Universidade Federal do Rio Grande do Sul
INF05516 - Semântica Formal - Turma U - Trabalho Final

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Linguagem L1 extendida

1 Sintaxe abstrata

$\Gamma \in$ Ambientes de Tipos
$\Gamma ::= \epsilon \mid (x : T, v) :: \Gamma$ onde $x \in$ Ident é o nome de uma variável, T é o tipo associado e $v \in e \cup$ None, indicando o valor associado ou sua ausência.

$T \in$ Types
$T ::=$ int $\mid$ bool $\mid T_1 \rightarrow T_2 \mid$ maybe $T \mid T$ list

$v \in$ Values
$v ::= n|b|\langle x, e, \rho \rangle|\langle f, x, e, \rho \rangle$

$\rho \in$ Env
$\rho ::= [] \mid \rho, x \rightarrow v$

$e \in$ L1
$e ::= n \mid b \mid e_1$ op $e_2 \mid$ if e_1 then e_2 else e_3
$\quad \mid x \mid e_1 \ e_2 \mid$ fn $x{:}T \Rightarrow e \mid$ let $x{:}T = e_1$ in e_2
$\quad \mid$ let rec $f{:}T_1 \rightarrow T_2 = ($fn $y{:}T_1 \Rightarrow e_1)$ in e_2
$\quad \mid$ nil $: T \mid e_1 :: e_2 \mid$ isempty $e \mid$ hd $e \mid$ tl e
$\quad \mid$ match e_1 with $\mid$ nil $\rightarrow e_2 \mid x{::}xs \rightarrow e_3$
$\quad \mid$ nothing $: T \mid$ just e
$\quad \mid$ match e_1 with $\mid$ nothing $\rightarrow e_2 \mid$ just $x \rightarrow e_3$

onde

$n \in$ conjunto de numerais inteiros
$b \in$ true, false
$x \in$ Ident
op $\in$ +, -, *, div, =, <, >, and, or, ...

2   Sistema de Tipos

$$\frac{}{\Gamma \vdash n : \text{int}} \ (\text{TInt}) \qquad \frac{}{\Gamma \vdash b : \text{bool}} \ (\text{TBool}) \qquad \frac{\Gamma(x) = T}{\Gamma \vdash x : T} \ (\text{T-Id}) \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \ (\text{TOp+})$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \text{bool}}{\Gamma \vdash e_1 \wedge e_2 : \text{bool}} \ (\text{TOp}\wedge) \qquad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int} \quad e_2 \neq 0}{\Gamma \vdash e_1 \text{ div } e_2 : \text{int}} \ (\text{TOpDiv})$$

$$\frac{\Gamma, x{:}T \vdash e : T'}{\Gamma \vdash (\text{fn } x{:}T \Rightarrow e) : T{\rightarrow}T'} \ (\text{TFn}) \qquad \frac{\Gamma \vdash e_1 : T{\rightarrow}T' \quad \Gamma \vdash e_2 : T}{\Gamma \vdash e_1 \ e_2 : T'} \ (\text{TApp})$$

$$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : T \quad \Gamma \vdash e_3 : T}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : T} \ (\text{T-If}) \qquad \frac{\Gamma \vdash e_1 : T \quad \Gamma, x{:}T \vdash e_2 : T'}{\Gamma \vdash \text{let } x{:}T = e_1 \text{ in } e_2 : T'} \ (\text{T-Let})$$

$$\frac{\Gamma, f{:}T_1{\rightarrow}T_2, y{:}T_1 \vdash e_1 : T_2 \quad \Gamma, f{:}T_1{\rightarrow}T_2 \vdash e_2 : T}{\Gamma \vdash \text{let rec } f{:}T_1{\rightarrow}T_2 = (\text{fn } y{:}T_1 \Rightarrow e_1) \text{ in } e_2 : T} \ (\text{T-LetRec}) \qquad \frac{}{\Gamma \vdash \text{nil} : \text{List } T} \ (\text{T-Nil})$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash e{::}\text{nil} : T \text{ List}} \ (\text{TConsNil}) \qquad \frac{\Gamma \vdash e_1 : T \quad \Gamma \vdash e_2 : T \text{ List}}{\Gamma \vdash e_1{::}e_2 : T \text{ List}} \ (\text{TCons}) \qquad \frac{\Gamma \vdash e : T \text{ List}}{\Gamma \vdash \text{isempty } e : \text{bool}} \ (\text{TIsempty})$$

$$\frac{\Gamma \vdash e : T \text{ List}}{\Gamma \vdash \text{hd}{::}e : T} \ (\text{THd}) \qquad \frac{\Gamma \vdash e : T \text{ List}}{\Gamma \vdash \text{tl}{::}e : T \text{ List}} \ (\text{TTl}) \qquad \frac{}{\Gamma \vdash \text{nothing} : \text{maybe } T} \ (\text{TNothing})$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \text{just } e : \text{maybe } T} \ (\text{TJust}) \qquad \frac{\Gamma \vdash e_1 : T \text{ List} \quad \Gamma \vdash e_2 : T' \quad \Gamma, x{:}T, xs{:}T \text{ List} \vdash e_3 : T'}{\Gamma \vdash \text{match } e_1 \text{ with } | \text{ nil} \rightarrow e_2 \ | \ x{::}xs \rightarrow e_3 : T'} \ (\text{TMatchList})$$

$$\frac{\Gamma \vdash e_1 : \text{Maybe } T \quad \Gamma \vdash e_2 : T' \quad \Gamma, x{:}T \vdash e_3 : T' \quad \Gamma, y{:}\text{Maybe } T \vdash e_4 : T'}{\Gamma \text{ match } e_1 \text{ with } | \text{ nothing} \rightarrow e_2 \ | \ \text{just } x \rightarrow e_3 : T'} \ (\text{TMatchMaybe})$$

3 Semântica Operacional Big-step

$$\rho \vdash n \Downarrow n \quad \text{(BNum)} \qquad \rho \vdash b \Downarrow b \quad \text{(BBool)} \qquad \frac{\rho \vdash e_1 \Downarrow \text{false} \quad \rho \vdash e_3 \Downarrow v}{\rho \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \quad \text{(BIfFalse)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{true} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow v} \quad \text{(BIfTrue)} \qquad \rho \vdash (\text{fn } x{:}T \Rightarrow e) \Downarrow \langle x, e, \rho \rangle \quad \text{(BFn)} \qquad \rho \vdash \text{nil} \Downarrow \text{nil} \quad \text{(BNil)}$$

$$\frac{\rho \vdash e_1 \Downarrow v' \quad \rho, x \to v' \vdash e_2 \Downarrow v}{\rho \vdash \text{let } x : T = e_1 \text{in } e_2 \Downarrow v} \quad \text{(BLet)} \qquad \frac{\rho \vdash e_1 \Downarrow v_1 \quad \rho \vdash e_2 \Downarrow v_2}{\rho \vdash e_1 :: e_2 \Downarrow v_1 :: v_2} \quad \text{(BCons)}$$

$$\frac{\rho \vdash e_1 \Downarrow n_1 \quad \rho \vdash e_2 \Downarrow n_2 \quad [\![n]\!] = [\![n_1]\!] + [\![n_2]\!]}{\rho \vdash e_1 + e_2 \Downarrow n} \quad \text{(BOp+)} \qquad \frac{\rho \vdash e_1 \Downarrow n_1 \quad \rho \vdash e_2 \Downarrow n_2 \quad n_2 \neq 0 \quad [\![n]\!] = \frac{[\![n_1]\!]}{[\![n_2]\!]}}{\rho \vdash e_1 \text{ div } e_2 \Downarrow n} \quad \text{(BOpDiv)}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle x, e, \rho' \rangle \quad \rho \vdash e_2 \Downarrow v' \quad \rho', x \to v' \vdash e \Downarrow v}{\rho \vdash e_1 e_2 \Downarrow v} \quad \text{(BApp)}$$

$$\frac{\rho, f \to \langle f, x, e_1, \rho \rangle \vdash e_2 \Downarrow v}{\rho \vdash \text{let rec } f : T \to T' = \text{fn } x : T \Rightarrow e_1 \text{ in } e_2 \Downarrow v} \quad \text{(BLetRec)}$$

$$\frac{\rho \vdash e_1 \Downarrow \langle f, x, e, \rho' \rangle \quad \rho \vdash e_2 \Downarrow v' \quad \rho', x \to v', \to \langle f, x, e, \rho' \rangle \vdash e \Downarrow v}{\rho \vdash e_1 e_2 \Downarrow v} \quad \text{(BAppRec)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{nil}}{\rho \vdash \text{isempty } e_1 \Downarrow \text{true}} \quad \text{(BIsemptyNil)} \qquad \frac{\rho \vdash e_1 \Downarrow v :: vs}{\rho \vdash \text{isempty } e_1 \Downarrow \text{false}} \quad \text{(BIsemptyCons)}$$

$$\frac{\rho \vdash e_1 \Downarrow v :: vs}{\rho \vdash \text{hd } e_1 \Downarrow v} \quad \text{(BHd)} \qquad \frac{\rho \vdash e_1 \Downarrow v :: vs}{\rho \vdash \text{tl } e_1 \Downarrow vs} \quad \text{(BTl)} \qquad \rho \vdash \text{nothing } \Downarrow \text{nothing} \quad \text{(BNothing)}$$

$$\frac{\rho \vdash e \Downarrow v}{\rho \vdash \text{just } e \Downarrow \text{just } v} \quad \text{(BJust)} \qquad \frac{\rho \vdash e_1 \Downarrow \text{nil} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with } | \text{ nil} \to e_2 | x :: xs \to e_3 \Downarrow v} \quad \text{(BMatchNil)}$$

$$\frac{\rho \vdash e_1 \Downarrow v :: vs \quad \rho \vdash e_3 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with } | \text{ nil} \to e_2 | x :: xs \to e_3 \Downarrow v} \quad \text{(BMatchCons)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{nothing} \quad \rho \vdash e_2 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with} \mid \text{nothing} \to e_2 \mid \text{just x} \to e_3 \Downarrow v} \quad \text{(BMatchNothing)}$$

$$\frac{\rho \vdash e_1 \Downarrow \text{just}(v) \quad \rho \vdash e_3 \Downarrow v}{\rho \vdash \text{match } e_1 \text{ with} \mid \text{nothing} \to e_2 \mid \text{just x} \to e_3 \Downarrow v} \quad \text{(BMatchJustx)}$$