Line of Sight MIMO

Marsona Panci
Mathematical Engineering - Statistical Learning

Information Theory 2023/24
Professor Maurizio Magarini

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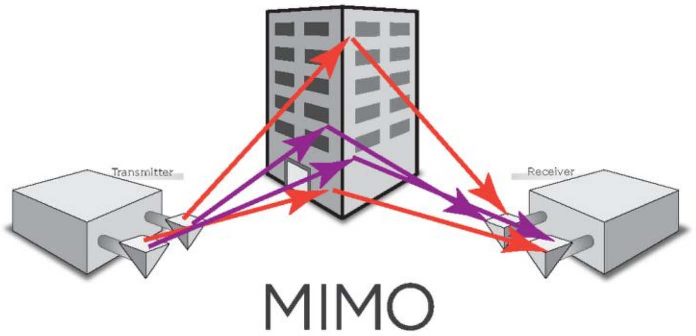
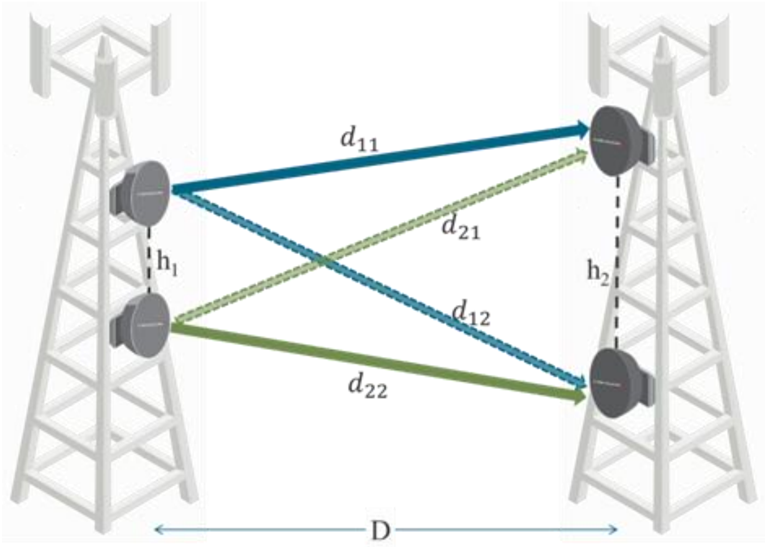
Introduction



A ***communication system*** with n_t transmitting antennas and n_r receiving antennas is commonly known as a multiple-input multiple-output (MIMO) system.

However, traditional MIMO techniques, which rely on the ***scattering of propagation parameters*** between multiple paths in non-line-of-sight conditions, are not applicable for microwave point-to-point radio links. These classical techniques ***will not be discussed here***.

Introduction



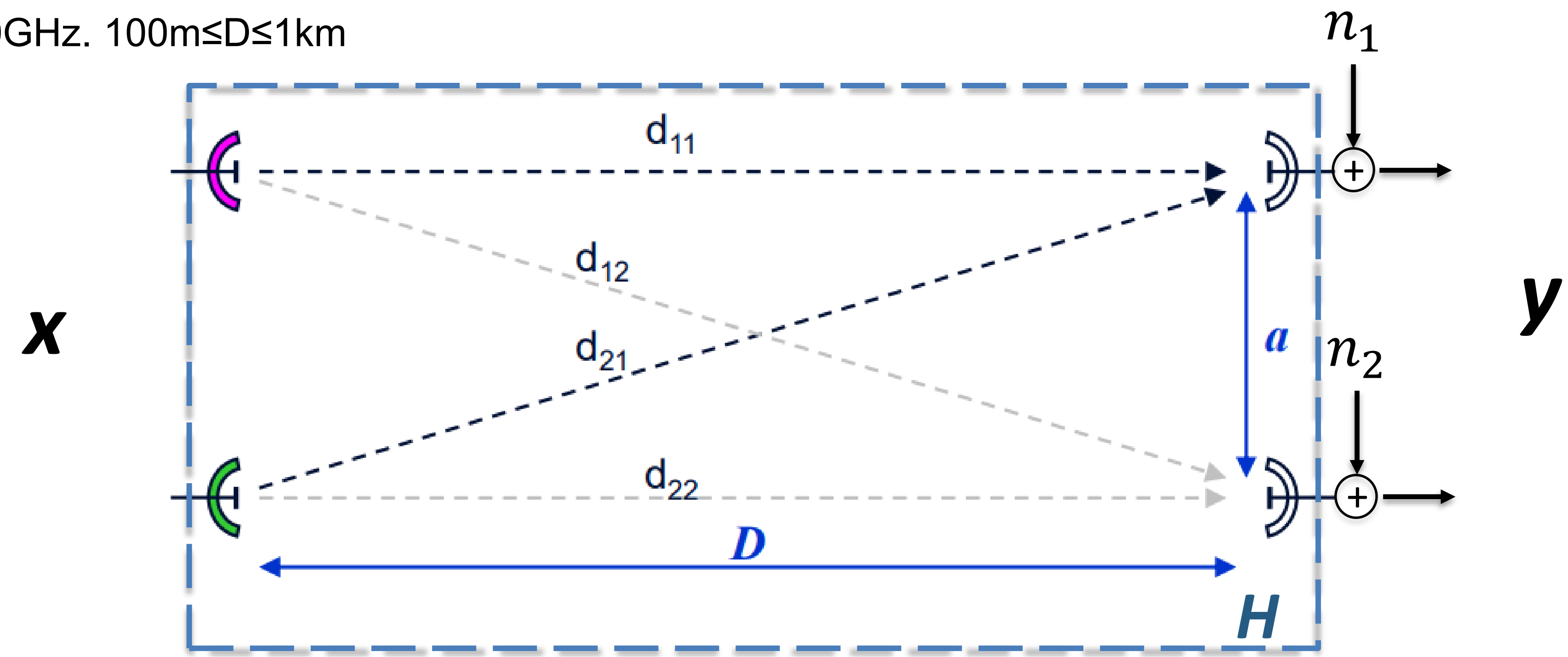
For microwave point-to-point radio links, the emphasis is on Line of Sight (LoS) MIMO:

- The transmitting and receiving stations must have ***direct visibility.***
- The propagation channel consists of a ***single dominant path***.

This specific focus is crucial for the effective application of MIMO in microwave point-to-point links.

LoS MIMO in Microwave Point-to-Point Links

1. Study capacity of each LoS MIMO of size N as a function of antennas spacing ($a < 10$ m) and hop Length (D) for these carrier frequencies (f):
   A. f=18GHz. 5km≤D≤20km
   B. f=38GHz. 2km≤D≤10km
   C. f=80GHz. 1km≤D≤5km
   D. f=115GHz. 500m≤D≤2km
   E. f=170GHz. 100m≤D≤1km



Mathematical Model

Channel equation:

$$\boldsymbol{y} = H\boldsymbol{x} + \boldsymbol{n}$$

where :

$\boldsymbol{y} \in \; C^{n_r};$
$\boldsymbol{x} \in \; C^{n_t} \;,$
$\boldsymbol{n} \sim \mathcal{CN}(\mathbf{0}\;, \boldsymbol{I}\sigma_n^2)$

$$H = \begin{bmatrix} e^{-jkd_{1,1}} & \cdots & e^{-jkd_{1,n_t}} \\ \vdots & \ddots & \vdots \\ e^{-jkd_{n_t,1}} & \cdots & e^{-jkd_{n_r,n_t}} \end{bmatrix}$$

Mathematical Model

Channel Capacity:

$$C = log_2\left(\det\left(I_{n_r} + \frac{\rho}{n_t}HH^H\right)\right)$$

n_t = *number of transmitting antennas*

n_r = *number of receiving antennas*

ρ = average received signal-to-noise ratio (SNR) at the input of the receiver

Where trasmitted power is equally divided among trasmitting antennas

Mathematical Model

$$HH^H = \begin{bmatrix} n_t & \cdots & \sum_{m=0}^{n_t-1} e^{-jk(d_{1,m}-d_{n_r,m})} \\ \vdots & \ddots & \vdots \\ \sum_{m=0}^{n_t-1} e^{-jk(d_{n_r,m}-d_{1,m})} & \cdots & n_t \end{bmatrix}$$

Minimum capacity

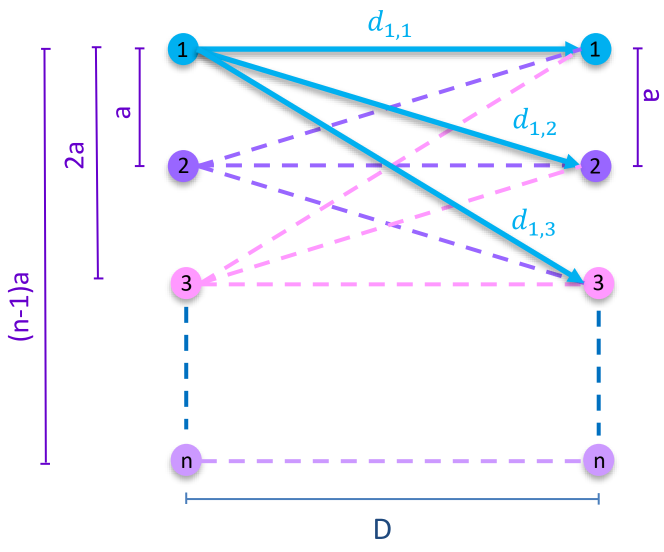
- Minimum capacity is obtained for $HH^H = n_t 1_{n_r}$
  - 1_{n_r} all ones $n_r \times n_t$ matrix.
  - This corresponds to an entirely correlated (rank-one) MIMO channel.

Maximum capacity

- Maximum capacity is obtained for $HH^H = n_t I_{n_r}$
  - I_{n_r} diagonal $n_r \times n_t$ matrix.
  - This corresponds to a perfectly orthogonal MIMO subchannels.

Symmetrical N x N model

To get the maximum capacity :
$$\langle h_k, h_l \rangle = \sum_{m=0}^{n-1} e^{-j\frac{2\pi}{\lambda}(d_{k,m}-d_{l,m})} = 0$$



- *Distance between transmitting and reciving antennas:*

$$d_{t,r} = \sqrt{D^2 + (t-r)^2 a^2}$$

  - t = *index of the transmitting antenna*
  - r = *index of the receiving antenna*

- *Since we are in the situation of* $a<<D$ *we can expand with Taylors series:*

$$d_{t,r} \approx D + \frac{(t-r)^2 a^2}{2D}$$

- *Which leads to:*

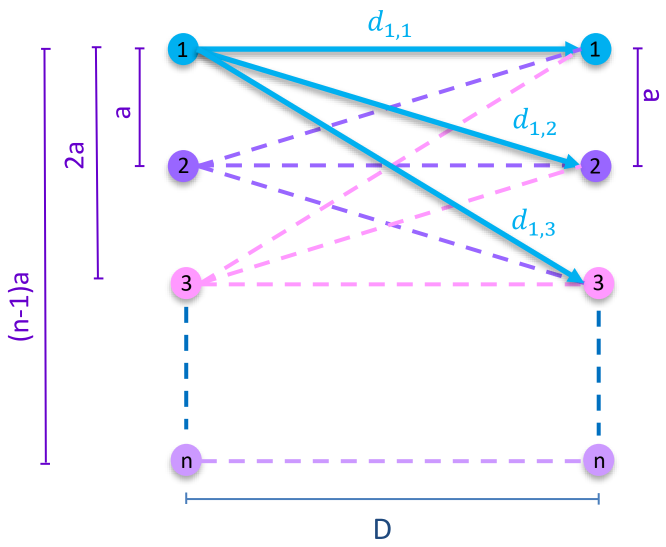
$$d_{k,m} - d_{l,m} \approx \frac{(k-m)^2 a^2}{2D} - \frac{(l-m)^2 a^2}{2D} \approx \frac{m(k-l)a^2}{D} = (k-l)(d_{1,m} - d_{2,m})$$

Ignoring the terms that are constant in the sum.

Symmetrical N x N model

To get the maximum capacity :

$$\langle h_k, h_l \rangle = \sum_{m=0}^{n-1} e^{-j\frac{2\pi}{\lambda}(d_{k,m}-d_{l,m})} = 0$$



- *Finnaly to get the first maximum we impose:*

$$\frac{2\pi}{\lambda}\left(d_{1,m} - d_{2,m}\right)n - \frac{2\pi}{\lambda}\left(d_{1,m+1} - d_{2,m+1}\right)n = 2\pi$$

which leads to:

$$a^2 = \frac{\lambda\, D}{n}$$

- *The condition relates wave length (λ), distance (D) and antenna spacing (a).*

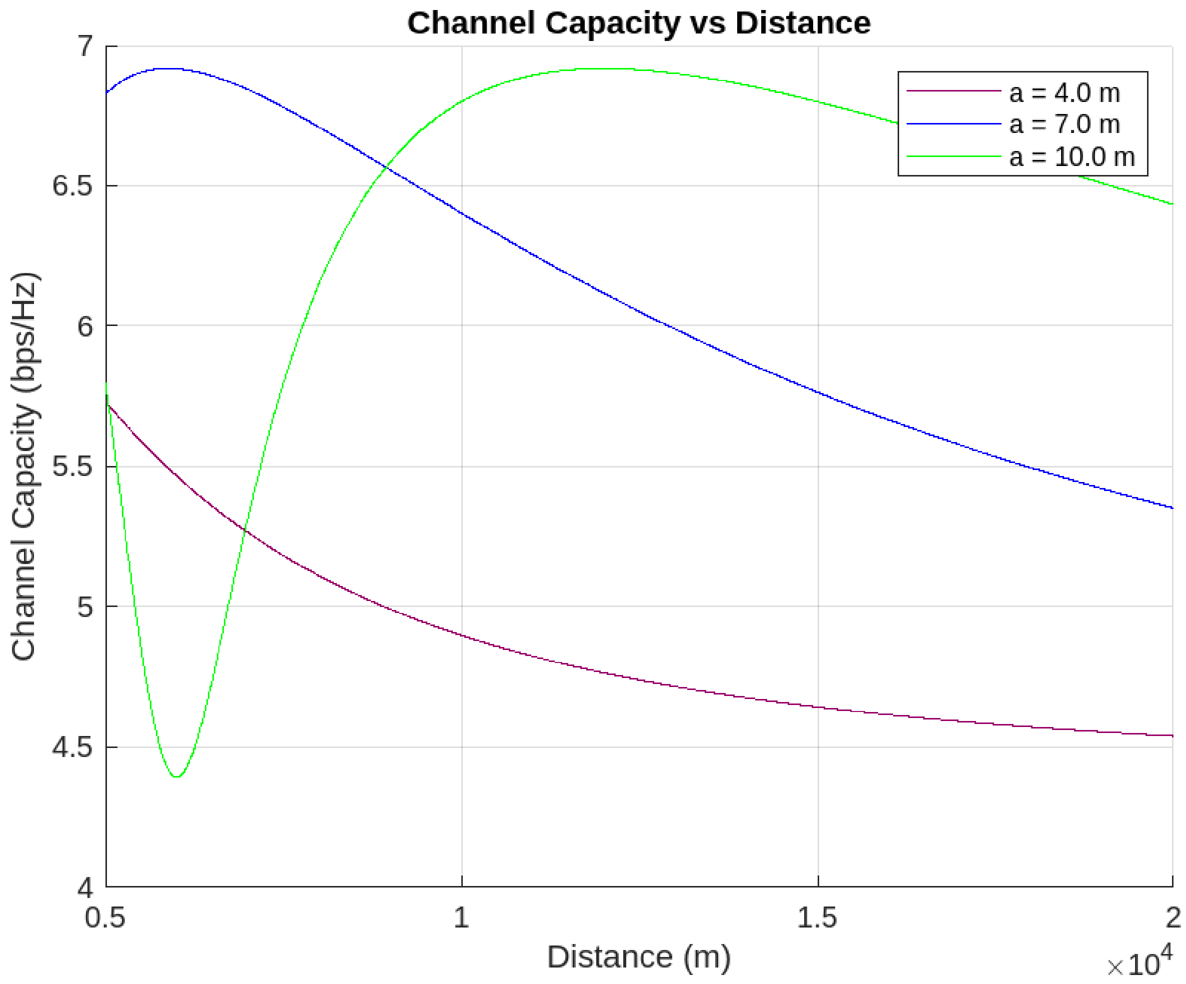
Computational Model

Assumptions:

- The system used for the graphs is a 2x2 MIMO, which does not lose generality.
- The average SNR (ρ) is fixed at 10 dB.
- Antenna spacing (a) is less than 10 meters.

Case of study

A. 18GHz – [5km, 20km]



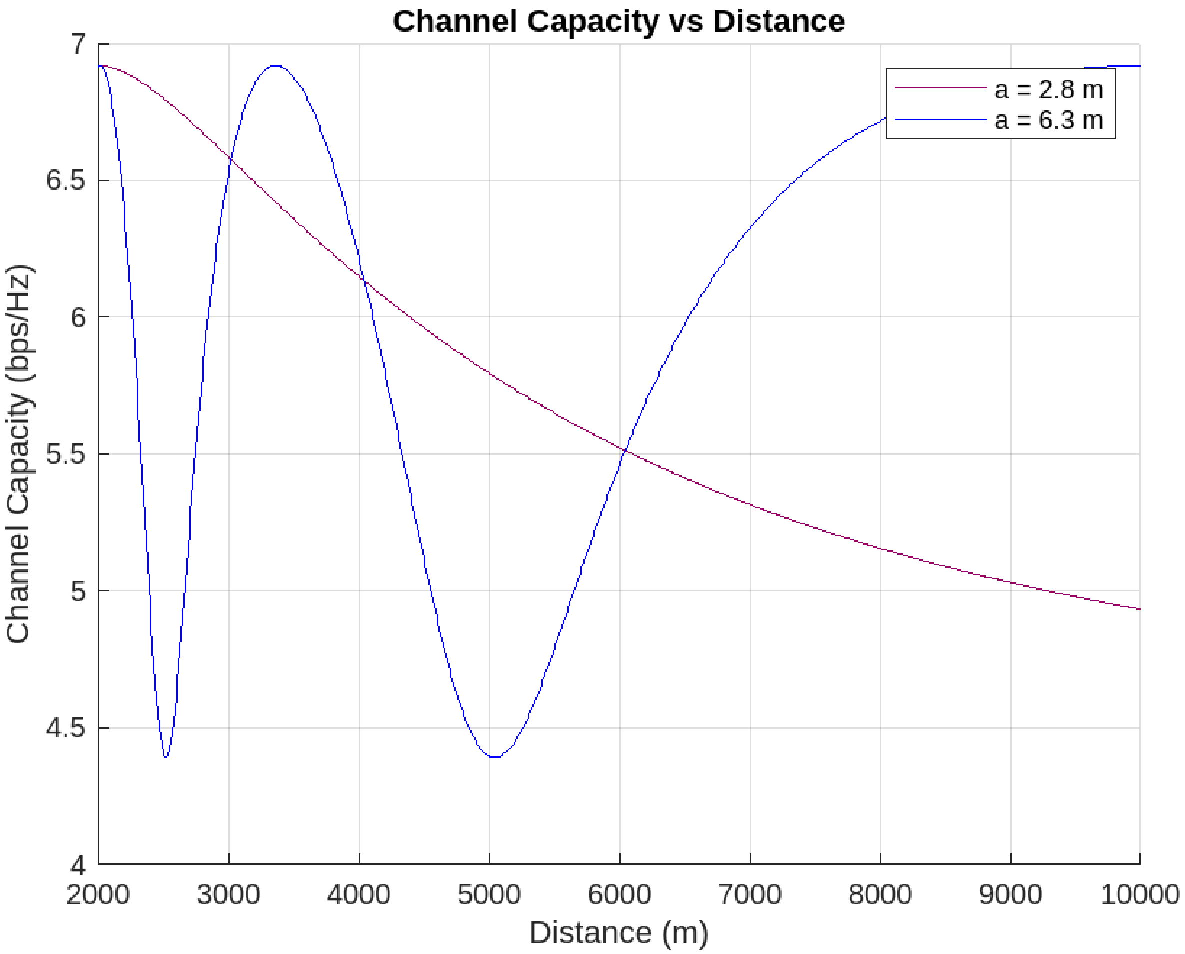
Analytical maximum of capacity in function of distance and antenna spacing:

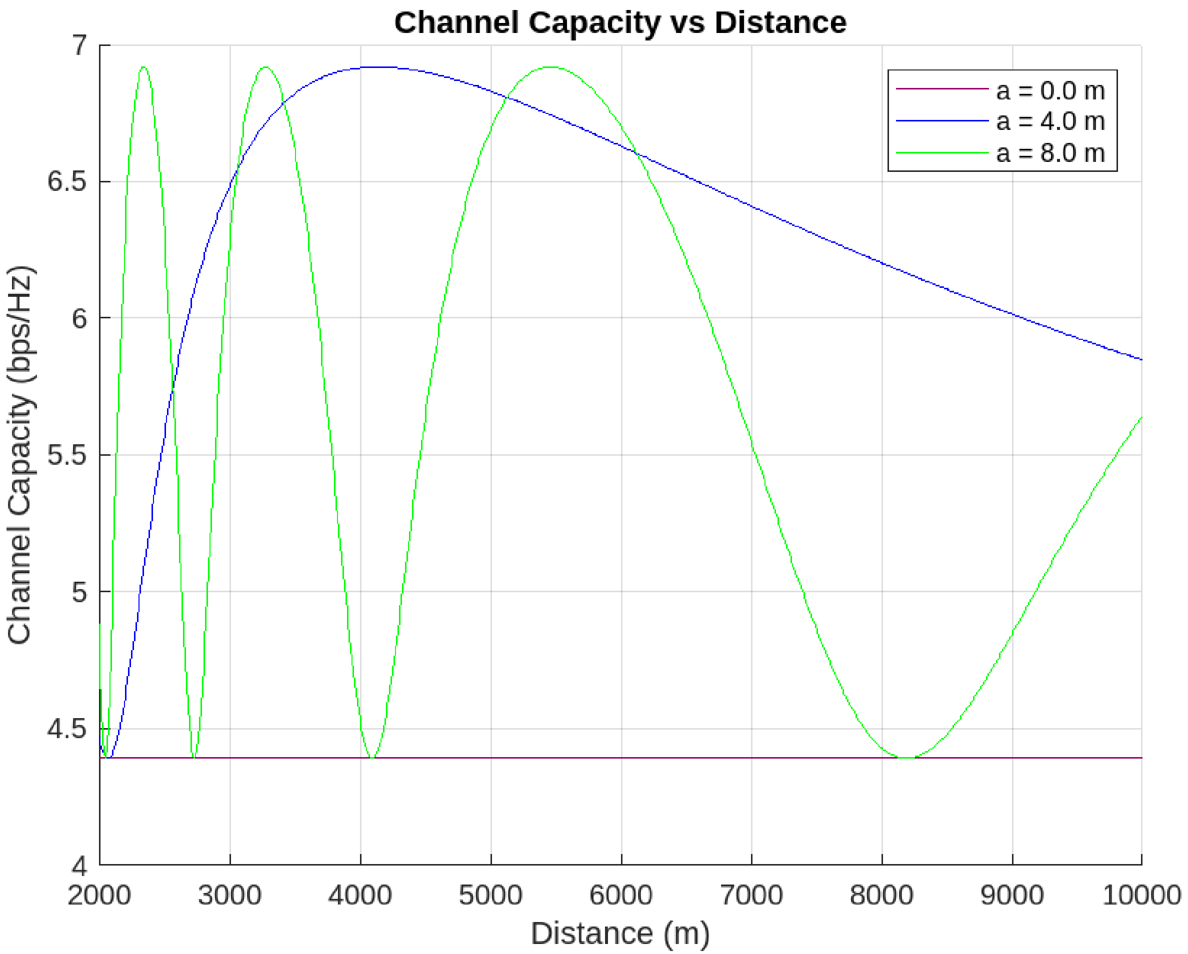
D		a
5 km	→	6.5 m
20 km	→	13 m > 10 m

- If antenna spacing is in the analytical range we are able to reach maximum capacity
- Else rule of thumb: lower the distance

Case of study

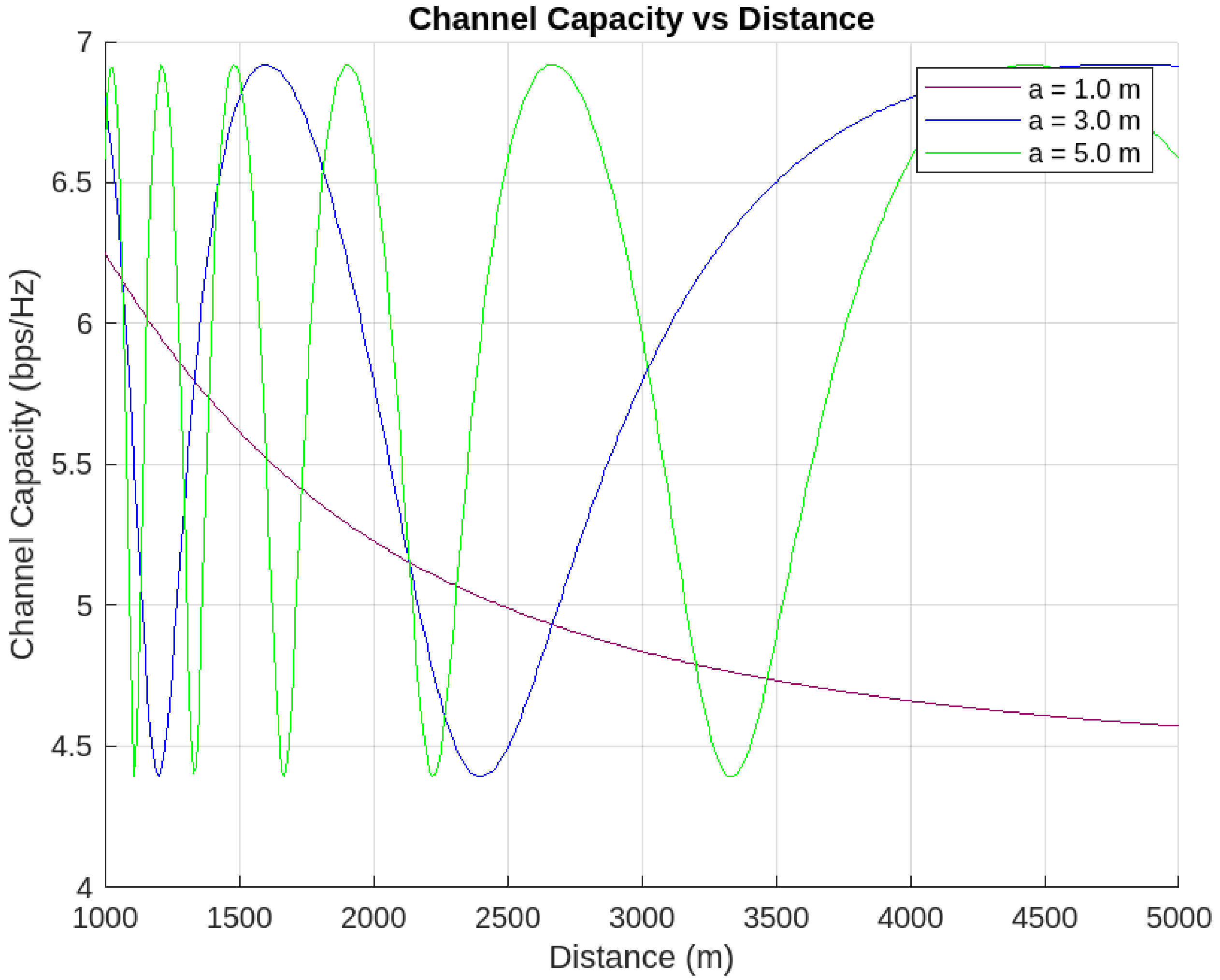
B. 38GHz – [2km, 10km]





Case of study

C. 80GHz – [1km, 5km]



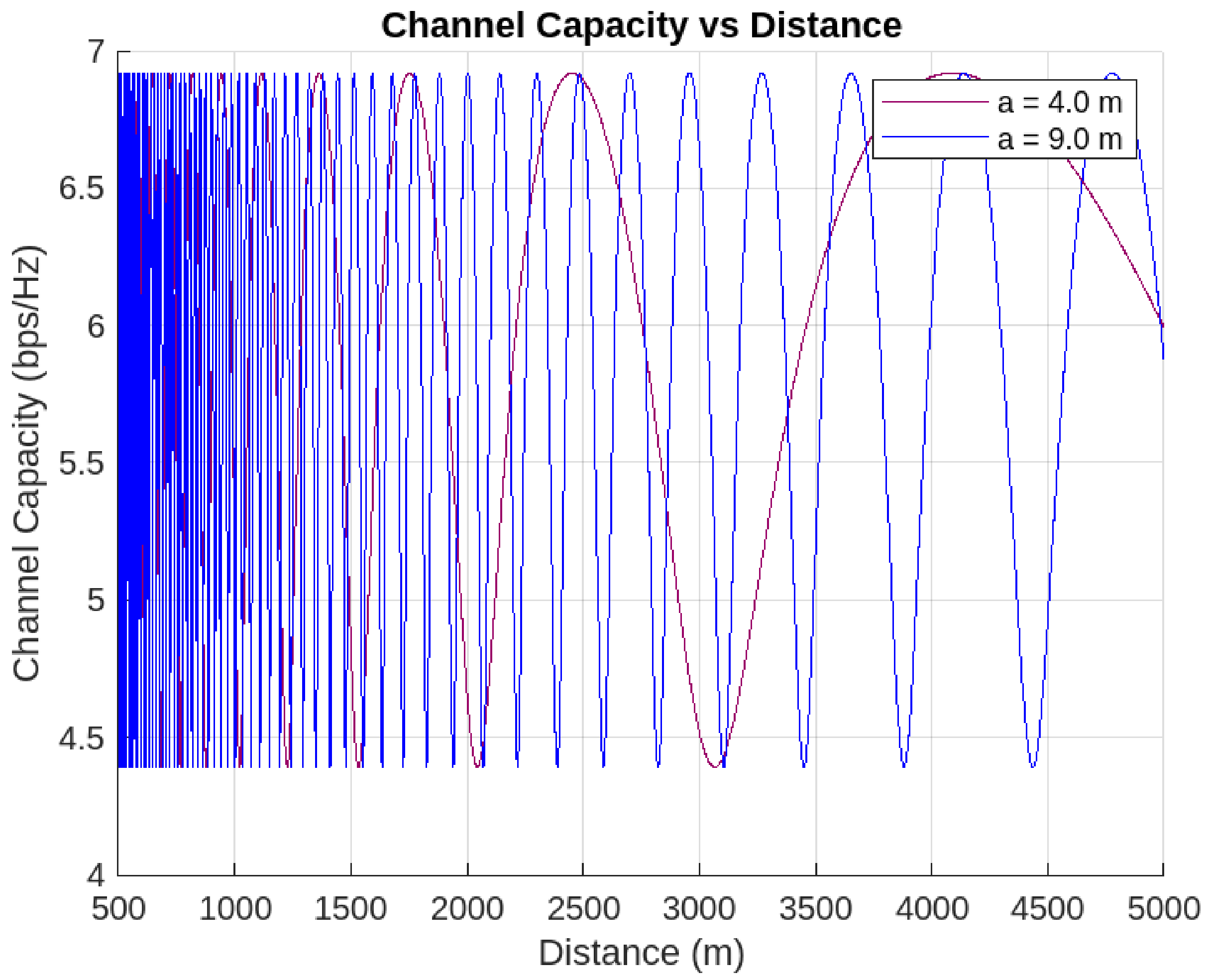
Higher frequency

↓

Lower antenna spacing

Case of study

D. 115GHz – [0.5km, 2km]



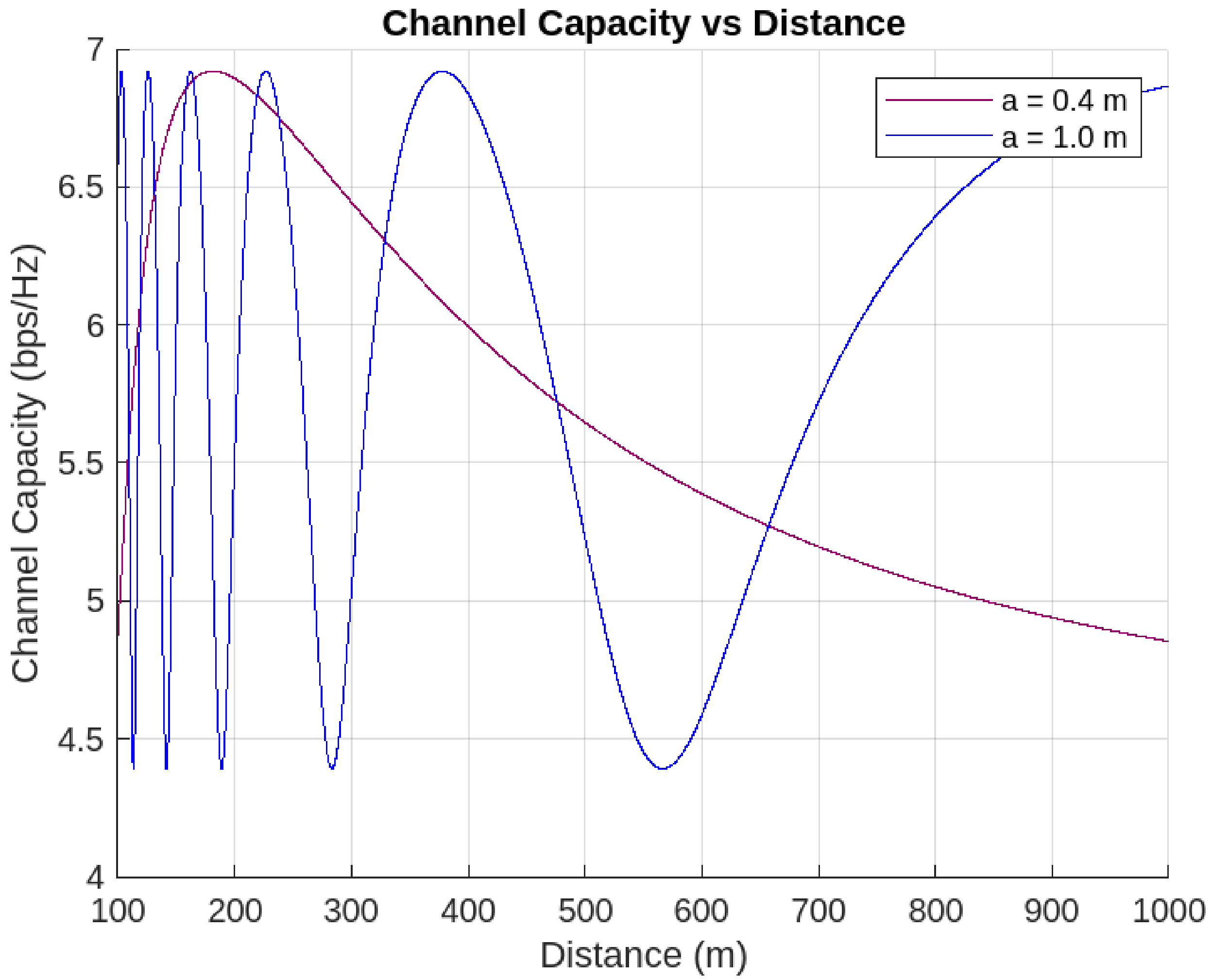
Analytical maximum of capacity in function of distance and antenna spacing:

D	a
0.5 km →	0.8 m
2 km →	1.6 m

Higher spacing → Increasing instability

Case of study

E. 170GHz – [0.1km, 1km]

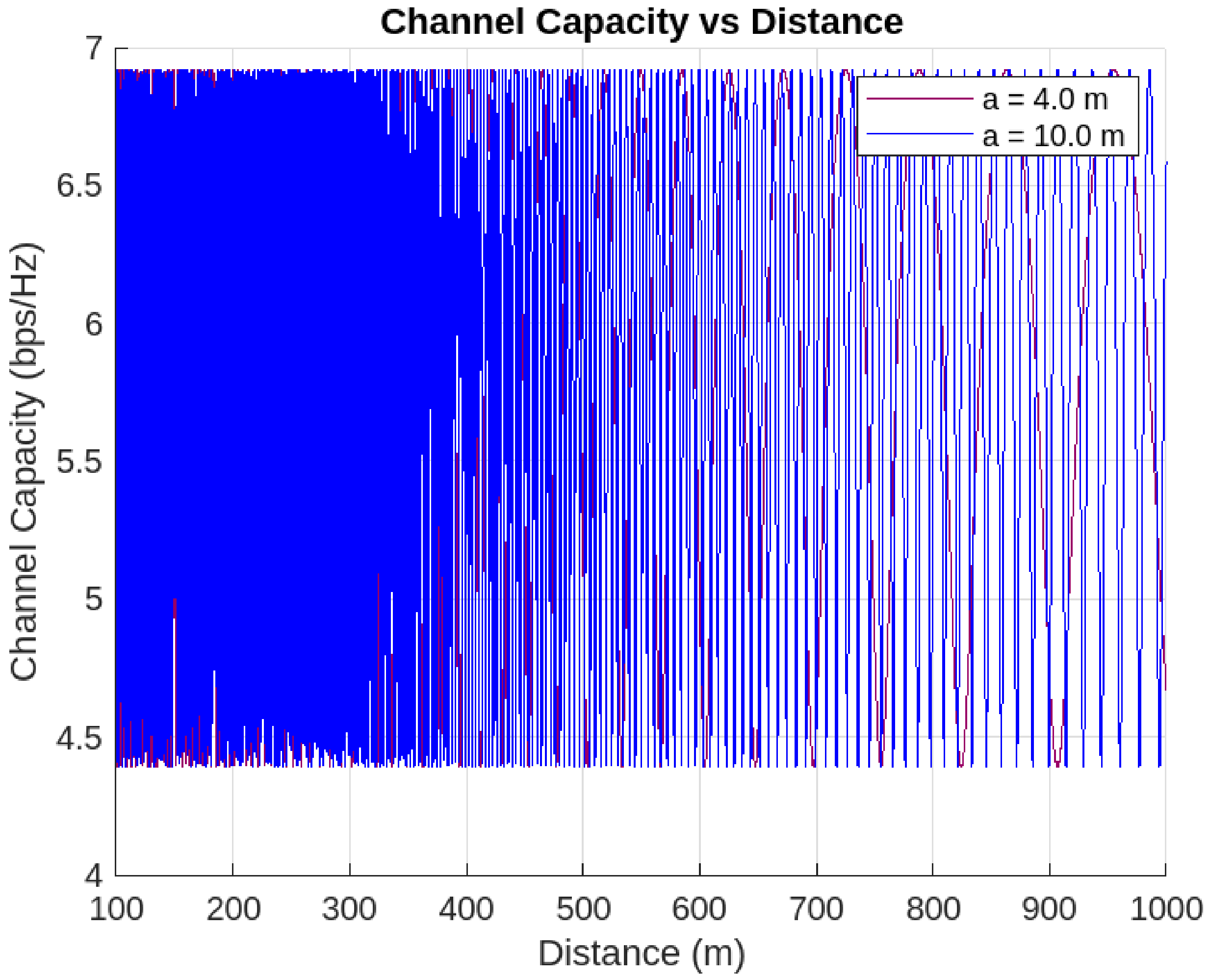


Analytical maximum of capacity in function of distance and antenna spacing:

D		a
0.1 km	→	0.3 m
1 km	→	0.9 m

Case of study

E. 170GHz – [0.1km, 1km]



Analytical maximum of capacity in function of distance and antenna spacing:

D	a
0.1 km →	0.3 m
1 km →	0.9 m

The channel capacity exhibits high sensitivity to variations in both the distance D and the antenna spacing a

Line of Sight MIMO in Microwave Point-to-Point Links

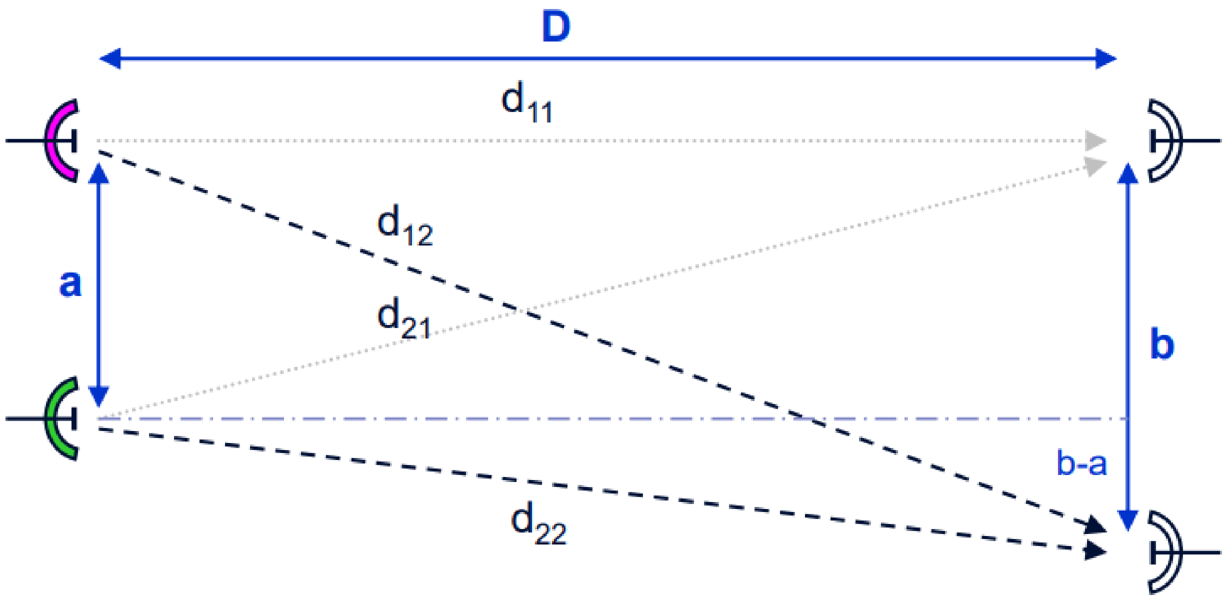
Due to the installation constraints the antenna spacing on the two sites can be different (a,b):

Study capacity of each Line-of-Sight MIMO of size N as a function of antennas spacing (a < 10 m and b < 10 m) and hop Length (D) for these carrier frequencies (f):

A. f=18GHz. 5km≤D≤20km

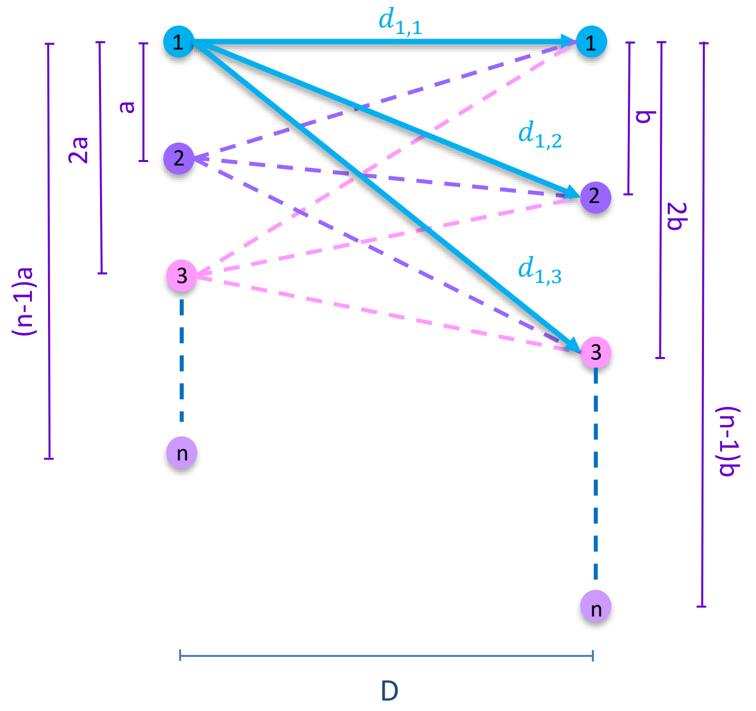
B. f=80GHz. 1km≤D≤5km

C. f=170GHz. 100m≤D≤1km



Asymmetrical N x N model – Different antenna spacing

To get the maximum capacity : $$\langle h_k, h_l \rangle = \sum_{m=0}^{n-1} e^{-j\frac{2\pi}{\lambda}(d_{k,m}-d_{l,m})} = 0$$



- *We should modify our distance introducing b (reciving antenna spacing $a < 10m$)*

$$d_{t,r} = \sqrt{D^2 + ((t-1)a - (r-1)b)^2}$$

which leads to:

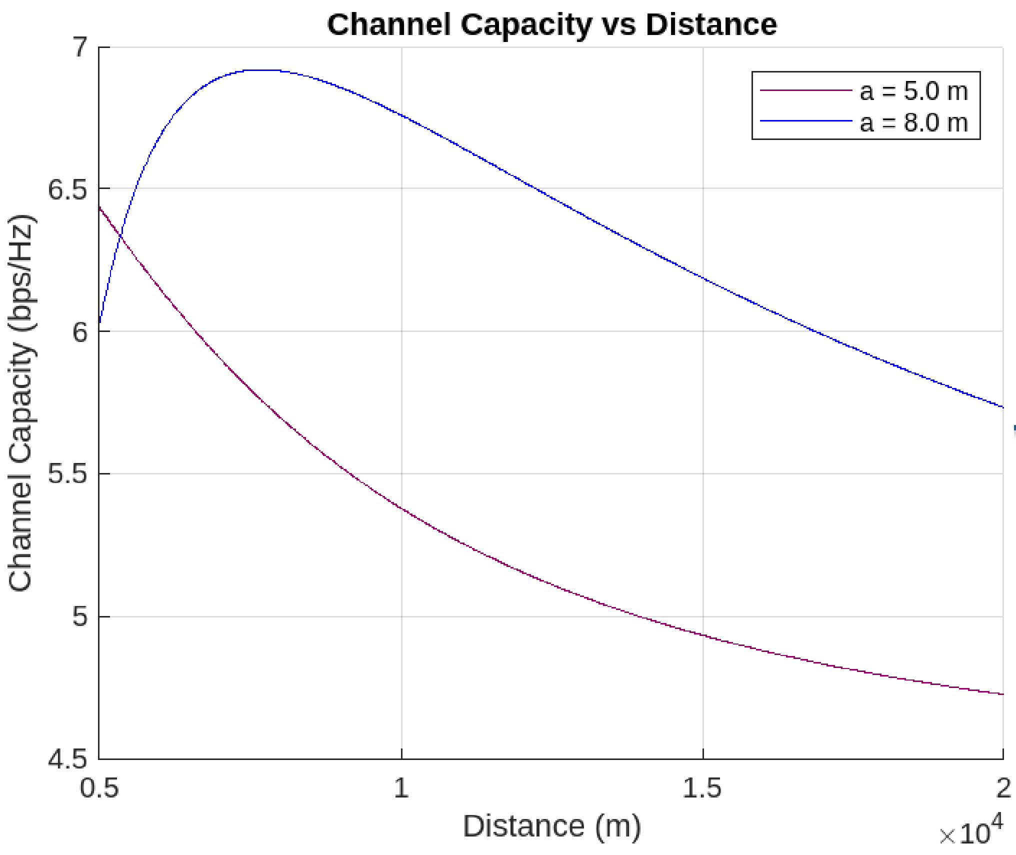
$$a \cdot b = \frac{\lambda D}{n}$$

- *The condition relates wave length, distance and antenna spacing of both trasmitter and reciver.*

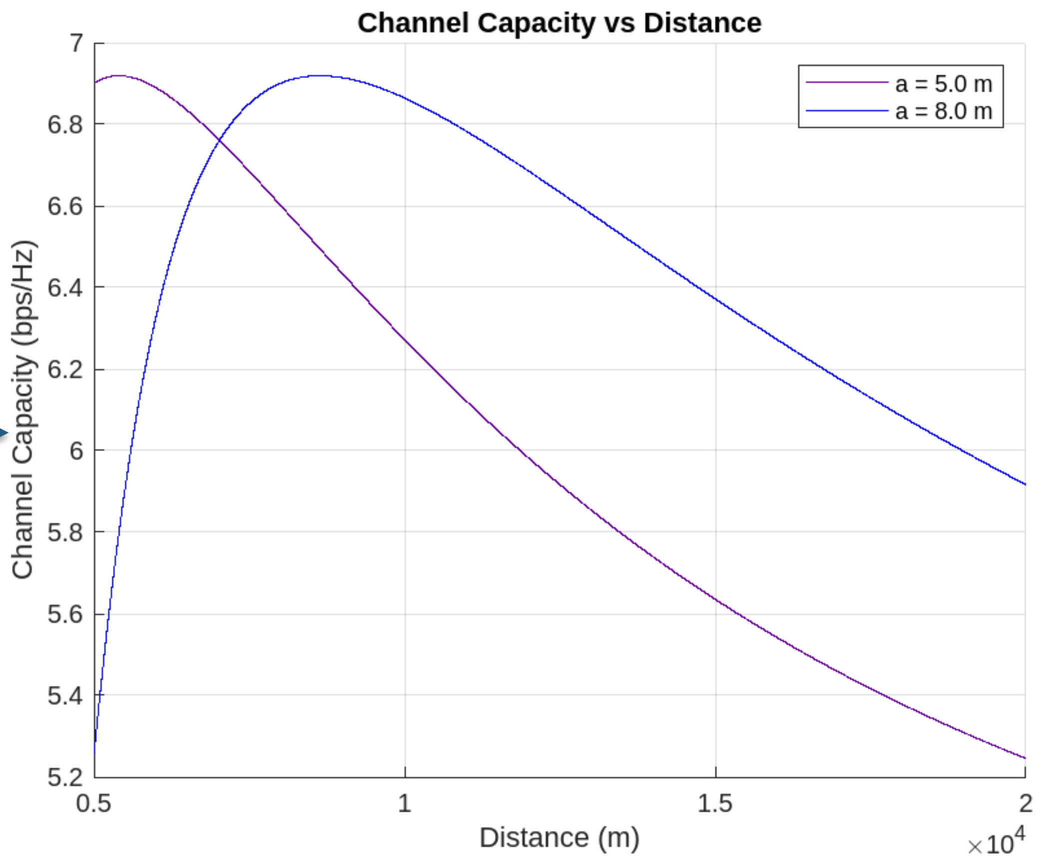
Case of study

A. 18GHz – [5km, 20km]

a = b



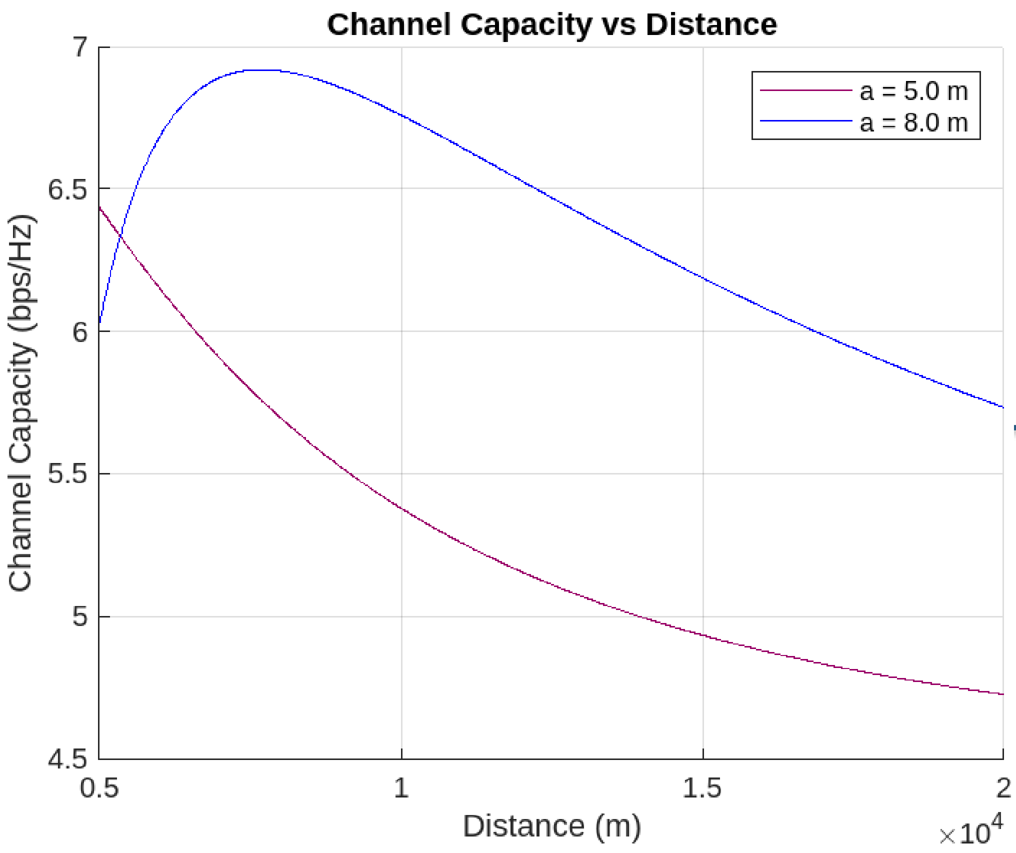
b = 9 m



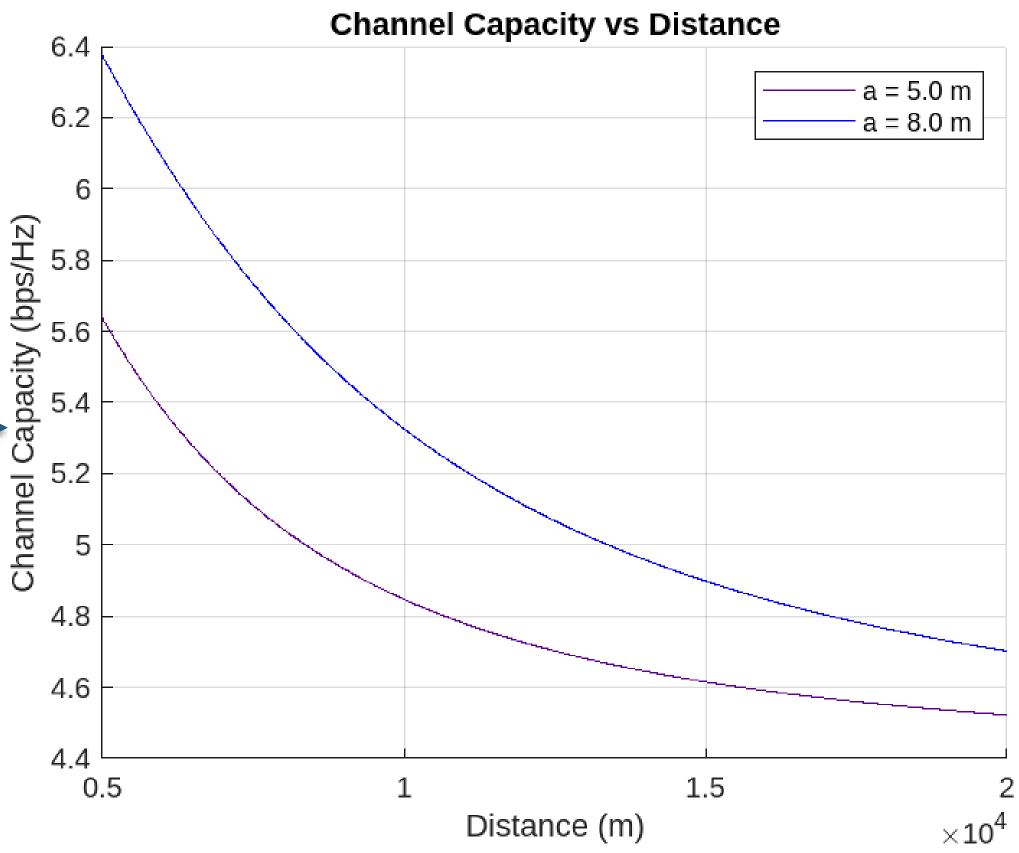
Case of study

A. 18GHz – [5km, 20km]

a = b



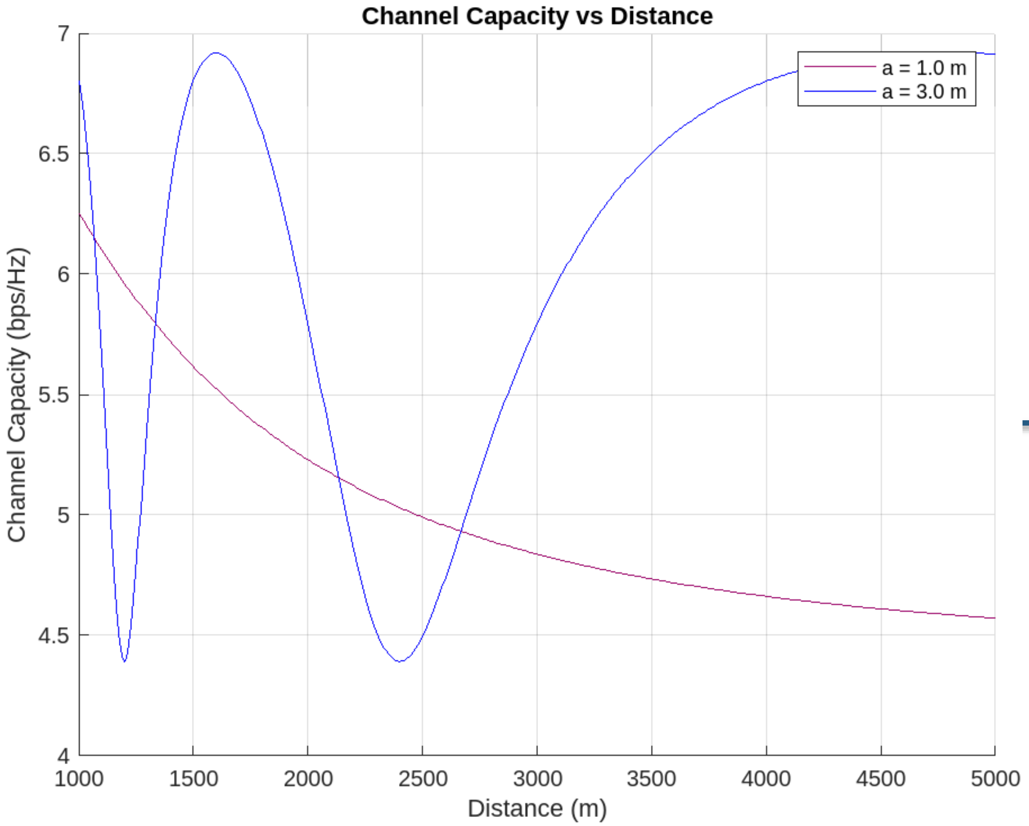
b = 3 m



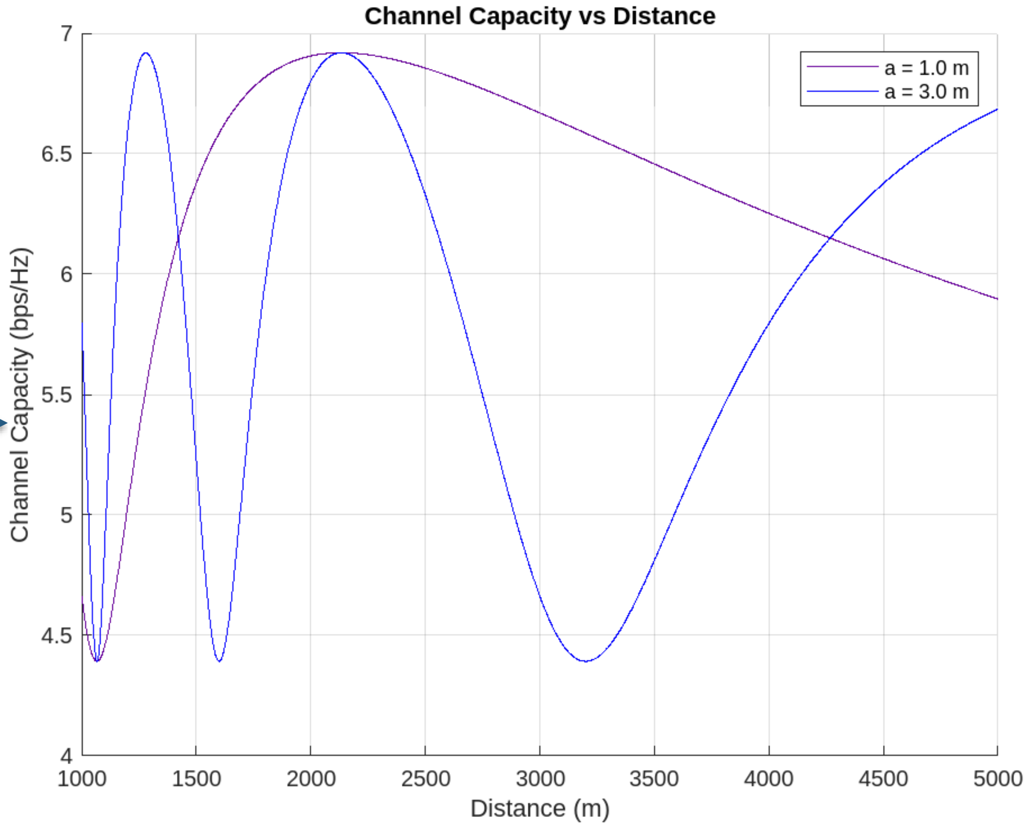
Case of study

B. 80GHz – [1km, 5km]

a = b



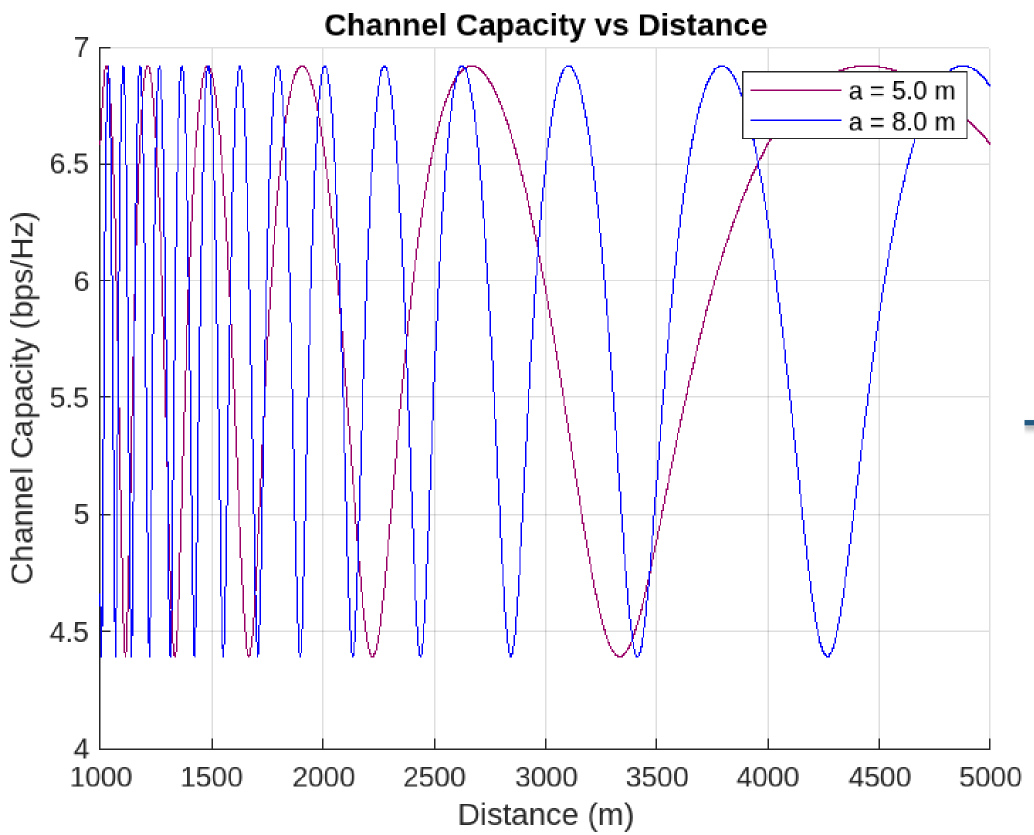
b = 4 m



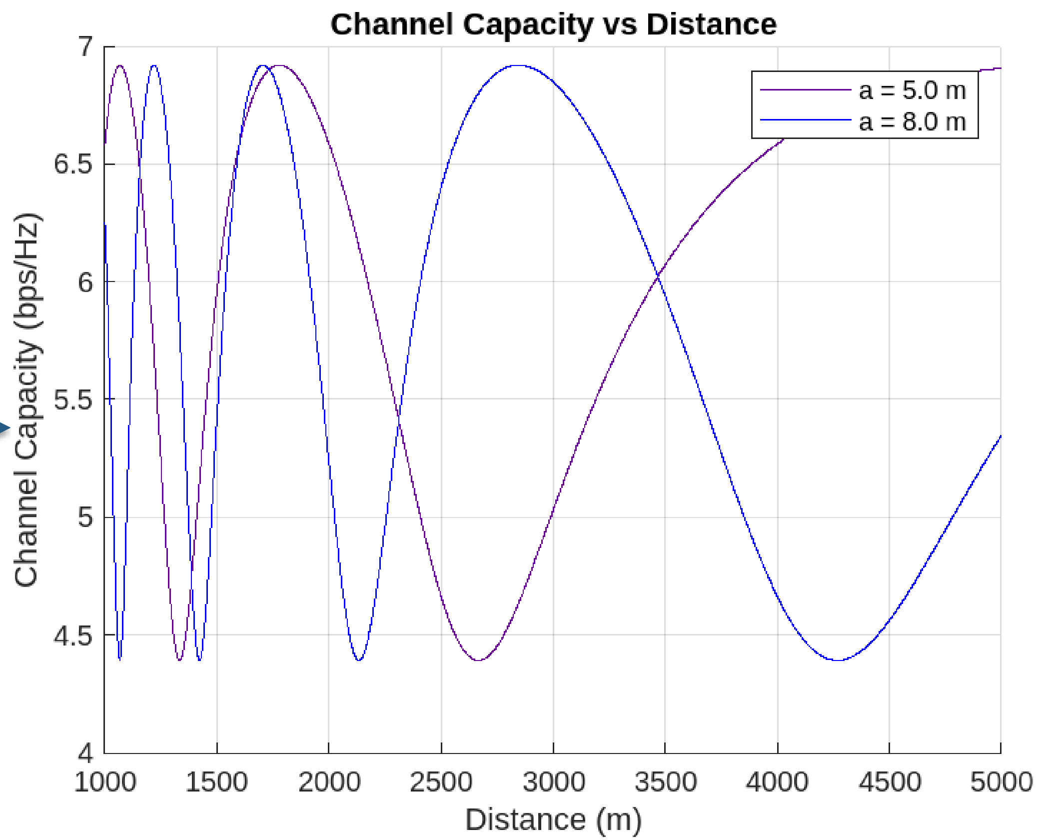
Case of study

B. 80GHz – [1km, 5km]

a = b



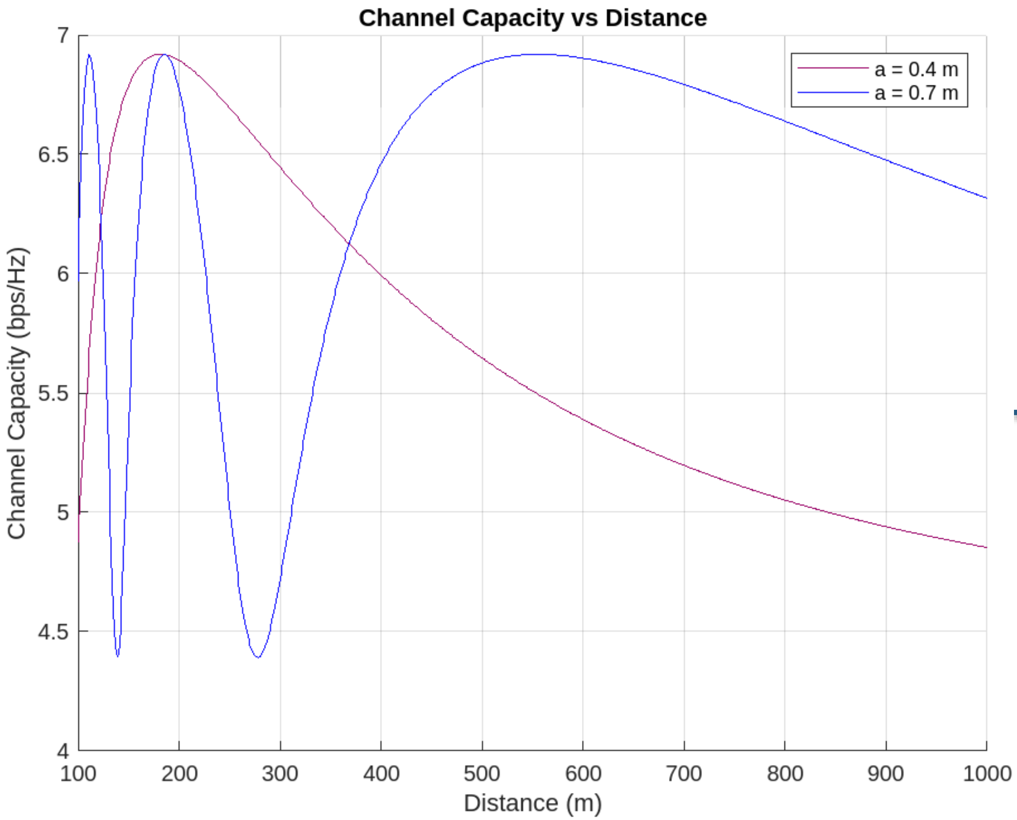
b = 2 m



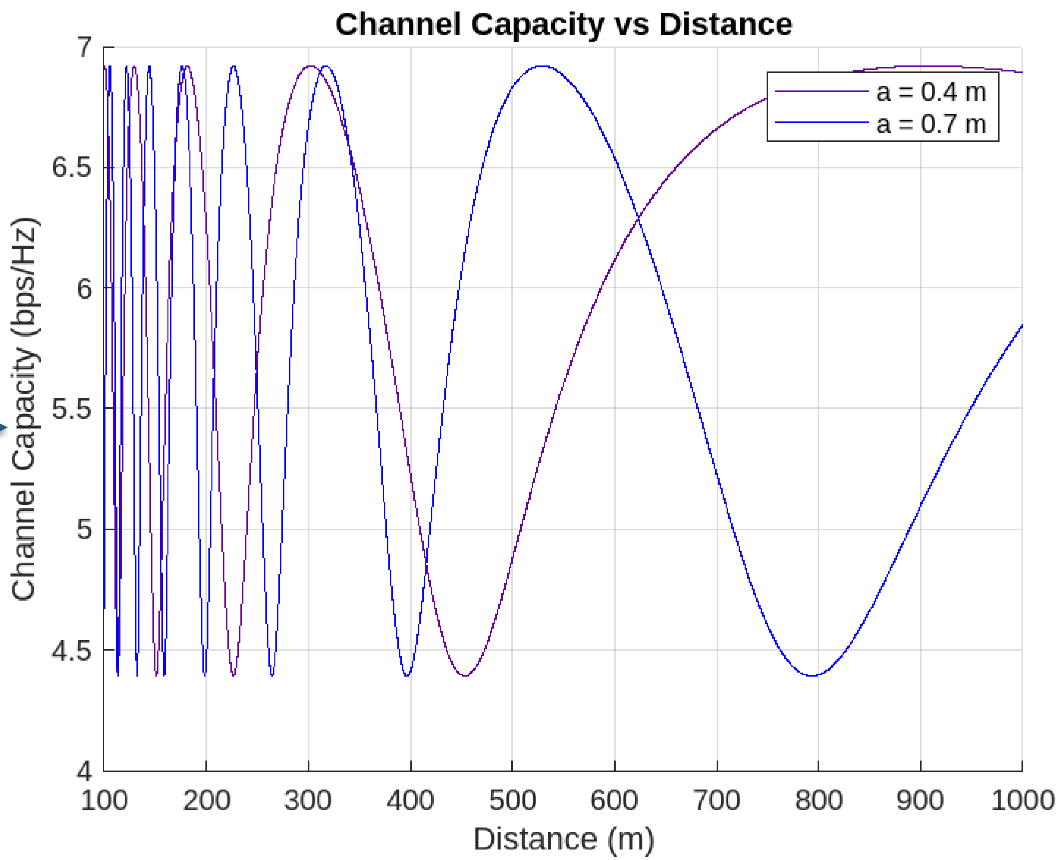
Case of study

C. 170GHz – [0.1km, 1km]

a = b



b = 2 m

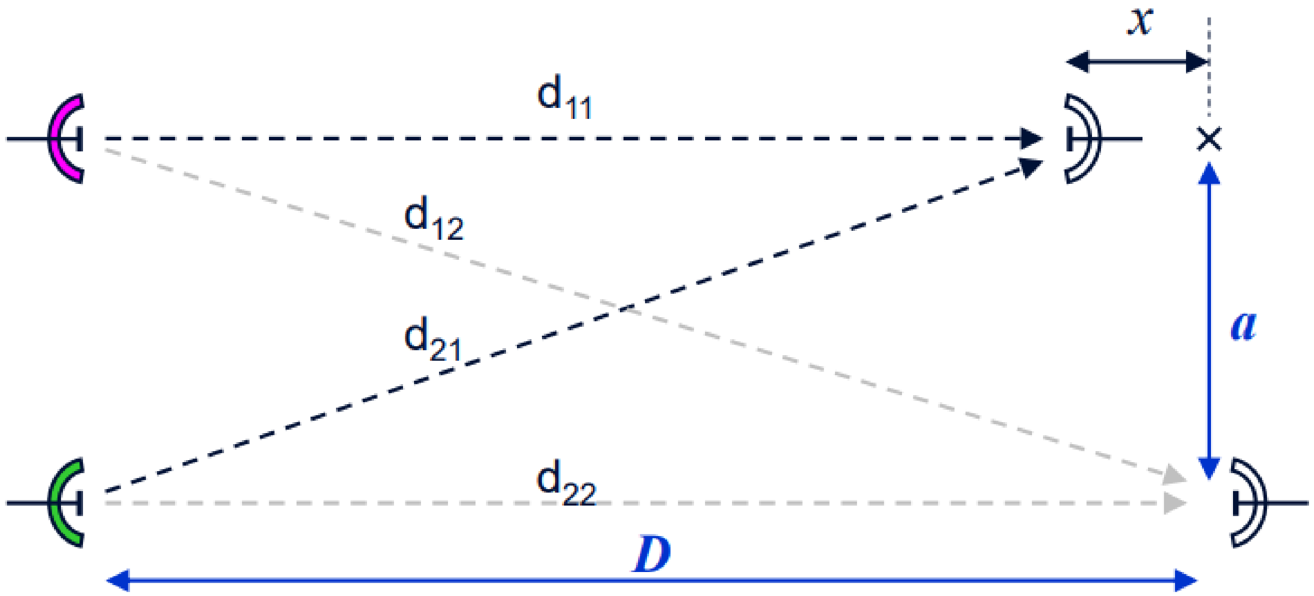


Line of Sight MIMO in Microwave Point-to-Point Links

Oscillations of the antenna tower induced by wind can result in a relative displacement x of the two MIMO antennas along the direction of propagation.

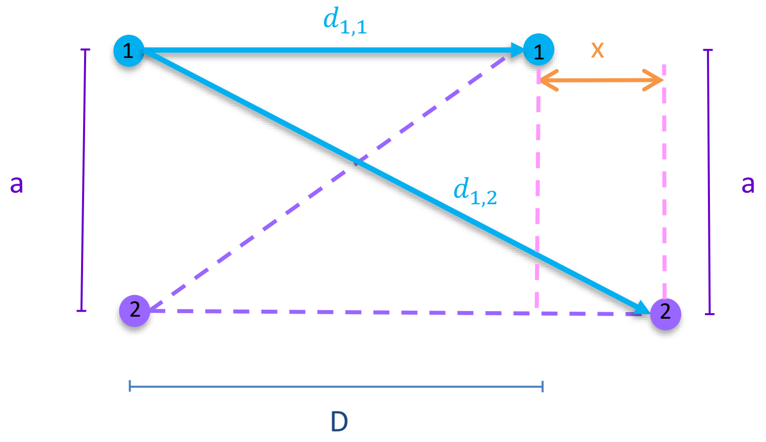
Study capacity of each Line-of-Sight MIMO of size N as a function of antennas spacing (a < 10 m), hop Length (D) and displacement (x < 1 m), for these carrier frequencies (f):

A. f=18GHz. 5km≤D≤20km
B. f=80GHz. 1km≤D≤5km
C. f=170GHz. 100m≤D≤1km



Asymmetrical 2 x 2 model – Displacement of antenna

To get the maximum capacity : $$\langle h_k, h_l \rangle = \sum_{m=0}^{1} e^{-j\frac{2\pi}{\lambda}(d_{k,m}-d_{l,m})} = 0$$



- *The distance will be (x = displacement < 1m)*

$$d_{t,r} = \sqrt{(D + (r-1)x)^2 + (t-r)^2 a^2}$$

Which leads to:

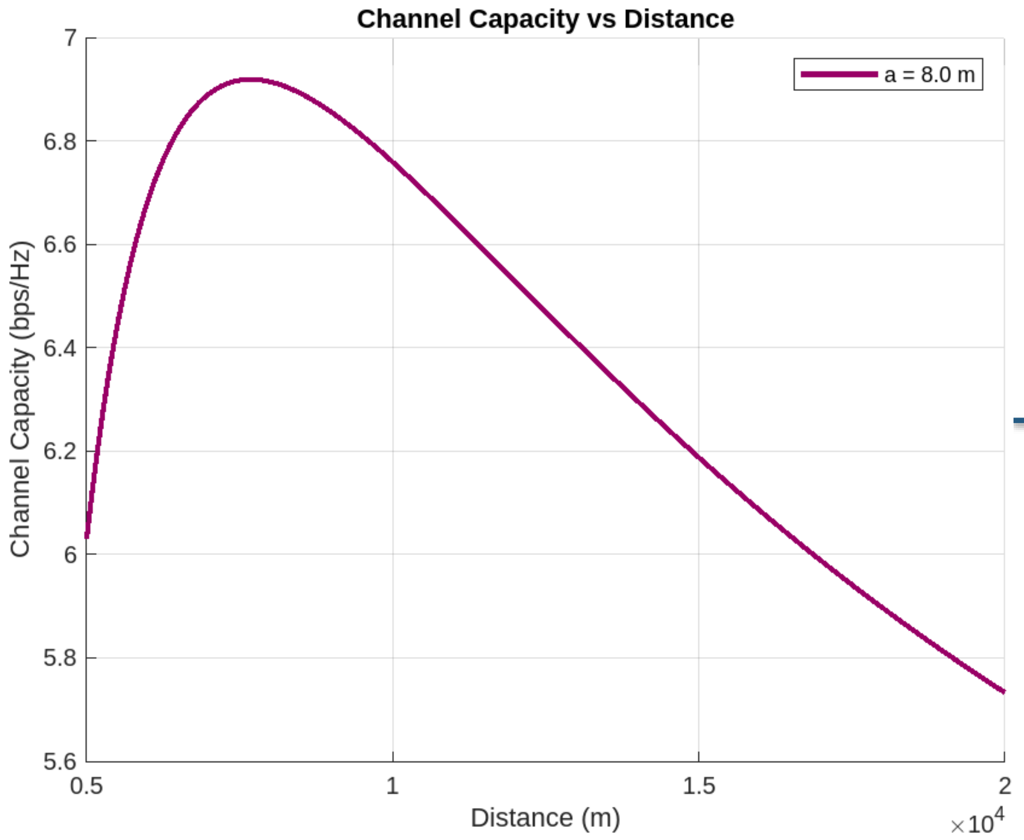
$$a^2 = \frac{\lambda\,(D + x)}{n}$$

- *The condition relates wave length, distance, antenna spacing and displacement.*

Case of study

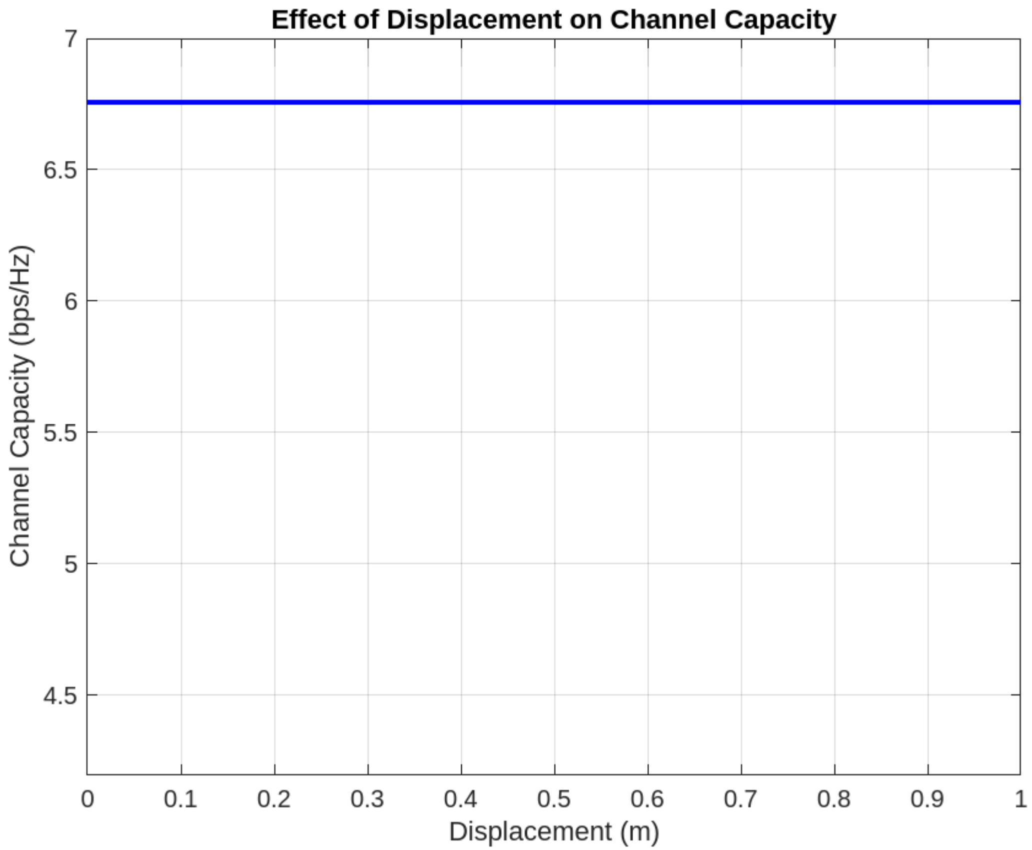
A. 18GHz – [5km, 20km]

x=0 m



Robust to small displacement

D = 10 Km

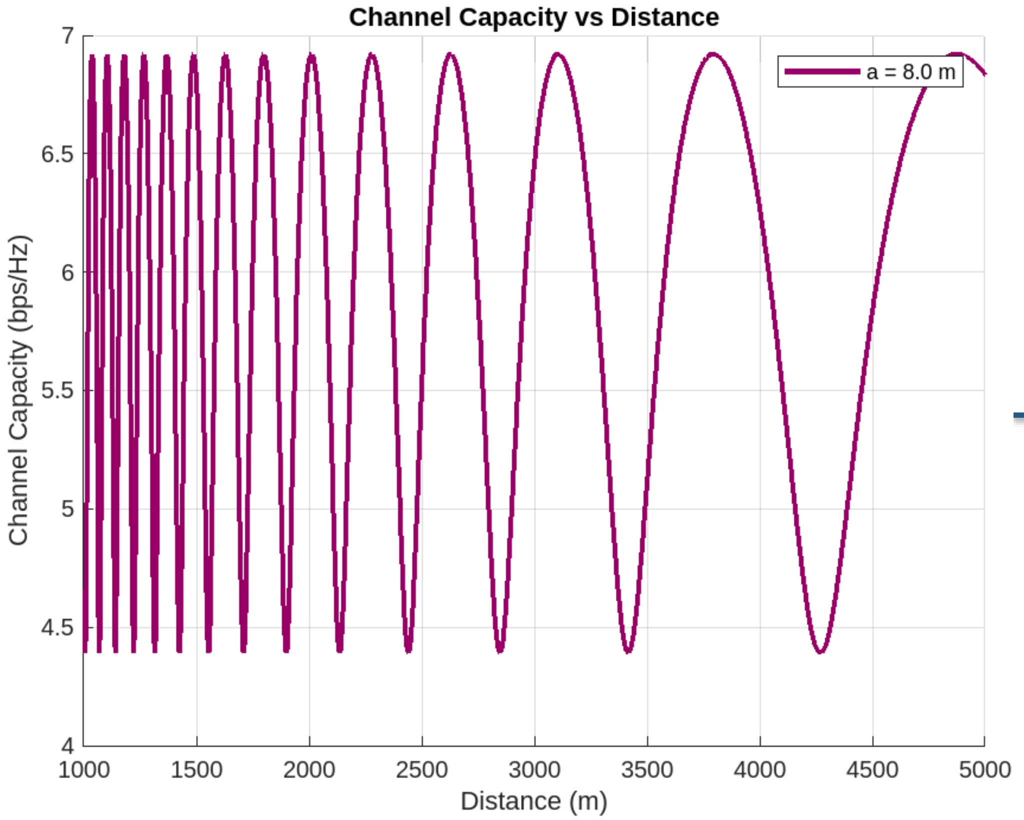


Case of study

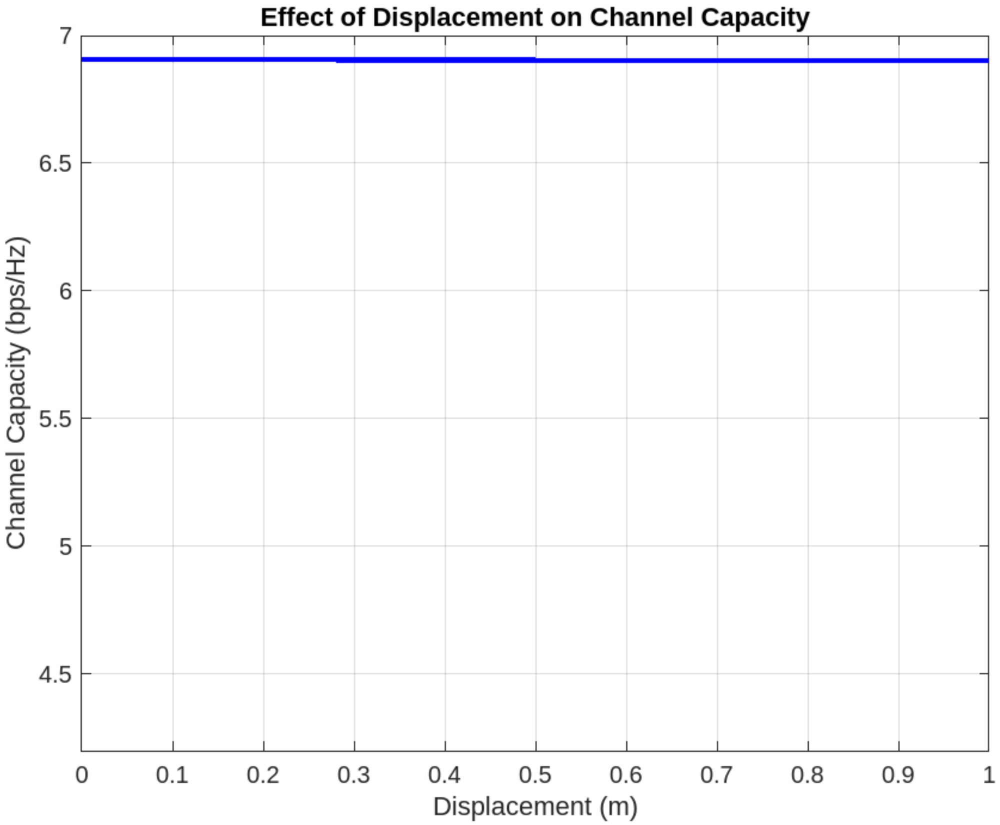
B. 80GHz – [1km, 5km]

x=0 m

D = 2 Km



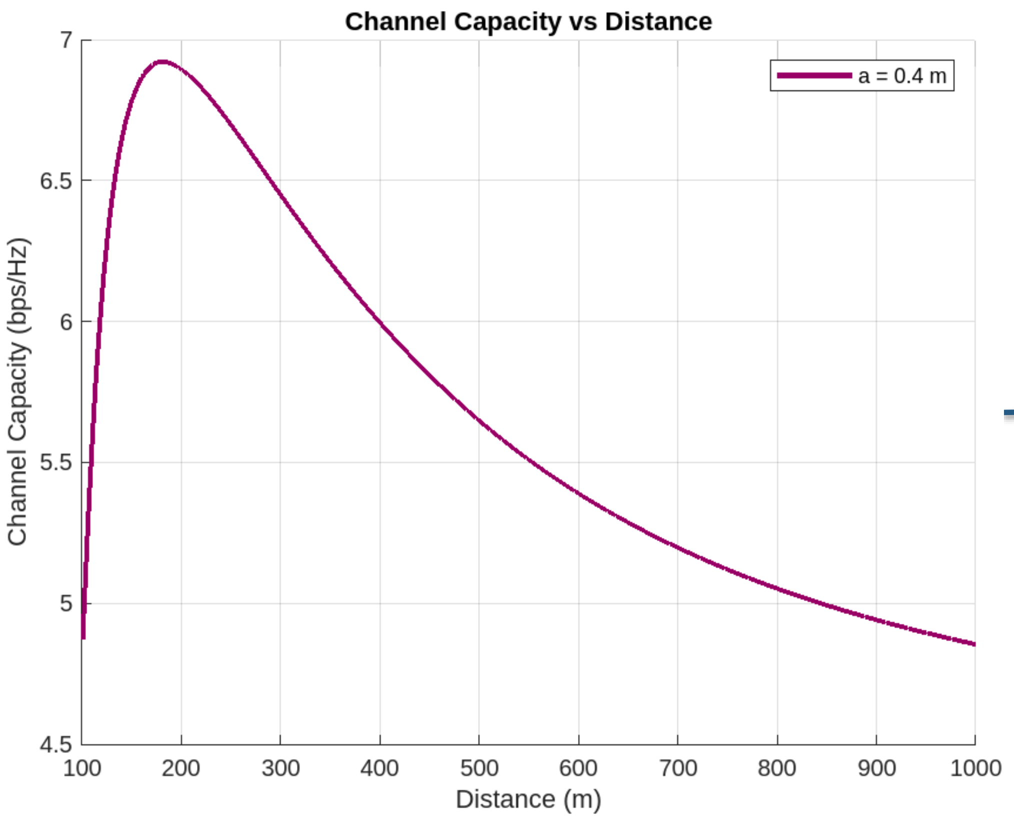
Robust to small displacement



Case of study

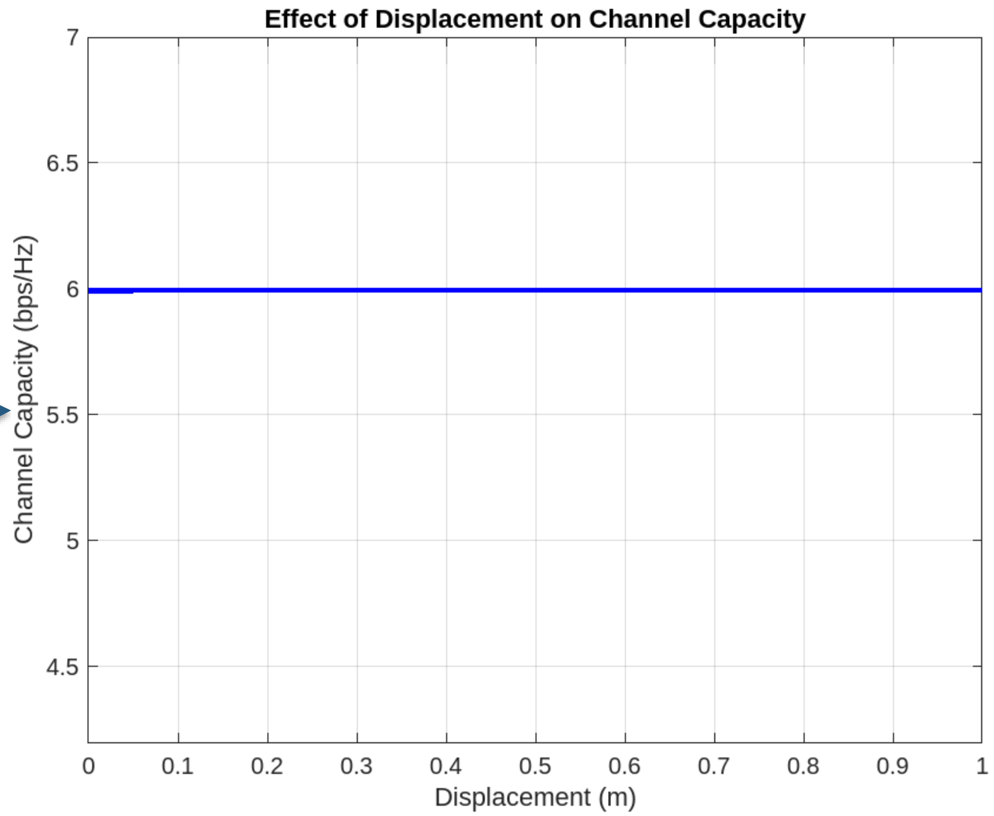
C. 170GHz – [0.1km, 1km]

x=0 m



Robust to small displacement

D = 400 m

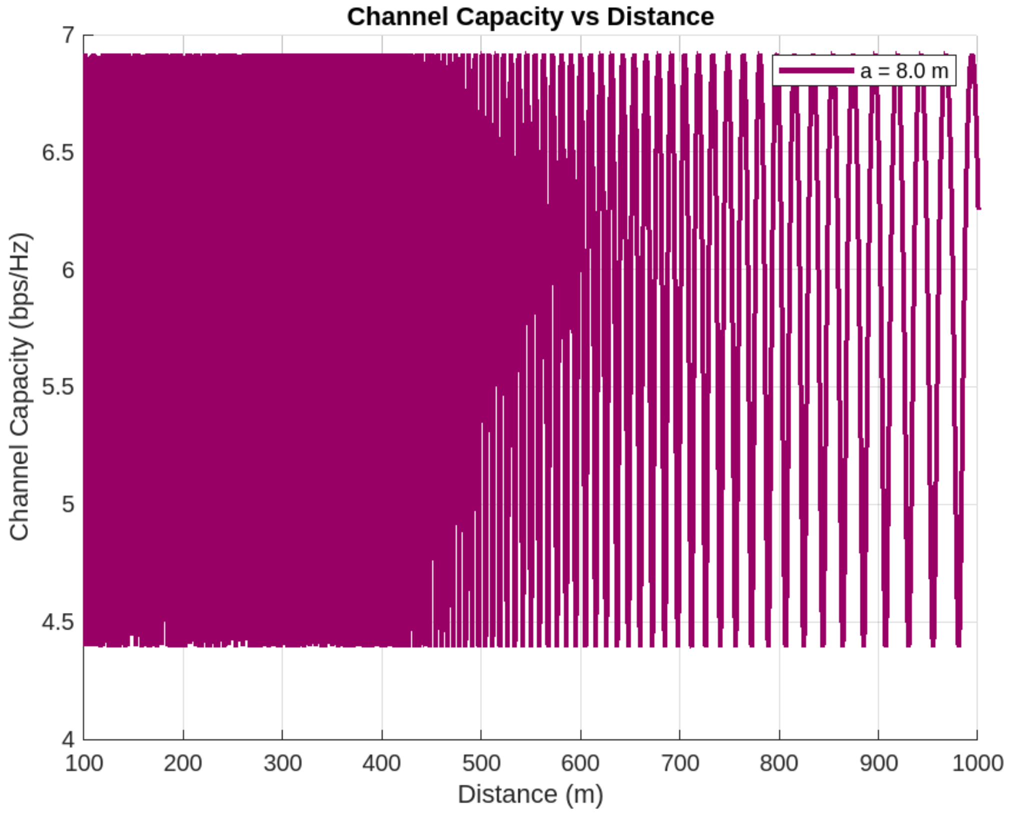


Case of study

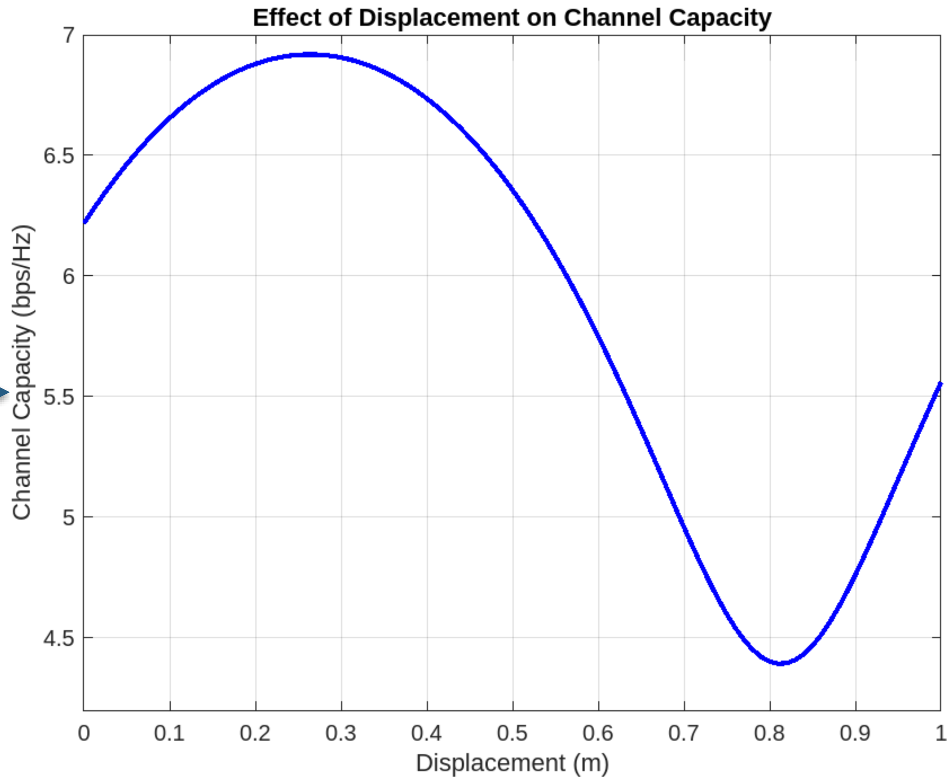
C. 170GHz – [0.1km, 1km]

x=0 m

D = 200 m



Very sensitive



Conclusions

- **Low frequencies:**
  - Requires high antenna spacing at both the transmitting and receiving antennas to achieve optimal capacity;
  - Capacity can be insensitive to differences in spacing of the receiving antenna on the order of meters;
  - Cover wide distances with constant capacity;
  - Are robust to antenna displacements.

- **High frequencies:**
  - Must have low antenna spacing at both the transmitting and receiving antennas to avoid high volatility in capacity;
  - Capacity can be very sensitive to differences in spacing of the receiving antenna on the order of meters;
  - Cover smaller distances with constant capacity;
  - can be sensitive to antenna displacements.

References

- Larsson P. Lattice array receiver and sender for spatially orthonormal MIMO communication, In Vehicular Technology Conference, volume 1, pages 192196, 2005.
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- P. F. Driessen and G. Foschini. On the capacity formula for multiple input-multiple output wireless channels: A geometric interpretation. IEEE Trans. Commun, 47(2):173 176, Feb 1999.
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THANK YOU FOR THE ATTENTION!

If you have any question don’t esitate to contact me:
marsona.panci@mail.polimi.it