Fluid Mechanics & Thermodynamics — Problem Set

Practice Sheet

1. Bernoulli's Equation in Pipe Flow

Problem Water flows through a horizontal pipe that narrows from diameter $D_1 = 10$ cm to $D_2 = 5$ cm. The pressure at section 1 is $P_1 = 200$ kPa and velocity is $v_1 = 2$ m/s. Find the pressure P_2 at the narrow section.

Given
- $D_1 = 0.10$ m, $D_2 = 0.05$ m
- $P_1 = 200 \times 10^3$ Pa, $v_1 = 2$ m/s
- $\rho_{water} = 1000$ kg/m^3
- Horizontal pipe $\Rightarrow z_1 = z_2$

Approach Apply **Continuity** ($A_1 v_1 = A_2 v_2$) and **Bernoulli** (incompressible, steady, inviscid):

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

Solution **Step 1:** Cross-sectional areas

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.10)^2}{4} = 7.854 \times 10^{-3} \text{ m}^2$$

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi(0.05)^2}{4} = 1.963 \times 10^{-3} \text{ m}^2$$

Step 2: Velocity at section 2 (Continuity)

$$v_2 = v_1 \frac{A_1}{A_2} = 2 \cdot \frac{7.854}{1.963} = 8 \text{ m/s}$$

Step 3: Pressure at section 2 (Bernoulli, $z_1 = z_2$)

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 200000 + 500(4 - 64)$$

$$P_2 = 200000 - 30000 = 170000 \text{ Pa}$$

Result: $P_2 = 170$ kPa

2. First Law of Thermodynamics

Problem A piston-cylinder device contains 0.5 kg of air initially at 300 K and 100 kPa. Heat is added until the temperature reaches 600 K. The process is isobaric. Calculate: (a) Work done by the gas, (b) Heat transfer, (c) Change in internal energy.

Given
- $m = 0.5$ kg, $T_1 = 300$ K, $T_2 = 600$ K
- $P = 100$ kPa (constant, isobaric)
- Air: $c_p = 1.005$ kJ/(kg·K), $c_v = 0.718$ kJ/(kg·K)
- $R_{air} = c_p - c_v = 0.287$ kJ/(kg·K)

Approach For isobaric process: $W = P\Delta V = mR\Delta T$. First Law: $Q = \Delta U + W$.

Solution **(a) Work done:**

$$W = mR(T_2 - T_1) = 0.5 \times 0.287 \times (600 - 300)$$

$$W = 0.5 \times 0.287 \times 300 = 43.05 \text{ kJ}$$

(b) Heat transfer (isobaric $\Rightarrow Q = mc_p\Delta T$):

$$Q = mc_p(T_2 - T_1) = 0.5 \times 1.005 \times 300 = 150.75 \text{ kJ}$$

(c) Change in internal energy:

$$\Delta U = mc_v(T_2 - T_1) = 0.5 \times 0.718 \times 300 = 107.70 \text{ kJ}$$

Result: $W = 43.05$ kJ, $Q = 150.75$ kJ, $\Delta U = 107.70$ kJ

3. Heat Transfer: Conduction

Problem A wall of thickness $L = 0.2$ m and thermal conductivity $k = 1.5$ W/(m·K) separates two rooms at temperatures $T_1 = 25$řC and $T_2 = -5$řC. The wall area is $A = 10$ m^2. Find the heat transfer rate through the wall.

Given
- $L = 0.2$ m, $k = 1.5$ W/(m·K), $A = 10$ m^2
- $T_1 = 25$řC, $T_2 = -5$řC $\Rightarrow \Delta T = 30$ K

Approach Fourier's Law of heat conduction (1D, steady state):

$$\dot{Q} = -kA\frac{dT}{dx} = kA\frac{T_1 - T_2}{L}$$

Solution

$$\dot{Q} = kA\frac{\Delta T}{L} = 1.5 \times 10 \times \frac{30}{0.2} = 2250 \text{ W}$$

Result: $\dot{Q} = 2250$ W $= 2.25$ kW

4. Fluid Statics: Hydrostatic Pressure

Problem A tank contains oil ($\rho_{oil} = 850$ kg/m^3) floating on water. The oil layer is 2 m thick, and there is 3 m of water below. Find the gauge pressure at the bottom of the tank.

Given
- $\rho_{oil} = 850$ kg/m^3, $h_{oil} = 2$ m
- $\rho_{water} = 1000$ kg/m^3, $h_{water} = 3$ m
- $g = 9.81$ m/s^2

Approach Hydrostatic pressure: $P = P_0 + \rho g h$. For layered fluids, sum contributions:

$$P_{gauge} = \rho_{oil} g h_{oil} + \rho_{water} g h_{water}$$

Solution

$$P_{gauge} = (850 \times 9.81 \times 2) + (1000 \times 9.81 \times 3)$$

$$P_{gauge} = 16677 + 29430 = 46107 \text{ Pa} \approx 46.1 \text{ kPa}$$

Result: $P_{gauge} = 46.1$ kPa

5. Carnot Cycle Efficiency

Problem A heat engine operates between reservoirs at $T_H = 800$ K and $T_C = 300$ K. The engine receives 1000 kJ of heat per cycle. Calculate: (a) Carnot efficiency, (b) Maximum work output, (c) Heat rejected.

Given
- $T_H = 800$ K, $T_C = 300$ K
- $Q_H = 1000$ kJ

Approach Carnot efficiency: $\eta_C = 1 - \frac{T_C}{T_H}$. Work: $W = \eta_C Q_H$. Rejected heat: $Q_C = Q_H - W$.

Solution **(a) Carnot efficiency:**

$$\eta_C = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{800} = 1 - 0.375 = 0.625 = 62.5\%$$

(b) Maximum work output:

$$W_{max} = \eta_C \cdot Q_H = 0.625 \times 1000 = 625 \text{ kJ}$$

(c) Heat rejected:

$$Q_C = Q_H - W = 1000 - 625 = 375 \text{ kJ}$$

Result: $\eta_C = 62.5\%$, $W_{max} = 625$ kJ, $Q_C = 375$ kJ

6. Reynolds Number & Flow Regime

Problem Water at 20°C ($\nu = 1.0 \times 10^{-6}$ m^2/s) flows through a pipe of diameter 5 cm at velocity 0.5 m/s. Determine the Reynolds number and flow regime.

Given
- $D = 0.05$ m, $v = 0.5$ m/s
- $\nu = 1.0 \times 10^{-6}$ m^2/s
- Transition criteria: Re < 2300 laminar, Re > 4000 turbulent

Solution

$$\text{Re} = \frac{vD}{\nu} = \frac{0.5 \times 0.05}{1.0 \times 10^{-6}} = \frac{0.025}{10^{-6}} = 25000$$

Result: Re $= 25000 \Rightarrow$ **Turbulent flow**

7. Projectile Motion (Classical Mechanics)

Problem A ball is thrown with initial velocity $v_0 = 20$ m/s at angle $\theta = 45$ř above horizontal. Find: (a) Maximum height, (b) Range, (c) Time of flight. Neglect air resistance.

Given
- $v_0 = 20$ m/s, $\theta = 45$ř, $g = 9.81$ m/s^2
- $v_{0x} = v_0 \cos\theta = 14.14$ m/s
- $v_{0y} = v_0 \sin\theta = 14.14$ m/s

Solution **(a) Maximum height:**

$$H = \frac{v_{0y}^2}{2g} = \frac{(14.14)^2}{2 \times 9.81} = \frac{200}{19.62} = 10.2 \text{ m}$$

(b) Range:

$$R = \frac{v_0^2 \sin(2\theta)}{g} = \frac{400 \times 1}{9.81} = 40.8 \text{ m}$$

(c) Time of flight:

$$T = \frac{2v_{0y}}{g} = \frac{2 \times 14.14}{9.81} = 2.88 \text{ s}$$

Result: $H = 10.2$ m, $R = 40.8$ m, $T = 2.88$ s

8. Work-Energy Theorem

Problem A 2 kg block slides down a frictionless incline of height 5 m. Find its speed at the bottom using conservation of energy.

Solution Conservation of mechanical energy: $E_i = E_f$

$$mgh + 0 = 0 + \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gh}$$

$$v = \sqrt{2 \times 9.81 \times 5} = \sqrt{98.1} = 9.9 \text{ m/s}$$

Result: $v = 9.9$ m/s (or $\sqrt{2gh}$ in general)

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