QBio II Homework 1

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1 Question 1

Given:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Solve $det(\lambda I - A) = 0$ to get eigenvalues and eigenvectors.

$$\lambda I - A = \begin{bmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{bmatrix}$$

$$det(\lambda I - A) = (\lambda - 2)^2 - 1 = 0$$

$$\Rightarrow \lambda_1 = 3, \lambda_2 = 1$$

With eigenvalues, we can calculate the eigenvectors by solving $(\lambda I - A)\mathbf{v} = 0$. Substitue $\lambda_1 = 3$ into the equation:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \mathbf{v} = 0$$

$$\Rightarrow \mathbf{v}_{\lambda_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Then, substitue $\lambda_2 = 1$ into the equation:

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \cdot \mathbf{v} = 0$$

$$\Rightarrow \mathbf{v}_{\lambda_2} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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