

MA5832: Data Mining & Machine Learning

Collaborate Week 1: Intro & Linear Algebra Review

Martha Cooper, PhD

JCU Masters of Data Science

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Housekeeping

- Collaborates = Tuesdays 6-7:30pm

For my Collaborate Sessions, you can get the [slides & R code](#) for each week on Github:

<https://github.com/MarthaCooper/MA8532>



Today's Goals

- Overview of topics covered in MA5832: Study Plan, Assessments & Expectations
- Vectors, Matrices & Linear Algebra
 - Matrix addition & multiplication
 - Computing the determinant for 2×2 and 3×3 matrices
 - Eigenvalues & Eigenvectors

MA5832: Data Mining & Machine Learning

MA5832 Study Plan

##	Weeks	Collaborate_Topics
## 1	1	MA5832 Overview & Linear Algebra
## 2	2	Probability & Optimisation
## 3	3	Tree based regression
## 4	4	Support Vector Machines
## 5	5	Neural Network
## 6	6	Capstone Q&A

Assessments

Time management is important!

Assessments 1 due 12/09/21 (25%) - *Week 2 topics*

Assessments 2 due 26/09/21 (35%) - *Week 3 & 4 topics*

Assessments 3 (Capstone) due 13/10/21 (40%) *Bring everything together*

Expectations

1. Independent study is **required**.

2. Extensions

- Read Section 4 of Course Outline
- Requests must be emailed to David **before** the deadline (unless it is an emergency)

3. Assessments 1 & 2 Submission Details

- Must be submitted in **PDF** format
- Can be written in **.Rmd or word** processor
- **Appendix with R code** must be attached at the end of the **same PDF** document.

4. Questions about collaborates?

- Email: **martha.cooper@jcu.edu.au**
- MA8510 Discussion board: **Friday & Saturday**

Vectors, Matrices & Linear Algebra

- Understand some basic concepts of linear algebra (revision...!)

Vectors

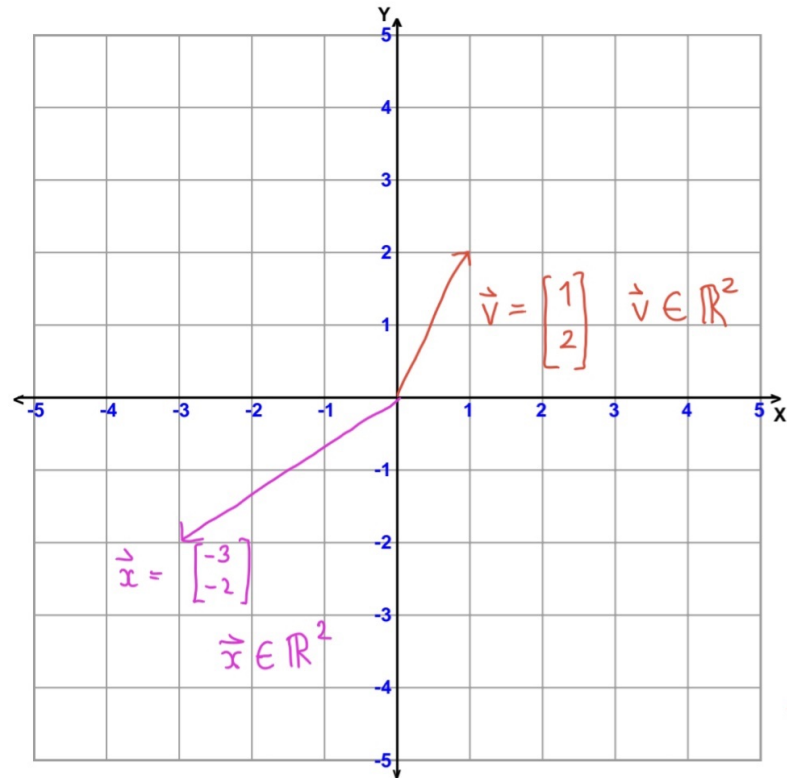
A **vector** is an array of numbers written as a column and enclosed by square brackets

$$\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

This vector, \mathbf{v} , contains m elements. If each element of \mathbf{v} is in \mathbb{R} , $\mathbf{v} \in \mathbb{R}^m$

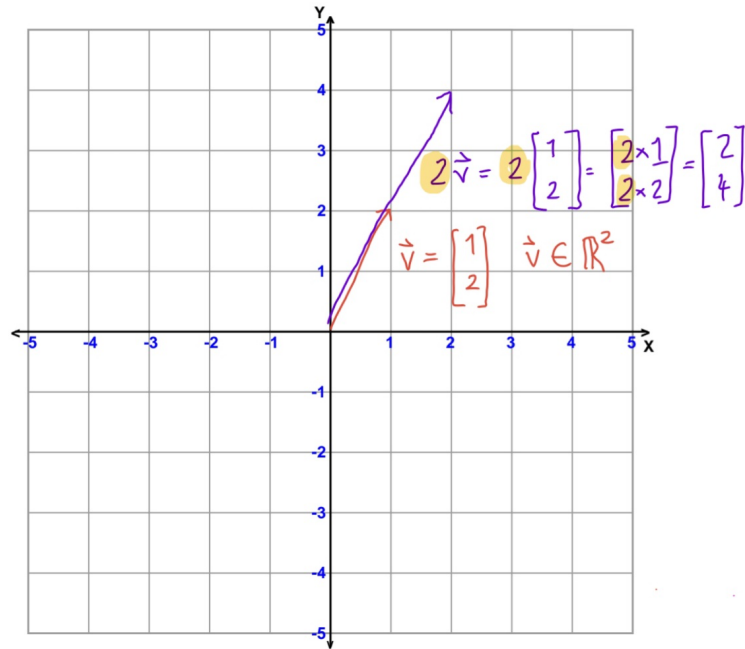
Vectors

Vectors can be represented geometrically



Scalars

A scalar is a number e.g. x



Vectors & Scalars in R

$$\mathbf{v} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

```
v <- c(3,5,2) #defining a vector  
v
```

```
## [1] 3 5 2
```

$$\mathbf{g} = 3\mathbf{v}$$

```
g <- 3*v #multiplying a vector by a scalar  
g
```

```
## [1] 9 15 6
```

Matrices

A **matrix** is a rectangular array of numbers, arranged in rows and columns. An $n \times p$ matrix has n rows and p columns

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{2,1} & \dots & x_{1,p} \\ x_{2,1} & x_{2,1} & \dots & x_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \dots & x_{n,p} \end{bmatrix}$$

If $x_{i,j} \in \mathbb{R}$, then $\mathbf{X} \in \mathbb{R}^{n \times p}$

- Rows = Samples/Observations
- Columns = Variables/Factors/Predictors

Setting up matrices in R

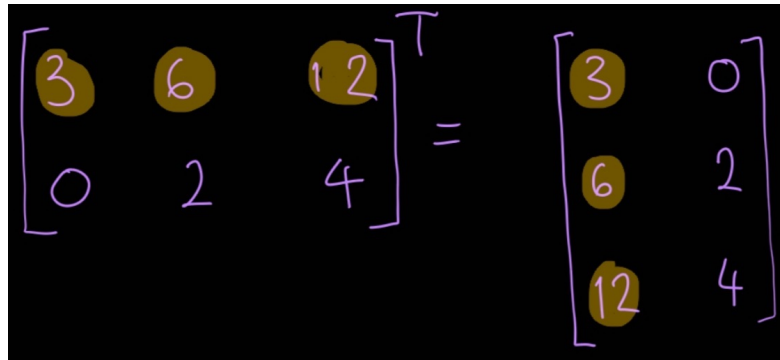
```
m <- matrix(c(1:9), nrow = 3, ncol = 3, byrow = T)
m
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
```

Matrix Concepts

Transpose

$$A^T$$



A handwritten equation on a black background showing the transpose of a 2x3 matrix. The original matrix is $\begin{bmatrix} 3 & 6 & 12 \\ 0 & 2 & 4 \end{bmatrix}$ with its elements written in white. The first row elements (3, 6, 12) are each circled in yellow. A superscript T is placed to the top right of the matrix. This is followed by an equals sign and the transposed matrix $\begin{bmatrix} 3 & 0 \\ 6 & 2 \\ 12 & 4 \end{bmatrix}$, where the first column elements (3, 6, 12) are also circled in yellow.

$$\begin{bmatrix} 3 & 6 & 12 \\ 0 & 2 & 4 \end{bmatrix}^T = \begin{bmatrix} 3 & 0 \\ 6 & 2 \\ 12 & 4 \end{bmatrix}$$

Matrix Concepts

Inverse

$$\mathbf{A}^{-1}$$

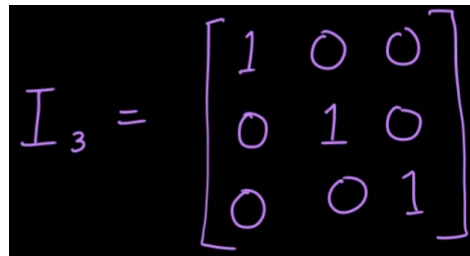
With numbers:

$$\frac{5}{1} \times \frac{1}{5} = 1$$

We can do the same thing with matrices:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$

Where \mathbf{I} is the Identity matrix - the 1 equivalent of a matrix

A handwritten equation on a black background showing the 3x3 identity matrix. The matrix is enclosed in large square brackets, and each element is written in purple ink. The diagonal elements are 1, and the off-diagonal elements are 0.
$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Addition

$$\mathbf{A} + \mathbf{B}$$

Add the numbers in the matching positions. (& subtraction is the same, because it is the addition of a negative matrix)

$$\mathbf{A} + (-\mathbf{B})$$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

Handwritten calculation: $3 + 4 = 7$ (with arrows pointing to the top-left elements of the matrices)

Note: The matrices must be the same size

Multiplying a matrix by a scalar

xA

$$2 \times \begin{bmatrix} 0 & 3 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 2 & -8 \end{bmatrix}$$

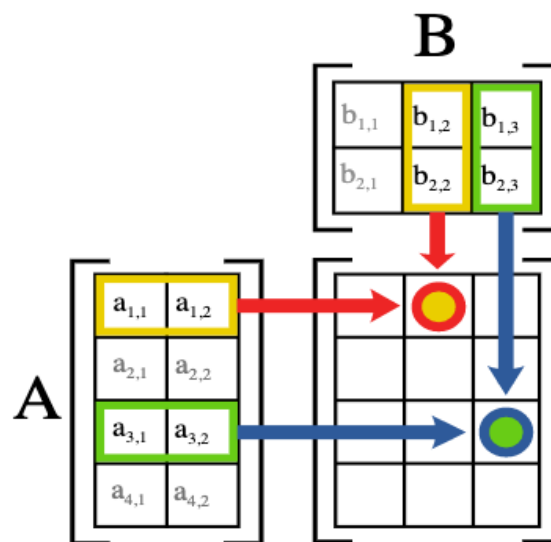
The diagram illustrates the scalar multiplication of a matrix. A scalar 2 is multiplied by a 2×2 matrix. The top row of the matrix is $[0 \ 3]$ and the bottom row is $[1 \ -4]$. The result is a 2×2 matrix with top row $[0 \ 6]$ and bottom row $[2 \ -8]$. Yellow arrows and text show the calculation for the top row: $2 \times 0 = 0$.

Matrix Multiplication

AB

Take the dot product

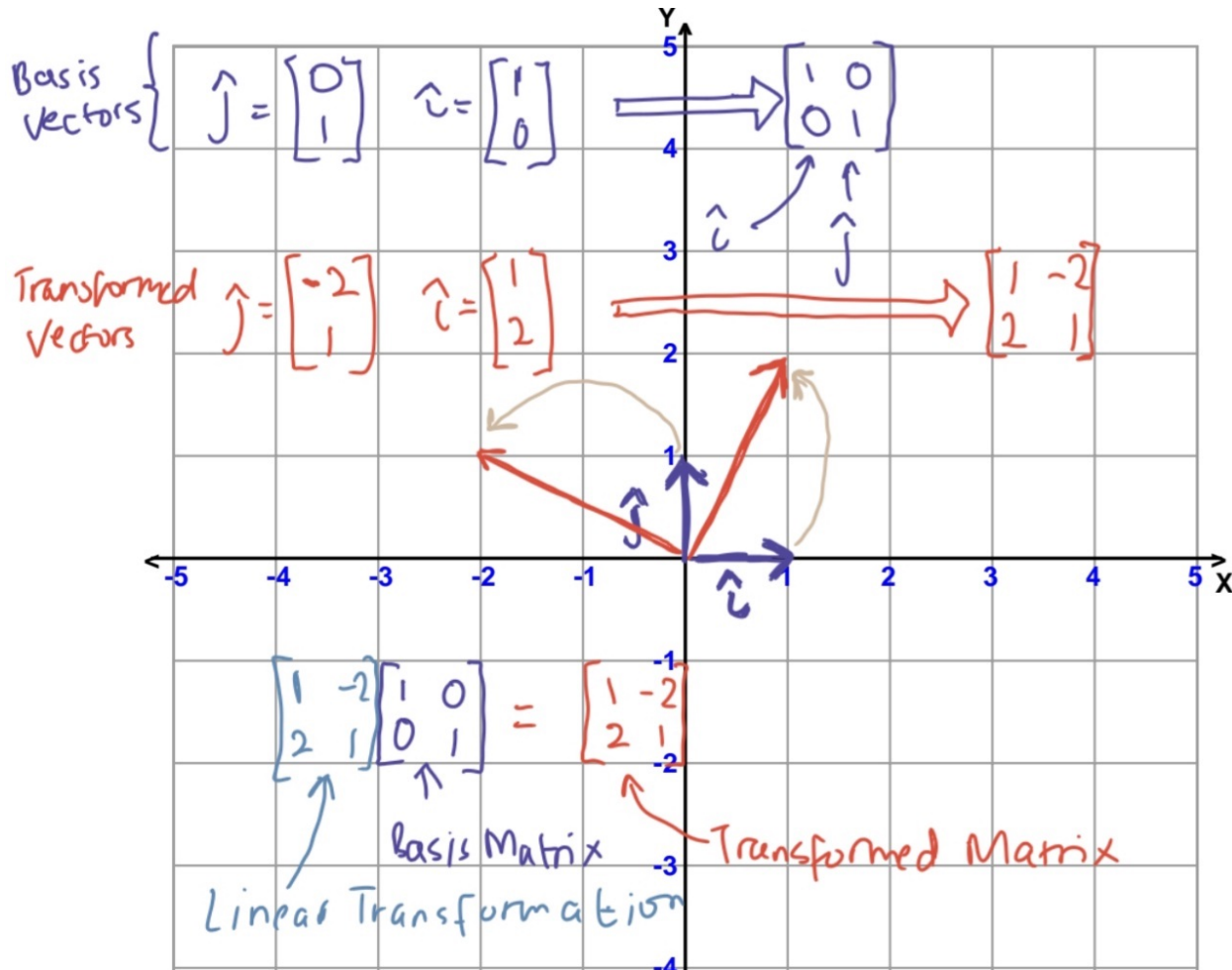
$$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 7 & 9 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 23 & 41 \end{bmatrix}$$
$$\begin{aligned} (1 \times 7) + (2 \times -1) &= 7 + (-2) = 5 \\ (1 \times 9) + (2 \times 0) &= 9 + 0 = 9 \\ (4 \times 7) + (5 \times -1) &= 28 + (-5) = 23 \\ (4 \times 9) + (5 \times 0) &= 36 + 0 = 36 \end{aligned}$$



The number of columns in the left matrix must equal the number of row in the right matrix

Visualising Matrix Multiplication

Matrix multiplication is a linear transformation which we can see geometrically

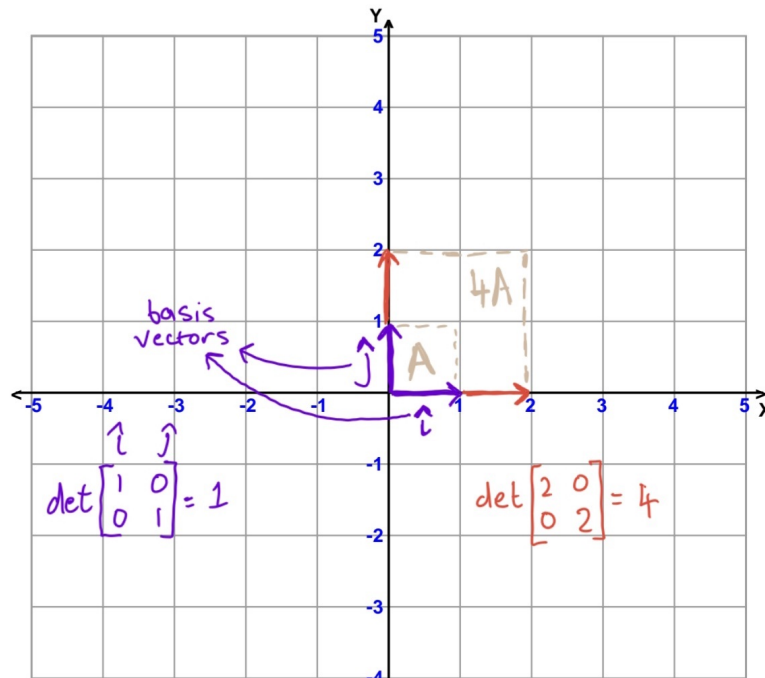


Summary of Addition and Multiplication

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}; \quad B = \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix}$$
$$A + B = \begin{bmatrix} a_{1,1} + b_{1,1} & a_{1,2} + b_{1,2} \\ a_{2,1} + b_{2,1} & a_{2,2} + b_{2,2} \end{bmatrix}$$
$$A \times B = \begin{bmatrix} a_{1,1} \times b_{1,1} + a_{1,2} \times b_{2,1} & a_{1,1} \times b_{1,2} + a_{1,2} \times b_{2,2} \\ a_{2,1} \times b_{1,1} + a_{2,2} \times b_{2,1} & a_{2,1} \times b_{1,2} + a_{2,2} \times b_{2,2} \end{bmatrix}$$
$$A \odot B = \begin{bmatrix} a_{1,1} \times b_{1,1} & a_{1,2} \times b_{1,2} \\ a_{2,1} \times b_{2,1} & a_{2,2} \times b_{2,2} \end{bmatrix}$$

Determinant of a Matrix

Determinant - describing how linear transformations change area or volume. Also useful for solving linear equations and changing variables integrals.



Computing the Determinant

2x2 matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad |A| = ad - bc$$
$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix} \quad |B| = 4 \times 8 - 6 \times 3$$
$$= 32 - 18$$
$$= 14$$

3x3 matrices

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$
$$|A| = a \cdot \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \cdot \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \cdot \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

Matrix Concepts in R

```
# a and b are two square matrices
```

```
a + b # Addition
```

```
a %*% b # Multiplication
```

```
t(a) # Transpose
```

```
det(a) # Determinant
```

```
solve(a) # Inverse
```

Test these for yourself by hand and then using R

```
h <- matrix(c(2,7,19,4), nrow = 2, byrow = F)
```

```
g <- matrix(c(1,3,4,2), nrow = 2, byrow = F)
```


Solving equations using matrices

Matrices make solving linear equations easier/faster

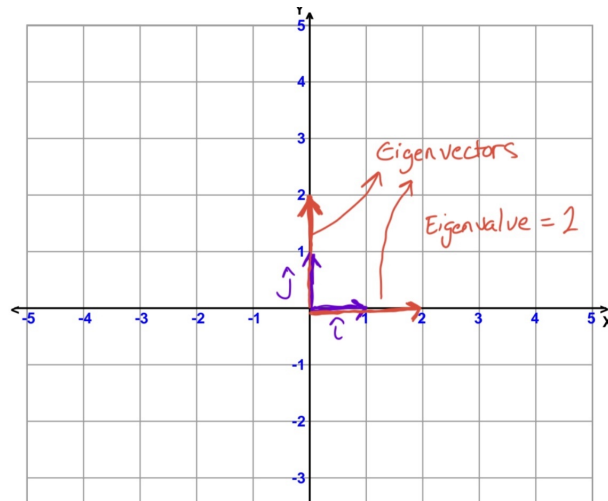
$$\begin{array}{rcl} x & + & y + z = 6 \\ & & 2y + 5z = -4 \\ 2x & + & 5y - z = 27 \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix} \dots$$

Eigenvalues and Eigenvectors

In (some) linear transformations, there are vectors that don't change direction and are only scaled (stretched or shrunk) within their own span.

Eigenvectors are the vectors that remain pointing in the same direction.

Eigenvalues are the scalars that the eigenvectors are stretch/shrunk by.



Eigenvectors and Eigenvalues

Almost all vectors change direction when multiplying a matrix, \mathbf{A} .

Eigenvectors, \mathbf{x} , are certain vectors that have the same direction as \mathbf{Ax} . The **Eigenvalue**, λ is the scalar by which \mathbf{x} is *stretched, shrunk, reversed* or *remained unchanged* when multiplied by \mathbf{A} .

$$\mathbf{Ax} = \lambda\mathbf{x}$$

We can find eigenvectors and eigenvalues of \mathbf{A} by setting the **determinant** of $\mathbf{A} - \lambda\mathbf{I}$ to be 0.

$$\det | (\mathbf{A} - \lambda\mathbf{I}) | = 0$$

Eigenvector and Eigenvalues in R

```
mat <- matrix(c(0.5, 0.5, 0.5, 0.5), byrow = T, nrow = 2)
mat
```

```
##      [,1] [,2]
## [1,]  0.5  0.5
## [2,]  0.5  0.5
```

```
eigen(mat)
```

```
## eigen() decomposition
## $values
## [1] 1 0
##
## $vectors
##      [,1]      [,2]
## [1,] 0.7071068 -0.7071068
## [2,] 0.7071068  0.7071068
```

Take what you have learned today and be able to:

- Perform linear regression using matrices (linear regression from scratch without using `lm`)
- Calculate eigenvalues and eigenvectors (by hand)

Extra reading

Very stuck?

- [Essence of Linear Algebra by 3Blue1Brown](#)
- [Maths is fun Intro to Matrices](#)
- [Lumen Learning Boundless Algebra Intro to Matrices](#)
- [Maths is Fun Eigenvectors and Eigenvalues](#)

Further Reading

- [Chapter 2 Deep Learning by Goodfellow, Bengio & Courville](#)

References

- [Chapter 2 Deep Learning by Goodfellow, Bengio & Courville](#)
- [Maths is fun Intro to Matrices](#)
- [Lumen Learning Boundless Algebra Intro to Matrices](#)
- [Dr Kelly Trinh MA5832 Week 1: Linear Algebra](#)

Slides

- [xaringan](#), [xaringantheme](#), [remark.js](#), [knitr](#), [R Markdown](#)