Circuit Analysis - Resistances, Voltage and Current Sources

Integrated Master Engineering Physics , Técnico, University of Lisbon

Laboratory Report 1

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1 Introduction

The aim of this laboratory assignment is to study a circuit composed exclusively by voltage sources (V_a and V_c), current sources (I_b and I_d), as well as resistors (R_1, R_2, R_4, R_5, R_6 and R_7). That being said, this circuit, which is shown in the image below, contains only linear components.

It was known beforehand that: $R_1 = 1039.3\ \Omega$, $R_2 = 2010.9\ \Omega$, $R_3 = 30411.2\ \Omega$, $R_4 = 4183.8\ \Omega$, $R_5 = 3095.8\ \Omega$, $R_6 = 2021.2\ \Omega$, $R_7 = 1022.3\ \Omega$, $V_a = 5.1798\ V$, $I_d = 0.0010044\ A$, $K_b = 0.0070875\ A/V$ and $K_c = 8185.8\ V/A$.

Therefore, the objective was to find the values of: V_0, V_1, V_2, V_3, V_4, V_5, V_6, V_7, I_b, I_c, I_1, I_2, I_3, I_4, V_b and V_c.

In order to make it easier to analyse the circuit, both meshes and nodes were labelled. N_0 was chosen as the ground node, $V_0 = 0\ V$. V_i is the voltage in node N_i and I_1 is the current that goes through mesh A counterclockwise. Likewise, I_2 goes through B, I_3 through C and I_4 through D, all anticlockwise. Besides that, in each resistance the direction of the current was arbitrarily chosen.

In Section 2, a theoretical analysis of the circuit is presented following two of the most common methods: the mesh method and the nodal method. In Section 3, the circuit is analysed by simulation. Finally, the comparison between both types of analysis and the conclusions of this study are in Section 4.

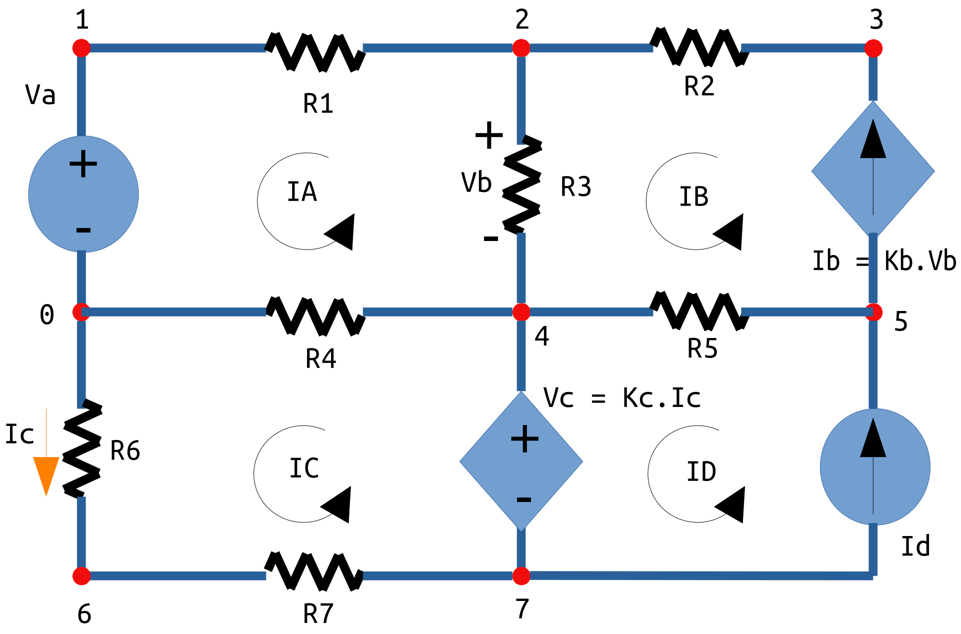

Figure 1: The circuit in study

2 Theoretical Analysis

2.1 Mesh Method

In the mesh method, equations are written for essential meshes, using Kirchhoff's voltage law (KVL), which states that "the directed sum of the potential differences (voltages) around any closed loop is zero". There are also additional equations.

In this method, as well as in the next one, Ohm's Law, $V = RI \Leftrightarrow I = \frac{V}{R}$, was used several times.

$$\begin{cases} R_6 I_3 + R_7 I_3 - V_c + R_4(I_3 - I_1) = 0 & \text{KVL for mesh } C \\ V_a - R_4(I_3 - I_1) - V_b + R_1 I_1 = 0 & \text{KVL for mesh } A \\ V_c = K_c I_c & \text{as given for voltage source } V_c \\ I_b = K_b V_b & \text{as given for current source } I_b \\ I_3 = I_c & \text{by inspection in mesh } C \\ I_4 = I_d & \text{by inspection in mesh } D \\ (I_2 - I_1) R_3 = V_b & \text{Ohm's Law for resistor } R_3 \\ I_2 = I_b & \text{by inspection in mesh } B \end{cases} \tag{1}$$

maybe explaining these equations a little better??
show system with matrix?

2.2 Nodal Method

In the nodal method, a set of equation is written based on Kirchhoff's current law (KCL), according to which "the algebraic sum of currents in a network of conductors meeting at a point is zero", which means that the sum of currents flowing into that node is equal to the sum of currents flowing out of that node. Plus, there are some additional equations.

mentioning the concept of super node?

$$\begin{cases} V_0 = 0 & \text{since } N_0 \text{ is the ground node} \\ V_1 - V_0 = V_a & \text{by inspection in voltage source } V_a \\ \frac{V_2-V_1}{R_1} + \frac{V_2-V_4}{R_3} + \frac{V_3-V_2}{R_2} = 0 & \text{KCL for node } N_2 \\ \frac{V_3-V_2}{R_2} - (V_2 - V_4)K_b = 0 & \text{KCL for node } N_3 \\ \frac{V_0-V_6}{R_6} - \frac{V_6-V_7}{R_7} = 0 & \text{KCL for node } N_6 \\ \frac{V_5-V_4}{R_5} + K_b(V_2 - V_4) - I_d = 0 & \text{KCL for node } N_5 \\ V_4 - V_7 = K_c\frac{V_0-V_6}{R_6} & \text{by inspection in voltage source } V_c \\ \frac{V_2-V_4}{R_3} + \frac{V_5-V_4}{R_5} + \frac{V_6-V_7}{R_7} - \frac{V_4-V_0}{R_4} - I_d = 0 & \text{KCL for node } N_4 \textbf{ is actually a supernode!} \end{cases} \tag{2}$$

maybe explaining these equations a little better??
show system with matrix?

2.3 Results

After solving a linear system of equations with Octave, it was found out that:

present values of V1 to V7, Ib, Ic, I1 to I4, Vb and Vc

Name	Value [A or V]
V_0	0.000000
V_1	5.179800
V_2	4.949130
V_3	4.481100
V_4	4.981968
V_5	8.811951
V_6	-1.958148
V_7	-2.948538

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3 Simulation Analysis

The aforementioned circuit was simulated using the ngspice simulator, which returned the following values:

4 Conclusion

In the end, one can say that the objectives defined for this laboratory session were met. The theoretical analysis, featuring both mesh and nodal methods, in which Octave maths tool was used, had results that proved to be consistent and coherent. As for the circuit simulation part with Ngspice, it was also well succeeded.

The results obtained through both methodologies were the same, as expected, since the circuit is straightforward and composed exclusively by linear components. As a consequence, the theoretical and the simulation models cannot differ. Were the components a bit more complex, they might have had some differences but that was not the case here.

Name	**Value [A or V]**
@gb[i]	-2.32745e-04
@id[current]	1.004395e-03
@r1[i]	2.219473e-04
@r2[i]	2.327451e-04
@r3[i]	-1.07978e-05
@r4[i]	-1.19077e-03
@r5[i]	-1.23714e-03
@r6[i]	9.688185e-04
@r7[i]	9.688185e-04
v(1)	5.179800e+00
v(2)	4.949130e+00
v(3)	4.481100e+00
v(4)	4.981968e+00
v(5)	8.811951e+00
v(6)	-1.95815e+00
v(7)	-2.94854e+00
v(8)	-1.95815e+00

Table 2: A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.