Quantitative Method

1. A 24 year old is using the following information to plan her retirement:

Current age	24
Expected retirement age	68
Life expectancy	93
Current annual expenditures	$30,000
Expected inflation rate of	
Current expenditures until retirement	3%
Expected return on investment	8%

She assumes her consumption expenditures will increase with the rate of inflation, 3 percent, until she retires. Upon retiring she will have end-of-year expenditures equal to her consumption expenditure at age 68. The minimum amount that she must accumulate by age 68 in orderto fund her retirement is closest to:

A. $928,000.
B. $1,176,000.
C. $1,552,000.

Answer: B

Her consumption spending (currently $30,000 annually) increases with the rate of inflation (3%) over the next 44 years until she retires. Her annual consumption spending at the time she retires will be $110,143.57 (PV = 30,000, %I = 3, N = 44, solve for FV). To support that level of spending for 25 years of retirement assuming an 8% return on her retirement account, she must accumulate $1,175,756 by her retirement date (PMT = 110,143.57, N = 25, %I = 8, solve for PV).

2.A graphic depiction of a continuous distribution that shows the left tail to be longer than the right tail is best described as having:

A. lepto-kurtosis.
B. positive skewness.
C. negative skewness.

Answer: C

A negatively skewed distribution appears as if the left tail has been pulled away from the mean.

3. The manager of a pension fund determines that during the past five years 85 percent of the stocks in the portfolio have paid a dividend and 40 percent of the stocks have announced a stock split. If 95 percent of the stocks have paid a dividend and/or announced a stock split, the joint probability of a stock paying a dividend and announcing a stock split is closest to:

A. 30%.
B. 45%.
C. 55%.

Answer: A

The probability that at least one of two events will occur is the sum of the probabilities of the separate events less the joint probability of the two events. P(A or B) = P(A) + P(B) – P(AB) 95% = 85% + 40% – P(AB); therefore P(AB) = 30%

4. A portfolio manager gathers the following information about three possible asset allocations:

Allocation	Expected annual return	Standard deviation of return
I13%	6%	
II	26%	14%
III	32%	20%

The manager's client has stated that her minimum acceptable return is 8 percent. Based on Roy's safety-first criterion, the most appropriate allocation is:

A. I

B. II

C. III

Answer: B

Roy's safety-first ratio = $[E(R_P) - R_L] / \sigma_P$ with the optimal portfolio having the highest ratio. The safety-first ratios for the three allocations are:

Allocation	Safety-first ratio
I	0.83
II	1.29
III	1.20

5. A sample of 100 observations drawn from a normally distributed population has a sample mean of 12 and a sample standard deviation of 4. Using the extract from the z-distribution given below, find the 95% confidence interval for the population mean. The 95% confidence interval is closest to:

Cumulative Probabilities for a Standard Normal Distribution

P(Z ≤ x) = N(x) for x ≥ 0 or P(Z ≤ z) = N(z) for z ≥ 0

x or z	0	0.01	0.02	0.03	0.04	.05	.06	.07	.08	.09
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817

A. 7.840 to 27.683

B. 11.216 to 12.784

C. 11.340 to 12.660

Answer = B

The 95% confidence interval uses $z_{0.025}$ as the reliability factor. The cumulative probability value closest to 0.975 provides the appropriate value of $z_{0.025}$ which is $z_{0.025}$ = 1.96. The confidence interval is formed as: $\overline{X} \pm z_{\frac{\alpha}{2}} ? \frac{s}{\sqrt{n}}$. In this problem, 12 ± 1.96 × (4 / √100) = 12 ± 1.96 × 0.4. Thus the confidence interval spans 11.216 to 12.784.

6. Using the sample results given below, drawn as 25 paired observations from their underlying distributions, test if the mean returns of the two portfolios differ from each other at the 1% level of statistical significance. Assume the underlying distributions of returns for each portfolio are normal and that their population variances are not known.

	Portfolio 1	Portfolio 2	Difference
Mean Return	17.00	21.25	4.25
Standard Deviation	15.50	15.75	6.25

t-statistic for 24 df and at the 1% level of statistical significance = 2.807
Based on the paired comparisons test of the two portfolios, the most appropriate conclusion is:
A. reject the hypothesis that the mean difference equals zero as the computed test statistic exceeds 2.807.
B. accept the hypothesis that the mean difference equals zero as the computed test statistic exceeds 2.807.
C. accept the hypothesis that the mean difference equals zero as the computed test statistic is less than 2.807.

Answer = A

The test statistic is: $\left(\overline{d} - \mu_{d0}\right) \Big/ S_d/\sqrt{n}$, where $\overline{d}$ is the mean difference, μ_{d0} is the hypothesized difference in the means, S_d is the sample standard deviation of differences, and n is the sample size. In this case, the test statistic equals: (4.25-0)/(6.25/√25)= 3.40. As 3.40 > 2.807, we reject the null hypothesis that the mean difference is zero.

7. The joint probability of returns, for securities A and B, are as follows:
Joint Probability Function of Security A and Security B Returns

(Entries are joint probabilities)	Return on security B=30%	Return on security B=20%
Return on security A= 25%	0.60	0
Return on security A= 20%	0	0.40

The covariance of the returns between securities A and B is closest to:
A. 3.
B. 12.

C. 24.

Answer: B

Expected return on security A = 0.6 × 25% + 0.4 × 20% = 15% + 8% = 23%

Expected return on security B = 0.6 × 30% + 0.4 × 20% = 18% + 8% = 26%

Cov (R_A,R_B) = 0.6 [(25 − 23)(30 −26)] +0.4 [(20 − 23)(20 − 26)]

= 0.6(2 × 4) + 0.4(−3 × −6) = 0.6(8) +0.4(18) = 4.8 + 7.2 = 12.

8. A two-tailed t-test of the null hypothesis that the population mean differs from zero has a p-value of 0.0275. Using a significance level of 5%, the most appropriate conclusion is:

A. reject the null hypothesis.

B. accept the null hypothesis.

C. the chosen significance level is too high.

Answer = A

The *p*-value is the smallest level of significance at which the null hypothesis can be rejected. In this case, the given *p*-value is less than the given level of significance and we reject the null hypothesis.

9. If the distribution of the population from which the samples are drawn is positively skewed, and given that the sample size is large, the sampling distribution of the sample means is most likely:

A. approximately normally distributed.

B. to have a variance equal to that of the entire population.

C. to have a mean smaller than the mean of the entire population.

Answer: A

The central limit theorem establishes that the sampling distribution of sample means will be approximately normal, will have a mean equal to the population mean, and will have a variance equal to the population variance divided by the sample size.

10. Which of the following statements about a normal distribution is least accurate? A normal distribution:

A. has an excess kurtosis of 3.

B. is completely described by two parameters.

C. can be the linear combination of two or more normal random variables.

Answer: A

A normal distribution has a kurtosis of 3. Its excess kurtosis (kurtosis – 3.0) equals zero.