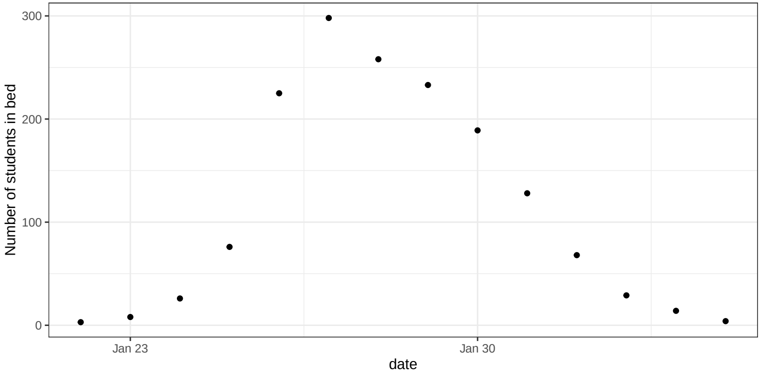
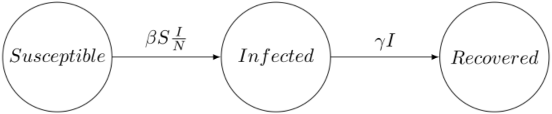
IV
Epidemiology model

1978 influenza outbreak in a British boarding school.

Data: daily number of students in bed.



Susceptible-Infected-Recovered (SIR) model



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\dot{S} &= -\beta S I / N \\
\dot{I} &= \beta S I / N - \gamma I \\
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\end{aligned}
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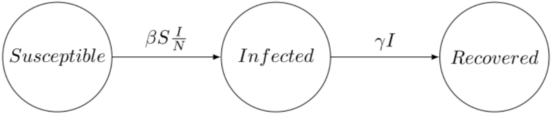

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β: average of number of contacts per person.

γ: rate of recovery of infected individuals.

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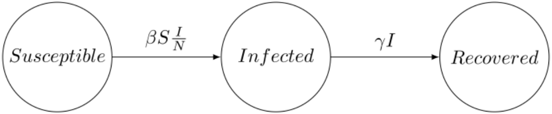

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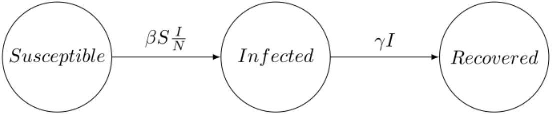

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Measurement model:

$p(y(t) \mid \beta, \gamma, \phi) = \text{negative-Binomial}(I(t), \phi)$

$\phi^{-1} \sim \text{exponential}(5)$.

Need additional blocks to fit this model:

functions: Here we'll construct a function that returns $\{\dot{S}, \dot{I}, \dot{R}\}$, which we can then pass to an ODE solver.

- `vector sir (real t, vector y, real beta, real gamma, int N) { ··· return dy_dt };`
- `t`: time
- `y`: the solution to the ODE, $y(t) = [S(t), I(t), R(t)]$

transformed parameters: Allows us to do manipulations on the parameters

- compute the inverted ϕ
- compute $I(t)$ by solving the ODE:
```
array[n_days] vector[3] y
  = ode_rk45(sir, y0, t0, ts, beta, gamma, N);
```
- y0: initial condition for $t = t_0$.
- ts: times at which we require a solution.

Exercise: *Write and fit the SIR model for the 1978 influenza outbreak.*

- *Check the standard diagnostics ($\widehat{R}$ and ESS) and examine the density and trace plots. Is the inference reliable?*
- *Do the posterior predictive checks: is the model accurately describing the data?*
- *How does the model change when using a Poisson likelihood rather than a negative binomial?*

For more discussion about this model (e.g. choice of priors, sensitivity tests), see [Grinsztajn et al., 2021].

References I

[Grinsztajn et al., 2021] Grinsztajn, L., Semenova, E., Margossian, C. C., and Riou, J. (2021).
Bayesian workflow for disease transmission modeling in stan.
Statistics in Medicine.