Computations on case 3 with Lagrange Multipliers

Fixed sample sizes in period 1 and 2

Set (and simplify) conditions

In[ ]:=
```
subst = {r11 → r1 / 2 , r01 → r1 / 2 , r23 → r3 / 2 , r03 → r3 / 2, r02 → r2 - r12 - r22 }
```

Out[ ]= $\left\{r11 \to \frac{r1}{2}, r01 \to \frac{r1}{2}, r23 \to \frac{r3}{2}, r03 \to \frac{r3}{2}, r02 \to -r12 + r2 - r22\right\}$

In[ ]:=
```
ex = {r1 → 0.1, r2 → 0.8, r3 → 0.1}
```

Out[ ]= $\{r1 \to 0.1, r2 \to 0.8, r3 \to 0.1\}$

Define terms to optimise

Note: *sigma*term1^(-1)/N* is the variance of the estimator of effect 1 (analogously *sigma*term2^(-1)/N* for effect 2). But since sigma and N are fixed, we simply work on term1 and term2 expressions. furthermore we set NT=1.

In[ ]:=
```
term1 = FullSimplify[(r11 * r01 / (r11 + r01)) + (r12 * r02 / (r12 + r02)) /. subst]
```

Out[ ]= $\frac{r1}{4} + r12 + \frac{r12^2}{-r2 + r22}$

In[ ]:=
```
term2 = FullSimplify[(r22 * r02 / (r22 + r02)) + (r23 * r03 / (r23 + r03)) /. subst]
```

Out[ ]= $r22 + \frac{r22^2}{r12 - r2} + \frac{r3}{4}$

In[ ]:=
```
constr = term1 - term2;
```

In[ ]:=
```
e1 = Solve[D[term1, r12] == l D[constr, r12], l]
e2 = Solve[D[term1, r22] == l D[constr, r22], l]
```

Out[ ]= $\left\{\left\{l \to \frac{(r12 - r2)^2 (2 r12 - r2 + r22)}{2 r12^3 - 5 r12^2 r2 + 4 r12 r2^2 - r2^3 + r12^2 r22 - 2 r12 r2 r22 + r2^2 r22 - r2 r22^2 + r22^3}\right\}\right\}$

Out[ ]= $\left\{\left\{l \to \frac{r12^2 (r12 - r2)}{r12^3 - r12^2 r2 + r12 r2^2 - r2^3 - 2 r12 r2 r22 + 4 r2^2 r22 + r12 r22^2 - 5 r2 r22^2 + 2 r22^3}\right\}\right\}$

In[ ]:=
```
e3 = e1[[1]][[1]][[2]] == e2[[1]][[1]][[2]]
```

Out[ ]= $$\frac{(r12 - r2)^2 (2 r12 - r2 + r22)}{2 r12^3 - 5 r12^2 r2 + 4 r12 r2^2 - r2^3 + r12^2 r22 - 2 r12 r2 r22 + r2^2 r22 - r2 r22^2 + r22^3} == \frac{r12^2 (r12 - r2)}{r12^3 - r12^2 r2 + r12 r2^2 - r2^3 - 2 r12 r2 r22 + 4 r2^2 r22 + r12 r22^2 - 5 r2 r22^2 + 2 r22^3}$$

In[ ]:= `sol = Solve[e3, {r12}][[2]]`

Out[ ]= $\left\{r12 \to \frac{r2\ (r2 - 2\ r22)}{2\ (r2 - r22)}\right\}$

In[ ]:= `solr22 = NSolve[term1 - term2 /. sol /. ex][[6]]`

Out[ ]= `{r22 → 0.234315}`

In[ ]:= `solr12 = r12 /. sol /. solr22 /. ex`

Out[ ]= `0.234315`

In[ ]:= `r2 - r22 - solr12 /. solr22 /. ex`

Out[ ]= `0.331371`

In[ ]:= `eq = FullSimplify[term1 - term2 /. sol] == 0`

Out[ ]= $\frac{1}{4}\left(r1 + \frac{\left(r2^2 - 4\ r2\ r22 + 2\ r22^2\right)\ \left(r2^2 - 2\ r2\ r22 + 2\ r22^2\right)^2}{r2^2\ (r2 - r22)^3} - r3\right) == 0$

Note that then the solutions are r22 satisfying "**eq**" and r12 when substituting r22 in "**sol**".

In[ ]:= `CForm[eq[[1]]]`

Out[ ]//CForm=

```
(r1 + ((Power(r2,2) - 4*r2*r22 + 2*Power(r22,2))*Power(Power(r2,2) - 2*r2*r22 + 2*Powe
```

In[ ]:= `CForm[sol[[1, 2]]]`

Out[ ]//CForm=

```
(r2*(r2 - 2*r22))/(2.*(r2 - r22))
```