Computations on case 3 with Lagrange Multipliers

Fixed sample sizes in period 1 and 2

Set (and simplify) conditions

```
In[ ]:= subst = {r11 → r1 / 2 , r01 → r1 / 2 , r23 →  r3 / 2 , r03 → r3 / 2, r02 → r2 - r12 - r22 }
```

$$\text{Out[ ]=}\ \left\{\text{r11} \to \frac{\text{r1}}{2}, \text{r01} \to \frac{\text{r1}}{2}, \text{r23} \to \frac{\text{r3}}{2}, \text{r03} \to \frac{\text{r3}}{2}, \text{r02} \to -\text{r12} + \text{r2} - \text{r22}\right\}$$

```
In[ ]:= ex = {r1 → 0.1, r2 → 0.8, r3 → 0.1}
```

$$\text{Out[ ]=}\ \{\text{r1} \to 0.1, \text{r2} \to 0.8, \text{r3} \to 0.1\}$$

Define terms to optimise

Note: *sigma*term1^(-1)/N* is the variance of the estimator of effect 1 (analogously *sigma*term2^(-1)/N* for effect 2). But since sigma and N are fixed, we simply work on term1 and term2 expressions. furthermore we set NT=1.

```
In[ ]:= term1 = FullSimplify[(r11 * r01 / (r11 + r01)) + (r12 * r02 / (r12 + r02)) /. subst]
```

$$\text{Out[ ]=}\ \frac{\text{r1}}{4} + \text{r12} + \frac{\text{r12}^2}{-\text{r2} + \text{r22}}$$

```
In[ ]:= term2 = FullSimplify[(r22 * r02 / (r22 + r02)) + (r23 * r03 / (r23 + r03)) /. subst]
```

$$\text{Out[ ]=}\ \text{r22} + \frac{\text{r22}^2}{\text{r12} - \text{r2}} + \frac{\text{r3}}{4}$$

```
In[ ]:= constr = term1 - term2;
```

```
In[ ]:= e1 = Solve[D[term1, r12] == l D[constr, r12], l]
        e2 = Solve[D[term1, r22] == l D[constr, r22], l]
```

$$\text{Out[ ]=}\ \left\{\left\{\text{l} \to \frac{(\text{r12} - \text{r2})^2\ (2\,\text{r12} - \text{r2} + \text{r22})}{2\,\text{r12}^3 - 5\,\text{r12}^2\,\text{r2} + 4\,\text{r12}\,\text{r2}^2 - \text{r2}^3 + \text{r12}^2\,\text{r22} - 2\,\text{r12}\,\text{r2}\,\text{r22} + \text{r2}^2\,\text{r22} - \text{r2}\,\text{r22}^2 + \text{r22}^3}\right\}\right\}$$

$$\text{Out[ ]=}\ \left\{\left\{\text{l} \to \frac{\text{r12}^2\ (\text{r12} - \text{r2})}{\text{r12}^3 - \text{r12}^2\,\text{r2} + \text{r12}\,\text{r2}^2 - \text{r2}^3 - 2\,\text{r12}\,\text{r2}\,\text{r22} + 4\,\text{r2}^2\,\text{r22} + \text{r12}\,\text{r22}^2 - 5\,\text{r2}\,\text{r22}^2 + 2\,\text{r22}^3}\right\}\right\}$$

```
In[ ]:= e3 = e1[[1]][[1]][[2]] == e2[[1]][[1]][[2]]
```

$$\text{Out[ ]=}\ \frac{(\text{r12} - \text{r2})^2\ (2\,\text{r12} - \text{r2} + \text{r22})}{2\,\text{r12}^3 - 5\,\text{r12}^2\,\text{r2} + 4\,\text{r12}\,\text{r2}^2 - \text{r2}^3 + \text{r12}^2\,\text{r22} - 2\,\text{r12}\,\text{r2}\,\text{r22} + \text{r2}^2\,\text{r22} - \text{r2}\,\text{r22}^2 + \text{r22}^3} == \frac{\text{r12}^2\ (\text{r12} - \text{r2})}{\text{r12}^3 - \text{r12}^2\,\text{r2} + \text{r12}\,\text{r2}^2 - \text{r2}^3 - 2\,\text{r12}\,\text{r2}\,\text{r22} + 4\,\text{r2}^2\,\text{r22} + \text{r12}\,\text{r22}^2 - 5\,\text{r2}\,\text{r22}^2 + 2\,\text{r22}^3}$$

In[ ]:=

In[ ]:=
```
sol = Solve[e3, {r12}][[2]]
```

Out[ ]=
$$\left\{\text{r12} \to \frac{\text{r2}\ (\text{r2} - 2\ \text{r22})}{2\ (\text{r2} - \text{r22})}\right\}$$

In[ ]:=
```
solr22 = NSolve[term1 - term2 /. sol /. ex][[6]]
```

Out[ ]=
$$\{\text{r22} \to 0.234315\}$$

In[ ]:=
```
solr12 = r12 /. sol /. solr22 /. ex
```

Out[ ]=
$$0.234315$$

In[ ]:=
```
r2 - r22 - solr12 /. solr22 /. ex
```

Out[ ]=
$$0.331371$$

In[ ]:=

In[ ]:=
```
eq = FullSimplify[term1 - term2 /. sol] == 0
```

Out[ ]=
$$\frac{1}{4}\left(\text{r1} + \frac{\left(\text{r2}^2 - 4\ \text{r2}\ \text{r22} + 2\ \text{r22}^2\right)\ \left(\text{r2}^2 - 2\ \text{r2}\ \text{r22} + 2\ \text{r22}^2\right)^2}{\text{r2}^2\ (\text{r2} - \text{r22})^3} - \text{r3}\right) == 0$$

In[ ]:=
```
CForm[eq[[1]]]
```

Out[ ]//CForm=
```
(r1 + ((Power(r2,2) - 4*r2*r22 + 2*Power(r22,2))*Power(Power(r2,2) - 2*r2*r22 + 2*Powe
```

In[ ]:=
```
CForm[sol[[1, 2]]]
```

Out[ ]//CForm=
```
(r2*(r2 - 2*r22))/(2.*(r2 - r22))
```