Computations on case 2

Fixed sample sizes in period 1 and 2 with ncc

Set (and simplify) conditions

Note that here we assume r1+r2=1, then r3=0

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\label{eq:continuous} $\inf[1]:= \mbox{ subst } = \{ \mbox{ r01 } - \mbox{ r1-r11 } \mbox{ , r02 } - \mbox{ } (1 - \mbox{ r1) } - \mbox{ r12 } - \mbox{ r22 } \}$$$$Out[1]:= \{ \mbox{ r01 } + \mbox{ r11 } - \mbox{ r11 } - \mbox{ r12 } - \mbox{ r22 } \}$$$$$$$$In[2]:= \mbox{ ex } = \{ \mbox{ r1 } \rightarrow \mbox{ 0.4} \}$$$$Out[2]:= \{ \mbox{ r1 } \rightarrow \mbox{ 0.4} \}$$$$$$$$$$$$$$}
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Define terms to optimise

Note: $sigma*term1^{(-1)/N}$ is the variance of the estimator of effect 1 (analogously $sigma*term2^{(-1)/N}$ for effect 2). But since sigma and N are fixed, we simply work on term1 and term2 expressions.

$$\begin{array}{l} \text{Out} [7] = \begin{array}{l} \displaystyle \frac{1}{2 \, r1 \, \left(-1 + r1 + r22\right)^2} \left(r22 \, \left(-1 + r1 + r22\right) \\ \\ \displaystyle \left(2 \, r1 - \frac{2 \, r1^{3/2} \, \left(-1 + r1 + 2 \, r22\right)}{\sqrt{\left(-1 + r1\right) \, \left(\left(-1 + r1\right) \, r1 - 4 \, r1 \, r11 + 4 \, r11^2\right) + 4 \, \left(-1 + r1\right) \, r1 \, r22 + 4 \, r1 \, r22^2} \right) \\ \\ r22 \, \left(r1^2 - 2 \, r11^2 + r1 \, \left(-1 + 2 \, r11 + 2 \, r22\right) - \\ \\ \sqrt{r1} \, \sqrt{\left(-1 + r1\right) \, \left(\left(-1 + r1\right) \, r1 - 4 \, r1 \, r11 + 4 \, r11^2\right) + 4 \, \left(-1 + r1\right) \, r1 \, r22 + 4 \, r1 \, r22^2} \right) + \\ \\ \left(-1 + r1 + r22\right) \, \left(r1^2 - 2 \, r11^2 + r1 \, \left(-1 + 2 \, r11 + 2 \, r22\right) - \\ \\ \sqrt{r1} \, \sqrt{\left(-1 + r1\right) \, \left(\left(-1 + r1\right) \, r1 - 4 \, r1 \, r11 + 4 \, r11^2\right) + 4 \, \left(-1 + r1\right) \, r1 \, r22 + 4 \, r1 \, r22^2} \right) \end{array} \right)$$

In[8]:= e2 = FullSimplify[D[term3, r11]]

$$\text{Dut[8]=} \quad \frac{\text{r22} \left(\text{r1} - 2 \text{ r11} + \frac{(-1 + \text{r1}) \sqrt{\text{r1}} (\text{r1} - 2 \text{r11})}{\sqrt{(-1 + \text{r1}) ((-1 + \text{r1}) \text{r1} - 4 \text{r1} \text{r1} + 4 \text{r11}^2) + 4 (-1 + \text{r1}) \text{r1} \text{r2} + 4 \text{r1} \text{r2} 2^2}}}{\text{r1} \left(-1 + \text{r1} + \text{r22} \right)} \right)$$

$$ln[9] = sol2 = Solve \left[\left\{ r22 \left(-1 + r1 + r22 \right) \right\} \right]$$

... Solve: There may be values of the parameters for which some or all solutions are not valid.

The solutions are then the following

0}, {r11, r22}];

$$In[26]:= sol2[11]$$

Out[26]=
$$\left\{r11
ightarrow rac{r1}{2}$$
 ,

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r22 \rightarrow 1-r1+\frac{1}{2} \sqrt{\left[4 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)+\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}+\frac{-22+65 \, r1^2+21 \, r1^3}{12 \, \left(-
                                                                                                         \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/\,\left(3\,\left(-1+r1\right)\right)
                                                                                                                                                                  1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296 \text{ r1}^5 - 303104 \text{ r1}^6 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 1204 + 12
                                                                                                                                                                                                               139 392 r1^7 - 34 560 r1^8 + 3456 r1^9 + \sqrt{(28311552 r1 - 467140608 r1^2 + (28311552 r1 - 467140608 r1^2 + (28311560 r1 - 467140608 r1^2 + (2831160 r1 - 46714060
                                                                                                                                                                                                                                                                   3588489216 r1^3 - 16978083840 r1^4 + 55228760064 r1^5 - 130682585088 r1^6 +
                                                                                                                                                                                                                                                                232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                                                                                                                                                                                                264\,790\,867\,968\,r1^{10}+162\,331\,361\,280\,r1^{11}-74\,439\,917\,568\,r1^{12}+24\,680\,595\,456
                                                                                                                                                                                                                                                                                  \left. \text{r1}^{13} - 5\,573\,836\,800\,\,\text{r1}^{14} + 764\,411\,904\,\,\text{r1}^{15} - 47\,775\,744\,\,\text{r1}^{16} \right) \right)^{1/3} \right) \, + \, \left. \left( \frac{1}{3} + \frac{1}
                                                                                                       \frac{1}{48 \times 2^{1/3} \ \left(-1+r1\right)} \left(1024-1536 \ r1-26112 \ r1^2+135040 \ r1^3-304896 \ r1^4+128112 \ r1^2+135040 \ r1^3-128112 \ r1^4+128112 \ r1^2+128112 \ r1^2+128112 \ r1^3+128112 \ 
                                                                                                                                                             391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                                                                                                                                             55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                                                                                                                                                               315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                                                                                                                               162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                                                                                                                                                               5\,573\,836\,800\,\,\mathtt{r1^{14}}\,+\,764\,411\,904\,\,\mathtt{r1^{15}}\,-\,47\,775\,744\,\,\mathtt{r1^{16}}\big)\,\Big)^{1/3}\Bigg|\,\,-\,
                                 \frac{1}{2} \sqrt{\left(8 \left(-1+r1\right)^2 + \frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right) - \frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)} - \frac{1}{2} \left(-1+r1\right)^2 + \frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right) - \frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)} - \frac{1}{2} \left(-1+r1\right)^2 + \frac{1}{4} \left(-1
                                                                                                              \left(2^{1/3} \, \left(4 - 40 \, \text{r1} + 129 \, \text{r1}^2 - 196 \, \text{r1}^3 + 154 \, \text{r1}^4 - 60 \, \text{r1}^5 + 9 \, \text{r1}^6\right)\right) \, \Bigg/ \, \left(3 \, \left(-1 + \text{r1}\right) + 100 \, \text{r1}^3 + 
                                                                                                                                                               (1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296 \text{ r1}^5 - 303104 \text{ r1}^6 + 391296 \text{ r1}^5 - 303104 \text{ r1}^6 + 391296 
                                                                                                                                                                                                               139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> + \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + (28311552 \text{ r})^2 + (2831152 \text{ r})^2 + (283
                                                                                                                                                                                                                                                                   3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + 55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 +
                                                                                                                                                                                                                                                                   232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                                                                                                                                                                                                   264790867968 \text{ r1}^{10} + 162331361280 \text{ r1}^{11} - 74439917568 \text{ r1}^{12} + 24680595456
                                                                                                                                                                                                                                                                                  \left. \text{r1}^{13} - 5\,573\,836\,800\,\,\text{r1}^{14} + 764\,411\,904\,\,\text{r1}^{15} - 47\,775\,744\,\,\text{r1}^{16} \right) \right)^{1/3} \right) - 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{13} + 10^{
                                                                                                         \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024-1536 \ r1-26112 \ r1^2+135040 \ r1^3-304896 \ r1^4+128112 \ r^2+128112 
                                                                                                                                                             391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                                                                             55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> + 232 168 882 176 r1<sup>7</sup> -
                                                                                                                                                                                                                 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                                                                                                                               162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                                                                                                                               5\,573\,836\,800\,r1^{14}+764\,411\,904\,r1^{15}-47\,775\,744\,r1^{16})\,\Big)^{1/3}+
                                                                                                              \left(-64 \, \left(-1+r1\right)^{3}+4 \, \left(-1+r1\right) \, \left(22-43 \, r1+21 \, r1^{2}\right)-6 \, \left(-4+11 \, r1-10 \, r1^{2}+3 \, r1^{3}\right)\right) \, \left(-64 \, \left(-1+r1\right)^{3}+4 \, \left(-1+r1\right)^{3}+4 \, r1^{3}\right) \, \left(-64 \, r1+r1\right)^{3}+4 \, r1^{3}+10 \, r1^{3}
                                                                                                                           \left(4\;\sqrt{\;\left(4\;\left(-1+r1\right)^{\;2}+\frac{1}{4}\;\left(-\,22+43\;r1-21\;r1^{2}\right)\;+\frac{-\,22\,+\,65\;r1-\,64\;r1^{2}\,+\,21\;r1^{3}}{12\;\left(-1+r1\right)}\;+\right.}\right.
                                                                                                                                                                                                               \left(2^{1/3} \left(4-40 \text{ r1}+129 \text{ r1}^2-196 \text{ r1}^3+154 \text{ r1}^4-60 \text{ r1}^5+9 \text{ r1}^6\right)\right)
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$$\begin{array}{l} \log(24) = & \mbox{sol} \left[\left[\left[1 \right] \right] \right] / \mbox{.} \mbox{sol} \left[\left[\left[1 \right] \right] \right] / \mbox{.} \mbox{sol} \left[\left[1 \right] \right] / \mbox{.} \mbox{.} \mbox{sol} \left[\left[1 \right] \right] / \mbox{.} \mbox{.} \mbox{sol} \left[\left[1 \right] \right] / \mbox{.} \mbox{.} \mbox{sol} \left[\left[1 \right] \right] / \mbox{.} \$$

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\frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}-\frac{1}{2}\right)}
                     \left(2^{1/3}\,\left(4-40\,\,\text{r1}+129\,\,\text{r1}^2-196\,\,\text{r1}^3+154\,\,\text{r1}^4-60\,\,\text{r1}^5+9\,\,\text{r1}^6\right)\,\right)\,\Bigg/
                          391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                  55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                5\,573\,836\,800\,\,\texttt{r1}^{\texttt{14}}\,+\,764\,411\,904\,\,\texttt{r1}^{\texttt{15}}\,-\,47\,775\,744\,\,\texttt{r1}^{\texttt{16}}\big)\,\Big)^{\texttt{1/3}}\bigg)\,\,-\,
                    \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left( 1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 128 + 128040 \right) + 128040 \ r1^3 - 128040 \
                                   304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                   \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + }
                                                  55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                  315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                  162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                 5\,573\,836\,800\,r1^{14}\,+\,764\,411\,904\,r1^{15}\,-\,47\,775\,744\,r1^{16}\big)\,\Big)^{1/3}\,+
                     \left(-64 \left(-1+r1\right)^{3}+4 \left(-1+r1\right) \left(22-43 \, r1+21 \, r1^{2}\right)\right)
                                  6 \left(-4 + 11 \, \text{r1} - 10 \, \text{r1}^2 + 3 \, \text{r1}^3\right)\right)
                          \left(4\,\sqrt{\,\left(4\,\left(-\,1+\,r\,1\right)^{\,2}\,+\,\frac{1}{4}\,\left(-\,22\,+\,43\,\,r\,1\,-\,21\,\,r\,1^{2}\right)\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12
                                                  \left(2^{1/3} \left(4-40 \text{ r1}+129 \text{ r1}^2-196 \text{ r1}^3+154 \text{ r1}^4-60 \text{ r1}^5+9 \text{ r1}^6\right)\right)
                                                       391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - }
                                                                                              16\,978\,083\,840\,r1^4 + 55\,228\,760\,064\,r1^5 - 130\,682\,585\,088\,r1^6 +
                                                                                              232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                              264 790 867 968 r1<sup>10</sup> + 162 331 361 280 r1<sup>11</sup> -
                                                                                              74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> - 5 573 836 800 r1<sup>14</sup> +
                                                                                             764 411 904 r1<sup>15</sup> - 47 775 744 r1<sup>16</sup>) )^{1/3} + \frac{1}{48 \times 2^{1/3} (-1 + r1)}
                                                    (1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296)
                                                                     r1^{5} - 303104 r1^{6} + 139392 r1^{7} - 34560 r1^{8} + 3456 r1^{9} + \sqrt{(28311552)}
                                                                                    r1 - 467140608 r1^2 + 3588489216 r1^3 - 16978083840 r1^4 +
                                                                                55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> + 232 168 882 176 r1<sup>7</sup> -
                                                                                315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                              5\,573\,836\,800\,r1^{14}+764\,411\,904\,r1^{15}-47\,775\,744\,r1^{16}))^{1/3}
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4 \, \text{r1}^2 \, \left( 1 - \text{r1} + \frac{1}{2} \, \sqrt{ \left( 4 \, \left( -1 + \text{r1} \right)^2 + \frac{1}{4} \, \left( -22 + 43 \, \text{r1} - 21 \, \text{r1}^2 \right) \right. + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^2 + 21 \, \text{r1}^3}{12 \, \left( -1 + \text{r1} \right)} \right. + \left( -22 + 43 \, \text{r1} - 21 \, \text{r1}^2 \right) + \left( -22 + 65 \, \text{r1} - 64 \, \text{r1}^2 + 21 \, \text{r1}^3 + 21 \, \text{r1}^
                                        \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                              391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                         162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                                         5\,573\,836\,800\,\,r1^{14}\,+\,764\,411\,904\,\,r1^{15}\,-\,47\,775\,744\,\,r1^{16}\big)\,\Big)^{1/3}\bigg)\,\,+
                                       \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left( 1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 128 + 128040 \right) + 128040 \ r1^3 - 128040 \
                                                         304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                         \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + }
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                         5\,573\,836\,800\,\,\texttt{r1}^{\texttt{14}}\,+\,764\,411\,904\,\,\texttt{r1}^{\texttt{15}}\,-\,47\,775\,744\,\,\texttt{r1}^{\texttt{16}}\big)\,\Big)^{\texttt{1/3}} \  \, -
                \frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}\right)}
                                         \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                              391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                         315\,144\,732\,672\,r1^8 + 329\,334\,128\,640\,r1^9 - 264\,790\,867\,968\,r1^{10} +
                                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                         5\,573\,836\,800\,r1^{14} + 764\,411\,904\,r1^{15} - 47\,775\,744\,r1^{16})
                                        \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1
                                                         304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                         55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> + 232 168 882 176 r1<sup>7</sup> -
                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                        5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
                                         \left(-64 \left(-1+r1\right)^{3}+4 \left(-1+r1\right) \left(22-43 \, r1+21 \, r1^{2}\right)\right)
                                                       6 \left(-4 + 11 \text{ r1} - 10 \text{ r1}^2 + 3 \text{ r1}^3\right)\right)
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\left(4\sqrt{\left(4\left(-1+r1\right)^{2}+\frac{1}{4}\left(-22+43\,r1-21\,r1^{2}\right)+\frac{-22+65\,r1-64\,r1^{2}+21\,r1^{3}}{12\,\left(-1+r1\right)}}\right.+
                                                                                                   \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                                                                                           391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                                                                 \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 467140608 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 358884889216 \text{ r1}^3 + 358884889216 \text{ r1}^3
                                                                                                                                                                     16 978 083 840 r1<sup>4</sup> + 55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> +
                                                                                                                                                                      232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                                                                                                      264 790 867 968 r1<sup>10</sup> + 162 331 361 280 r1<sup>11</sup> -
                                                                                                                                                                      74439917568 \text{ r1}^{12} + 24680595456 \text{ r1}^{13} - 5573836800 \text{ r1}^{14} +
                                                                                                                                                                     764 411 904 r1<sup>15</sup> - 47 775 744 r1<sup>16</sup>) \right)^{1/3} + \frac{1}{48 \times 2^{1/3} (-1 + r1)}
                                                                                                      (1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296)
                                                                                                                                 r1^{5} - 303104 r1^{6} + 139392 r1^{7} - 34560 r1^{8} + 3456 r1^{9} + \sqrt{(28311552)}
                                                                                                                                                       r1 - 467140608 r1^2 + 3588489216 r1^3 - 16978083840 r1^4 +
                                                                                                                                                 55\,228\,760\,064\,r1^5 - 130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                                                                                315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                                                                 162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                                                                                               4 \, \text{r1} \, \left(1 - \text{r1} + \frac{1}{2} \, \sqrt{\left(4 \, \left(-1 + \text{r1}\right)^{\, 2} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) \right.} + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^{\, 2} + 21 \, \text{r1}^{\, 3}}{12 \, \left(-1 + \text{r1}\right)} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^{\, 2} + 21 \, \text{r1}^{\, 3}}{12 \, \left(-1 + \text{r1}\right)} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 +
                                                               \left(2^{1/3} \left(4-40 \text{ r1}+129 \text{ r1}^2-196 \text{ r1}^3+154 \text{ r1}^4-60 \text{ r1}^5+9 \text{ r1}^6\right)\right)
                                                                     \left(3 \; \left(-1 + r1\right) \; \left(1024 - 1536 \; r1 - 26112 \; r1^2 + 135040 \; r1^3 - 304896 \; r1^4 + 126112 \; r1^2 + 135040 \; r1^3 + 126112 \; r1^4 + 1261
                                                                                                           391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                          \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840)}
                                                                                                                                         r1^4 + 55228760064 r1^5 - 130682585088 r1^6 + 232168882176
                                                                                                                                         r1^7 - 315144732672 r1^8 + 329334128640 r1^9 - 264790867968
                                                                                                                                        r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} + 24680595456
                                                                                                                                        \texttt{r1}^{13} - 5\,573\,836\,800\,\texttt{r1}^{14} + 764\,411\,904\,\texttt{r1}^{15} - 47\,775\,744\,\texttt{r1}^{16} \big) \, \Big)^{1/3} \bigg) \,\, + \,\, \\
                                                             \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1
                                                                                     304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                                                    \sqrt{(28311552\,r1-467140608\,r1^2+3588489216\,r1^3-16978083840\,r1^4+}
                                                                                                          55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                                                          315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                          162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                         5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
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\frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}-\frac{1}{2}}\right)}
                    \left(2^{1/3}\,\left(4-40\,\text{r1}+129\,\text{r1}^2-196\,\text{r1}^3+154\,\text{r1}^4-60\,\text{r1}^5+9\,\text{r1}^6\right)\right)\,\Bigg/
                        391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                               \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840)}
                                                                 r1^4 + 55228760064 r1^5 - 130682585088 r1^6 + 232168882176
                                                                 r1^7 - 315144732672 r1^8 + 329334128640 r1^9 - 264790867968
                                                                r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} + 24680595456
                                                                 \texttt{r1}^{13} - 5\,573\,836\,800\,\texttt{r1}^{14} + 764\,411\,904\,\texttt{r1}^{15} - 47\,775\,744\,\texttt{r1}^{16} \big) \, \Big)^{1/3} \bigg) \, - \\
                  \frac{1}{48\times2^{1/3}\ (-1+r1)}\left(1024-1536\ r1-26\ 112\ r1^2+135\ 040\ r1^3-1248\times2^{1/3}\right)
                                 304\,896\,r1^4 + 391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                 \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + }
                                               55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                               315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                              162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                              5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
                    \left(-64 \left(-1+r1\right)^{3}+4 \left(-1+r1\right) \left(22-43 \, r1+21 \, r1^{2}\right)\right)
                                6 \left(-4 + 11 \text{ r1} - 10 \text{ r1}^2 + 3 \text{ r1}^3\right)\right)
                        \left(4\sqrt{\left(4\left(-1+r1\right)^{2}+\frac{1}{4}\left(-22+43\,r1-21\,r1^{2}\right)+\frac{-22+65\,r1-64\,r1^{2}+21\,r1^{3}}{12\,\left(-1+r1\right)}}\right.+
                                               \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\left/\right.
                                                    3 (-1 + r1) (1024 - 1536 r1 - 26112 r1^2 + 135040 r1^3 - 126112 r1^2 + 135040 r1^2 + 126112 r1^2 + 135040 r1^2 + 126112 r1^2 + 135040 r1^2 + 126112 r1^2 +
                                                                          304\,896\,r1^4 + 391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 -
                                                                          34\,560\,\mathrm{r1}^8 + 3456\,\mathrm{r1}^9 + \sqrt{(28\,311\,552\,\mathrm{r1} - 467\,140\,608\,\mathrm{r1}^2 + (28\,311\,552\,\mathrm{r1} - 467\,140\,608\,\mathrm{r1}^2)}
                                                                                        3588489216 r1^3 - 16978083840 r1^4 + 55228760064 r1^5 -
                                                                                        130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                        315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968
                                                                                            r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} +
                                                                                        24\,680\,595\,456\,r1^{13}\,-\,5\,573\,836\,800\,r1^{14}\,+\,764\,411\,904\,r1^{15}\,-\,
                                                                                      47775744 \text{ r1}^{16}))^{1/3} + \frac{1}{48 \times 2^{1/3} (-1 + \text{r1})} \left(1024 - 1536 \text{ r1} - \frac{1}{1024} + \frac{1}{1
                                                            26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296 \text{ r1}^5 -
                                                            467 140 608 r1<sup>2</sup> + 3 588 489 216 r1<sup>3</sup> - 16 978 083 840 r1<sup>4</sup> +
                                                                          55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                          315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                          162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                          5 573 836 800 r1<sup>14</sup> + 764 411 904 r1<sup>15</sup> - 47 775 744
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$$|r1^{16}\rangle\rangle^{1/3}$$

ln[23]:= 1 - r1 - sol2[11][2][2] - sol[1][2] /. sol2[11][2]

$$\begin{array}{c} \log (3)^2 - \frac{1}{2} \sqrt{ \left(4 \ (-1+r1)^2 + \frac{1}{4} \left(-22 + 43 \ r1 - 21 \ r1^2 \right) + \frac{-22 + 65 \ r1 - 64 \ r1^2 + 21 \ r1^3}{12 \ (-1 \ r1)} + \frac{2^{1/3}}{4} \left(4 \ 40 \ r1 + 129 \ r1^2 \ 196 \ r1^3 + 154 \ r1^4 \ 60 \ r1^5 + 9 \ r1^6 \right) \right) / }{ \left(3 \ (-1+r1) \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135 \ 040 \ r1^3 - 304 \ 896 \ r1^4 + 391 \ 296 \ r1^5 - 303 \ 104 \ r1^6 + 139 \ 392 \ r1^7 - 34 \ 560 \ r1^8 + 3456 \ r1^9 + \sqrt{ (28 \ 311552 \ r1 - 467 \ 140 \ 608 \ r1^2 + 3588 \ 489 \ 216 \ r1^3 - 169 \ 808 \ 840 \ r1^4 + 55 \ 228 \ 760 \ 604 \ r1^5 - 130 \ 682 \ 585 \ 888 \ r1^6 + 232 \ 168 \ 882 \ 176 \ r1^2 - 315 \ 144 \ 732 \ 672 \ r1^8 + 329 \ 334 \ 128 \ 640 \ r1^2 - 264 \ 790 \ 867 \ 968 \\ - \ r1^{13} - 5573 \ 836 \ 800 \ r1^{14} + 764 \ 411904 \ r1^3 - 47775 \ 744 \ r1^{16} \right) \right)^{1/3} \right) + \\ \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135 \ 640 \ r1^3 - 304 \ 896 \ r1^4 + 391 \ 296 \ r1^5 - 303 \ 164 \ r1^6 + 139 \ 392 \ r1^7 - 34 \ 560 \ r1^8 + 3456 \ r1^9 + \sqrt{ (28 \ 311552 \ r1 - 467 \ 140 \ 608 \ r1^2 + 3588 \ 489 \ 216 \ r1^3 - 169 \ 883 \ 840 \ r1^4 + 391 \ 917 \ 568 \ r1^2 + 329 \ 334 \ 128 \ 640 \ r1^3 - 264 \ 790 \ 867 \ 968 \ r1^{10} + 162 \ 331 \ 361 \ 280 \ r1^{13} - 7775 \ 744 \ r1^{16} \right) \right)^{1/3} \right) + \\ \frac{1}{2} \sqrt{ \left(8 \ (-1+r1)^2 + \frac{1}{4} \left(-22 + 43 \ r1 - 21 \ r1^2 \right) - \frac{-22 + 65 \ r1 - 64 \ r1^2 + 21 \ r1^3}{12 \ (-1+r1)} - \frac{1}{4} \left(-22 + 43 \ r1 - 21 \ r1^2 \right) - \frac{-22 + 65 \ r1 - 64 \ r1^2 + 21 \ r1^3}{12 \ (-1+r1)} - \frac{1}{4} \left(-22 + 43 \ r1 - 21 \ r1^2 \right) - \frac{-22 + 65 \ r1 - 64 \ r1^2 + 21 \ r1^3}{12 \ (-1+r1)} - \frac{1}{4} \left(-22 + 33 \ r1 - 26 \ 112 \ r1^2 + 135 \ 640 \ r1^3 - 304 \ 896 \ r1^4 + 391 \ 296 \ r1^5 - 303 \ 140 \ r1^6 + 139 \ 392 \ r1^7 - 34 \ 560 \ r1^8 + 3456 \ r1^9 + \sqrt{ (28 \ 311552 \ r1 - 467 \ 140 \ 608 \ r1^2 + 358 \ 849 \ 216 \ r1^3 - 304 \ 896 \ r1^4 + 391 \ 296 \ r1^5 - 303 \ 140 \ r1^6 + 139 \ 392 \ r1^7 - 34 \ 560 \ r1^8 + 3456 \ r1^9 + \sqrt{ (28 \ 311552 \ r1 - 467 \ 140 \ 608 \ r1^4 + 314 \ 704 \ 707 \ 707 \ 707 \ 707 \ 707 \ 707 \$$

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\left(4\sqrt{\left(4\left(-1+r1\right)^{2}+\frac{1}{4}\left(-22+43\,r1-21\,r1^{2}\right)+\frac{-22+65\,r1-64\,r1^{2}+21\,r1^{3}}{12\,\left(-1+r1\right)}}\right.+
                                                          \left(2^{1/3} \, \left(4 - 40 \, \text{r1} + 129 \, \text{r1}^2 - 196 \, \text{r1}^3 + 154 \, \text{r1}^4 - 60 \, \text{r1}^5 + 9 \, \text{r1}^6\right)\right) \, \Bigg/
                                                              391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                           55\,228\,760\,064\,r1^5 - 130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                                           315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                           162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                          5\,573\,836\,800\,\,\text{r1}^{14}\,+\,764\,411\,904\,\,\text{r1}^{15}\,-\,47\,775\,744\,\,\text{r1}^{16}\big)\,\Big)^{1/3}\bigg)\,\,+\,
                                                        \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left( 1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 304896 \right)
                                                                               r1^4 + 391296 r1^5 - 303104 r1^6 + 139392 r1^7 - 34560 r1^8 + 3456 r1^9 +
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                          315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                          162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                          5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
\frac{1}{2 \text{ r1}} \left[ \text{r1} - \text{r1}^2 - \sqrt{\text{r1}} \right] \sqrt{\left[ \text{r1} - 2 \text{ r1}^2 + \text{r1}^3 - 4 \text{ r1} \left[ 1 - \text{r1} + \frac{1}{2} \right] \sqrt{\left[ 4 \left( -1 + \text{r1} \right)^2 + \frac{1}{2} \right]} \right]} \right]
                                                                       \frac{1}{4} \left(-22 + 43 \text{ r1} - 21 \text{ r1}^2\right) + \frac{-22 + 65 \text{ r1} - 64 \text{ r1}^2 + 21 \text{ r1}^3}{12 \left(-1 + \text{r1}\right)} + 
                                                                         \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                                                               3 (-1+r1) (1024-1536 r1-26112 r1^2+135040 r1^3-304896 r1^4+136040 r1^3-1604896 r1^4+1604896 r1^4+160486 r1^4+160
                                                                                                           391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                          \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + 3688489216 \text{ r1}^4 + 36884889216 \text{ r1}^4 + 36884889216 \text{ r1}^4 + 36884889216 \text{ r1}^4 + 36884889216 \text{ r1}^4 + 368848889216 \text{ r1}^4 + 36884889216 \text{ r1}^4 + 36888889216 \text{ r1}^4 + 36888889216 \text{ r1}^4 + 3688
                                                                                                                           55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                                                           315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                                           162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                                           5\,573\,836\,800\,\,\texttt{r1}^{\texttt{14}}\,+\,764\,411\,904\,\,\texttt{r1}^{\texttt{15}}\,-\,47\,775\,744\,\,\texttt{r1}^{\texttt{16}}\big)\,\Big)^{\texttt{1/3}}\bigg)\,\,+\,
                                                                         \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \ \Big( 1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^3 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^2 + 126112 \
                                                                                          55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                                                           315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                           162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                           5\,573\,836\,800\,\,\texttt{r1}^{\texttt{14}}\,+\,764\,411\,904\,\,\texttt{r1}^{\texttt{15}}\,-\,47\,775\,744\,\,\texttt{r1}^{\texttt{16}}\big)\,\Big)^{\texttt{1/3}} \  \, -
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\frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}-\frac{1}{2}}\right)}
                          \left(2^{1/3} \, \left(4 - 40 \, \text{r1} + 129 \, \text{r1}^2 - 196 \, \text{r1}^3 + 154 \, \text{r1}^4 - 60 \, \text{r1}^5 + 9 \, \text{r1}^6\right)\right) \, \bigg/ \, \left(3 \, \left(-1 + \text{r1}\right) + 100 \, \text{r1}^6\right)
                                               1024 - 1536 r1 - 26112 r1^2 + 135040 r1^3 - 304896 r1^4 + 391296 r1^5 - 1204 r1^5 + 1204 r1^5 - 1204 r1^5 + 1204
                                                                467\,140\,608\,r1^2+3\,588\,489\,216\,r1^3-16\,978\,083\,840\,r1^4+55\,228\,760\,064
                                                                                         r1^5 - 130682585088 r1^6 + 232168882176 r1^7 - 315144732672 r1^8 +
                                                                                  329\,334\,128\,640\,r1^9 - 264\,790\,867\,968\,r1^{10} + 162\,331\,361\,280\,r1^{11} -
                                                                                  74439917568 \text{ r1}^{12} + 24680595456 \text{ r1}^{13} - 5573836800 \text{ r1}^{14} +
                                                                                 764\,411\,904\,r1^{15}\,-\,47\,775\,744\,r1^{16}\big)\,\Big)^{1/3}\bigg)\,-\,\frac{1}{48\times 2^{1/3}\,\,(\,-\,1\,+\,r1)}
                             (1024 - 1536 \, \text{r1} - 26112 \, \text{r1}^2 + 135040 \, \text{r1}^3 - 304896 \, \text{r1}^4 + 391296 \, \text{r1}^5 - 10040 \, \text{r1}^3 + 10
                                             467\,140\,608\,r1^2+3\,588\,489\,216\,r1^3-16\,978\,083\,840\,r1^4+55\,228\,760\,064
                                                                     r1^{5} - 130682585088 r1^{6} + 232168882176 r1^{7} - 315144732672 r1^{8} +
                                                                329\,334\,128\,640\,r1^9 - 264\,790\,867\,968\,r1^{10} + 162\,331\,361\,280\,r1^{11} -
                                                                74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> - 5 573 836 800 r1<sup>14</sup> +
                                                              764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})^{1/3} + (-64(-1+\text{r1})^3 +
                                           4 (-1+r1) (22-43 r1 + 21 r1^2) - 6 (-4+11 r1 - 10 r1^2 + 3 r1^3))
                                 \left(4\,\sqrt{\,\left(4\,\left(-\,1+\,r\,1\right)^{\,2}\,+\,\frac{1}{4}\,\left(-\,22\,+\,43\,\,r\,1\,-\,21\,\,r\,1^{2}\right)\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1\,-\,64\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,65\,\,r\,1^{\,2}\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r\,1\right)}\,+\,\frac{-\,22\,+\,21\,\,r\,1^{\,3}}{12\,\left(-\,1\,+\,r
                                                                \left(2^{1/3} \, \left(4 - 40 \, \text{r1} + 129 \, \text{r1}^2 - 196 \, \text{r1}^3 + 154 \, \text{r1}^4 - 60 \, \text{r1}^5 + 9 \, \text{r1}^6\right)\right) \, \Bigg/
                                                                      391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                     \sqrt{(28\,311\,552\,r1-467\,140\,608\,r1^2+3\,588\,489\,216\,r1^3-}
                                                                                                                        16\,978\,083\,840\,r1^4+55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+
                                                                                                                       232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                                                        264 790 867 968 r1<sup>10</sup> + 162 331 361 280 r1<sup>11</sup> -
                                                                                                                       74439917568 r1^{12} + 24680595456 r1^{13} - 5573836800 r1^{14} +
                                                                                                                       764 411 904 r1<sup>15</sup> - 47 775 744 r1<sup>16</sup>))<sup>1/3</sup>) + \frac{1}{48 \times 2^{1/3} (-1 + r1)}
                                                                  (1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296)
                                                                                         r1^{5} - 303104 r1^{6} + 139392 r1^{7} - 34560 r1^{8} + 3456 r1^{9} + \sqrt{(28311552)}
                                                                                                           r1 - 467140608 r1^2 + 3588489216 r1^3 - 16978083840 r1^4 +
                                                                                                      55\,228\,760\,064\,r1^5 - 130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                                      315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                    162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                    5\,573\,836\,800\,r1^{14}+764\,411\,904\,r1^{15}-47\,775\,744\,r1^{16}\big)\,\Big)^{1/3}\,\bigg|\,\,\bigg|\,\bigg|\,\bigg|\,+
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4 \, \text{r1}^2 \, \left( 1 - \text{r1} + \frac{1}{2} \, \sqrt{ \left( 4 \, \left( -1 + \text{r1} \right)^2 + \frac{1}{4} \, \left( -22 + 43 \, \text{r1} - 21 \, \text{r1}^2 \right) \right. + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^2 + 21 \, \text{r1}^3}{12 \, \left( -1 + \text{r1} \right)} \right. + \left( -22 + 43 \, \text{r1} - 21 \, \text{r1}^2 \right) + \left( -22 + 65 \, \text{r1} - 64 \, \text{r1}^2 + 21 \, \text{r1}^3 + 21 \, \text{r1}^
                                        \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                              391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                         162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                                         5\,573\,836\,800\,\,r1^{14}\,+\,764\,411\,904\,\,r1^{15}\,-\,47\,775\,744\,\,r1^{16}\big)\,\Big)^{1/3}\bigg)\,\,+
                                       \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left( 1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 128 + 128040 \right) + 128040 \ r1^3 - 128040 \
                                                         304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                         \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + }
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                         5\,573\,836\,800\,\,\texttt{r1}^{\texttt{14}}\,+\,764\,411\,904\,\,\texttt{r1}^{\texttt{15}}\,-\,47\,775\,744\,\,\texttt{r1}^{\texttt{16}}\big)\,\Big)^{\texttt{1/3}} \  \, -
                \frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}\right)}
                                         \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                              391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                                                         55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                                                                         315\,144\,732\,672\,r1^8 + 329\,334\,128\,640\,r1^9 - 264\,790\,867\,968\,r1^{10} +
                                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                         5\,573\,836\,800\,r1^{14} + 764\,411\,904\,r1^{15} - 47\,775\,744\,r1^{16})
                                        \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1
                                                         304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                         55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> + 232 168 882 176 r1<sup>7</sup> -
                                                                         315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                         162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                        5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
                                         \left(-64 \left(-1+r1\right)^{3}+4 \left(-1+r1\right) \left(22-43 \, r1+21 \, r1^{2}\right)\right)
                                                       6 \left(-4 + 11 \text{ r1} - 10 \text{ r1}^2 + 3 \text{ r1}^3\right)\right)
```

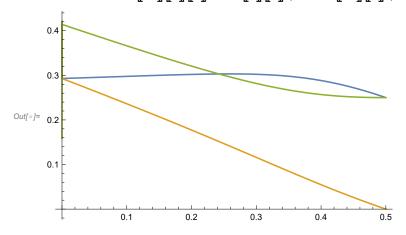
```
\left(4\sqrt{\left(4\left(-1+r1\right)^{2}+\frac{1}{4}\left(-22+43\,r1-21\,r1^{2}\right)+\frac{-22+65\,r1-64\,r1^{2}+21\,r1^{3}}{12\,\left(-1+r1\right)}}\right.+
                                                                                                    \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\Bigg/
                                                                                                            391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                                                                                                                                 \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 467140608 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 3588489216 \text{ r1}^3 + 35884889216 \text{ r1}^3 + 358884889216 \text{ r1}^3 + 358884889216 \text{ r1}^3
                                                                                                                                                                      16 978 083 840 r1<sup>4</sup> + 55 228 760 064 r1<sup>5</sup> - 130 682 585 088 r1<sup>6</sup> +
                                                                                                                                                                       232 168 882 176 r1<sup>7</sup> - 315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> -
                                                                                                                                                                       264 790 867 968 r1<sup>10</sup> + 162 331 361 280 r1<sup>11</sup> -
                                                                                                                                                                       74439917568 \text{ r1}^{12} + 24680595456 \text{ r1}^{13} - 5573836800 \text{ r1}^{14} +
                                                                                                                                                                      764 411 904 r1<sup>15</sup> - 47 775 744 r1<sup>16</sup>) \right)^{1/3} + \frac{1}{48 \times 2^{1/3} (-1 + r1)}
                                                                                                       (1024 - 1536 \text{ r1} - 26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296)
                                                                                                                                 r1^{5} - 303104 r1^{6} + 139392 r1^{7} - 34560 r1^{8} + 3456 r1^{9} + \sqrt{(28311552)}
                                                                                                                                                        r1 - 467140608 r1^2 + 3588489216 r1^3 - 16978083840 r1^4 +
                                                                                                                                                 55\,228\,760\,064\,r1^5 - 130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                                                                                315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                                                                 162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                                                                                               4 \, \text{r1} \, \left(1 - \text{r1} + \frac{1}{2} \, \sqrt{\left(4 \, \left(-1 + \text{r1}\right)^{\, 2} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) \right.} + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^{\, 2} + 21 \, \text{r1}^{\, 3}}{12 \, \left(-1 + \text{r1}\right)} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{-22 + 65 \, \text{r1} - 64 \, \text{r1}^{\, 2} + 21 \, \text{r1}^{\, 3}}{12 \, \left(-1 + \text{r1}\right)} + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1} - 21 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 + 43 \, \text{r1}^{\, 2}\right) + \frac{1}{4} \, \left(-22 +
                                                              \left(2^{1/3} \left(4-40 \text{ r1}+129 \text{ r1}^2-196 \text{ r1}^3+154 \text{ r1}^4-60 \text{ r1}^5+9 \text{ r1}^6\right)\right)
                                                                     \left(3 \; \left(-1 + r1\right) \; \left(1024 - 1536 \; r1 - 26112 \; r1^2 + 135040 \; r1^3 - 304896 \; r1^4 + 126112 \; r1^2 + 135040 \; r1^3 + 126112 \; r1^4 + 1261
                                                                                                            391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                                                                                           \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840)}
                                                                                                                                         r1^4 + 55228760064 r1^5 - 130682585088 r1^6 + 232168882176
                                                                                                                                         r1^7 - 315144732672 r1^8 + 329334128640 r1^9 - 264790867968
                                                                                                                                        r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} + 24680595456
                                                                                                                                        \texttt{r1}^{13} - 5\,573\,836\,800\,\texttt{r1}^{14} + 764\,411\,904\,\texttt{r1}^{15} - 47\,775\,744\,\texttt{r1}^{16} \big) \, \Big)^{1/3} \bigg) \,\, + \,\, \\
                                                             \frac{1}{48 \times 2^{1/3} \ (-1+r1)} \left(1024 - 1536 \ r1 - 26112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^2 + 135040 \ r1^3 - 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1^3 + 135040 \ r1^3 + 126112 \ r1
                                                                                     304\,896\,r1^4+391\,296\,r1^5-303\,104\,r1^6+139\,392\,r1^7-34\,560\,r1^8+3456\,r1^9+
                                                                                    \sqrt{(28311552\,r1-467140608\,r1^2+3588489216\,r1^3-16978083840\,r1^4+}
                                                                                                           55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                                                           315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                                                           162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                                                                                          5\,573\,836\,800\,\,r1^{14}\,+\,764\,411\,904\,\,r1^{15}\,-\,47\,775\,744\,\,r1^{16}\big)\,\Big)^{1/3}\bigg|\,\,-\,47\,775\,744\,\,r1^{16}\Big)\,\Big)^{1/3}
```

```
\frac{1}{2}\sqrt{\left(8 \left(-1+r1\right)^2+\frac{1}{4} \left(-22+43 \, r1-21 \, r1^2\right)-\frac{-22+65 \, r1-64 \, r1^2+21 \, r1^3}{12 \, \left(-1+r1\right)}-\frac{1}{2}}\right)}
                    \left(2^{1/3}\,\left(4-40\,\text{r1}+129\,\text{r1}^2-196\,\text{r1}^3+154\,\text{r1}^4-60\,\text{r1}^5+9\,\text{r1}^6\right)\right)\,\Bigg/
                        391 296 r1<sup>5</sup> - 303 104 r1<sup>6</sup> + 139 392 r1<sup>7</sup> - 34 560 r1<sup>8</sup> + 3456 r1<sup>9</sup> +
                                               \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840)}
                                                                 r1^4 + 55228760064 r1^5 - 130682585088 r1^6 + 232168882176
                                                                 r1^7 - 315144732672 r1^8 + 329334128640 r1^9 - 264790867968
                                                                 r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} + 24680595456
                                                                 \texttt{r1}^{13} - 5\,573\,836\,800\,\,\texttt{r1}^{14} + 764\,411\,904\,\,\texttt{r1}^{15} - 47\,775\,744\,\,\texttt{r1}^{16} \big) \, \Big)^{1/3} \bigg) \, - \\
                  \frac{1}{48\times2^{1/3}\ (-1+r1)}\left(1024-1536\ r1-26\ 112\ r1^2+135\ 040\ r1^3-1248\times2^{1/3}\right)
                                 304\,896\,r1^4 + 391\,296\,r1^5 - 303\,104\,r1^6 + 139\,392\,r1^7 - 34\,560\,r1^8 + 3456\,r1^9 +
                                 \sqrt{(28311552 \text{ r1} - 467140608 \text{ r1}^2 + 3588489216 \text{ r1}^3 - 16978083840 \text{ r1}^4 + }
                                               55\,228\,760\,064\,r1^5-130\,682\,585\,088\,r1^6+232\,168\,882\,176\,r1^7-
                                               315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                              162\,331\,361\,280\,r1^{11} - 74\,439\,917\,568\,r1^{12} + 24\,680\,595\,456\,r1^{13} -
                                              5573836800 \text{ r1}^{14} + 764411904 \text{ r1}^{15} - 47775744 \text{ r1}^{16})
                    \left(-64 \left(-1+r1\right)^{3}+4 \left(-1+r1\right) \left(22-43 \, r1+21 \, r1^{2}\right)\right)
                                6 \left(-4 + 11 \text{ r1} - 10 \text{ r1}^2 + 3 \text{ r1}^3\right)\right)
                        \left(4\sqrt{\left(4\left(-1+r1\right)^{2}+\frac{1}{4}\left(-22+43\,r1-21\,r1^{2}\right)+\frac{-22+65\,r1-64\,r1^{2}+21\,r1^{3}}{12\,\left(-1+r1\right)}}\right.+
                                               \left(2^{1/3}\,\left(4-40\,r1+129\,r1^2-196\,r1^3+154\,r1^4-60\,r1^5+9\,r1^6\right)\right)\,\left/\right.
                                                    3 (-1 + r1) (1024 - 1536 r1 - 26112 r1^2 + 135040 r1^3 - 126112 r1^2 + 135040 r1^2 + 126112 r1^2 + 135040 r1^2 + 126112 r1^2 + 135040 r1^2 + 126112 r1^2 +
                                                                          304896 \text{ r1}^4 + 391296 \text{ r1}^5 - 303104 \text{ r1}^6 + 139392 \text{ r1}^7 -
                                                                          34\,560\,\mathrm{r1}^8 + 3456\,\mathrm{r1}^9 + \sqrt{(28\,311\,552\,\mathrm{r1} - 467\,140\,608\,\mathrm{r1}^2 + (28\,311\,552\,\mathrm{r1} - 467\,140\,608\,\mathrm{r1}^2)}
                                                                                        3588489216 r1^3 - 16978083840 r1^4 + 55228760064 r1^5 -
                                                                                        130\,682\,585\,088\,r1^6 + 232\,168\,882\,176\,r1^7 -
                                                                                        315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968
                                                                                            r1^{10} + 162331361280 r1^{11} - 74439917568 r1^{12} +
                                                                                        24\,680\,595\,456\,r1^{13}\,-\,5\,573\,836\,800\,r1^{14}\,+\,764\,411\,904\,r1^{15}\,-\,
                                                                                       47775744 \text{ r1}^{16}))^{1/3} + \frac{1}{48 \times 2^{1/3} (-1 + \text{r1})} \left(1024 - 1536 \text{ r1} - \frac{1}{1024} + \frac{1}{1
                                                            26112 \text{ r1}^2 + 135040 \text{ r1}^3 - 304896 \text{ r1}^4 + 391296 \text{ r1}^5 -
                                                            467 140 608 r1<sup>2</sup> + 3 588 489 216 r1<sup>3</sup> - 16 978 083 840 r1<sup>4</sup> +
                                                                          55228760064 \text{ r1}^5 - 130682585088 \text{ r1}^6 + 232168882176 \text{ r1}^7 -
                                                                          315 144 732 672 r1<sup>8</sup> + 329 334 128 640 r1<sup>9</sup> - 264 790 867 968 r1<sup>10</sup> +
                                                                          162 331 361 280 r1<sup>11</sup> - 74 439 917 568 r1<sup>12</sup> + 24 680 595 456 r1<sup>13</sup> -
                                                                          5 573 836 800 r1<sup>14</sup> + 764 411 904 r1<sup>15</sup> - 47 775 744
```

$$|r1^{16}\rangle\rangle^{1/3}\rangle\rangle\rangle\rangle\rangle^{2} + 4 r1 r11 - 4 r1^{2} r11 - 4 r11^{2} + 4 r1 r11^{2}\rangle$$

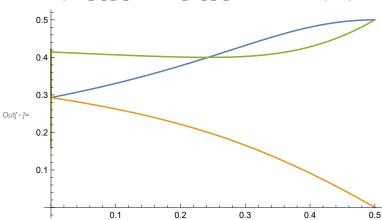
Plot solutions in period 2 with respect to r1

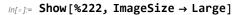
 $\label{eq:local_$ $1-r1-sol2[11][2][2]-sol[1][2]/.sol2[11][2]/.r11 \rightarrow r1/2}, {r1, 0, .5}]$

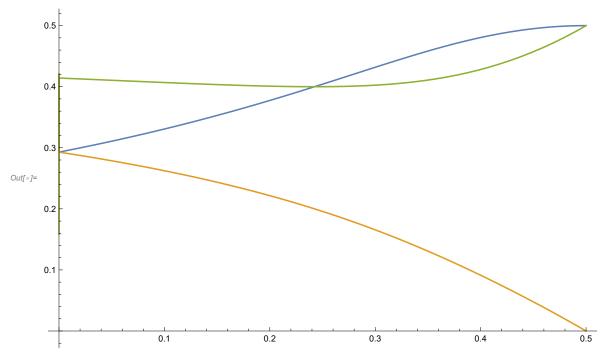


 $ln[\circ] := Plot[\{ sol2[11][2][2] / (1 - r1) ,$

 $(sol[1][2] /. sol2[11][2] /. r11 \rightarrow r1 / 2) / (1 - r1), 1 - sol2[11][2][2] / (1 - r1) - r1 / 2) / (1 - r1) /$ (sol[1][2]] /. sol2[11][2]] /. $r11 \rightarrow r1 / 2) / (1 - r1)}, {r1, 0, .5}]$







Numerical example:

```
ln[-]:= sol2 /. \{r1 \rightarrow 0.1\}
Out[\circ]= \{ \{ r22 \rightarrow 0.875048 - 0.110736 i \} \}
               \{\, r22 \rightarrow \textbf{0.875048} \, + \, \textbf{0.110736} \,\, \dot{\mathbbm{1}} \, \} , \, \{\, r22 \rightarrow \textbf{0.297642} \, \} , \, \{\, r22 \rightarrow \textbf{1.55226} \, \} \, \}
 ln[*]:= sol /. sol2 /. {r1 \rightarrow 0.1}
Out[*]= \{ \{ r12 \rightarrow 0.00069231 + 0.104757 \ i \} \},
               \{\,\text{r12} \rightarrow \text{0.00069231} \, - \, \text{0.104757 i} \, \} \, , \, \, \{\,\text{r12} \rightarrow \text{0.236194} \, \} \, , \, \, \{\,\text{r12} \rightarrow - \, \text{0.662421} \, \} \, \}
```

Import solutions to R

CForm[sol2[11][2][2]];

```
In[*]:= CForm[sol[1][2]]
```

Out[@]//CForm=

```
 (\texttt{r1} - \texttt{Power}\,(\texttt{r1,2}) - \texttt{Sqrt}\,(\texttt{r1}) \, * \texttt{Sqrt}\,(\texttt{r1} - \, 2 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, \texttt{Power}\,(\texttt{r1,3}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r11} \, - \, 4 \, * \, \texttt{Power}\,(\texttt{r1,2}) \, + \, 4 \, * \, \texttt{r1} \, * \, \texttt{r1} \, + \, 4 \, 
                                                                                                                                                                                                           4*r1*Power(r22,2)))/(2.*r1)
```