10/10

Foundations of Statistical Modeling

March 6, 2024

0.0.1 Assignment 2

Protogene Hahirwabayo

Question 1 the universe Ω consists of you, two dice with 6 sides and numbers ranging from 1 to 6, one coinsand a table. the random variable (RV) function X for throwing two dice and one coin.

The sample space Ω can be described as the set of all possible outcomes, where each outcome is an ordered triple (x, y, h), where x and y are the outcomes of the two dice, and h is the outcome of the coin which consist of "Head","Tails"

Ω={(d1,d2,c) d1,d2 {1,2,3,4,5,6},c {"Heads","Tails"}}

Now, let's define the RV function X:

X:Ω→R

X((d1,d2,c))=d1.d2.c

Here, X represents the sum of the outcomes of the two dice and one coin

a) Two Dice and One Coin The universe Ω consists of all possible outcomes from throwing two six-sided dice and one coin:

Ω={(d1,d2,c) d1{1,2,3,4,5,6},d2{1,2,3,4,5,6},c {H,T}}

The RV function X that captures the total of the dice rolls and the coin outcome might be represented as:

X:Ω→S

X((d1,d2,c))=(d1,d2,c)

Data Value Space (S) The Data Value Space S is directly derived from the possible outcomes, represented by the same structure as Ω

S={(d1,d2,c) d1 {1,2,3,4,5,6},d2 {1,2,3,4,5,6},c {Heads,Tails}}

Elements in S

To calculate the number of elements in S

There are 6 outcomes for the first die. There are 6 outcomes for the second die. There are 2 outcomes for the coin.

S =6×6×2=72 elements

So, the corrected Data Value Space S contains 72 elements, each a pair of the two dice rolls and the coin flip outcome.

b) One Die and One Coin Universe(Ω) = one die and the coin:

Ω ={(d,c) d {1,2,3,4,5,6},c {Heads,Tails}}

Adjusted RV-function (X):

X :Ω →S

X ((d,c))=(d,c)

Adjusted Data Value Space (S)

S ={(d,c) d {1,2,3,4,5,6},c {Heads,Tails}} = {1,2,3,4,5,6} × {Heads,Tails}}

S =6×2=12 elements

Question 2

1. Raw Input: Grayscale photographic images of handwritten numbers, with each image having a resolution of 200 by 200 pixels.
2. Pixel Intensity: Each pixel's intensity is measured on a grayscale where 0 corresponds to white, 1 corresponds to black, and values in between 0 and 1 correspond to varying shades of gray.
3. Pixel Position: The position of each pixel is identified by its coordinates in the image, where the coordinates are given by (x, y) pairs, with x , y { 0 , 1 , . . . , 199 } x,y {0,1,...,199}.

RV-Function:

The RV-function for this scenario maps each pixel in the image to a value in the data value space S.

f:P→S

where P is the set of all pixels in the image, and S is the data value space that includes both the grayscale intensity and the position of each pixel.

Data Value Space S:

To describe the grayscale intensity and exact location of each pixel, the data value space S can be defined as a combination of grayscale values and pixel coordinates.

S=G×X×Y

1. G=[0,1] represents the grayscale intensity of a pixel, with 0 being white, 1 being black, and values in between representing shades of gray.
2. X={0,1,...,199} and Y={0,1,...,199} represent the x and y coordinates of a pixel in the image, respectively.

Therefore, an element s S is a triplet(g,x,y), where g is the grayscale value, and(x,y) are the coordinates of the pixel in the image. This representation allows for each pixel's intensity and location to be uniquely identified and processed by the handwriting recognition system.