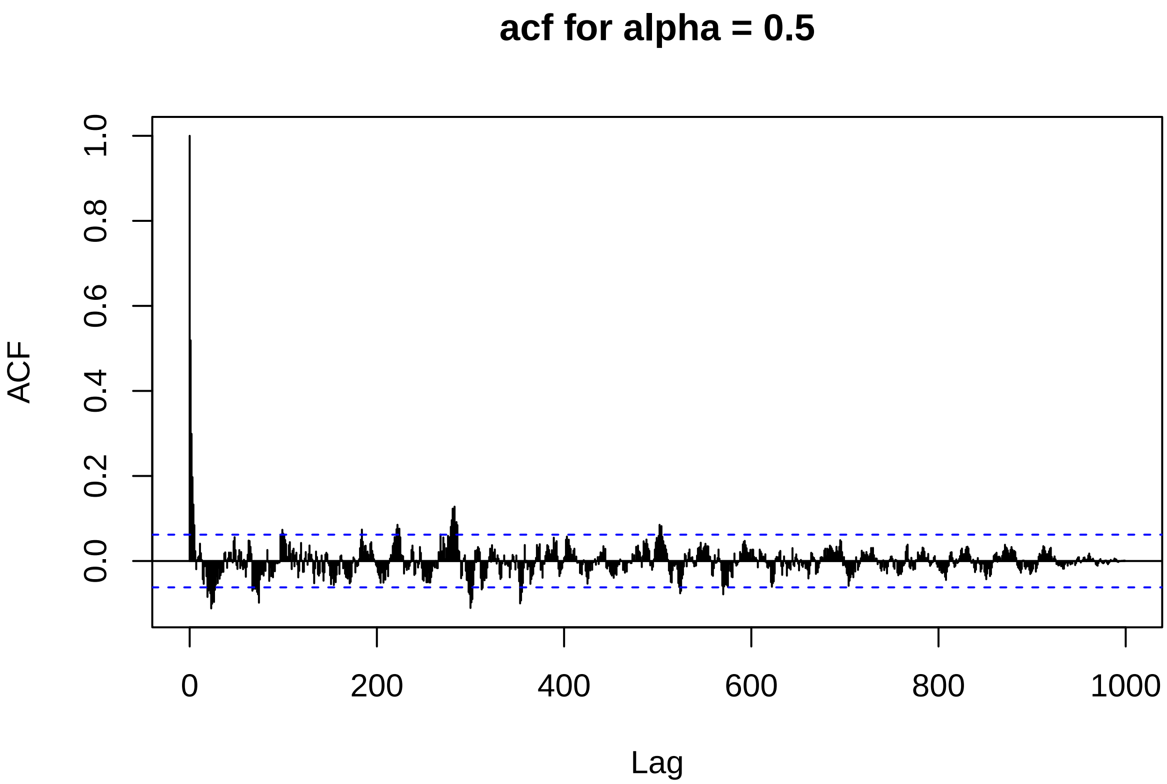

```r
acf(q3c_sim_ts_6, main = "acf for alpha = -0.01")
```



As $alpha \to 0$, the decay of autocorrelation coefficient becomes more rapid. As we can see that when $alpha \to 0$, the time series becomes more stationary.

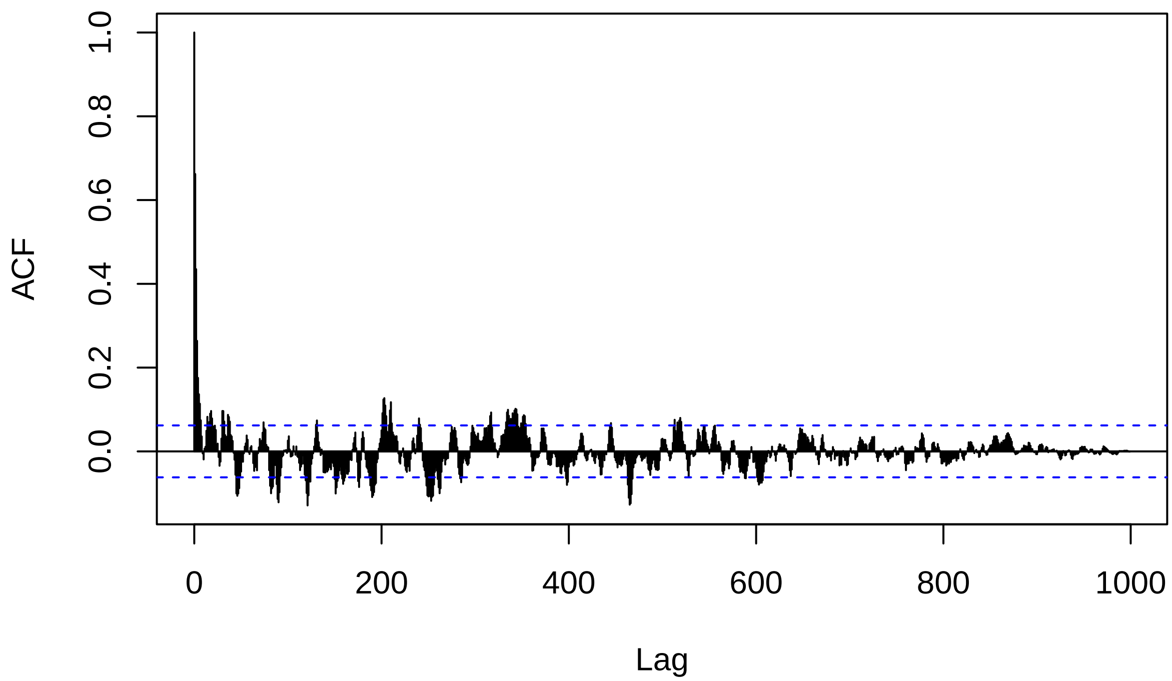
d)

```
q3d_sim_ts_1 = ts(arima.sim(n = 1000, list(ar = c(0.5), sd = 0.1)))
q3d_sim_ts_2 = ts(arima.sim(n = 1000, list(ar = c(0.7), sd = 0.1)))
q3d_sim_ts_3 = ts(arima.sim(n = 1000, list(ar = c(0.9), sd = 0.1)))
q3d_sim_ts_4 = ts(arima.sim(n = 1000, list(ar = c(0.99), sd = 0.1)))

acf(q3d_sim_ts_1,lag.max = 999, main = "acf for alpha = 0.5")
```



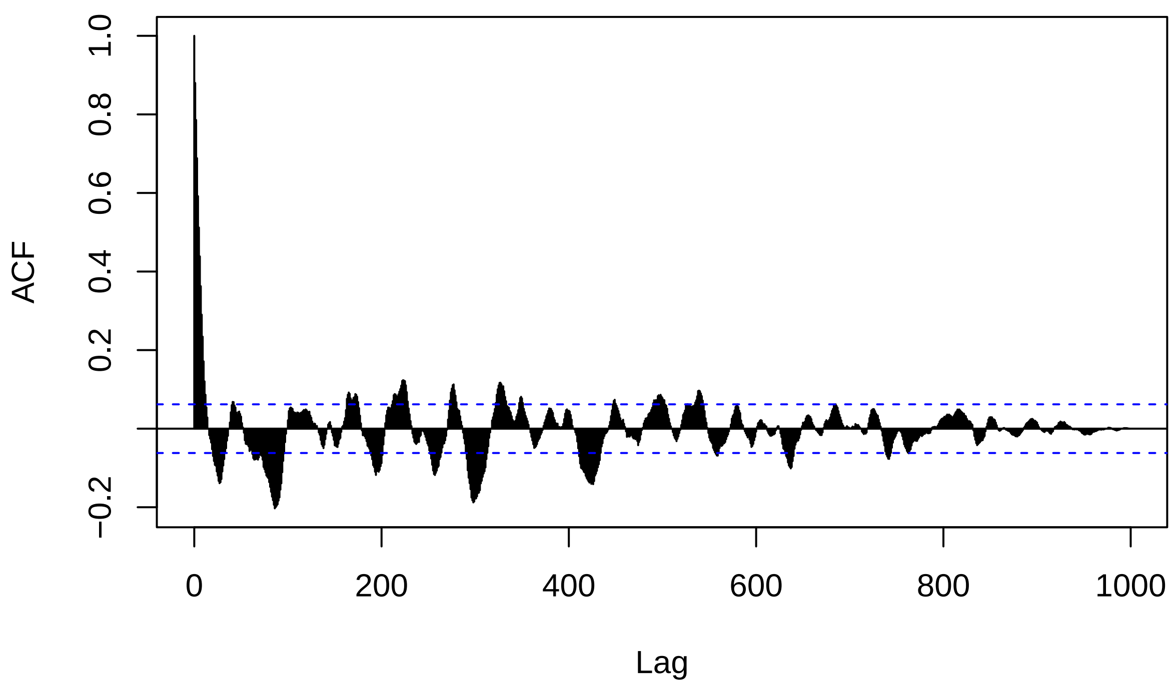

```
acf(q3d_sim_ts_2,lag.max = 999, main = "acf for alpha = 0.7")
```

acf for alpha = 0.7

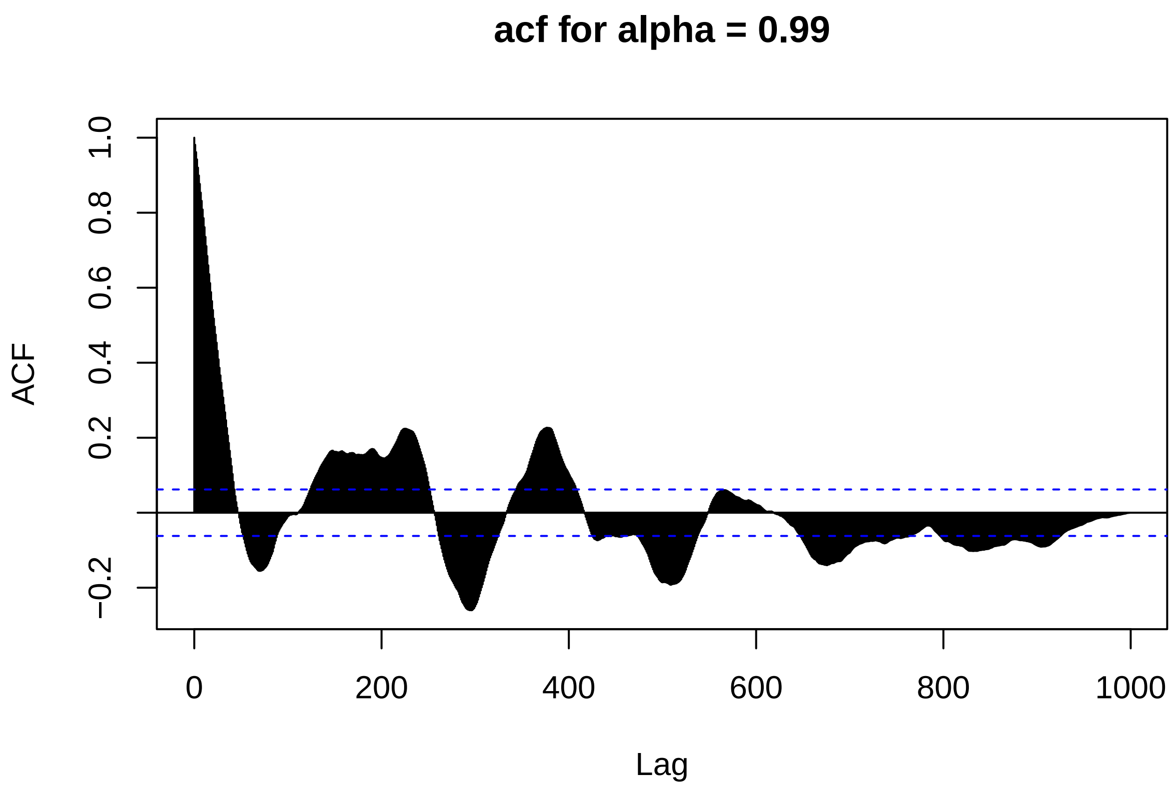



```r
acf(q3d_sim_ts_3,lag.max = 999, main = "acf for alpha = 0.9")
```

acf for alpha = 0.9

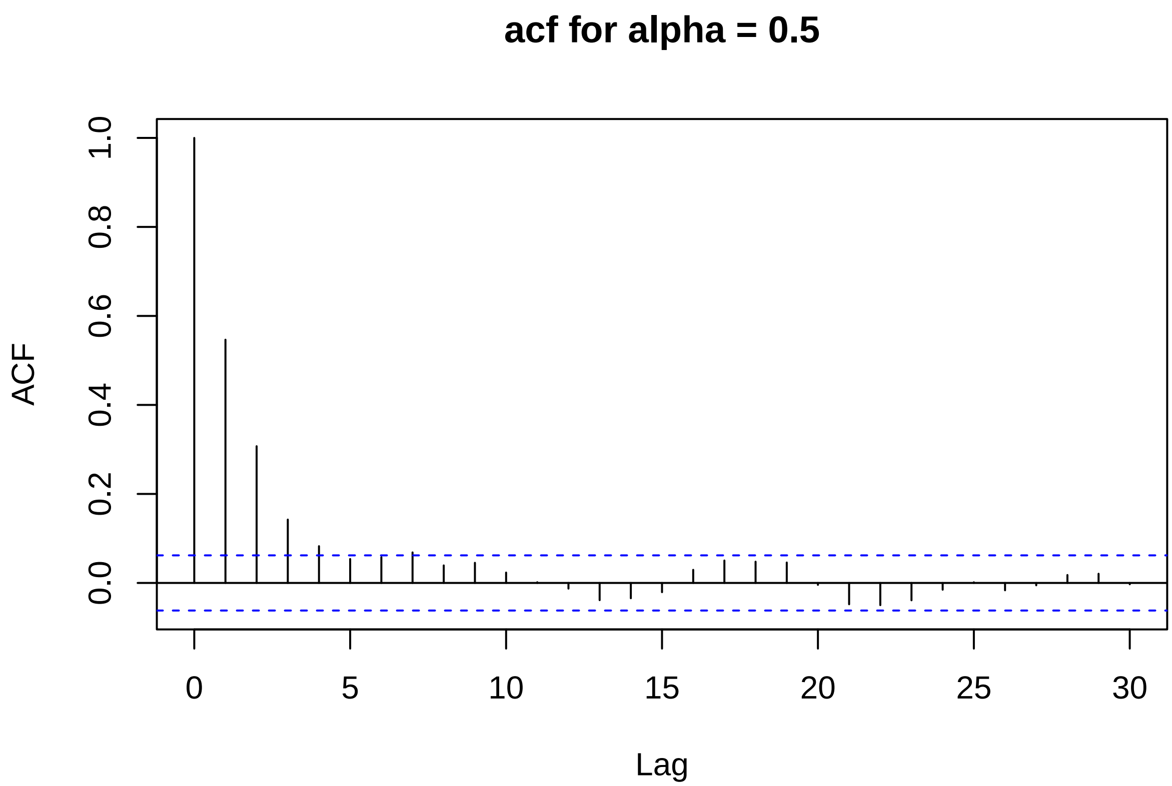


```r
acf(q3d_sim_ts_4,lag.max = 999,  main = "acf for alpha = 0.99")
```

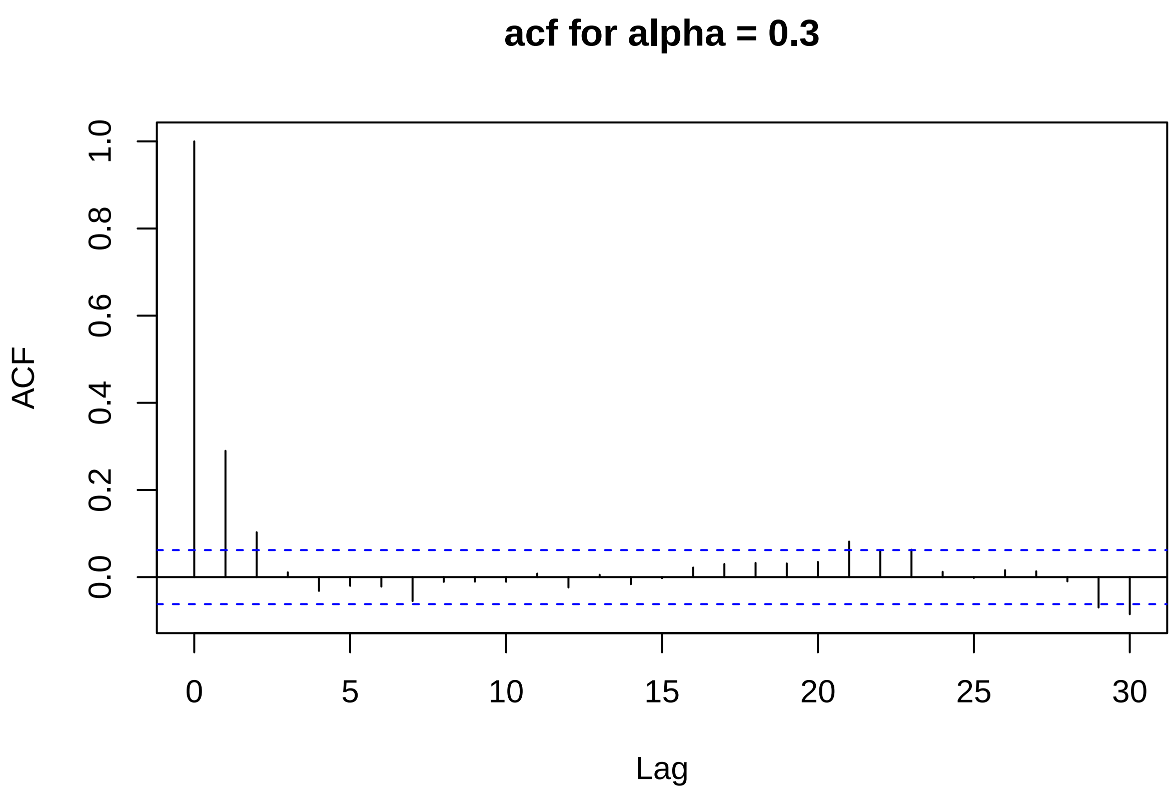


When $\alpha \to 1$, we observe that the decay of autocorrelation coefficients become more and more slowly. The time series data exhibit a stronger and stronger positive temporal correlation in the data. The time series become more non-stationary.

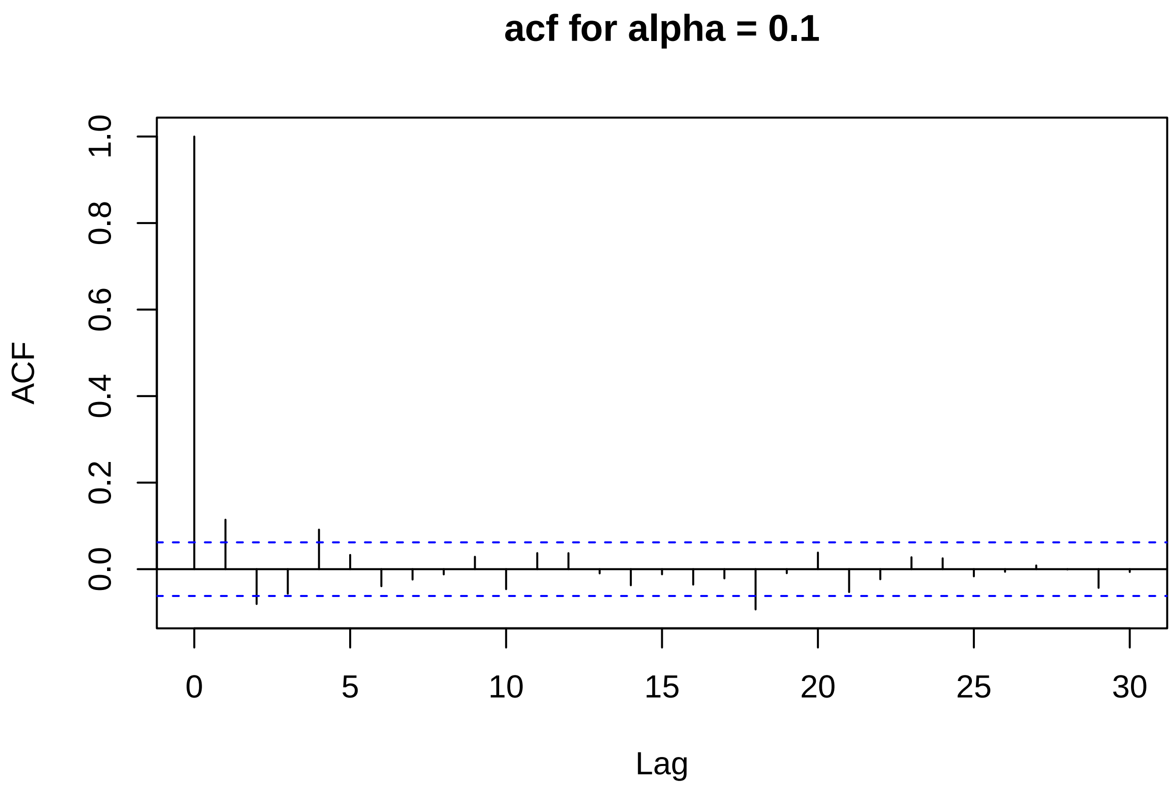
```r
q3d_sim_ts_5 = ts(arima.sim(n = 1000, list(ar = c(0.5), sd = 0.1)))
q3d_sim_ts_6 = ts(arima.sim(n = 1000, list(ar = c(0.3), sd = 0.1)))
q3d_sim_ts_7 = ts(arima.sim(n = 1000, list(ar = c(0.1), sd = 0.1)))
q3d_sim_ts_8 = ts(arima.sim(n = 1000, list(ar = c(0.01), sd = 0.1)))

acf(q3d_sim_ts_5, main = "acf for alpha = 0.5")
```



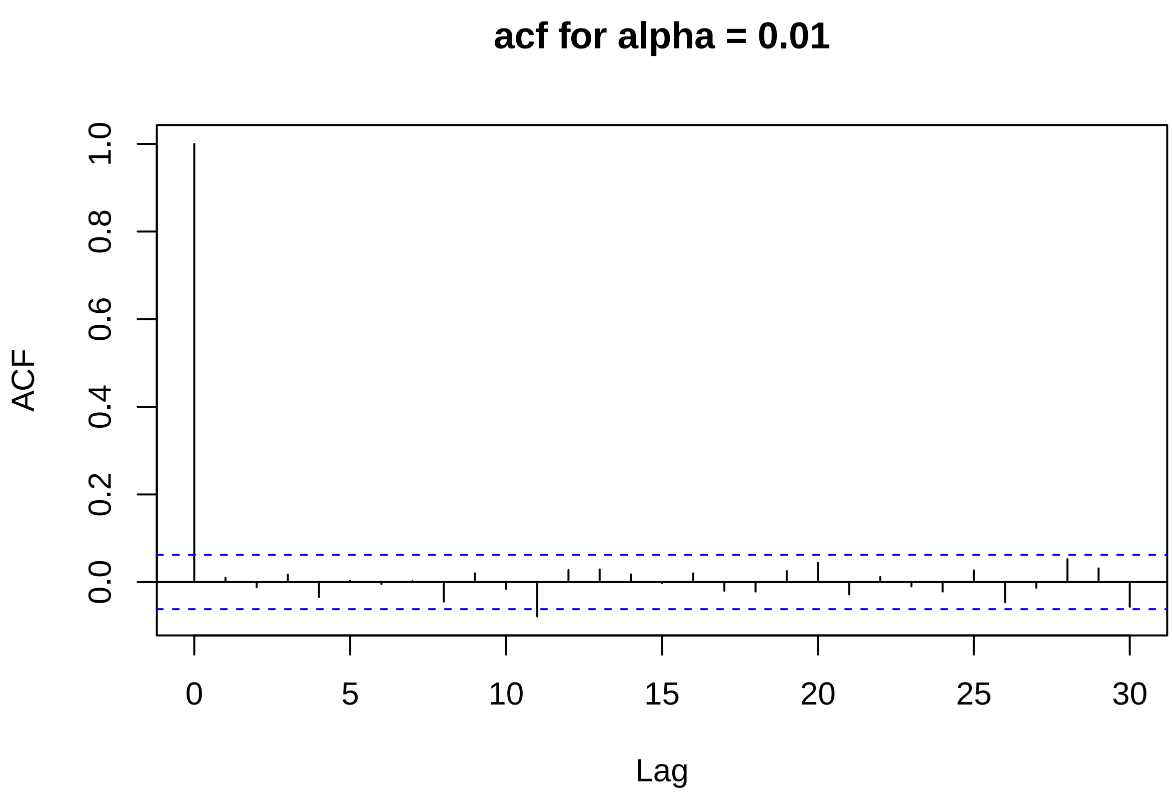

```
acf(q3d_sim_ts_6, main = "acf for alpha = 0.3")
```



```r
acf(q3d_sim_ts_7, main = "acf for alpha = 0.1")
```




```r
acf(q3d_sim_ts_8, main = "acf for alpha = 0.01")
```



As $alpha \to 0$ from $\alpha = 0.5$, we observe similar acfs as in part c), the decay of autocorrelation coefficient becomes more rapid. As we can see that when $alpha \to 0$, the time series becomes more stationary.