Supplementary Material of “Optimal allocation strategies in platform trials”

Optimisation of the sum of variances

In[•]:=
```
subst = {n11 → r1 * (1 - p01) * N , n01 → r1 * p01 * N,
   n12 → r2 * N - n02 - n22 , n03 → r3 * p03 * N, n23 → r3 * (1 - p03) * N};
substp = {n12 → r2 * N * p12, n22 → r2 * N * p22, n02 → r2 * N * p02, r3 → 1 - r1 - r2};
```

In[•]:=
```
term1 =
 FullSimplify[((n11 * n01 / (n11 + n01)) + (n12 * n02 / (n12 + n02))) / N /. subst /. substp]
term2 =
 FullSimplify[(n22 * n02 / (n22 + n02) / N) + (n23 * n03 / (n23 + n03) / N) /. subst /. substp]
```

Out[•]= $-((-1+p01)\ p01\ r1)+\frac{p02\ (-1+p02+p22)\ r2}{-1+p22}$

Out[•]= $\frac{(-1+p03)\ p03\ (p02+p22)\ (-1+r1)+(-1+p03)\ p03\ p22\ r2+p02\ ((-1+p03)\ p03+p22)\ r2}{p02+p22}$

In[•]:=
```
f[p02_, p22_, p01_, p03_, r2_] := FullSimplify[term1 + term2]
```

In[•]:=
```
(*Set constraints*)
constraints = 0 ≤ p02 ≤ 1 && 0 ≤ p22 ≤ 1 && 0 ≤ p01 ≤ 1 && 0 ≤ p03 ≤ 1 && r2 > 0 && r1 > 0
(*Calculate the derivatives*)
dfdp02 = D[f[p02, p22, p01, p03, r2], p02]
dfdp22 = D[f[p02, p22, p01, p03, r2], p22]
dfdp01 = D[f[p02, p22, p01, p03, r2], p01]
dfdp03 = D[f[p02, p22, p01, p03, r2], p03]
```

Out[•]= $0 \le p02 \le 1\ \&\&\ 0 \le p22 \le 1\ \&\&\ 0 \le p01 \le 1\ \&\&\ 0 \le p03 \le 1\ \&\&\ r2 > 0\ \&\&\ r1 > 0$

Out[•]= $\frac{p02\ r2}{-1+p22}+\frac{(-1+p02+p22)\ r2}{-1+p22}+\frac{(-1+p03)\ p03\ (-1+r1)+((-1+p03)\ p03+p22)\ r2}{p02+p22}-\frac{(-1+p03)\ p03\ (p02+p22)\ (-1+r1)+(-1+p03)\ p03\ p22\ r2+p02\ ((-1+p03)\ p03+p22)\ r2}{(p02+p22)^2}$

Out[•]= $\frac{p02\ r2}{-1+p22}-\frac{p02\ (-1+p02+p22)\ r2}{(-1+p22)^2}+\frac{(-1+p03)\ p03\ (-1+r1)+p02\ r2+(-1+p03)\ p03\ r2}{p02+p22}-\frac{(-1+p03)\ p03\ (p02+p22)\ (-1+r1)+(-1+p03)\ p03\ p22\ r2+p02\ ((-1+p03)\ p03+p22)\ r2}{(p02+p22)^2}$

Out[•]= $-((-1+p01)\ r1)-p01\ r1$

Out[•]= $\frac{1}{p02+p22}((-1+p03)\ (p02+p22)\ (-1+r1)+p03\ (p02+p22)\ (-1+r1)+p02\ (-1+2\ p03)\ r2+(-1+p03)\ p22\ r2+p03\ p22\ r2)$

In[•]:=
```
dfdp02s = Simplify[dfdp02]
```

Out[•]= $\frac{\left(2\ p02^3+2\ (-1+p22)\ p22^2+2\ p02\ p22\ (-1+2\ p22)+p02^2\ (-1+5\ p22)\right)\ r2}{(-1+p22)\ (p02+p22)^2}$

In[]:=
```
dfdp22s = Simplify[dfdp22]
```

Out[]=
$$-\frac{p02^2\,(1+p02)\,(-1+p02+2\,p22)\,r2}{(-1+p22)^2\,(p02+p22)^2}$$

In[]:=
```
dfdp01s = Simplify[dfdp01]
```

Out[]= $r1 - 2\,p01\,r1$

In[]:=
```
dfdp03s = Simplify[dfdp03]
```

Out[]= $(-1+2\,p03)\,(-1+r1+r2)$

In[]:=
```
(*Solve for critical points*)
criticalPoints = NSolve[
  {dfdp02s == 0, dfdp22s == 0, dfdp01s == 0, dfdp03s == 0, constraints}, {p02, p22, p01, p03}]
```

Out[]=
$$\left\{\left\{p02 \to \boxed{0.414214 \text{ if } r2>0\,\&\&\,r1>0},\ p22 \to \boxed{0.292893 \text{ if } r2>0\,\&\&\,r1>0},\ p01 \to \boxed{0.5 \text{ if } r2>0\,\&\&\,r1>0},\ p03 \to \boxed{\frac{-1.+r1+r2}{-2.+2.\,r1+2.\,r2} \text{ if } r2>0\,\&\&\,r1>0}\right\}\right\}$$

In[]:=
```
(*Evaluate the function at critical points*)
min = MinimalBy[criticalPoints, f[p02, p22, r2] /. # &]
```

Out[]=
$$\left\{\left\{p02 \to \boxed{0.414214 \text{ if } r2>0\,\&\&\,r1>0},\ p22 \to \boxed{0.292893 \text{ if } r2>0\,\&\&\,r1>0},\ p01 \to \boxed{0.5 \text{ if } r2>0\,\&\&\,r1>0},\ p03 \to \boxed{\frac{-1.+r1+r2}{-2.+2.\,r1+2.\,r2} \text{ if } r2>0\,\&\&\,r1>0}\right\}\right\}$$

In[]:=
```
Solve[{dfdp02s == 0, dfdp22s == 0, dfdp01s == 0, dfdp03s == 0, constraints},
 {p02, p22, p01, p03}]
```

Out[]=
$$\left\{\left\{p02 \to \boxed{-1+\sqrt{2} \text{ if } r2>0\,\&\&\,r1>0},\ p22 \to \boxed{\frac{-\left(-1+\sqrt{2}\right)^2 r2+\left(-1+\sqrt{2}\right)^4 r2}{-2\left(-1+\sqrt{2}\right)^2 r2-2\left(-1+\sqrt{2}\right)^3 r2} \text{ if } r2>0\,\&\&\,r1>0},\ p01 \to \boxed{\frac{1}{2} \text{ if } r2>0\,\&\&\,r1>0},\ p03 \to \boxed{\frac{-1+r1+r2}{-2+2\,r1+2\,r2} \text{ if } r2>0\,\&\&\,r1>0}\right\}\right\}$$