9/10

Foundation of Statistical Modeling Assignment 4_Protogene Hahirwabayo

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1. Probability Space, Conditional probability Ω = {HH, HT, TH, TT} where heads ar H and T represents tail 1. so the probability of each outcome is 1/4

a) Tossing the first coin and finds "tail", what will be the probability to finds tail fo the other coin {TT} Ω = {HT,TT} so the probability when first coin shows "tail So,P(A)= P(first coin shows"tail")= 2/4 = 1/2

the probability of getting "tail" when tossing the second coin, given that the first one showed "tail":

1. P(B|A) = P(second coin shows "tail" | first coin shows "tail")
2. P(B|A)= Number of outcomes in Ω where second coin shows "tail" / Total number of outcomes in Ω

P(B|A) = 1/2 The probability of getting "tail" when tossing the second coin, given that the first coin shows "tail", is 1/2.

b) Tossing 2 coins and knowing that one of them shows "tail" Ω = {HT,TH,TT} when one of the coins shows "tail"

P(A) = P(at least one coin shows "tail") = 3/4

the probability that the other coin also shows "tail", given that one of them shows "tail"

1. P(B|A) = P(other coin shows "tail" | at least one coin shows "tail")
2. P(B|A) = Number of outcomes in Ω where both coins show "tail" / Total number of outcomes in Ω

P(B|A) = 1/3 The probability that the other coin also shows "tail", given that one of them shows "tail", is 1/3.

2. Probability Space, Independence

1. Probability Space:
   - Sample space S = {1, 2, 3, 4, 5, 6}, representing the possible outcomes of rolling a fair six-sided die.
   - Each outcome is equally likely, so the probability of each outcome is 1/6.
2. Events:
   - Event A: Subset of S containing even numbers {2, 4, 6}.
   - Event B: Subset of S containing numbers smaller than or equal to 4 {1, 2, 3, 4}.

- Event C: Subset of S containing numbers 2 and 3 {2, 3}.

Now, let's analyze the independence of these events:

a) Events A and B:
   - P(A) = 3/6 = 1/2 (there are 3 even numbers out of 6)
   - P(B) = 4/6 = 2/3 (there are 4 numbers smaller than or equal to 4 out of 6)
   - P(A B) = P({2, 4}) = 2/6 = 1/3 (there are 2 even numbers smaller than or equal to 4)
   - Since P(A B) = P(A) * P(B), events A and B are independent.

b) Events A and C:
   - P(A) = 1/2 (as calculated above)
   - P(C) = 2/6 = 1/3 (there are 2 numbers in the subset C out of 6)
   - P(A C) = P({2}) = 1/6 (only one number is common in A and C)
   - Since P(A C) P(A) * P(C), events A and C are dependent.

c) Events B and C:

A and C are independent

   - P(B) = 2/3 (as calculated above)
   - P(C) = 1/3 (as calculated above)
   - P(B C) = P({2, 3}) = 2/6 = 1/3 (there are 2 numbers common in B and C)
   - Since P(B C) P(B) * P(C), events B and C are dependent.

Summary: - Events A and B are independent. - Events A and C, and events B and C are dependent.

3. Conditional Probability The conditional probability of an event Y occurring given that event X has already occurred is defined as:

P(Y X)= P(X)/ P(X Y)

Statement given : P(X A,Y B,Z C,W D)=P(X A)P(Y B X A)P(Z C X A,Y B)P(W D X A,Y B,Z C)

break down each term using conditional probabilities

1. P(X A) represents the probability of X being in A.
2. P(Y B X A) represents the probability of Y being in B given that X is in A.
3. P(Z C X A,Y B) represents the probability of Z being in C given that X is in A and Y is in B.
4. P(W D X A,Y B,Z C) represents the probability of W being in D given that X is in A, Y is in B, and Z is in C.

If the statement is correct, then the product of these probabilities should equal the joint probability of all events occurring.

So, the statement is correct if:

P(X A,Y B,Z C,W D)=P(X A)P(Y B X A)P(Z C X A,Y B)P(W D X A,Y B,Z C)

This is a fundamental property of conditional probability known as the chain rule. Therefore, the statement is correct.

4. Probability Space, Conditional Probability Probability Space:

For the coin toss:

1. Sample space S_coin = {1, 2}, where 1 represents “head” and 2 represents “tail”.

For the dice throw:

2. Sample space S_dice = {1, 2, 3, 4, 5, 6}, representing the possible outcomes of rolling a fair six-sided die. Each outcome is equally likely, so the probability of each outcome is 1/2 for the coin and 1/6 for the die.

Events:

Event A: The coin shows “head”.

Event B: The dice shows 5 all k times.

Now, let’s calculate the conditional probability using Bayes’ theorem: P(A B)= P(B A) P(A) / P(B)

- P(A) is the probability of the coin showing “head”, which is 1/2.
- P(B A) is the probability of the dice showing 5 all k times given that the coin shows “head”.

Since each throw of the fair die is independent of the coin toss, this probability is (1/6)^k .

- P(B) is the probability of the dice showing 5 all k times, regardless of the coin toss. This is the sum of probabilities of getting 5 in k throws, considering all possible values of k.

Given that we know the dice shows 5 all k times, but we don’t know k,P(B)is the sum of probabilities of getting 5 in 1 throw, 2 throws, 3 throws, …, up to k throws, each multiplied by the probability of k tosses

let’s denote p as the probability of getting 5 in one throw of the fair die. so p=1/6.

P(B)= ^∞ k=1 (p$^{k).(1/2)}$k = ^∞ k=1 (1/6.1/2)^k

This is an infinite geometric series with common ratio 1/12 and first term 1/12. The sum of an infinite geometric series is a/1-r, where a is the first term and r is the common ratio.

P(B) = (1/6.1/2)/(1-1/12)= (1/12)/(11/12)=1/11

so

P(A B) = (1/6)^k.(1/2)/(1/11) = (11/12).(1/6)^k   =6/7

This gives us the probability that the coin showed “head” given that the dice shows 5 all k times.

5. Probability Space, Conditional Probability

a) Three fair dice are rolled. If you find that two of them show a 3, what is then the probability that the remaining dice shows a 3, as well?

For this case, the events as follows:

- A: The first die shows a 3.
- B: The second die shows a 3.
- C: The third die shows a 3.

We want to find P(C A B), the probability that the third die shows a 3 given that the first two dice show a 3.

Since all three dice are rolled independently,P(C A B)=P(C)= 1/6.

Each die has an equal chance of showing any number, including 3, so the result is simply 1/6

b) Three fair dice are rolled. If you know that any two of them show a 3 (but you do not know which ones of the three dice), what is then the probability that the remaining dice shows a 3, as well, if you do not know which dice that is?

In this case, the probability is still the same,P(C)= 1/6 , because the information about which dice show a 3 is not relevant. Each die has the same chance of showing a 3, regardless of the outcomes of the other dice.

c) Now you take a single fair die and roll it 2 times, recording each time the result, getting thereby a time series. If you find that all 2 times you recorded it you got a 3, what is then the probability that the next time the die shows a 3, again?

Here, each roll of the die is independent, so the probability of getting a 3 on the next roll is the same as the probability of getting a 3 on any single roll, which is 1/6.The previous outcomes do not affect the outcome of the next roll since the die is fair.

Therefore, the probability remains 1/6.