Exercise: Metropolis MCMC in a harmonic potential

Write a Metropolis-based MCMC sampling algorithm to sample positions in the x/y-plane where we assume that the random walker moves in the potential ϕ (given on the right).

$$\phi(x, y) = \frac{x^2 + y^2}{2}$$

To this aim, we start at a given position close to the origin and move according to Metropolis rule, (on the right), where the the trial

$$(x_n, y_n) = \begin{cases} (x_{\text{trial}}, y_{\text{trial}}), & \text{if accepted} \\ (x_{n-1}, y_{n-1}), & \text{else} \end{cases}$$

positions are randomly and symmetrically chosen around the current position in the Markov chain.

The conditional probability to accept a trial configuration, which requires you to evaluate ϕ at the trial and the current position, is given below:

$$\mathbb{A}\left((x_{\text{trial}}, y_{\text{trial}})|(x_{n-1}, y_{n-1})\right) = \min\left\{1, \frac{\exp\left(-\phi(x_{\text{trial}}, y_{\text{trial}})\right)}{\exp\left(-\phi(x_{n-1}, y_{n-1})\right)}\right\}$$