NOVA SCHOOL OF SCIENCE & TECHNOLOGY

MEEC/MIEEC

Radio Frequency Electronics

Low Noise Amplifier - Part I

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1 Introduction

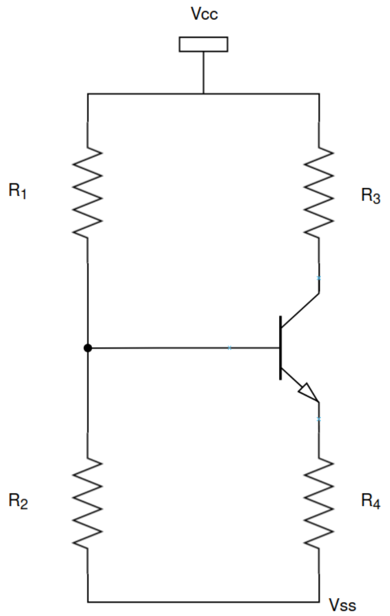
This report presents the design and analysis of a Low Noise Amplifier (LNA) designed to operate in the ISM (Industrial, Scientific and Medical) band. In modern telecom systems, LNAs play a crucial role in dealing with low amplitude signals at high frequencies with high data rates. During the development of this project, advanced RF design concepts were explored, including the use of adaptive loops for impedance, stability, gain and noise within the specified frequency range (3-6 GHz). The challenges faced, and the solutions adopted are detailed in this report.

The aim of this report is to comprehensively document the process of designing and analyzing a Low Noise Amplifier (LNA) for operation in the ISM band, with a focus on achieving critical performance specifications. The circuit will have to be designed following certain detailed specifications, starting by designing a suitable polarization network for the transistor, taking into account the effects of encapsulation.

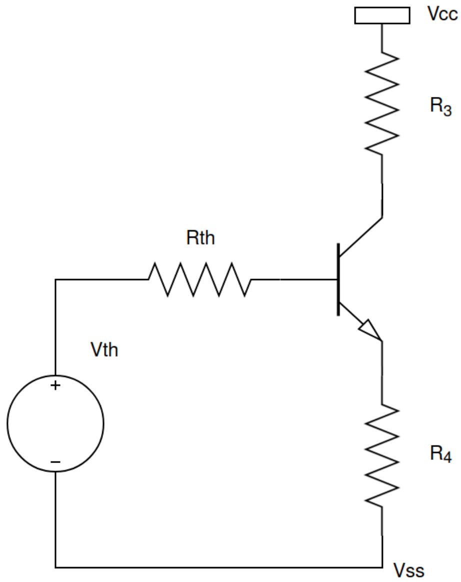
2 Design of the LNA

2.1 Transistor Bias Network

The DC bias point of a transistor directly influences its small-signal S-parameters, and hence the gain, noise figure and stability of the LNA. This makes this step crucial. Figure 1 shows the biasing circuit and its Thévenin equivalent used to simplify analysis.



(a) Transistor DC biasing circuit.



(b) Bias circuit equivalent circuit.

Figure 1: Transistor DC biasing circuit and its Thévenin equivalent.

As shown in Figure 1b the Thévenin equivalent is given by the equations 1, replacing the R_1, R_2 voltage divider.

$$\begin{aligned} R_{TH} &= R_1 // R_2 \\ V_{TH} &= V_{cc} \frac{R_2}{R_1 + R_2} \end{aligned} \tag{1}$$

Using Kirchhoff voltage law, the equations 2 are derived, the first starts at V_{TH} goes through R_{TH}, V_{BE} and R_4. The second goes from V_{CC} through R_3, V_{CE} and R_4.

$$\begin{cases} 0 = V_{TH} - I_b \cdot R_{TH} - V_{BE} - I_E \cdot R_4 \\ 0 = V_{CC} - R_3 \cdot I_C - V_{CE} - I_E \cdot R_4 \end{cases} \tag{2}$$

Solving the system of equations, assuming fixed values for R_2 and R_4, originates the equations 3.

$$
\begin{aligned}
R_1 &= \frac{R_2\left(-I_C R_4 \beta - I_C R_4 - V_{BE}\beta + V_{CC}\beta\right)}{I_C R_2 + I_C R_4 \beta + I_C R_4 + V_{BE}\beta} \\
R_3 &= \frac{-I_C R_4 \beta - I_C R_4 + V_{CC}\beta - V_{CE}\beta}{I_C \beta}
\end{aligned} \tag{3}
$$

The Table 1, shows the provided values for the biasing circuit and the fixed values for R_2 and R_4.

Table 1: Transistor biasing parameters

Parameter	**Value**
R_2	$1\,\mathrm{k\Omega}$
R_4	$100\,\Omega$
β	72.534
I_C	$9\,\mathrm{mA}$
V_{CC}	$10\,\mathrm{V}$
V_{BE}	$1\,\mathrm{V}$
V_{CE}	$5\,\mathrm{V}$

Resulting in $R_1 = 4\,\mathrm{k\Omega}$ and $R_3 = 454\,\Omega$.

2.2 S-parameters with packaging effects

The diagram of the LNA is shown in Figure 7, where the LNA has arbitrary input and output impedances different from $50\,\Omega$ and reflection coefficients.

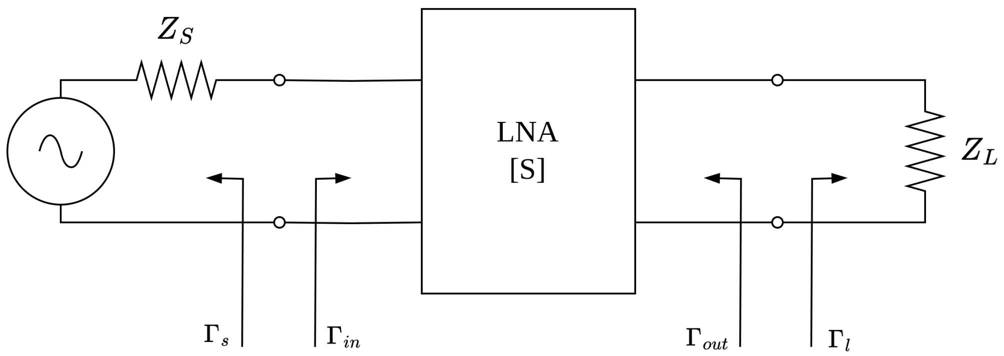


Figure 2: LNA diagram with reflection coefficients and no matching networks.

With the biasing circuit designed, the next step was to simulate the S-parameters of the transistor in LTSpice. The S-parameters were taken for a frequency range of $1\,\mathrm{GHz}$ to $10\,\mathrm{GHz}$, Figure 3 shows the S-parameters of the transistor without any matching network.

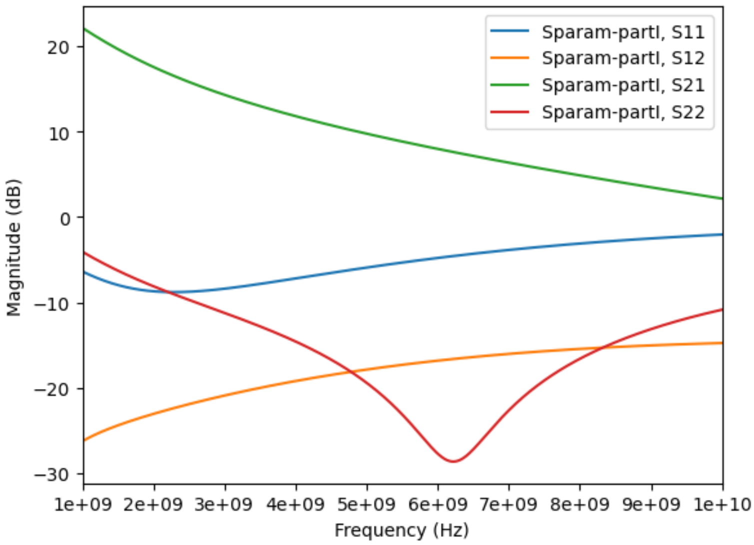


Figure 3: S-parameters of the transistor, for the range of frequencies, without matching network.

2.3 Stability

Ensuring that the LNA remains stable is critical for reliable operation. The network is unconditionally stable for a frequency if for any source impedance value, $|\rho_{in}| < 1$ and for the load impedance $|\rho_{out}| < 1$. Below, the stability analysis is performed using the S-parameters obtained in the previous step to calculate the stability factors, ins this case, the K and Δ factors and the μ factor.

Defining Δ as:

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21} \tag{4}$$

Where S_{11}, S_{22}, S_{12} and S_{21} are the S-parameters of the LNA.
And defining K as:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12} \cdot S_{21}|} \tag{5}$$

The stability conditions can be summarized as follows:

- $K > 1$ and $|\Delta| < 1 \rightarrow$ unconditionally stable
- $K > 1$ and $|\Delta| > 1$ or $K < 1 \rightarrow$ potentially unstable or always unstable

Another criteria is the μ factor, defining μ as:

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta S_{11}^*| + |S_{12} \cdot S_{21}|}$$

If $\mu > 1 \rightarrow$ unconditionally stable In addition, it can be said that larger values of μ imply greater stability.

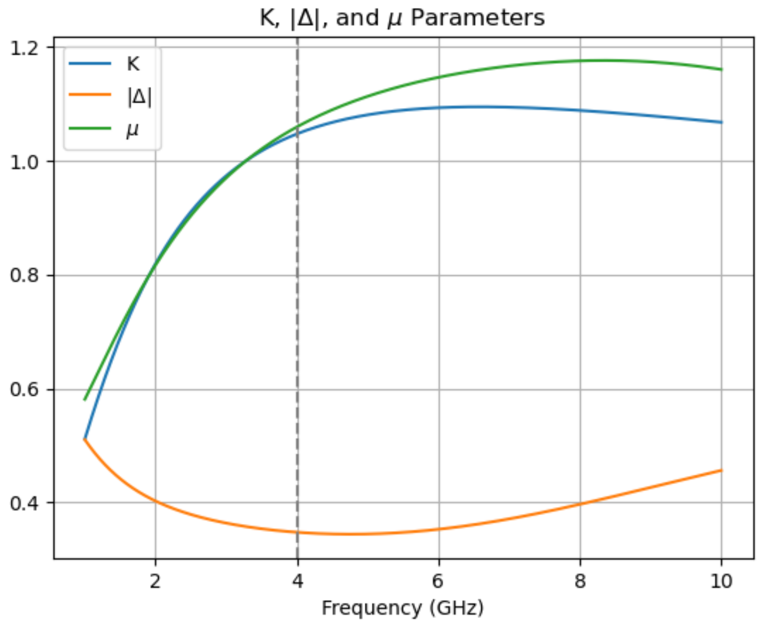


Figure 4: Stability factors K, Δ and μ for the range of frequencies.

Figure 4, shows that the LNA is stable for frequencies above 3.1 GHz and above 10 GHz loses stability again.

At this stage another important figure is the Maximum Available Gain, MAG, which for the bilateral case can be expressed as the equation 6.

$$MAG = \left|\frac{S_{21}}{S_{12}}\right| \cdot \left[K - \sqrt{K^2 - 1}\right] \tag{6}$$

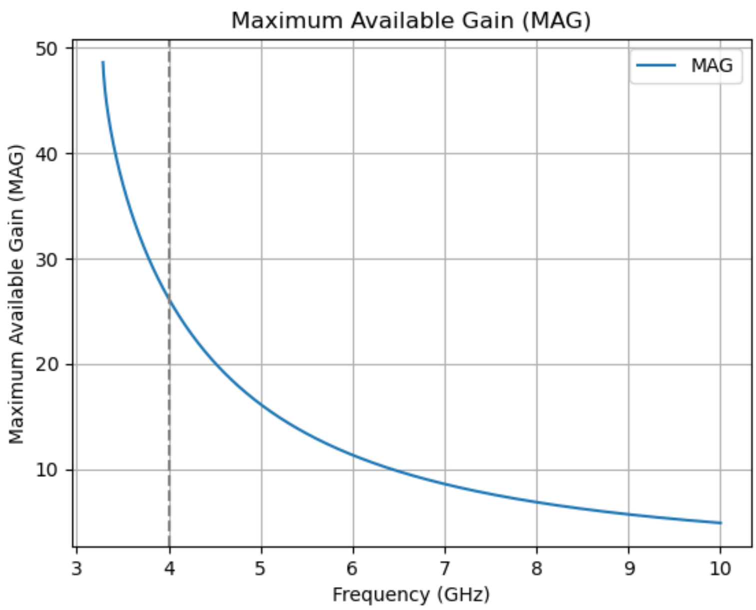


Figure 5: Maximum Available Gain for the range of frequencies.

Now having the full picture of the LNA characteristics, an operating frequency can be decided. The frequency chosen was the one that maximizes the gain while maintaining stability. In this case, the chosen was 4 GHz, where the MAG is $26,13$ and the stability factors are $K = 1,05$, $\Delta = 0,35$ and $\mu = 1,06$, the summary of the stability parameters is shown in Table 2.

Table 2: Stability parameters for the chosen frequency.

Parameter	**Value**
Chosen Frequency	4 GHz
$\|\Delta\|$	$0,35$
k	$1,05$
μ	$1,06$
MAG	$26,13$

The stability circles in the Smith Chart for the input and output are shown in Figure 6, where is possible to see that at 4GHz the LNA is stable for all the source and load impedances.

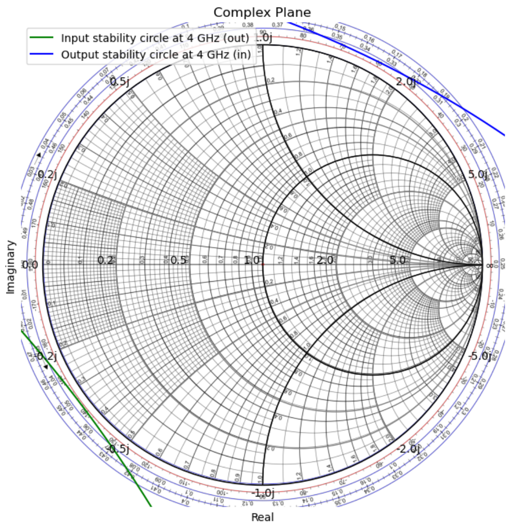


Figure 6: Stability circles for the input and output of the LNA.

2.4 Input and output matching networks for Maximum Gain

The adaptation for maximum gain is done using the line impedance transformation method. The input and output matching networks are designed to transform the input and output impedances of the transistor to the desired values, which are $50\,\Omega$ in this case. In the Smith Chart, the matching is done with inductors and capacitors or lines and stubs.

In Figure 7 the diagram of the LNA is shown, where is possible to see the reflection coefficients, the input and output matching networks, the source and load impedances Z_0.

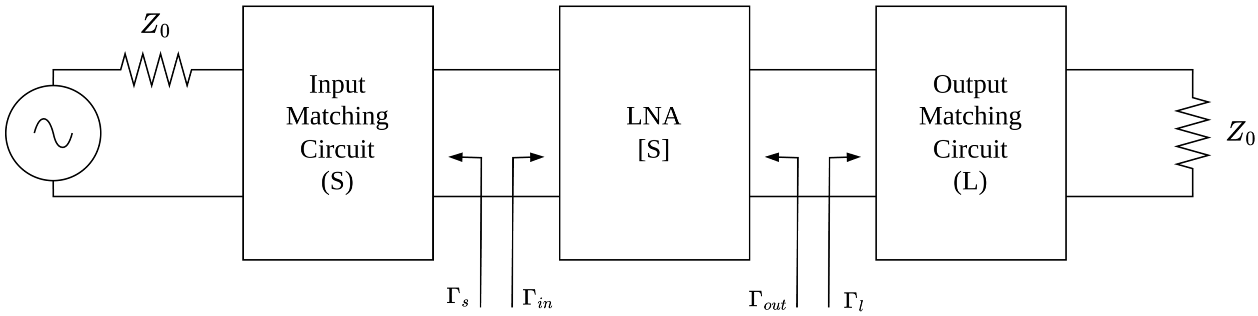


Figure 7: LNA diagram with matching networks and reflection coefficients.

Observing the diagram, the input and output matching networks are designed to transform the input and output impedances of the transistor to $50\,\Omega$, corresponding to the source and load impedances Z_0.

The maximum power transfer from the input matching network to the transistor will occur when:

$$\Gamma_{in} = \Gamma_s^* \tag{7}$$

And the maximum power transfer from the transistor to the output matching network will occur when:

$$\Gamma_{out} = \Gamma_l^* \tag{8}$$

where Γ_{in} and Γ_{out} are the reflection coefficients at the input and output of the transistor, respectively, and Γ_s and Γ_l are the reflection coefficients at the source and load impedances, respectively. Using the equations obtain in [1], the expressions that result in the reflection coefficients at the input and output of the transistor are:

$$B_1 = 1 + |S_{11}^2| - |S_{22}|^2 - |\Delta|^2 \tag{9}$$

$$B_2 = 1 + |S_{22}^2| - |S_{11}|^2 - |\Delta|^2 \tag{10}$$

$$C_1 = S_{11} - \Delta S_{22}^* \tag{11}$$

$$C_2 = S_{22} - \Delta S_{11}^* \tag{12}$$

Finally, the reflection coefficients at the source and load of the transistor are given by:

$$\Gamma_S = \frac{B_1 \pm \sqrt{B-1^2 - 4|C_1|^2}}{2C_1} \tag{13}$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B-2^2 - 4|C_2|^2}}{2C_2} \tag{14}$$

In Table 3 the reflection coefficients at the input and output of the transistor are shown, where the values are calculated using the S-parameters obtained in the previous step.

Table 3: Reflection coefficients at the input and output of the transistor.

Parameter	Value
Γ_S	$-0.61339711 - 0.4649262j$
Γ_L	$0.29177619 + 0.6148401j$
B_1	1.03559444
B_2	0.72244362
C_1	$-0.39891088 + 0.30235572j$
C_2	$0.144066 - 0.30358047j$
S_{11}	$-0.33990673 + 0.27537675j$
S_{12}	$0.06042986 - 0.17647029j$
S_{21}	$0.09296393 + 0.05839295j$
S_{22}	$2.52965222 + 2.95848991j$

With the values of the reflection coefficients at the source and load, the normalized impedances at the source and load can be calculated using the equations 15 and 16.

$$z_S = \frac{1 + \Gamma_S}{1 - \Gamma_S} \tag{15}$$

$$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L} \tag{16}$$

Where Z_0 is the desired impedance, in this case $50\,\Omega$. The resulting normalized impedances at the source and load are shown in Table 4, where the values are calculated using the reflection coefficients obtained in the previous step.

Table 4: Impedances at the source and load of the transistor.

Parameter	Value
z_S	$0.14457529 - 0.32982769j$
z_L	$0.6103145 + 1.39798452j$

The Smith Chart representation of the normalized impedances at the source and load is shown in Figure 8, where the normalized impedances are represented as points.

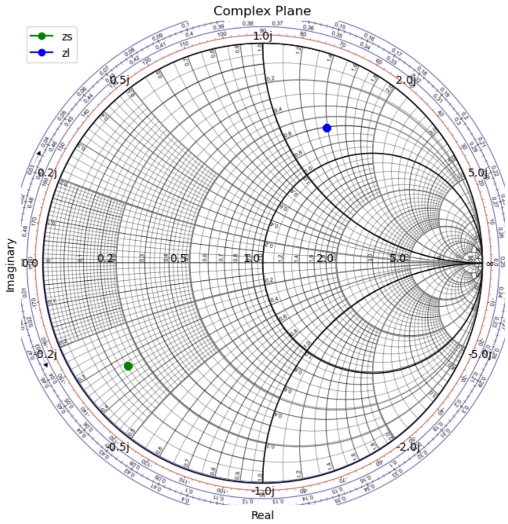


Figure 8: Normalized impedances at the source and load of the transistor in the Smith Chart.

2.4.1 Matching with lumped elements

The matching networks were first designed using the Smith chart, which allows for the visualization of the impedance transformation. A second analysis was done with an analytic approach using a Python script in order to have more accurate results. The input and output impedances of the LNA are transformed to $50\,\Omega$ using a combination of inductors and capacitors. The values of the components are also calculated using the equations for impedance transformation.

Results with the Smith Chart:

The matching using the Smith Chart for the input and output are shown in Figures 9 and 11, where the input and output impedances of the transistor are transformed to $50\,\Omega$ using a combination of inductors and capacitors, in this case an L-section matching network.

The adaptation mesh for the input was done with a shunt capacitor and a series inductor and the equivalent circuit is shown in Figure 10. The values of the components were also calculated using the equations for impedance transformation as a form of validation.

The adaptation mesh for the output is done with a series inductor and a shunt inductor and the equivalent circuit is shown in Figure 12. The values of the components were also calculated using the equations for impedance transformation as a form of validation.

After retrieving the values of the in-between impedances Z_a from the Smith Chart, it is possible to obtain the values of the components using the following equations [1]:

Series inductor:

$$\begin{aligned} z_L = z_2 - z_1 = jx \quad (x > 0) \\ L = \frac{xZ_0}{\omega} \end{aligned} \tag{17}$$

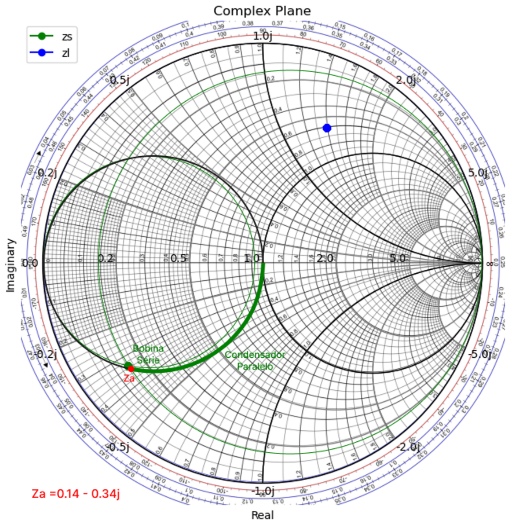


Figure 9: Smith chart for input matching with lumped elements.

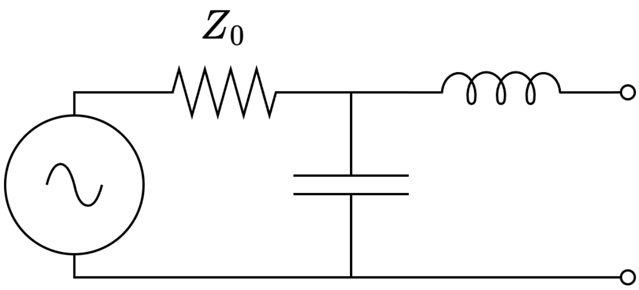


Figure 10: Matching circuit for input.

Shunt Inductor:

$$y_L = y_2 - y_1 = -jb \quad (b > 0)$$
$$L = \frac{Z_0}{b\omega} \tag{18}$$

Series Capacitor:

$$z_C = z_2 - z_1 = -jx \quad (x > 0)$$
$$C = \frac{1}{xZ_0\omega} \tag{19}$$

Shunt Capacitor:

$$y_C = y_2 - y_1 = jb \quad (b > 0)$$
$$C = \frac{b}{Z_0\omega} \tag{20}$$

From input in-between impedance $Z_a = 0,14 - 0,34j$ and the configuration chosen in the smith chart represented in Figure 10, the values of the components are calculated as follows:

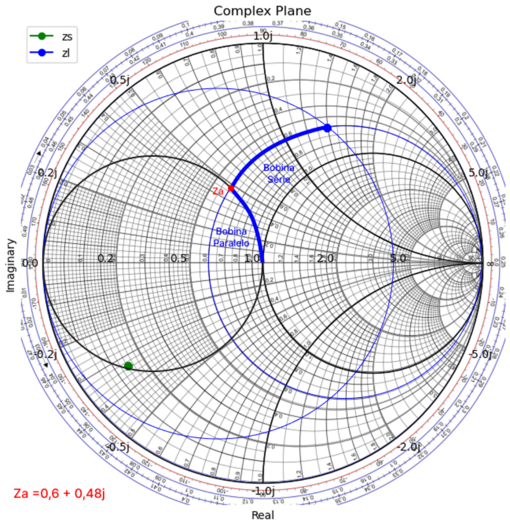


Figure 11: Smith chart for output matching with lumped elements.

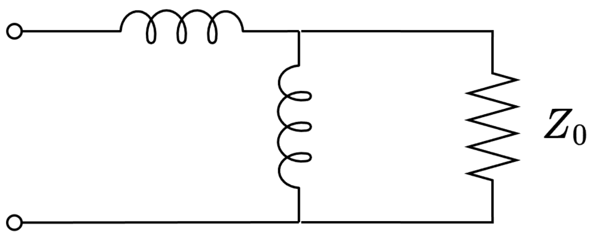


Figure 12: Matching circuit for output.

Shunt Capacitor:

$$\begin{aligned} y_C = y_a - y_0 = 1 + 2,5j - 1 = 2,5j \\ b = 2,5 \\ C_{in} = \frac{2,5}{2\pi 4 10^9 Z_0} = 1,989\,\text{pF} \end{aligned} \tag{21}$$

Series Inductor:

$$\begin{aligned} z_L = z_s - z_a = 0,14 - 0,33j - 0.14 - 0,34j = 0,01j \\ x = 0,01 \\ L_{in} = \frac{0,01 \cdot Z_0}{2\pi 4 10^9} = 198,9\,\text{fH} \end{aligned} \tag{22}$$

From output in-between impedance $Z_a = 0,6 + 0,48j$ and the configuration chosen in the Smith Chart represented in Figure 12, the values of the components are calculated as follows:

Shunt Inductor:

$$\begin{aligned} y_L = y_a - y_0 = 1 - 0.81 - 1 = -0,81j \\ b = 0,81 \\ L_{1out} = \frac{Z_0}{0,81 \cdot 2\pi 4 \cdot 10^9} = 2,46\,\text{nH} \end{aligned} \tag{23}$$

Series Inductor:

$$\begin{aligned} z_L = z_l - z_a = 0,6 + 1,4j - 0,6 + 0,48 = 0,92j \\ x = 0,92 \\ L_{2out} = \frac{0,92 \cdot Z_0}{2\pi 4 \cdot 10^9} = 1,83\,\mathrm{nH} \end{aligned} \tag{24}$$

Results with analytic approach:

The matching using the analytic approach was done using a Python script (Appendix A) that calculates the values of the components based on the input and output impedances of the transistor. The equations used in the script are the following [1]:

Matching for z inside the $1 + jx$ circle:

$$B = \frac{x_l \pm \sqrt{rl/z_0}\sqrt{r_l^2 + x_l^2 - z_0 r_l}}{r_l^2 + x_l^2} \tag{25}$$

$$X = \frac{1}{B} + \frac{x_l z_0}{r_l} - \frac{z_0}{B r_l} \tag{26}$$

For this case the network is designed with a series reactance and a shunt suspectance, where B is the component suspectance and X the component reactance, as shown in Figure 13.

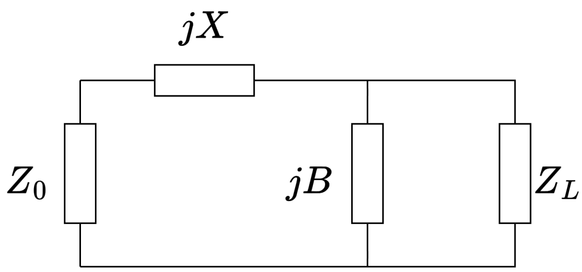


Figure 13: Matching circuit for z inside the $1 + jx$ circle.

Matching for z outside the $1 + jx$ circle:

$$X = \pm\sqrt{r_l(z_0 - r_l)} - x_l \tag{27}$$

$$B = \pm\frac{\sqrt{(z_0 - rl)/r_l}}{z_0} \tag{28}$$

For this case the network is designed with a shunt suspectance and a series reactance, as shown in Figure 14.

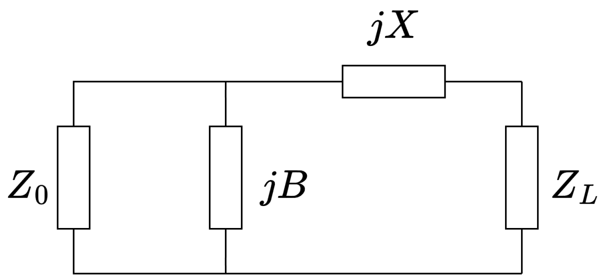


Figure 14: Matching circuit for z outside the $1 + jx$ circle.

With B being the component suspectance and X the component reactance, r_l the real part of the impedance z and x_l the imaginary part of the normalized impedance z.

The results of the matching networks using the analytic approach are shown in Table 5, where the values of the components are calculated based on the input and output impedances of the transistor.

Table 5: Values of the components for the matching networks using the analytic approach.

Parameter	Value
C_{in}	1.93417775 pF
L_{in}	43.4243305 pH
L_{1out}	2.4877816 nH
L_{2out}	1.8095882 nH

Comparing the values obtained from the Smith Chart and the analytic approach, it is possible to see that the values are very similar, with a difference of less than 5% in all cases. This validates the results obtained from the Smith Chart.

The resulting circuit, with the final values, is shown in Figure 15.

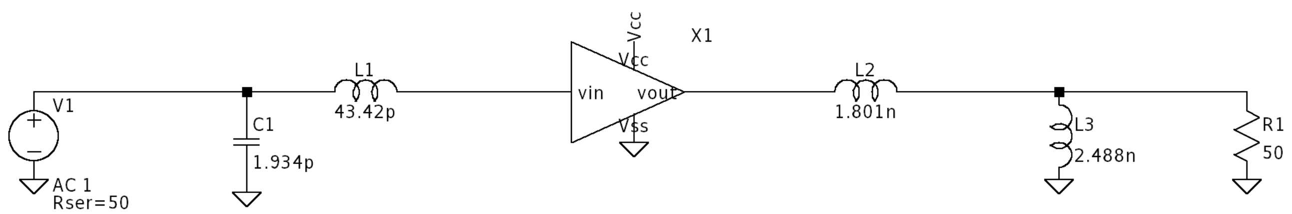


Figure 15: Matching circuit for input and output with final values.

2.4.2 Matching lines and stubs

The matching networks were also designed using transmission lines and stubs, this type of adaptation allows greater frequencies (more than 1 GHz) in real conditions. The input and output impedances of the transistor were transformed to $50\,\Omega$ using a combination of transmission lines and stubs. The values were obtain with the Smith Chart and with an analytic approach using a Python script (Appendix A) that calculates the values of the components based on the input and output impedances of the transistor.

Results with the Smith Chart:

The matching using the Smith Chart for the input and output are shown in Figures 16 and 18.

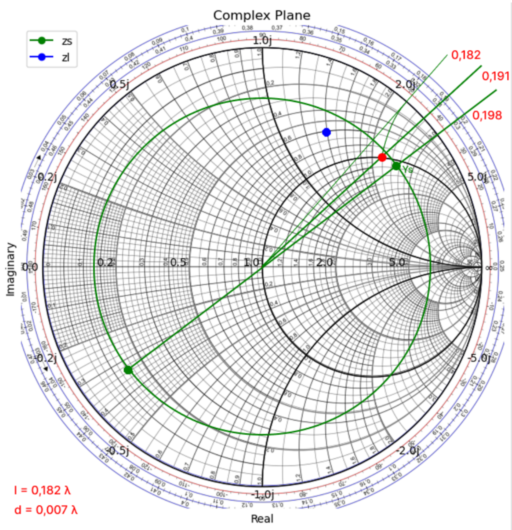


Figure 16: Smith chart for input matching with lines and stubs

The adaptation mesh for the input was done with an open circuit shunt stub and a series line, as shown in Figure 17.

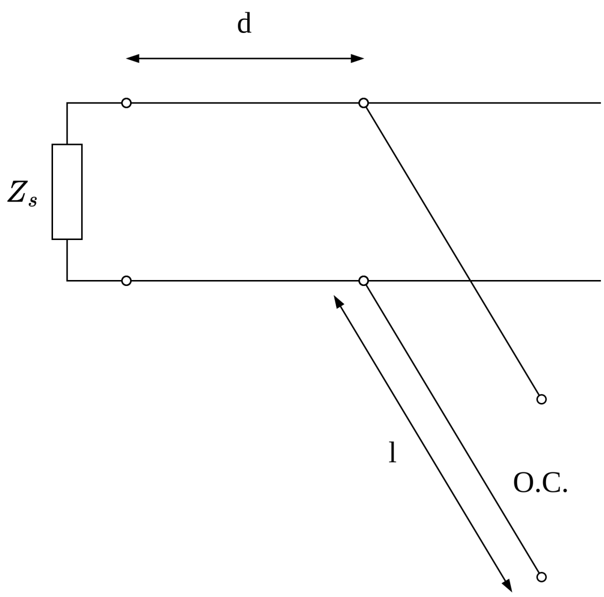


Figure 17: Matching circuit for input with lines and stubs

The values obtain from the Smith Chart for the input matching network are shown in Table 6.

Table 6: Values of the components for the matching networks using the Smith Chart with lines and stubs.

Parameter	Value
l	$0,182\lambda$
d	$0,007\lambda$

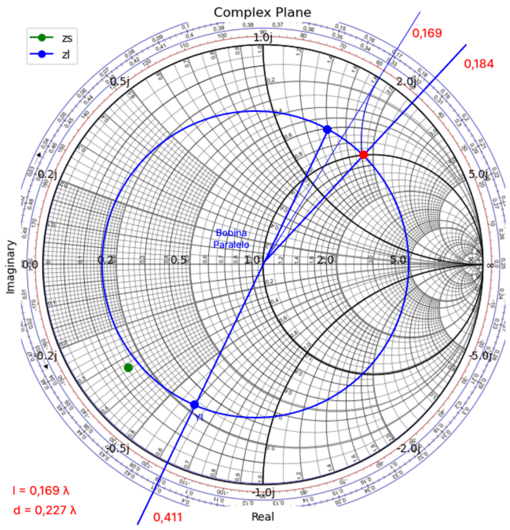


Figure 18: Matching circuit for input with lines and stubs

The adaptation mesh for the output is done with a series line and an open circuit shunt stub as shown in Figure 19.

The values obtain from the Smith Chart for the output matching network are shown in Table 7.

Table 7: Values of the components for the matching networks using the Smith Chart with lines and stubs.

Parameter	Value
l	$0,169\lambda$
d	$0,227\lambda$

Results with analytic approach:

The expressions used in the Python script to calculate the values of the components are the following [1]:

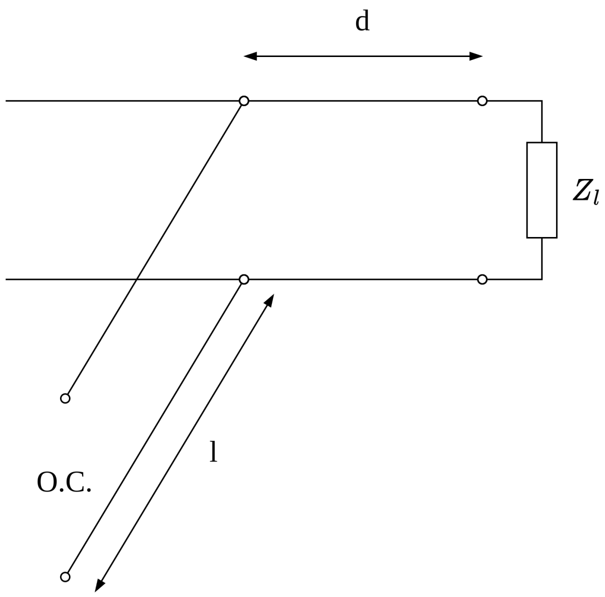


Figure 19: Matching circuit for output with lines and stubs

Considering an impedance Z:

$$Y = G + jB = \frac{1}{Z} \tag{29}$$

Where G is the conductance and B is the susceptance and can be calculated as:

$$G = \frac{R_l(1+t^2)}{R_L^2 + (X_L + Z_0 t)^2} \tag{30}$$

$$B = -\frac{R_l^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]} \tag{31}$$

Where R_l is the real part of the impedance, X_l is the imaginary part of the impedance, Z_0 is the characteristic impedance of the line and t is the length of the line in wavelengths.

If R_L is different from Z_0, the equation for t can be expressed as:

$$t = \frac{X_L \pm \sqrt{R_L[(Z_0 - R_L)^2 + X_L^2]/Z_0}}{R_L - Z_0} \tag{32}$$

If $R_L = Z_0$, the equation for t can be expressed as:

$$t = \frac{-X_L}{2Z_0} \tag{33}$$

Thus, the solutions for the line lengths d normalized to the wavelength are:

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi}\tan^{-1} t & \text{for } t \geq 0 \\ \frac{1}{2\pi}\left(\pi + \tan^{-1} t\right) & \text{for } t < 0 \end{cases} \tag{34}$$

Now to find the open-circuited stub length l normalized to the wavelength, the following equations are used:

$$\frac{l}{\lambda} = \frac{-1}{2\pi} tan^{-1} \frac{B}{Y_0} \tag{35}$$

To calculate the wavelength λ in the line, the following equation is used:

$$\lambda = \frac{v}{f} \tag{36}$$

And the velocity v in the line is given by:

$$v = \frac{c}{\sqrt{\epsilon_{eff}}} \tag{37}$$

The effective permittivity ϵ_{eff} is calculated as:

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{12h}{w}\right)^{-0.5} \tag{38}$$

where $w = 1,5$mm and $h = 800$µm that correspond to the characteristics of the microstrip line used in the design.

The results obtain from the analytic approach are shown in Table 8. It is possible to observe that the values of the components are very similar to the ones obtained from the Smith Chart, with a difference of less than 5% in all cases. This validates the results obtained from the Smith Chart.

Table 8: Values of the components for the matching networks using the analytic approach with lines and stubs.

Parameter	**Value**
v	$171739780,4465274\,\mathrm{m\,s^{-1}}$
ϵ_{eff}	$3,051410966570356$
λ	$42,90158139038047\,\mathrm{mm}$
l_{in}	$8,0649\,\mathrm{mm}$
d_{in}	$157,9\,\mu\mathrm{m}$
l_{out}	$7,3823\,\mathrm{mm}$
d_{out}	$9,6999\,\mathrm{mm}$

The final circuit is shown in Figure 20, where the input and output matching networks are designed using a combination of transmission lines and stubs.

2.4.3 Constant Gain circles

A Python script (Appendix A) was made, in line with [2], to calculate the constant gain circles for the input and output matching networks and plot it in the Smith Chart, which are shown in Figure 21. The constant gain circles are used to visualize the gain of the LNA for

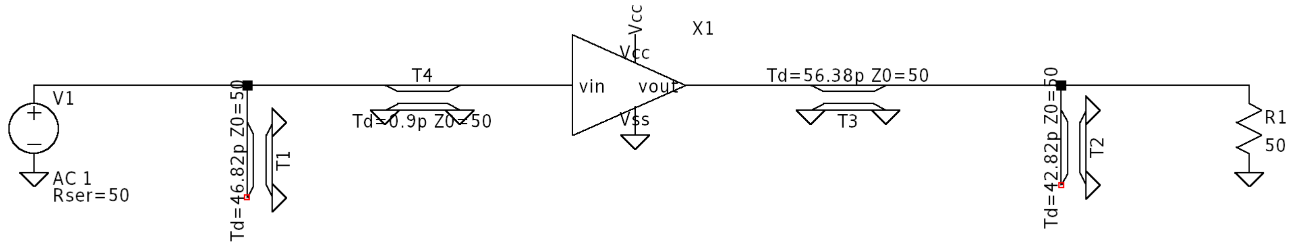


Figure 20: Matching circuit for input and output with values.

different source and load impedances. For maximum gain, the source and load impedances should be located on the crosses that represent the maximum gain circles.

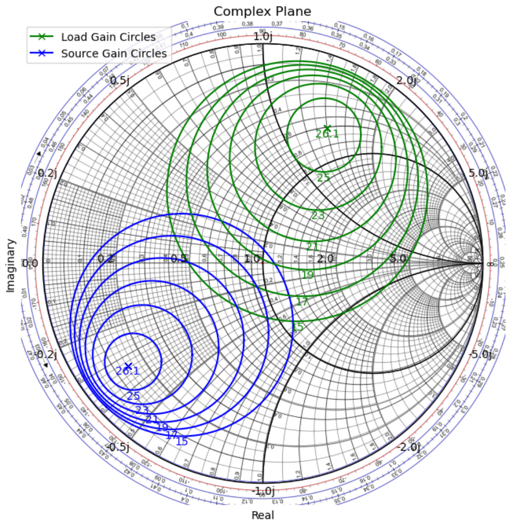


Figure 21: Constant gain circles for the input matching network.

2.5 Input and output matching networks for Minimum Noise

After designing the LNA for maximum gain, the next step is to design the input and output matching networks for minimum noise. The goal is to minimize the noise figure (F).

The noise figure is a measure of how much noise the LNA adds to the signal. The lower the noise figure, the better the LNA performs in terms of noise. And it can be defined as [2]:

$$F = \frac{P_{No}}{P_{Ni} G_A} \tag{39}$$

Where P_{No} is the output available noise power, P_{Ni} is the input a¡vailable noise power due to R_n and G_A is the available gain of the LNA.

The available gain of the LNA is defined as:

$$G_A = \frac{P_{So}}{P_{Si}} \tag{40}$$

Where P_{So} is the output available signal power and P_{Si} is the input available signal power.

Thus, rewriting equation 39 in terms of the available gain, the noise figure can be expressed as:

$$F = \frac{P_{Si}/P_{Ni}}{P_{So}/P_{No}} = \frac{SNR_i}{SNR_o} \tag{41}$$

When working with scattering and noise parameters, the noise figure can be expressed as [2]:

$$F = F_{min} + \frac{r_n}{g_s}|y_s - y_{opt}|^2 \tag{42}$$

Where F_{min} is the minimum noise figure, r_n is the normalized noise resistance, g_s is the normalized source conductance and y_s is the normalized source admittance. The y_{opt} is the optimal source admittance that minimizes the noise figure to the value F_{min}.

The following noise parameters where obtain in cadence from the transistor model without any matching network, and compiled in Table 9.

Table 9: Noise parameters of the transistor.

Parameter	**Value**
F_{min}	$2,171$
r_n	$9.505\,\Omega$
g_s	$1.11479037\,\mathrm{S}$
y_s	$1.11479037 + 2.54323358j\,\mathrm{S}$
y_{opt}	$0.02750192 + 0.00928327j\,\mathrm{S}$
Γ_s	$-0.1889065597 - 0.1585114245j$

Once again a Python script (Appendix A) was made to calculate and plot the constant input noise figure circles in the Smith Chart, which are shown in Figure 22. The constant noise figure circles are used to visualize the noise figure of the LNA for different source and load impedances. For minimum noise, the source and load impedances should be located on the points that represent the minimum noise figure circles.

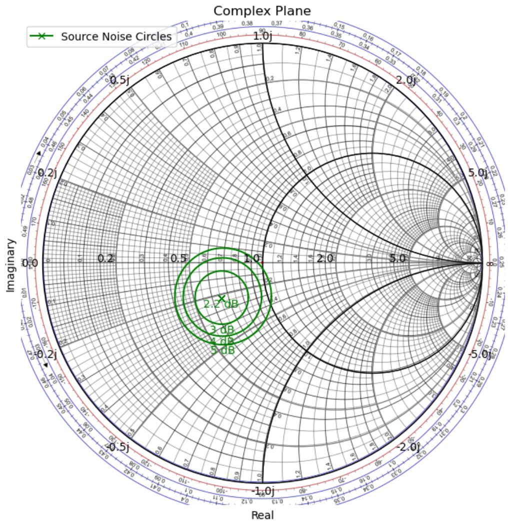


Figure 22: Constant noise figure circles for the input matching network.

In order to do the impedance matching for minimum noise, the output reflection coefficient has to be calculated with the following equation [3]:

$$\Gamma_L = (\frac{s_{22} + (s_{12}s_{21}\Gamma_s)}{1 - s_{11}\Gamma_s})^* \tag{43}$$

Now for the sake of simplicity, the reflection coefficients were transformed to the normalized impedances using Equations 15 and 16.

In table 10 the normalized impedances at the source and load for minimum noise are shown, where the values are calculated using the reflection coefficients obtained in the previous step.

Table 10: Normalized impedances at the source and load for minimum noise.

Parameter	**Value**
z_S	$0.652838 - 0.2203652j\ \Omega$
z_L	$1.07281071 + 0.65421904j\Omega$

In figure 23 the normalized impedances at the source and load for minimum noise are shown in the Smith Chart.

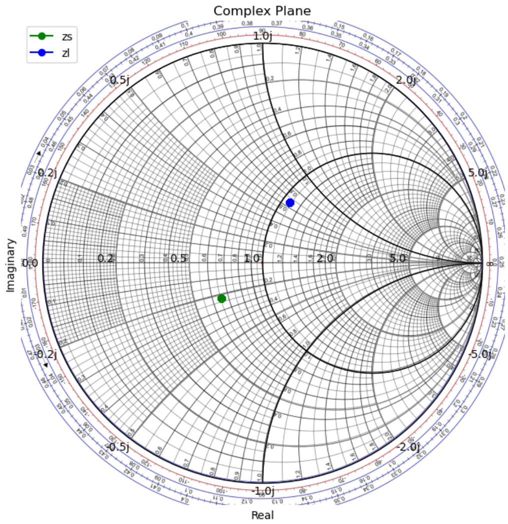


Figure 23: Normalized impedances at the source and load for minimum noise in the Smith Chart.

Since the Python script (Appendix A) for calculating the matching networks has already been verified in the previous section, in this section the matching networks were calculated using it, without the use of the Smith Chart. The equations used in the script are the same as in the previous section, but with the values of the normalized impedances at the source and load for minimum noise.

2.5.1 Matching with lumped elements for minimum noise

Using the same matching circuits as in the previous sections, depicted in Figures 10 and 12, the values of the components were calculated using the Equations 25 to 28. The results of the matching networks for minimum noise using the analytic approach are shown in Table 11, where the values of the components are calculated based on the input and output impedances of the transistor for minimum noise.

Table 11: Values of the components for the matching networks for minimum noise using the analytic approach.

Parameter	Value
C_{in} (shunt)	0.5798506944259898 pF
L_{in} (series)	0.5083089871809204 nH
L_{1out} (shunt)	2.25632568 nH
L_{2out} (series)	1.36538599 nH

In Figure 24 the final circuit with the values of the components for minimum noise is shown, where the input and output matching networks are designed using a combination of inductors and capacitors.

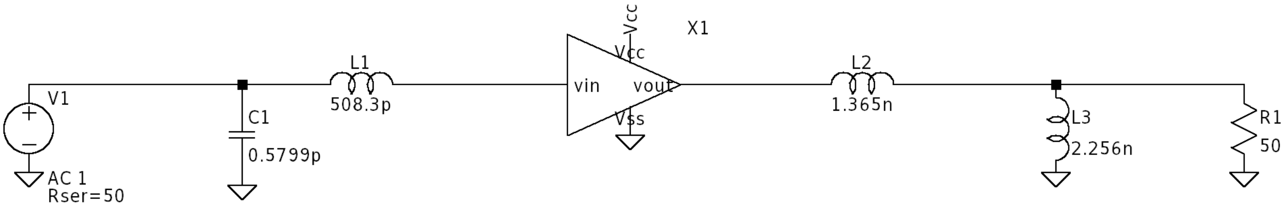


Figure 24: Matching circuit for input and output with final values for minimum noise.

2.5.2 Matching lines and stubs for minimum noise

Using the same matching circuits as in the previous sections, depicted in Figures 17 and 19, the values of the components were calculated using the Equations 34 to 35 in the Python script (Appendix A).

The results of the matching networks for minimum noise using the analytic approach are shown in Table 12.

Table 12: Values of the components for the matching networks for minimum noise using the analytic approach with lines and stubs.

Parameter	Value
l_{in}	$3,21\,\text{mm}$
d_{in}	$2,13\,\text{mm}$
l_{out}	$3,87\,\text{mm}$
d_{out}	$2,47\,\text{mm}$

The final circuit with the values of the components for minimum noise is shown in Figure 25, where the input and output matching networks are designed using a combination of transmission lines and stubs.

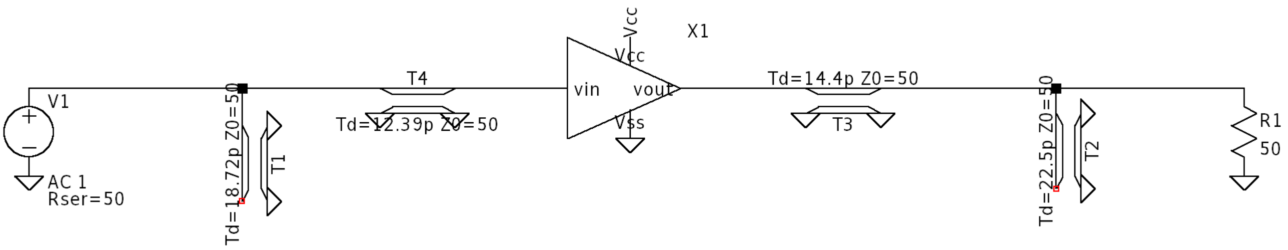


Figure 25: Matching circuit for input and output with values for minimum noise.

2.6 Gain-Noise Optimization

To optimize the LNA for both gain and noise, a trade-off between the two parameters is necessary. The goal is to find a balance between the maximum gain and the minimum noise figure. To obtain this result the input gain circles and the input noise circles were plotted in the Smith Chart and analyzed, the plot is shown in Figure 26.

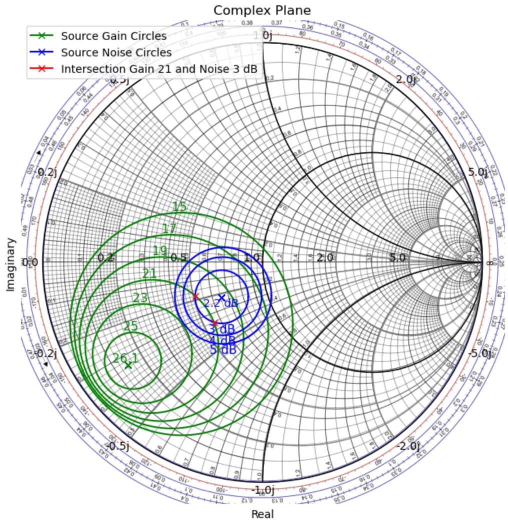


Figure 26: Gain and noise circles for the input matching network.

The intersection of the gain and noise circles represents the optimal point for the LNA, where the gain is maximized ($G = 21$) while the noise figure is minimized ($F = 3$dB).

In table 13 the values of the intersection points of the 21 gain circle and the 3 dB noise figure circle are shown, where the values were calculated using the Python script (Appendix A).

Table 13: Values of the intersection points of the gain and noise circles.

Parameter	Value
Γ_{A1}	$-0.30695536 - 0.15142555j\ \Omega$
Γ_{A2}	$-0.22141437 - 0.27183862j\ \Omega$
z_{A1}	$0.51000409 - 0.17495104j\ \Omega$
z_{A2}	$0.56016594 - 0.34723135j\Omega$

For the following calculations $\Gamma_s = \Gamma_{A1}$ and $z_{A1} = z_s$ will be used.

Once again in order do obtain a bilateral matching network, the output reflection coefficient has to be calculated with the equation 43. Withthe reflection coefficient value, it is possible to obtain the normalized impedance at the load using the equation 16.

In table 14 the normalized impedances at the source and load and the reflection coefficients for gain-noise optimization are shown.

Table 14: Normalized impedances at the source and load for gain-noise optimization.

Parameter	Value
z_S	$0.51000409 - 0.17495104j\ \Omega$
z_L	$0.95353092 + 0.74623035j\Omega$
Γ_S	$-0.30695536 - 0.15142555j$
Γ_L	$0.10657803 + 0.34127875j$

In Figure 27 the normalized impedances at the source and load for gain-noise optimization are shown in the Smith Chart.

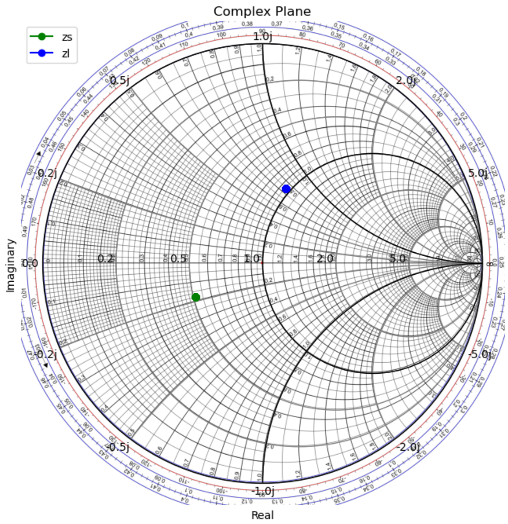


Figure 27: Normalized impedances at the source and load for gain-noise optimization in the Smith Chart.

2.6.1 Matching with lumped elements for gain-noise optimization

Using the same matching circuits as in the previous sections, depicted in Figures 10 and 12, the values of the components were calculated using the Equations 25 to 28 in the Python script (Appendix A).

The results of the matching networks for gain-noise optimization using the analytic approach are shown in Table 15.

Table 15: Values of the components for the matching networks for gain-noise optimization using the analytic approach.

Parameter	Value
C_{in} (shunt)	0.779402735 pF
L_{in} (series)	0.64596288 nH
L_{1out} (shunt)	9.00487954 nH
L_{2out} (series)	1.06497606 nH

In Figure 28 the final circuit with the values of the components for gain-noise optimization is shown.

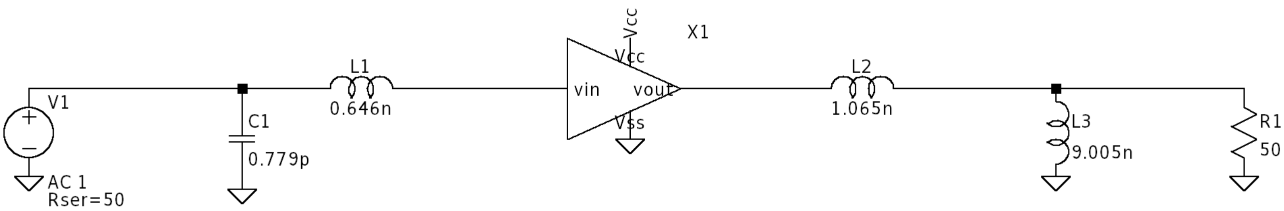


Figure 28: Matching circuit for input and output with final values for gain-noise optimization.

2.6.2 Matching lines and stubs for gain-noise optimization

Once again were used the same matching circuits as in the previous sections, depicted in Figures 17 and 19, the values of the components were calculated using the Equations 34 to 35 in the Python script (Appendix A).

The results of the matching networks for gain-noise optimization using the analytic approach are shown in Table 16.

Table 16: Values of the components for the matching networks for gain-noise optimization using the analytic approach with lines and stubs.

Parameter	Value
l_{in}	$4,299$ mm
d_{in}	$2,605$ mm
l_{out}	$4,462$ mm
d_{out}	$10,511$ mm

The final circuit with the values of the components for gain-noise optimization is shown in Figure 29, where the input and output matching networks are designed using a combination of transmission lines and stubs.

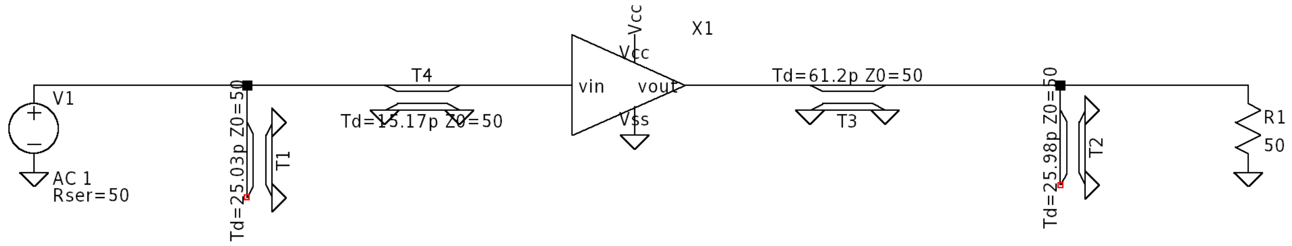


Figure 29: Matching circuit for input and output with values for gain-noise optimization.

3 Simulation

In this section, the simulations of the LNA and the corresponding matching networks will be presented.

3.1 Validation of the LNA design

First, using the LTSpice, the T502 transistor was added and the parasites capacitance and inductance of the package were considered, resulting in the simulated real transistor circuit in Figure 30.

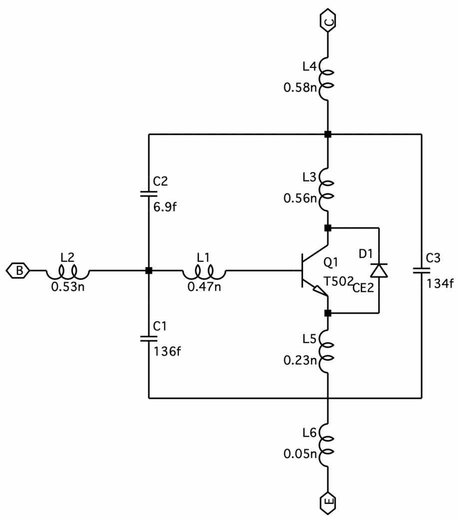


Figure 30: Transistor with package effects

After this, the biasing circuit of the transistor for the required parameters is present in Figure 31. explicar a existencia de bobinas e condensadores

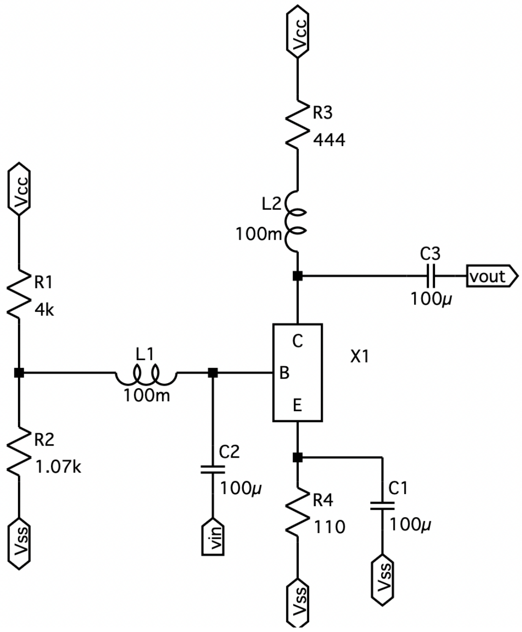


Figure 31: Biasing circuit simulated

After simulating the operation point of the previous circuit, the result for the required parameters is shown in Figure 32.

Figure 32: Result of the operation point simulation

After analyzing the previous graphic, it is possible to confirm that result parameters of the biasing circuit are within the required.

At the same time, the same circuit was implemented in Cadence, to ensure greater reliability of the results obtained. So, this circuit can be seen in Figure ??.

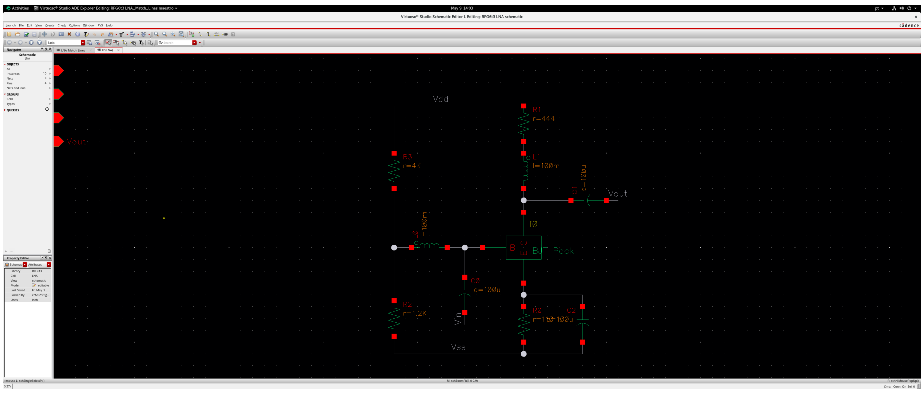


Figure 33: Biasing circuit simulated in Cadence

So the matching networks for the source and the load, both with an impedance of 50Ω, can be implemented.

3.2 Input and output matching networks design simulation

As mentioned before, the first step in designing a matching network in an LNA is to know the S-parameters of the amplifier circuit, so, switching to an AC analysis of the LNA circuit, and adding both the source and the load, the simulated circuit is shown in Figure 34.

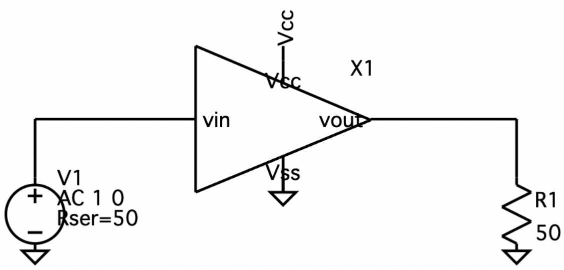


Figure 34: Circuit for the S-parameters

The resulting S-parameters can be seen in Figure 35.

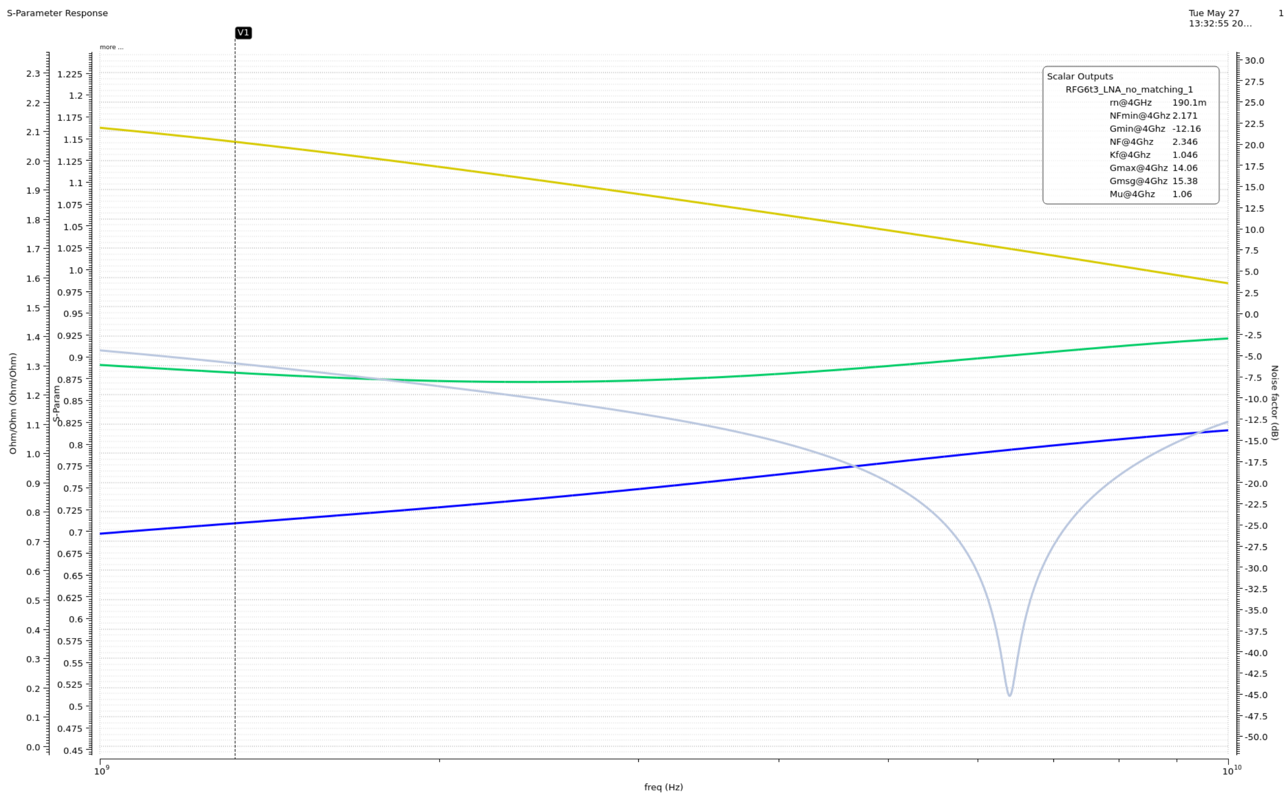


Figure 35: S-parameters for the LNA circuit

After assuming a working frequency of $4GHz$, the resulting matching networks using capacitors and inductors is exhibit in Figure 36.

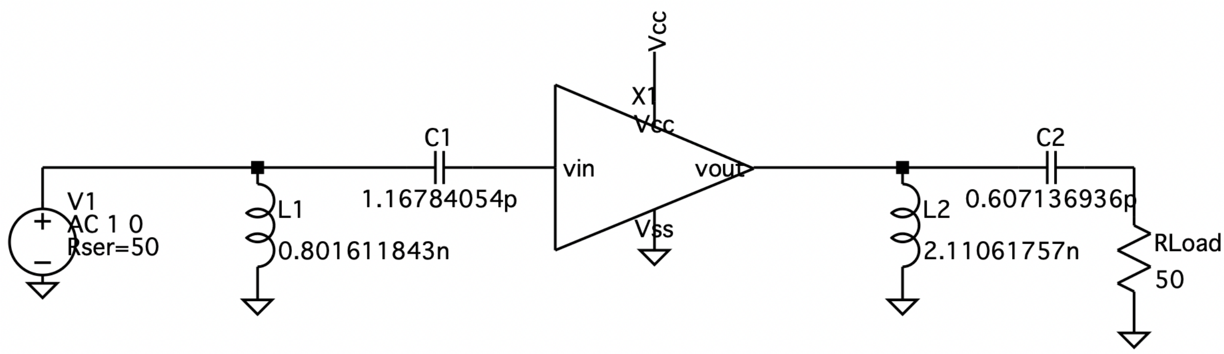


Figure 36: Matching networks using inductors and capacitors

Using again the same AC analysis, the S-parameters of new circuit using the capacitors and inductors matching networks is displayed in Figure 37.

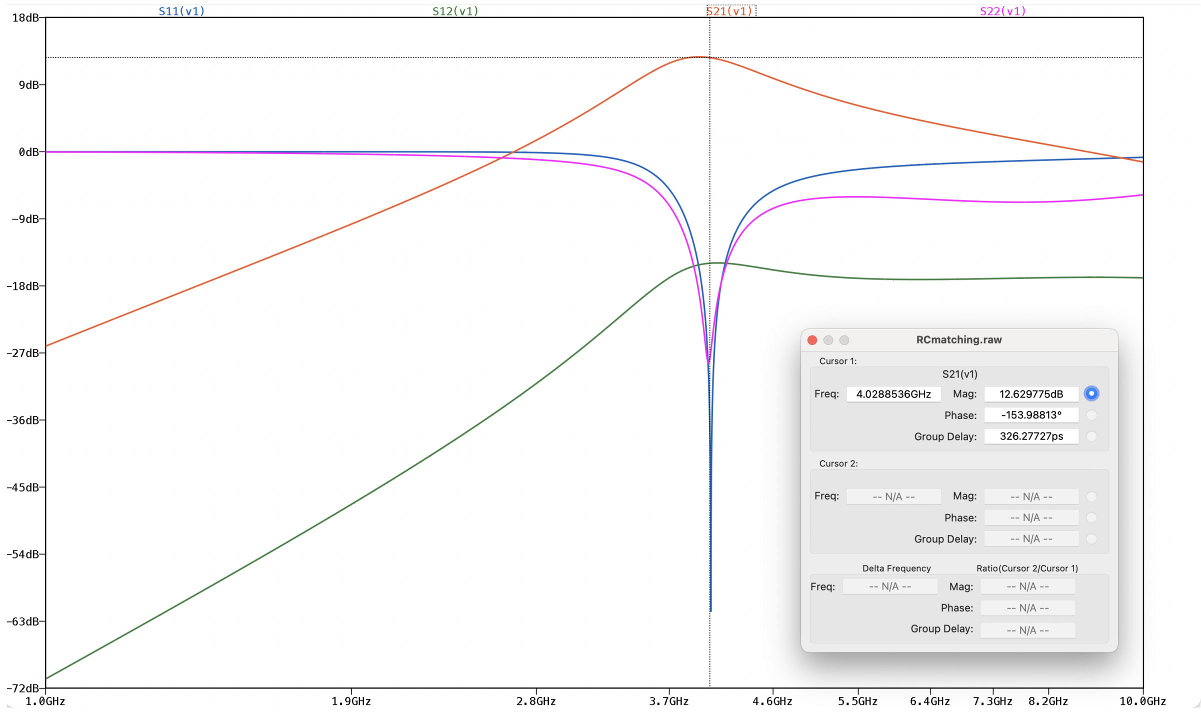


Figure 37: S-parameters for the matching networks using inductors and capacitors

After looking at the graphic above, it is possible to conclude that this matching network is working properly, since there is a sharp drop in both $S11$ and $S22$ ate the desired frequency of $4GHz$ as well as a high point in the $S21$ curve.

In cadence, the circuit for the matching using inductors and capacitors is the following.

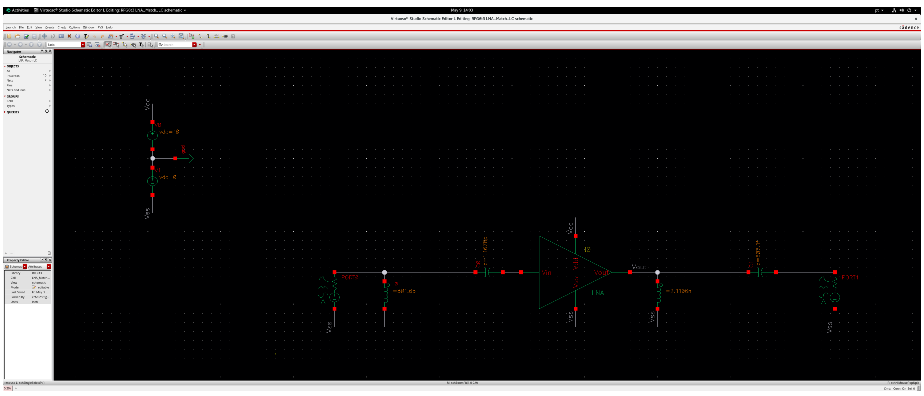


Figure 38: Matching networks using inductors and capacitors in Cadence

Simulating, the S-parameters of this circuit with the mentioned matching networks is visible in Figure 39.

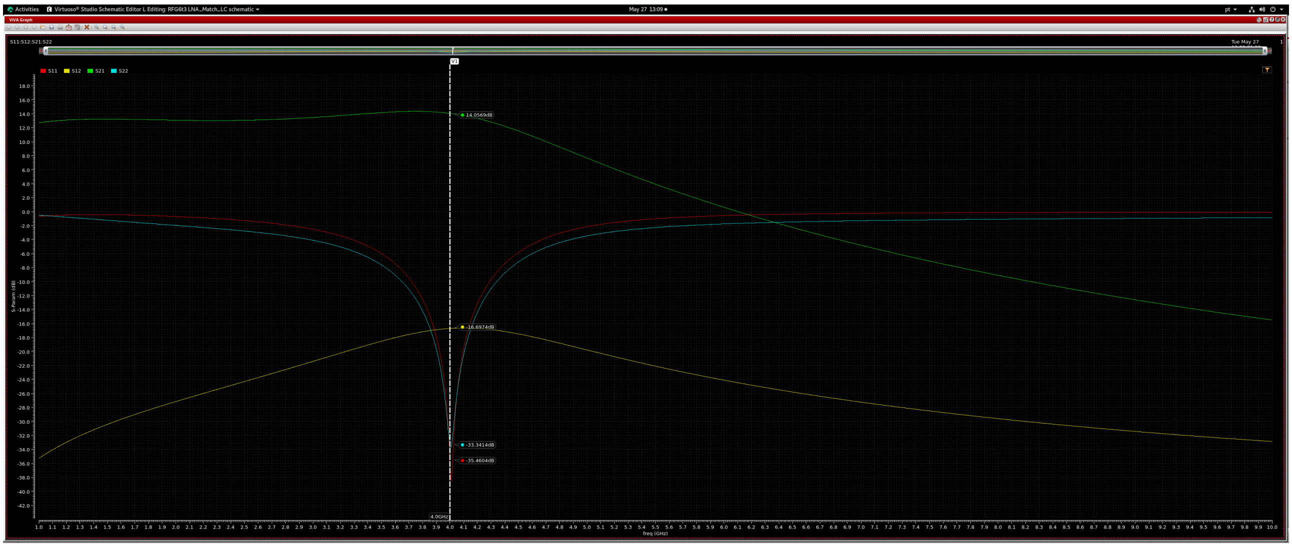


Figure 39: S-parameters for the matching networks using inductors and capacitors in Cadence

Passing to the matching networks using transmission lines and stubs, and using the same working frequency, the new LNA circuit is shown in Figure 40.

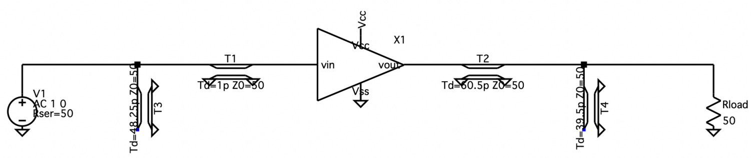


Figure 40: Matching networks using transmission lines and stubs

The S-parameters for this new matched circuit can be seen in Figure 41.

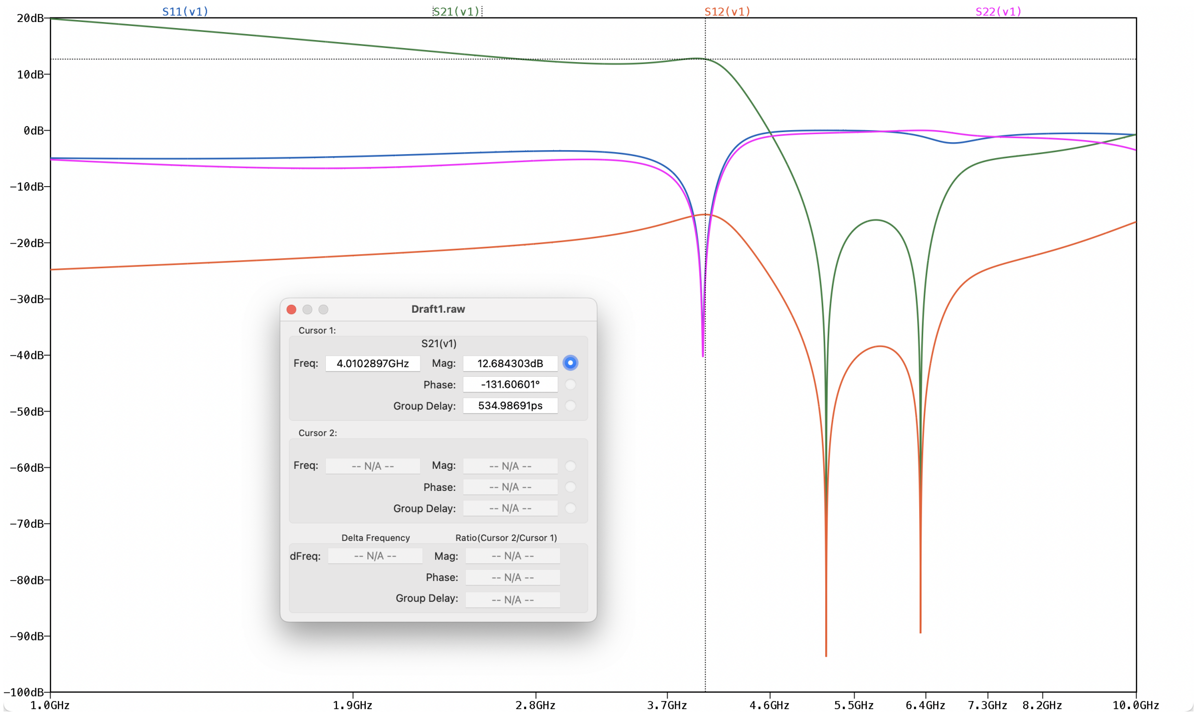


Figure 41: S-parameters for the matching networks using transmission lines and stubs

Similarly to the first matching networks, the drop in both $S11$ and $S22$ can also be seen, as well as the same approximated value of $S21$, so, this matching network using transmission lines and stubs is correctly working.

In cadence, the circuit for the matching using transmission lines and stubs is the following.

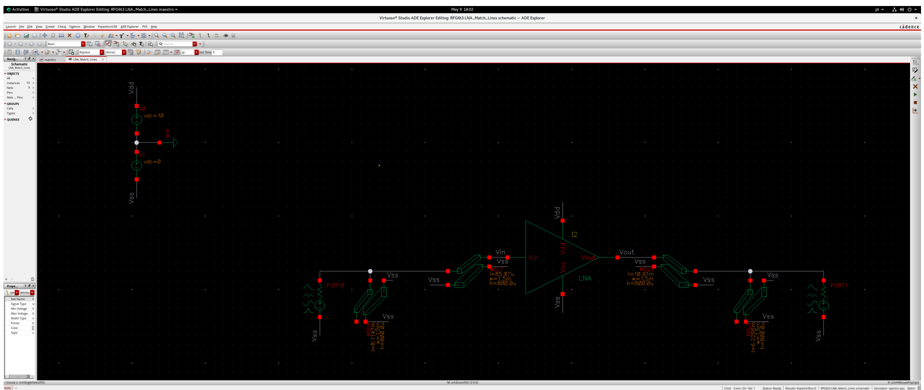


Figure 42: Matching networks using transmission lines and stubs in Cadence

Simulating, the S-parameters of this circuit with the mentioned matching networks is visible in Figure ??.

3.3 Noise

To simulate the noise of each matching network, the Cadence was used, resulting in the graphic shown in Figure 43 for the inductors and capacitors.

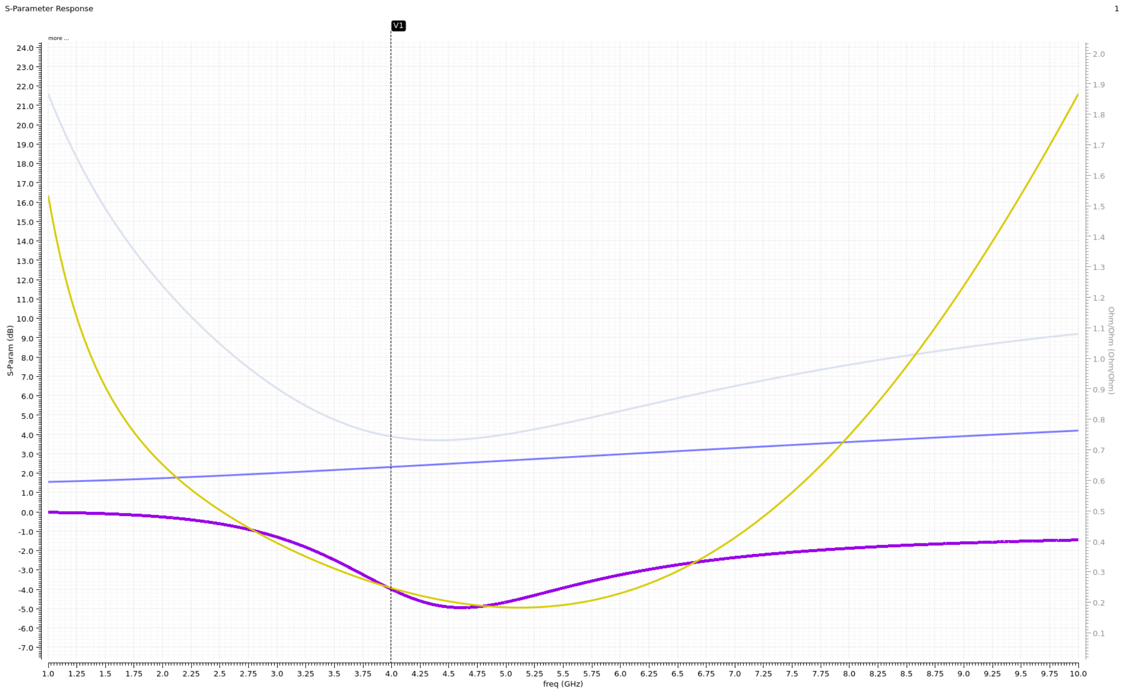


Figure 43: Noise for the matching networks using capacitors and inductors in Cadence

And for transmission lines and stubs in Figure 44.

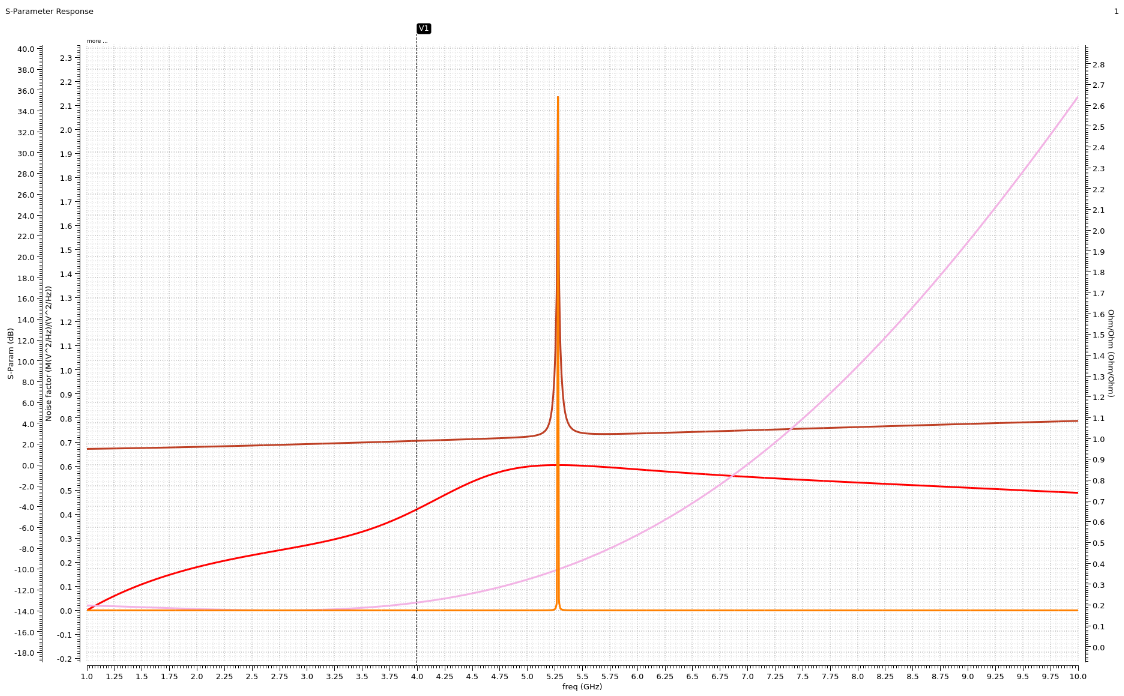


Figure 44: Noise for the matching networks using transmission lines and stubs in Cadence

3.4 Simulation without matching networks

simulação LTSpice
simulação cadence

3.5 Simulation for Maximum Gain Adaptation

simulação LTSpice
simulação cadence

3.6 Simulation for Minimum Noise Adaptation

simulação cadence

3.7 Simulation for Noise-Gain Adaptation

simulação cadence

4 Conclusion

The Cadence simulation were not what was expected because of an error in the initial transistor package circuit that effected the remaining project, but in LTspice, after correcting this mistake, all results were the expected for both matching networks. In second phase, the mistake will be corrected in Cadence as well.

References

[1] D. M. Pozar, *Microwave engineering*, 4th ed. John Wiley and Sons, Inc, 2012.

[2] G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, 2nd ed. Prentice Hall, 1997.

[3] L. Oliveira, "Class slides," 2025.

A Appendix A: Python Script