TÉCNICO LISBOA

Circuit Analysis - Resistances, Voltage and Current Sources

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Laboratory Report 1

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Contents

1 Introduction

The aim of this laboratory assignment is to study the circuit shown below assuming the lumped-elements approximation. It is composed of voltage sources (an independent one, V_a, and a linear current-controlled one, V_c), current sources (an independent one, I_d, and a linear voltage-controlled one, I_b) and resistors (R_1, R_2, R_3, R_4, R_5, R_6 and R_7). All components of this circuit are linear and time-invariant.

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N_0 was chosen as the ground node, $V_0 = 0\ V$. V_i is the voltage in node N_i $(i = 1, 2, ..., 7)$ and I_i is the current that goes through mesh i counterclockwise $(i = A, B, C, D)$.

In Section 2, a theoretical analysis of the circuit is presented following two of the most common methods: the mesh method and the nodal method (whereas in the latter it is to find

	Original Values	Approximate Values (Octave e Ngspice)
$R_1(\Omega)$	1.03930068064	1039.3
$R_2(\Omega)$	2.0109087471	2010.9
$R_3(\Omega)$	3.04124242148	3041.2
$R_4(\Omega)$	4.18383562625	4183.8
$R_5(\Omega)$	3.09583437009	3095.8
$R_6(\Omega)$	2.02117084711	2021.2
$R_7(\Omega)$	1.02226630661	1022.3
$V_a(V)$	5.17979967502	5.1798
$I_d(A)$	1.00439545365	0.0010044
$K_b(S)$	7.08750963899	0.0070875
$K_c(\Omega)$	8.18575062147	8185.8

Table 1: Significant figures considered by Octave and Ngsipe- approximation

the potential in each node). In the mesh method, the goal is to find the values of the currents in each essential mesh. The equations written in order to do this are applications of Kirchhoff's voltage law, or KVL (which states that "the directed sum of the potential differences (voltages) around any closed loop is zero") to essential meshes or determined by inspection of the circuit. If more equations are needed (because KVL couldn't be applied to one of the meshes), a super-mesh equation is written. In the nodal method, the goal is to determine all node potentials, with the set of equations written being based on Kirchhoff's current law, or KCL, ("the algebraic sum of currents in a network of conductors meeting at a point is zero") applied to the nodes. If more equations are needed, super-node equations are used. In this part, Octave was used to solve all systems of (linear) equations. In Section 3, the circuit is analysed by simulation using ngspice. The declaration of all components in ngspice is very straightforward, with the exception of the current-controlled voltage source, which requires that an independent voltage source through which the control current passes be indicated. So, an imaginary 0V voltage source must be added in between R_6 and R_7 (positive terminal connected to R_6, negative to R_7), since through it would pass the control current I_c. Finally, the comparison between both types of analysis and the conclusions of this study are in Section 4.

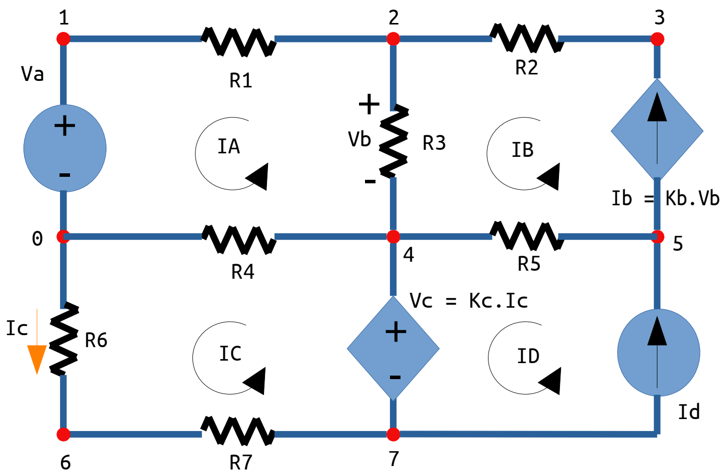

Figure 1: The circuit in analysis

2 Theoretical Analysis

2.1 Mesh Method

In this method, as well as in the next one, Ohm's Law, $V = RI \Leftrightarrow I = \frac{V}{R}$, is implicitly used several times.

$$\begin{cases} V_a - R_4(I_C - I_A) - V_b + R_1 I_A = 0 & (1) \\ R_6 I_C + R_7 I_C - V_c + R_4(I_C - I_A) = 0 & (2) \\ I_B = I_b & (3) \\ I_D = I_d & (4) \\ V_c = K_c I_c & (5) \\ I_b = K_b V_b & (6) \\ I_C = I_c & (7) \\ (I_B - I_A)R_3 = V_b & (8) \end{cases} \tag{1}$$

KVL was applied to meshes A and C to obtain equations (1) and (2) respectively. KVL can't be applied to meshes B and D, due to the current sources, so instead equations (3) and (4) were written, by direct inspection. Because the previous equations involve dependent sources, equations (5) to (8) were determined, allowing us to relate the output of said sources to the mesh currents. Equations (5) and (6) are given, equation (7) is determined by inspection of mesh C and, lastly, equation (8) is the application of Ohm's law to R_3, with $(I_B - I_A)$ being the current that goes through that same resistor.

Mesh Currents	Value [A]
I_A	-0.000222
I_B	-0.000233
I_C	0.000969
I_D	0.001004

Table 2: Mesh Current values determined through mesh analysis. The results or positive or negative depending on if the currents go in the direction indicated in the introductory drawing or not

2.2 Nodal Method

$$\begin{cases} \frac{V_2-V_1}{R_1} + \frac{V_2-V_4}{R_3} + \frac{V_3-V_2}{R_2} = 0 & (1) \\ \frac{V_3-V_2}{R_2} - (V_2 - V_4)K_b = 0 & (2) \\ \frac{V_5-V_4}{R_5} + K_b(V_2 - V_4) - I_d = 0 & (3) \\ \frac{V_0-V_6}{R_6} - \frac{V_6-V_7}{R_7} = 0 & (4) \\ V_0 = 0 & (5) \\ V_1 - V_0 = V_a & (6) \\ V_4 - V_7 = K_c \frac{V_0-V_6}{R_6} & (7) \\ \frac{V_2-V_4}{R_3} + \frac{V_5-V_4}{R_5} + \frac{V_6-V_7}{R_7} - \frac{V_4-V_0}{R_4} - I_d = 0 & (8) \end{cases} \tag{2}$$

In equations (1), (2), (3) and (4) we have KCL applied to nodes N_2, N_3, N_5 and N_6, respectively. Using Ohm's law, one knows that $\frac{V_2-V_1}{R_1}$ is the current that goes through resistor R_1, $\frac{V_3-V_2}{R_2}$ the one that goes through R_2, $\frac{V_2-V_4}{R_3}$ is the one through R_3, $\frac{V_5-V_4}{R_5}$ is the current through R_5, $\frac{V_0-V_6}{R_6}$ is the one through R_6 and $\frac{V_6-V_7}{R_7}$ the one through R_7. I_d is the current imposed by current source I_d while $V_2 - V_4)K_b$ is the one imposed by I_b. KCL can't be applied to nodes N_0, N_1, N_4 and N_7. Instead, equations (5) to (8) are written. Equation (5) refers to N_0 being chosen as the ground node and equation (6) is reached by inspection in voltage source V_a. Equation (7) is determined by inspection in voltage source V_c, with $V_c = K_c I_c$ being the voltage imposed by voltage source V_c, and I_c being the current that goes through resistor R_6, hence it is equal to $\frac{V_0 - V_6}{R_6}$. Besides that, $V_c = V_4 - V_7$. Finally, because nodes N_4 and N_7 are connected by a current source, a super-node equation, (8), can be written by algebraically summing all currents flowing into or out of these nodes except the current related to the aforementioned source that connects these two nodes.

Node Potential	Value [V]
V_0	0.000000
V_1	5.179800
V_2	4.949130
V_3	4.481100
V_4	4.981968
V_5	8.811951
V_6	-1.958148
V_7	-2.948538

Table 3: Node potentials determined through nodal analysis.

2.3 Comparing both methods

Mesh Current	Mesh Method [A]	Method Comparison Equations [A]
I_A	-0.000222	-0.000222
I_B	-0.000233	-0.000233
I_C	0.000969	0.000969
I_D	0.001004	0.001004

Table 4: Mesh Currents determined through mesh analysis (same as in table 2) and through the comparison equations based on the nodal potentials determined by nodal analysis

After solving a linear system of equations with Octave, it was found out that the results obtained through both methods match. This confirms that they are equivalent.

$$\begin{cases} I_A = \frac{V_2 - V_1}{R_1} & (1) \\ I_B = \frac{V_3 - V_2}{R_2} & (2) \\ I_C = -\frac{V_7 - V_6}{R_7} & (3) \\ I_D = -\frac{V_4 - V_5}{R_5} + I_B & (4) \end{cases} \tag{3}$$

Using the values obtained through the nodal analysis for the potentials, the value for the current through each essential mesh was calculated and then compared to the one obtained through the mesh method as shown in the table below.

In the previous equations Ohm's law is applied to a resistor in the respective essential mesh. In equation (4), since resistor R_5 belongs to two essential meshes, B and D, the current that goes through it is the difference between the current in each of those meshes.

3 Simulation Analysis

The aforementioned circuit was simulated using the ngspice simulator, which returned the values in the table below:

Name	**Value [A or V]**
@gb[i]	-2.32745e-04
@id[current]	1.004395e-03
@r1[i]	2.219473e-04
@r2[i]	2.327451e-04
@r3[i]	-1.07978e-05
@r4[i]	-1.19077e-03
@r5[i]	-1.23714e-03
@r6[i]	9.688185e-04
@r7[i]	9.688185e-04
v(1)	5.179800e+00
v(2)	4.949130e+00
v(3)	4.481100e+00
v(4)	4.981968e+00
v(5)	8.811951e+00
v(6)	-1.95815e+00
v(7)	-2.94854e+00
v(8)	-1.95815e+00

Table 5: A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In the end, one can say that the objectives defined for this laboratory session were met. Through the theoretical analysis, all node potentials and mesh currents were determined, with the results proving to be consistent and coherent. Meanwhile, the circuit simulation returned values for the node potentials (and some branch currents) which perfectly matched those determined through nodal analysis. As shown in subsection 2.3, you can calculate all mesh currents utilizing the node potentials and you'll obtain the same values as the ones obtained through mesh analysis. Thus, we can say that we obtained the same results through the simulation that we did in the theoretical analysis (this was to be expected as the circuit is very straightforward, being composed entirely of time-invariant linear components).