Judge input Hidden

Omni-MATH

Problem

Call a positive integer almost-square if it can be written as $a \cdot b$, where a and b are integers and $a \leq b \leq \frac{4}{3} a$. How many almost-square positive integers are less than or equal to 1000000? Your score will be equal to $25-65 \frac{|A-C|}{\min(A, C)}$.

Reference answer

130348

Solution

[...] Let's assume that it's relatively unlikely that a number is almost-square in more than one way. Then the number of almost square numbers less than n will be approximately [...] 160000. We can do better, though. For example, we summed $\frac{a}{3}$ all the way up to $\sqrt{n}$, but we are really overcounting here because when a is close to $\sqrt{n}$, $a \cdot b$ will be less than n only when $b \leq \frac{n}{a}$, as opposed to $b \leq \frac{4 a}{3}$. So we should really be taking the sum [...], so 143840 would be the best guess if we were to use this strategy. In reality, we would want to round down a bit in both cases, since we are overcounting (because numbers could be square-free in multiple ways), so we should probably answer something like 140000. A final refinement to our calculation (and perhaps easier than the previous one), is to assume that the products $a \cdot b$ that we consider are randomly distributed between 1 and n, [...] Giving us an answer of 134000, which is very close to the correct answer. The actual answer was found by computer, using the following C++ program: [...] 130348.

Metadata

Id: 2501 · Domain: Number Theory · Difficulty: 5.25 · Source: HMMT

Model

Model generation

143498

Token use

Reasoning tokens: 31296 – Output tokens: 12

Omni-Judge

Inputs: problem, model generation, reference answer

Extracted final answer

143498

Judge: Incorrect

The student's answer of 143498 is incorrect in comparison to the reference answer, which is 130348. This means that the student counted almost-square positive integers less than or equal to 1000000 as more instances than the correct number. The discrepancy between these two counts indicates a difference in how they defined and enumerated almost-square numbers.