The Role of Wage Expectations in the Labor Market

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Abstract

The standard search and matching model does not reproduce some key aspects of the US labor market, in particular, the high volatility in vacancies and unemployment and the null contemporaneous correlation between the vacancy-unemployment ratio and labor productivity from 1990-2020. In addition, I document that survey wage expectations and rational wage expectations covary differently with labor productivity. I formally reject the hypothesis that this is compatible with rational expectations. This paper develops a search and matching model applied to the business cycle with internally rational agents. Even though agents hold subjective expectations about wages, they behave rationally given these expectations. The inclusion of learning significantly improves the model's fit with US data compared to its rational expectations counterpart. During expansionary periods, agents underestimate future wages amplifying the effect of productivity shocks on the labor market. In light of this model, certain countercyclical unemployment insurance policy rules may lead to instability in the belief system, making them undesirable.

Keywords:Internal Rationality, Wage Expectations, Labor Market, Subjective Expectations, Belief Shock.

JEL Classification: E24,E32,E83.

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1 Introduction

The Search and Matching Model (DMP) has become the standard equilibrium unemployment theory. However, several studies question the model's ability to accurately represent labor market fluctuations in the United States.[1] In particular, the standard DMP struggles to replicate observed fluctuations in the labor market and the propagation of productivity shocks. Targeting the ratio of standard deviations between labor market variables and productivity has been the focus of much research. However, the near zero correlation between productivity and labor market tightness post-1990 has been largely neglected in the literature. In this paper, I show that a DMP model is able to reproduce these observations if one allows for small deviations from rational expectations (RE).

I study how to introduce internal rationality (IR) in a DMP model.[2] I relax the standard assumption that agents have perfect knowledge about the wage function obtained from the standard Nash bargaining process. Agents have limited foresight and can not perfectly predict the outcome of wage bargaining, instead workers and firms have subjective beliefs about wages, and they maximize their objective functions subject to their constraints. I call such agents "internal rational" because they know all internal aspects of their problem and maximize their respective objective functions given their knowledge about the wage process. I consider systems of beliefs implying only a small deviation from rational expectations (RE), and that match some aspects of survey wage expectations. The model has a self-referential mechanism: shifts in beliefs about future returns to labor affect current wages, and agents use realized wages to update their beliefs. This generates an additional source of dynamics that helps to match the data. Framing the model under IR provides a microfoundation to previous adaptive learning papers on unemployment.[3]

Moreover, I present a formal econometric test of the null hypothesis that survey evidence is consistent with RE, and demonstrate that the hypothesis of rational wage expectations is rejected by the survey data. This adds another puzzle for the standard version of DMP. The datasets used for this analysis are sourced from the European Commission's professional forecasters and the New York Federal Reserve's panel data on workers' expectations. A notable aspect of this test is its capacity to offer insights into the reasons behind the failure of the RE hypothesis: the failure arises because survey expectations and rational expectations covary differently with the labor productivity. This finding is used to discipline expectations in the model of IR.

To quantitatively evaluate the learning and the RE models, I consider how well they match labor market moments. I use formal structural estimation based on simulated moments (MSM), adapting the results of Duffie and Singleton (1990) to estimate some parameters. Subsequently, I conduct a formal test to determine whether the

[1] See Shimer (2005), Hall (2005), Fujita and Ramey (2003), Costain and Reiter (2008).

[2] See Adam and Marcet (2011).

[3] See Schaefer and Singleton (2018) and Di Pace et al. (2021).

model statistics significantly deviate from their empirical counterparts. The learning model offers a more accurate representation of U.S. data compared to RE. A key finding is the model's capacity to yield a low contemporaneous correlation between labor market tightness and productivity, coupled with elevated relative volatilities in the labor market. For instance, it produces relative volatilities of unemployment and the vacancy-unemployment ratio that are 7.7 and 10.85 times higher than those generated under rational expectations, respectively. Most models under RE require wages to exhibit minimal responsiveness to productivity variations to achieve such volatility. This results in a wage volatility that is less than that of productivity, a scenario inconsistent with empirical data. In my approach, wages are not rigid, they are influenced by both productivity fluctuations and agents' expectations, enabling the model to exhibit a wage volatility that slightly surpasses that of productivity. Additionally, the model generates the positive correlation between wage forecast error and productivity found in surveys, a relationship that is non-existent under rational expectations.

The reduction in the correlation between labor market tightness and labor productivity stems from the additional source of variability introduced by learning, which affects job creation conditions. In a RE framework, labor market tightness solely depends on current productivity, yielding a correlation nearly equal to one. However, with IR, labor market tightness is influenced not only by productivity but also by the time-varying coefficients determining wage expectations, thereby reducing the aforementioned correlation.

Furthermore, learning introduces an endogenous amplification of productivity shocks in the labor market due to the slower adaptation of wage expectations, a result of the constant gain learning algorithm. Hiring decisions are contingent upon firms' projections of future profits per hire, requiring an estimation of the future marginal product and wages over an indefinite horizon. For instance, after a positive productivity shock, IR firms expect lower future wages compared to RE firms. Under RE, firms know perfectly how wages correlate with productivity, whereas in the IR model, they do not know exactly how changes in productivity translate into changes in wages instead, they learn about this relationship. The productivity shock generates a negative impact on the forecast error, updating the expectation downwards. In subsequent periods, agents revise their beliefs in response to changes in market opportunities. This causes wage expectations to be lower compared to RE for a while. It will take some periods to adjust its expectations upward, and in the meanwhile, firms will post more jobs, so that for a while the response of unemployment is contrary to the needed adjustment.

The quantitative model is next used to assess the welfare implications of the current US unemployment insurance (UI). The UI programs, in United States, become more generous during economic downturns. This issue has gained renewed attention given the recent recession. I find that in an economy where agents learn about wages, the welfare costs are significantly higher compared to a RE model, and also, the policy introduces relatively more uncertainty in the economy. Additionally, I found that such policy may destabilize the macroeconomic system when agent learn, specially if the UI is linked to unemployment. Policymakers should steer clear of rules that induce to instability.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 tests the RE assumption with data from professional forecasters and consumer. Section 4 describes the the model. Section 5 presents the calibration of the model and summarizes the main results. Section 6 studies welfare properties of some labor market policies. Section 7 performs some robustness exercises. Lastly, section 8 concludes.

2 Related Literature

This model aligns with efforts to solve the Shimer puzzle in the search and matching model literature. Two solutions stand out in the literature. (I) Change in wage formation, in wage formation, as suggested by Shimer (2005), Hall (2005), Gertler and Trigari (2009), where wages don't fully adjust to productivity shifts, spurring job creation. (II) Calibration changes, as proposed by Hagedorn and Manovskii (2008), enhance firm bargaining power and unemployment benefits, inducing endogenous wage rigidities. Yet, these methods face critiques, and there is no consensus in the literature about how to solve the puzzle.[4] Although these models generate volatility in the labor market, they fall short in explaining the near-zero correlation between labor market metrics and productivity, and the slightly higher wage volatility relative to productivity. Departing slightly from Rational Expectations (RE), I introduce more rigid expectations rather than rigid wages. To the best of my knowledge, this is the first paper to propose a model that is able to generate high volatility in the labor market, a subdued correlation between vacancy-unemployment ratio and labor productivity, flexible wages, and a rationale for wage expectation surveys.

Some recent papers study DMP departing from full information rational expectations (FIRE). For example Morales-Jiménez (2022) and Menzio (2022). In this papers, workers misperceive the true process for productivity. Moreover, workers are assumed to know the mapping from productivity to wages. In these models, agents still require immense knowledge of market behavior. Alternatively, I endow agents with uncertainty regarding how wages are linked to productivity. This fact is tested using wage expectation surveys. My model adeptly addresses the observed correlation between the forecast error of wages and productivity documented in surveys. This is achieved by showcasing a significantly reduced covariance between wage expectations and productivity, as compared to what is implied by rational expectations. In contrast, Morales-Jiménez (2022) and Menzio (2022) do not consider surveys of workers to test the productivity hypothesis.

[4] These approaches were criticized by Pissarides (2009), Haefke et al. (2013), Mortensen and Nagypal (2007) and Costain and Reiter (2008). There are more solutions to generate volatility in the labor market; see Costain and Reiter (2008), Silva and Toledo (2009), Reiter (2007), Menzio (2005) among others.

This paper extends the adaptive learning literature, with applications outlined in Evans and Honkapohja (2012), Bullard and Mitra (2002) and Eusepi and Preston (2011). Recently, the introduction of a standard adaptive learning approach in the search and matching model has been studied. Schaefer and Singleton (2018) find that when agents make one-step-ahead forecast of labor market tightness, the learning model struggles to capture labor market volatility. Conversely, Di Pace et al. (2021) find that when agents use a misspecified model for wage expectations, while it amplifies labor market dynamics, it overstates wage fluctuations and does not appreciably adjust the correlation between labor market variables and productivity. Di Pace et al. (2021) is the paper most akin to mine. The main difference lies in the way agents form wage expectations. In my paper, agents use productivity directly to form wage expectations, a fact I test with survey data, while in their paper they form wage expectations using an autoregressive model, implying that agents have an inaccurate model to form such expectations. This paper builds on the adaptive learning literature, but maintains the rationality of the agents. Importantly, it is also specific about beliefs system that the agents have in the economy.[5] Both these modelling features are the hallmark of the Internal Rationality framework developed by Adam and Marcet (2011). This approach has not been applied to the search and matching model before and can provide a micro-foundation for adaptive learning models.

A large literature studies the optimality of UI policies including Fredriksson and Holmlund (2001), Coles and Masters (2006), Lehmann and Van der Linden (2007), Landais et al. (2010), Mitman and Rabinovich (2015); among others.[6] I quantify the policy bias in the cost-benefit calculation of unemployment policies that depends on the state of the economy in job creation when using a RE model instead of a learning model. Results show that the cost/benefit of unemployment benefits on job creation is significantly underestimated in rational expectations models.

A vibrant literature has recently developed studying the behavior of expectation surveys. Some papers show that there is a significant discrepancy between the expectations implicit in the macroeconomic model under RE and the expectations coming from the survey data; see Conlon et al. (2018), Greenwood and Shleifer (2014), Adam et al. (2017), Malmendier and Nagel (2016), Coibion and Gorodnichenko (2012),Coibion and Gorodnichenko (2015); among others. I applied the statistical test proposed by Adam et al. (2017) to see whether the data support the rational expectation assumption regarding the formation of future wages. I show that neither workers, firms, nor professional forecasters form wage expectations following rational expectations. Moreover, the test provides clues about why the RE

[5]The adaptive learning literature does not specify what agents' views are on the evolution of macro-variables. They only equip them with a recursion, which tracks some moments of the variable. If beliefs are not fully specified in the model, then why, exactly, agents must form expectations according to a given recursion and how this relates to rational behaviour is unclear.

[6]Optimal benefit levels strikes a balance between insurance and incentives, providing insurance against unemployment risk and providing firms with incentives for vacancy creation. I do not address the mention tradeoff, but only highlight the importance of the RE assumption in predicting effects of UI on job creation.

hypothesis fails, which I used as a guide for modeling expectations.

3 Wages and Wage Forecast

Wage expectations play an important role in the labor market decisions. In the search and matching framework, they affect the match surplus and therefore, current wages and also, the hiring decisions made by firms. In the standard DMP model, workers and firms bargain about the wage and the equilibrium wage equation is known by them. More precisely, all agents are assumed to know the mapping from observed productivity shocks to equilibrium wages. This "complete information" assumption is commonly made, although rarely proven, because expectations are very rarely observed.[7]

This section shows that forecast of wages are inconsistent with the notion that agents hold rational wage expectations. I present a formal econometric test following Adam et al. (2017) showing that expectations and RE covary differently with the labor productivity. To run the test, Section 3.1 employs survey data from professional forecasts provided by the European Commission, while Section 3.2 uses data from the Survey of Consumer Expectations (SCE) Labor Market Survey by the New York Fed, which contains workers' wage expectations. In both datasets, the observed covariance between wage expectations and productivity is significantly lower than the one implied by rational expectations.

3.1 Professional Forecasters

This section conducts a test for rational expectations using survey data that comprises the average forecast of annual wage growth in the United States, reported by the European Commission for the period of 1999 to 2020.[8]

Let $E_t^{\mathcal{S}}$ denote the agent's subjective expectation operator based on information up to time t, which can differ from the rational expectation operator E_t. Let $\hat{w}_{t+2}$ denote the two-period ahead realized annual growth of wages, and let s_k be a measure of agent's subjective beliefs regarding future growth of wages that are possibly subject to measurement error, ν_t, obtained from survey data. Therefore, $s_{t+2} = E_t^{\mathcal{S}}(\hat{w}_{t+2}) + \nu_t$ represents an estimate of the agents' subjective beliefs about annual wage growth two semesters ahead. Given the forecast horizon of professional forecasters, t+2 stands for 2 semesters.

$$\hat{w}_{t+2} = c^R + b^R \hat{y}_t + \nu_t, \tag{1}$$
$$s_{t+2} = c^E + b^E \hat{y}_t + \epsilon_t, \tag{2}$$

[7] Conlon et al. (2018), using the Survey of Consumer Expectations (SCE) Labor Market Survey from the New York FED, found a significant correlation between labor force's revisions of wage offer forecasts and their forecast errors. This finding supports the existence of information rigidities in forming expectations about future wage offers.

[8] The forecast is reported twice a year in Autumn and Spring. They just report the average forecast. Link reports: https://ec.europa.eu/economy_finance/publications/european_economy/forecasts/index_en.htm

Indep. variable	b^R	b^E	P-value H$_0: b^R = b^E$	P-value H$_0: b^R \geq b^E$
$\hat{y}_t$	0,75**	0.15*	0,0487	0,025
	(2,35)	(1,9)		
$\hat{y}_{t-1}$	0,78***	0.12	0,0234	0,0202
	(2,71)	(1,18)		

Table 1: RE test

*Note: ***,**,* denote sig. at 1%, 5% and 10% levels, respectively. t statistics in parentheses. The Table presents the results of the test $b^R = b^E$. The third row shows the results of the test where I include the independent variable with a lag of half a year. The p-values for the test are constructed using Monte-Carlo simulations. The number of observations is 42.*

where $\hat{y}$ represents annual productivity growth. Under the null hypothesis of RE $(H_0 : E_t = E_t^{\mathcal{S}})$, if $\hat{y}_t$ is in the informational set of agents for time period t, the prediction error must be orthogonal to $\hat{y}_t$. $\hat{b}^R$ and $\hat{b}^E$ must be estimates of the same regression coefficient because $b^R = b^E$. If coefficients across equations are different I reject RE. [9]

Table (1) shows the result of the test. Column 4 shows the p-values.[10] Additionally, column 5 shows the p-values for the one-sided test. As a robustness exercise, in the third row, I report the results when the test is performed with annual productivity growth lagged. The results provide evidence against the notion that survey expectations of wages are compatible with RE. This rejection arises because survey expectations and rational expectations covary differently with the labor productivity. Therefore, the forecast error of wages is correlated with productivity growth.

An intriguing observation emerges from the data: during recessions, professional forecasters tend to overestimate wage growth, whereas during expansionary periods, they underestimate it. For instance, amidst the Great Recession, forecasters predicted an average annual wage growth of 0.99%. In contrast, the actual average annual growth for that period experienced a decline of 3.4%. Between Q1-2011 and Q3-2016, a period of economic expansion, the pattern reversed. Forecasts anticipated a growth of 0.76%, yet the actual realization was an impressive 2.51%. Such disparities in wage growth predictions could potentially account for the pronounced

[9] Coibion and Gorodnichenko (2012, 2015) bring evidence in favor of information rigidity in expectation formation described by a significant correlation between forecast revisions and forecast error.

$$s_{t+2/t} - s_{t+2/t-1} = c + b(\hat{w}_{t+2} - s_{t+2/t}) + \epsilon_t$$

Using my data, b=-0.08, the non-significance can be due to the fact that the measurement error of the survey data makes the explanatory variables correlated with the residual and gives a bias b.

[10] The p-values are constructed using a small sample correction procedure. To construct the p-values for the test I rely on Monte-Carlo simulations rather than on asymptotic results. Please refer to Section 2 and Appendix A.3 of Adam et al. (2017) for additional details of the test.

b^1	b^2	Is $\hat{y}_t$ included?	R^2
-	0,094***	No	0,09
	(2,57)		
0,204**	0,149***	Yes	0,16
(1,96)	(1,97)		

Table 2: RE test

*Note: ***,**,* denote sig. at 1%, 5% and 10% levels, respectively. t statistics in parentheses. The Table presents the results of regression 3. The first row shows the results of the regression when I do not include productivity growth as independent variable. The p-values are constructed using Monte-Carlo simulations. The number of observations is 42.*

fluctuations observed in the labor market. For example, during an expansionary period, a firm that anticipates lower future wages might be inclined to post more job vacancies.

Di Pace et al. (2021) posits that agents rely on an autoregressive models to shape wage expectations.[11] This implies that agents do not use directly productivity to form wage expectations. To test that assumption, I run the following regression:

$$\hat{s}_{t+2} = c + b^1\hat{y}_t + b^2\hat{w}_t + \varepsilon_t. \tag{3}$$

Table 2 reveals that productivity remains a significant factor in forecasting wage growth, even after accounting for the realized wage growth.

3.2 Consumer Expectations

The data on consumer expectations is sourced from the Survey of Consumer Expectations (SCE) by the Federal Reserve Bank of New York. For the Rational Expectation test, I utilize two datasets: (1) the SCE, which includes detailed demographic information of participants, and (2) the SCE Labor Market Survey.[12] The latter dataset comprises two primary sections: (I) the "Experiences" section, capturing labor market outcomes such as recent wage offers, search behavior, and job satisfaction, and (II) the "Expectations" section, recording expectations regarding wage offers, job transitions, and retirement.

The panel data enables me to explore how individual expectations align with realizations over the subsequent 4-month period, offering insights into the accuracy and formation of expectations in the labor market. Each interview date is denoted by the subscript t. Respondents are surveyed quarterly for up to a year, and each

[11] Di Pace et al. (2021) employ the same survey data up to 2018Q3. However, rather than employing regression (1) and (2) to test Rational Expectations (RE), they examine the correlation between the forecast error, $\hat{w}_{t+2} - s_{t+2}$, and GDP growth. While this is a valid approach, as supported by Coibion and Gorodnichenko (2012, 2015), it does not allow for an exploration into the potential association between GDP growth and the forecast of wage growth.

[12] Details can be found at https://www.newyorkfed.org/microeconomics/databank.html.

respondent is identified by the subscript i. To calculate forecast errors and conduct a statistical test, respondents must participate in at least two consecutive surveys. I focus on data from November 2014 onwards, a time when the survey began including questions about current and anticipated job offers.[13]

The distinction of the rational expectations test conducted in this section, compared to the one proposed by Adam et al. (2017)), lies in the nature of the forecast: agents are predicting their own wage offers rather than aggregate economic variables.

Let $E_t^{\mathcal{S},i}$ denote agent i's subjective expectation operator based on information up to time t, which can differ from the rational expectation operator E_t^i. Let w_{t+1}^i denote the realized wage offer that the agent receives four months ahead, and let s_k^i be a measure of agent i's subjective beliefs regarding future wage offers that are possibly subject to measurement error, ν_t^i obtained from survey data. Therefore, $s_{t+1}^i = E_t^{\mathcal{S},i}(w_{t+1}^i) + \nu_t^i$ represents an estimate of agent i's subjective beliefs about his/her wage offer four months ahead. Given the expectation horizon in the Labor Market Survey, *t+1* stands for four months.

$$w_{t+1}^i = a + \delta\hat{y}_t + \sum_{n=1}^{N} \alpha_n X_{i,n} + \epsilon_{i,t}, \tag{4}$$

$$s_{t+1}^i = a^e + \delta^e\hat{y}_t + \sum_{n=1}^{N} \alpha_n^e X_{i,n} + \mu_{i,t}. \tag{5}$$

Where $\hat{y}$ represents quarterly productivity growth and N is the number of control variables such as income, age, race, numeracy, gender, location, education, type of industry, search effort, and employment status (dummies and categorical variables).

Table (3) shows the results of the test under different specifications of the independent variable. The last column presents the p-values of the test.

The results presented in Table (3) demonstrate that the null hypothesis is rejected in all cases, implying that rational expectations do not find empirical support. Aggregate labor productivity growth correlates with the forecast error of wage offers at a significance level of less than 0.020. Therefore, the forecast error is not orthogonal to productivity growth.

A similar result is evident among professional forecasters. The correlation between actual wages and productivity surpasses that between wage expectations and productivity. Additionally, it appears that, on average, the labor force does not fully incorporate the impact of productivity when forming their beliefs about future wages.

This empirical observation does not necessarily mean that workers are irrational.

[13] Section A.2 of the Appendix provides a detailed overview of the survey data and clarifies the assumptions I adopted.

Table 3: RATIONAL EXPECTATIONS TEST, $H_0 : \delta = \delta^e$

	δ	δ^e	p-value $H_0 : \delta = \delta^e$
Ind. variables: y_t	3,24* (1,729)	0,208 (1,768)	0,017
Ind. variables: y_{t-2}	3,62** (1,694)	0,55 (0,532)	0,011
Ind. variables: y_{t-4}	2,85** (1,467)	-0,47 (0,195)	0,009

Note: Robust standard errors in parenthesis.***,**,* denote sig. at 1%, 5% and 10% levels, respectively. The number of observations are 740. The regression in the second row uses productivity growth with two lags (six-month lagged) as an independent variable instead of contemporaneous productivity growth. The regression of the third row uses productivity growth with 4 lags (one year lagged) as independent variable of contemporaneous productivity growth.

One plausible explanation could be that each worker is privy to the time series data of their own wages, but lacks access to the comprehensive panel data that includes wage offers for a broad spectrum of workers. As a result, when a worker conducts a regression of their personal wage offers against aggregate productivity, the derived coefficient may lack statistical significance due to the limited sample size. Consequently, workers might discount aggregate labor productivity as a non-informative factor in predicting their future wages.

The next section spells out the microfundations of a DMP model under IR where wage expectations are formed using a adaptative approach. Therefore, forecasting errors are not supposed to be necessarily orthogonal to the variables agents observe when making the predictions.

4 The Model

I propose a model featuring labor market search and matching friction as in Mortensen and Pissarides (1994) applied to the business cycle. Under the standard setting of RE, agents understand how productivity maps to wages. Instead, I assume the lack of common knowledge of general equilibrium wage mapping and equip agents with a fully specified system of beliefs. Agents form their expectations about the future path of wages based on their respective perceived law of motion (PLM) and update their beliefs as new information becomes available. Given their expectations, agents take optimal decisions. Two shocks can hit the economy: a productivity shock and a shock that affects the agents' beliefs about their expected wages. At the start of a period, shocks occur. Agents forecast future wages, influencing employment surplus of workers, firms' hiring surplus and vacancy decisions. Should a match occur, wages are then bargained over. The period concludes with certain jobs destroyed exogenously.

4.1 The Labor Market

Following the standard literature, this economy is characterized by frictions in the labor market. There is a time-consuming and costly process of matching workers and job vacancies, which is capture by a standard constant returns to scale matching function *m(u,v)* where u denotes the unemployment rate and v is the vacancy rate. I refer to $\theta_t = \frac{v_t}{u_t}$ as the market tightness at time t. Hence, the rate at which unemployed workers find a jobs, $f(\theta)$, and vacancies are filled $q(\theta)$ depend of the vacancy-unemployment ratio, where $f(\theta) = \theta q(\theta)$ and $f(\theta)' > 0$, $q(\theta)' < 0$. The unemployment rate increases when jobs are destroyed at a exogenous rate, λ, and decreases when workers find jobs. Thus,employment evolves according

$$n_{t+1} = (1-\lambda)n_t + q(\theta_t)v_t. \tag{6}$$

The labor productivity takes the form of stationary AR(1) in logs:

$$\ln(y_t) = (1-\rho)\ln(\overline{y}) + \rho\ln(y_{t-1}) + \epsilon_t,\ 0 < \rho < 1. \tag{7}$$

Where $\epsilon_t \sim N(0, \sigma^2)$ and ρ measures the persistence.

4.2 Worker's Problem

There is a continuum of identical, risk neutral workers with total measure one and an infinite horizon. These workers can either be employed or unemployed in each period.

An employed worker earns a wage w_t at t, and faces a probability λ of losing his job in the subsequent period. Conversely, an unemployed worker receives unemployment benefits b and has a probability $f(\theta_t)$ of finding a job in the next period. The wage process, w_t, and the tightness of the labor market, θ_t, are given by individual workers. Individual workers have nothing to choose, whether they are employed or not is determined exogenously. The primary calculation where their expectations will play a role is the net surplus of the match that is used to bargain the wage with the firm if the match is realized. This surplus is the difference between the value of being employed and unemployed.

Deriving the standard surplus of the worker hides many assumptions that I wish to bring out in this section. The worker surplus depends on expectations and expectations are determined with a probability measure $\mathcal{P}^w$. The definition of $\mathcal{P}^w$ depends on exactly how much workers are assumed to know about the equilibrium process for n, θ and w and about the properties of these variables. So, I start with a general definition of $\mathcal{P}^w$ that is consistent with the above setup and that encompasses a number of standard equilibrium concepts that are found in the literature. This will be useful, first, to unveil some assumptions in the adaptive learning literature that are often not explicitly stated and it will allow me to extend those equilibrium concepts. Then, I obtain step by step some familiar derivations in the literature and explain how each derivation depends on an increasing amount of assumptions. This provides a clear comparison of the IR equilibrium studied in the paper with RE and with some adaptive learning versions of the model.

4.2.1 A generic worker problem under Internal Rationality

Consider first the case where I do not make any assumption about the relation between workers' beliefs and actual equilibrium. The next subsection will cover the case of RE as well as the case of Bayesian/RE.

If workers are rational, at the very least, the state space for the measure $\mathcal{P}^w$ has to contain the payoff relevant variables for individual workers that are beyond the agent's control, therefore $\mathcal{P}^w$ puts probabilities on sequences $\{(w, \theta, n)^t\}_{t=0}^{\infty}$.[14] $(w, \theta, n)^t$ is the usual notation describing sequences up to t, and it is understood that $E_t^{\mathcal{P}^w}$ responds to the usual definition meaning "conditional expectation given $(w, \theta, n)^t$".

Following, I state the first assumption on beliefs

Assumption 1. The belief system $\mathcal{P}^w$ is Markov up to a state vector m. More precisely,

$$\begin{aligned} Prob^{\mathcal{P}^w}(w_t, \theta_t, n_t \mid (w, \theta, n)^{t-1}) &= \mu(m_{t-1}), \\ m_t &= g(m_{t-1}, y_t, w_t, \theta_t, n_t). \end{aligned} \tag{8}$$

For some given functions μ, g conformable to their arguments and for a vector m_t that contains θ_t, w_t, n_t. In standard IR models, m will also contain variables that in the workers' mind summarize the best forecast of future wages, as is the case in the main sections of this paper.

Now, I can formulate the value functions for the worker.

The present value of working for an agent is as follows:

$$\mathcal{W}(m_t) = w_t + \beta E_t^{\mathcal{P}^w} \left[(1-\lambda)\mathcal{W}(m_{t+1}) + \lambda \mathcal{U}(m_{t+1})\right]. \tag{9}$$

On the other hand, workers can be unemployed. The present value of unemployment is given by:

$$\mathcal{U}(m_t) = b + \beta E_t^{sw}[f(\theta_t)\mathcal{W}(m_{t+1}) + (1 - f(\theta_t))\mathcal{U}(m_{t+1})]. \tag{10}$$

Where $\mathcal{W}$ and $\mathcal{U}$ are time-invariant functions.

It may seem that this is enough to arrive at a standard equation for worker's surplus, $\mathcal{W} - \mathcal{U}$. But since I have not given any market knowledge to agents, they still do not necessarily know the equilibrium process of θ unless I make the following additional assumption.

[14]From the point of view of probability theory I should also state that the probabilities $\mathcal{P}^w$ are defined on the sets of a sigma algebra of the mentioned sequence space, but since, it is obvious how to set this up and it does not have an impact on any application to search models we will not mention sigma algebras anywhere else in the paper.

Assumption 2. Individual workers have a model that forecast correctly the true evolution of θ. Formally, $Prob^{\mathcal{P}^w}(\theta_t = \theta | m_t = m) = Prob^{\mathcal{P}}(\theta_t = \theta | m_t = m)\ \forall(\theta, m)$.

Only under all these assumptions I get the workers' share of the total surplus is:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = w_t - b + \beta\left(1 - \lambda - f(\theta_t)\right) E_t^{\mathcal{P}^w}\left(\mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right). \quad (11)$$

This equation would be satisfied when agents learn about wages, as long as Assumptions 1-2 hold. Learning problem remains hidden in the belief structure $\mathcal{P}^w$. In section 4.4, I provide an explicit system of beliefs $\mathcal{P}^w$.

4.2.2 The individual problem under RE

Assume now that agents are endowed with the knowledge that wages are a function of the productivity, y_t, that is I include in (8) an equation giving w_t as an exact function of y_t. This is summarize in assumption 3.

Assumption 3. The system of equations (8) includes

$$w_t = \mu_w(y_t). \quad (12)$$

In addition, assume that agents know the law of motion of productivity, i.e. they know equation (7). In this paper, I focus on the RE equilibrium that takes the form of the fundamental or minimum state variable solution (MSV).[15] With these additional assumptions then, indeed, we have that $m_t = (y_t)$. In this case, market wages carry only redundant information. This allows to exclude wages from the state space without loss of generality.

Additionally, I have to assume the following.

Assumption 4. Agents' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_w(y_t)$.

Then workers have RE.

4.3 Firms Problem

Consider an economy populated by a mass of infinity firms. Firms' revenues are $y_t n_t$, where n_t and y_t are exogenous to the firms. The productivity, y_t follows a AR(1) process (7). Firms pay a total of $w_t n_t$ at t, the wage process is taken as given by firms. Each period firms choose the number of vacancies v to post at a constant ongoing cost c. Their period-t profits, Π_t, is $y_t n_t - w_t n_t - c v_t$.

[15]While there may be RE equilibria contradicting this assumption, with added lags in wage determination, Campbell (1994) shows that the RE solution has w_t as an ARMA(2,1) process. Adhering to McCallum (1983), I select the minimal state variable set that's indispensable for a solution.

A key feature of equilibria will be the firms' expected discounted profits from period t onwards, given by

$$\Pi_t \equiv E_t^{\mathcal{P}^f}\left(\sum_{j=0}^{\infty}\beta^j[y_{t+j}n_{t+j} - w_{t+j}n_{t+j} - cv_{t+j})]\right), \tag{13}$$

where $\mathcal{P}^f$ is the firm' probability measure about relevant future variables.

To derive the standard job creation condition, it is common in the literature to appeal to dynamic programming to write Π_t in a forward recursive form. In the next subsection, I set out the necessary assumptions to derive this equation. The definition of $\mathcal{P}^f$ depends on exactly how much firms are assumed to know about the equilibrium process for w, y, θ, n, and about the properties of these variables. Therefore, as in the workers problem, I start with a general definition of $\mathcal{P}^f$ and derive step by step some familiar derivations and explain how each derivation depends on a large amount of assumptions.

4.3.1 A generic firm problem under Internal Rationality

This subsection will cover the case of RE as well as the case of Bayesian/RE for the firms' problem. I do not make any assumption about the process for equilibrium variables nor about the relation between firms' beliefs and actual equilibrium.

The state space for the measure $\mathcal{P}^f$ has to contain all payoff-relevant variables for individual firms. Hence, $\mathcal{P}^f$ puts probabilities on sequences $\{(w,\theta,y)^t\}_{t=0}^{\infty}$. $(w,\theta,y)^t$ is the sequences up to t. $E_t^{\mathcal{P}^f}$ in (13) represents the "conditional expectation given $(w,\theta,y)^t$".

Since Π_t is still a function of the whole sequence $(w,\theta,y)^t$, to obtain a recursive formulation, I need to add assumptions 1 of the workers' problem, that set that the belief system is a Markov up to a state vector, together with the transversality condition, $E_t^{\mathcal{P}^f}\beta^j\Pi_{t+j} \to 0$ as $j \to \infty$ almost surely in $\mathcal{P}^f$.

Additionally, to set the problem and derive the job creation condition, I have to add *assumption* 2 from the worker' problem, that firms forecast correctly the labor market tightness, and the following *assumption* 5.[16]

Assumption 5. Individual firms know the law of motion of n.

Taking into account previous assumptions, firms make contingent plans for vacancy posting subject to the evolution of employment (6). Now, I can state the maximization problem of the firms:

$$\Pi(m_t) = \max_{v_t \geq 0} \quad y_t n_t - w_t n_t - cv_t + E_t^{\mathcal{P}^f}\ \Pi(m_{t+1}) \tag{14}$$

[16]In Garcia-Rodriguez and Pinilla-Torremocha (2021), we relax assumption 2 in the DMP model.

subject to

$$n_{t+1} = (1-\lambda)n_t + q(\theta_t)v_t. \tag{15}$$

Below, I will specify the probability measure through some perceived law of motion describing the firm's view about the evolution of (w_t, y_t) over time, together with a prior distribution about the parameters governing this law of motion. Optimal behavior will then entail learning about these parameters, in the sense that agents update their posterior beliefs about the unknown parameters in the line of new wage, and productivity observations. For the moment, this learning problem remains hidden in the belief structure $\mathcal{P}^f$.

Optimality Conditions. The firm's optimal plan is characterized by the first order condition, together with the envelop condition with respect to n_t.

$$E_t^{\mathcal{P}^f} \mathcal{J}_{t+1} = \frac{c}{\beta q(\theta_t)}, \tag{16}$$

$$\mathcal{J}_t = y_t - w_t + \beta(1-\lambda)E_t^{\mathcal{P}^f} \mathcal{J}_{t+1}. \tag{17}$$

where $\mathcal{J}_t = \frac{\partial \Pi(m_t)}{\partial n_t}$ represents the marginal value of having an additional worker employed at the firm. Therefore, equation (17) gives the surplus of the firm coming from a match. Combining (16) and (17) and iterated forward, I come up with the job creation condition

$$\frac{c}{q(\theta_t)} = E_t^{\mathcal{P}^f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j \left[\frac{y_{t+j} - w_{t+j}}{1-\lambda} \right]. \tag{18}$$

This equation would be satisfied when agents learn about wages, as long as all previous assumptions hold. Therefore, in this case I have that the usual job creation condition, but I still need a generic m in (16) and (17).

4.3.2 The individual firm problem under RE

Analogous to section 4.2.2 of the worker's problem, assume now that firms are endowed with some knowledge of how wages are formed, i.e., assumption 4 holds. Also, firms know the law of motion of y and firms' beliefs are correct, that is, in equilibrium $\mathbf{w}_t = \mu_f(y_t)$.

Then, firms have RE.

4.4 Agents' Belief System

Once one departs from rational expectations, beliefs become part of the microfoundations of the model. Previous sections left open how $\mathcal{P}^w$ and $\mathcal{P}^f$ incorporate wage beliefs. In this section, I introduce a fully specified probability measure $\mathcal{P}$ and derive the optimal belief updating equation it implies. For simplicity, I assume that this part of beliefs is common to $\mathcal{P}^w$ and $\mathcal{P}^f$.[17] Nevertheless, agents may not know that this is true prior to wage bargaining. It is important to understand how agents view the wage process to specify an internally consistent rational agent model. The belief system of internally rational agents requires that they do not make obvious mistakes while learning.

Agents have the following perceived law of motion (PLM) which they use to make forecast of wages:

$$\begin{aligned} w_t &= d_t^c + d_t^y y_{t-1} + \epsilon_t, \\ D_t &= D_{t-1} + \nu_t. \end{aligned} \tag{19}$$

Where $D_t = [d_t^c \;\; d_t^y]$. Shocks $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other. This PLM considers a fundamental or minimal state variable solution with unobserved coefficients.

Consider the case where agents' prior beliefs are centered at the REE with the prior variance $\sigma_{D,0}^2$:

$$D_0 \sim \mathcal{N}(D^{REE}, \sigma_{D,0}^2 I_2). \tag{20}$$

Where $\sigma_{D,0}^2$ is set to the steady-state Kalman filter variance. Note that the agents' beliefs (36) encompass the REE of the model. In particular, when agents believe $\sigma_\nu^2 = 0$ and assign probability 1 to $D_0 = D^{REE}$, I have that $D_t = D^{REE}$ for all $t \geq 0$ and wages are given by RE equilibrium wages in all periods. Alternatively, if (20) is combined with a belief that σ_ν^2 is small, even though the resulting dynamics of the economy are not going to be precisely given by REE, it will be close to REE.

Agents' posterior beliefs at any time t are given by

$$D_t \sim \mathcal{N}(\hat{D}_t, \sigma_{D,t}^2 I_2). \tag{21}$$

Given that agents are rational, they update $\hat{D}_t$ according to the recursive least squares (RLS) algorithm:

$$\begin{aligned} \hat{D}_t &= \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[\mathbf{w}_{t-1} - \hat{D}_{t-1}' z_{t-1}] + \epsilon_t^\beta, \\ R_t &= R_{t-1} + \gamma(z_{t-1} z_{t-1}' - R_{t-1}). \end{aligned} \tag{22}$$

Where $\hat{D}_t = [\hat{d}_t^c \;\; \hat{d}_t^y]'$ represent the estimated coefficients, R_t denotes the moment matrix for $z_{t-1} = [1 \;\; y_{t-1}]$ and $\mathbf{w}_t$ denotes the realized previous wage. $\epsilon_t^\beta \sim \mathcal{N}(0_{2,1}, \sigma^{\beta 2} I_2)$ is a shock to wage beliefs and γ denotes the steady state Kalman gain $\in$ (0,1) that determines the rate at which older observations are discounted.[18] Strictly speaking,

[17] In the section 7.3, I build a version that allow workers and firms have a different belief system for wages.

[18] The variable w_t is *not* introduced with a delay in the estimation of $\hat{D}$, is a standard assumption in the learning literature. This approach conveniently avoids the simultaneous determination of forecasts and endogenous variables. As proved by Marcet and Sargent (1989a), this does not alter the asymptotic results obtained in the following as compared to the algorithm allowing for simultaneity.

given the above information structure the Kalman filter requires $\sigma^{\beta 2} = 0$. This shock to beliefs can be interpreted as additional information about ν_t available to agents or as a departure from fully rational belief formation.

These beliefs constitute a small deviation from RE beliefs in the limiting case with vanishing innovation to the random walk process. Agents' prior uncertainty then vanishes, and the optimal gain goes to zero. As a result, one recovers the RE equilibrium value for wages.

4.5 Wage Bargaining

Wages are negotiated according to a Nash bargaining process. Each agent calculates its respective surplus from its problem, taking into account its system of beliefs of wages, before going to the bargaining process. The wage w_t maximizes the joint surplus of a match between workers and firms,

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha}\, \mathcal{J}_t^{1-\alpha} \tag{23}$$

where α represents the bargaining power of the worker. The first order condition of this problem gives the standard sharing rule that characterizes the optimal split of the aggregate surplus,

$$(1-\alpha)(\mathcal{W}(m_t) - \mathcal{U}(m_t)) = \alpha(\mathcal{J}_t). \tag{24}$$

Assuming that agents know that (24) holds in expectations, the equilibrium wage mapping $\mathbf{w}_t$ is given by

$$\mathbf{w}_t = \alpha(y_t + c\theta_t) + (1-\alpha)b. \tag{25}$$

Since agents do not hold rational wage expectations, I need to distinguish between the stochastic process for equilibrium wages $\mathbf{w}_t$ and agents' perceived wage process w_t. The wage equation is the weighted average of the marginal product of employment, the cost of replacing the worker, and the opportunity cost of working, b. Labor market tightness is a function of expectations; therefore, expectations play an important role in determining wages in equilibrium.

4.6 Equilibrium Dynamics under Learning

Under internal rationality, the solution of the model is summarized by (18), (25) and (22). It follows from (36) and (7) that beliefs about wages k periods ahead are given by

$$E_t^{\mathcal{P}}(w_{t+k}) = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{k-1}) + \rho^{k-1} y_t). \tag{26}$$

Inserting equation (26) into equation (18) and, then the resulting one into (25), one can write the actual law of motion (ALM) of wages as follows:

$$\mathbf{w}_t = T_c(\hat{d}_t^c \;\; \hat{d}_t^y) + T_y(\hat{d}_t^y) y_{t-1} + T_\epsilon(\hat{d}_t^y)\epsilon_t, \tag{27}$$

where T_c, T_y and T_ϵ are functions of the estimated coefficients of the PLM.[19] T_c, T_y and T_ϵ represent the coefficients of the the equilibrium wage equation and therefore, implicitly defines the mapping from the PLM to the ALM. The interpretation of the ALM is that describes the stochastic process followed by wages if forecasts are made under the fixed rule given by the PLM. To formulate the T-mapping, $T(\hat{D}) = (T_c, T_y)$, I following the method of Marcet and Sargent (1989b) and Evans and Honkapohja (2012). This function maps the agents' perceptions about wage coefficients $(\hat{D})$ to their realized values $(T(\hat{D}))$. The T-mapping is not know to agents.

The fixed point of this mapping is the REE of the model.

Definition: A rational expectations equilibrium is a matrix $D = [d^c, d^y]$ that satisfies $D = T(D)$. Thus a rational expectations equilibrium is a fixed point of the mapping T. Let me denote such equilibrium by $d^{c,RE}$ and $d^{y,RE}$.

T-mapping determines the evolution of beliefs in the transition to long-run equilibrium. The fact that agents learn about D_t introduces a different dynamic behavior. In particular, if firms believe that wages are going to be high tomorrow, this expectation will be transmitted to the actual realized wage through (27), and wages respond to this belief. This is a key feature of self-referential learning models that are absent in Bayesian learning models. Wage expectations affect realized wages, and agents use wages to update their expectations and so on.

Intuitively, the reason learning matters is the following. The higher the wage expectation, the lower the number of vacancies that firms open up, because their expected profits are lower. This makes the labor market tighter, which in turn reduces the probability of finding a job. When firms and workers negotiate wages, -through the bargaining process- in the presence of lower expected profits and a lower probability of finding a job, wages tend to fall. Figures (5) and (6) shows the $T_y(\hat{d}_t^c \hat{d}_t^{y,RE})$ and $T_y(\hat{d}_t^y)$, respectively, represented by the dashed line, which are linear decreasing functions. Values of the coefficient $\hat{d}_t^c$ and $\hat{d}_t^y$, on the right hand side of the fixed point, which is the intersection between the 45 degree line and the T-mapping, indicate that agents expect wages above their realization and vice versa. The negative slopes of the dashed lines reflects the negative relationship between wage expectations and wages present in the model.

Because the agent's equation of wages can differ from the truth, his beliefs evolve over time. To understand the dynamic behavior of $\hat{D}$, it helps to analyze whether the learning rule induces instability in the state evolution. Using the theorems of Sargent and Williams (2005), if g is small enough, to analyze local stability, I need to check the following condition, known as E-stability condition.[20] Accordingly, the stability of the systems (22) is governed by the following ordinary differential

[19]For exact formula for Φ^c, Φ^y and Φ^ϵ and the derivations see Appendix C.

[20]If g is small enough, the local stability conditions are the same than assuming decreasing gain, $g = \frac{1}{t-1}$.

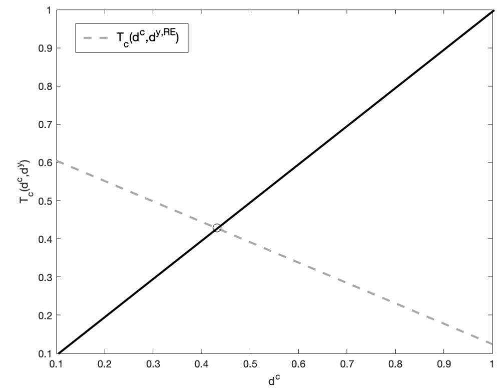

Figure 1: Operator T-mapping for the constant coefficient

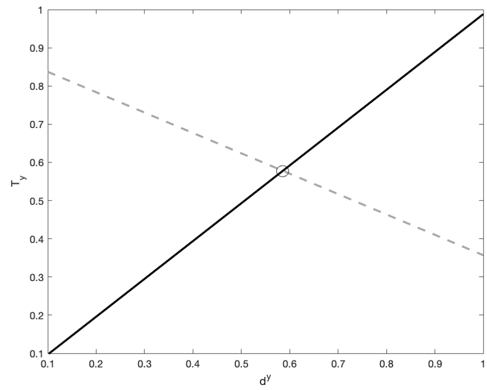

Figure 2: Operator T-mapping for the productivity coefficient

Note: The dashed line represents the operator, T-mapping for each coefficient . The thick black line represents the 45 degree line. The ellipse represents the RE point -which is the fixed point of the T-mapping. In the first subplot, I assume that the coefficient of the productivity is at its RE point. T-mapping is obtained under the calibration of the learning model especified in section 5.1.

equations (o.d.e.):

$$\begin{bmatrix} \dot{\hat{d}}^c \\ \dot{\hat{d}}^y \end{bmatrix} = \begin{bmatrix} T_c(\hat{d}_t^c \ \hat{d}_t^c) - \hat{d}^c \\ T_y(\hat{d}_t^y) - \hat{d}^y \end{bmatrix}. \tag{28}$$

For local stability, I need all eigenvalues of Ω are less than 0 in real part:

$$\Omega = \frac{\partial [T(D) - D]}{\partial D}\bigg|_{D=D^{RE}} < 0. \tag{29}$$

The eigenvalues are real and negative, because the derivative of the T-mapping with respect to D is negative as one can see in figures (5) and (6), so that the condition for local stability of the learning mechanisms is satisfied. Therefore, one may expect constant gain models fluctuate around the REE, and least squared learning would converge.

4.7 Is the structure of Internal Rationality logically inconsistent? An heterogeneous approach

In the context of Internal Rationality (IR), the equilibrium wage function is solely dependent on productivity. This dependency induces a singularity in the objective density across wages and productivity. Given this scenario, a pertinent question arises: would the awareness of this singularity enable internally rational agents to accurately discern the equilibrium wage function through deductive reasoning? In other words, if agents are aware that productivity is the sole source of fundamental disturbances, does internal rationality inherently imply external rationality?

The answer to this question turns out to be 'no'. In this section, I consider some sources of heterogeneity to highlight that an individual agent would not be able to infer the wage function from observations and her own behavior.

I consider a model in which firms are heterogeneous in some parameter values. Consider the previous RBC search and matching model with firms heterogeneous in the cost of opening a vacancy, c^j and their discount factor $\beta^{F,j}$, but they face the same productivity y_t that follows an AR(1) process. The values of the pair $(c^j, \beta^{F,j})$ are drawn from exogenously specified, possibly time-varying distribution. When solving their optimal problem, agents know their own values of $(c^j, \beta^{F,j})$. Therefore, the job creation condition for a firm endowed with $(c^j, \beta^{F,j})$ is as follows:

$$\frac{c^j}{q(\theta_t)} = E_t^{sf} \sum_{z=1}^{\infty} \left[\beta^{F,j}(1-\lambda)\right]^z \left[\frac{y_{t+z} - w_{t+z}}{1-\lambda}\right]. \tag{30}$$

Due to the fact that workers are homogeneous and there is no heterogeneity in productivity, there is no dispersion in wages. The equilibrium can be characterized by a degenerate distribution of wages arising from a bilateral bargaining problem between each firm and the average worker. The wage equation is represented by

$$\mathbf{w}_t = \alpha y_t + (1-\alpha)\left(b + \beta^W m\left(\int_j \frac{v_t^j}{u_t} dj\right) E_t^w \frac{\partial W_{t+1}}{\partial n_{t+1}}\right) \tag{31}$$

Wages are a function of aggregate vacancies. Equivalent, the previous equation can be written as follows

$$\mathbf{w}_t = \int_j \overline{\Phi}^{c,j} dj + \int_j \overline{\Phi}^{y,j} y_{t-1} dj + \int_j \overline{\Phi}^{\epsilon,j} \epsilon_t dj. \tag{32}$$

Assume that firms know that workers know the process of productivity and how to map productivity in their future surplus and the distribution of idiosyncratic parameters across firms. In, this case the firm can perfectly map productivity into the wage.

Instead, firms can know the process of productivity, know that workers form expectations in the right way, and still are not enough to know perfectly how productivity maps to the aggregate level of wages. In addition, I have to assume that firms know the distribution of the vacancy cost and discount factor across firms at each point in time.[21] From this example, I can conclude that it is logically consistent to assume that agents are rational and do not have perfect knowledge of the mapping between productivity onto wages. All I need to assume is that firms do not know the distribution of other firms' vacancy costs and utilities when they have to make the decision of posting vacancies, how the average worker forms its expectations at

[21] In appendix C, you can see the structural form of the parameters Φ. $\overline{\Phi}$ are the parameters Φ evaluated at the RE point.

each point in time, or both.

In fact, section 7.4 extends the model to allow for discrepancies in the way workers and firms form expectations about wages. In that model, I arrive at an equilibrium wage equation that is different from the one obtained in the main paper.

5 Quantitative Analysis

In macroeconomics, search and matching models are essential tools for evaluating a range of labor market policies, both existing and prospective. Therefore, it's crucial for the selected model to accurately reflect observed moments in the data. However, the textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the US economy in response to productivity shocks of plausible magnitude. Additionally, the model demonstrates a lack of propagation, evidenced by an almost 1 contemporaneous correlation between the vacancy-unemployment ratio and productivity, a stark contrast to the near-zero correlation observed in empirical data. The model proposed by Di Pace et al. (2021) does not account for the latter fact.

This section evaluates the quantitative performance of the search and matching model with subjective wage beliefs. I formally estimate and test the model using a mixed strategy calibration that includes the Method of Simulated Moments (MSM). Testing helps me to focus on the ability of the model to explain the specific moments of the data described in Table 6.

5.1 Estimation of the Model

This section describes the calibration/estimation of the model parameters. The parameterization strategy is threefold. The model has 11 parameters: a subset is selected from the literature, another subset is picked from the US data, and the rest is estimated following the Method of Simulated Moments (MSM). [22]

Specifically, the vector $\hat{Z} = [\beta, \lambda, \alpha, \overline{y}, \nu]$ is obtained directly from the literature. I normalize time to one-quarter. Following the literature, I assume that the matching function is Cobb-Douglas. Without loss of generality, the steady state of productivity is normalized to 1. The value of the discount factor β is set to generate an annual real interest rate of approximately 5%. The value of the separation rate is set following Shimer (2005), who suggests a quarterly separation rate of 0.10. Hence, on average, jobs last for approximately 2.5 years.

I set the value of the elasticity of the matching function at 0.5 in line with the literature. This value lies within the plausible interval of [0.5 0.7] as surveyed by Petrongolo and Pissarides (2001). Following Hosios (1990), I set the bargaining power of

[22] This constitutes another difference with respect to the paper of Di Pace et al. (2021), they do not use MSM to estimate some parameters of the model.

Parameter	Description	Value	Source
β	discount factor	0.99	r=0.05
λ	separation rate	0.10	Shimer (2005)
α	bargaining power worker	0.50	Hosios rule: $\alpha = 1 - \nu$
ν	elasticity of matching function	0.50	standard
$\overline{y}$	steady state productivity	1.00	Normalization
σ_ϵ	st. dev. of productivity shocks	0.0058	Data
ρ	persistency of productivity	0.73	Data

Table 4: Calibrated quarterly parameters from literature and data

the worker to 0.5. Using US data, I set the standard deviation and persistence of the productivity process to match the empirical behavior of labor productivity from 1990 to 2020. I find a quarterly autocorrelation and standard deviation of 0.7518 and 0.0058, respectively.

Defining $Z = [c, A, g, \sigma^\beta, b]$ as the vector of parameters to be estimated using an extension of the Simulated Method of Moments. These parameters are estimated to match the first 11 moments reported in table 6, are standardly used in the search and matching literature to summarize the main features of the labor market.[23] The MSM estimator is given by

$$\min_{Z} \quad (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z))' \hat{\Sigma}_{\mathcal{S}}^{-1} (\hat{\mathcal{S}} - \tilde{\mathcal{S}}(Z)) \ . \tag{33}$$

where $\tilde{\mathcal{S}}(Z)$ is the vector of empirical moments to be matched, $\hat{\mathcal{S}}$ is the model moments counterpart and $\hat{\Sigma}_{\mathcal{S}}$ is the weighting matrix, which determines the relative importance of each statistic deviation from its target. I use a diagonal weighting matrix whose diagonal is composed of the inverse of the estimated variances of the data statistics.[24] Model-implied statistics are generated through a Montecarlo experiment with 1000 realizations. I formally test the hypothesis that any individual model statistics differ from its empirical counterpart.

The calibrated gain is inside the values found in the literature, which range from 0.002-0.05. Additionally, when compared against wage forecasts from the European Commission, the estimated gain is 0.086.[25] Meanwhile, the standard deviation of the belief shock, introduced in a variant of the learning model, is notably conservative. It is markedly smaller than the empirical standard deviation, which I estimated using

[23] I include functions of moments, instead of pure moments. I target 13 functions of moments. See appendix D for more details.

[24] In practice the estimated variances of the data moments, $\hat{\mathcal{S}}$ is used. The variances are obtained using a Newey-West estimator and the delta method as in Adam et al. (2016).

[25] Learning in the model is about wage level, therefore I have transformed the annual wage growth forecasts into de-trended levels to estimate the gain, ensuring the forecast generated by the European Commission remains parallel to forecasts implied by the model's learning mechanism. I estimate the gain parameter using a nonlinear least squares to minimize the distance between expectations implied by a constant gain algorithm and the survey expectations.

the survey of the European Commission, standing at 0.01. These values estimated by survey data can be interpreted as upper bounds.

Parameter	Description	Values Learning	Values RE
c	cost of open a vacancy	0.45	1.30
A	efficiency matching technology	0.97	1.10
g	constant gain	0.009	0.00
σ^{β}	Std. wage belief shocks	0.0009	0.00
b	unemployment benefits	0.75	0.80

Table 5: Estimated quarterly parameters from SMM

5.2 Statistical Properties

In this section, the estimation results are reported. Table 6 contains statistics from the US labor market data and those implied by the model under rational expectations and learning dynamics.[26] The sample length of one simulation is T=120 quarters. I simulate the model 10,000 times and report the mean values of the statistics of interest as deviations from the steady state, facilitating comparison to earlier studies.[27] The statistics considered are the relative standard deviation of each labor market variable with respect to the standard deviation of labor productivity, the correlation between each labor variable and labor market tightness, the latter's autocorrelation and wages, and the Beveridge curve represented by the correlation between unemployment and vacancies. Furthermore, the last two rows of Table 6 contrast the non-targeting coefficients from regressions (5) and (6) in Section 3, I employed to test Rational Expectations using forecast data sourced from Professional Forecasters at the European Commission, with those obtained by running the same regressions within the learning and rational expectations models. The second column in Table 6 reports the labor market moments from the data. The third and fourth columns present the moments and t-statistics of the learning model, respectively, while the fifth and sixth columns provide those of the RE model.

The simplest version of the DMP model with learning performs remarkably well quantitatively. The model statistics pass almost all the t-tests. It can generate a low contemporaneous correlation between labor market tightness and productivity, together with the high relative volatilities in the labor market, solving the two puzzles, the propagation and the amplification puzzle. This is achieved without generating rigid wages. This represents a significant success, being problematic for the standard real business cycle model. Job creation is driven by the difference between the expected productivity and the expected cost of labor in new matches. In my

[26] The sources for the data can be found in Appendix E.

[27] The initial values of the employment, n_t, unemployment, u_t, productivity, y_t and wages w_t needed to initialize the algorithm are set to the steady state values. The initial value R is given by $R_0 = T^{-1} z_T' z_T$ where T is 155 quarters that represent a pre-sample period before 1990-Q1. The initial D_0 are set to the RE values, that is $d^{c,RE} = 0.4359$ $d^{y,RE} = 0.5869$ under the learning calibration.

Moment's Symbol	Data	Learning Model	t-stat	RE model Re-est	t-stat
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	8.591	1.622	0.767	5.400
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	16.162	-1.587	1.773	6.176
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	22.105	0.664	2.426	5.673
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.972	-1.022	0.741	4.328
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.145	-0.429	0.991	-2.400
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.948	3.956	0.981	0.261
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.968	-1.102	-0.894	-7.969
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.949	-0.480	0.991	-0.600
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.786	2.348	0.618	4.454
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.785	1.302	0.703	3.840
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.844	-4.134	-0.791	-6.771
b^E	0.15	0.39	-2.14	0.62	-4.20
b^R	0.75	0.85	0.35	0.74	0.035

Table 6: Labor Market Statistics

Note: Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. b^R *is the coefficientes of regression (5) and* b^E *is the coefficientes of regression (6) running in Section 3. Survey data: European Commision from 1990-2020. t-ratios are defined as (data moment-model moment)/(estimated standard deviation of the model moment).*

model, the learning mechanism makes wage expectations less responsive to changes in productivity, and this generates the amplification. Additionally, the equilibrium wage is now a function not only of productivity but also of expectations. This additional dynamics generated by learning about D as described in section 4.6, lead to wages fluctuating more extensively than productivity.

Furthermore, the model gives an explanation for the fact that the labor market tightness is not strongly correlated with productivity. This phenomenon occurs because the model introduces a novel source of fluctuations stemming from wage learning. The labor market tightness is not only a function of productivity but also of the estimated coefficients of wage expectations. Additionally, the model also aligns more closely with the coefficient b^E of regression (6), consistent with the values found in survey data from professional forecasters. This is achieved, even though this coefficient was not directly targeted. The model isn't without its limitations. Large t-ratios of certain moments highlight areas of improvement, though it's important to consider the model's simplicity compared to others within the DMP literature.

I posit a scenario where agents are learning about two coefficients influencing wages and introduce a belief shock. There might be speculations on the specific coefficient driving these results or debates on whether the outcomes are attributed to learning or merely the shock in expectations. In section 7 dedicated to robustness, I demonstrate that the predominant factor is indeed the learning about the coefficient that goes with productivity. This re-calibrated model with just learning about d_t^y, achieves a relative standard deviation of the vacancy-unemployment ratio and unemployment at 19.9 and 5.22, respectively, and reduces the correlation between the vacancy-unemployment ratio and productivity to 0.39.

The key to understanding where the volatility in the learning model of d_t^y comes from, lies in the job creation equation (18). This equation is a function of the discounted presented value of profits, the difference of the infinite sums of expected revenues (Θ_y) and expected labor costs (Θ_w). That different can be written as:

$$\Theta_y - \Theta_w = \underbrace{C}_{R1} + \underbrace{\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho} y_t}_{R2} + \underbrace{\left(\frac{1}{1-\beta(1-\lambda)\rho} - \frac{1}{1-\beta(1-\lambda)}\right)\hat{d}_t^y}_{R3}, \tag{34}$$

where $C = \frac{1-d^{c,RE}}{1-\beta(1-\lambda)} - \frac{\rho}{1-\beta(1-\lambda)\rho}$ is a constant.[28] Under RE the volatility in the discounted presented value of profits just come from R2. R3 is constant, so the volatility of that term is zero under RE, $var(\Theta_y - \Theta_w)^{RE} = (\frac{\rho - d^{y,RE}}{1-\beta(1-\lambda)\rho})^2 var(y_t)$. The solutions that tries to solve the volatility puzzle keeping RE, try to make the coefficient $d^{y,RE}$ smaller to increase $\frac{\rho-\hat{d}_t^y}{1-\beta(1-\lambda)\rho} y_t$. My mechanism does not operate in that way. With a small standard deviation of $\hat{d}_t^y$, R3 can add a significant volatility to the discounted presented value of profits, due to the large value of the difference that multiplies $\hat{d}_t^y$. Moreover, the small deviations of RE coming from the learning of

[28]See Appendix B for the details.

$\hat{d}_t^y$ not just make R3 volatile, but also generate a higher volatility in R2. For example, under the proposed calibration R2 is 3 times more volatile than in the RE model.

The Effects of a Productivity Shock. To develop more the intuition on the role

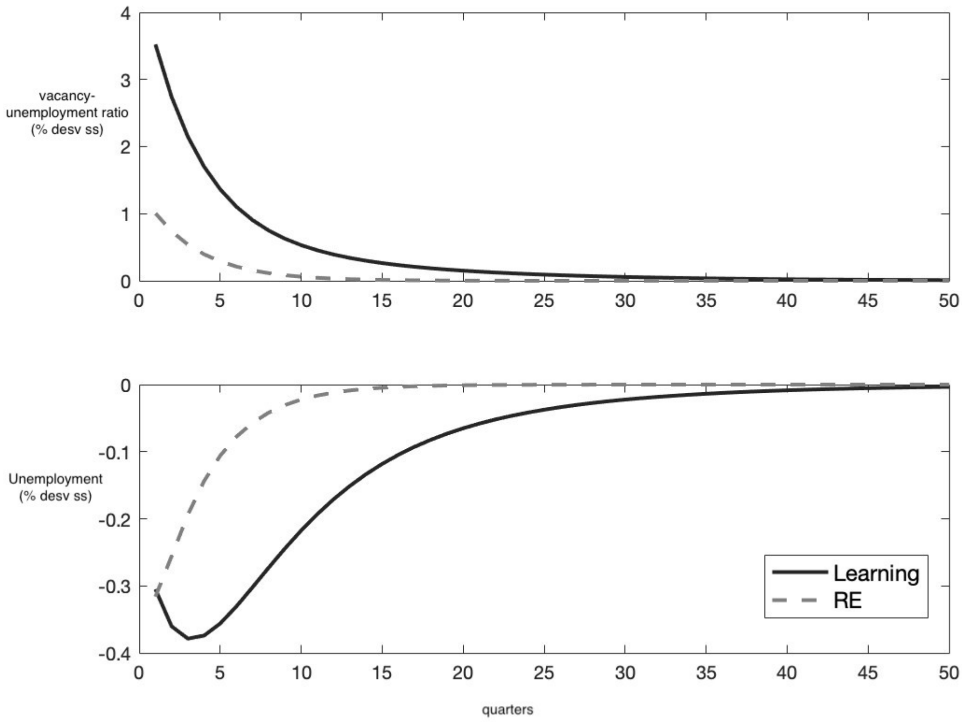

Figure 3: Impulses Responses to a positive Productivity Shock

Note: Impulse response functions of labor market tightness and unemployment following a one standard deviation positive productivity shock. The dashed line represents the learning model (calibrated under the RE model), while the solid line represents the RE model. The horizontal axis displays the number of quarters after the shock.

of wage expectations in labor market fluctuations, consider the model's impulse response functions of the labor market tightness, unemployment and the estimated coefficient of wages expectations, d_t^y to a positive standard deviation productivity shock. The impulse response functions of the labor market tightness and unemployment are expressed in percentage deviations from steady state.

Figure 3 reports the median impulse response functions of the labor market tightness and unemployment. It illustrates a pronounced impact of a productivity shock on the labor market under a learning framework compared to Rational Expectations (RE). Figure 4 shows the dynamics following for one of estimated coefficients that determined wage expectations, d_t^y after a positive productivity shock. Under learning, the productivity shock leads to revisions in wage beliefs that commence the period after the disturbance. Subsequent dynamics are largely driven by revisions to wage beliefs. Particularly, after the shock, firms think that the wages will be lower compared

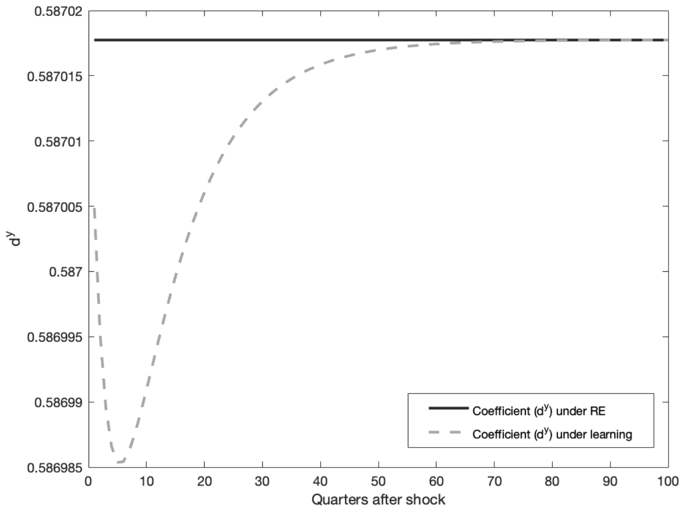

Figure 4: Impulses Responses to a positive Productivity Shock

Note: Impulse response functions of coefficient $\hat{d}_t^y$ *following a one standard deviation positive productivity shock. The dashed line represents the estimated time-varying coefficient under the learning model (using the RE model calibration), and the solid line represents the constant coefficient* $\hat{d}^{y,RE}$ *under the RE model. The horizontal axis displays the number of quarters following the shock.*

to the RE economy. For a number of periods, the coefficient undergoes a downward adjustment because the forecasting error is negative, i.e., $w_{t-1} < d^{c,RE} + \hat{d}_t^y y_{t-1}$, until some given period. However, after a certain point, economic recovery ensues. Firms start the process of upwardly adjusting their expectations, $w_{t-1} > d^{c,RE} + \hat{d}_t^y y_{t-1}$ until they align with the RE benchmark. During this adjustment phase, firms' expectations temporarily deviate from the rational expectations framework as they adapt to integrating productivity changes into wage-setting. This adaptive period, where firms anticipate comparatively lower wages than under RE, prompts them to post more vacancies, leading to a decrease in unemployment. Additionally, it takes time to converge to the steady state, so the positive effect of the productivity shock in the labor market is more persistent.

6 Labor Market Policies and Welfare

Introduce learning as in surveys has more reliable and robust macroeconomic implications. However, arguably, an even more important value of the new models lies in their usefulness for analyzing policy. Given that the learning model reproduces the dynamics of the U.S. labor market data remarkably well, the next step is to analyze the effects of some labor market policies using such model. The goal is to provide policymakers with a more accurate understanding of the potential costs and benefits of these policies and highlight the importance of considering the impact of expectations on labor market outcomes. Learning could amplify the effects of a given policy. If policy makers do not take this effect into account, they may obtain biased estimates and perhaps incur a large cost to the economy after implementation. In the following section, I evaluate the differences in welfare and the standard deviation of unemployment when policymakers use the RE model versus the learning model to assess certain labor market policies. The standard deviation of unemployment can be considered a measure of uncertainty in the economy.

To quantify the welfare effects, I use the compensating variation method. This method calculates the number of consumption units that I should give to the representative individual of the economy, uniformly period after period, so that he or she would be indifferent between the economy subject to the base policy and the economy subject to the reform. The welfare measure in these comparisons, λ, is defined from

$$E_0[\sum_{t=0}^{\infty}\beta^t(1+\lambda)c_t] = E_0[\sum_{t=0}^{\infty}\beta^t c_t^R], \tag{35}$$

where c_t is the aggregate consumption under the benchmark case and c_t^R is the aggregate consumption under a particular experiment. If $\lambda > 0$ there is a welfare gain; otherwise, there is a welfare loss.

6.1 Asymmetric countercyclical UI Policy

Although unemployment insurance (UI) in principle remains constant regardless of labor market conditions, the United States adjusts its generosity during economic downturns. For instance, during the 2007-2011 labor market downturn, the weekly benefit amount increased by \$25. More recently, during the pandemic, interventions such as the FPUC, which offered a weekly supplement in addition to full social security benefits, were implemented. Papers that try to look at the impact of these policies on the economy, for instance Schwartz (2013), they assume that agents have RE. It is important to analyze whether the fact that agents learn about wages amplifies the effects of such a policy.

I consider two rule-based systems that link the level of UI benefits to either GDP, denote by z, or unemployment, and vary the elasticity of the response to changes in these variables. The gdp-based rule is the following:

$$b_t = b - \phi\tilde{z}_{t-1}1_{\tilde{z}_{t-1}<0},$$

where b is the calibrated benchmark UI of each respective model, and ϕ represents the elasticity of UI with respect to gdp, that is, the percentage increase in UI for each percent drop in gdp with respect to their steady-state value, $\tilde{y}$. The UI is financed using taxes proportional to wages. Alternatively, the rule can be linked to unemployment rather than gdp as follows:

$$b_t = b + \phi\tilde{u}_t 1_{\tilde{u}_t<0}.$$

Notice that now, in the IR economy with the introduction of the time-varying UI, if agents internalize such policy in their expectations, the PLM of wages becomes:

$$\begin{aligned} w_t = d_t^c + d_t^y y_{t-1} + d_t^b b_{t-1} + \epsilon_t, \\ D_t = D_{t-1} + \nu_t. \end{aligned} \tag{36}$$

Where $D_t = [d_t^c \;\; d_t^y \;\; d_t^b]$. Shocks $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{3,1}, \sigma_\nu^2 I_3)$ are independent of each other. Such PLM incorporates additional learning on how unemployment benefits are mapped to wages. The RE economy in this context is the fixed point of the T-mapping implied by the above PLM.

Tables 7 and 8 showcase how λ values and unemployment standard deviation change when contrasting two policy rules against an economy where UI remains stable throughout business cycles. In both cases, the models predict decreased welfare and increased unemployment volatility. These results arise from the more generous UI benefits, which lead to elevated wages and reduced anticipated profits, thereby diminishing firms' motivation to create new job openings. It is noteworthy, however, that the outcomes diverge based on the chosen model and rule.

	Learning		RE	
ϕ	$\lambda(\%)$	std$(\tilde{u}_t)$	$\lambda(\%)$	std$(\tilde{u}_t)$
0	0	0.083	0	0.007
0.5	-0.1	0.09	-0.47	0.017
1	-0.44	0.117	-0.83	0.026
1.25	-2.78	0.20	-1.00	0.032

Table 7: GDP-based rule

Note: Values of λ, compared with the benchmark economy where UI is constant over the business cycle and the standard deviation of unemployment in an economy that undergoes UI reform, utilizing the GDP-based rule. The calibration used for each model is in table 4 and 5.

The Learning model appears to be more sensitive to changes in ϕ than the RE model, especially when $\phi > 1$. This implies that economies with agents that learn about wages may experience more pronounced welfare reductions when UI benefits become more reactive to changes in GDP. In specific terms, with ϕ at 1.25, the welfare reduction in the Learning model hits -2.78%, whereas the RE model shows a milder reduction of -1.00%. This stark contrast underlines the learning model's heightened sensitivity and indicates a potential policy consideration.

Additionally, the standard deviation of unemployment escalates with ϕ in both models, suggesting that tying UI to GDP fluctuations introduces more uncertainty into the economy. Again, the learning model's reactions are considerably more accentuated, drawing attention to the intensified outcomes when agents are learning wage expectations.

	Learning		RE	
ϕ	$\lambda(\%)$	$\text{std}(\tilde{u}_t)$	$\lambda(\%)$	$\text{std}(\tilde{u}_t)$
0	0	0.083	0	0.007
0.03	-1.31	0.09	-0.04	0.0079
0.05	-2.27	0.10	-0.1	0.0085
0.08	-3.80	0.14	-0.54	0.014

Table 8: Unemployment-based rule

Note: Values of λ, compared with the benchmark economy where UI is constant over the business cycle and the standard deviation of unemployment in an economy that undergoes UI reform, utilizing the Unemployment-based rule. The calibration used for each model is in table 4 and 5.

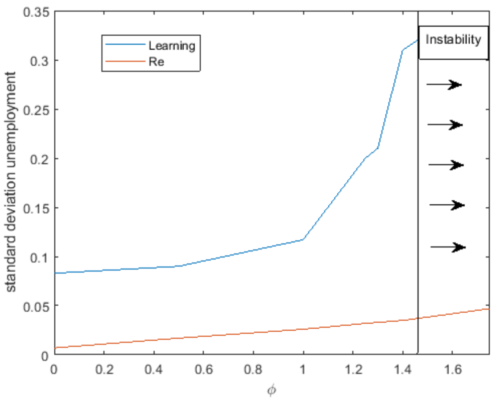

Figure 5: GDP-based rule

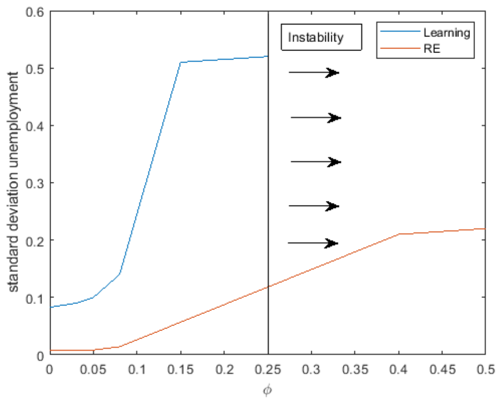

Figure 6: Unemployment-based rule

Note: The figure present the stability and instability (regions with arrows) for different values of ϕ. Y axis show the standard deviation of the unemployment associated at each value of ϕ. The stability of the system is given by (29).

When UI benefits are tethered to unemployment rates, the potential detriment to overall welfare is evident, and becomes concerning if the linkage is too responsive. To illustrate, the Learning model exhibits a welfare reduction from -1.31% at $\phi = 0.03$ to -3.80% at $\phi = 0.08$. In comparison, the RE model remains relatively stable against changes in ϕ. Unemployment's standard deviation generally amplifies with rising ϕ. The pronounced volatility observed in the learning model underscores the importance of acknowledging expectation formation methods among agents. This variability between models serves as an essential insight for policymakers: comprehending real-world expectation formulation is pivotal in understanding policy repercussions.

When expectations enter explicitly in model, certain policy rules may be associated with instability of the REE. Making UI responds to GDP and unemployment might on the other hand introduce additional volatility in the economy which might destabilize the system. Policimakers should only advocate policy rules which induce E-stable REE. Figure X shows the values of ϕ under the two rules that induce to instability. While small reactions to GDP might not always lead to instability, policymakers should exercise caution when tying UI to unemployment, given its inherent volatility.

6.2 Symmetric procyclical UI Policy

In an economy where agents learn, the existing US unemployment insurance system results in notable welfare costs due to its impact on job creation.[29] Can pro-cyclical unemployment benefits smooth cyclical fluctuations in unemployment and deliver substantial welfare gains? The current section seeks to answer that.

Unemployment fluctuations are driven by expectations. Policymakers might adjust unemployment benefits in response to economic indicators like unemployment or GDP to influence firms' expectations.

I explore two policy rules: a linear response to lagged GDP (rule I in table 9) and a linear response to unemployment (rule II in table 9). To grasp their implications, I study three scenarios: (I) an economy with rational expectations, (II) one where agents learn about wages and internalize the UI policy, and (III) where they learn about wages but disregard the UI policy

I. GDP linear rule	$\mathrm{b}_t = b + \phi \tilde{y}_{t-1}$
II. Unemployment linear rule	$\mathrm{b}_t = b - \phi \tilde{u}_t$

Table 9: Unemployment Benefit Policy rules

Note: $\tilde{y}_{t-1}$ and $\tilde{u}_t$ represent deviations from the steady state. b is the unemployment benefits estimated from section 4.1.

The prior analysis assumed agents internalize the UI policy's response to unemployment or GDP during recessions. Thus, when forecasting wages, they'd factor in government adjustments to UI levels. However, it's plausible agents might overlook the government's business cycle reactions due to unclear policy communication or skepticism about government commitment. In this section, additionally, I explore the impact of such an information gap, assuming agents disregard this rule in their PLM, even as the government adjusts based on GDP or unemployment deviations.[30]

[29] Note that in this simplified model, workers are risk neutral, so I do not take into account the positive effects of unemployment benefit as insurance for workers against the risk of unemployment. The optimal policy of this model without risk adverse workers is zero IU.

[30] In this case, the agents' PLM do not incorporates unemployment benefits, as in the benchmark model: $w_t = d_t^{f,c} + d_t^{f,y} y_{t-1} + \epsilon_t^f$.

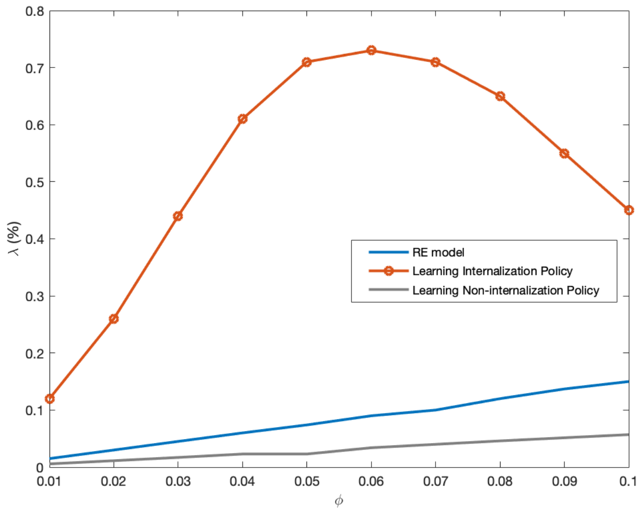

Figure 7: Welfare Implications of UI GDP linear rule

Note: The figure shows the welfare gains λ as defined in equation (31) for different combinations of coefficients for GDP using rule I defined in table (9). Welfare gains were computed as averages over 1000 simulations, each including 120 time periods.

Figure 7 depicts the welfare benefits of UI rules based on GDP from table 9 for varying GDP response coefficients. When agents internalize the policy during wage learning, substantial welfare improvements result. The policy impacts both wages and wage expectations. In expansions, subsidies decrease to boost job creation, while in recessions, the UI rises. Yet without such internalization, welfare gains shrink, mirroring gains under rational expectations. The welfare gain's relationship to GDP-rule has an inverted U-shape, indicating excessive GDP reactions can diminish welfare as UI rises in expansions.

Figure 8 displays welfare benefits from UI rules based on unemployment, as detailed in Table 9, across various unemployment response coefficients. While linear and symmetrical UI responses to unemployment could potentially add volatility, figure 8 indicates stability for smaller reactions. Within this stability zone, left of the dotted line, findings align with those of the GDP-based rule. When agents internalize and learn this policy, the potential welfare gains substantially outpace those under rational expectations or non-internalization scenarios.

7 Robustness

In this section, I examine the performance of the learning mechanism under alternative assumptions and extensions. All tables I refer to in Section F.1 in the Appendix.

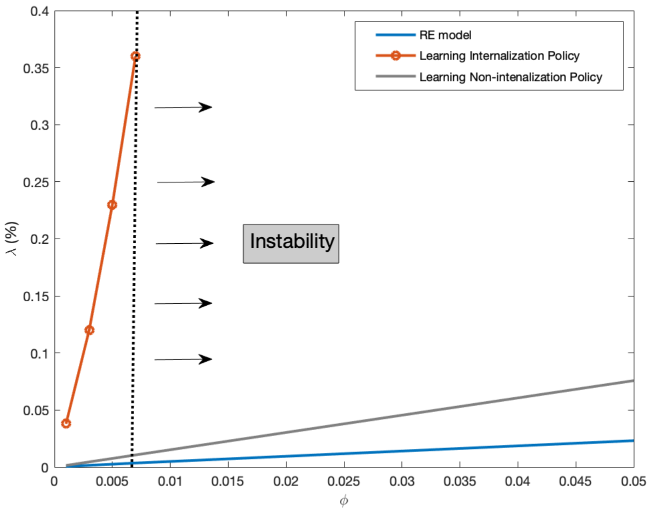

Figure 8: Welfare Implications of UI Unemployment linear rule

Note: The figure shows the welfare gains λ as defined in equation (31) for different combinations of coefficients for unemployment, using rule II defined in table (9). Welfare gains have been computed as averages over 1000 simulations, each one including 120 periods.

7.1 Asymmetric Perceived Law of Motions

In this section, I extend the standard search and matching frictions to account for differences in how both workers and firms anticipate future wages. The survey of Consumer Expectations indicates that workers on average do not internalize the effect of aggregate labor productivity on the formation of such beliefs.

To incorporate this fact, I have adjusted the system of beliefs of the worker. Particularly, I have introduced a new Assumption 2, which simplifies the model.[31]

Assumption 2. Individual workers perceived that the labor market tightness θ is constant over time, therefore $E_t^{\mathcal{P}^w}(\theta_{t+k}) = \overline{\theta}$.

Consequently, based on this assumption, workers consistently perceive the job-finding probability as a constant value, $\overline{f}$ over time.[32] When these assumptions are considered, the resultant workers' share of total surplus can be expressed as follows:

$$\mathcal{W}(m_t) - \mathcal{U}(m_t) = \sum_{j=1}^{\infty} \beta^{j-1}(1 - \lambda - \overline{f})^{j-1} E_t^{sw}[w_{t+j-1} - b]. \tag{37}$$

[31] This assumption is made to find a closed form solution if equilibrium wages when firms and workers form wage expectations differently.

[32] This assumption is in line with the empirical fact documented by Balleer et al. (2021). Using the Survey of Consumer Expectations, they find that workers do not update their expected labor market transition probabilities. Therefore, they find no empirical evidence of learning about labor market transition probabilities over the life cycle.

Firms adopt a perceived law of motion for wages that aligns with the minimal state variable, the one used in the main paper.

$$\begin{aligned} w_t &= d_t^{f,c} + d_t^{f,y} y_{t-1} + \epsilon_t^f, \\ D_t^f &= D_{t-1}^f + \nu_t. \end{aligned} \tag{38}$$

Where $D_t^f = [d_t^f \;\; d_t^y]$. Shocks $\epsilon_t^f \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $\nu_t \sim \mathcal{N}(0_{2,1}, \sigma_\nu^2 I_2)$ are independent of each other.

In contrast, workers do not use productivity to form wage expectations in line with the documented fact. Therefore, I assume workers believe that wages follow an unobserved component model of the following form:

$$\begin{aligned} w_t &= d_t^{w,c} + \epsilon_t^w, \\ d_t^{w,c} &= d_{t-1}^{w,c} + u_t. \end{aligned} \tag{39}$$

Shocks $\epsilon_t^w \sim \mathcal{N}(0, \sigma_\epsilon^2)$ and $u_t \sim \mathcal{N}(0, \sigma_u^2)$ are independent of each other. The previous setup defines a filtering problem in which agents need to decompose observed wages into its persistent and transitory elements.

The estimation takes place using the recursive least squares (RLS) algorithm. Agents estimate equations (39) and (38) and update their coefficient estimates for every period as new data become available. Workers' and firms' beliefs evolve according to the following schemes, respectively:

$$\hat{d}_t^{w,c} = \hat{d}_{t-1}^{w,c} + \gamma[\mathbf{w}_{t-1} - \hat{d}_{t-1}^{w,c}] + \epsilon_t^\beta \tag{40}$$

$$\begin{aligned} \hat{D}_t^f &= \hat{D}_{t-1}^f + \gamma R_t^{-1} z_{t-1}[\mathbf{w}_{t-1} - \hat{D}_{t-1}^{f'} z_{t-1}] + \epsilon_t^\beta \\ R_t &= R_{t-1} + \gamma(z_{t-1} z_{t-1}^{'} - R_{t-1}) \end{aligned} \tag{41}$$

Where $D_t^f = [d_t^{f,c} \;\; d_t^{f,y}]'$ and $z_{t-1} = [1 \;\; y_{t-1}]$. ϵ_t^β is a shock to wage beliefs (sentiment shock), $\mathbf{w}_{t-1}$ denotes the realized previous wage, and γ denotes the constant gain $\in$ (0,1).

The actual law of motion of wages stemming from prior assumptions and the bargaining process is the following:

$$\mathbf{w}_t = [T_c(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c}) \;\; T_y(\hat{d}_t^{f,y})][1 \;\; y_{t-1}]' + C_\epsilon \epsilon_t, \tag{42}$$

where $T_c(\hat{d}_t^{f,c} \;\; \hat{d}_t^{w,c})$ and $T_y(\hat{d}_t^{f,y})$ represents the T-mapping. I follow the method of Marcet and Sargent (1989a) and Evans and Honkapohja (2012) to formulate the function T-mapping that maps the agents' expectations - D=$[\hat{d}_t^{f,c} \;\; \hat{d}_t^{f,y} \;\; \hat{d}_t^{w,c}]$' - to their realized values. The T-mapping obtained in this section is different from that described in the main article.[33]

[33] For T-mapping details, see Appendix D.

When the forecast model of some agents is misspecified, the natural limit of adaptive learning dynamics is called restricted perception rational expectation equilibrium (RP-REE). To this end, I apply the theory of Marcet and Sargent (1989a). In this version of the model, the worker makes decisions using a misspecified model to form wage expectations. In other words, the worker does not use one relevant state variable for forecasting.

Formally, there exists an n x 1 state vector z_t. Let z_{it} be any n_i x 1 vector $z_{it} = e_i z_t$, where $1 \geq n_i \geq n$ and e_i are the selector matrices for $i = w, f$. There are two types of agents, firms and workers, types f and w, which use $z_{ft} = z_t$ and $z_{wt} = e_w z_t$, respectively. In my environment, the state and the noise of the model at t are specified as

$$z_t = \begin{bmatrix} 1 \\ y_t \end{bmatrix}, \epsilon_t. \tag{43}$$

Firm behaves competitively, it forecasts w_t using $z_{ft} = [1 \;\; y_t]'$. On the other hand, worker behaves competitive as well. However, to forecast w_t, he uses a subset of z_t such that $z_{wt} = 1$. Under the described settings, the operator that determines the REE of my model is related to, but distinct from the described T-mapping. The restricted perception of one of the agents alters the relevant operator.

If the ALM of w_t is (42), then the linear least-squares projection of w_t on z_{t-1} for each agent is given by

$$E(w_{it}|z_{it-1}) = S_i(D) z_{it-1}, \tag{44}$$

where

$$S_i(D) = T(D)[M_{z_i}(D)^{-1} M_{z_i.z}(D)]', \;\; for \;\; i = f, w. \tag{45}$$

Where $M_{z_i}(D) = E z_{it} z_{it}'$ and $M_{z_i.z}(D) = E z_{it} z_t'$, $i = f, w$. Notice that for the firm, $S_f(D) = T(D)$. The operator $S_i(D)$ maps the perceptions D=[$\hat{d}_t^{f,c} \;\; \hat{d}_t^{f,y} \;\; \hat{d}_t^{w,c}$]'coefficients $(T(D), \; S_w(D))$. The S-mapping determines the evolution of beliefs in transition to the Restricted Perception long-run equilibrium (RP-REE).

I now advance the following definition.

Definition: A Restricted Perception Rational Expectation Equilibrium is a vector D=[$\hat{d}_t^{f,c} \;\; \hat{d}_t^{f,y} \;\; \hat{d}_t^{w,c}$]' that satisfies D=S(D).

Thus the rational expectations equilibrium or the long-run equilibrium of this economy is a fixed point of the mapping S.[34] Let me denote such equilibrium D^{RPREE}. Notice that this concept of a rational expectations equilibrium is relative to the fixed information sets z_{wt} and z_{ft} specified by the model builder.

[34]For exact formula for S, and the derivations see Appendix D.

Agent's equation of wages can differ from the truth, his beliefs evolve over time. In this case, the stability of systems (40) and (41) is governed by the following ordinary differential equations (o.d.e.):

$$\begin{bmatrix} \dot{\hat{d}}^{f,c} \\ \dot{\hat{d}}^{f,y} \\ \dot{\hat{d}}^{w,c} \end{bmatrix} = \begin{bmatrix} T_c(\hat{d}_t^{f,c} \ \hat{d}_t^{w,c}) - \hat{d}^{f,c} \\ T_y(\hat{d}_t^{f,y}) - \hat{d}^{f,y} \\ S_w(\hat{d}_t^{f,c} \ \hat{d}_t^{w,c} \ \hat{d}_t^{f,y}) - \hat{d}^{w,c} \end{bmatrix}. \tag{46}$$

Figure 9 describes the phase diagram of this economy. The intersection between the 3 planes is the RPREE.[35]

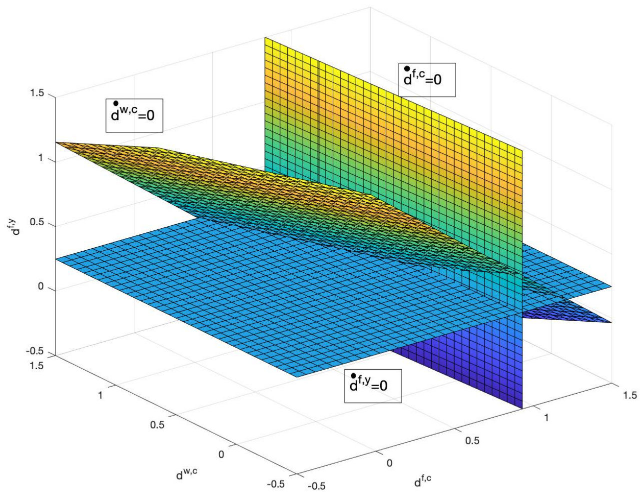

Figure 9: Phase Diagram

Table 13 shows the statistics coming from the simulation of the models. The calibration of the model is summarized in Tables (14) and (15). The learning model with a lower gain than the learning model in the main paper, is able to match very well the moments in the data. It is able to generate fluctuations in the labor market, account for the low correlation between the vacancy-unemployment ratio and productivity, together with flexible wages. Using t-statistics derived from asymptotic theory, I cannot reject the hypothesis that any of the individual model moments differ from the moments in the data in the estimated earning with asymmetric PLMs. Therefore, it performs slightly better than the learning model present in the main paper. Each time a productivity shock hits the economy, the worker becomes pessimistic

[35]For local stability, I need all eigenvalues of Ω are less than 0 in real part:

$$\Omega = \left. \frac{\partial[S(D) - D]}{\partial D} \right|_{D=D_f} < 0. \tag{47}$$

(optimistic), which affects the realized wage and generates a mistake in the enterprise's wage forecasts. This mechanism endogenously causes agents to deviate from rational expectations and propagate productivity shocks.

Additionally, to explores whether the model has the potential to quantitatively address the fact that I found in the survey of consumers. In each iteration, I carry out a similar test to the one carried out in the empirical part using the theoretical wage expectations of workers.[36] The productivity coefficient is not statistically significant on wage expectations 88% of the time at 5% significant level. Moreover, I reject the null hypothesis that coefficients are equal across regressions of the realize wages and wage' expectations 98% of the time.

7.2 Alternative Calibration

I assess the robustness of the expectation channel proposed in the paper to introducing changes in the calibration of some parameter values. Results are collected in the Table 11.

First, I examine the performance of the baseline model under RE with the solution proposed by Hagedorn and Manovskii (2008) in order to compare both solutions under the same framework. According to them, the standard DMP model is unable to match the data because of an erroneous parametrization of two parameters: the instantaneous utility of being unemployment and workers' bargaining power. With a higher calibrated value for unemployment benefits closed to the steady-state of wages ($b = 0.955$), and a low bargaining power of the worker, close to zero ($\alpha = 0.05$), the model generates endogenous wage rigidities.[37] This can be seen in the significant drop in the RE value of the coefficient that goes with productivity in the linear wage equation, $d^{y,RE}$. Under the learning model 1, that coefficient moves around 0.58, while under the calibration of Hagedorn and Manovskii (2008) it is reduced to 0.11. The rigid wages increase second moment of the vacancy-unemployment ratio. However, the RE model, under Hagedorn and Manovskii (2008) calibration, fails in generating a relative standard deviation between wages and productivity higher than 1. Also, is no able to decrease significantly the correlation between the vacancy-unemployment ratio and productivity.

Second, I simulate the learning model 1 under the calibration of Shimer (2005), keeping the gain parameter equal to their estimated value reported in Table 5, third column. This exercise is running in order to check that the amplification is coming from the learning process instead of an alternative calibration of other parameters

[36]Regressions run with the model to check RE:

$$\tilde{w}_{t+1} = a^1 + b^1\tilde{w}_{t-1} + \delta_t^1\tilde{y}_t + \epsilon_t \tag{48}$$

$$E_t\tilde{w}_{t+1} = a^2 + b^2\tilde{w}_{t-1} + \delta_t^2\tilde{y}_t + \nu_t \tag{49}$$

[37]The calibration of the remaining parameters follows table 4 and the fifth column of table 5.

such us a higher bargaining power of the worker, $\alpha = 0.72$ or lower unemployment benefits, $b = 0.4$. Column 5 of Table 11 shows that the model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model.

7.3 Learning about d_t^y

In the main paper, I introduce a framework where agents learn about two coefficients that impact wages and incorporate a belief shock. This might spark speculation about which specific coefficient yields these results or raise questions regarding whether the outcomes stem from the learning process or merely from the shock to expectations.

In this subsection, I evaluate the performance of the learning model when agents learn about how productivity correlates with wages, represented in the model by d_t^y. In this case, d_t^c is fixed at the RE value, and I exclude belief shocks during the model's simulation.

Table 12 displays the moments derived from this learning model along with the respective t-statistics. The model that learns about d_t^y demonstrates impressive quantitative performance. The model statistics pass many of the t-tests. It can generate a low contemporaneous correlation between labor market tightness and productivity, together with the high relative volatilities in the labor market, solving the two puzzles, the propagation and the amplification puzzle. This represents a significant success. For instance, the model can generate unemployment's relative volatility that is 7.7 times greater than that produced under rational expectations.

7.4 Learning about the constant

In the previous subsection, I argue that the primary factor behind labor market fluctuations in the learning model is the learning process about the coefficient governing the relationship between wages and productivity in the wage linear equation, denoted as productivity in the linear equation of wages, d^y.

Subsequently, I investigate the performance of the learning model when agents are provided with the rational expectations (RE) coefficient $d_t^y = d^{y,RE}\ \forall t$, with their learning focused solely on the constant term of the wage linear equation, d^c. I also make an assumption that the model environment is devoid of belief shocks to singularly emphasize the impact of learning about the constant. The findings, as presented in Column 7 of Table 11, reveal that the revised model yields a relative standard deviation of labor market tightness, which is 2.3 times higher than what's observed with the RE model. Nevertheless, it is almost 5 times lower compared to learning about d_t^y. Notably, the model effectively reduces the contemporaneous correlation between labor market tightness and labor productivity.

7.5 Information Assumption

In the baseline model, I assume that agents do not observe period wages at the time they make their forecasts. This is a standard assumption in the learning literature to avoid the simultaneous determination of forecast and endogenous variables. I will move away from that assumption, and I will assume that the forecast of wages, the decision of vacancies and realized wages are determined simultaneously. Consequently, agents beliefs evolve according to the following scheme:

$$\begin{aligned} \hat{D}_t &= \hat{D}_{t-1} + \gamma R_t^{-1} z_{t-1}[w_t - \hat{D}'_{t-1} z_t], \\ R_t &= R_{t-1} + \gamma (z_{t-1} z'_{t-1} - R_{t-1}). \end{aligned} \tag{50}$$

Where $\hat{D}_t^f = [\hat{d}_t^c \ \hat{d}_t^y]'$ represent the estimated coefficients and $z_{t-1} = [1 \ y_t]$. Note that in this case, $(w_t - \hat{D}'_{t-1} z_t)$ is the most recent forecast error.

As you can see in Table 11, the re-calibrated model still delivers the amplification of the labor market tightness, and a lower correlation between vacancy-unemployment ratio and productivity compared to the RE version of the model. However, this is achieved at the cost of making wages too volatile.

7.6 Sentiment shocks under RE

In this article, I claim that the key model that would solve the puzzle in the labor market is the combination of learning with expectation shocks. A question that may arise is whether the learning mechanism is necessary or whether the same results can be achieved by maintaining rational expectations and adding a sentiment shock.

To address this question, I do two exercise: (1) I simulate the RE model under the same calibrated parameter values than in the main paper and I introduce sentiment shock with the same standard deviation than in the learning model with sentiment shocks. (2) I simulate the RE model and I estimate the standard deviation of the expectation shock to match 2 moments, the amplification and the propagation.

Table 17 reports the results coming from the two exercises. Three things are observed: (I) the model under rational expectations with the same shock as in the learning version generates a volatility almost four times lower than the previous model and a higher correlation. (II) The introduction of this shock in RE generates negative autocorrelations in the labor market variables. (III) To generate the same relative volatility between labor market tightness and productivity as in learning, the volatility of the shock needs to be approximately multiplied by a factor of four. But the higher the volatility of the shock, the higher the negative autocorrelation of the unemployment vacancy ratio.

It seems that this shock does not operate in an economy where agents do not make small mistakes, as it leads to negative autocorrelations in the labor market.

8 Conclusions

A simple search and matching model applied to the business cycle is able to quantitatively replicate a number of important labor market facts in US, provided that one slightly relaxes the assumption that agents perfectly know how wages are formed in the market. I assume that agents are internally rational, in the sense that they formulate their doubts about market outcomes using a consistent set of subjective beliefs about wages and behave optimally given this set of beliefs. The system of beliefs is internally consistent in the sense that it specifies a proper joint distribution of wages and fundamentals at all dates. Moreover, the perceived distribution of wage behavior, although different from the true distribution, is nevertheless close to it and the discrepancies are hard to detect.

In such a setting, optimal behavior implies that agents learn about equilibrium wage process from past wage behavior. This gives rise to a self-referential model of learning about wages. I document that the relation between wage expectations and wages is negative in this model. Higher wage expectations will lead to larger drops in wages.

There are some facts in the labor market that appear puzzling from the RE viewpoint, and many papers question the quantitative consistency of the search and matching models. Sticky wages has gained attention among the literature to solve the puzzling behavior. However as Pissarides (2009) and Haefke et al. (2013) have point out, this mechanisms in matching models is difficult to justify on empirical ground. The learning model performs remarkably well, despite its simplicity. It generates fluctuations in the labor market variables, and it is not subject to the previous critics, in the sense that, the learning approach does not generate rigid wages. Moreover, RE is not supported by survey data in the formation of wage expectations. My result suggest that learning about wage behavior may be a crucial ingredient in understanding labor market volatility.

The finding that large labor market fluctuations can result from optimizing agents with subjective beliefs is also relevant from a policy perspective. As I show in the last part of the paper, If policy makers rely on RE models instead of IR ones, they can get bias estimates for effects of policies related to unemployment benefits. Also, it will be interesting include risk averse agents in the model, and take into account such channel, to get a non-zero optimal policy for unemployment benefits. Therefore, computing the optimal policy of unemployment benefits under internal rationality, and see if such optima policy is time dependent with respect to business cycle, appears to be an interesting avenue for further research.

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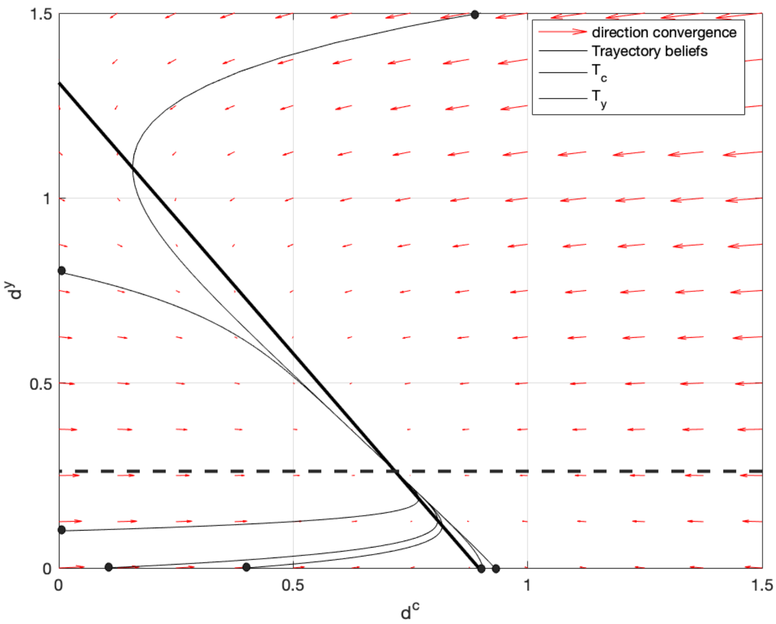
Figure 10: Phase Diagram



The figure plots the trajectories for beliefs about the output gap and its actual realisations given different initial conditions. The thick black line represents the combination of beliefs hatd^c and hatd^y such that $\dot{\hat{d}}^c = 0$. The dashed line the represents the values of hatd^y such that $\dot{\hat{d}}^y = 0$.

A Survey Data

A.1 Business Leaders Survey

A.1.1 Survey design

In this section, I analyze the expectations of firms about future wages. For that, I use the Business Leaders Survey that is a monthly survey conducted by the Federal Reserve Bank of New York that asks service firms in its district - which includes New York State, upstate New Jersey, and Fairfield County, Connecticut - about recent and expected trends in key business indicators. Service sector participants respond to a questionnaire and report on a range of indicators, both in terms of recent and expected changes.

The survey is sent on the first business day of each month to the same pool of about 150 business executives, usually the president or CEO, in the region's service sector. In a typical month, about 100 responses are received by around the tenth of the month when the survey closes.

A.1.2 Questions and description of the data

I am interested in the questions regarding wages paid to the company's average worker. I am focus in two variables: wpcdina - current wages - this variable tells me how wages have changed in the last 3 months on average, and wpfdina - future wages - lets me know how companies expect wages to change in the next 6 months. Both variables are express in diffusion indexes -the difference between the percentage of firms that report an increase of wages minus the percentage of firms that report a decrease.

Before running the test, I have to deal with a problem of different horizons between the diffusion index of wage realization and the one for wage expectation. To solve this problem and keep things simple, assume that firms have in mind that wages are generated by the following process:

$$w_t = w_t^p + \epsilon_t, \tag{51}$$

$$w_t^p = w_{t-1}^p + \nu_t. \tag{52}$$

Where w_t^p is a persistent component and ϵ_t is a transitory component. Persistent component depends on the past persistent component and on a shock ν_t. Both shocks are independently and normally distributed with zero mean. Under this assumption, $E_t(w_{t+2}) = E_t(w_{t+1})$.

A.1.3 RE test

In this part, I construct a test to verify whether the assumption of RE holds in the data. I test if firms are rational on average when they form expectations about wages. Moreover, this test allow me to know how the firms form their expectations and which variables are important for the determination of them. Particularly, I

b^R	b^E	p-value $\text{H}_0 : b^R = b^E$	p-value $\text{H}_0 : b^R \leq b^E$
0,0098 (0,0882)	0,16 (1,7378)	0,039	0,00427

Table 10: RE test firms

Note: *t statistics HAC covariance estimator in parentheses. This table presents the results of the test (54). Monthly data from January 2007 to March 2022.*

want to test if aggregate productivity in New York State, New Jersey, and Connecticut is used by the firms on average, when they form their expectations.

To be rational, expectations have to efficiently use the available information. Forecasting errors, in RE models, have to be orthogonal to all information that was available and relevant to the agents at the moment of making forecasts. The realizations and expectations for each agent should identically incorporate the information contained in his/her past realizations.

Let E_t^P denote firms' subjective expectations operator based on information up to time t, which can differ from the rational expectations operator E_t. Let $\tilde{w}_{t+1}$ denote the realized wages that the firms paid 3 months ahead, and let s_k be a measure of firms' subjective beliefs regarding future wages paid to their company's average worker that are possibly subject to measurement error, obtained, from survey data. Therefore, $s_{t+1} = E_t^P(\tilde{w}_{t+1})$ represents an estimate of firms' subjective beliefs about their wage paid 3 months ahead. Given the expectations horizon in the Business Leaders Survey, t+1 stands for 3 months -quarterly measure-.

$$w_{t+1} = a^R + \delta_t^R Y_{t-1} + \epsilon_t, \tag{53}$$

$$s_{t+1} = a^E + \delta_t^E Y_{t-1} + \nu_t. \tag{54}$$

Where Y represents the quarterly labor productivity growth in New York State, New Jersey, and Connecticut.

Under the null hypothesis of the information structure of RE ($H_0 : E_t = E_t^P$), $\hat{\delta}_R = \hat{\delta}_E$ must be estimates of the same regression coefficient, because $d_E = d_R$ under RE. Under the null hypothesis, the coefficients should equal. If coefficients across equations are different, we reject RE. Under RE, if Y_{t-1} is in the informational set of the agents for the time period t, the prediction error must be orthogonal to Y_{t-1}.

Table 10 shows the result of the test. Column 3 shows the p-values for the test. Additionally, column 4 shows the p-values for the one-sided test. The results provide evidence against the notion that firm survey expectations of wages are compatible with RE. The null hypothesis is rejected for the considered period, implying that rational expectations with respect to real wages do not provide empirical support in that period. The forecast error of wages is correlated with the productivity. It can be seen that firms on average underestimate the effect of productivity in the formation of wage expectations.

A.2 Survey of Consumer Expectations

A.2.1 Description of the Data

The survey data on expectations comes from the Survey of Consumer Expectations conducted by the Federal Reserve Bank of New York. To conduct the Rational Expectation test, I use two data sets: (1) the Survey of Consumer Expectations which report information on many demographic variables of the participants, and (2) the Survey of Consumer Expectations (SCE) Labor Market Survey.[38] The participants in the Labor Market Survey are members who participate in an SCE monthly survey in the prior three months. Since respondents are on the SCE panel for a maximum of 12 months, they end up participating in one, two or three labor market surveys during their tenure in the panel.

The SCE Labor Market Survey has two main sets of questions: (I) an "Experiences" category that takes data on labor market outcomes, such us wage offers received in the past 4 months, search behavior, reservation wages, job satisfaction and (II) "Expectations" category, which takes data on expectations related to job offer wage expectations, expected job transitions, and retirement.

The panel data enables me to explore how each individual's expectations relate to realizations in the next 4-months period, which allows me to assess the accuracy of expectations and how individuals form their expectations in the labor market. The data from the Labor Market Survey covers the waves from March 2014 to March 2020. The date on which each interview was conducted is represented by the subscript t. Individuals are surveyed every four months for up to one year, and I will identify each individual with the subscript i. In the sample, $26,01\%$ took one labor market survey, $33,29\%$ took two surveys, and $40,71\%$ took three surveys. To compute the forecast error for each agent and carry out a statistical test, agents must participate in at least two consecutive surveys. Therefore, I focus on the last two groups.

It is important to clarify the data assumptions that I made. I turn the variables of salary offers and expectations into real terms, the base period of March 2014, using the consumer price index (CPI) for All Urban Consumers from the Federal Reserve Bank of St. Louis. [39] I transform the annual earnings of offers and expectations into

[38] See https://www.newyorkfed.org/microeconomics/databank.html for details.

[39] Due to the fact that the survey is four-monthly, I transformed the quarterly data into four-monthly data using interpolation methods. Source: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CPIAUCSL, June 14, 2021.

hourly earnings, taking into account whether the contract is part-time or full-time. If people work full-time, we divide earnings by 2080 (52 weeks times 40 hours), and if people work part-time, we divide earnings by 1040 (52 weeks, 20 hours). With respect to beliefs, if anyone has received only part-time offers, we assume that her/his beliefs are about part-time work; otherwise, we assume that her/his beliefs are about full-time work. I drop respondents whose revision in beliefs between surveys or the gap between the realizations and the previous period's expectation is greater (lower) than quartile 99 (quartile 1). Finally, I focus on the data from November 2014 onward, when questions about current job offers and expectations about future offers were added to the survey.

A.2.2 Main Questions

Question NL2 give me the realized wage offer of each agent, w_t^i, where t denotes four-month period. It asks participants the annual salary of the three best offers they received in the last 4 months, and whether they were full-time or part-time offers. More precisely, the question is the following:

What was the annual salary of this job offer? An was it for a full-time or part-time job?/ Thinking about 3 best job offers that you received in the last 4 months. What was their annual salary? And were they for a full-time or a part-time job? Note the best offer is the offer you would be most likely to accept.

Each agent can report at most 3 offers; therefore, I calculate and average offer for each agent.

On the other hand, question OO2a, allow me to know the expected wage offer of each agent, $E_t^i(w_{t+1}^i)$, where $t+1$ represents the next four-month period. It asks respondents reporting a non-zero percent chance of receiving a job offer about the average salary of the offers they may receive within the coming four months. Particularly, the question is the following:

Think about the job offer that you may receive within the coming four months. Roughly speaking, what do you think the average annual salary for these offers will be for the first year?

B Data Source

The time series are presented as seasonally adjusted quarterly series. The period covered is from 1990-Q1 to 2020-Q1, a total of 277 quarters.

Unemployment: U.S. Bureau of Labor Statistics, Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series/UNEMPLOY`, October 20, 2022.

Vacancies: To get the vacancy level, I combine two sources. Barnichon, Regis.

2010. "Building a composite Help-Wanted Index", retrieved from `https://sites.google.com/site/regisbarnichon/data` and U.S. Bureau of Labor Statistics, Job Openings [JTS10000000JOL], retrieved from `https://data.bls.gov/timeseries/JTS10000000JOL`, October 20, 2022.

Labor productivity: U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Output per Job for All Employed Persons [PRS85006163], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/PRS85006163`, October 20, 2022.

Wages: U.S. Bureau of Economic Analysis, Gross domestic income: Compensation of employees, paid: Wages and salaries [A4102C1Q027SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/A4102C1Q027SBEA`, October 12, 2022.

Consumer Price Index: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items in U.S. City Average [CPIAUCSL], retrieved from FRED, Federal Reserve Bank of St. Louis;
`https://fred.stlouisfed.org/series/CPIAUCSL`, October 12, 2022.

C Theoretical Model

C.1 Equilibrium equations

To determine the labor market tightness of the economy, I have to start with the job creation condition:

$$\frac{c}{q(\theta_t)} = E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^j [\frac{y_{t+j} - w_{t+j}}{1-\lambda}] \tag{55}$$

Plugging the expectations of productivity and wages; $E_t y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t w_{t+j} = \hat{d}_t^c + \hat{d}_t^y((1-\rho^{j-1}) + \rho^{j-1} y_t)$, the labor market tightness can be writte as follows:

$$\theta_t = (\frac{A\beta}{c})^{\frac{1}{1-\nu}} [\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}, \tag{56}$$

where Θ_y represents the present discount revenues and Θ_w the present discount labor costs. These two can be writte as

$$\begin{aligned} \Theta_y &= \frac{1}{1-\beta(1-\lambda)} + \frac{\rho}{1-\beta(1-\lambda)\rho}(y_t - 1) \\ \Theta_w &= \frac{\hat{d}_t^c + \hat{d}_t^y}{1-\beta(1-\lambda)} + \frac{\hat{d}_t^y}{1-\beta(1-\lambda)\rho}(y_t - 1). \end{aligned} \tag{57}$$

Therefore, vacancies are determine by

$$v_t = u_t(\frac{A\beta}{c})^{\frac{1}{1-\nu}} [\Theta_y - \Theta_w]^{\frac{1}{1-\nu}}. \tag{58}$$

C.2 Wages

Wages are negotiated according to a Nash bargaining process. The wage maximizes the joint surplus of a match between workers and firms. The maximization problem is the following:

$$\max_{w_t} \quad [\mathcal{W}(m_t) - \mathcal{U}(m_t)]^{\alpha} \mathcal{J}_t^{1-\alpha} \tag{59}$$

where α is the workers' bargaining power. The first order condition is as follows:

$$\begin{aligned} \alpha (\mathcal{W} - \mathcal{U})^{\alpha-1} (\mathcal{J}_t)^{1-\alpha} + (\mathcal{W} - \mathcal{U})^{\alpha} (1-\alpha) (\mathcal{J}_t)^{-\alpha} (-1) = 0, \\ \alpha (\mathcal{W} - \mathcal{U})^{\alpha-1} (\mathcal{J}_t)^{1-\alpha} = (\mathcal{W} - \mathcal{U})^{\alpha} (1-\alpha) (\mathcal{J}_t)^{-\alpha}, \\ \alpha (\mathcal{J}_t) = (1-\alpha) (\mathcal{W} - \mathcal{U}). \end{aligned} \tag{60}$$

Therefore, the following equalities are satisfied:

$$\mathcal{W} - \mathcal{U} = \alpha S_t, \tag{61}$$

$$\mathcal{J}_t = (1-\alpha) S_t. \tag{62}$$

Where S_t is the total surplus of the match.

$$S_t = (\mathcal{W} - \mathcal{U}) + \mathcal{J}_t. \tag{63}$$

Plugging the surpluses into (60), I come up with:

$$\begin{aligned} \alpha(\Big[y_t - w_t + \beta \Big[(1-\lambda) E_t^{\mathcal{P}^f} (\mathcal{J}_{t+1}))\Big] = \\ (1-\alpha) \left[w_t - b + \beta \left[(1-\lambda - f(\theta_t)) E_t^{\mathcal{P}^w} \mathcal{W}(m_{t+1}) - \mathcal{U}(m_{t+1})\right]\right]. \end{aligned} \tag{64}$$

Assuming that agents belief that (62) and (63) hold in expectations,

$$\begin{aligned} \alpha \Big[y_t - w_t + \beta E_t^{\mathcal{P}^f} \left[(1-\lambda)((1-\alpha) S_{t+1})\right]\Big] = \\ (1-\alpha) \left[w_t - b + \beta E_t^{\mathcal{P}^w} \left[(1-\lambda - f(\theta_t))(\alpha S_{t+1})\right]\right] \end{aligned} \tag{65}$$

Doing some algebra, I come up with

$$\begin{aligned} (1-\alpha) w_t - (1-\alpha) b + (1-\alpha) \beta E_t^{\mathcal{P}^w} \left[(1-\lambda - f(\theta_t))(\alpha S_{t+1})\right] = \\ \alpha y_t - \alpha w_t + \alpha \beta E_t^{\mathcal{P}^f} \left[(1-\lambda)((1-\alpha) S_{t+1})\right] \end{aligned} \tag{66}$$

Let's assume that $E_t^{\mathcal{P}^f} = E_t^{\mathcal{P}^w} = E_t^{\mathcal{P}}$,

$$w_t = \alpha y_t + (1-\alpha) b + \beta f(\theta_t) \alpha E_t^{\mathcal{P}} (S_{t+1})(1-\alpha).$$

Finally, if both agents know that the FOC of firms hold, $(1-\alpha)\beta E_t^{\mathcal{P}}(S_{t+1}) = \frac{c\theta_t}{f(\theta_t)}$, I come up with the following expression:

$$\begin{aligned} w_t &= \alpha y_t + (1-\alpha) b + f(\theta_t) \alpha \frac{c\theta_t}{f(\theta_t)} \\ &= \alpha (y_t + c\theta_t) + (1-\alpha) b. \end{aligned} \tag{67}$$

C.3 T-mapping

First all, I linearize the job creation condition applying a first-order Taylor polynomial of this equation at the steady state $\theta = \overline{\theta}$, $w = \overline{w}$ and $y = \overline{y} = 1$. The job creation condition is represented by the following equation:

$$\frac{c}{\beta q(\theta_t)} = E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}] \tag{68}$$

I take the first-order Taylor polynomial of each component of the previous equation:

$$\frac{c}{\beta q(\theta_t)} = \frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) \tag{69}$$

$$E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} y_{t+j} = \frac{1}{1-\beta(1-\lambda)} + E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} (y_{t+j} - 1) \tag{70}$$

$$E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} w_{t+j} = \frac{\overline{w}}{1-\beta(1-\lambda)} + E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} (w_{t+j} - \overline{w}) \tag{71}$$

Therefore, I can write equation (68) as

$$\frac{c}{\beta q(\overline{\theta})} - \frac{c}{\beta q(\overline{\theta})^2} \frac{\partial q(\overline{\theta})}{\partial \overline{\theta}} (\theta_t - \overline{\theta}) = E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}], \tag{72}$$

$$\theta_t = \overline{\theta} + \phi E_t^{sf} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}]. \tag{73}$$

where $\phi = \frac{\beta q(\overline{\theta})^2}{c(q'(\overline{\theta}))}$. I plug the previous equation into the wage equation, I come up with

$$w_t = \alpha \left(y_t + c \left(\overline{\theta} + \phi E_t^{\mathcal{P}f} \sum_{j=1}^{\infty} [\beta(1-\lambda)]^{j-1} [y_{t+j} - w_{t+j}] \right) \right) + (1-\alpha) b. \tag{74}$$

Taking into account the expectation; $E_t^{\mathcal{P}} y_{t+j} = (1-\rho^j) + \rho^j y_t$ and $E_t^{\mathcal{P}} w_{t+j} = \hat{d}_t^c + \hat{d}_t^y ((1-\rho^{j-1}) + \rho^{j-1} y_t)$, I come up with:

$$w_t = \Phi^c + \Phi^y y_{t-1} + \Phi^\epsilon \epsilon_t, \tag{75}$$

where

$$\Phi^c = \alpha \left[c\overline{\theta} + \frac{c\phi}{1-\beta(1-\lambda)} \left[1 - (\hat{d}_t^c + \hat{d}_t^y) \right] + (1-\rho) - \rho \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)\rho} \right] \right] + (1-\alpha) b,$$

$$\Phi^y = \rho \left[\alpha + \phi \alpha c \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)} \right] \right],$$

$$\Phi^\epsilon = \left[\alpha + \phi \alpha c \left[\frac{\rho - \hat{d}_t^y}{1-\beta(1-\lambda)} \right] \right]. \tag{76}$$

C.4 Method of moments

$$\min_{\theta} \quad (\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta))'\hat{\Sigma}_{\mathcal{S}}^{-1}(\hat{\mathcal{S}}_i - \tilde{\mathcal{S}}_i(\theta)) \ . \tag{77}$$

Where $\hat{\Sigma}_{\mathcal{S}}$ is the varianza of the moments.

C.4.1 The Statistics and Moment Functions

This section gives explicit expressions for the statistics function S(.) and the moment functions *h(.)*.

The undrerlying sample moments needed to construct statistics of interes are:

$$\hat{M}_N = \frac{1}{N}\sum_{t=1}^{N} h(y_t), \tag{78}$$

where $h(.)$ is defined as $\begin{bmatrix} \tilde{v}_t \\ \tilde{u}_t \\ \tilde{\theta}_t \\ \tilde{y}_t \\ \tilde{w} \\ \tilde{v}_t^2 \\ \tilde{u}_t^2 \\ \tilde{\theta}_t^2 \\ \tilde{y}_t^2 \\ \tilde{w}_t^2 \\ \tilde{v}_t\tilde{\theta}_t \\ \tilde{u}_t\tilde{\theta}_t \\ \tilde{y}_t\tilde{\theta}_t \\ \tilde{w}_t\tilde{\theta}_t \\ \tilde{v}_t\tilde{u}_t \\ \tilde{\theta}_t\tilde{\theta}_{t-1} \\ \tilde{w}_t\tilde{w}_{t-1} \end{bmatrix}$

The 13 statistics I consider can be expresed as functions of the moments as follows:

$$
\mathcal{S}(M) = \begin{bmatrix} \sigma_{\tilde{v}} \\ \sigma_{\tilde{u}} \\ \sigma_{\tilde{\theta}} \\ \sigma_{\tilde{y}} \\ \sigma_{\tilde{w}} \\ \rho(\tilde{v}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{y}_t, \tilde{\theta}_t) \\ \rho(\tilde{w}_t, \tilde{\theta}_t) \\ \rho(\tilde{u}_t, \tilde{v}_t \\ \rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t) \\ \rho(\tilde{w}_{t-1}, \tilde{w}_t) \end{bmatrix} = \begin{bmatrix} \sqrt{M_6 - M_1^2} \\ \sqrt{M_7 - M_2^2} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_9 - M_4^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{11} - M_1 M_3}{\sqrt{(M_6 - M_1^2)(M_8 - M_3^2)}} \\ \frac{M_{12} - M_2 M_3}{\sqrt{(M_7 - M_2^2)(M_8 - M_3^2)}} \\ \frac{M_{13} - M_4 M_3}{\sqrt{(M_9 - M_4^2)(M_8 - M_3^2)}} \\ \frac{M_{14} - M_5 M_3}{\sqrt{(M_{10} - M_5^2)(M_8 - M_3^2)}} \\ \frac{M_{15} - M_1 M_2}{\sqrt{(M_2 - M_1^2)(M_7 - M_2^2)}} \\ \sqrt{M_8 - M_3^2} \\ \sqrt{M_{10} - M_5^2} \\ \frac{M_{17} - M_3^2}{S_{11}^2} \\ \frac{M_{18} - M_5^2}{S_{12}^2} \end{bmatrix}, \tag{79}
$$

where M_i drenotes the ith element of M.

I compute the t-statistics for a particular statistic i as follows:

$$
\sqrt{N} \frac{\mathcal{S}_i - \mathcal{S}_i^M}{\hat{\sum}_{\mathcal{S}}}, \tag{80}
$$

where $\mathcal{S}_i$ are the i statistic of the data and $\mathcal{S}_i^M$ is the i statistic coming from the model. $\hat{\sum}_{\mathcal{S}}$ is the variance for the sample statistics $\mathcal{S}$:

$$
\hat{\sum}_{\mathcal{S}} = \frac{\partial S}{\partial M'} \hat{\mathcal{S}}_w \frac{\partial S'}{\partial M} \tag{81}
$$

I can test if the ability of the model to explain individual moments using t-statistics based on formal asymptotic distribution:

$$
\sqrt{N} \frac{\mathcal{S} - \mathcal{S}^M}{\hat{\sum}_{\mathcal{S}}} \to N(0,1). \tag{82}
$$

D Asymmetric Perceived Law of Motions

D.1 Bargainig

Wages are negotiated according to a Nash bargaining process. The wage maximizes the joint surplus of a match between workers and firms. The FOC of the problem is:

$$
\alpha\left(\mathcal{J}_t\right) = (1-\alpha)\left(\mathcal{W}(m_t) - \mathcal{U}(m_t)\right)
$$

$\alpha(\frac{1}{1-\beta(1-\lambda)\rho}y_t+\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}-\frac{\beta(1-\lambda)\rho}{1-\beta(1-\lambda)\rho}-w_t-\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}\hat{d}_t^{f,c}-\hat{d}_t^{f,y}[\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}+\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}(y_t-1)] = (1-\alpha)(w_t+\frac{1}{1-\beta(1-\lambda-\bar{f})}[\beta(1-\lambda-\bar{m})\hat{d}_t^{w,c}-b]),$

$\mathrm{w}_t = \alpha[\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}(1-\hat{d}_t^{f,c})+\frac{1-\rho}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}]+(1-\alpha)[\frac{b}{1-\beta(1-\lambda-\bar{f})}-\frac{\beta(1-\lambda-\bar{m})}{1-\beta(1-\lambda-\bar{f})}\hat{d}_t^{w,c}]+ ...$

$...+\alpha\rho[\frac{1}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)d_t^{\hat{f},y}}{1-\beta(1-\lambda)\rho}]y_{t-1}+\alpha[\frac{1}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)d_t^{\hat{f},y}}{1-\beta(1-\lambda)\rho}]\epsilon_t.$

Therefore,

$$T_c = \alpha[\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)}(1-\hat{d}_t^{f,c})+\frac{1-\rho}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)}{1-\beta(1-\lambda)\rho}]+ ...$$

$$...+(1-\alpha)[\frac{b}{1-\beta(1-\lambda-\bar{f})}-\frac{\beta(1-\lambda-\bar{f})}{1-\beta(1-\lambda-\bar{f})}\hat{d}_t^{w,c}], \tag{83}$$

$$T_y = \alpha\rho[\frac{1}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}], \tag{84}$$

$$C_\epsilon = \alpha[\frac{1}{1-\beta(1-\lambda)\rho}-\frac{\beta(1-\lambda)\hat{d}_t^{f,y}}{1-\beta(1-\lambda)\rho}]. \tag{85}$$

D.2 S-mapping

In this section, I derive the mapping S. Firstly, I will derive the operator S for the worker, the agent that have a misspeficied model in mind to form the expectations of wages. Following Marcet and Sargent (1989a) the formula is the following:

$$S_w(D) = T(D)[M_{z_w}(D)^{-1}M_{z_w.z}(D)]', \tag{86}$$

where $T(D) = [T_c \; T_y]$, $M_{z_w}(D) = Ez_wz_w' = 1$ and $M_{z_w.z}(D) = Ez_wz' = E[1 \; y]'$. Assuming that the expectation of y is set to 1, $S_w(D) = T_c + T_y\bar{y}$.

On the other hand, given the fact that the firm has the right model in mind to form expectations, $T_f(D) = T(D)$. Therefore,

$$S(D) = \begin{bmatrix} T(D) \\ S_w(D)] \end{bmatrix} = \begin{bmatrix} T_c \\ T_y \\ T_c + T_y\frac{\bar{y}}{1-\rho} \end{bmatrix}.$$

The operator S governs the dynamics of $D = [d_t^{f,c} \; d_t^{w,c} \; d_t^{f,y}]$.

D.3 Tables of Summary Statistics of Robustness

	Data	Learning[1] model	RE model[2] Hagedorn and Manovskii	Learning[3] Shimer	Learning[4] Simultaneous (Re-est)	Learning[5] Constant (Re-est)
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	19.891	15.63	16.02	19.31	4.00
		(1.228)	(2.31)	(2.21)	(1.37)	(5.27)
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.314	0.10	1.38	6.43	0.73
		(1.834)	(7.11)	(1.54)	(-20.42)	(4.37)
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.389	0.998	0.21	0.11	0.65
		(-0.998)	(-2.42)	(-0.57)	(-0.35)	(-1.60)

Table 11: Summary Statistics: Alternative calibration, learning constant coefficient and infortantion assumption

Note: ***1.*** *Learning model about d_t^y with productivity shocks, $d_t^c = d^{c,RE}$. Calibration follows tables 4 and 5.* ***2.*** *Calibration follows tables 4 and 5 of the RE model except for parameters $b = 0.955$ and $\alpha = 0.05$.* ***3.*** *Calibration follows tables 4 and 5 of the learning model, expect for parameters $b = 0.4$ and $\alpha = 0.72$.* ***4.*** *Calibration follows tables 4 and the estimated coefficients coming from SMM are: $c = 0.9$, $A = 0.6$, $g = 0.051$ and $b = 0.09$.* ***5.*** *Calibration follows tables 4 and the estimated coefficients coming from SMM are: $c = 0.8105$, $A = 0.5737$, $g = 0.02$ and $b = 0.75$.The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.*

Moment's	Data	Learning Model[1]	t-stat	RRE model	t-stat
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	5.220	3.250	0.767	5.400
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	16.240	-1.628	1.773	6.176
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	19.891	1.228	2.426	5.673
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.314	1.838	0.741	4.328
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.389	-0.998	0.991	-2.400
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.964	2.163	0.981	0.261
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.966	-1.283	-0.894	-7.969
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.863	-0.234	0.991	-0.600
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.797	1.935	0.618	4.454
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.763	1.987	0.703	3.840
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.875	-2.612	-0.791	-6.991
$b^R - b^E$	0.60	0.24	-	0.12	-

Table 12: Labor Market Statistics

Note: *1. Learning model of d_t^y with productivity shocks, $d_t^c = d^{c,RE}$. Calibration follows tables 4 and the estimated coefficients coming from SMM are: $c = 0.6$, $A = 1$, $g = 0.05$ and $b = 0.7$.* ***5.****Data moments are computed over the period 1990Q1: 2020Q1. Moments have been computed as averages over 1000 simulations. t-ratios are defined as (data moment-model moment)/ S.E of data moment. $b^R - b^E$ represents the difference in the coefficients coming from reegressions 5 and 6*

	Data	Learning Model	t-stat	RP-RRE model	t-stat
$\sigma_{\tilde{u}}/\sigma_{\tilde{y}}$	11.952	8.814	1.515	0.310	5.621
$\sigma_{\tilde{v}}/\sigma_{\tilde{y}}$	13.221	13.918	-0.376	0.762	6.721
$\sigma_{\tilde{\theta}}/\sigma_{\tilde{y}}$	24.713	22.360	0.599	1.026	6.030
$\sigma_{\tilde{w}}/\sigma_{\tilde{y}}$	1.737	1.682	0.238	1.059	2.944
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.046	-0.199	0.993	-2.404
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.989	-0.594	0.983	0.072
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.970	-0.897	-0.892	-8.161
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.705	0.216	0.993	-0.605
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.907	-1.747	0.999	-2.653
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.840	-0.417	1.000	-5.415
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.923	-0.200	-0.793	-6.651

Table 13: Labor Market Statistics. Asymmetric learning

Note: *The calibration for the two models is described in tables 14 and 15. The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.*

Variable	description	value	source
β	discount factor	0,99	Kyndland & Prescott (1982): r=0,04.
λ	separation rate	0,1	Shimer (2005).
1-α	bargaining power firm	0,5	Standard
ν	elasticity of matching function	0,5	Hosios rule (1990): $\alpha = 1 - \nu$.
b	unemployment benefit	0,4	Shimer (2005).
$\tilde{y}$	steady state productivity	1	Normalization.
σ_ϵ	st. dev. of productivity shocks	0,0058	Calibrated
ρ	persistency of productivity	0,7318	Calibrated

Table 14: Calibrated quarterly parameters. Asymmetric Learning

Variable	Description	Values (Learning)	Values (RRPE)
c	cost of open a vacancy	0,24	0,195
A	efficiency matching technology	0,63	0,543
g	constant gain	0,011	-
σ^β	Std. wage belief shocks	0,928*10^{-3}	-

Table 15: Estimated quarterly parameters from SMM. Asymmetric Learning

	Data	Learning Model[1]	t-stat	Learning Model[2] Productivity (Re-est)	t-stat
$\sigma_{\tilde{u}/\sigma_{\tilde{y}}}$	11.952	7.461	2.168	8.621	1.608
$\sigma_{\tilde{v}/\sigma_{\tilde{y}}}$	13.221	16.850	-1.958	15.183	-1.058
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	24.713	23.401	0.334	21.569	0.801
$\sigma_{\tilde{w}/\sigma_{\tilde{y}}}$	1.737	1.912	-0.765	1.958	-0.963
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	-0.040	0.097	-0.319	0.152	-0.447
$\rho(\tilde{v}_t, \tilde{\theta}_t)$	0.984	0.984	-0.018	0.950	3.719
$\rho(\tilde{u}_t, \tilde{\theta}_t)$	-0.980	-0.907	-6.743	-0.980	0.051
$\rho(\tilde{w}_t, \tilde{\theta}_t)$	0.780	0.947	-0.475	0.954	-0.495
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0.941	0.825	-1.217	0.830	1.713
$\rho(\tilde{w}_{t-1}, \tilde{w}_t)$	0.826	0.831	-0.141	0.808	0.572
$\rho(\tilde{u}_t, \tilde{v}_t)$	-0.927	-0.817	-5.460	-0.873	-2.720

Table 16: Learning about wages and productivity

Note: *1. Learning model of d_t^y and d_t^c with productivity and sentiment shocks. 2. Learning model of d_t^y, d_t^c, a_t^c and a_t^y with productivity and sentiment shocks. Calibration of learning model 1 follows table 4 and the forth column of table 5. Calibration of learning model 2 follows table 4 and the estimated parameters coming from SMM are: $c = 0.4$, $A = 0.5$, $b = 0.7$, $g = 0.02$ and $\sigma^{\beta} = 0.0016$. The moments of the data are calculated for the period 1990Q1: 2020Q1. The moments are calculated as averages of 1,000 simulations. The t-statistics are defined as (data moment-model moment)/E.S. of the data moment.*

	RE* No sent. shocks	RE (1) sent. shocks	RE (2) sent. shocks
$\sigma_{\tilde{\theta}/\sigma_{\tilde{y}}}$	1.75	6.60	24.16
$\rho(\tilde{y}_t, \tilde{\theta}_t)$	1.00	0.31	0.09
$\rho(\tilde{\theta}_{t-1}, \tilde{\theta}_t)$	0,70	-0.27	-0.35

Table 17: Belief Shocks in the RE model

**Calibration follows tables 4 and 5 of the RE model. The moments are calculated as averages of 1,000 simulations.*