An Empirical Comparison of Supervised Learning Algorithms

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Abstract

A number of supervised learning methods have been introduced in the last decade. Unfortunately, the last comprehensive empirical evaluation of supervised learning was the Statlog Project in the early 90's. We present a large-scale empirical comparison between ten supervised learning methods: SVMs, neural nets, logistic regression, naive bayes, memory-based learning, random forests, decision trees, bagged trees, boosted trees, and boosted stumps. We also examine the effect that calibrating the models via Platt Scaling and Isotonic Regression has on their performance. An important aspect of our study is the use of a variety of performance criteria to evaluate the learning methods.

1. Introduction

There are few comprehensive empirical studies comparing learning algorithms. STATLOG is perhaps the best known study (King et al., 1995). STATLOG was very comprehensive when it was performed, but since then new learning algorithms have emerged (e.g., bagging, boosting, SVMs, random forests) that have excellent performance. An extensive empirical evaluation of modern learning methods would be useful.

Learning algorithms are now used in many domains, and different performance metrics are appropriate for each domain. For example Precision/Recall measures are used in information retrieval; medicine prefers ROC area; Lift is appropriate for some marketing tasks, etc. The different performance metrics measure different tradeoffs in the predictions made by a classifier, and it is possible for learning methods to perform well on one metric, but be suboptimal on other metrics. Because of this it is important to evaluate algorithms on a broad set of performance metrics.

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This paper presents results of a large-scale empirical comparison of ten supervised learning algorithms using eight performance criteria. We evaluate the performance of SVMs, neural nets, logistic regression, naive bayes, memory-based learning, random forests, decision trees, bagged trees, boosted trees, and boosted stumps on eleven binary classification problems using a variety of performance metrics: accuracy, F-score, Lift, ROC Area, average precision, precision/recall break-even point, squared error, and cross-entropy. For each algorithm we examine common variations, and thoroughly explore the space of parameters. For example, we compare ten decision tree styles, neural nets of many sizes, SVMs with many kernels, etc.

Because some of the performance metrics we examine interpret model predictions as probabilities and models such as SVMs are not designed to predict probabilities, we compare the performance of each algorithm both before and after calibrating its predictions with Platt Scaling and Isotonic Regression.

The empirical results are surprising. To preview: prior to calibration, bagged trees, random forests, and neural nets give the best average performance across all eight metrics and eleven test problems. Boosted trees, however, are best if we restrict attention to the six metrics that do not require probabilities. After calibration with Platt's Method, boosted trees predict better probabilities than all other methods and move into first place overall. Neural nets, on the other hand, are so well calibrated to begin with that they are hurt slightly by calibration. After calibration with Platt's Method or Isotonic Regression, SVMs perform comparably to neural nets and nearly as well as boosted trees, random forests and bagged trees. Boosting full decision trees dramatically outperforms boosting weaker stumps on most problems. On average, memory-based learning, boosted stumps, single decision trees, logistic regression, and naive bayes are not competitive with the best methods. These generalizations, however, do not always hold. For example, boosted stumps and logistic regression, which perform poorly on average, are the best models for some metrics on two of the test problems.

2. Methodology

2.1. Learning Algorithms

We attempt to explore the space of parameters and common variations for each learning algorithm as thoroughly as is computationally feasible. This section summarizes the parameters used for each learning algorithm, and may safely be skipped by readers who are easily bored.

SVMs: we use the following kernels in SVMLight (Joachims, 1999): linear, polynomial degree 2 & 3, radial with width {0.001,0.005,0.01,0.05,0.1,0.5,1,2}. We also vary the regularization parameter by factors of ten from 10^{-7} to 10^{3} with each kernel.

ANN we train neural nets with gradient descent backprop and vary the number of hidden units {1,2,4,8,32,128} and the momentum {0,0.2,0.5,0.9}. We halt training the nets at many different epochs and use validation sets to select the best nets.

Logistic Regression (LOGREG): we train both unregularized and regularized models, varying the ridge (regularization) parameter by factors of 10 from 10^{-8} to 10^{4}.

Naive Bayes (NB): we use Weka (Witten & Frank, 2005) and try all three of the Weka options for handling continuous attributes: modeling them as a single normal, modeling them with kernel estimation, or discretizing them using supervised discretization.

KNN: we use 26 values of K ranging from $K = 1$ to $K = |trainset|$. We use KNN with Euclidean distance and Euclidean distance weighted by gain ratio. We also use distance weighted KNN, and locally weighted averaging. The kernel widths for locally weighted averaging vary from 2^0 to 2^{10} times the minimum distance between any two points in the train set.

Random Forests (RF): we tried both the Breiman-Cutler and Weka implementations; Breiman-Cutler yielded better results so we report those here. The forests have 1024 trees. The size of the feature set considered at each split is 1,2,4,6,8,12,16 or 20.

Decision trees (DT): we vary the splitting criterion, pruning options, and smoothing (Laplacian or Bayesian smoothing). We use all of the tree models in Buntine's IND package (Buntine & Caruana, 1991): BAYES, ID3, CART, CART0, C4, MML, and SMML. We also generate trees of type C44LS (C4 with no pruning and Laplacian smoothing), C44BS (C44 with Bayesian smoothing), and MMLLS (MML with Laplacian smoothing). See (Provost & Domingos, 2003) for a description of C44LS.

Bagged trees (BAG-DT): we bag 100 trees of each type described above. With **boosted trees (BST-DT)** we boost each tree type as well. Boosting can overfit, so we consider boosted trees after 2,4,8,16,32,64,128,256,512,1024 and 2048 steps of boosting. With **boosted stumps (BST-STMP)** we boost single level decision trees generated with 5 different splitting criteria, each boosted for 2,4,8,16,32,64,128,256,512,1024,2048,4096,8192 steps.

With LOGREG, ANN, SVM and KNN we scale attributes to 0 mean 1 std. With DT, RF, NB, BAG-DT, BST-DT and BST-STMP we don't scale the data. In total, we train about 2000 different models in each trial on each problem.

2.2. Performance Metrics

We divide the eight performance metrics into three groups: threshold metrics, ordering/rank metrics and probability metrics.

The threshold metrics are accuracy (ACC), F-score (FSC) and lift (LFT). See (Giudici, 2003) for a description of Lift Curves. Usually ACC and FSC have a fixed threshold (we use 0.5). For lift, often a fixed percent, p, of cases are predicted as positive and the rest as negative (we use $p = 25\%$). For thresholded metrics, it is not important how close a prediction is to a threshold, only if it is above or below threshold.

The ordering/rank metrics depend only on the ordering of the cases, not the actual predicted values. As long as ordering is preserved, it makes no difference if predicted values fall between 0 and 1 or 0.89 and 0.90. These metrics measure how well the positive cases are ordered before negative cases and can be viewed as a summary of model performance across all possible thresholds. The rank metrics we use are area under the ROC curve (ROC), average precision (APR), and precision/recall break even point (BEP). See (Provost & Fawcett, 1997) for a discussion of ROC from a machine learning perspective.

The probability metrics, squared error (RMS) and cross-entropy (MXE), interpret the predicted value of each case as the conditional probability of that case being in the positive class. See (Caruana & Niculescu-Mizil, 2004) for a more detailed description of the eight performance metrics.

2.3. Comparing Across Performance Metrics

Performance metrics such as accuracy or squared error have range $[0, 1]$, while others (lift, cross entropy) range from 0 to p where p depends on the data set. For some metrics lower values indicate better perfor-

mance. For others higher values are better. Metrics such as ROC area have baseline rates that are independent of the data, while others such as accuracy have baseline rates that depend on the data. If baseline accuracy is 0.98, an accuracy of 0.981 probably is not good performance, but on another problem the Bayes optimal rate might be 0.60 and achieving an accuracy of 0.59 might be excellent performance.

To permit averaging across metrics and problems, performances must be placed on comparable scales. We do this by scaling performance for each problem and metric from 0 to 1, where 0 is baseline performance and 1 is Bayes optimal. Since the Bayes optimal rate cannot be estimated on real problems, we use the best observed performance as a proxy. The following baseline model is used: predict p for every case, where p is the percent of positives in the data.[1] Performances are normalized to $[0, 1]$, where 0 is baseline and 1 represents best performance. If a model performs worse than baseline, its normalized score will be negative.

One disadvantage of normalized scores is that recovering raw performances requires knowing the performances at the top and bottom of the scale, and as new best models are found the top of the scale may change. The performances that define the top and bottom of the scales for each problem and metric are available at `www.cs.cornell.edu\~caruana` to allow comparison with our results.

2.4. Calibration Methods

Some of the learning algorithms we examine are not designed to predict probabilities. For example the outputs of an SVM are normalized distances to the decision boundary. And naive bayes models are known to predict poorly calibrated probabilities because of the unrealistic independence assumption.

A number of methods have been proposed for mapping predictions to posterior probabilities. Platt (1999) proposed transforming SVM predictions to posterior probabilities by passing them through a sigmoid. Platt's method also works well for boosted trees and boosted stumps (Niculescu-Mizil & Caruana, 2005). A sigmoid, however, might not be the correct transformation for all learning algorithms.

Zadrozny and Elkan (2002; 2001) used a more general calibration method based on Isotonic Regression (Robertson et al., 1988) to calibrate predictions from SVMs, naive bayes, boosted naive bayes, and decision trees. Isotonic Regression is more general in that the only restriction it makes is that the mapping function be isotonic (monotonically increasing). A standard algorithm for Isotonic Regression that finds a piecewise constant solution in linear time, is the pair-adjacent violators (PAV) algorithm (Ayer et al., 1955).

To calibrate models, we use the same 1000 points validation set that will be used for model selection.

[1]For F-Score, baseline is calculated by predicting 1 for all cases since predicting p can yield 0 F-Score.

Table 1. Description of problems

PROBLEM	#ATTR	TRAIN SIZE	TEST SIZE	%POZ
ADULT	14/104	5000	35222	25%
BACT	11/170	5000	34262	69%
COD	15/60	5000	14000	50%
CALHOUS	9	5000	14640	52%
COV_TYPE	54	5000	25000	36%
HS	200	5000	4366	24%
LETTER.P1	16	5000	14000	3%
LETTER.P2	16	5000	14000	53%
MEDIS	63	5000	8199	11%
MG	124	5000	12807	17%
SLAC	59	5000	25000	50%

2.5. Data Sets

We compare the algorithms on 11 binary classification problems. ADULT, COV_TYPE and LETTER are from the UCI Repository (Blake & Merz, 1998). COV_TYPE has been converted to a binary problem by treating the largest class as the positive and the rest as negative. We converted LETTER to boolean in two ways. LETTER.p1 treats "O" as positive and the remaining 25 letters as negative, yielding a very unbalanced problem. LETTER.p2 uses letters A-M as positives and the rest as negatives, yielding a well balanced problem. HS is the IndianPine92 data set (Gualtieri et al., 1999) where the difficult class Soybean-mintill is the positive class. SLAC is a problem from the Stanford Linear Accelerator. MEDIS and MG are medical data sets. COD, BACT, and CALHOUS are three of the datasets used in (Perlich et al., 2003). ADULT, COD, and BACT contain nominal attributes. For ANNs, SVMs, KNNs, and LOGREG we transform nominal attributes to boolean (one boolean per value). Each DT, BAG-DT, BST-DT, BST-STMP, RF, and NB model is trained twice, once with transformed attributes and once with the original ones. See Table 1 for characteristics of these problems.

3. Performances by Metric

For each test problem we randomly select 5000 cases for training and use the rest of the cases as a large final test set. We use 5-fold cross validation on the

5000 cases to obtain five trials. For each trial we use 4000 cases to train the different models, 1000 cases to calibrate the models and select the best parameters, and then report performance on the large final test set. We would like to run more trials, but this is a *very* expensive set of experiments. Fortunately, even with only five trials we are able to discern interesting differences between methods.

Table 2 shows the normalized score for each algorithm on each of the eight metrics. For each problem and metric we find the best parameter settings for each algorithm using the 1k validation sets set aside by cross-validation, then report that model's normalized score on the final test set. Each entry in the table averages these scores across the five trials and eight test problems. The second column tells if model predictions were calibrated. A "–" means the model predictions were not calibrated – they are the raw model predictions. (The one exception is SVMs, where distances to the separating hyperplane are linearly scaled to [0,1] before computing the three probability metrics.) A "PLT" or "ISO" in the second column indicates that the model predictions were scaled after the model was trained using Platt Scaling or Isotonic Regression, respectively. These scaling methods were discussed in Section 2.4. In the table, higher scores always indicate better performance.

The second to last column, MEAN, is the mean normalized score over the eight metrics, eleven problems, and five trials. The models in the table are sorted by the mean normalized score in this column. For now, ignore the last column, OPT-SEL. This column will be discussed later in this section.

In the table, the algorithm with the best performance on each metric is **boldfaced**. Other algorithm's whose performance is not statistically distinguishable from the best algorithm at $p = 0.05$ using paired t-tests on the 5 trials are *'ed.[2] Entries in the table that are neither bold nor starred indicate performance that *is significantly lower* than the best models at $p = 0.05$.[3]

Averaging across all eight metrics, the strongest models are calibrated boosted trees, calibrated random forests, bagged trees, PLT-calibrated SVMs and neural nets. If not calibrated, the best models overall are bagged trees, random forests, and neural nets. With or without calibration, the poorest performing models are naive bayes, logistic regression, and decision trees. Memory-based methods (e.g. KNN) are remarkably unaffected by calibration, but exhibit mediocre overall performance. Boosted stumps, even after calibration, also have mediocre performance, and do not perform nearly as well as boosted full trees.

Looking at individual metrics, boosted trees, which have poor squared error and cross-entropy prior to calibration, dominate the other algorithms on these metrics after calibration. Bagged trees, random forests and neural nets also predict good probabilities. Interestingly, calibrating neural nets with PLT or ISO hurts their calibration. If neural nets are trained well to begin with it is better not to adjust their predictions.

Boosted trees also have excellent performance on ordering metrics: area under the ROC, average precision, and the precision/recall break-even point.[4] Random forests and bagged trees have similar performance as boosted trees on these metrics. The neural nets and SVMs also order cases extremely well.

On metrics that compare predictions to a threshold: accuracy, F-score, and Lift, the best models are calibrated or uncalibrated random forests, calibrated boosted trees, and bagged trees with or without calibration. We are not sure why ISO-calibration significantly improves the F-Score of all models except NB, including well-calibrated models such as neural nets and bagged trees.

The last column, OPT-SEL, is the mean normalized score for the eight metrics when model selection is done by cheating and looking at the final test sets. The means in this column represent the best performance that could be achieved with each learning method if model selection were done optimally. We present these numbers because parameter optimization is more critical (and more difficult) with some algorithms than with others. For example, bagging works well with most decision tree types and requires little tuning, but neural nets and SVMs require careful parameter selection. As expected, the mean normalized scores in the

[2]Performing this many independent t-tests, each at $p = 0.05$, is problematic. Some differences that are labeled significant in the table probably are not truly significant at $p = 0.05$. We considered applying a more stringent experiment-wise p-value that takes into account the number of tests performed, but the strong correlations between performances on different metrics, and on calibrated and uncalibrated models, makes this problematic as well, so we decided to keep it simple. Most of the differences in the table are significant well beyond $p = 0.05$. Doing the t-tests at $p = 0.01$ adds few additional stars to the table.

[3]Note that it is possible for the difference between the scores 0.90 and 0.89 to be statistically significant, and yet for the same 0.90 score to be statistically indistinguishable from a poorer score of 0.88 if the variance of the 0.88 score is higher than the variance of the 0.89 score.

[4]PLT calibration does not change the ordering predicted by models, so it does not affect these metrics. ISO calibration, however, can introduce ties in the predicted values that may affect performance on the ordering metrics.

Table 2. Normalized scores for each learning algorithm by metric (average over eleven problems)

MODEL	CAL	ACC	FSC	LFT	ROC	APR	BEP	RMS	MXE	MEAN	OPT-SEL
BST-DT	PLT	.843*	.779	**.939**	**.963**	**.938**	.929*	**.880**	**.896**	**.896**	**.917**
RF	PLT	.872*	.805	.934*	.957	.931	**.930**	.851	.858	.892	.898
BAG-DT	–	.846	.781	.938*	.962*	.937*	.918	.845	.872	.887*	.899
BST-DT	ISO	.826*	.860*	.929*	.952	.921	.925*	.854	.815	.885	.917*
RF	–	**.872**	.790	.934*	.957	.931	**.930**	.829	.830	.884	.890
BAG-DT	PLT	.841	.774	.938*	.962*	.937*	.918	.836	.852	.882	.895
RF	ISO	.861*	**.861**	.923	.946	.910	.925	.836	.776	.880	.895
BAG-DT	ISO	.826	.843*	.933*	.954	.921	.915	.832	.791	.877	.894
SVM	PLT	.824	.760	.895	.938	.898	.913	.831	.836	.862	.880
ANN	–	.803	.762	.910	.936	.892	.899	.811	.821	.854	.885
SVM	ISO	.813	.836*	.892	.925	.882	.911	.814	.744	.852	.882
ANN	PLT	.815	.748	.910	.936	.892	.899	.783	.785	.846	.875
ANN	ISO	.803	.836	.908	.924	.876	.891	.777	.718	.842	.884
BST-DT	–	.834*	.816	**.939**	**.963**	**.938**	.929*	.598	.605	.828	.851
KNN	PLT	.757	.707	.889	.918	.872	.872	.742	.764	.815	.837
KNN	–	.756	.728	.889	.918	.872	.872	.729	.718	.810	.830
KNN	ISO	.755	.758	.882	.907	.854	.869	.738	.706	.809	.844
BST-STMP	PLT	.724	.651	.876	.908	.853	.845	.716	.754	.791	.808
SVM	–	.817	.804	.895	.938	.899	.913	.514	.467	.781	.810
BST-STMP	ISO	.709	.744	.873	.899	.835	.840	.695	.646	.780	.810
BST-STMP	–	.741	.684	.876	.908	.853	.845	.394	.382	.710	.726
DT	ISO	.648	.654	.818	.838	.756	.778	.590	.589	.709	.774
DT	–	.647	.639	.824	.843	.762	.777	.562	.607	.708	.763
DT	PLT	.651	.618	.824	.843	.762	.777	.575	.594	.706	.761
LR	–	.636	.545	.823	.852	.743	.734	.620	.645	.700	.710
LR	ISO	.627	.567	.818	.847	.735	.742	.608	.589	.692	.703
LR	PLT	.630	.500	.823	.852	.743	.734	.593	.604	.685	.695
NB	ISO	.579	.468	.779	.820	.727	.733	.572	.555	.654	.661
NB	PLT	.576	.448	.780	.824	.738	.735	.537	.559	.650	.654
NB	–	.496	.562	.781	.825	.738	.735	.347	-.633	.481	.489

cheating column (OPT-SEL) tend to be higher than the mean normalized scores when selection is done using 1k validation sets because model selection using the validation sets does not always select the model with the best performance on the final test set.

Comparing the MEAN and OPT-SEL columns, selection using 1k validation sets yields on average about 0.023 decrease in normalized score compared to optimal selection. As expected, high variance models have the biggest drop in performance. Neural nets, for example, which have relatively high variance, lose substantial performance when selecting models with 1k validation sets, while other models such as random forests, which have small variance, lose very little performance when selection is done using 1k validation sets. SVM's have variance between RF and ANNs, and thus lose more than RF, but less than ANN. Boosted trees also have relatively high variance, but their overall performance after PLT or ISO calibration is so strong that they remain the best model overall even when selection is done using 1k validation sets.

4. Performances by Problem

Table 3 shows the normalized score for each algorithm on each of the 11 test problems. Each entry is an average over the eight performance metrics and five trials when selection is done using 1k validation sets.

As the No Free Lunch Theorem suggests, there is no universally best learning algorithm. Even the best models (calibrated boosted trees, random forests and bagged trees) perform poorly on some problems, and models that have poor average performance perform well on a few problems or metrics. For example, the best models on ADULT are calibrated boosted stumps, random forests and bagged trees. Boosted trees perform much worse. Bagged trees and random forests also perform very well on MG and SLAC. On MEDIS, the best models are random forests, neural nets and logistic regression. The only models that never exhibit excellent performance on any problem are naive bayes and memory-based learning.

Boosting full decision trees yields better performance

Table 3. Normalized scores of each learning algorithm by problem (averaged over eight metrics)

MODEL	CAL	COVT	ADULT	LTR.P1	LTR.P2	MEDIS	SLAC	HS	MG	CALHOUS	COD	BACT	MEAN
BST-DT	PLT	**.938**	.857	**.959**	**.976**	.700	.869	**.933**	.855	**.974**	**.915**	.878*	**.896***
RF	PLT	.876	.930	.897	.941	**.810**	.907*	.884	.883	.937	.903*	.847	.892
BAG-DT	–	.878	.944*	.883	.911	.762	.898*	.856	**.898**	.948	.856	**.926**	.887*
BST-DT	ISO	.922*	.865	.901*	.969	.692*	.878	.927	.845	.965	.912*	.861	.885*
RF	–	.876	.946*	.883	.922	.785	.912*	.871	.891*	.941	.874	.824	.884
BAG-DT	PLT	.873	.931	.877	.920	.752	.885	.863	.884	.944	.865	.912*	.882
RF	ISO	.865	.934	.851	.935	.767*	**.920**	.877	.876	.933	.897*	.821	.880
BAG-DT	ISO	.867	.933	.840	.915	.749	.897	.856	.884	.940	.859	.907*	.877
SVM	PLT	.765	.886	.936	.962	.733	.866	.913*	.816	.897	.900*	.807	.862
ANN	–	.764	.884	.913	.901	.791*	.881	.932*	.859	.923	.667	.882	.854
SVM	ISO	.758	.882	.899	.954	.693*	.878	.907	.827	.897	.900*	.778	.852
ANN	PLT	.766	.872	.898	.894	.775	.871	.929*	.846	.919	.665	.871	.846
ANN	ISO	.767	.882	.821	.891	.785*	.895	.926*	.841	.915	.672	.862	.842
BST-DT	–	.874	.842	.875	.913	.523	.807	.860	.785	.933	.835	.858	.828
KNN	PLT	.819	.785	.920	.937	.626	.777	.803	.844	.827	.774	.855	.815
KNN	–	.807	.780	.912	.936	.598	.800	.801	.853	.827	.748	.852	.810
KNN	ISO	.814	.784	.879	.935	.633	.791	.794	.832	.824	.777	.833	.809
BST-STMP	PLT	.644	**.949**	.767	.688	.723	.806	.800	.862	.923	.622	.915*	.791
SVM	–	.696	.819	.731	.860	.600	.859	.788	.776	.833	.864	.763	.781
BST-STMP	ISO	.639	.941	.700	.681	.711	.807	.793	.862	.912	.632	.902*	.780
BST-STMP	–	.605	.865	.540	.615	.624	.779	.683	.799	.817	.581	.906*	.710
DT	ISO	.671	.869	.729	.760	.424	.777	.622	.815	.832	.415	.884	.709
DT	–	.652	.872	.723	.763	.449	.769	.609	.829	.831	.389	.899*	.708
DT	PLT	.661	.863	.734	.756	.416	.779	.607	.822	.826	.407	.890*	.706
LR	–	.625	.886	.195	.448	.777*	.852	.675	.849	.838	.647	.905*	.700
LR	ISO	.616	.881	.229	.440	.763*	.834	.659	.827	.833	.636	.889*	.692
LR	PLT	.610	.870	.185	.446	.738	.835	.667	.823	.832	.633	.895	.685
NB	ISO	.574	.904	.674	.557	.709	.724	.205	.687	.758	.633	.770	.654
NB	PLT	.572	.892	.648	.561	.694	.732	.213	.690	.755	.632	.756	.650
NB	–	.552	.843	.534	.556	.011	.714	-.654	.655	.759	.636	.688	.481

than boosting stumps only on seven problems. Occasionally boosted stumps perform very well, but sometimes they perform very poorly so their average performance is low. On ADULT, when boosting trees, the first iteration of boosting hurts the performance of *all* tree types, and never recovers in subsequent rounds. When this happens even single decision trees outperform their boosted counterparts. Bagged trees and random forests, however, consistently outperform single trees on all problems. Bagging and random forests are "safer" than boosting, even on the metrics for which boosting yields the best overall performance.

5. Bootstrap Analysis

The results depend on the choice of problems and metrics. What impact might selecting other problems, or evaluating performance on other metrics, have on the results? For example, neural nets perform well on all metrics on 10 of 11 problems, but perform poorly on COD. If we hadn't included the COD problem, neural nets would move up 1-2 places in the rankings.

To help evaluate the impact of the choice of problems and metrics we performed a bootstrap analysis. We randomly select a bootstrap sample (sampling with replacement) from the original 11 problems. For this sample of problems we then randomly select a bootstrap sample of 8 metrics from the original 8 metrics (again sampling with replacement). For this bootstrap sample of problems and metrics we rank the ten algorithms by mean performance across the sampled problems and metrics (and the 5 folds). This bootstrap sampling is repeated 1000 times, yielding 1000 potentially different rankings of the learning methods.

Table 4 shows the frequency that each method ranks 1st, 2nd, 3rd, etc. The 0.228 entry for boosted trees in the column for 2nd place, tells us that there is a 22.8% chance that boosted decision trees would have placed 2nd in the table of results (instead of 1st) had we selected other problems and/or metrics.

The bootstrap analysis complements the t-tests in Tables 2 and 3. The results suggest that if we had selected other problems/metrics, there is a 58% chance that boosted decision trees would still have ranked 1st

Table 4. Bootstrap Analysis of Overall Rank by Mean Performance Across Problems and Metrics

MODEL	1ST	2ND	3RD	4TH	5TH	6TH	7TH	8TH	9TH	10TH
BST-DT	0.580	0.228	0.160	0.023	0.009	0.000	0.000	0.000	0.000	0.000
RF	0.390	0.525	0.084	0.001	0.000	0.000	0.000	0.000	0.000	0.000
BAG-DT	0.030	0.232	0.571	0.150	0.017	0.000	0.000	0.000	0.000	0.000
SVM	0.000	0.008	0.148	0.574	0.240	0.029	0.001	0.000	0.000	0.000
ANN	0.000	0.007	0.035	0.230	0.606	0.122	0.000	0.000	0.000	0.000
KNN	0.000	0.000	0.000	0.009	0.114	0.592	0.245	0.038	0.002	0.000
BST-STMP	0.000	0.000	0.002	0.013	0.014	0.257	0.710	0.004	0.000	0.000
DT	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.616	0.291	0.089
LOGREG	0.000	0.000	0.000	0.000	0.000	0.000	0.040	0.312	0.423	0.225
NB	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.284	0.686

overall, and only a 4.2% chance of seeing them rank lower than 3rd place. Random forests would come in 1st place 39% of the time, 2nd place 53% of the time, with little chance (0.1%) of ranking below third place.

There is less than a 20% chance that a method other than boosted trees, random forests, and bagged trees would rank in the top three, and no chance (0.0%) that another method would rank 1st—it appears to be a clean sweep for ensembles of trees. SVMs probably would rank 4th, and neural nets probably would rank 5th, but there is a 1 in 3 chance that SVMs would rank after neural nets. The bootstrap analysis clearly shows that MBL, boosted 1-level stumps, plain decision trees, logistic regression, and naive bayes are not competitive *on average* with the top five models on these problems and metrics when trained on 5k samples.

6. Related Work

STATLOG is perhaps the best known study (King et al., 1995). STATLOG was a very comprehensive study when it was performed, but since then important new learning algorithms have been introduced such as bagging, boosting, SVMs, and random forests. LeCun et al. (1995) presents a study that compares several learning algorithms (including SVMs) on a handwriting recognition problem using three performance criteria: accuracy, rejection rate, and computational cost. Cooper et al. (1997) present results from a study that evaluates nearly a dozen learning methods on a real medical data set using both accuracy and an ROC-like metric. Lim et al. (2000) perform an empirical comparison of decision trees and other classification methods using accuracy as the main criterion. Bauer and Kohavi (1999) present an impressive empirical analysis of ensemble methods such as bagging and boosting. Perlich et al. (2003) conducts an empirical comparison between decision trees and logistic regression. Provost and Domingos (2003) examine the issue of predicting probabilities with decision trees, including smoothed and bagged trees. Provost and Fawcett (1997) discuss the importance of evaluating learning algorithms on metrics other than accuracy such as ROC.

7. Conclusions

The field has made substantial progress in the last decade. Learning methods such as boosting, random forests, bagging, and SVMs achieve excellent performance that would have been difficult to obtain just 15 years ago. Of the earlier learning methods, feedforward neural nets have the best performance and are competitive with some of the newer methods, particularly if models will not be calibrated after training.

Calibration with either Platt's method or Isotonic Regression is remarkably effective at obtaining excellent performance on the probability metrics from learning algorithms that performed well on the ordering metrics. Calibration dramatically improves the performance of boosted trees, SVMs, boosted stumps, and Naive Bayes, and provides a small, but noticeable improvement for random forests. Neural nets, bagged trees, memory based methods, and logistic regression are not significantly improved by calibration.

With excellent performance on all eight metrics, calibrated boosted trees were the best learning algorithm overall. Random forests are close second, followed by uncalibrated bagged trees, calibrated SVMs, and uncalibrated neural nets. The models that performed poorest were naive bayes, logistic regression, decision trees, and boosted stumps. Although some methods clearly perform better or worse than other methods *on average*, there is significant variability across the problems and metrics. Even the best models sometimes perform poorly, and models with poor average

performance occasionally perform exceptionally well.

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