

Omni-MATH

Problem

Let a_1, a_2, \dots, a_n be a permutation of $1, 2, \dots, n$. Among all possible permutations, find the minimum of $\sum_{i=1}^n \min(a_i, 2i-1)$.

Reference answer

$$\sum_{i=1}^n \min(n+1-i, 2i-1)$$

Metadata

Id: 56 · Domain: Algebra · Difficulty: 8
· Source: China Team Selection Test

Model

Model generation

Let $t_i = 2i-1$. For any $i < j$ with $t_i \leq t_j$ and $a_i \leq a_j$, swapping a_i and a_j does not increase $\min(a_i, t_i) + \min(a_j, t_j)$, since $\min(a_i, t_i) + \min(a_j, t_j) \leq \min(a_j, t_i) + \min(a_i, t_j)$. Thus an optimal permutation has a_i nonincreasing as i increases (i.e., pair largest numbers with smallest thresholds). Hence the minimum is $S_{\min} = \sum_{i=1}^n \min(n-i+2, 2i-1)$. Let $k = \left\lfloor \frac{n+2}{3} \right\rfloor$. Then for $i < k$, $2i-1 \leq n-i+1$; for $i \geq k$, $n-i+1 \leq 2i-1$. Therefore $S_{\min} = \sum_{i=1}^{k-1} (2i-1) + \sum_{i=k}^n (n-i+1) = (k-1)2 + \frac{(n-k+1)(n-k+2)}{2}$. This simplifies to $S_{\min} = \frac{n(n+1)}{3}$ if $n \equiv 0$ or $2 \pmod{3}$, $S_{\min} = \frac{n(n+1)+1}{3}$ if $n \equiv 1 \pmod{3}$, i.e., $S_{\min} = \left\lfloor \frac{n(n+1)}{3} \right\rfloor$. An attaining permutation is $a_i = n-i+1$. $\left\lfloor \frac{n(n+1)}{3} \right\rfloor$

Token use

Reasoning tokens: 5504 – Output tokens: 373

Judge

Inputs: problem, model generation, reference answer

Extracted final answer

$$\left\lfloor \frac{n(n+1)}{3} \right\rfloor$$

Omni-Judge: Incorrect

Although both the student's answer and reference answer are expressions for a summation, they differ in their mathematical structures. The student's expression includes the ceiling function (indicated by $\lceil \dots \rceil$), whereas the reference answer directly evaluates to a sum without any additional functions applied.

The main difference between these two results stems from how they handle the condition for $i \geq k$. In the reference solution, it is directly incorporated into the summation as part of the min function ($\min(n+1-i, 2i-1)$). However, in the student's solution, this condition is used to derive a specific formula involving the ceiling function.

As a result, despite sharing some conceptual similarities, these expressions do not have the same meaning due to their distinct mathematical formulations. Thus, the student's answer does not capture the correct interpretation of the reference answer.

GPT-5 mini: Correct

The response's initial expression $\sum \min(n+1-i, 2i-1)$ matches the provided correct_answer exactly. The response then simplifies this sum to the closed form $\lfloor n(n+1)/3 \rfloor$ (equivalently the piecewise form given), which is algebraically equivalent to that sum (verified by the $k = \lfloor (n+2)/3 \rfloor$ partition leading to the stated piecewise values). Therefore the extracted final answer is equivalent to the correct_answer.