

1. [30 points] Write a function that computes the exact value of $n!$.
 - (a) The function must be named `factorial` verbatim.
 - (b) The function must take a single non-negative integer as input and return a single integer as output. Note that $0! = 1$.
 - (c) Do not use Stirling's formula—use the exact definition of factorial.
 - (d) Do NOT use the factorial function from any library—you must write your own!

2. [30 points] Write a function that determines if a positive integer is a decimal narcissistic number. A positive, N -digit integer x written in base-10 (decimal), represented by

$$x = d_{N-1} \times 10^{N-1} + d_{N-2} \times 10^{N-2} + \dots + a_2 \times 10^2 + d_1 \times 10^1 + d_0 \times 10^0,$$

is decimal narcissistic if

$$d_{N-1}^N + d_{N-2}^N + \dots + d_2^N + d_1^N + d_0^N = x.$$

For example, all positive, single-digit integers less than ten are decimal narcissistic. Also, 153 is decimal narcissistic since

$$153 = 1 \times 10^2 + 5 \times 10^1 + 3 \times 10^0 = 1^3 + 5^3 + 3^3.$$

- (a) The function must be named `is_narcissistic` verbatim.
 - (b) The function must take a single non-negative integer as input and return a boolean `True/False` as output.
3. [30 points] Using your `is_narcissistic` function (do NOT rewrite or copy/paste your code), write a function that returns the first N decimal narcissistic numbers starting with 1.
 - (a) The function must be named `find_narcissistic` verbatim.
 - (b) The function must take a single non-negative integer N as input and return a list of the N integers.

4. [30 points] Write a function that estimates the value of π to twelve decimal places using the fixed-point iteration,

$$x_{n+1} = x_n + \sin(x_n),$$

using the initial guess $x_0 = 3$.

- (a) The function must be named `compute_pi` verbatim.
 - (b) The function will not take any parameters and will return a floating-point number as output.
 - (c) Do NOT use the definition of π from any library—you must write your own!