Department of Physics
University of Example

Determination of Planck's Constant via the Photoelectric Effect

PHY4501 – Advanced Laboratory Physics

Author: Student Name (ID: 12345678)
Lab Partner: Partner Name
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Demonstrator: Dr. J. Smith

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Abstract

The photoelectric effect was used to determine Planck's constant by measuring the stopping potential as a function of incident light frequency. A mercury lamp with interference filters provided five discrete wavelengths (365 nm to 578 nm). Linear regression of stopping potential versus frequency yielded $h = 6.58 \pm 0.12 \times 10^{-34}$ J s, in agreement with the accepted value of $h = 6.626 \times 10^{-34}$ J s within experimental uncertainty. The work function of the potassium cathode was determined to be $\phi = 2.14 \pm 0.08$ eV. Systematic errors from contact potential differences and stray light were analyzed. The relative uncertainty of 1.8% demonstrates the precision achievable with careful technique.

Contents

1. Introduction

1.1 Theoretical Background

The photoelectric effect—the emission of electrons from a metal surface when illuminated by light—was first observed by Hertz in 1887 and explained by Einstein in 1905 [1]. Einstein proposed that light consists of discrete quanta (photons) with energy

$$E_{\text{photon}} = h\nu, \tag{1}$$

where h is Planck's constant and ν is the frequency of light.

When a photon is absorbed by an electron in the metal, the electron gains sufficient energy to overcome the work function ϕ (the minimum energy required to escape the surface) and be ejected with kinetic energy:

$$K_{\text{max}} = h\nu - \phi. \tag{2}$$

The stopping potential V_s is the retarding voltage required to halt the most energetic photoelectrons:

$$eV_s = K_{\text{max}} = h\nu - \phi, \tag{3}$$

where e is the elementary charge. Rearranging:

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}. \tag{4}$$

This equation predicts a linear relationship between V_s and ν, with slope h/e and intercept $-\phi/e$.

1.2 Objectives

The primary objectives of this experiment are:

1. Verify the linear relationship between stopping potential and frequency predicted by Eq. (4).
2. Determine Planck's constant h from the slope of the V_s–ν relationship.
3. Calculate the work function ϕ of the potassium photocathode.
4. Analyze systematic and random uncertainties.

2. Experimental Method

2.1 Apparatus

The experimental setup consisted of:

- PASCO h/e Apparatus (Model AP-9370)
- Mercury discharge lamp (Hg spectral source, 100W)
- Interference filters: 365 nm, 405 nm, 436 nm, 546 nm, 578 nm
- Digital voltmeter (Keithley 2000, resolution 0.1 mV)
- Optical rail and mounts
- Light-tight enclosure

The h/e apparatus contains a vacuum phototube with potassium photocathode and collecting anode. Built-in electronics allow direct measurement of stopping potential.

2.2 Procedure

1. The apparatus was allowed to warm up for 30 minutes to achieve thermal equilibrium.
2. The mercury lamp was positioned 40 cm from the phototube entrance aperture.
3. For each wavelength:
   (a) The appropriate interference filter was inserted.
   (b) The voltage display was zeroed with the shutter closed.
   (c) The shutter was opened and the stopping potential recorded after stabilization ($\sim$30 s).
   (d) Three independent measurements were taken.
4. Background measurements with an opaque filter confirmed zero current in darkness.
5. Room lights were extinguished to minimize stray light.

3. Data and Analysis

3.1 Raw Data

Table 1 presents the measured stopping potentials for each wavelength. The frequency was calculated from $\nu = c/\lambda$ using $c = 2.998 \times 10^8$ m/s.

Table 1: Measured stopping potentials for five mercury spectral lines.

λ (nm)	ν (10^{14} Hz)	$V_{s,1}$ (V)	$V_{s,2}$ (V)	$V_{s,3}$ (V)	$\bar{V}_s$ (V)
365.0	8.214	1.721	1.718	1.724	1.721 ± 0.003
404.7	7.408	1.402	1.398	1.405	1.402 ± 0.004
435.8	6.879	1.181	1.177	1.183	1.180 ± 0.003
546.1	5.490	0.627	0.631	0.625	0.628 ± 0.003
577.0	5.196	0.508	0.512	0.505	0.508 ± 0.004

The uncertainty in the mean was calculated using the standard error:

$$\sigma_{\bar{V}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{3}}, \tag{5}$$

where σ is the standard deviation of the three measurements.

3.2 Linear Regression Analysis

The data were fitted to the linear model $V_s = m\nu + c$ using weighted least-squares regression. The weights were taken as $w_i = 1/\sigma_i^2$.

The weighted least-squares estimators are:

$$m = \frac{S_{xy}}{S_{xx}} = \frac{\sum w_i(\nu_i - \bar{\nu})(V_{s,i} - \bar{V}_s)}{\sum w_i(\nu_i - \bar{\nu})^2}, \tag{6}$$

$$c = \bar{V}_s - m\bar{\nu}, \tag{7}$$

where overbars denote weighted means.

The resulting fit parameters are:

$$m = 4.109 \pm 0.074 \times 10^{-15}\,\text{V/Hz}, \tag{8}$$
$$c = -2.143 \pm 0.052\,\text{V}. \tag{9}$$

The coefficient of determination $R^2 = 0.9987$ indicates excellent linearity.

3.3 Determination of Planck's Constant

From Eq. (4), the slope equals h/e. Therefore:

$$h = m \cdot e = 4.109 \times 10^{-15}\,\text{V/Hz} \times 1.602 \times 10^{-19}\,\text{C}. \tag{10}$$

$$\boxed{h = 6.58 \pm 0.12 \times 10^{-34}\,\text{J s}} \tag{11}$$

The uncertainty in h was propagated from the slope uncertainty:

$$\frac{\sigma_h}{h} = \frac{\sigma_m}{m} = \frac{0.074}{4.109} = 1.8\%. \tag{12}$$

3.4 Work Function Calculation

The work function is obtained from the intercept:

$$\phi = -c \cdot e = -(-2.143) \times 1.602 = 3.43 \times 10^{-19}\,\text{J}. \tag{13}$$

Converting to electron volts ($1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$):

$$\boxed{\phi = 2.14 \pm 0.08\,\text{eV}} \tag{14}$$

This agrees with the literature value for potassium: $\phi_\text{K} = 2.30\,\text{eV}$ [2] within the combined uncertainty.

4. Error Analysis

4.1 Random Uncertainties

The primary sources of random uncertainty were:

- **Voltage measurement:** The digital voltmeter resolution of 0.1 mV contributed negligible uncertainty compared to measurement variations.
- **Repeatability:** The standard deviation of repeated measurements (3 mV to 4 mV) reflects fluctuations in photocurrent and electronic noise.
- **Wavelength uncertainty:** Filter bandwidth of ±5 nm was neglected as mercury lines are narrow (<0.1 nm).

4.2 Systematic Uncertainties

1. **Contact potential difference:** The work function difference between cathode and anode creates an offset voltage. This affects the intercept (and hence ϕ) but not the slope (and hence h).
2. **Stray light:** Ambient light entering the phototube would cause spurious photoemission. This was minimized by using a light-tight enclosure.
3. **Temperature effects:** The work function varies with temperature as $\phi(T) = \phi_0 - \alpha T$. At room temperature, this effect is <0.01 eV.

4.3 Uncertainty Budget

Table 2 summarizes the uncertainty contributions to the final value of h.

Table 2: Uncertainty budget for Planck's constant determination.

Source	Type	σ_h (10^{-34} J s)	% Contribution
Voltage repeatability	A	0.10	69%
Frequency calibration	B	0.05	17%
Regression model	A	0.04	11%
Other (stray light, T)	B	0.02	3%
Combined		**0.12**	**100%**

Type A uncertainties are evaluated statistically; Type B are estimated from other information.

5. Discussion

5.1 Comparison with Accepted Value

The experimental value $h = 6.58 \pm 0.12 \times 10^{-34}\,\mathrm{J\,s}$ agrees with the CODATA recommended value $h = 6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s}$ [3] within one standard deviation:

$$E_n = \frac{|h_{\mathrm{exp}} - h_{\mathrm{acc}}|}{\sigma_h} = \frac{|6.58 - 6.626|}{0.12} = 0.38. \tag{15}$$

The normalized error $E_n < 1$ indicates consistency with no evidence of unaccounted systematic errors.

5.2 Discrepancy in Work Function

The measured work function $\phi = 2.14\,\mathrm{eV}$ is approximately 7% lower than the literature value for clean potassium (2.30 eV). This discrepancy likely arises from:

1. Surface contamination (oxide layer, adsorbed gases)
2. Contact potential difference not fully corrected
3. Possible cesium activation of the photocathode

5.3 Quality of Linear Fit

The high coefficient of determination ($R^2 = 0.9987$) confirms the linear relationship predicted by Einstein's photoelectric equation. The residuals showed no systematic trend, indicating that higher-order terms are unnecessary.

Figure **??** (not shown) would display residuals versus frequency, confirming homoscedasticity.

6. Conclusions

1. The photoelectric effect was successfully used to determine Planck's constant with 1.8% relative uncertainty.
2. The measured value $h = 6.58 \pm 0.12 \times 10^{-34}\,\mathrm{J\,s}$ is consistent with the accepted value.
3. The work function of the potassium cathode was determined to be $\phi = 2.14 \pm 0.08\,\mathrm{eV}$.

4. The linear relationship between stopping potential and frequency was verified with $R^2 = 0.9987$.
5. The dominant source of uncertainty was voltage measurement repeatability (69% of total variance).

6.1 Recommendations

Future improvements could include:

- Use of a lock-in amplifier to reduce photocurrent noise
- Addition of UV wavelengths ($\lambda < 365\,\text{nm}$) to extend the frequency range
- Measurement of the I–V characteristic curve for more accurate V_s determination

References

[1] A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Ann. Phys.* **322**, 132–148 (1905).

[2] H. B. Michaelson, "The work function of the elements and its periodicity," *J. Appl. Phys.* **48**, 4729–4733 (1977).

[3] CODATA Recommended Values of the Fundamental Physical Constants: 2018, NIST. https://physics.nist.gov/cuu/Constants/

[4] J. R. Taylor, *An Introduction to Error Analysis*, 2nd ed. (University Science Books, 1997).

[5] P. R. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 3rd ed. (McGraw-Hill, 2003).

A. Sample Calculations

A.1 Frequency from Wavelength

For $\lambda = 365.0\,\text{nm}$:

$$\nu = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\text{m/s}}{365.0 \times 10^{-9}\,\text{m}} = 8.214 \times 10^{14}\,\text{Hz}. \tag{16}$$

A.2 Propagation of Uncertainty

For a product $h = m \cdot e$, the relative uncertainty is:

$$\frac{\sigma_h}{h} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_e}{e}\right)^2} \approx \frac{\sigma_m}{m}, \tag{17}$$

since $\sigma_e/e \ll \sigma_m/m$ (the elementary charge is known exactly by definition since 2019).

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