Report NO4LSP

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INTRODUZIONE

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Model

The function described in this problem is the following

$$F(\mathbf{x}) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x)$$
$$f_k(\mathbf{x}) = x_k - \frac{x_{k+1}^2}{10}, \quad 1 \le k < n$$
$$f_n(\mathbf{x}) = x_n - \frac{x_1^2}{10}$$

where n denotes the dimensionality of the input vector $\mathbf{x}$.
The starting point for the minimization is the vector $\mathbf{x}_0 = [2, 2, \ldots, 2]$.

To be able to say something more about the behaviour of the problem is useful to look at the gradient of the function $F(\mathbf{x})$ and at its Hessian matrix.

$$\nabla F(\mathbf{x}) = \begin{bmatrix} \frac{\partial F}{\partial x_1}(\mathbf{x}) \\ \vdots \\ \frac{\partial F}{\partial x_k}(\mathbf{x}) \\ \vdots \\ \frac{\partial F}{\partial x_n}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1}\frac{1}{2}\left[f_n^2 + f_1^2\right](\mathbf{x}) \\ \vdots \\ \frac{\partial}{\partial x_k}\frac{1}{2}\left[f_{k-1}^2 + f_k^2\right](\mathbf{x}) \\ \vdots \\ \frac{\partial}{\partial x_n}\frac{1}{2}\left[f_{n-1}^2 + f_n^2\right](\mathbf{x}) \end{bmatrix} = \begin{bmatrix} -\frac{x_1}{5}\left(x_n - \frac{x_1^2}{10}\right) + \left(x_1 - \frac{x_1^2}{10}\right) \\ \vdots \\ -\frac{x_k}{5}\left(x_{k-1} - \frac{x_k^2}{10}\right) + \left(x_k - \frac{x_{k+1}^2}{10}\right) \\ \vdots \\ -\frac{x_n}{5}\left(x_{n-1} - \frac{x_n^2}{10}\right) + \left(x_n - \frac{x_1^2}{10}\right) \end{bmatrix}$$

Due to the particular structure of the function, the Hessian matrix as a sparse structure, with only 3 diagonals different from zero. The non-zero elements are the following:

$$\frac{\partial^2 F}{\partial x_k^2}(\mathbf{x}) = -\frac{1}{5}x_{k-1} - \frac{3}{50}x_k^2 + 1, \quad 1 < k \le n$$
$$\frac{\partial^2 F}{\partial x_k \partial x_{k+1}}(\mathbf{x}) = -\frac{1}{5}x_{k+1}, \quad 1 \le k < n$$
$$\frac{\partial^2 F}{\partial x_k \partial x_{k-1}}(\mathbf{x}) = -\frac{1}{5}x_k, \quad 1 < k \le n$$

$$\frac{\partial^2 F}{\partial x_1^2}(\mathbf{x}) = -\frac{1}{5}x_n - \frac{3}{50}x_1^2 + 1,$$
$$\frac{\partial^2 F}{\partial x_n \partial x_1}(\mathbf{x}) = -\frac{1}{5}x_1$$
$$\frac{\partial^2 F}{\partial x_1 \partial x_n}(\mathbf{x}) = -\frac{1}{5}x_n$$

We can now easily notice that the gradient of the function is null when all the components of the vector $\mathbf{x}$ are equal to 0. The Hessian matrix is positive definite, so the point $\mathbf{x} = \mathbf{0}$ is a minimum of the function $F(\mathbf{x})$. Because of the definition of the function, 0 is the lowest value that the function can assume, so the minimum is global.

Nealder Mead Method

	avg fbest	avg num of iters	avg time of exec (sec)	n failure	avg roc
10	1.0000e-04	2.1864e+02	4.5116e+00	0.0000e+00	NaN
25	0.0000e+00	1.6804e+03	3.1524e+01	0.0000e+00	NaN
50	2.9039e+01	1.4007e+04	2.6917e+02	1.0000e+01	NaN

Figura 1: Resultats obtained by running the symplex method on the function $F(\mathbf{x})$.

Modified Newton Method

	avg fbest	avg gradf_norm	avg num of iters	avg time of exec (sec)	n failure	avg roc
1000	6.7091e-10	3.6631e-05	4.0000e+00	1.6874e-02	0.0000e+00	1.9391e+00
10000	2.4479e-17	6.9969e-09	4.0000e+00	1.4366e-02	0.0000e+00	2.0053e+00
100000	8.1020e-21	1.2730e-10	5.0000e+00	1.9585e-01	0.0000e+00	2.0026e+00

Figura 2: Resultats obtained by running the Modified Newton Method on the function $F(\mathbf{x})$ using the exact derivatives.

CONCLUSIONI