Project 1: Insurance Model

Simulation and Resampling

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1 Introduction

Humans have always wanted to feel security, wellness and happiness, that is why the origin of insurance dates back to 4,000 years ago.

An insurance is a contract through which a policy holder obtain protection against some financial loss or service (such a medical treatment).

The policy holder usually pays a monthly or annual premium in exchange of the contract and he can report a claim if he suffers some damage. The cost of the claims depends on the type of the service/product covered and can vary depending on the scale.

Usually, insurance companies have high fluctuation of clients that join or leave the company constantly. Therefore, if the client stays in the company only during a fraction of time, he will pay a fraction of the premium.

In this project we have been required to create a simulation program which determines the mean and the standard deviation of the final positive capital after some given time in an insurance company using the inputs of few financial arguments such as initial capital, number of clients, number of claims, cost of the claims, etc.

In addition, we have to obtain the fraction of positive simulation paths.

In this case, the cost of the claims is modeled according to a Log-Normal distribution with parameters: $\mu = 5$ and $\sigma = 1.5$.

Also, new clients enroll in the company according to an homogeneous Poisson process with yearly rate ν and remain in the company according to an exponential distribution of yearly rate μ.

The number of claims are generated according to an independent homogeneous Poisson process with yearly rate λ.

On the other hand, the following parameters of the model are known:

- Initial capital: c_0
- Initial number of insured clients: n_0
- Insurance's annuity premium: a
- Path's length (years): t_l
- Claims rate λ, departure rate μ and enrollment rate ν
- Number of simulations (paths): m

2 Methodology

As we already mentioned, the insurance model is defined by three different processes:

- Clients enrollment, modeled by an homogeneous Poisson process of rate ν.

- Time that clients remain in the company defined by an exponential distribution of rate μ.
- Number of claims generated per client, modeled by an homogeneous Poisson process of rate λ.

We know than an homogeneous Poisson process of rate θ is a counting process $\{N_t, t \geq 0\}$ where $N_t \sim \text{Poisson}(\theta t)$, $\forall t > 0$. However, this process can also be defined by the time between events. This interarrival times follow an exponential distribution of rate θ.

That is, we can model the previous processes using solely the time between events. Where we have to consider each event per client ($\times n$) in the case of claims and departures. Such that

- Time until a new client enrolls $\sim \text{Exp}(\nu)$
- Time until a new claim occurs $\sim \text{Exp}(n\lambda)$
- Time until a client departs $\sim \text{Exp}(n\mu)$

This allows for a new approach: **time until a new event occurs**. Assuming all three random variables to be independent then the minimum time until an event occurs is given by $\text{Exp}(\nu + n\lambda + n\mu)$.

Once an event has occurred we need to establish the type of event. Such that the probability of an event is its rate parameter divided by the sum of parameters $(\nu + n\lambda + n\mu)$. Therefore, using cumulative probabilities, we generate a random probability u from a uniform distribution on the interval $[0, 1]$ where

- A new client enrolls if $u < \frac{\nu}{\nu+n\lambda+n\mu}$
- A client departs if $u < \frac{\nu+n\mu}{\nu+n\lambda+n\mu}$
- A client generates a claim if $u < 1 - \frac{\nu+n\mu}{\nu+n\lambda+n\mu}$

3 Algorithm

The structure of the algorithm is as follows

Algorithm 1 Insurance Risk Model

```
function INSURANCE SIMULATION
    for simulation in iterations do
        initialize default values for simulation
        while cumulative time < t_l and not broken do
            generate time of event
            update cumulative time
            charge clients up to cumulative time
            if cumulative time does not exceed t_l then
                determine type of event
                if event is new client then
                    add new client
                else if event is lost client then
                    remove client
                else
                    generate cost of a claim and update capital
                    if capital < 0 then
                        broken = True
            else
                charge clients the remaining time to t_l
    obtain positive paths and its proportion
     return mean and standard deviation of positive paths and its proportion
```

Where the final capital of the company for each simulation is stored in an array. As soon as the company enters bankruptcy (*broken* flag) that specific simulation is halted in order to track positive and negative paths.

Every client is charged proportionally once an event occurs. This allows clients to be charged accordingly to the time they remain at the company. The payment of any client to complete the whole period will be divided but paid in full at the end, whereas any part-time client will only pay the corresponding part up to the time they depart or from the time they join.

4 Conclusions

This program is a good choice to test a business case, in that way, we have to run it with an example of real data:

- Initial capital: 10,000 euros
- Initial number of insured clients: 50
- Insurance's annuity premium: 1,000 euros
- Path's length (years): 20 years
- Claims rate: *2*, departure rate: *2*, and enrollment rate: *30*
- Number of simulations (paths): 10,000

The results obtained for this case are: 56,217.1 euros of final capital with an standard error of 22,660.22 euros, where we attain 55% of positive paths.

We can conclude that for this or similar scenarios we could obtain around 35-78,000 euros of benefit after 20 years of business, however, this program has some limitations in the business scope such as the limit of claims per year, the yearly variation of premiums (raises when the accident rates of users are higher or reductions when the users has good behaviours), etc.

With all those considerations, surely, we could build a more efficient algorithm to simulate the reality in a more accurate way.

5 Bibliography

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