Reputation-based Ranking Systems and their Resistance to Bribery

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***Abstract*—We study bribery resistance properties in two classes of reputation-based ranking systems, where the rankings are computed by weighting the rates given by users with their reputations. In the first class, the rankings are the result of the aggregation of all the ratings, and all users are provided with the same ranking for each item. In the second class, there is a first step that clusters users by their rating pattern similarities, and then the rankings are computed cluster-wise. Hence, for each item, there is a different ranking for distinct clusters. We study the setting where the seller of each item can bribe users to rate the item, if they did not rate it before, or to increase their previous rating on the item. We model bribing strategies under these ranking scenarios and explore under which conditions it is profitable to bribe a user, presenting, in several cases, the optimal bribing strategies. By computing dedicated rankings to each cluster, we show that bribing, in general, is not as profitable as in the simpler without clustering. Finally, we illustrate our results with experiments using real data.**

I. Introduction

The evolution of contemporary society towards an information economy boosted the development of e-commerce. The fast pace of information spreading around the world has been reconfiguring our social interactions. The social networks and online *fora* instigated the exchange of opinions and the online word of mouth (WOM), which in turn gained the potential to drive e-commerce sales, see [1], [2] and [3], for instances.

Nowadays, the importance of reviews and rankings became paramount for sellers, as the visibility and the sales numbers are related with them, [4], [5] and [6]. Further, studies pointed out that, in several cases, online reviews may be more influential than traditional marketing, see [7]. This strong influence increased the attempts to manipulate them, [8], and it fostered the need for designing ranking systems robust to spam and attacks, as in [9], [10] and references therein. The companies invest money to convince users to vouch for their products/services, either by giving samples of them so that users can comment on them or by directly paying users to provide positive feedback on their products and negative on competitor ones, see [11]. Aware of the importance and influence of bribing to manipulate rankings, we model this phenomenon and characterize it quantitatively, so that its impact can be better understood. We mitigate the impact of such behaviors, by showing that reputation-based ranking systems using clusters are, in general, more robust to bribery.

Previous work. The influence of individual decisions on global properties in network-based rating systems was studied in several works. In [12], the authors investigate how to turn a product into a tendency among users by changes on a social network. In [13], the authors explore how to design an impartial mechanism for peer review to mitigate the effect of a reviewer interfering with the likelihood of its work being accepted.

In [10], the authors proposed a reputation-based ranking system that clusters users by their ranking pattern similarities, also exploring this idea for recommendation systems in [14]. The authors showed that, by doing so, their approach is more robust to both spamming users and users trying to attack the ranking system to change the ranking of a set of items.

The authors of [15] analyzed the resistance of two ranking systems, one that simply averages the ratings of users (AA), another that takes into account the influence network of a given user, using the AA to compute the ranking for each network. They showed that the AA ranking system is bribable, and, in particular, bribing users who did not rate is profitable. When considering social networks of users, they show that the bribery effect is diminished. Their work assumes a fixed set of users with only one item to rate. The AA does not capture a possible multimodal ratings' behavior, as noticed in [16]. This motivated us to study bribing in reputation-based ranking systems and to explore the case where users are clustered in groups which, intuitively, must lessen the bribing effect.

Our contribution. Here, we study the resistance to bribery of a class of ranking systems that assign reputations to users. We show that a ranking system computing the items rank by the weighted average of users ratings with their reputations is bribable since users that rated the item with a reputation above the users' average reputation are bribable. By clustering users by their rating pattern and assigning possibly different rankings for the same item for each cluster, we increase the bribery resistance of the ranking system. This makes the ranking system in [10] much more robust to bribing. Further, a user is bribable if its reputation is larger than the average reputation of the users that rate the item. This bound increases in the clustering scenario, since within each cluster the number of users that rated the item is smaller than the non-clustering scenario. Our model also applies to evaluate marketing strategies, where a company is willing to invest money to augment its sales, either increasing the users base or boosting positive reviews.

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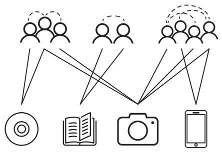


Fig. 1: Bipartite/multipartite graph representation of users and items with edges interconnecting them weighted by the users' ratings for items, not considering/considering the dashed links.

II. Preliminaries and definitions

Here, we set up the notational conventions and definitions. Reputation-based ranking systems assign weights to users to aggregate their rankings on given items. Here, we discuss two classes of ranking systems, namely, the *bipartite reputation-based ranking systems* (BRS) and the *multipartite reputation-based ranking systems* (MRS), see [10] for instances.

Given a *set of items* I, suppose that any user u, among the *set of users* U, can assign a *rating*, R_{ui}, to any item $i \in I$. Further, for each item, i, we attribute it a score that is a weighted average of its ratings. In this setting, the reputation-based ranking systems attribute a *reputation* score, c_u, to every user $u \in U$. The *ranking*, r_i, for each item, i, is then computed as a function of the reputations of the users who rated the item and their ratings to the item. Here, we assume, without loss of generality, that both reputations and rankings take values in $]0, 1]$.

A BRS iteratively computes the ranking of each item, using a weighted average, whose coefficients are the users' reputations, updated considering the discrepancy of the item's ranking and the user's rating on that item. We repeat the process until convergence. In this setting, every user, $u \in U$, has access to the same ranking, r_i, for each item, $i \in I$. Schematically, we represent these ranking systems as a bipartite graph, in which a set of vertices corresponds to users and the other to items, see [17], and see [9] or [10] for a definition. The edges connecting vertices are weighted by the ratings that users gave to items and only connect users to items, see Figure 1, not considering the dashed lines.

An MRS takes two steps. In the first step, the users are clustered by their rating pattern similarities. In the second step, the ranking of each item is iteratively computed, as in the BRS case, only using information from each cluster, producing (possibly) different rankings for different clusters. Since the ranking of an item may differ from cluster to cluster, every user on the same cluster accesses the local ranking of a given item, in the case where the item was rated by, at least, one user belonging to that cluster. If for a given item, within a given cluster, no user rated that item, the available ranking for that item is the weighted average of that item's rankings among clusters where the item was rated. Each user can link to several items, by edges weighted by its ratings, as before. Now, we allow for edges between users (encoding similarities between them), with clusters of connected users, forming a multipartite graph, see Figure 1, and see [17] for the definition. For details and convergence of this ranking systems class see [10]. In the BRS, the *ranking* of the item i is computed by $r_i = \frac{1}{\alpha} \sum_{u \in U_i} c_u R_{ui}$, where $\alpha = \sum_{u \in U_i} c_u$, and U_i denotes the set of users that rated item $i \in I$, where I is the set of all users.

For the MRS, let $\mathcal{M}_1, \ldots, \mathcal{M}_N$ be a partition of the set of users U into N disjoint groups of users, that is, $U = \bigcup_{n=1}^{N} \mathcal{M}_n$ and, for $m \neq n$, $\mathcal{M}_m \bigcap \mathcal{M}_n = \varnothing$. We denote the set of items rated by users from cluster $\mathcal{M}_n$ by $I^{\mathcal{M}_n}$, with $I^{\mathcal{M}_n} = \bigcup_{u \in \mathcal{M}_n} I_u$, where I_u is the set of items rated by user u. The set of users in the cluster $\mathcal{M}_n$ that rated item i is denoted by $U_i^{\mathcal{M}_n}$, where $U_i^{\mathcal{M}_n} = U_i \cap \mathcal{M}_n$. Now, the ranking is computed independently for each cluster as $r_i^{\mathcal{M}_n} = \frac{1}{\alpha} \sum_{u \in U_i^{\mathcal{M}_n}} c_u R_{ui}$, $\alpha = \sum_{u \in U_i^{\mathcal{M}_n}} c_u$. Note that for users belonging to a cluster $\mathcal{M}_n$, the displayed ranking of item i can be one of the following two possibilities: (i) the ranking of the item for that cluster $r_i^{\mathcal{M}_n}$, whenever there are users in the cluster that rated item i; (ii) otherwise, the ranking of item i is the weighted average of the rankings of i for the clusters with users that rated item i, that is, $\bar{r}_i = \sum_{n \in \mathcal{X}_i} |U_i^{\mathcal{M}_n}| r_i^{\mathcal{M}_n} \big/ \sum_{n \in \mathcal{X}_i} |U_i^{\mathcal{M}_n}|$, where $\mathcal{X}_i = \{m : i \in I^{\mathcal{M}_m} \text{ and } m = 1, \ldots, N\}$. In what follows, for a set of users $U' \subseteq U$, $\bar{c}_{U'} = \sum_{u \in U'} c_u \big/ |U'|$.

Suppose the seller of item i has an initial wealth proportional to the item's ranking and to the number of customers that rated the item. To boost the sales of i, the seller may invest his resources (*wealth*) to promote the item's popularity so that users like it more, and/or to expand his consumer base by making people buy it, like it, but not necessarily love it. Here, we model this setting assuming that the popularity is an increasing function of the ranking of the item, r_i, and supposing that the number of consumers that bought the item is an increasing function of the number of users that rated the product, $|U_i|$.

We define the *reward function* or *wealth*, in the BRS and MRS, for the seller of item i as $J_i = |U_i| \, r_i$ and $\bar{J}_i = \sum_{n \in \mathcal{X}_i} J_i^{\mathcal{M}_n}$, respectively, where $J_i^{\mathcal{M}_n} = |U_i^{\mathcal{M}_n}| r_i^{\mathcal{M}_n}$.

We define the *strategy* of the seller of item i as a vector $\sigma^i \in \mathcal{S}_i$, with size $|U|$, where the u-th entry is the value of the invested wealth to convince user u to increase his rating by ρ_u, and $S_i \subseteq [0, 1]^{|U|} \backslash \{\mathbf{0}\}$, where $\mathbf{0}$ is the null strategy that does not bribe any user. If user u rated item i with R_{ui}, then $\rho_u \leqslant 1 - R_{ui}$. If $\rho_u = 0$ this means the seller does not try to persuade user u to rate or to change his rating on item i.

For the seller of item i, we denote by $\Xi_i = \{\sigma^i \in \mathcal{S}_i : \sigma^i(u) = \rho_u = 0 \text{ for all } u \notin U_i\}$ the set of strategies that consists, exclusively, in bribing users that already rated the item i. Analogously, we denote by $\bar{\Xi}_i = \mathcal{S}_i \backslash \Xi_i = \{\sigma^i \in \mathcal{S}_i : \sigma^i(u) = \rho_u = 0 \text{ for all } u \in U_i\}$, the set of strategies of bribing users that did not rate item i. We say that a bribing strategy σ^i is an *elementary strategy* if for some user $u \in U$ we have that $\sigma_u^i > 0$ and, for all $v \in U$ with $v \neq u$, $\sigma_v^i = 0$. To easy notation, instead of denoting by $\sigma^i(u)$ the strategy of seller of item i to bribe user u, we write σ_u^i. Further, the wealth spent by playing strategy σ^i is given by $\|\sigma^i\|_1 = \sum_{u \in U} \sigma_u^i$.

After strategy σ^i, the wealth of seller i becomes $J_{\sigma^i} = |U_{\sigma^i}| \, r_{\sigma^i} - \sum_{u \in U_{\sigma_i}} \rho_u$, with $\bar{J}_{\sigma^i} = \sum_{n \in \mathcal{X}_i} \left| U_{\sigma^i}^{\mathcal{M}_n} \right| r_{\sigma^i}^{\mathcal{M}_n} - \sum_{u \in U_{\sigma_i}} \rho_u$, respectively, for the BRS and the MRS, where r_{σ_i} is the new value of r_i after σ^i.

The *profit* of playing the strategy σ^i is $\pi_{\sigma^i} = J_{\sigma^i} -$

J_i and $\bar{\pi}_{\sigma^i} = \bar{J}_{\sigma^i} - \bar{J}_i$, respectively, for the BRS and MRS.

III. Bribing in ranking systems

Here we study the resistance to bribery of reputation-based ranking systems. To simplify the analysis, we assume a fixed assignment of reputations to users. First, we describe the set of decomposable bribing strategies.After, we find the conditions for the strategies to be profitable. Lastly, we compare BRS with MRS and show that, by using clusters, MRS is more robust to bribery.

A. Properties of strategies and its profit in the BRS

First, we investigate what particular conditions allow us to decompose a strategy into elementary ones. We start by considering the case where item $i \in I$ sellers bribe users that already rated the item, proving that all strategies bribing several users at once are decomposable into several elementary ones.

Proposition 1. *Let $u, v \in U_i$ be two users that rated the item $i \in I$. If two strategies, σ_u^i and σ_v^i, consist in bribing users to change their ratings from R_{ui} and R_{vi} to $R_{ui} + \rho_u$ and $R_{vi} + \rho_v$, respectively, then we have that $\pi_{\sigma_u^i + \sigma_v^i} = \pi_{\sigma_u^i} + \pi_{\sigma_v^i}$.*

Proof: When the seller of item i plays the strategy σ_u^i, the ranking of item i changes according to $r_{\sigma_u^i} = r_i + \alpha^{-1} c_u \rho_u$. Thus, an elementary strategy's profit is:

$$\pi_{\sigma_u^i} = |U_i|\, r_{\sigma_u^i} - \rho_u - |U_i|\, r_i = \left(\frac{c_u}{\bar{c}_{U_i}} - 1\right)\rho_u. \quad (1)$$

The profit of the sum of strategies is given by: $\pi_{\sigma_u+\sigma_v} = |U_i|\, r_{\sigma_u^i+\sigma_v^i} - (\rho_u + \rho_v) - |U_i| r_i = \pi_{\sigma_u} + \pi_{\sigma_v}$, the profits' sum of elementary strategies, $\pi_{\sigma_u^i}$ and $\pi_{\sigma_v^i}$. ■

Now, we consider the case when a seller opts to bribe users that did not rate the item i.

Proposition 2. *Consider a user that did not rate the item i,* i.e.*, $u \notin U_i$, and any other user $v \in U$. The strategy that is to bribe both users, u and v, does not carry the same profit as the sum of the profits of bribing each user,* i.e.*, $\pi_{\sigma_u^i+\sigma_v^i} \neq \pi_{\sigma_u^i} + \pi_{\sigma_v^i}$, unless both elementary strategies have zero profit.*

Proof: If both users did not rate the item i, their strategies change the ranking of the product in the same way $r_{\sigma_u^i} = \frac{\sum_{v \in U_i} c_v R_{vi} + c_u \rho_u}{\alpha + c_u} = \frac{\alpha r_i + c_u \rho_u}{\alpha + c_u}$. Thus, we have

$$\pi_{\sigma_u^i} = (\alpha - |U_i|\, c_u)\,(r_i - \rho_u)/(\alpha + c_u). \quad (2)$$

Hence, the profit for the sum of strategies, $\sigma_v^i + \sigma_u^i$, is $\pi_{\sigma_u^i+\sigma_v^i} = |U_{\sigma_u^i+\sigma_v^i}| r_{\sigma_u^i+\sigma_v^i} - (\rho_u + \rho_v) - |U_i|\, r_i = \frac{\alpha + c_u}{\tilde{\alpha}} \pi_{\sigma_u^i} + \frac{\alpha + c_v}{\tilde{\alpha}} \pi_{\sigma_v^i} + \frac{1}{\tilde{\alpha}}(\rho_u - \rho_v)(c_u - c_v)$, where $\tilde{\alpha} = \alpha + c_u + c_v$. To have a positive profit of the sum of strategies that is equal to the sum of the profits of each elementary strategy, we need the following conditions to hold $\frac{\alpha + c_u}{\tilde{\alpha}} = \frac{\alpha + c_v}{\tilde{\alpha}} = 1$ and $(\rho_u - \rho_v)(c_u - c_v) = 0$, this implies $c_u = c_v = \tilde{\alpha} - \alpha > 1$, which contradicts the fact that $c_u, c_v > 0$. However, in the case that $c_u = c_v = \bar{c}_{U_i}$ the sum of the strategies' profit (each being zero) is zero.

The case where one of the users to be bribed did not rate the item, $v \notin U_i$, but the other user did, $u \in U_i$, yields a profit

$$\begin{aligned}
\pi_{\sigma_u^i+\sigma_v^i} &= \left|U_{\sigma_v^i}\right| r_{\sigma_u^i+\sigma_v^i} - (\rho_u + \rho_v) - |U_i|\, r_i \\
&= \frac{|U_i|}{\alpha + c_v} c_v(\rho_v - r_i) + \frac{|U_i|}{\alpha + c_v} c_u \rho_u \\
&\quad + \frac{\alpha}{\alpha + c_v}(r_i - \rho_v) + \frac{1}{\alpha + c_v}\rho_u(c_u - c_v) - \frac{\alpha}{\alpha + c_v}\rho_u \\
&= \frac{\alpha}{\alpha + c_v}\pi_{\sigma_u^i} + \pi_{\sigma_v^i} + \frac{1}{\alpha + c_v}\rho_u(c_u - c_v),
\end{aligned} \quad (3)$$

which carries the same conclusion as above. ■

As we noted in the previous proof, special conditions on the users' reputation make the profit zero, hence decomposable into elementary strategies. We discuss this in the next result.

Proposition 3. *Pick an item $i \in I$. Consider the following case: The seller of i bribes users that already rated the item i, $u, v \in U_i$, and all the users have the same reputation $c_u = c_v = \bar{c}_{U_i}$. In this case, the strategy is not profitable, and the sum of the elementary strategies is zero, $\pi_{\sigma_u^i+\sigma_v^i} = \pi_{\sigma_u^i} + \pi_{\sigma_v^i} = 0$.*

Proof: For the strategies composition, the profit is given by (3), with $c_v = c_w = \bar{c}_{U_i}$, thus $\pi_{\sigma_v^i+\sigma_w^i} = 0$. σ_v^i has profit given by (1), with $c_v = \bar{c}_{U_i}$, thus $\pi_{\sigma_v^i} = 0$. σ_w^i has profit given by (2), where $c_w = \bar{c}_{U_i}$, hence $\pi_{\sigma_w^i} = 0$. ■

Next, we analyze strategies regarding the profit they carry, because we want to classify users into bribable and non-bribable ones, based on their reputation. We assume that all information is publicly available to sellers, both users' ratings and reputations. First, we analyze bribing users that already rated the item.

Proposition 4. *If user v rated item i, $v \in U_i$, a valid strategy, $\sigma^i \in \Xi_i$, s.t. $\|\sigma^i\|_1 = \sigma_v^i = \rho_v$, is profitable if $c_v > \bar{c}_{U_i}$.*

Proof: Since, $\rho_v > 0$, the profit of such strategy, σ_v^i is given by (1), which is positive whenever $c_v > \frac{\alpha}{U_i} = \bar{c}_{U_i}$. ■

We obtain, as a corollary of Proposition 4, the result of Lemma 2 in [15], if $v \in U_i$, and $c_v = c_u = \bar{c}_{U_i}$ for all $u \in U_i$ (the ranking is given by the arithmetic average) then $\pi_{\sigma_v^i} = 0$.

Suppose item i seller wants to bribe a user who did not rate the item. We study what conditions make this action profitable.

Proposition 5. *Let $v \notin U_i$, the strategy σ_v^i is profitable whenever one of the following holds:*
1) $c_v < \bar{c}_{U_i}$ and $\rho_v < r_i$, or 2) $c_v > \bar{c}_{U_i}$ and $\rho_v > r_i$.

Proof: The result follows from (2). ■

Note that this marks a difference from the work in [15], where, in Example 1, the authors showed that a user that did not rate the item can be bribed and it always increase the wealth.

Further, notice that the result of Proposition 5 means that, in case 1), if a seller bribes a user (that did not rate item i) that has reputation below the average then the bribing value, ρ_v, must be smaller than r_i. This happens because, the effect of bringing a new rater to the set of raters increases the wealth, as long we do not pay a high price, ρ_v, since the reputation of the user is smaller, henceforth the effect on the rating is small.

In the case where the bribed user has a reputation above the average, case 2), its effect on the ranking of the item is large, so bribing with a value below the ranking degrades it, thus lessening the wealth. Hence, if the ranking is computed by the AA, it is not profitable to bribe a user that did not rate the item.

B. Optimal Strategies in the BRS

Here, we investigate what is the optimal investment strategy that the seller of item i should use to increase his initial wealth, by influencing the opinion of customers. First, we consider two simpler cases where a vendor either tries to change the opinion of users that already rated item i or tries to persuade users that did not rate it. Then, we analyze the more complex case when a seller influences both raters and non-raters. We obtain the optimal bribing strategies in closed form.

To model these problems, we consider a common set up. The seller of item i has an initial wealth of J_i, and we consider two reference customers, u and v, with reputations $c_u > c_v$. We compute the profit per amount of invested wealth, $\frac{\pi_\sigma}{\|\sigma\|_1}$, so we can design the optimal bribing strategy.

a) Bribing users that already rated item i: Let us consider the case where the seller wants to bribe users that already rated item i, *i.e.* $u \in U_i$. We formulate this problem as

$$\text{maximize: } \pi_{\sigma^i}, \quad \text{subject to: } \|\sigma^i\|_1 \leqslant J_i, \quad \sigma^i \in \Lambda_i, \quad (4)$$

where $\Lambda_i = \Xi_i$. As we show in Proposition 4, to have a positive profit $\pi_{\sigma_u^i}$, when bribing user u, we need to have $c_u > \frac{\alpha}{|U_i|} = \bar{c}_{U_i}$. Therefore, we do not consider strategies that bribe users, v, s.t. $c_v < \bar{c}_{U_i}$, since it would not increase the wealth, J_i.

Let $c_u > c_v > \bar{c}_{U_i}$, we look into the profit per unit of invested resources, $\pi_{\sigma_u^i}/\rho_u - \pi_{\sigma_v^i}/\rho_v = (c_u - c_v)/\bar{c}_{U_i} > 0$. Hence, the profit per unit of invested wealth is larger for user u than for user v. The optimal strategy is then: to bribe users by decreasing order of their reputation, investing all the available wealth until either the exhaustion of available profitable users ($c_u > \bar{c}_{U_i}$) or the depletion of funds.

b) Bribing users that did not rate the item i before: Suppose that the seller of item i wants to bribe users that did not rate the item, *i.e.*, $u \notin U_i$. We formulate this problem as (4) with $\Lambda_i = \bar{\Xi}_i$. Let users $u, v \notin U_i$ be s.t. $c_u > c_v$, and let $\alpha = \sum_{w \in U_i} c_w$, $\gamma = \frac{|U_i| c_u - \alpha}{c_u + \alpha}$ and $\delta = \frac{|U_i| c_v - \alpha}{c_v + \alpha}$. The profit is given by (2), hence, we have, for user u and v, $\frac{\rho_u - r_i}{c_u + \alpha}\left(|U_i|\, c_u - \alpha\right)$ and $\frac{\rho_v - r_i}{c_v + \alpha}\left(|U_i|\, c_v - \alpha\right)$, respectively. The difference of profits is $(\rho_u - r_i)\gamma - (\rho_v - r_i)\delta$, and hence for the same amount of wealth spent, $\pi_{\sigma_u^i}/(\rho_u - r_i) > \pi_{\sigma_v^i}/(\rho_v - r_i)$, because $\gamma > \delta$.

Again, the optimal strategy is to bribe users by decreasing order reputation, investing all the available wealth until either the exhaustion of profitable users ($c_u > \bar{c}_{U_i}$) or funds.

c) General case: Again, under the same conditions for the seller of item i, we now consider that all users, $u \in U$, are bribable. The problem of finding the best bribing strategy is (4) with $\Lambda_i = \mathcal{S}_i = \Xi_i \cup \bar{\Xi}_i$. Next, we investigate when it is better to bribe a user $u \in U_i$ or a non-rater user $v \notin U_i$. For this, we consider the profit change rate, which are $\pi_{\sigma_u^i}/\rho_u = \delta$ and $\pi_{\sigma_v^i}/(\rho_u - r_i) = \gamma$, respectively. In the case, $c_u \geqslant c_v$ we always have $\delta \geqslant \gamma$. In the other case, $c_u < c_v$, we have $\gamma < \delta$ whenever either $\bar{c}_{U_i} < 1/|U_i|$ and $c_u < \alpha$, or $\bar{c}_{U_i} \geqslant 1/|U_i|$. Again, the optimal strategy consists in ordering bribable users by decreasing reputation for each of the sets U_i and $U \backslash U_i$, and start allocating wealth first to U_i.

C. Properties of strategies and its profit in MRS

Now, we explore the profit of bribing on the MRS case. To simplify the analysis, we assume that, when a user is bribed and changes his rating for an item, his reputation keeps unchanged. This assumption is not unrealistic since not only whenever the user has rated several items its reputation's change is small if only one of his ratings changes, but also because in real systems the re-computation of the reputations is often performed only from time to time. We assume that the users' ratings and reputations are publicly available, but the network of users, i.e., the clusters' partition is private.

Proposition 6 (Bribing a user in a cluster that already rated the item)**.** *Suppose that $v \in U_i^{\mathcal{M}_s}$, for some cluster $s \in \{1, \ldots, N\}$. If $c_v > \bar{c}_{U_i^{\mathcal{M}_s}}$, then any $\sigma_v \in \Xi_v$ is profitable.*

Proof: Following the same steps as in the proof of Proposition 4, replacing U_i by $U_i^{\mathcal{M}_s}$, we have that $\bar{\pi}_{\sigma_v^i} = \bar{J}_{\sigma_v^i} - \bar{J}_i = \rho_v(c_v/\bar{c}_{U_i^{\mathcal{M}_s}} - 1) > 0$. ■

This result is Proposition 4 applied to $\mathcal{M}_s$.

Proposition 7 (Bribing a user in a cluster to rate a non-rated item in the cluster)**.** *Suppose that $v \in \mathcal{M}_s$, for a cluster $s \in \{1, \ldots, N\}$, and consider an item, i, that was not rated by any member of the cluster, that is $i \notin I^{\mathcal{M}_s}$. In this case, any $\sigma_v \in \Xi_v$ is non-profitable.*

Proof: Since $|U_i^{\mathcal{M}_s}| = 0$, then $\bar{\pi}_{\sigma_v^i} = \sum_{m \in \mathcal{X}_i} |U_i^{\mathcal{M}_m}| r_i^{\mathcal{M}_m} + (|U_i^{\mathcal{M}_s}| + 1)\frac{c_v \rho_v}{c_v} - \rho_v - \sum_{m \in \mathcal{X}_i} |U_i^{\mathcal{M}_m}| r_i^{\mathcal{M}_m} = 0$. ■

Proposition 8 (Bribing a user in a cluster to rate an item that he did not rate before, but $i \in I^{\mathcal{M}_s}$)**.** *Suppose that we want to bribe a user that did not rate item i and the user belongs to a cluster where some user already rated item i, in other words, $v \in \mathcal{M}_s$, $v \notin U_i^{\mathcal{M}_s}$ and $i \in I^{\mathcal{M}_s}$. The strategy σ_v^i is profitable whenever one of the following holds:*
1) $c_v < \bar{c}_{U_i^{\mathcal{M}_s}}$ and $\rho_v < r_i^{\mathcal{M}_s}$, 2) $c_v > \bar{c}_{U_i^{\mathcal{M}_s}}$ and $\rho_v > r_i^{\mathcal{M}_s}$.

Proof: By an adaptation of (2), the profit of σ_v^i is $\bar{\pi}_{\sigma_v^i} = (|U_i^{\mathcal{M}_s}| + 1) r_{\sigma_v^i}^{\mathcal{M}_s} - \rho_v - |U_i^{\mathcal{M}_s}| r_i^{\mathcal{M}_s} = (\alpha - |U_i^{\mathcal{M}_s}| c_v)\frac{r_i^{\mathcal{M}_s} - \rho_v}{\alpha + c_v}$, where $\alpha = \sum_{u \in U_i^{\mathcal{M}_s}} c_u$. It is profitable if 1) or 2) holds. ■

D. Optimal Strategies in Multipartite RS

Next, we study the optimal bribing strategies for the MRS, as we did in Section III-B for the BRS. Again, we consider three scenarios: (i) bribing users that rated the item; (ii) bribing users that did not rate the item; (iii) bribing users from the set of all users. We compute the close form of the optimal strategies for some cases, for the others LP can be used.

To model these problems we assume that the seller of item i disposes of an initial wealth given by $\bar{J}_i$, and we consider two reference customers, u and v, with reputations s.t. $c_u > c_v$.

a) Bribing users that rated item i: Consider that the seller wants to bribe users that already rated item i, *i.e.* $u \in U_i$, *i.e.*

$$\text{maximize: } \bar{\pi}_{\sigma^i}, \quad \text{subject to: } \|\sigma^i\|_1 \leqslant \bar{J}_i, \quad \sigma^i \in \Upsilon_i, \qquad (5)$$

where $\Upsilon_i = \Xi_i$. There are two cases to explore: (i) both users are in the same cluster; (ii) each user is in a different cluster. (i) Suppose that $u, v \in \mathcal{M}_s$ are two users that already rated item i. By Proposition 6, to have a positive profit $\bar{\pi}_{\sigma^i_u}$, when bribing user u, we need to have $c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$. Thus, we do not consider strategies that bribe a user, v, s.t. $c_v < \bar{c}_{U_i^{\mathcal{M}_s}}$, because it would not increase the wealth, $\bar{J}_i$.

Let $c_u > c_v > c_{U_i^{\mathcal{M}_s}}$, we compute the profit per unit of invested resources, $\bar{\pi}_{\sigma^i_u}/\rho_u - \bar{\pi}_{\sigma^i_v}/\rho_v = (c_u - c_v)/\bar{c}_{U_i^{\mathcal{M}_s}} > 0$. Thus, the profit per unit of invested wealth is larger for user u. Hence, as we obtained for the BRS, the optimal strategy is: to bribe users by decreasing reputation, investing all the wealth until either the lack of available profitable users ($c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$) or the exhaustion of funds to bribe profitable users.
(ii) When each reference user belongs to distinct clusters, $u \in \mathcal{M}_s$, $v \in \mathcal{M}_t$ and $s \neq t$, we have that if $|U_i^{\mathcal{M}_s}| \geqslant |U_i^{\mathcal{M}_t}|$, then the profit per unit of invested wealth ($\bar{\pi}_{\sigma^i_u}/\rho_u$ versus $\bar{\pi}_{\sigma^i_v}/\rho_v$) is larger for user u. If $|U_i^{\mathcal{M}_s}| < |U_i^{\mathcal{M}_t}|$ then the profit per unit of invested wealth is larger for user u if $|U_i^{\mathcal{M}_s}| > (c_u - c_v)^{-1}$ and $|U_i^{\mathcal{M}_t}| < (|U_i^{\mathcal{M}_s}|c_u - 1)/c_v$, and larger for user v, otherwise.

b) Bribing users that did not rate the item i: Under the same conditions for item i seller, suppose that he wants to bribe users that did not rate i, *i.e.* $u \notin U_i$. We formulate this as (5) with $\Upsilon_i = \bar{\Xi}_i$. Recalling Proposition 7, we only need to explore the case where the seller of item i wants to bribe users belonging to clusters with users that already rated the item, clusters m s.t. $i \in I^{\mathcal{M}_m}$, otherwise the profit is zero. Let users $u, v \in \mathcal{M}_s$ and $u, v \notin U_i$ be s.t. $c_u > c_v$, and let $\alpha = \sum_{w \in U_i^{\mathcal{M}_s}} c_w$, $\gamma = \frac{|U_i^{\mathcal{M}_s}|c_u - \alpha}{c_u + \alpha}$ and $\delta = \frac{|U_i^{\mathcal{M}_s}|c_v - \alpha}{c_v + \alpha}$. By Proposition 8, we have that the profits for bribing users u and v are $\frac{\rho_u - r_i^{\mathcal{M}_s}}{c_u + \alpha}(|U_i^{\mathcal{M}_s}|c_u - \alpha)$ and $\frac{\rho_v - r_i^{\mathcal{M}_s}}{c_v + \alpha}(|U_i^{\mathcal{M}_s}|c_v - \alpha)$, respectively. The difference of profits is $(\rho_u - r_i^{\mathcal{M}_s})\gamma - (\rho_v - r_i^{\mathcal{M}_s})\delta$, hence, for the same amount of spent wealth, $\bar{\pi}_{\sigma^i_u}/(\rho_u - r_i) > \bar{\pi}_{\sigma^i_v}/(\rho_v - r_i^{\mathcal{M}_s})$, because $\gamma > \delta$.

Again, the optimal strategy is to bribe users by decreasing order reputation, investing all the available wealth until either the exhaustion of profitable users ($c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$) or funds.

In the case both users are in distinct clusters and did not rate item i, we cannot derive simple conditions and we need to solve a LP for each instance.

c) General case: Under the same conditions for item i seller, we consider that all users, $u \in U$, can be bribed. The problem of finding the best bribing strategy is written as (5) with $\Upsilon_i = \mathcal{S}_i = \Xi_i \cup \bar{\Xi}_i$. Next, we investigate when it is better to bribe a user $u \in U_i^{\mathcal{M}_s}$ or a non-rater user $v \notin U_i^{\mathcal{M}_s}$. The result is the adaptation of the one for the general case in III-B. We consider the profit change rate, which are $\frac{\bar{\pi}_{\sigma^i_u}}{\rho_u} = \delta$ and $\frac{\bar{\pi}_{\sigma^i_v}}{\rho_u - \bar{r}_i} = \gamma$, respectively. In the case, $c_u \geqslant c_v$ we always have $\delta \geqslant \gamma$. In the other case, $c_u < c_v$, we have $\gamma < \delta$ whenever either $\bar{c}_{U_i^{\mathcal{M}_s}} < 1/|U_i^{\mathcal{M}_s}|$ and $c_u < \alpha$, or $\bar{c}_{U_i} \geqslant 1/|U_i^{\mathcal{M}_s}|$. Again, the optimal strategy is to order bribable users by decreasing reputation for each of the sets $U_i^{\mathcal{M}_s}$ and $U \backslash U_i^{\mathcal{M}_s}$, and start allocating wealth to $U_i^{\mathcal{M}_s}$ and, afterward, to $U \backslash U_i^{\mathcal{M}_s}$. If the reference users are in different clusters, we cannot draw simple conditions, and we need to solve the LP for each instance.

E. Bipartite RS vs. Multipartite RS

Here, we compare the profits obtained in the MRS case and BRS, for same conditions. In the case where the user rated the item, we have the following result:

Proposition 9. *Suppose that the seller of item i wants to bribe a user v that already rated the item,* i.e. $v \in U_i$. *Let the user v be in cluster $\mathcal{M}_s$, then the profit is larger in the BRS, $\bar{\pi}_{\sigma^i} < \pi_{\sigma_i}$, if and only if $\bar{c}_{(U_i \backslash U_i^{\mathcal{M}_s})} < \bar{c}_{U_i^{\mathcal{M}_s}}$, the average of the reputations in $(U_i \backslash U_i^{\mathcal{M}_s})$ and $U_i^{\mathcal{M}_s}$, respectively.*

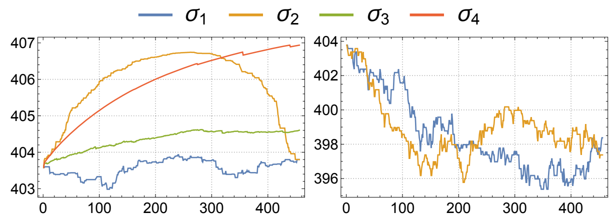
Proof: By definition, $\bar{\pi}_{\sigma^i} < \pi_{\sigma_i}$ is the same as $\left(\frac{|U_i^{\mathcal{M}_s}|c_v}{\sum_{u \in U_i^{\mathcal{M}_s}} c_u} - 1\right)\rho_v < \left(\frac{|U_i|c_v}{\sum_{u \in U_i} c_u} - 1\right)\rho_v$, which is equivalent to $|U_i^{\mathcal{M}_s}| \sum_{u \in U_i} c_u < |U_i| \sum_{u \in U_i^{\mathcal{M}_s}} c_u$. Noticing that $U_i = U_i^{\mathcal{M}_s} \cup (U_i \backslash U_i^{\mathcal{M}_s})$, we can rewrite it as $|U_i^{\mathcal{M}_s}|(\sum_{u \in U_i^{\mathcal{M}_s}} c_u + \sum_{u \in U_i \backslash U_i^{\mathcal{M}_s}} c_u) < (|U_i^{\mathcal{M}_s}| + |U_i \backslash U_i^{\mathcal{M}_s}|) \sum_{u \in U_i^{\mathcal{M}_s}} c_u$. This is $\bar{c}_{(U_i \backslash U_i^{\mathcal{M}_s})} < \bar{c}_{U_i^{\mathcal{M}_s}}$. ■

Hence, there are cases where bribing a user in MRS is more profitable than in BRS. Since the clusters' partition is assumed to be unknown for the sellers, they cannot determine the users that verify the previous condition. Unlike users' reputations that are often public. Now, we compare the profit of bribing a user that did not rate the item i in the case the bribed user v belongs to a network where no users rated the item, $v \in \mathcal{M}_s$ and $i \notin I^{\mathcal{M}_s}$. In this case, bribing user v in MRS yields zero profit, but in BRS the strategy can be profitable, as we showed in Proposition 5. In the case that the bribed user did not rate the item, but he belongs to a cluster where some user rated the item, we cannot draw simple conditions as in the previous cases. We need to check for each concrete case which one is the most profitable.

IV. Simulations

Here, we illustrate the main results of the paper with real data, using the 5-core version of "Amazon Instant Video" data set, [18], as in [10], with 5130 users, 1685 items, 37126 ratings, and where each user rated, at least, 5 items.

We simulate bribing strategies for the seller of the most rated item (455 ratings). This item allows us to have more data to explore (the results would be similar for other items). We study the effect of four strategies in BRS and MRS, which are: σ_1 – bribe users that rated the item, by a random order; σ_2 – bribe users that rated the item, by decreasing reputation; σ_3 – bribe users uniformly at random, from all users (only for BRS); σ_4 – bribe users in decreasing order of reputation (only for BRS).



(a) (b)

Fig. 2: Profit of bribing strategies of the most rated item's sellers in (a) BRS (σ_1 – σ_4), and (b) MRS (σ_1 and σ_2).

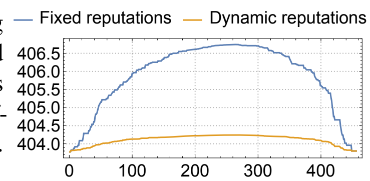


Fig. 3: Profit of bribing strategy σ_2 in BRS, fixed users' reputations versus reputations recomputed after each user being bribed.

In Figure 2 (a) and (b), we show the results of different bribing strategies for BRS and MRS, respectively. The steps where the rewards are constant, in Figures 2 (a) and (b), represent choosing users that already rated the item with the maximum allowed rating. For the BRS, Figure 2 (a), after bribing the same users in strategies σ_1 and σ_2, both strategies yield the same profit, as stated in Proposition 1. Finally, the strategy σ_3 of Figure 2 (a) is to bribe users, from the set of all users, by decreasing reputation. As expected, bribing users among the ones who rated the item and are more influential (have a larger reputation) results in a faster increase of reward, whereas random bribing among the item's raters has an expected profit close to zero, and does not increase wealth. The strategy σ_4 is the most profitable, but only after a certain number of users in comparison to strategy σ_2. For all strategies, the profit is positive. In the MRS scenario, Figure 2 (b), for σ_1 and σ_2, the wealth is strictly smaller at the end of the bribing strategy. This illustrates the fact that the MRS is more robust to bribery than the BRS, which meets the discussion in Section III-E. Lastly, we apply strategy σ_2 to the BRS, assuming that the users' reputations are fixed, or without this restriction (each time a user is bribed, both rankings and reputations updated) as in [10]. The results of this experiment are depicted in Figure 3. In Figure 3, we see that the reputations of the bribed users decrease therefore the impact of their ratings is smaller as well their profits than when the reputations are fixed.

V. Conclusions and Future Work

We model bribing in two reputation-based ranking systems. The first does not aggregate users, while the second clusters users by their similarities and presents a dedicated ranking of items for each cluster. In both settings, we show which users to bribe to get positive profit, and we show that clustering users decrease the number of profitable bribing strategies. We illustrate our results with a real world dataset. In future work, we would like to study the interactions between bigger and smaller players, and the scenario where sellers bribe users to decrease a competitor item's ranking through a game theory model with the sellers as players. Another aspect we want to explore and incorporate into the bribery analysis is the impact on the profit of strategies of dynamic reputations that changed when the ratings change. Therefore studying new conditions to design profitable bribing strategies.

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