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RADIO FREQUENCY ELECTRONICS

Low Noise Amplifier - Part I

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Contents

1	Introduction	3
2	Design of the LNA	3
2.1	Transistor Bias Network	3
2.2	S-parameters with packaging effects	4
2.3	Stability	5
2.4	Input and output matching networks for Maximum Gain	8
2.4.1	Matching with lumped elements	11
2.4.2	Matching lines and stubs	16
3	Simulation	16
3.1	Validation of the LNA design	16
3.2	Input and output matching networks design simulation	18
3.3	Noise	23
4	Conclusion	25
A	Appendix A: Python Script	27

List of Figures

1	Transistor DC biasing circuit and its Thévenin equivalent.	3
2	LNA diagram with reflection coefficients and no matching networks.	5
3	S-parameters of the transistor, for the range of frequencies, without matching network.	5
4	Stability factors K , Δ and μ for the range of frequencies.	6
5	Maximum Available Gain for the range of frequencies.	7
6	Stability circles for the input and output of the LNA.	8
7	LNA diagram with matching networks and reflection coefficients.	8
8	Normalized impedances at the source and load of the transistor in the Smith Chart.	10
9	Smith chart for input matching with lumped elements.	11
10	Matching circuit for input.	12
11	Smith chart for output matching with lumped elements.	12
12	Matching circuit for output.	13
13	Matching circuit for z inside the $1 + jx$ circle.	14
14	Matching circuit for z outside the $1 + jx$ circle.	15
15	Matching circuit for input and output with final values.	15
16	Matching circuit for input and output with values	16
17	Transistor with package effects	17
18	Biassing circuit simulated	17
19	Result of the operation point simulation	18
20	Biassing circuit simulated in Cadence	18
21	Circuit for the S-parameters	19
22	S-parameters of the LNA	19
23	Matching networks using inductors and capacitors	19
24	S-parameters for the matching networks using inductors and capacitors	20
25	Matching networks using inductors and capacitors in Cadence	20
26	S-parameters for the matching networks using inductors and capacitors in Cadence	21
27	Matching networks using transmission lines and stubs	21
28	S-parameters for the matching networks using transmission lines and stubs	22
29	Matching networks using transmission lines and stubs in Cadence	22
30	S-parameters for the matching networks using transmission lines and stubs in Cadence	23
31	Noise for the matching networks using capacitors and inductors in Cadence	24
32	Noise for the matching networks using transmission lines and stubs in Cadence	24

1 Introduction

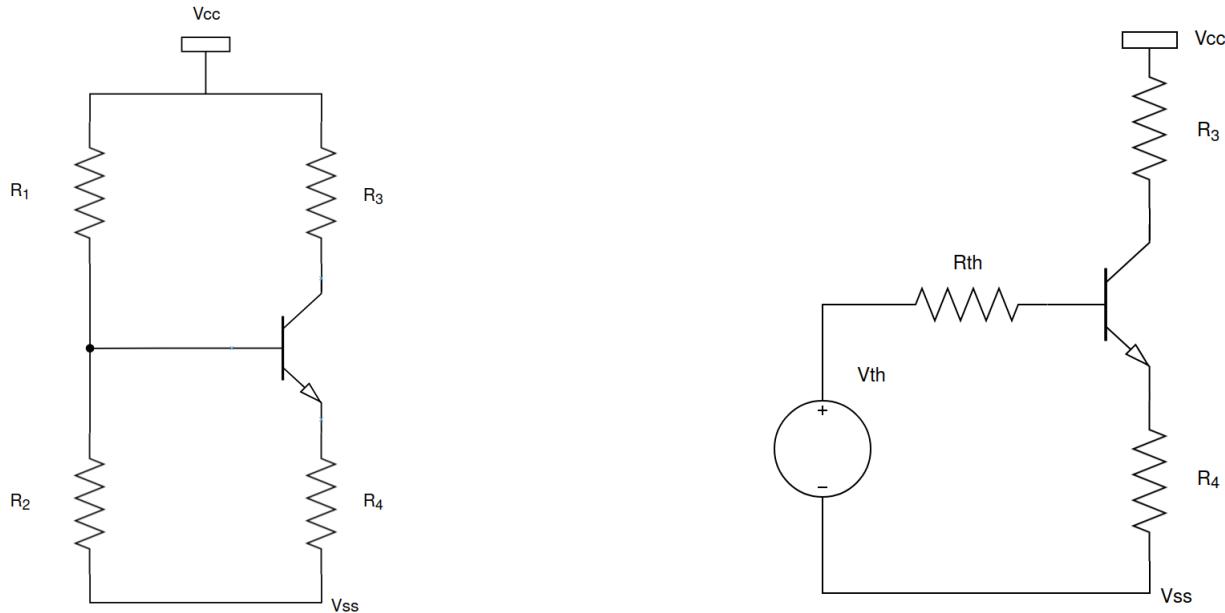
This report presents the design and analysis of a Low Noise Amplifier (LNA) designed to operate in the ISM (Industrial, Scientific and Medical) band. In modern telecom systems, LNAs play a crucial role in dealing with low amplitude signals at high frequencies with high data rates. During the development of this project, advanced RF design concepts were explored, including the use of adaptive loops for impedance, stability, gain and noise within the specified frequency range (3-6 GHz). The challenges faced, and the solutions adopted are detailed in this report.

The aim of this report is to comprehensively document the process of designing and analyzing a Low Noise Amplifier (LNA) for operation in the ISM band, with a focus on achieving critical performance specifications. The circuit will have to be designed following certain detailed specifications, starting by designing a suitable polarization network for the transistor, taking into account the effects of encapsulation.

2 Design of the LNA

2.1 Transistor Bias Network

The DC bias point of a transistor directly influences its small-signal S-parameters, and hence the gain, noise figure and stability of the LNA. This makes this step crucial. Figure 1 shows the biasing circuit and its Thévenin equivalent used to simplify analysis.



(a) Transistor DC biasing circuit.

(b) Bias circuit equivalent circuit.

Figure 1: Transistor DC biasing circuit and its Thévenin equivalent.

As shown in Figure 1b the Thévenin equivalent is given by the equations 1, replacing the R_1, R_2 voltage divider.

$$\begin{aligned} R_{TH} &= R_1 // R_2 \\ V_{TH} &= V_{cc} \frac{R_2}{R_1 + R_2} \end{aligned} \quad (1)$$

Using Kirchhoff voltage law, the equations 2 are derived, the first starts at V_{TH} goes through R_{TH} , V_{BE} and R_4 . The second goes from V_{CC} through R_3 , V_{CE} and R_4 .

$$\begin{cases} 0 = V_{TH} - I_b \cdot R_{TH} - V_{BE} - I_E \cdot R_4 \\ 0 = V_{CC} - R_3 \cdot I_C - V_{CE} - I_E \cdot R_4 \end{cases} \quad (2)$$

Solving the system of equations, assuming fixed values for R_2 and R_4 , originates the equations 3.

$$\begin{aligned} R_1 &= \frac{R_2 (-I_C R_4 \beta - I_C R_4 - V_{BE} \beta + V_{CC} \beta)}{I_C R_2 + I_C R_4 \beta + I_C R_4 + V_{BE} \beta} \\ R_3 &= \frac{-I_C R_4 \beta - I_C R_4 + V_{CC} \beta - V_{CE} \beta}{I_C \beta} \end{aligned} \quad (3)$$

The Table 1, shows the provided values for the biasing circuit and the fixed values for R_2 and R_4 .

Table 1: Transistor biasing parameters

Parameter	Value
R_2	1 kΩ
R_4	100 Ω
β	72.534
I_C	9 mA
V_{CC}	10 V
V_{BE}	1 V
V_{CE}	5 V

Resulting in $R_1 = 4 \text{ k}\Omega$ and $R_3 = 454 \Omega$.

2.2 S-parameters with packaging effects

The diagram of the LNA is shown in Figure 7, where the LNA has arbitrary input and output impedances different from 50Ω and reflection coefficients.

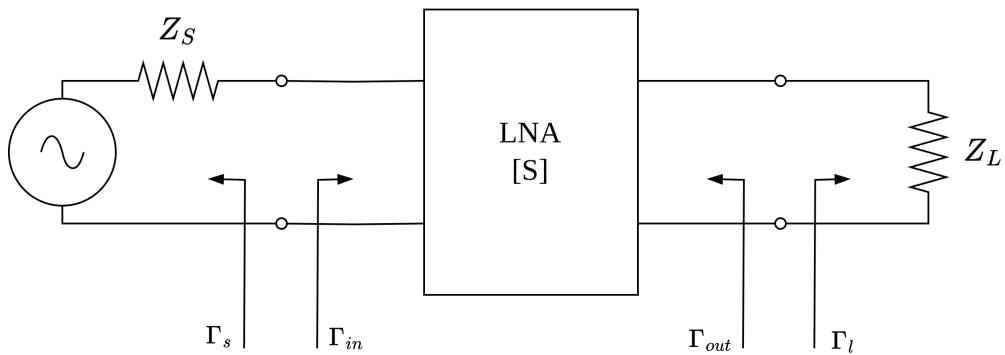


Figure 2: LNA diagram with reflection coefficients and no matching networks.

With the biasing circuit designed, the next step was to simulate the S-parameters of the transistor in LTSpice. The S-parameters were taken for a frequency range of 1 GHz to 10 GHz, Figure 3 shows the S-parameters of the transistor without any matching network.

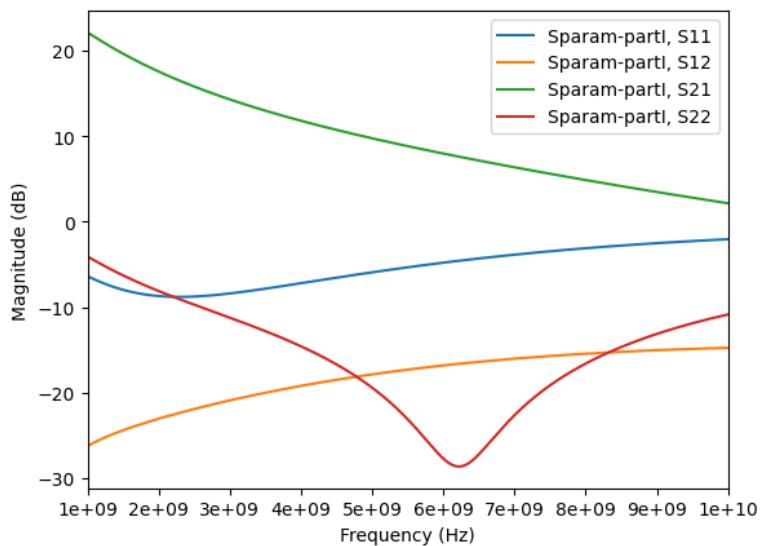


Figure 3: S-parameters of the transistor, for the range of frequencies, without matching network.

2.3 Stability

Ensuring that the LNA remains stable is critical for reliable operation. The network is unconditionally stable for a frequency if for any source impedance value, $|\rho_{in}| < 1$ and for the load impedance $|\rho_{out}| < 1$. Below, the stability analysis is performed using the S-parameters obtained in the previous step to calculate the stability factors, in this case, the K and Δ factors and the μ factor.

Defining Δ as:

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21} \quad (4)$$

Where S_{11} , S_{22} , S_{12} and S_{21} are the S-parameters of the LNA.
And defining K as:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12} \cdot S_{21}|} \quad (5)$$

The stability conditions can be summarized as follows:

- $K > 1$ and $|\Delta| < 1 \rightarrow$ unconditionally stable
- $K > 1$ and $|\Delta| > 1$ or $K < 1 \rightarrow$ potentially unstable or always unstable

Another criteria is the μ factor, defining μ as:

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12} \cdot S_{21}|}$$

If $\mu > 1 \rightarrow$ unconditionally stable In addition, it can be said that larger values of μ imply greater stability.

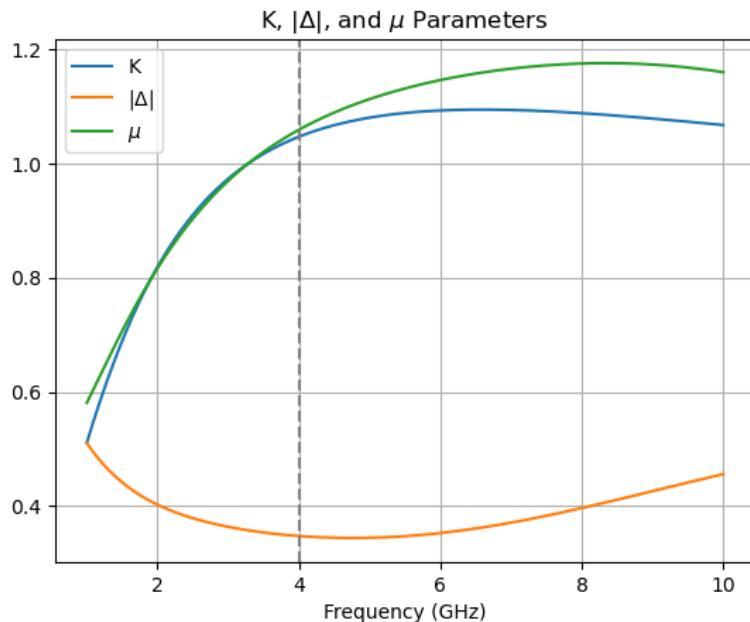


Figure 4: Stability factors K , Δ and μ for the range of frequencies.

Figure 4, shows that the LNA is stable for frequencies above 3.1 GHz and above 10 GHz loses stability again.

At this stage another important figure is the Maximum Available Gain, MAG , which for the bilateral case can be expressed as the equation 6.

$$MAG = \left| \frac{S_{21}}{S_{12}} \right| \cdot \left[K - \sqrt{K^2 - 1} \right] \quad (6)$$

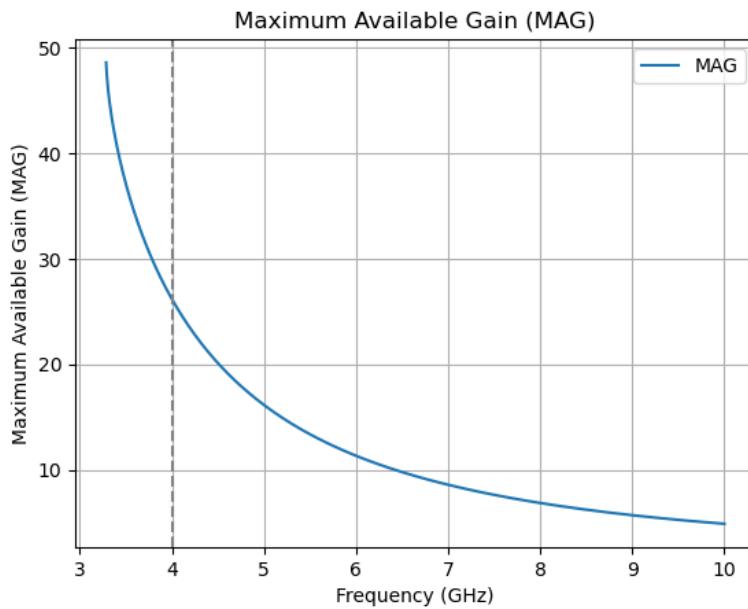


Figure 5: Maximum Available Gain for the range of frequencies.

Now having the full picture of the LNA characteristics, an operating frequency can be decided. The frequency chosen was the one that maximizes the gain while maintaining stability. In this case, the chosen was 4 GHz, where the *MAG* is 26, 13 and the stability factors are $K = 1, 05$, $\Delta = 0, 35$ and $\mu = 1, 06$, the summary of the stability parameters is shown in Table 2.

Table 2: Stability parameters for the chosen frequency.

Parameter	Value
Chosen Frequency	4 GHz
$ \Delta $	0, 35
k	1, 05
μ	1, 06
<i>MAG</i>	26, 13

The stability circles in the Smith Chart for the input and output are shown in Figure 6, where is possible to see that at 4GHz the LNA is stable for all the source and load impedances.

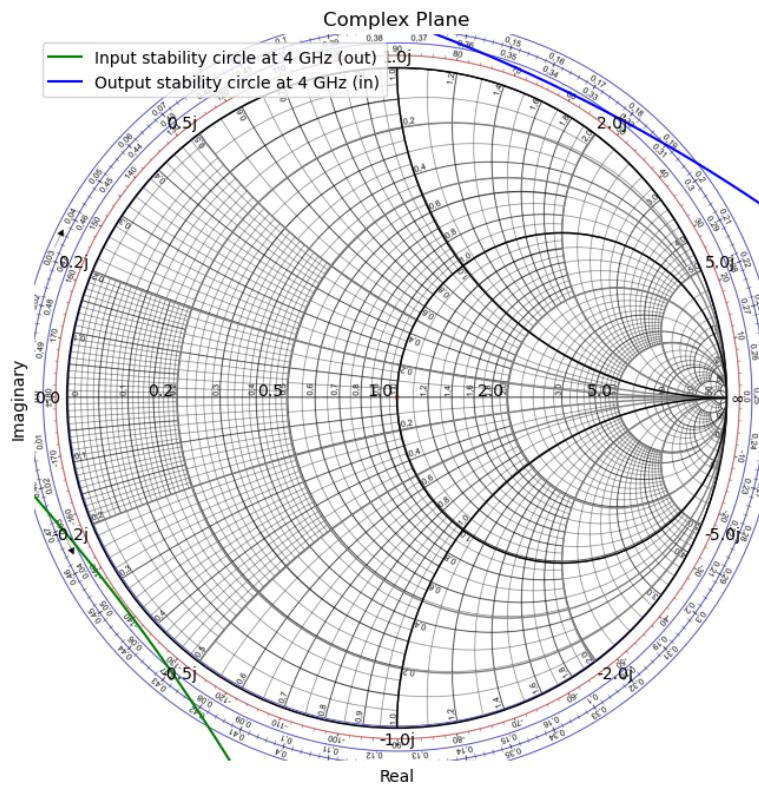


Figure 6: Stability circles for the input and output of the LNA.

2.4 Input and output matching networks for Maximum Gain

The adaptation for maximum gain is done using the line impedance transformation method. The input and output matching networks are designed to transform the input and output impedances of the transistor to the desired values, which are 50Ω in this case. In the Smith Chart, the matching is done with inductors and capacitors or lines and stubs.

In Figure 7 the diagram of the LNA is shown, where is possible to see the reflection coefficients, the input and output matching networks, the source and load impedances Z_0 .

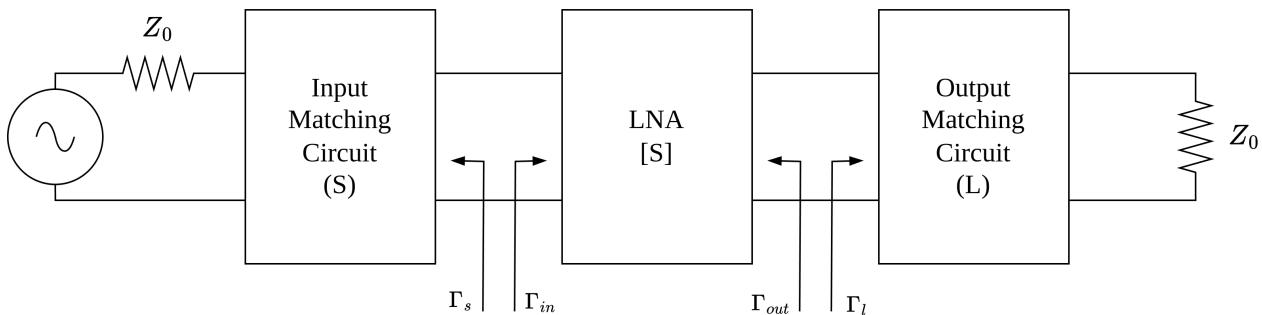


Figure 7: LNA diagram with matching networks and reflection coefficients.

Observing the diagram, the input and output matching networks are designed to transform the input and output impedances of the transistor to 50Ω , corresponding to the source and

load impedances Z_0 .

The maximum power transfer from the input matching network to the transistor will occur when:

$$\Gamma_{in} = \Gamma_s^* \quad (7)$$

And the maximum power transfer from the transistor to the output matching network will occur when:

$$\Gamma_{out} = \Gamma_l^* \quad (8)$$

where Γ_{in} and Γ_{out} are the reflection coefficients at the input and output of the transistor, respectively, and Γ_s and Γ_l are the reflection coefficients at the source and load impedances, respectively. Using the equations obtain in [1], the expression that result in the reflection coefficients at the input and output of the transistor are:

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad (9)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \quad (10)$$

$$C_1 = S_{11} - \Delta S_{22}^* \quad (11)$$

$$C_2 = S_{22} - \Delta S_{11}^* \quad (12)$$

Finally, the reflection coefficients at the source and load of the transistor are given by:

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \quad (13)$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \quad (14)$$

In Table 3 the reflection coefficients at the input and output of the transistor are shown, where the values are calculated using the S-parameters obtained in the previous step.

Table 3: Reflection coefficients at the input and output of the transistor.

Parameter	Value
Γ_S	$-0.61339711 - 0.4649262j$
Γ_L	$0.29177619 + 0.6148401j$
B_1	1.03559444
B_2	0.72244362
C_1	$-0.39891088 + 0.30235572j$
C_2	$0.144066 - 0.30358047j$
S_{11}	$-0.33990673 + 0.27537675j$
S_{12}	$0.06042986 - 0.17647029j$
S_{21}	$0.09296393 + 0.05839295j$
S_{22}	$2.52965222 + 2.95848991j$

With the values of the reflection coefficients at the source and load, the normalized impedances at the source and load can be calculated using the equations 15 and 16.

$$z_S = \frac{1 + \Gamma_S}{1 - \Gamma_S} \quad (15)$$

$$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L} \quad (16)$$

Where Z_0 is the desired impedance, in this case 50Ω . The resulting normalized impedances at the source and load are shown in Table 4, where the values are calculated using the reflection coefficients obtained in the previous step.

Table 4: Impedances at the source and load of the transistor.

Parameter	Value
z_S	$0.14457529 - 0.32982769j$
z_L	$0.6103145 + 1.39798452j$

The Smith Chart representation of the normalized impedances at the source and load is shown in Figure 8, where the normalized impedances are represented as points.

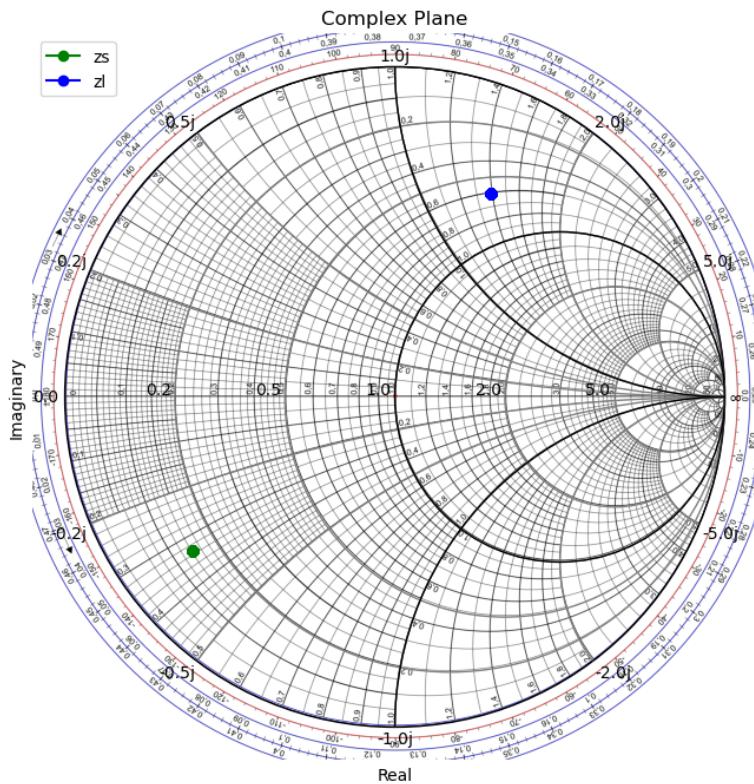


Figure 8: Normalized impedances at the source and load of the transistor in the Smith Chart.

2.4.1 Matching with lumped elements

The matching networks were first designed using the Smith chart, which allows for the visualization of the impedance transformation. A second analysis was done with an analytic approach using a Python script in order to have more accurate results. The input and output impedances of the LNA are transformed to 50Ω using a combination of inductors and capacitors. The values of the components are also calculated using the equations for impedance transformation.

Results with the Smith Chart:

The matching using the Smith Chart for the input and output are shown in Figures 9 and 11, where the input and output impedances of the transistor are transformed to 50Ω using a combination of inductors and capacitors, in this case a L-section matching network.

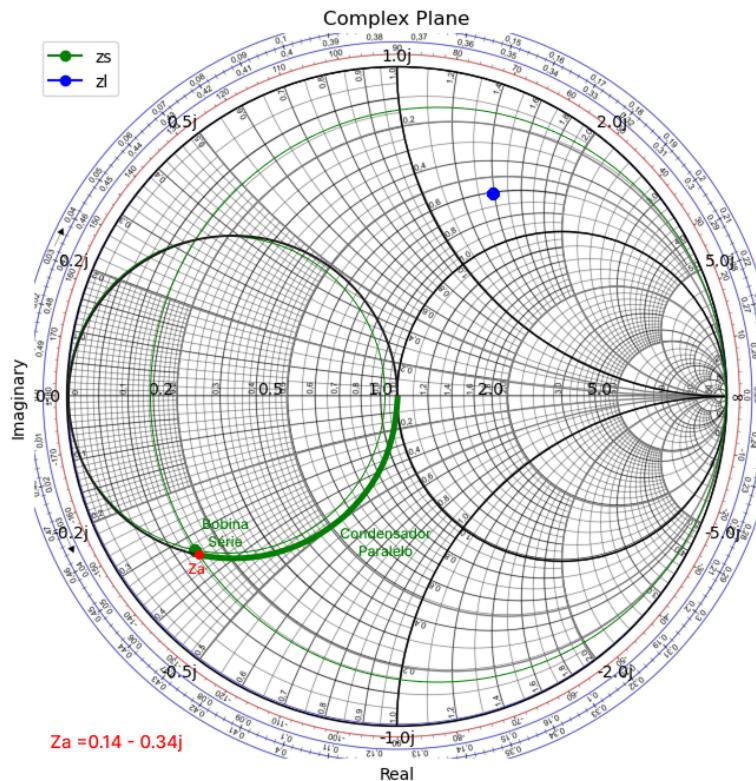


Figure 9: Smith chart for input matching with lumped elements.

The adaptation mesh for the input was done with a shunt capacitor and a series inductor and the equivalent circuit is shown in Figure 10. The values of the components were also calculated using the equations for impedance transformation as a form of validation.

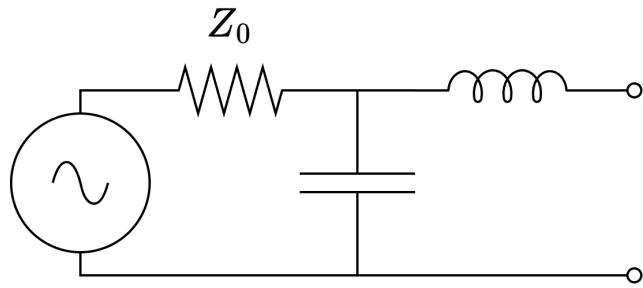


Figure 10: Matching circuit for input.

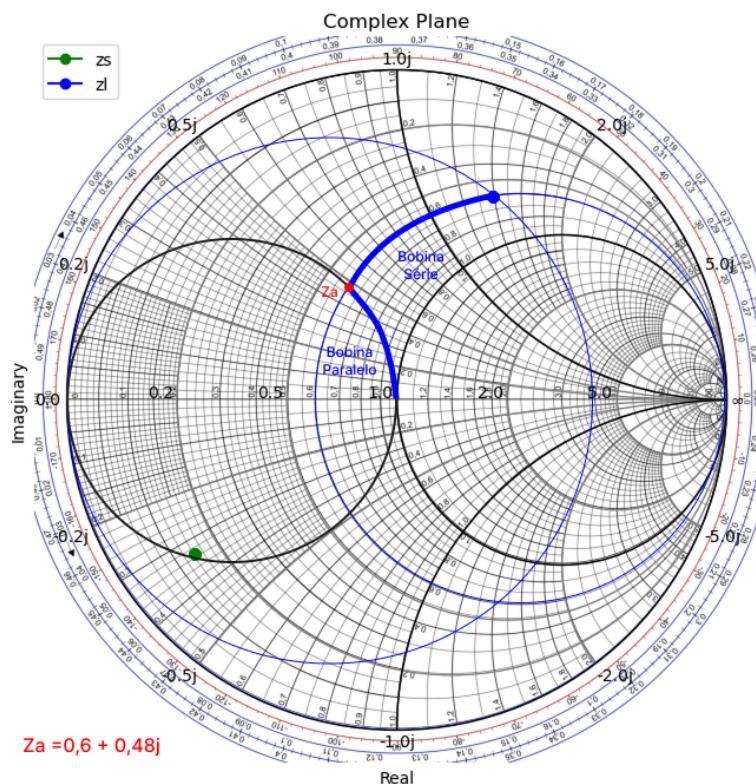


Figure 11: Smith chart for output matching with lumped elements.

The adaptation mesh for the output is done with a series inductor and a shunt inductor and the equivalent circuit is shown in Figure 12. The values of the components were also calculated using the equations for impedance transformation as a form of validation.

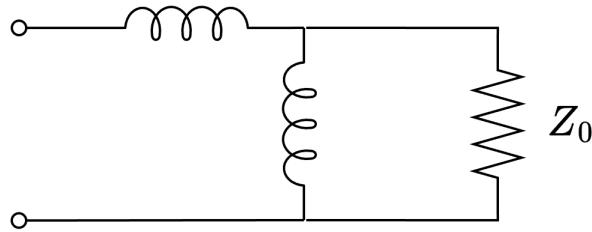


Figure 12: Matching circuit for output.

After retrieving the values of the in-between impedances Z_a from the Smith Chart, it is possible to obtain the values of the components using the following equations [1]:

Series inductor:

$$\begin{aligned} z_L &= z_2 - z_1 = jx \quad (x > 0) \\ L &= \frac{xZ_0}{\omega} \end{aligned} \tag{17}$$

Shunt Inductor:

$$\begin{aligned} y_L &= y_2 - y_1 = -jb \quad (b > 0) \\ L &= \frac{Z_0}{b\omega} \end{aligned} \tag{18}$$

Series Capacitor:

$$\begin{aligned} z_C &= z_2 - z_1 = -jx \quad (x > 0) \\ C &= \frac{1}{xZ_0\omega} \end{aligned} \tag{19}$$

Shunt Capacitor:

$$\begin{aligned} y_C &= y_2 - y_1 = jb \quad (b > 0) \\ C &= \frac{b}{Z_0\omega} \end{aligned} \tag{20}$$

From input in-between impedance $Z_a = 0,14 - 0,34j$ and the configuration chosen in the smith chart represented in Figure 10, the values of the components are calculated as follows:

Shunt Capacitor:

$$\begin{aligned} y_C &= y_a - y_0 = 1 + 2,5j - 1 = 2,5j \\ b &= 2,5 \\ C_{in} &= \frac{2,5}{2\pi 410^9 Z_0} = 1,989 \text{ pF} \end{aligned} \tag{21}$$

Series Inductor:

$$\begin{aligned} z_L &= z_s - z_a = 0,14 - 0,33j - 0,14 - 0,34j = 0,01j \\ x &= 0,01 \\ L_{in} &= \frac{0,01 \cdot Z_0}{2\pi 410^9} = 198,9 \text{ fH} \end{aligned} \tag{22}$$

From output in-between impedance $Z_a = 0, 6 + 0, 48j$ and the configuration chosen in the Smith Chart represented in Figure 12, the values of the components are calculated as follows:

Shunt Inductor:

$$\begin{aligned} y_L &= y_a - y_0 = 1 - 0,81 - 1 = -0,81j \\ b &= 0,81 \\ L_{1out} &= \frac{Z_0}{0,81 \cdot 2\pi 4 \cdot 10^9} = 2,46 \text{ nH} \end{aligned} \quad (23)$$

Series Inductor:

$$\begin{aligned} z_L &= z_l - z_a = 0,6 + 1,4j - 0,6 + 0,48 = 0,92j \\ x &= 0,92 \\ L_{2out} &= \frac{0,92 \cdot Z_0}{2\pi 4 \cdot 10^9} = 1,83 \text{ nH} \end{aligned} \quad (24)$$

Results with analytic approach:

The matching using the analytic approach was done using a Python script (Appendix A) that calculates the values of the components based on the input and output impedances of the transistor. The equations used in the script are the following [1]:

Matching for z inside the $1 + jx$ circle:

$$B = \frac{x_l \pm \sqrt{rl/z_0} \sqrt{r_l^2 + x_l^2 - z_0 r_l}}{r_l^2 + x_l^2} \quad (25)$$

$$X = \frac{1}{B} + \frac{x_l z_0}{r_l} - \frac{z_0}{B r_l} \quad (26)$$

For this case the network is designed with a series reactance and a shunt susceptance, where B is the component susceptance and X the component reactance, as shown in Figure 13.

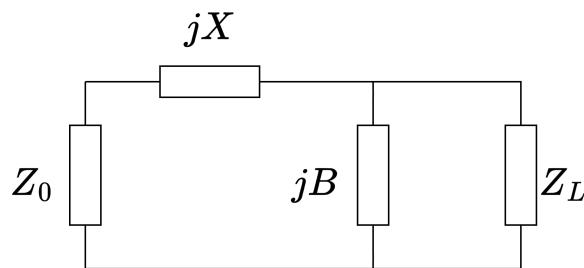


Figure 13: Matching circuit for z inside the $1 + jx$ circle.

Matching for z outside the $1 + jx$ circle:

$$X = \pm \sqrt{r_l(z_0 - r_l)} - x_l \quad (27)$$

$$B = \pm \frac{\sqrt{(z_0 - r_l)/r_l}}{z_0} \quad (28)$$

For this case the network is designed with a shunt susceptance and a series reactance, as shown in Figure 14.

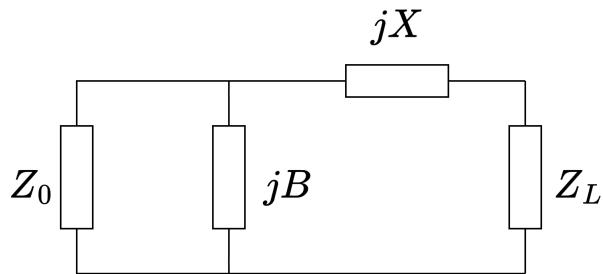


Figure 14: Matching circuit for z outside the $1 + jx$ circle.

With B being the component susceptance and X the component reactance, r_l the real part of the impedance z and x_l the imaginary part of the normalized impedance z .

The results of the matching networks using the analytic approach are shown in Table 5, where the values of the components are calculated based on the input and output impedances of the transistor.

Table 5: Values of the components for the matching networks using the analytic approach.

Parameter	Value
C_{in}	1.93417775 pF
L_{in}	0,434243305 pH
L_{1out}	2.4877816 nH
L_{2out}	1.8095882 nH

Comparing the values obtained from the Smith Chart and the analytic approach, it is possible to see that the values are very similar, with a difference of less than 5% in all cases. This validates the results obtained from the Smith Chart.

The resulting circuit, with the final values, is shown in Figure 15.

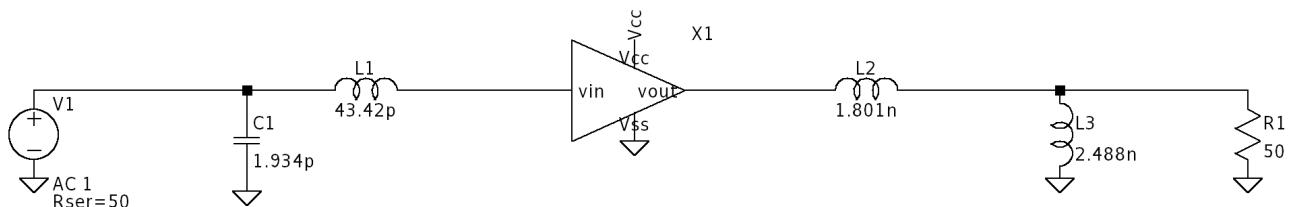


Figure 15: Matching circuit for input and output with final values.

2.4.2 Matching lines and stubs

The matching networks were also designed using transmission lines and stubs, this type of adaptation allows greater frequencies (more than 1 GHz) in real conditions. The input and output impedances of the transistor were transformed to 50Ω using a combination of transmission lines and stubs. The values of the components were also calculated using the equations for impedance transformation to validate the results.

The matching using the Smith Chart for the input and output are shown in Figures ?? and ??.

The adaptation mesh for the input was done with an open circuit shunt stub and a series line.

The adaptation mesh for the output is done with a series line and an open circuit shunt stub.

The final circuit is shown in Figure 16, where the input and output matching networks are designed using a combination of transmission lines and stubs.

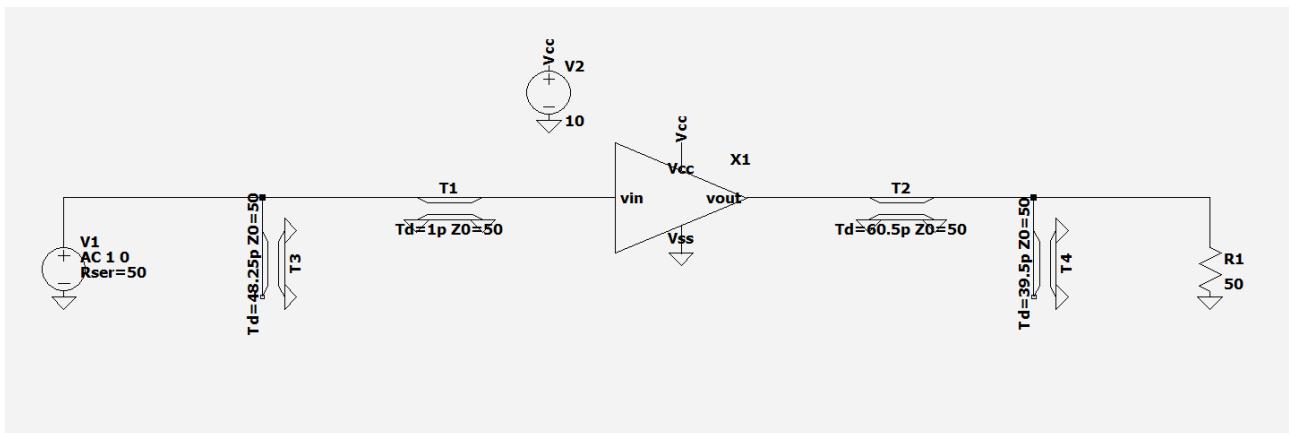


Figure 16: Matching circuit for input and output with values

3 Simulation

In this section, the simulations of the LNA and the corresponding matching networks will be presented.

3.1 Validation of the LNA design

First, using the LTSpice, the T502 transistor was added and the parasites capacitance and inductance of the package were considered, resulting in the simulated real transistor circuit in Figure 17.

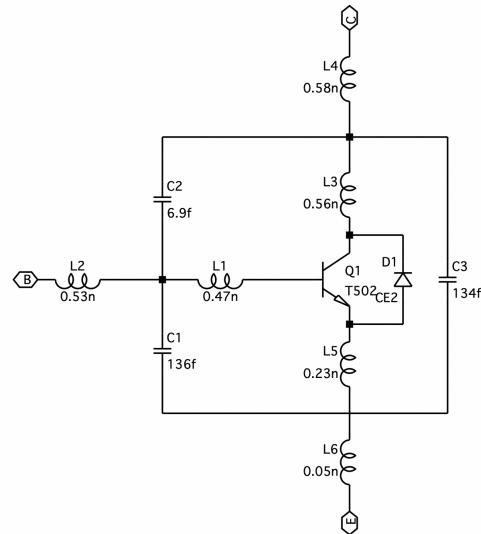


Figure 17: Transistor with package effects

After this, the biasing circuit of the transistor for the required parameters is present in Figure 18. [explicar a existencia de bobinas e condensadores](#)

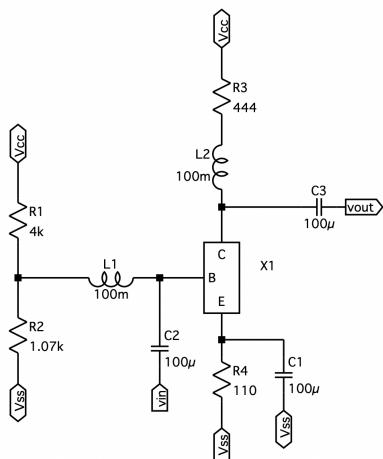


Figure 18: Biasing circuit simulated

After simulating the operation point of the previous circuit, the result for the required parameters is shown in Figure 19.

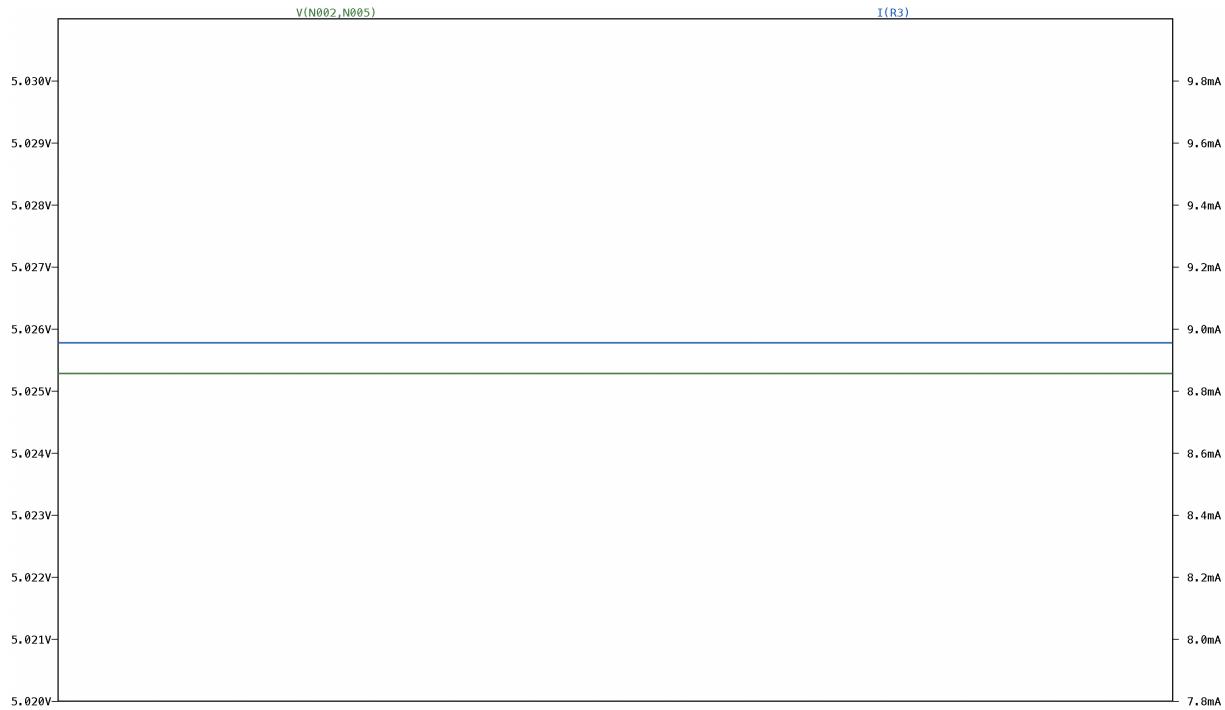


Figure 19: Result of the operation point simulation

After analyzing the previous graphic, it is possible to confirm that result parameters of the biasing circuit are within the required.

At the same time, the same circuit was implemented in Cadence, to ensure greater reliability of the results obtained. So, this circuit can be seen in Figure ??.

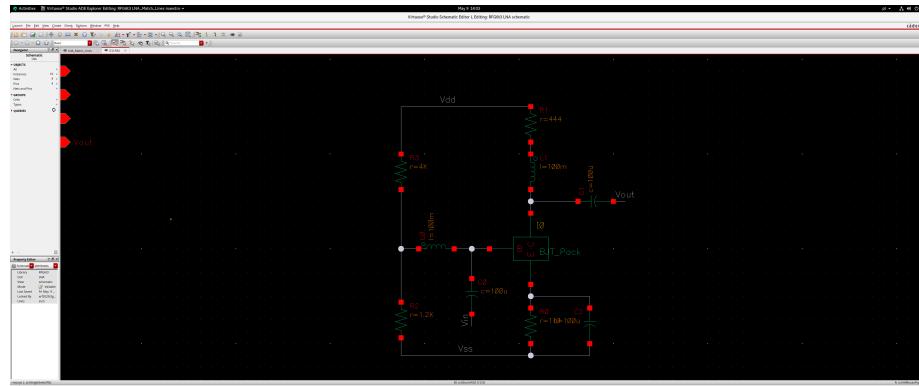


Figure 20: Biasing circuit simulated in Cadence

So the matching networks for the source and the load, both with an impedance of 50Ω , can be implemented.

3.2 Input and output matching networks design simulation

As mentioned before, the first step in designing a matching network in an LNA is to know the S-parameters of the amplifier circuit, so, switching to an AC analysis of the LNA circuit,

and adding both the source and the load, the simulated circuit is shown in Figure 21.

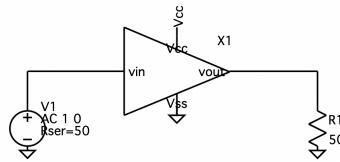


Figure 21: Circuit for the S-parameters

The resulting S-parameters can be seen in Figure 22.

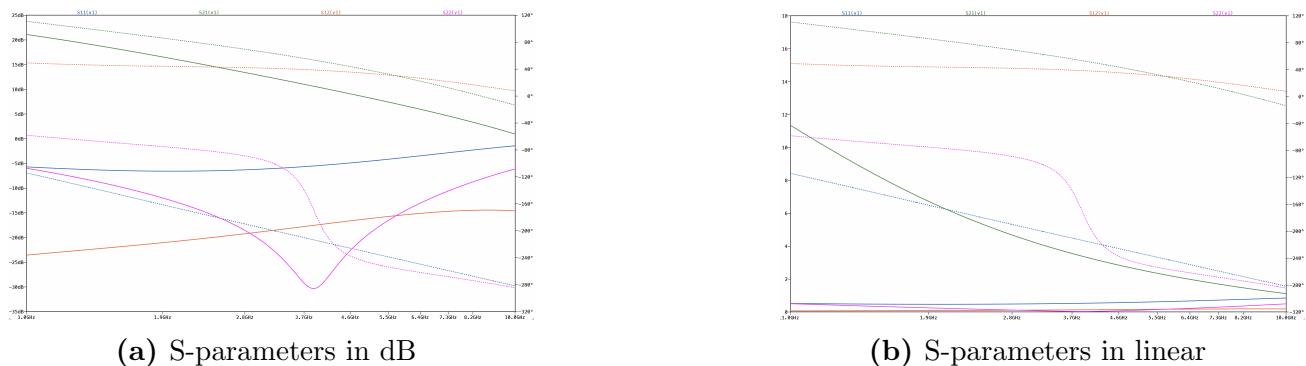


Figure 22: S-parameters of the LNA

After assuming a working frequency of 4GHz, the resulting matching networks using capacitors and inductors is exhibit in Figure 23.

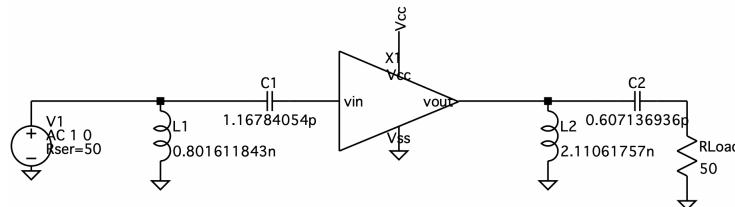


Figure 23: Matching networks using inductors and capacitors

Using again the same AC analysis, the S-parameters of new circuit using the capacitors and inductors matching networks is displayed in Figure 24.

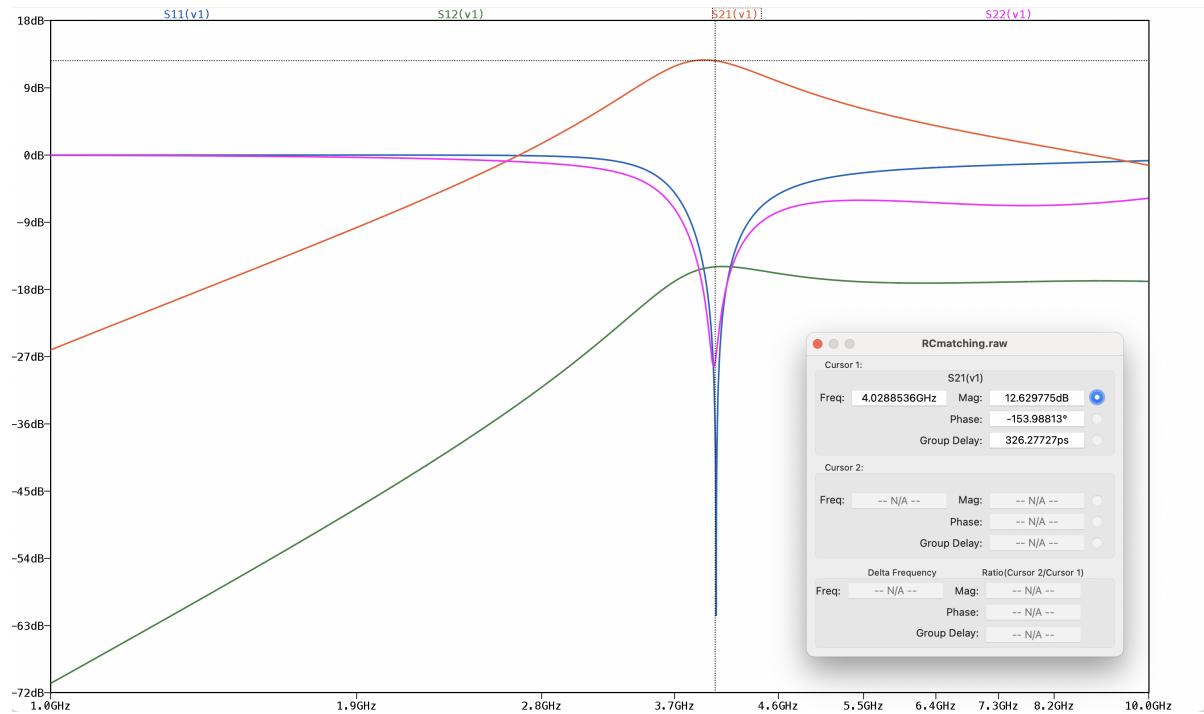


Figure 24: S-parameters for the matching networks using inductors and capacitors

After looking at the graphic above, it is possible to conclude that this matching network is working properly, since there is a sharp drop in both $S11$ and $S22$ at the desired frequency of $4GHz$ as well as a high point in the $S21$ curve.

In cadence, the circuit for the matching using inductors and capacitors is the following.

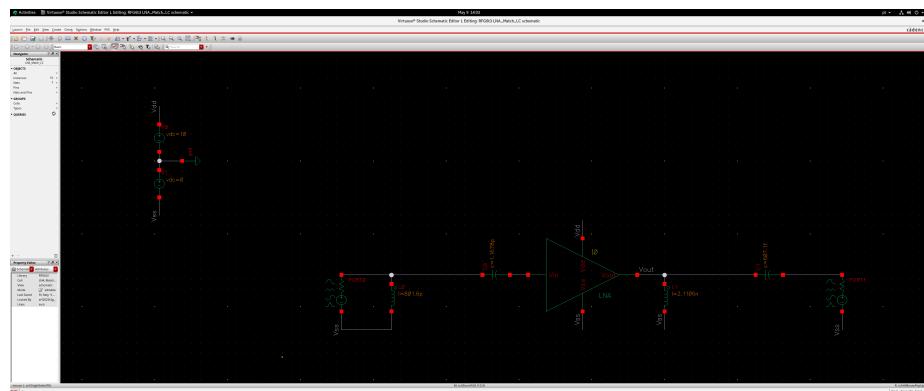


Figure 25: Matching networks using inductors and capacitors in Cadence

Simulating, the S-parameters of this circuit with the mentioned matching networks is visible in Figure 26.

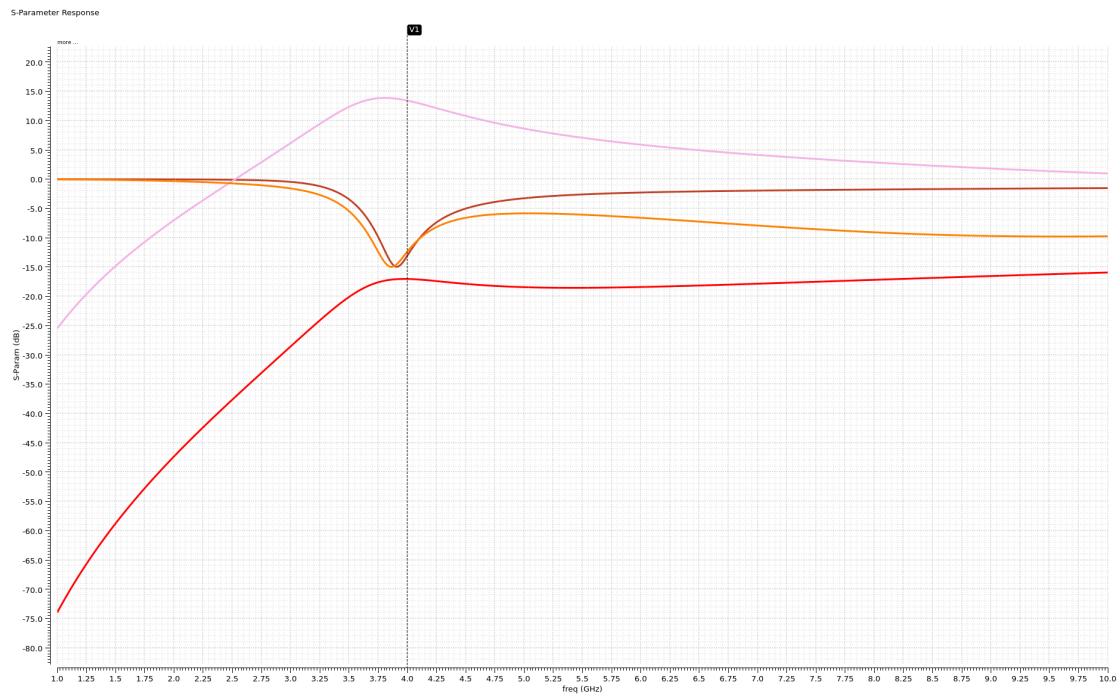


Figure 26: S-parameters for the matching networks using inductors and capacitors in Cadence

Passing to the matching networks using transmission lines and stubs, and using the same working frequency, the new LNA circuit is shown in Figure 27.

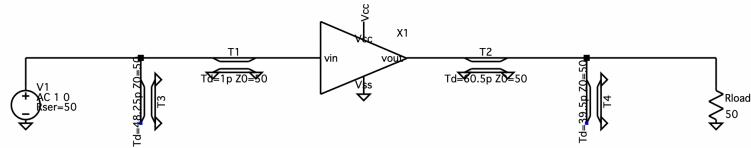


Figure 27: Matching networks using transmission lines and stubs

The S-parameters for this new matched circuit can be seen in Figure 28.

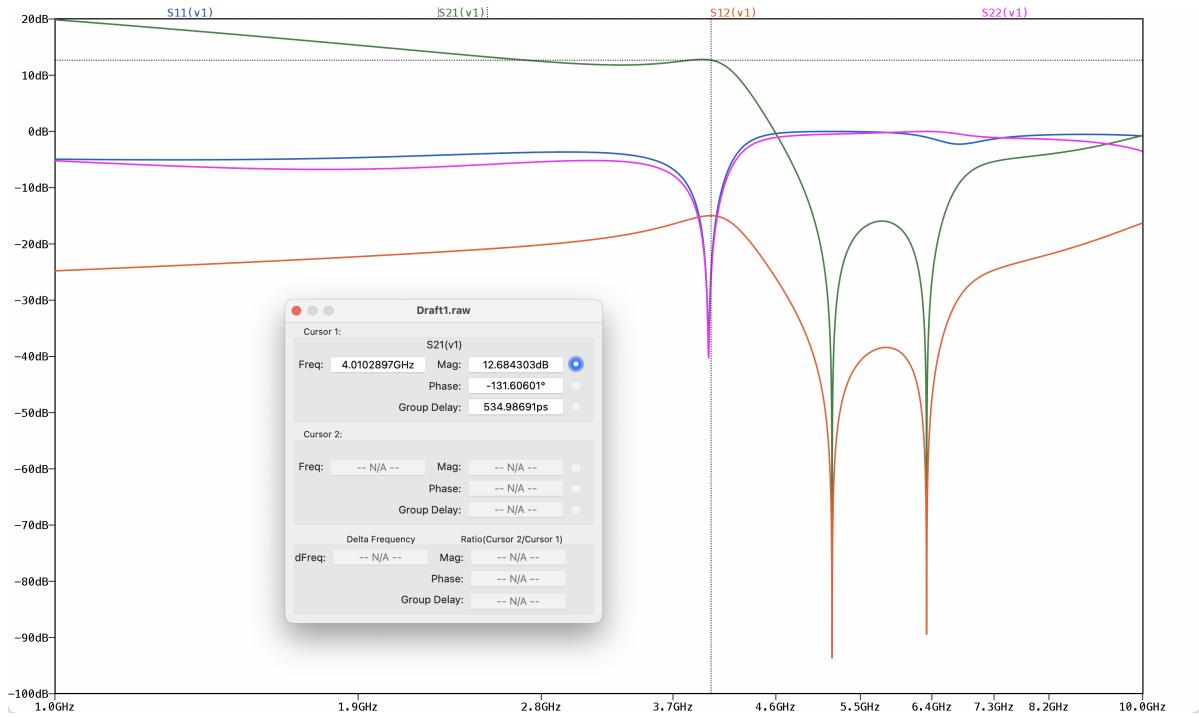


Figure 28: S-parameters for the matching networks using transmission lines and stubs

Similarly to the first matching networks, the drop in both S_{11} and S_{22} can also be seen, as well as the same approximated value of S_{21} , so, this matching network using transmission lines and stubs is correctly working.

In cadence, the circuit for the matching using transmission lines and stubs is the following.



Figure 29: Matching networks using transmission lines and stubs in Cadence

Simulating, the S-parameters of this circuit with the mentioned matching networks is visible in Figure 30.

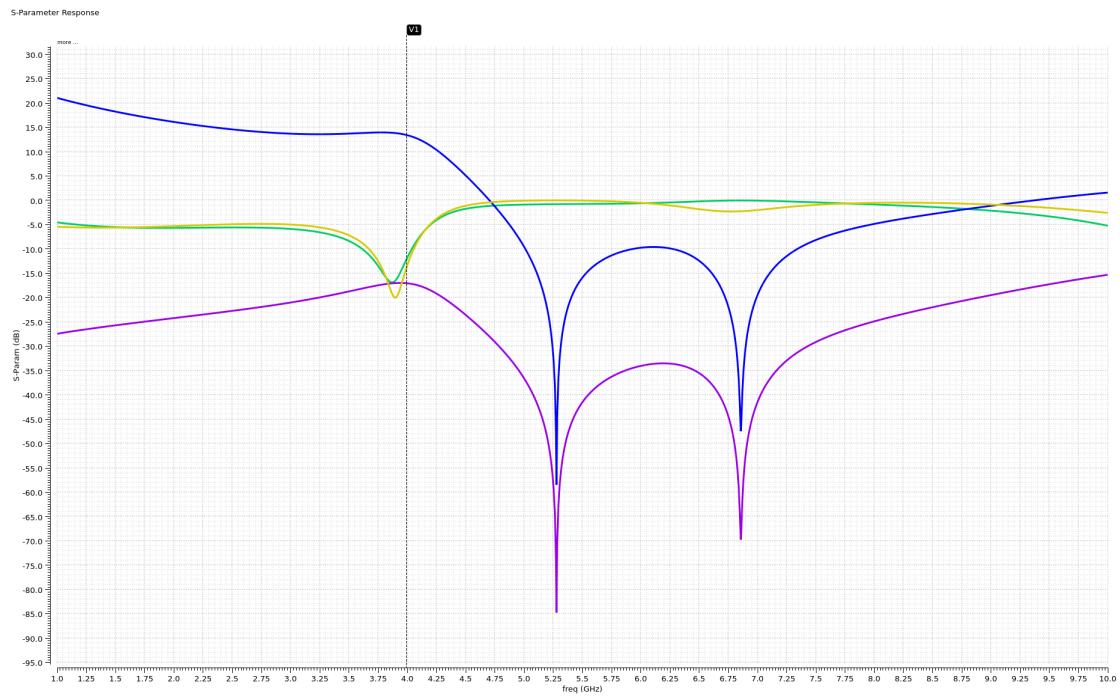


Figure 30: S-parameters for the matching networks using transmission lines and stubs in Cadence

3.3 Noise

To simulate the noise of each matching network, the Cadence was used, resulting in the graphic shown in Figure 31 for the inductors and capacitors.

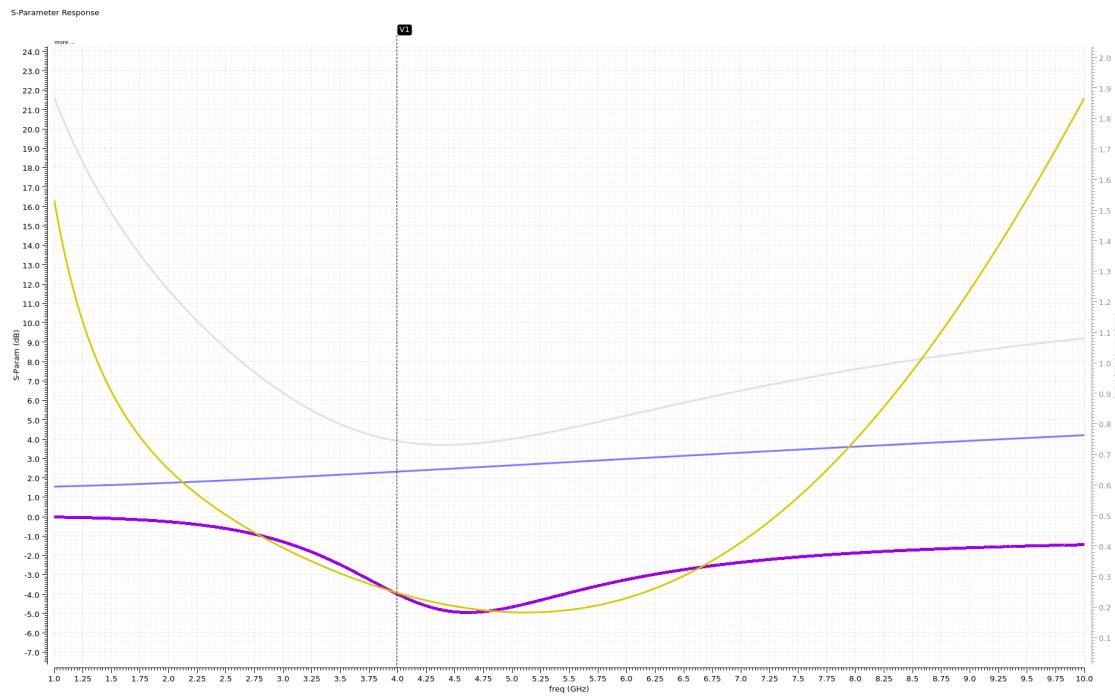


Figure 31: Noise for the matching networks using capacitors and inductors in Cadence

And for transmission lines and stubs in Figure 32.

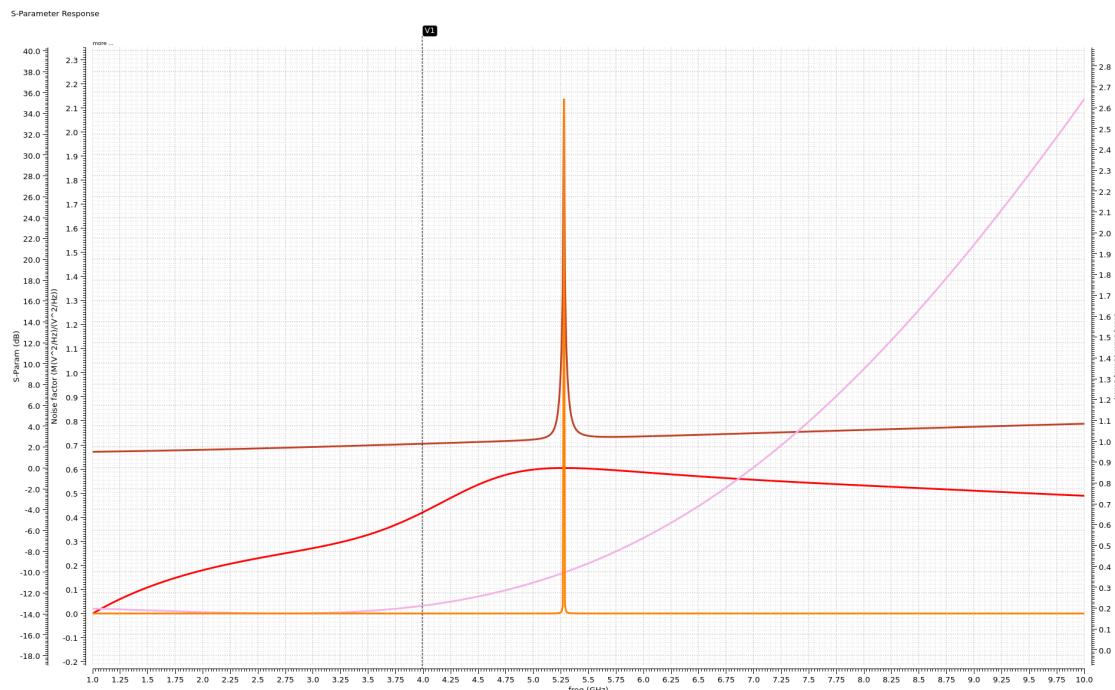


Figure 32: Noise for the matching networks using transmission lines and stubs in Cadence

4 Conclusion

The Cadence simulation were not what was expected because of an error in the initial transistor package circuit that effected the remaining project, but in LTspice, after correcting this mistake, all results were the expected for both matching networks. In second phase, the mistake will be corrected in Cadence as well.

References

- [1] D. M. Pozar, *Microwave engineering*, 4th ed. John Wiley and Sons, Inc, 2012.

A Appendix A: Python Script