Quantitative Macroeconomics: Project 3

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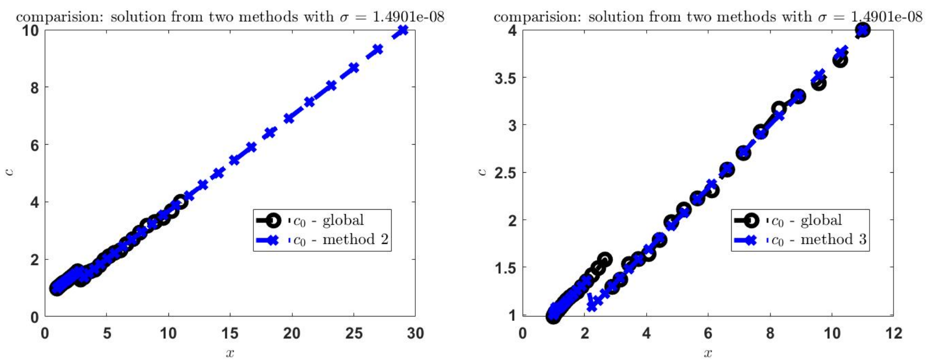
PROBLEM A

1. Run the code to compare the solution methods

The figures below plot the results from the three solution methods and compare them to each other. It is clear that the third method has difficulties with the discontinuity of the policy function.



(a) Methods 1 and 2 (b) Methods 1 and 3

Figure 1: Consumption policy functions at t=0

2. Explain the solution methods

The three solution methods vary in how they solve the backwards iteration for periods 1 and 0, as the last period has already been solved in a common manner beforehand. Therefore, the three methods are different ways of computing the choice specific policy functions for the two periods where the labour choice is continuous.

1 *Global solution method:* this method evaluates the value function on the grid we created to discretize consumption, then finds the maximum point of this value function. Finally it searches for an optimum point in the neighbourhood of the maximum using golden search (an extremum finding method which successively narrows the values of the interval according to the golden ratio).

2 *Discrete continuous exogenous grid method:* create the exogenous savings grid, then solve the household problem on that savings grid. Then, check monotonicity on the endogenous cash on hand grid, and correct the results if necessary.

3 *Exogenous grid method (rootfinding):* the last method uses a univariate solver (fzero, which implements Brent's method) to solve for consumption by finding roots on the FOC based on some initial guess for cash on hand. Then it checks if the resulting consumption values satisfy the budget constraint, and if not it makes $c_t = x_t$. As a last step it computes the corresponding value function.

3. Compare to the closed form solution in period 1

We can compare the closed form solution for c_1 to the analytical one by adding some code to the function which computes the closed form solutions, *funclosed()*. From the results provided in the code for the case with no taste shocks (c_1 is already computed when we have taste shocks) we see that:

$$a_2 = x_1 + 1 - 2c_0$$

And because from the definition of cash on hand, $x_1 = a_2 - c_1$, we get that the analytical solution for consumption in the first period is $c_1 = 2c_0 - 1$.

Adding that to the function and slightly modifying the previous code so that it compares the analytical and numerical solutions for t=1 instead of t=0, I obtain the following plots for all three methods presented in Figure 2. In all three methods consumption is larger in the analytical than in the numerical case, contrasting with the overlapping results which we obtained for c_0.

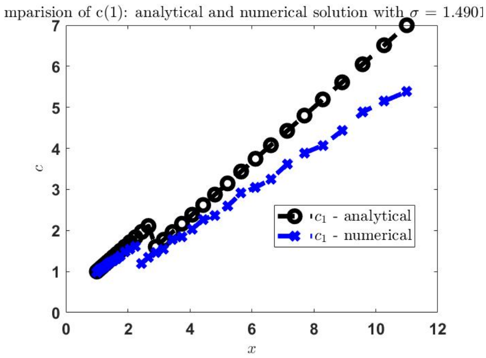
However, because we initially set the variance of both shocks equal to $\sqrt{eps} \simeq 0$ that would imply we are in a deterministic case just like in the handout, and we should have equal consumption in all periods for the analytical solution without taste shocks.

4. Running times

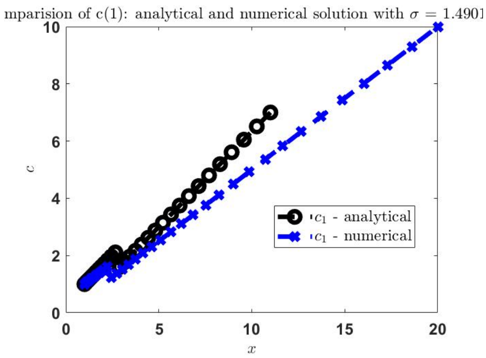
The running times of the three different methods vary greatly:

- Method 1 = 26.48 seconds
- Method 2 = 1.85 seconds
- Method 3 = 18.87 seconds

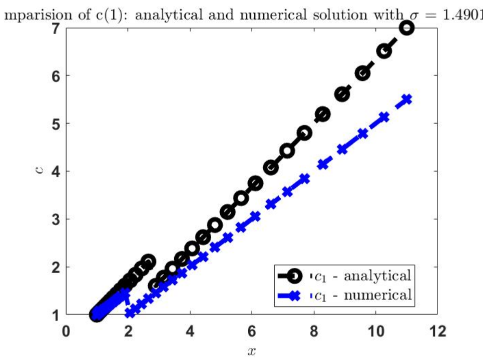
Method 2 is by far the fastest, being 10 times quicker to get a solution than method 3. The global method is the slowest. That is probably due to



(a) Method 1



(b) Method 2



(c) Method 3

Figure 2: Analytical and numerical solutions for c_1

the endogenous grid method being much faster than both the exogenous grid and the global method, where the golden search in order to find a maximum slows down the computation.

5. Method 3 problems

It is clear when looking at panel B in figure 1 that the third solution method is not able to replicate the discontinuity in the consumption policy function resulting from the discrete choice on labour participation. That is possibly due to the use of a univariate solver (*fzero*) to find the solution for consumption, because it does not work as well with the non-convexities resulting from the introduction of a discrete choice in the last period as the other methods do.

6. Increase the income shock variance

Increasing the variance of the income shocks results in the policy functions for consumption being bounded by the borrowing constraint at the beginning, so that we can see different slopes on the policy function for values of consumption and savings below 1.

7. Increase the taste shock variance

When making $\sigma = 1$, the resulting policy functions are almost linear for all three methods. That shows that for a sufficiently large variance of the taste shocks, the non-concave regions around the kinks of the value function are smoothed out, making the value function become globally concave and getting rid of the discontinuity in the policy functions.

PROBLEM B

Problem 1: household problem

Explain the code:

The code is divided into four main sections:

1 <u>Calibration:</u> all parameters are defined, the permanent component of income is created (wage=1, pension=0.4). The survival rates are set up like in the previous project and total population is computed and normalized to 1. In addition to that, the Markov chain for the possible states of the income shock is also computed.

2 <u>Household model:</u> the problem of the household is solved here using the endogenous grid method. To do that, after initializing empty grids and decision rules for the relevant variables, a constant grid on savings

is generated. Then an exogenous grid for cash at hand is generated and used on the terminal conditions to get c_T, a_T and the value function and its derivative. In a backwards iteration loop for j, η and each point in the grid of x, we get the expected income for the next period, use it to determine the maximum amount of x_{t+1}, then interpolate on the value function's derivative to get the value function. Finally, compute the right hand side of the euler equation and invert it to obtain the consumption policy function. As a last step, it also checks for the optimal decisions at the minimum value of cash on hand, computes assets for every point in the grid and updates the value functions.

3 Aggregation: this step consists of computing the cross sectional distributions of the mode and using them to get the aggregate values of each variable (c,a). To do so, it creates a transfer function (TT) for each state using the Markov chain and it also creates the transfer distribution (ϕ). After checking that the distribution adds up to 1, the aggregate variables are calculated:

$$A = \Sigma totpop\phi(j, \eta, a)s(a)$$
$$C = \Sigma totpop\phi(j, \eta, a)c$$

4 Average life cycle profiles: this part creates profiles for assets, consumption, income, labour income and the value function averages over the life time, which are later on plotted.

Compare func_hh to the exogenous grid method in the lecture notes:

The household problem in *func_hh* is solved using the endogenous grid method instead of the exogenous one. The main differences are that the endogenous method creates a constant grid on savings and also an exogenous grid for cash at hand in T (which is substituted by an endogenous one once we have characterized consumption in the backwards iteration process) while the exogenous one only creates a grid on cash on hand. That implies that the way of solving for consumption is different, as in the endogenous grid method we do not need a nonlinear rootfinder (like *fzero*) to solve for it as consumption only appears in the right hand side of the euler equation. We can solve it by interpolating on Λ_x, like the code does when defining *cfun*. That contrasts with the exogenous grid method, where a non-linear root finder is required. The rest of the process is similar for both methods.

As already mentioned, the main advantage of working with a grid on savings is that we only have consumption present once in the euler equation and it does not require a univariate solver to solve for it. That makes the computation much faster. It also implies that we know the minimum point of the savings grid should be given by $\underline{s} = -\frac{\phi}{1+r}$ and that will save us some time in not needing to check if the budget constraint is satisfied for the value of consumption we find, as it is already implicit in our definition of $\underline{s}$. In terms of disadvantages: interpolation in the endogenous grid method is more complicated, as the cash on hand grid is exogenous and not necessarily formed by equally spaced points. It becomes specially complicated when dealing with large state spaces and non-convexities.

Implement the exogenous grid method:

I implemented the exogenous grid method by modifying the provided code in the file *B1EXOGM*.

Aggregation function:

The objects TT and ϕ are the transition function and distribution, and we use them in order to get the aggregates of the variables of interest. I modified the code to make their computation faster in the file *B1fun_aggr*. Merging the two loops reduces the running time from 10.6 seconds to 9.9, and adding a break for the case where $\phi(j-1, x, \eta) = 0$ reduces it further to 9.1 seconds.

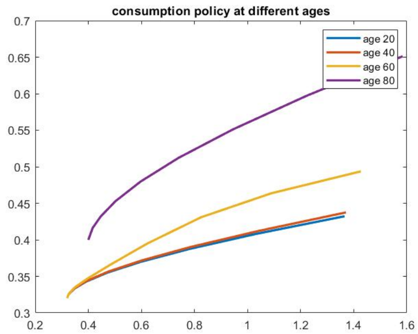
Solve under different settings:

The different suggested settings imply whether there is idiosyncratic labour income risk (high or low income shocks) and survival risk (with some probability you die before T). The default setting of the code makes both of those risks present in the model, while the suggested alternative settings here combine these possibilities. The resulting consumption policies for the different settings are presented in the figure below.

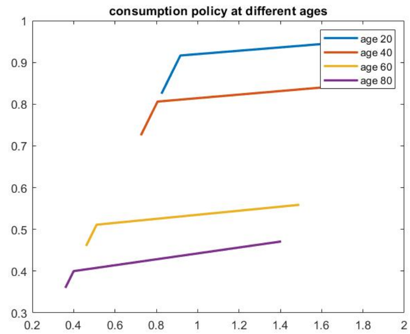
In option 1 (opt_det = true, opt_nosr = true) there is no risk of any kind, so it is a deterministic model. The resulting consumption policy functions for different ages are such that all ages hit the borrowing constraint (and therefore are linear with two different slopes), and due to the lack of income risk the young agents consume more than the old ones, as they get more income and ...

In option 2 (opt_det = false, opt_nosr = true) there is labour income risk but no survival rates, such that everyone lives until T but those who work face uncertainty over their labour income. That results in older agents consuming more than the young ones, although those aged 80 still hit the borrowing constraint. Both ages of working-aged agents represented in the graph consume exactly the same, as they need to save more in order to insure against the income shocks that they may face.

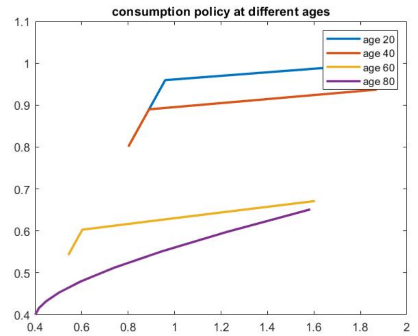
In option 3 (opt_det = true, opt_nosr = false), on the other hand, there is no idiosyncratic labour income but there is survival risk: everyone that works get a certain amount of income but everyone might die before reaching T. This results in consumption policies where the young ones again consume more than the old (they do not need to save to insure against income risk) but where the only age that does not hit the borrowing constraint is 80, as they are the ones who know for sure that they will die next period and they consume exactly as much as they have in terms of cash in hand.



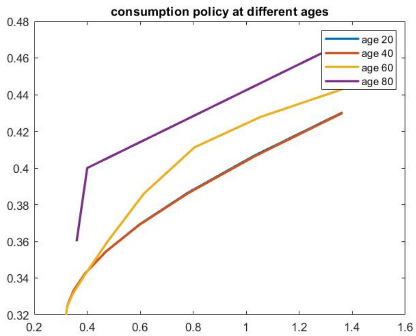
(a) opt_det = false, opt_nosr = false



(b) opt_det = true, opt_nosr = true



(c) opt_det = true, opt_nosr = false



(d) opt_det = false, opt_nosr = true

Figure 3: Consumption policies with different settings on risk

When lowering the coefficient of risk aversion (θ), the slope of the consumption policy function increases: less risk averse individuals save less in order to insure against the possible risks they face, and as a result they consume more. This change is especially noticeable for the older generation. Increasing θ has the opposite effect: consumption policy decreases as a result of increased savings.

Problem 2: general equilibrium

Embed the code into a GE model:

To extend the code into a general equilibrium model (*B2GEq*), I added a loop which iterates over guesses of the interest rate until the guess and the value resulting from the model are very close.

First, I define my initial guess for the rate of return (ret = 0.02) and I create a function called *func_olg* which follows the following steps to solve for an interest rate which makes the assets market clear:

1 Solve for the wage using the FOCs from the firm's problem

2 With a specified replacement rate, compute the tax rate on income ($\tau = \frac{rr*retired}{workers+rr*retired}$)

3 Solve the household problem. To do this, I use the same function as in the partial equilibrium but now I use the guess of the interest rate, the wage obtained from the firm's problem and the tax. That implies that the income is now different, as both wages and pensions are endogenous and taxed.

4 Aggregate using the same function as in the partial equilibrium, modified to consider the same changes I introduced in the household problem (wages, guessed interest rate, taxes)

5 Using aggregate assets into the FOC of the firm's problem, obtain an estimate for the rate of return

Finally, check for convergence of the guess and estimate: if $||r_m - \tilde{r_m}|| < \varepsilon$ there is convergence and the iteration can be stopped, otherwise the guess of the rate of return should be updated ($r_{m+1} = \omega \tilde{r_m} + (1-\omega) r_m$ and we repeat the process from step 1.

Dynamic inefficiency:

A General equilibrium OLG economy with idiosyncratic risk is dynamically inefficient because it requires $r < \rho$ in order for the equilibrium to exist, which results in capital stock being higher than in the complete markets case. This over-saving is a result of wanting to insure against the idiosyncratic risk as well as a precautionary savings motive.

Welfare analysis:

To include the consumption equivalent variation as a measure of the welfare effect of the change in pension policy I run the GE code for the cases where the replacement rate is 0 and 0.6 and save the value function for the newborns (j = 1) in both cases on a txt file, which I then use in a separate file (*B2CEV*) to compute the CEV. The consumption equivalent variation can be computed in the following way:

$$g(a,\eta) = (\frac{V_1(a,\eta)}{V_0(a,\eta)})^{\frac{1}{1-\theta}} - 1$$

Where V_0 corresponds to the case before the policy change (rr = 0, no pensions) and V_1 to the situation after the change (rr = 0.6).

The CEV values I obtain for the different shocks and asset values range from around 1100 to 2800, increasing as assets increase for those who receive the high shock and with concave shape for those with the low shock. That means that the households benefit greatly from the transition to the new policy, especially those hit by the high shock.

Changing theta influences the results: for a higher value of theta we get negative rates of return when there are no pensions, and the introduction of pensions has a much larger effect for all values of the shock and the assets, which makes sense for more risk averse households. On the other hand, decreasing theta to 1.1 has interesting effects: it starts off with small negative effects for both shocks for the asset poor agents but it becomes positive as we look at the effect on households with more assets. In this case, the introduction of the pension system has a larger effect for those faced by the low shock, which is consistent with the fact that they are less risk averse and do not save as much to face possible negative shocks.

In order to decompose the welfare effects from the policy into those resulting from insurance against risk and general equilibrium effects, I modify the partial equilibrium code (now *B2CEVpartialeq*) to include the tax system and resulting wages and pensions, as well as use the fixed wages and interest rates that we got from the GE problem with rr=0. From there I save the value function of the youngest generation and compute the CEV with that as V_1 and V_0 from the initial steady state in GE. That results in a CEV which represents the welfare gains arising from insuring against risk, and which is larger than the overall gains for those who are asset poor and lower than the overall gains for the asset rich. That means that the asset poor agents have a negative gain from the GE effect while that is positive for the asset rich households.

Gini coefficient:

By constructing the distribution of assets in the GE economy (in the general GE file) as $a = A * \Phi_a$ and using the provided *ginindex* function I obtain the following Gini coefficient of the model:

It is clear from the figure that the model has important levels of inequality: the 20% poorest households in the population barely hold any income at all, while the top 10% richest households hold around 40% of the total income in the economy. This replicates quite closely what is seen in real data.

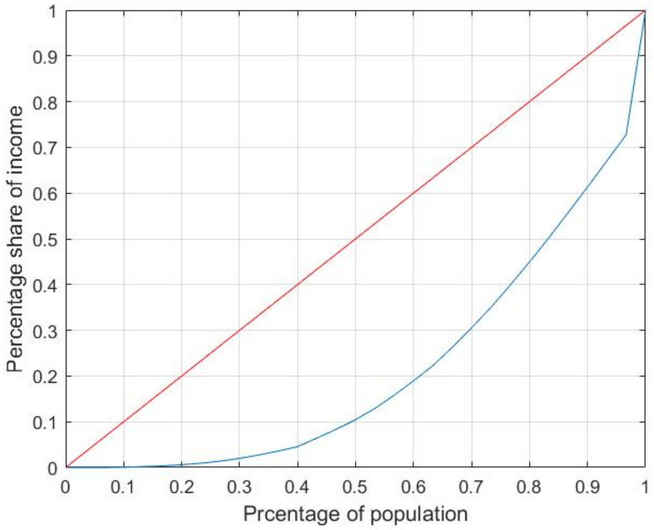


Figure 4: Gini index of wealth distribution

Problem 3: transitional dynamics

Here is pseudocode on how I would solve the general equilibrium OLG model with idiosyncratic risk where a PAYG system is introduced:

1 Define a maximum period T, where the transition from initial to final steady state has been completed.

2 Solve for the initial and final steady states using the corresponding tax (resulting from rr = 0 for the ISS and rr = 0.6 for the FSS). To do that, I would use the same code I already have for GE, but would need to run it twice save the results for both cases.

3 Make an initial guess of the rate of return in the transition (r_t^0) from a linear interpolation between the rates of return in the initial and final steady states.

4 In each iteration (m), do the following:

a Calculate wages for the transition (w_t^m) from the firm's FOC, based on the guess for r_t^m.

b Using r_t^m, w_t^m, τ_t solve the household problem with a backwards loop (t=T:1, j=jn:1), starting from the solution of the final steady state.

c Using the policy functions ($a_{t+1}^m(t, j, a_t, \eta_t)$) and the transition matrix ($\pi(\eta'|\eta)$), compute the transition laws and use them to obtain the distributions for all periods, by iterating forwards in t from the know initial distribution ($\phi_0(j)$). Use the distribution to obtain the aggregate capital supply of the households.

d Apply the market clearing condition of the assets market ($\tilde{K}_t^m = A_t^m$) for all t and use it on the firm's FOCs to update the rate of return: $\tilde{r}_t^m$ for the whole transition.

e Check convergence: if $||\tilde{r}^m - r^m|| < \varepsilon$ then we can stop the iteration process, if not update the initial guess to be $r_t^{m+1} = \gamma\tilde{r}_t^m + (1-\gamma)r_t^m$ and repeat from (a).

5 Check if the market clearing condition holds ($||A_t - K_t|| < \varepsilon$). If it does not, increase T and repeat from step (1).

PROBLEM C

On the optimal provision of social insurance: progressive taxation versus education subsidies in general equilibrium - Krueger and Ludwig (2016)

The paper computes the optimal tax and education policy transitions in an economy where progressive taxes provide social insurance against idiosyncratic wage risk but distort the education decisions of households (because it discourages high earning households from accumulating human capital in order to become asset-rich). Tertiary education subsidies mitigate the effect of this distortions, by reducing the college wage premium and having important redistributive effects. This implies that progressive taxes combined with education subsidies are potential policy substitutes for social insurance. They point out how a full characterization of the transition path between steady states is crucial for policy evaluation, as the transitional costs of the policies need to be taken into account in order to choose the optimal degree of tax progressivity.

They consider a benchmark model where the skilled and unskilled labour are imperfect substitutes in production. The maximization of the social welfare function in the transition results in an education subsidy of 150 percent of the tuition cost and a tax system with only moderate progressivity. So even in a setting where workers are perfect substitutes and transitional dynamics are not taken into account, the progressive taxation and education subsidies provide improved social insurance (more progressive than the US status quo system). They also result in a substantial welfare gain due to the fact that per capita output and consumption increase as a result of higher education levels and better skill distribution.

When we take into account the transitional costs into the optimal policy reform, the results are still large education subsidies but less progressive taxes in order to avoid the short run recession induced by higher marginal tax rates, which result in substantial welfare gains. This last result remains the same if we go back to considering the differently skilled labour as imperfect substitutes, but it implies that in steady state progressive taxation is not very large either, because the fall in college wage premium resulting from the subsidies acts as a substitute for redistributive tax progressivity.

Main results:

1 The explicit consideration of transitional dynamics in the analysis of education finance reform models with endogenous human capital accumulation is potentially very important for optimal tax design. That implies that time-varying policies over the transition would provide larger welfare gains.

2 The general equilibrium wage effects can turn education subsidies and progressive taxes from policy complements to substitutes. Both are potentially effective in creating a more equally distributed consumption and lifetime welfare.