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Multi-objective Evolutionary Algorithm for Calculus of Variations

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Abstract -

Keywords -

Calculus of variations; Brachistochrone; MOEA; Genetic Algorithm; Reinforcement Learning

1 Introduction

The mathematician Johann Bernoulli posed a question and invited mathematicians of the time to solve it. The question is: What should be the shape of the curve so a bead will reach the lower point in the least time?. Suppose that a bead is placed on the wire at the higher point and allowed to slide under gravity, starting from rest and assuming no friction. Five prominent mathematicians of the time solved the problem, namely Johann and Jakob Bernoulli, Leibniz, l'Hospital, and Newton. They showed that the solution is also the cycloid, and gave the cycloid its name brachistochrone, which is Greek for shortest-time.

The calculus of variations is one of the classical branches of mathematics. The problem of the Brachistochrone had a very strong influence on the development of the calculus of variations. Applications of variational principles also occur in elasticity, electromagnetic theory, aerodynamics, the theory of vibrations, quantum electrodynamics, and other areas in engineering and science.

This paper is about experiments done to analyze the level of optimal behavior achieve by a combination of reinforcement learning and genetic algorithm. In this specific case, to resolve the brachistochrone problem, specifically, the experiment is expect to solve the classic brachistochrone problem without having any prior knowledge of math or physics.

2 Brachistochrone Problem

3 Reinforcement Learning

One of the challenges that arise in reinforcement learning, and not in other kinds of learning, is the trade-off between exploration and exploitation. To obtain a lot of reward, a reinforcement learning agent must prefer actions that it has tried in the past and found to be effective in producing reward. But to discover such actions, it has to try actions that it has not selected before. The agent has to exploit what it has already experienced in order to obtain reward, but it also has to explore in order to make better action selections in the future.

3.1 Formulating the five Characteristics

Agent, Action, Environment, State, Reward.

4 Genetic Algorithm Operators

Genetic algorithms and their hybrids are increasingly being applied to produce near-optimal solutions to difficult optimization problems.

4.1 References

References to the book [1]

5 Conclusion

References

[1] Bernard Dacorogna. *Introduction to the Calculus of Variations*. Imperial College Press, Distributed by World Scientific, 2004.