

# MA5832: Data Mining & Machine Learning

## Collaborate Week 1: Intro & Linear Algebra Review

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# Housekeeping

- Collaborates = **Thursdays 6-7:30pm**

For my Collaborate Sessions, you can get the **slides & R code** for each week on Github:

<https://github.com/MarthaCooper/MA8532>



# Today's Goals

- Overview of topics covered in MA5832: Study Plan, Assessments & Expectations
- Vectors, Matrices & Linear Algebra
  - Matrix addition & multiplication
  - Computing the determinant for  $2 \times 2$  and  $3 \times 3$  matrices
  - Eigenvalues & Eigenvectors

# MA5832: Data Mining & Machine Learning

# MA5832 Study Plan

<del>###</del>	Weeks	Collaborate_Topics
<del>###</del> 1	1	MA5832 Overview & Linear Algebra
<del>###</del> 2	2	Probability & Optimisation
<del>###</del> 3	3	Tree based regression
<del>###</del> 4	4	Support Vector Machine & Assessment 1 Q&A
<del>###</del> 5	5	Neural Network
<del>###</del> 6	6	Assessment 2 & Capstone Q&A

# Assessments

*Time management is important!*

**Assessments 1** due Sunday Week 2 (25%) - *Week 2 topics*

**Assessments 2** due Sunday Week 4 (35%) - *Week 3 & 4 topics*

**Assessments 3 (Capstone)** due Wednesday Week 7 (40%)

# Expectations

1. This is a masters course. Independent study is **required**.
2. Extensions
  - Read Section 4 of Course Outline
  - Requests must be emailed to Kelly **before** the deadline (unless it is an emergency)
3. Assessments 1 & 2 Submission Details
  - Must be submitted in **PDF** format
  - Can be written in **.Rmd or word** processor
  - **Appendix with R code** must be attached at the end of the **same PDF** document.
4. Questions about collaborates?
  - Email: **martha.cooper@jcu.edu.au**
  - MA8510 Discussion board: **Friday & Saturday**

# Vectors, Matrices & Linear Algebra

- Understand some basic concepts of linear algebra (revision...!)



# Vectors

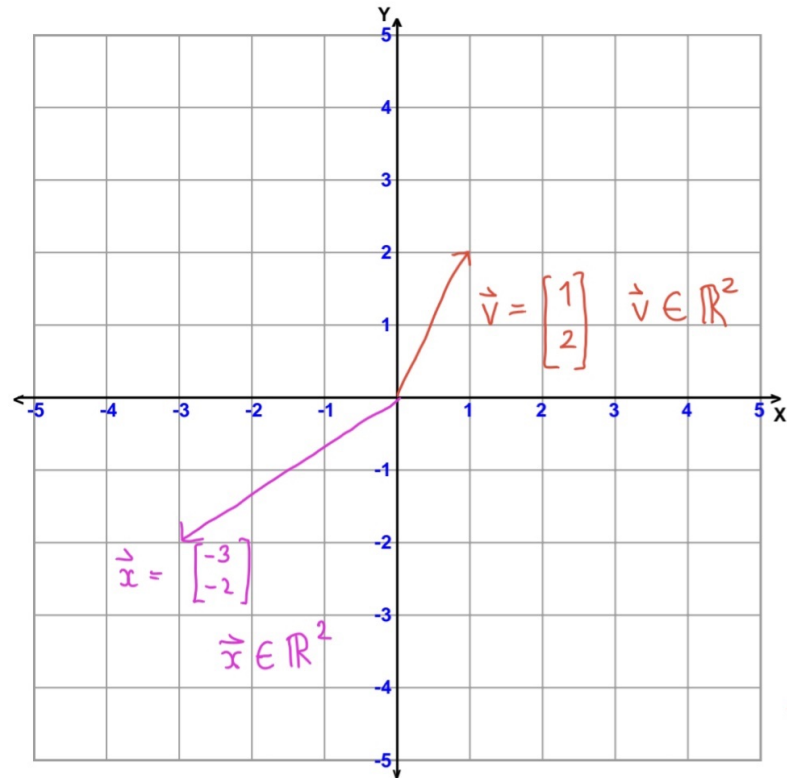
A **vector** is an array of numbers written as a column and enclosed by square brackets

$$\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

This vector,  $\mathbf{v}$ , contains  $m$  elements. If each element of  $\mathbf{v}$  is in  $\mathbb{R}$ ,  $\mathbf{v} \in \mathbb{R}^m$

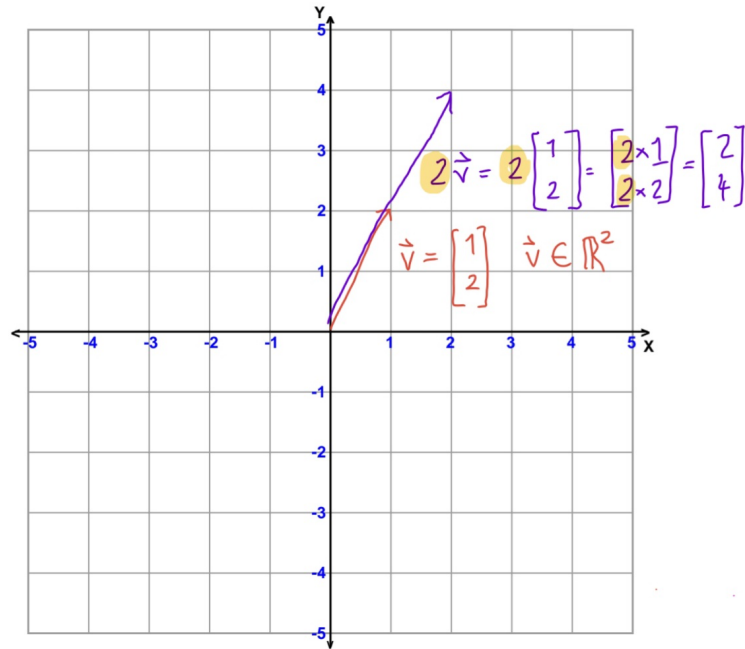
# Vectors

Vectors can be represented geometrically



# Scalars

A scalar is a number e.g.  $x$



# Vectors & Scalars in R

$$\mathbf{v} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

```
v <- c(3,5,2) #defining a vector  
v
```

```
## [1] 3 5 2
```

$$\mathbf{g} = 3\mathbf{v}$$

```
g <- 3*v #multiplying a vector by a scalar  
g
```

```
## [1] 9 15 6
```

# Matrices

A **matrix** is a rectangular array of numbers, arranged in rows and columns. An  $n \times p$  matrix has  $n$  rows and  $p$  columns

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{2,1} & \dots & x_{1,p} \\ x_{2,1} & x_{2,1} & \dots & x_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \dots & x_{n,p} \end{bmatrix}$$

If  $x_{i,j} \in \mathbb{R}$ , then  $\mathbf{X} \in \mathbb{R}^{n \times p}$

- Rows = Samples/Observations
- Columns = Variables/Factors/Predictors

# Setting up matrices in R

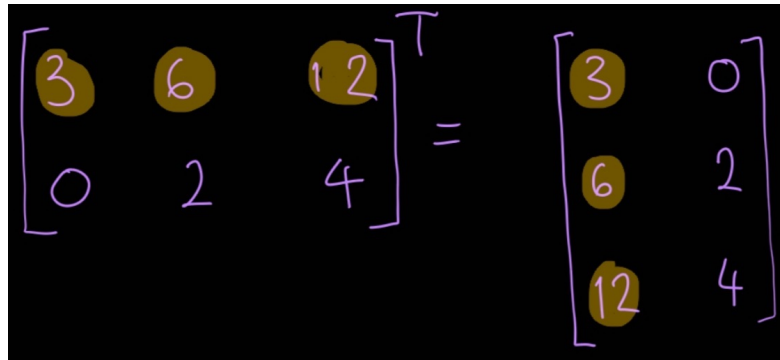
```
m <- matrix(c(1:9), nrow = 3, ncol = 3, byrow = T)
m
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
```

# Matrix Concepts

Transpose

$$A^T$$



A handwritten equation on a black background showing the transpose of a 2x3 matrix. The original matrix is  $\begin{bmatrix} 3 & 6 & 12 \\ 0 & 2 & 4 \end{bmatrix}$  with its elements written in white. The first row elements (3, 6, 12) are each circled in yellow. A superscript  $T$  is written to the top right of the first row. This is followed by an equals sign and the transposed matrix  $\begin{bmatrix} 3 & 0 \\ 6 & 2 \\ 12 & 4 \end{bmatrix}$ , where the first column elements (3, 6, 12) are circled in yellow.

$$\begin{bmatrix} 3 & 6 & 12 \\ 0 & 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & 0 \\ 6 & 2 \\ 12 & 4 \end{bmatrix}$$

# Matrix Concepts

## Inverse

$$\mathbf{A}^{-1}$$

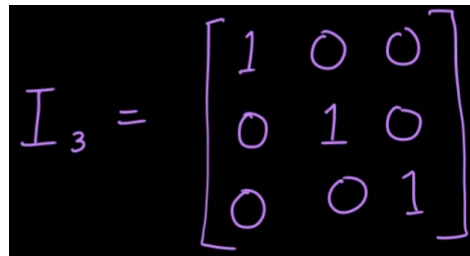
With numbers:

$$\frac{5}{1} \times \frac{1}{5} = 1$$

We can do the same thing with matrices:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$

Where  $\mathbf{I}$  is the Identity matrix - the 1 equivalent of a matrix

A handwritten equation showing the 3x3 identity matrix. The text is written in purple on a black background. It consists of the symbol I with a subscript 3, followed by an equals sign, and then a 3x3 matrix enclosed in large square brackets. The matrix has 1s on the main diagonal (top-left, middle-middle, bottom-right) and 0s in all other positions.
$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Matrix Addition

$$\mathbf{A} + \mathbf{B}$$

Add the numbers in the matching positions. (& subtraction is the same, because it is the addition of a negative matrix)

$$\mathbf{A} + (-\mathbf{B})$$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

Handwritten calculation:  $3 + 4 = 7$  (with arrows pointing to the top-left elements of the matrices)

Note: The matrices must be the same size

# Multiplying a matrix by a scalar

$xA$

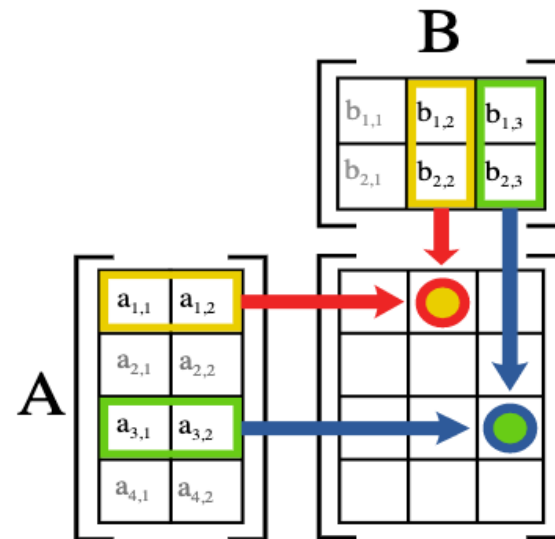
$$2 \times \begin{bmatrix} 0 & 3 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 2 & -8 \end{bmatrix}$$

# Matrix Multiplication

$AB$

Take the dot product

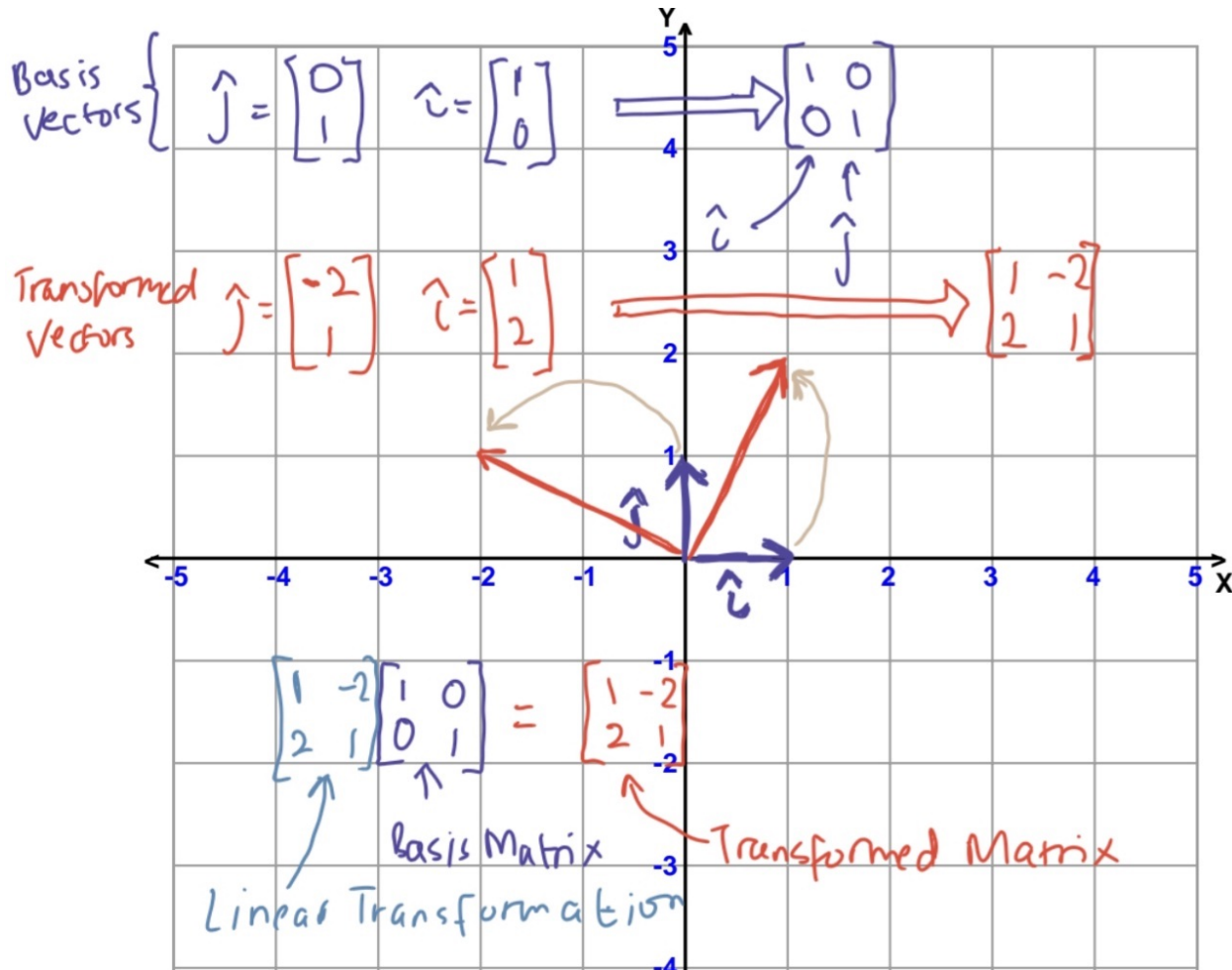
$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 7 & 9 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 23 & 41 \end{bmatrix}$$
$$\begin{aligned} (1 \times 7) + (2 \times -1) &= 7 + (-2) = 5 \\ (1 \times 9) + (2 \times 0) &= 9 + 0 = 9 \\ (4 \times 7) + (5 \times -1) &= 28 + (-5) = 23 \\ (4 \times 9) + (5 \times 0) &= 36 + 0 = 36 \end{aligned}$$



The number of columns in the left matrix must equal the number of row in the right matrix

# Visualising Matrix Multiplication

Matrix multiplication is a linear transformation which we can see geometrically

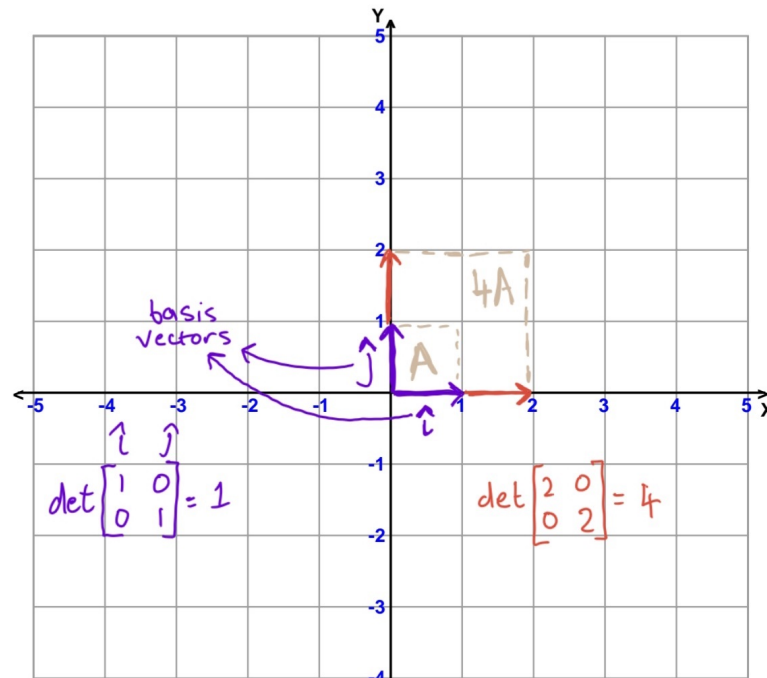


# Summary of Addition and Multiplication

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}; \quad B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$$
$$A + B = \begin{bmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{bmatrix}$$
$$A \times B = \begin{bmatrix} a_{1,1} \times b_{1,1} + a_{1,2} \times b_{2,1} & a_{1,1} \times b_{1,2} + a_{1,2} \times b_{2,2} \\ a_{2,1} \times b_{1,1} + a_{2,2} \times b_{2,1} & a_{2,1} \times b_{1,2} + a_{2,2} \times b_{2,2} \end{bmatrix}$$
$$A \odot B = \begin{bmatrix} a_{1,1} \times b_{1,1} & a_{1,2} \times b_{1,2} \\ a_{2,1} \times b_{2,1} & a_{2,2} \times b_{2,2} \end{bmatrix}$$

# Determinant of a Matrix

**Determinant** - describing how linear transformations change area or volume. Also useful for solving linear equations and changing variables integrals.



# Computing the Determinant

*2x2 matrices*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad |A| = ad - bc$$
$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix} \quad |B| = 4 \times 8 - 6 \times 3$$
$$= 32 - 18$$
$$= 14$$

*3x3 matrices*

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$
$$|A| = a \cdot \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

# Matrix Concepts in R

```
# a and b are two square matrices
```

```
a + b # Addition
```

```
a %*% b # Multiplication
```

```
t(a) # Transpose
```

```
det(a) # Determinant
```

```
solve(a) # Inverse
```

Test these for yourself by hand and then using R



# Solving equations using matrices

Matrices make solving linear equations easier/faster

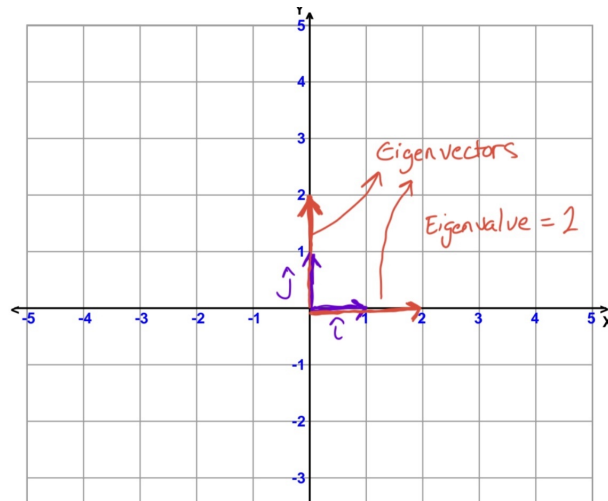
$$\begin{array}{rcl} x + y + z & = & 6 \\ & 2y + 5z & = -4 \\ 2x + 5y - z & = & 27 \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix} \dots$$

# Eigen-things

In (some) linear transformations, there are vectors that don't change direction and are only scaled (stretched or shrunk) within their own span.

**Eigenvectors** are the vectors that remain pointing in the same direction.

**Eigenvalues** are the scalars that the eigenvectors are stretch/shrunk by.



# Eigenvectors and Eigenvalues

Almost all vectors change direction when multiplying a matrix,  $\mathbf{A}$ .

**Eigenvectors**,  $\mathbf{x}$ , are certain vectors that have the same direction as  $\mathbf{Ax}$ . The **Eigenvalue**,  $\lambda$  is the scalar by which  $\mathbf{x}$  is *stretched, shrunk, reversed* or *remained unchanged* when multiplied by  $\mathbf{A}$ .

$$\mathbf{Ax} = \lambda \mathbf{x}$$

We can find eigenvectors and eigenvalues of  $\mathbf{A}$  by setting the **determinant** of  $\mathbf{A} - \lambda \mathbf{I}$  to be 0.

$$\det | (\mathbf{A} - \lambda \mathbf{I}) | = 0$$

# Take what you have learned today and be able to:

- Perform linear regression using matrices (linear regression from scratch without using `lm`)
- Calculate eigenvalues and eigenvectors (by hand)

# Extra reading

Very stuck?

- [Essence of Linear Algebra by 3Blue1Brown](#)
- [Maths is fun Intro to Matrices](#)
- [Lumen Learning Boundless Algebra Intro to Matrices](#)
- [Maths is Fun Eigenvectors and Eigenvalues](#)

Further Reading

- [Chapter 2 Deep Learning by Goodfellow, Bengio & Courville](#)

# References

- Chapter 2 Deep Learning by Goodfellow, Bengio & Courville
- Maths is fun Intro to Matrices
- Lumen Learning Boundless Algebra Intro to Matrices
- Dr Kelly Trinh Week 1: Linear Algebra

## Slides

- xaringan, xaringantheme, remark.js, knitr, R Markdown