STAT 443: Lab 5

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Question 1

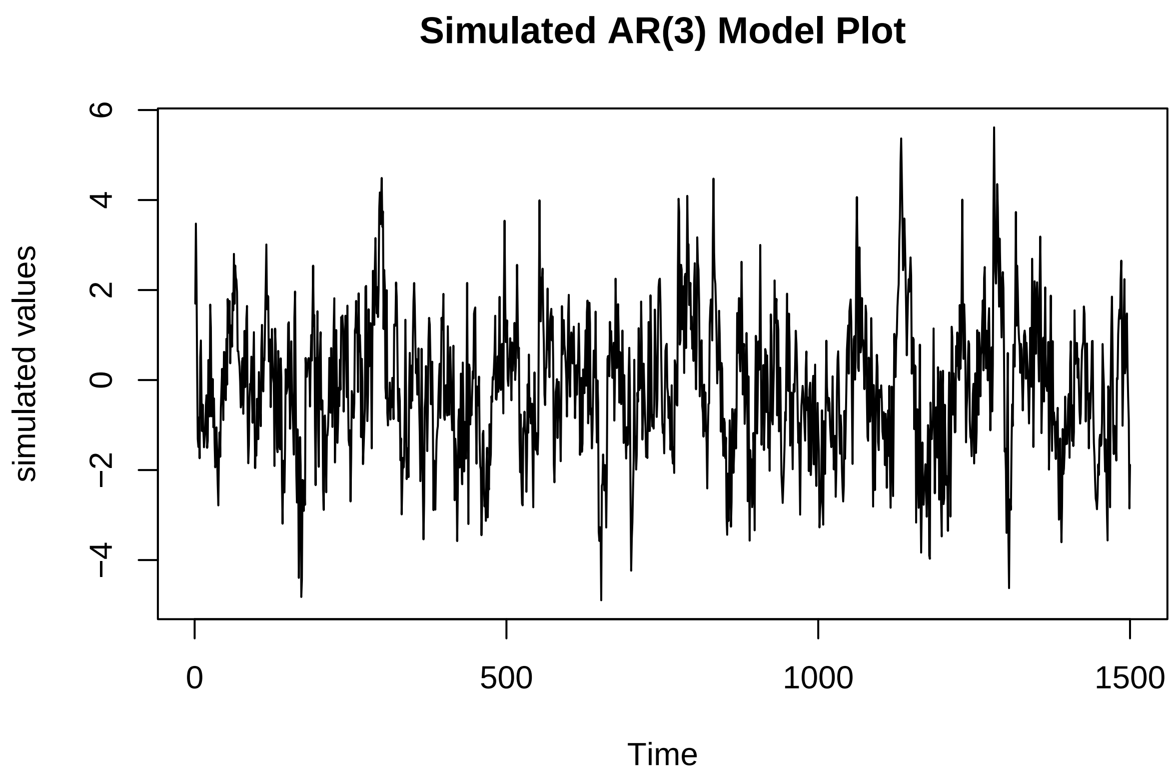
We can rewrite the process X_t in the form of $Z_t = X_t - 0.8BX_t + \frac{1}{3}B^2X_t - \frac{0.6}{\sqrt{3}}B^3X_t$, looking at the degree of its characteristic polynomial, we can see this process is a AR process of order 3, hence $AR(3)$ process.

Question 2

We could first look at the time series plot to see if the process is stationary or not. If it appears to be stationary, then we look at its acf plot. If the acf plot shows a gradual exponential decay or "sin-function" like decay, we think this is likely an AR process. Then we look at the partial ACF plot to determine its order. We want to look at the value of pacf at each lag, and stops at a lag when the pacf values following this lag all fall within the $\pm 2/\sqrt{n}$ bound.

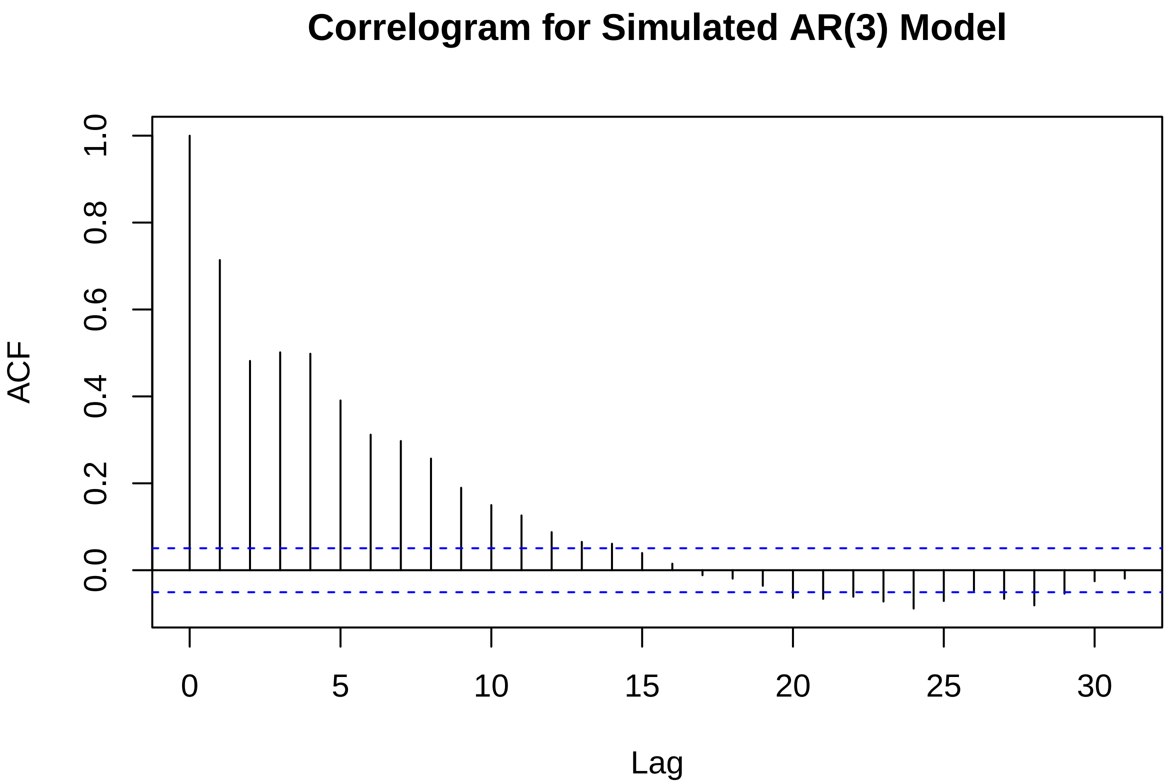
Question 3

```
set.seed(23456)
sim_series <- arima.sim(list(ar = c(0.8,-1/3,0.6/sqrt(3))),
                        n = 1500)
plot(sim_series,
     ylab = "simulated values",
     main = "Simulated AR(3) Model Plot")
```



Question 4

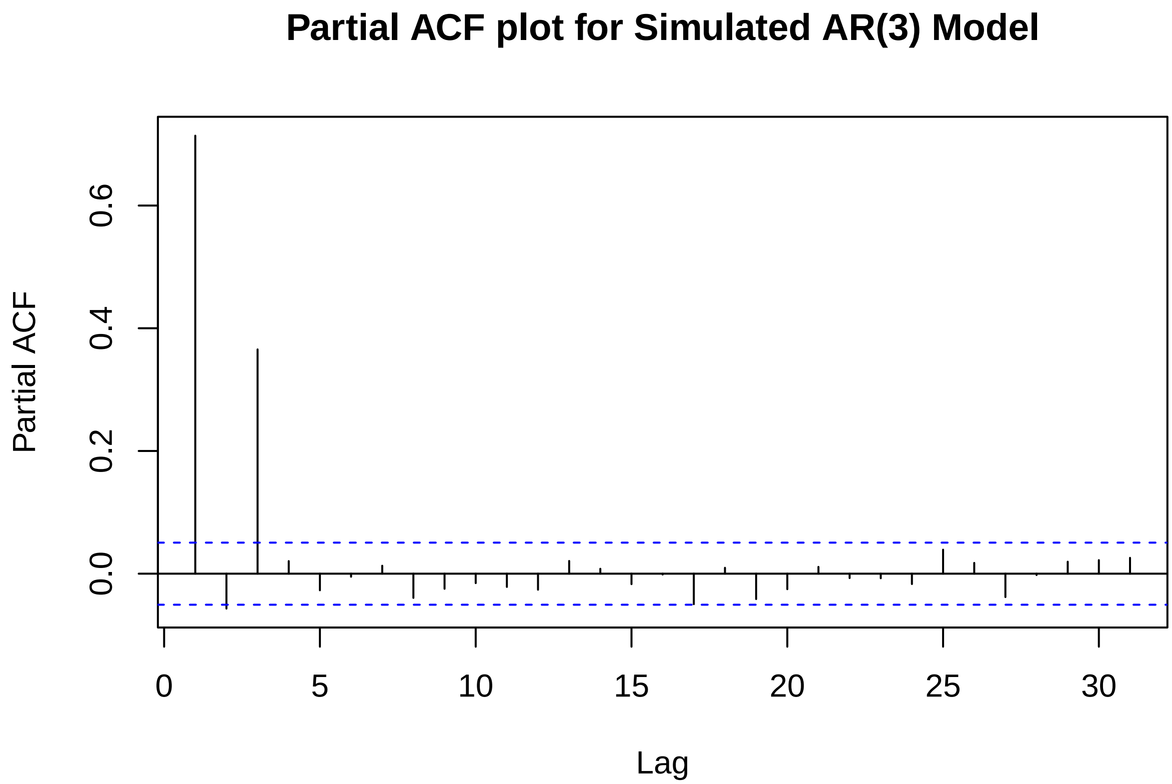
```
acf(sim_series,
    main = "Correlogram for Simulated AR(3) Model")
```



The values of auto-correlation at different lags show a gradual decay which matches the expected behaviour for an AR model.

Question 5

```
pacf(sim_series,
    main = "Partial ACF plot for Simulated AR(3) Model")
```



Yes, it behaves as expected. We observe that the partial ACF values becomes non-significant after lag 3. Therefore we think this model is a AR model of order 3.

Question 6

```
ARMA_CSS_ML <- arima(sim_series,
                    order = c(3,0,0),
                    seasonal = c(0,0,0),
                    include.mean = FALSE,
                    method = "CSS-ML")
ARMA_ML <- arima(sim_series,
                    order = c(3,0,0),
                    seasonal = c(0,0,0),
                    include.mean = FALSE,
                    method = "ML")
ARMA_CSS <- arima(sim_series,
                    order = c(3,0,0),
                    seasonal = c(0,0,0),
                    include.mean = FALSE,
                    method = "CSS")
result <- rbind(True_values = c(0.8,round(-1/3,3),round(0.6/sqrt(3),3),0.8^2),
                 CSS_ML = c(0.775,-0.330,0.368,0.996),
                 ML = c(0.775,-0.330,0.368,0.996),
                 CSS = c(0.774,-0.330,0.367,0.994))
colnames(result) <- c("a1","a2","a3","sigma^2")
kable(result)
```

	a1	a2	a3	sigmaˆ2
True_values	0.800	-0.333	0.346	0.640
CSS_ML	0.775	-0.330	0.368	0.996
ML	0.775	-0.330	0.368	0.996
CSS	0.774	-0.330	0.367	0.994

i) I decided to fit a $ARMA(3, 0, 0)$ model. Looking at the plot for the simulated time series, it appears this series is stationary, so I did not include a non-zero mean.

ii) Looking at the above result, the estimated values for $\alpha_1, \alpha_2, \alpha_3$ are fairly close to the the true parameter. While the all three methods over-estimated the σ^2.