STAT 443: Assignment 1

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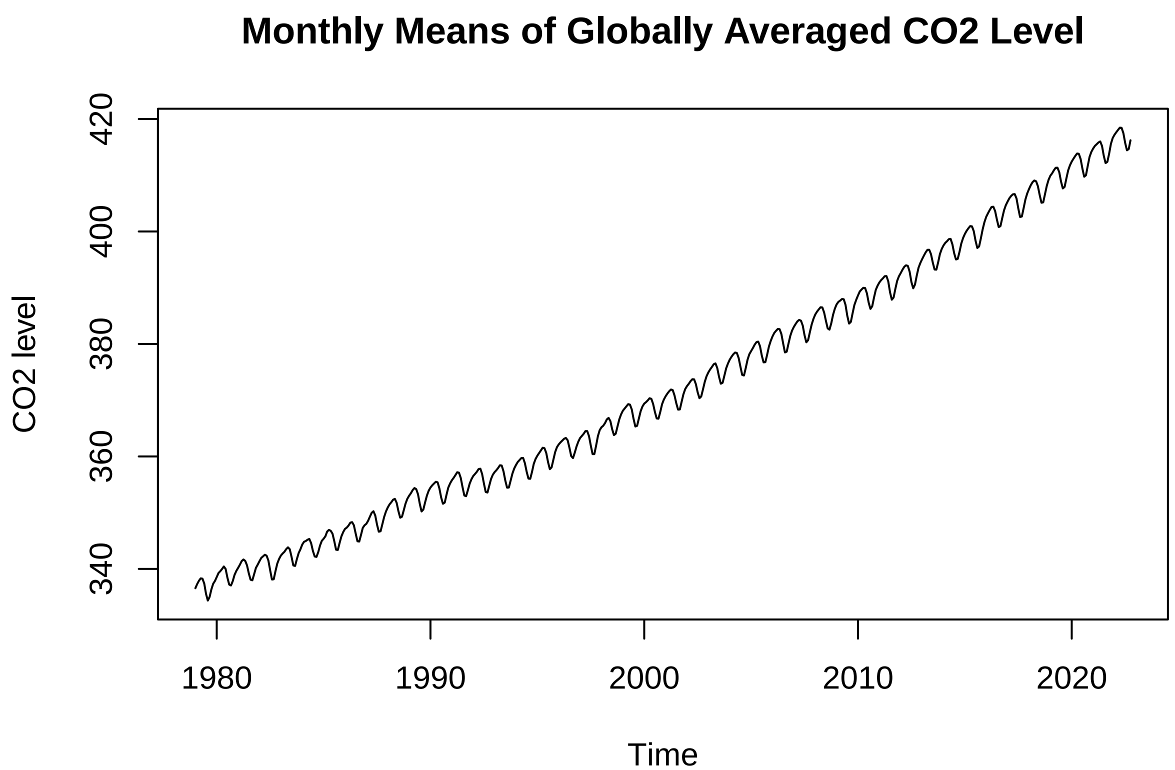
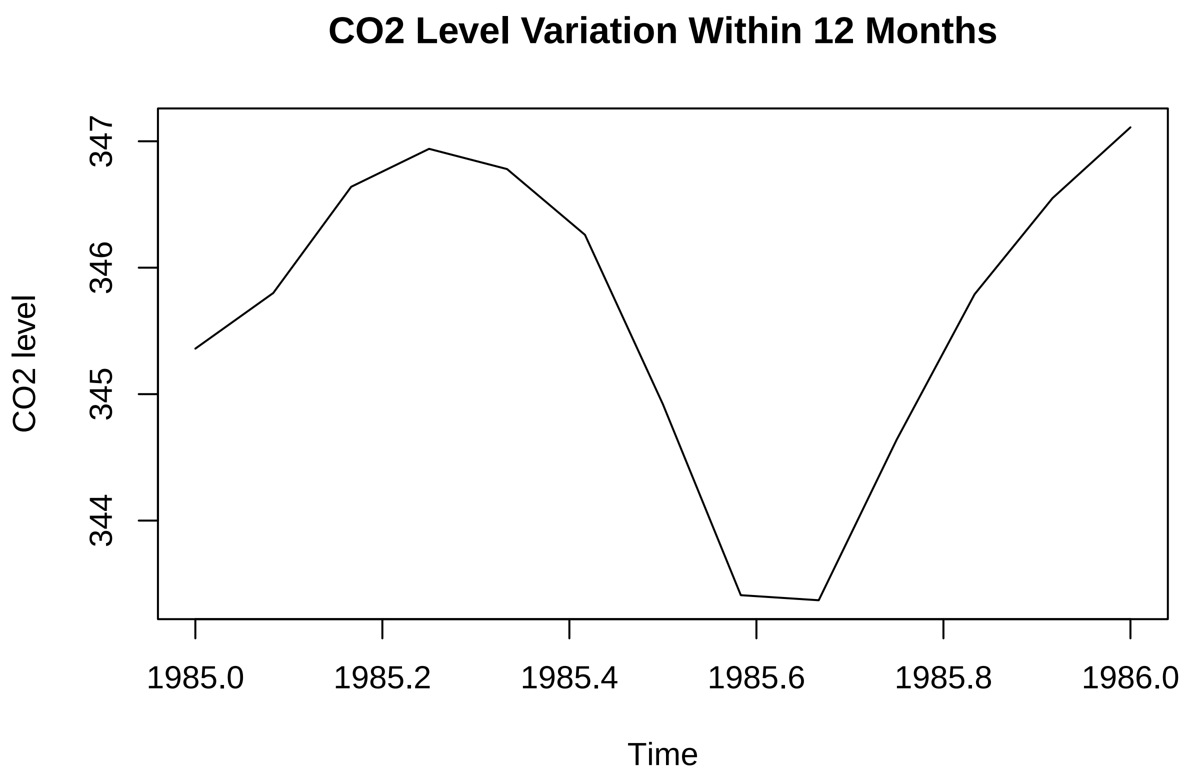
Question 1

a)

```
co2 <- read.csv("co2_mm_gl.csv", header = TRUE, skip = 55)
co2_ts <- ts(co2[,4], start = c(1979,1),frequency = 12)
plot(co2_ts,
     main = "Monthly Means of Globally Averaged CO2 Level",
     ylab = "CO2 level")
```




```
plot(window(co2_ts, start = c(1985,1),end = c(1986,1)),
     main = "CO2 Level Variation Within 12 Months",
     ylab = "CO2 level")
```



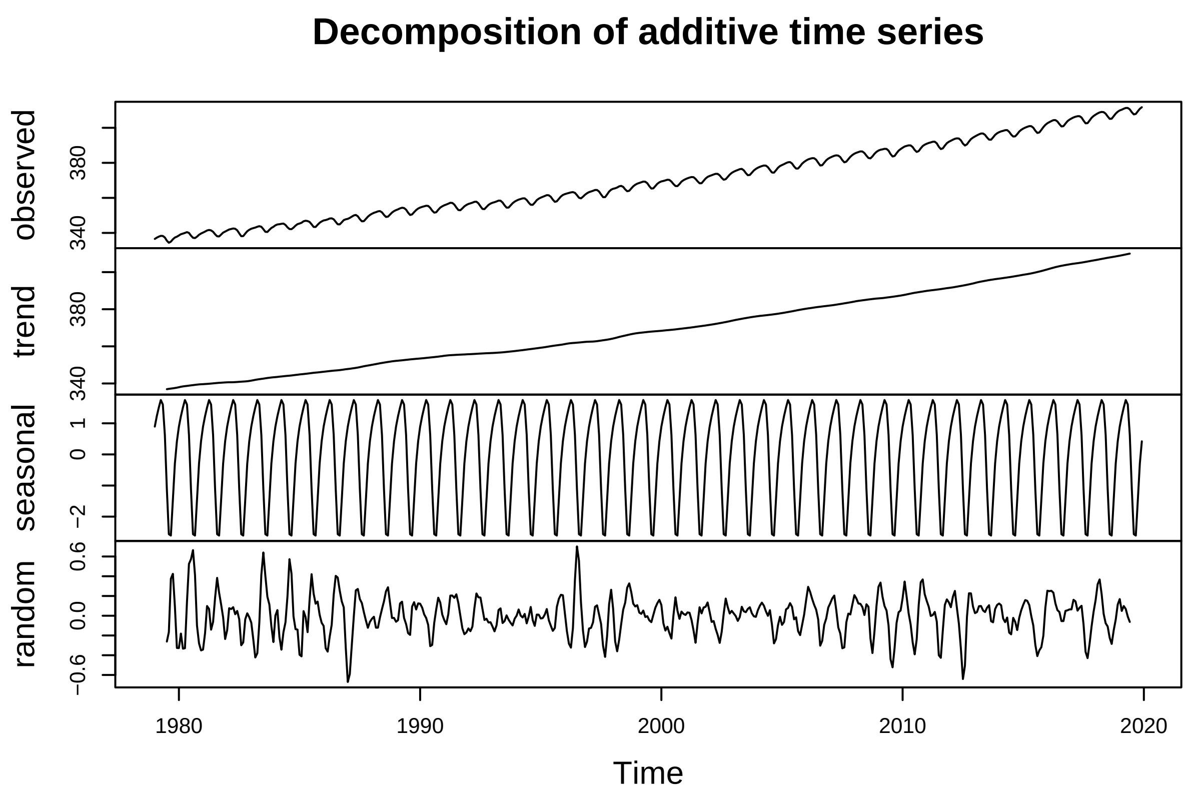
i) The above time series have a clear **upward trend** such that the monthly means of globally averaged CO2 level is increasing every year despite some variations.

ii) There appears to be **seasonal variations** in the monthly CO2 level as when we restrict the plot to display the average CO2 level over a 12-month period, we can clear see the CO2 level is high around March and December, low around June.**An additive model is more suitable** as we can see the seasonal effect remains constant over time and the error is also constant over time, therefore an additive model would be more appropriate than a multiplicative model.

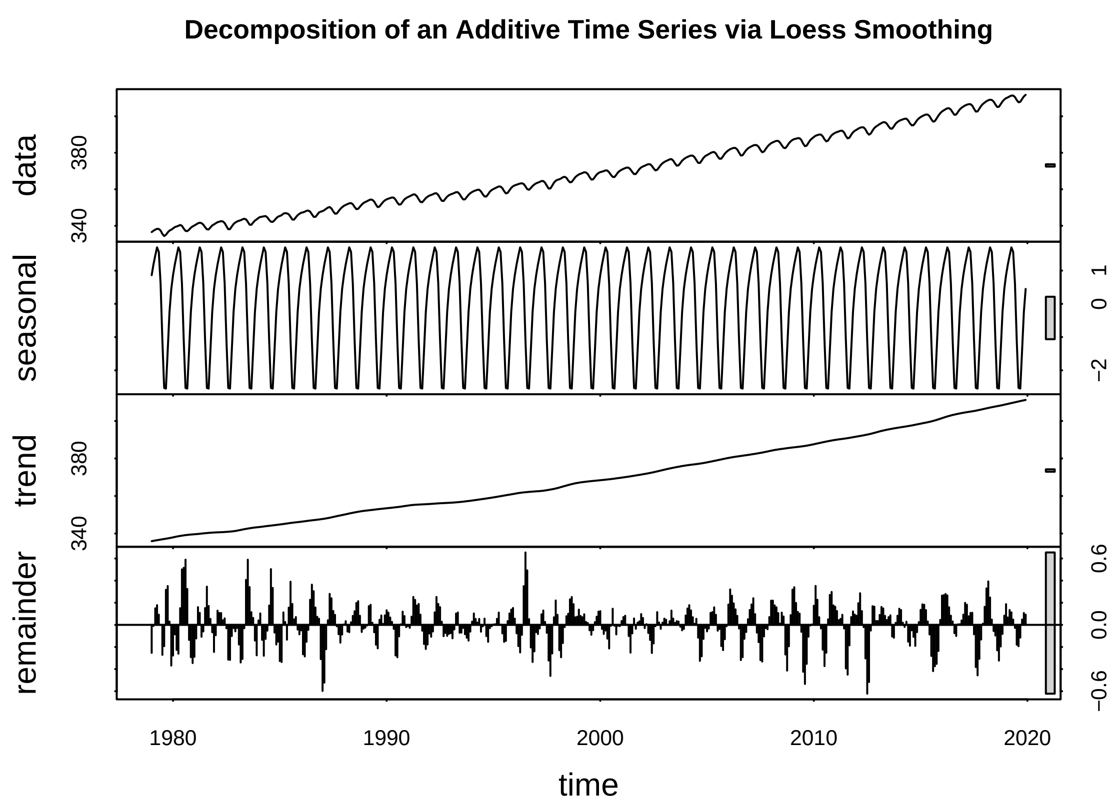
iii) **No**, the series have a clear upward increasing trend therefore it is not stationary.

b)

```
co2_train <- window(co2_ts,start = c(1979,1),end = c(2019,12),frequency = 12)
co2_test <- window(co2_ts, start = c(2020,1), end = c(2022,10), frequency = 12)

co2_train_decom <- decompose(co2_train, type = "additive")
plot(co2_train_decom)
```




```r
co2_train_loess <- stl(co2_train,s.window = "periodic")
plot(co2_train_loess,
     main = "Decomposition of an Additive Time Series via Loess Smoothing")
```



c)

```r
# MA method
ma_trend <- co2_train_decom$trend
ma_seas <- co2_train_decom$seasonal
ma_error <- co2_train_decom$random
# Loess smoothing
loess_trend <- co2_train_loess$time.series[,"trend"]
loess_seas <- co2_train_loess$time.series[,"seasonal"]
loess_error <- co2_train_loess$time.series[,"remainder"]
# creating data frame
lm_data <- data.frame(ma_trend = ma_trend,
                      loess_trend = loess_trend,
                      time = c(1:length(co2_train)))
# Fitted Models
lm_ma <- lm(ma_trend~time, data = lm_data)
lm_loess <- lm(loess_trend~time, data = lm_data)
summary(lm_ma)
```

```
##
## Call:
## lm(formula = ma_trend ~ time, data = lm_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9933 -1.6701 -0.3312  1.2281  4.7441
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.329e+02  1.819e-01  1830.0   <2e-16 ***
## time        1.488e-01  6.433e-04   231.3   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.953 on 478 degrees of freedom
##   (12 observations deleted due to missingness)
## Multiple R-squared:  0.9911, Adjusted R-squared:  0.9911
## F-statistic: 5.349e+04 on 1 and 478 DF,  p-value: < 2.2e-16
```

```r
summary(lm_loess)
```

```
##
## Call:
## lm(formula = loess_trend ~ time, data = lm_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.0771 -1.7471 -0.3894  1.2344  4.9349
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.329e+02  1.832e-01  1817.6   <2e-16 ***
## time        1.491e-01  6.439e-04   231.5   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.028 on 490 degrees of freedom
## Multiple R-squared:  0.9909, Adjusted R-squared:  0.9909
## F-statistic: 5.361e+04 on 1 and 490 DF,  p-value: < 2.2e-16
```

i) Fitted model using MA method:

$$\hat{m_t} = 0.03329 + 0.1488 * t$$

Fitted model using loess smoothing method:

$$\hat{m_t} = 0.03329 + 0.1491 * t$$

ii) The trend component is significant at 95% confidence level under both method.

iii) I think the trend component is a good predictor because the trend is linear over the entire time range of the time series data looking at the trend component plot in part b); second, from the output of the linear models we observe that $R^2 > 0.99$f ro both models. This indicates that majority of the variation within the time series is explained by the trend component.

iv)

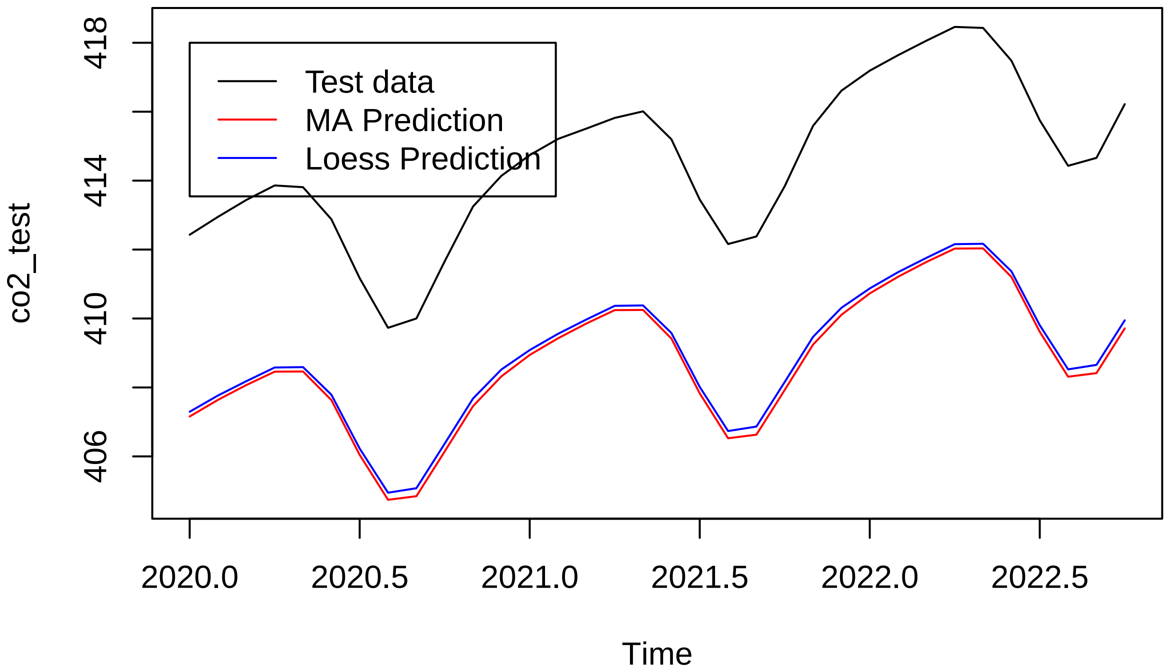
```
t_test <- data.frame(time = max(lm_data$time) + c(1:length(co2_test)))
ma_trend_pred <- predict(lm_ma, newdata = t_test)
loess_trend_pred <- predict(lm_loess, newdata = t_test)

# estimate seasonal effect
seasonals <- tibble(ma_sea_hat = ma_seas,
                        loess_sea_hat = loess_seas,
                        season = rep(1:12,41)) %>%
  group_by(season) %>%
  summarise(shat_ma = mean(ma_sea_hat),
            shat_loess = mean(loess_sea_hat))

ma_pred <- ts(ma_trend_pred + seasonals$shat_ma,
              start = c(2020,1),
              frequency = 12)
```

```
## Warning in ma_trend_pred + seasonals$shat_ma: longer object length is not a
## multiple of shorter object length
```

```
loess_pred <- ts(loess_trend_pred + seasonals$shat_loess,
                 start = c(2020,1),
                 frequency = 12)
```

```
## Warning in loess_trend_pred + seasonals$shat_loess: longer object length is not
## a multiple of shorter object length
```

```r
plot(co2_test, ylim = c(min(co2_test,ma_pred,loess_pred), max(co2_test,ma_pred,loess_pred)))
lines(ma_pred,col = "red")
lines(loess_pred, col = "blue")
legend(2020,
       418,
       legend = c("Test data", "MA Prediction", "Loess Prediction"),
       lty = c("solid","solid","solid"),
       col = c("black", "red","blue"))
```




```r
# MSPE
MSPE <- tibble(ma_pred = ma_pred,
               loess_pred = loess_pred,
               observed = co2_test) %>%
  mutate(sqr_diff_ma = (observed - ma_pred)^2,
         sqr_diff_loess = (observed - loess_pred)^2) %>%
  summarise(mspe_ma = mean(sqr_diff_ma),
            mspe_loess = mean(sqr_diff_loess))
MSPE
```

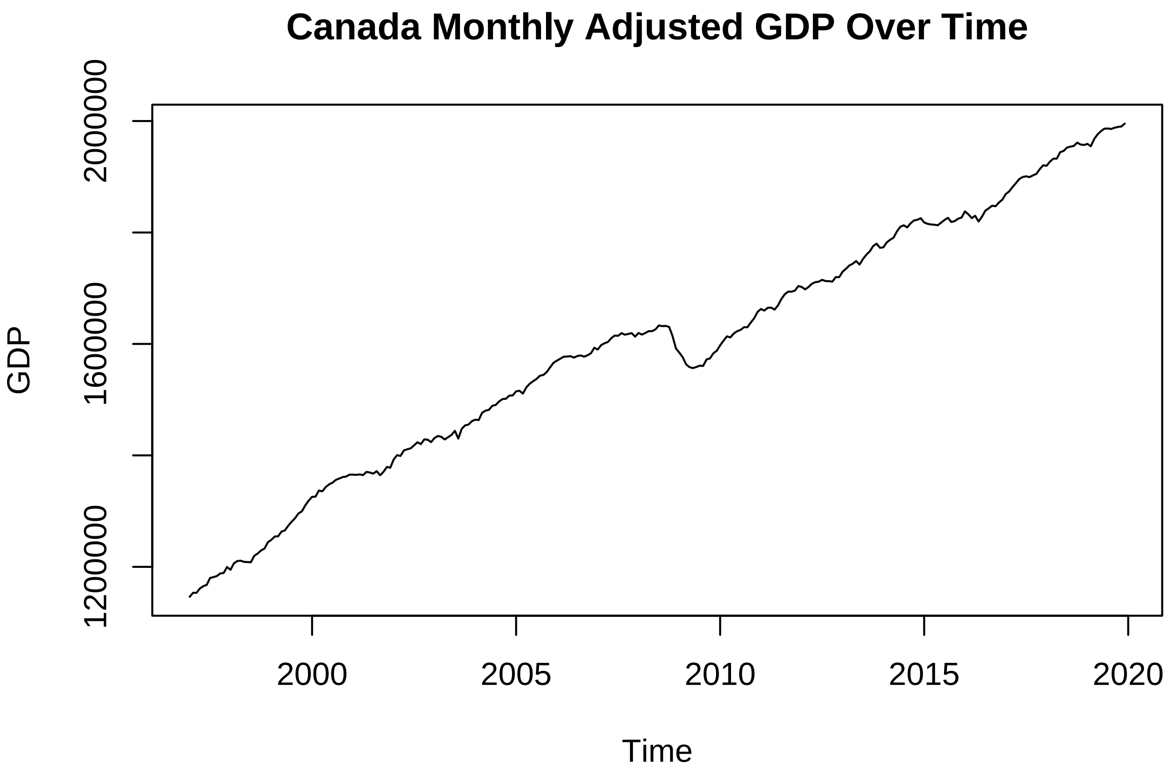
```
## # A tibble: 1 x 2
##   mspe_ma mspe_loess
##     <dbl>      <dbl>
## 1    34.1       32.1
```

Looking at the above table, the loess smoothing method result in lower MSPE on the test data. Therefore I would recommend to use the prediction model from the loess smoothing decomposition.

Question 2

a)

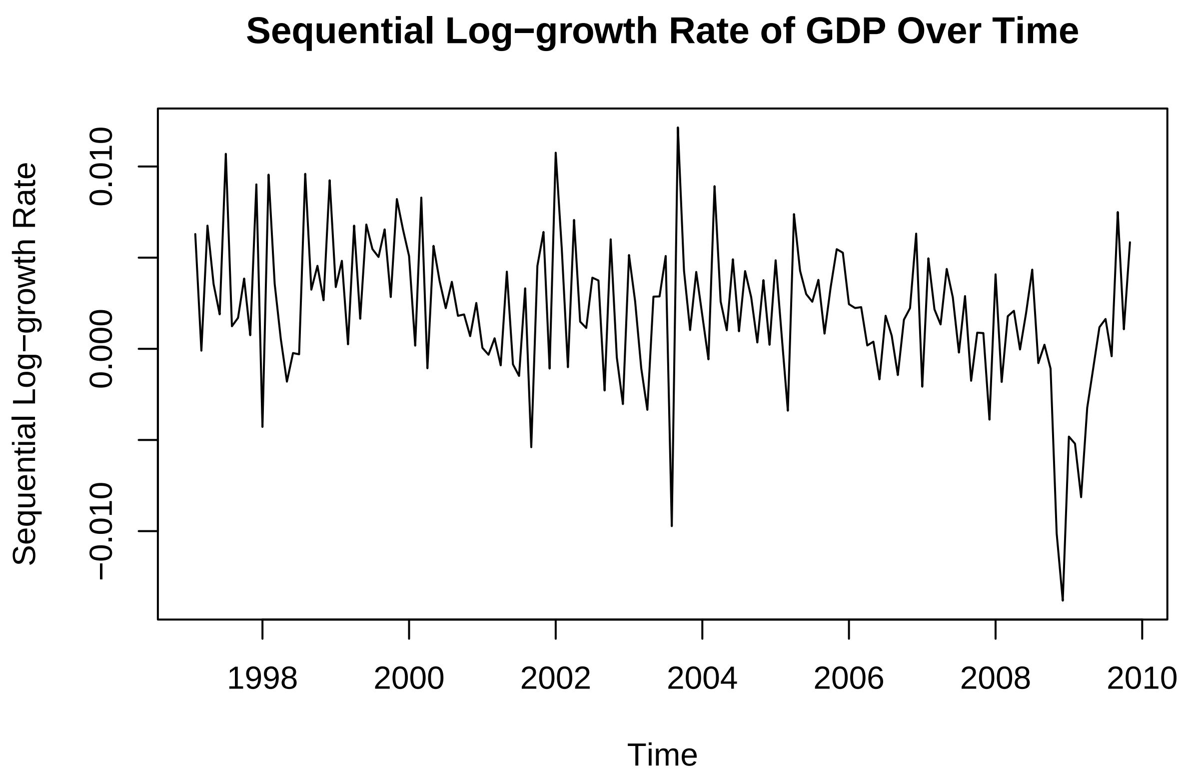
```r
gdp <- read_csv("CanandaGDP-2.csv",
                skip = 1,
                col_names = c("Time", "GDP"),
                show_col_types = FALSE)
gdp_ts <- ts(gdp[,2],start = c(1997,1), end = c(2019,12), frequency = 12)
plot(gdp_ts, main = "Canada Monthly Adjusted GDP Over Time")
```



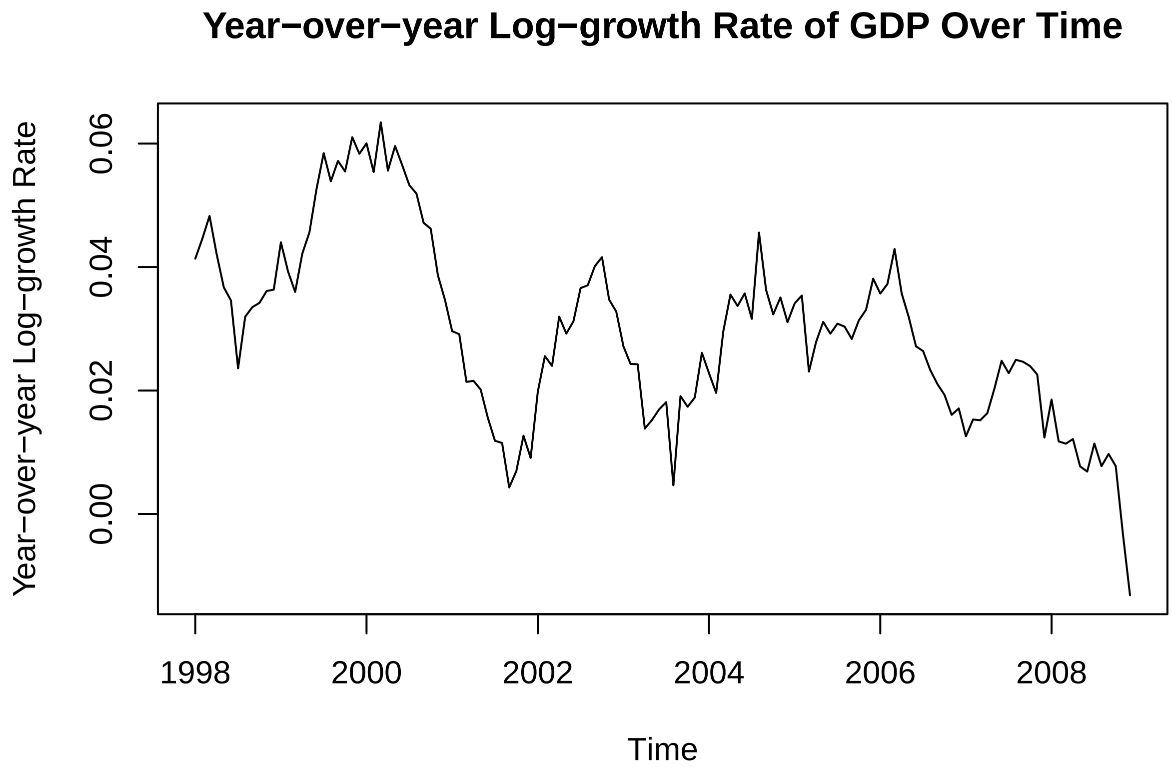
This time series is likely to be non-stationary because the present of a clear upward trend.

b)

```r
n = length(gdp$GDP)
seq_log_growth <- ts(log((gdp$GDP[-1])/(gdp$GDP[-n])),
                     start = c(1997,2),
                     end = c(2009,11),
                     frequency = 12)
plot(seq_log_growth,
     ylab = "Sequential Log-growth Rate",
     main = "Sequential Log-growth Rate of GDP Over Time")
```




```r
year_over_year_log_growth <- ts(log((gdp$GDP[-c(1:12)])/(gdp$GDP[-c((n-11):n)])),
                                start = c(1998,1),
                                end = c(2008,12),
                                frequency = 12)
plot(year_over_year_log_growth,
     ylab = "Year-over-year Log-growth Rate",
     main = "Year-over-year Log-growth Rate of GDP Over Time")
```

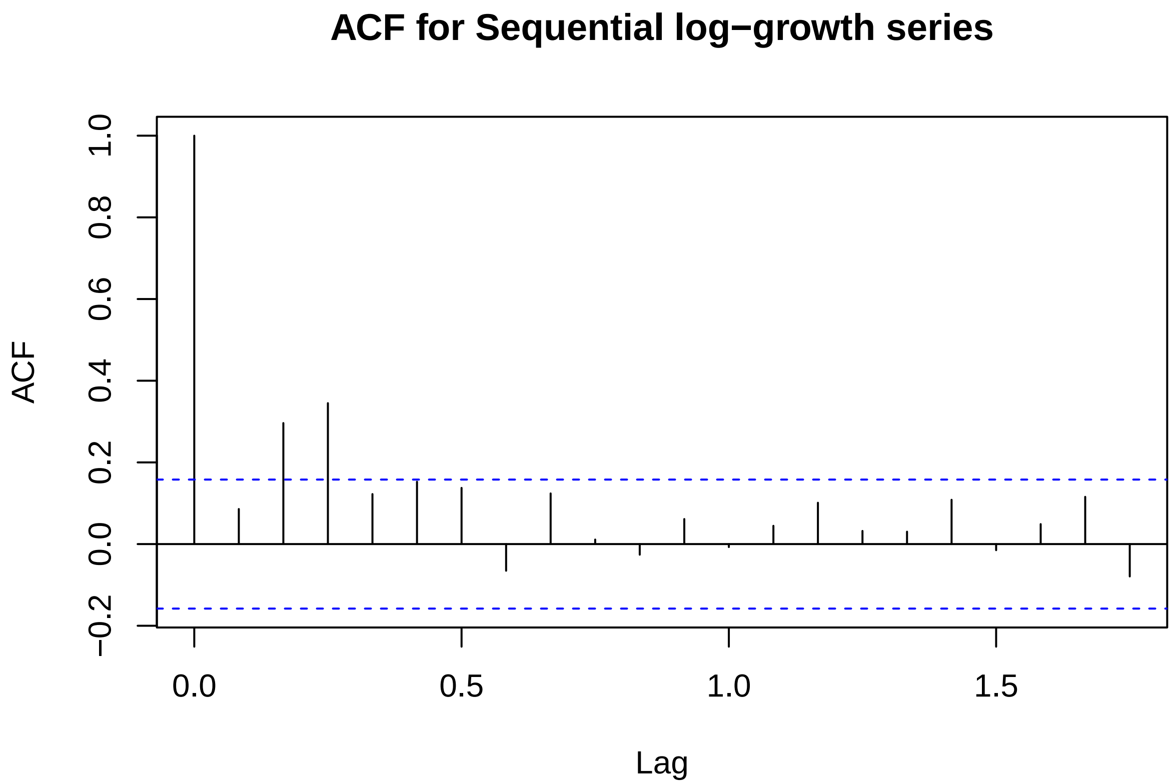


i) Looking at the above plots of log-growth rate series, the sequential log-growth rate series appear to be stationary because there is no apparent trend and the mean of log-growth rate appears to be constant. Whereas the year-over-year log-growth rate series appear to be non-stationary because the presence of a downward trend in log-growth rate over time.

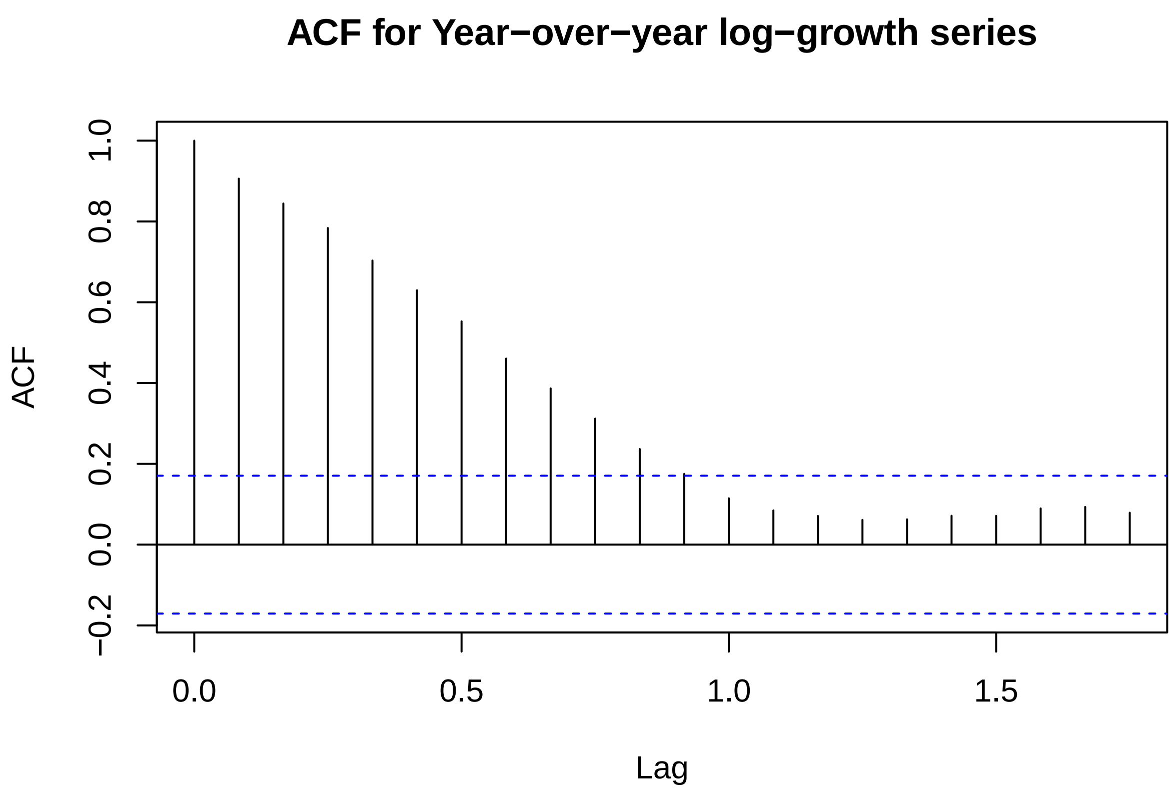
ii) The sequential log-growth rate definition is preferable, this is because the sequential log-growth transformation results in a new time series that is stationary.

iii)

```
acf(seq_log_growth,main = "ACF for Sequential log-growth series")
```




```r
acf(year_over_year_log_growth, main = "ACF for Year-over-year log-growth series")
```

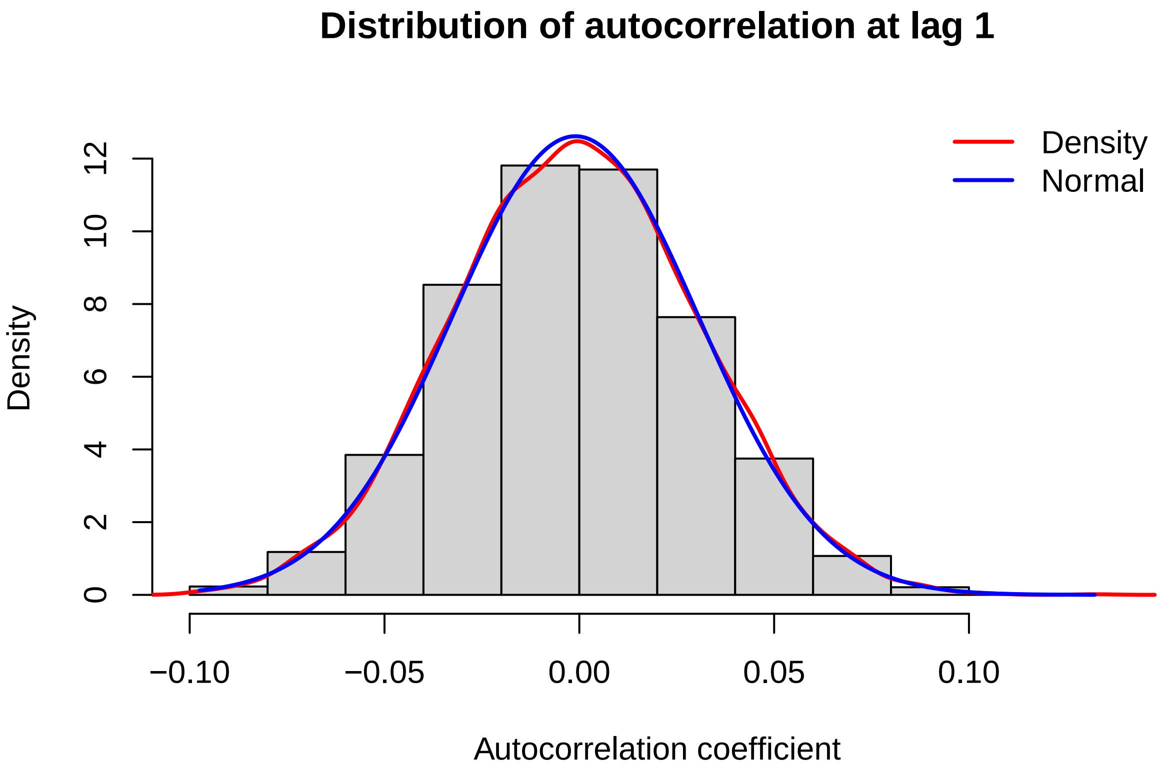


i) In the correlogram for sequential log-growth series, we observe the auto-correlation decays rapidly with lag and approaches toward 0, and most of the auto-correlations fall within the $\pm 2/\sqrt{n}$ bound. This suggests that there is likely no trend present in the series and the series is stationary.

ii) In the correlogram for year-over-year log-growth series, we observe that the auto-correlation decays slowly with lag. All of the auto-correlations are positive and many exceed the $\pm 2/\sqrt{n}$ bound. This suggests that there is strong positive temporal dependence or a upward trend present in the series and the series is likely non-stationary.

Question 3

```r
m = 5000
n = 1000
rhos <- c(rep(0,m))
for (i in 1:m) {
  sim_ts <- ts(rnorm(n))
  sim_ts_acf <- acf(sim_ts, lag.max = 1, plot = FALSE)
  rhos[i] <- sim_ts_acf$acf[2]
}

x <- seq(min(rhos), max(rhos), length = n)
f <- dnorm(x, -1/n, sqrt(1/n))
hist(rhos,
     freq = FALSE,
     ylim = c(0,13),
     xlab = "Autocorrelation coefficient",
     main = "Distribution of autocorrelation at lag 1")
lines(density(rhos), col = "red",lwd = 2)
lines(x, f, col = "blue", lwd = 2)
legend("topright",
       c("Density", "Normal"),
       box.lty = 0,
       lty = 1,
       col = c("red", "blue"),
       lwd = c(2, 2))
```



Yes, the empirical sampling distribution and the asymptotic normal density agrees. As we can see from the above plot, the empirical sampling density (red line) is very close to the theoretical value (blue line).