Introduction to the course "Optimization Methods and Game Theory"

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Optimization Methods and Game Theory
Master of Science in Artificial Intelligence and Data Engineering
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Preliminary informations

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Question time: by appointment (send e-mail)

Course schedule

- tuesday 10.30 - 13.30 , Room ADII1
- thursday 14 - 16 , Room ETR F2

Course material

- Microsoft Teams platform: Team 696AA 22/23 - Optimization Methods and Game Theory [WAI-LM] (Slides of the lectures)
- https://elearn.ing.unipi.it/course/view.php?id=3049

Aim of the course

Study optimization methods for data analysis and decision problems

Main tool

Optimization problems defined by

$$\min\ (max)\ \{f(x):\ x \in X\} \qquad (P)$$

- $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function
- $X \subseteq \mathbb{R}^n$ is the constraints set or feasible region
- If $X \equiv \mathbb{R}^n$ then (P) is said to be unconstrained

(P) can be considered as a decision problem where X is the set of all the admissible (feasible) decisions x and $f(x)$ is the value of the decision x (for example, a cost or a gain).

In general X is defined by constraint functions

- $X = \{x \in \mathbb{R}^n : g(x) \leq 0, h(x) = 0\}$
- $g(x) = (g_1(x), \ldots, g_m(x))$, where $g_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$ are the inequality constraints functions
- $h(x) = (h_1(x), \ldots, h_p(x))$, where $h_j : \mathbb{R}^n \to \mathbb{R}$, $j = 1, \ldots, p$ are the equality constraints functions

$$\begin{cases} \min \ f(x) \\ g(x) \leq 0 \\ h(x) = 0 \end{cases}$$

We will generally consider minimization problems since

$$\max\{f(x) : \ x \in X\} = -\min\{-f(x) : \ x \in X\}.$$

It is of fundamental importance to analyze the properties of (P) under suitable assumptions on X and on the involved functions:

- convexity
- differentiability

Outline of the program

- Preliminaries of convex analysis
- Optimization problems: existence of optima, optimality conditions, duality
- Solution methods for optimization problems:
  - gradient and conjugate gradient method
  - Newton and quasi-Newton methods, derivative-free methods
  - active-set, penalty, logarithmic barrier methods
- Applications to machine learning:
  - Supervised machine learning: optimization models for classification and regression problems
  - Unsupervised machine learning: clustering problems
- Multiobjective (or vector) optimization problems:
  - Pareto and weak Pareto optimal solutions
  - existence, optimality conditions, scalarization approach, goal method
- Non-cooperative game theory:
  - zero-sum finite games: Nash Equilibrium (NE), existence, min-max theorem
  - non zero-sum finite games: existence, optimality conditions, algorithms
  - convex games: existence of NE, optimality conditions, merit functions
- Exercise sessions with MATLAB software

MATLAB

You can download and install MATLAB on your laptop using the Campus License paid by University of Pisa, see:

Link for Matlab installation

- https://unipi.it/matlab
- "Accedi per iniziare"
- Recall that in order to install Matlab it is necessary to use your istitutional mail, namely, ...@studenti.unipi.it, and not any e-mail address.
- In particular, install the optimization toolbox.

Bibliography

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