Circuit Analysis - Resistances, Voltage and Current Sources

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Laboratory Report 1

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Contents

1 Introduction

The aim of this laboratory assignment is to study a circuit composed exclusively by voltage sources (V_a and V_c), current sources (I_b and I_d), as well as resistors (R_1, R_2, R_4, R_5, R_6 and R_7). That being said, this circuit, which is shown in the image below, contains only linear components.

It was known beforehand that: $R_1 = 1039.3\ \Omega$, $R_2 = 2010.9\ \Omega$, $R_3 = 30411.2\ \Omega$, $R_4 = 4183.8\ \Omega$, $R_5 = 3095.8\ \Omega$, $R_6 = 2021.2\ \Omega$, $R_7 = 1022.3\ \Omega$, $V_a = 5.1798\ V$, $I_d = 0.0010044\ A$, $K_b = 0.0070875\ A/V$ and $K_c = 8185.8\ V/A$.

N_0 was chosen as the ground node, $V_0 = 0\ V$. V_i is the voltage in node N_i and I_i ($i = 1, 2, ..., 7$) is the current that goes through mesh i counterclockwise ($i = A, B, C, D$).

In Section 2, a theoretical analysis of the circuit is presented following two of the most common methods: the mesh method and the nodal method (in the former the objective was to find the values of the current in each essential mesh, whereas in the latter it was to find the potential in each node). In Section 3, the circuit is analysed by simulation. Finally, the comparison between both types of analysis and the conclusions of this study are in Section 4.

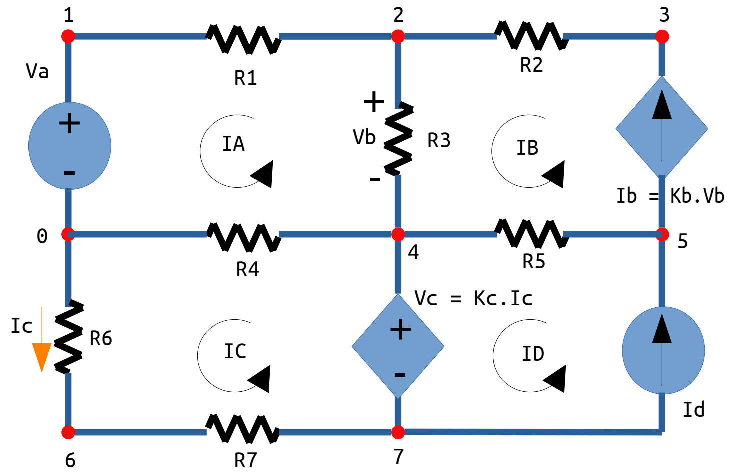

Figure 1: The circuit in study

2 Theoretical Analysis

2.1 Mesh Method

In the mesh method, equations are written for essential meshes, using Kirchhoff's voltage law (KVL), which states that "the directed sum of the potential differences (voltages) around any closed loop is zero". There are also additional equations.

In this method, as well as in the next one, Ohm's Law, $V = RI \Leftrightarrow I = \frac{V}{R}$, was used several times.

$$\begin{cases} R_6 I_3 + R_7 I_3 - V_c + R_4(I_3 - I_1) = 0 & (1) \\ V_a - R_4(I_3 - I_1) - V_b + R_1 I_1 = 0 & (2) \\ V_c = K_c I_c & (3) \\ I_b = K_b V_b & (4) \\ I_3 = I_c & (5) \\ I_4 = I_d & (6) \\ I_2 = I_b & (7) \\ (I_2 - I_1) R_3 = V_b & (8) \end{cases} \tag{1}$$

KVL was used for meshes C and A in equations (1) and (2) respectively. As for equations (3) and (4), they were based on the information given for voltage source V_c and current source I_b. Equations (5), (6) and (7) were by inspection meshes C and D. Lastly, in equation (8) it was known that the difference of potential between the terminals of resistor R_3 was V_b, but because of Ohm's law it is also equal to $(I_2 - I_1)R_3$, $(I_2 - I_1)$ being the current that goes through that same resistor.

2.2 Nodal Method

In the nodal method, a set of equation is written based on Kirchhoff's current law (KCL), according to which "the algebraic sum of currents in a network of conductors meeting at a point is zero", which means that the sum of currents flowing into that node is equal to the sum of currents flowing out of that node. Plus, there are some additional equations.

Name	Value [A or V]
I_A	-0.000222
I_B	-0.000233
I_C	0.000969
I_D	0.001004

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

$$\begin{cases} V_0 = 0 & (1) \\ V_1 - V_0 = V_a & (2) \\ \frac{V_2-V_1}{R_1} + \frac{V_2-V_4}{R_3} + \frac{V_3-V_2}{R_2} = 0 & (3) \\ \frac{V_3-V_2}{R_2} - (V_2 - V_4)K_b = 0 & (4) \\ \frac{V_0-V_6}{R_6} - \frac{V_6-V_7}{R_7} = 0 & (5) \\ \frac{V_5-V_4}{R_5} + K_b(V_2 - V_4) - I_d = 0 & (6) \\ V_4 - V_7 = K_c\frac{V_0-V_6}{R_6} & (7) \\ \frac{V_2-V_4}{R_3} + \frac{V_5-V_4}{R_5} + \frac{V_6-V_7}{R_7} - \frac{V_4-V_0}{R_4} - I_d = 0 & (8) \end{cases} \tag{2}$$

Equation (1) refers to N_0 being the ground node and equation (2) is reached by inspection in voltage source V_a. In equations (3), (4), (5) and (6) we have KCL for nodes N_2, N_3, N_6 and N_5, respectively. Using Ohm's law, one knows that $\frac{V_2-V_1}{R_1}$ is the current that goes through resistor R_1, $\frac{V_3-V_2}{R_2}$ the one that goes through R_2, $\frac{V_2-V_4}{R_3}$ through R_3, $\frac{V_5-V_4}{R_5}$ through R_5, $\frac{V_0-V_6}{R_6}$ through R_6, $\frac{V_6-V_7}{R_7}$ through R_7.

Name	Value [A or V]
V_0	0.000000
V_1	5.179800
V_2	4.949130
V_3	4.481100
V_4	4.981968
V_5	8.811951
V_6	-1.958148
V_7	-2.948538

Table 2: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

maybe explaining these equations a little better??

2.3 Comparing both methods

After solving a linear system of equations with Octave, it was found out that.

3 Simulation Analysis

The aforementioned circuit was simulated using the ngspice simulator, which returned the following values:

Name	Value [A or V]
@gb[i]	-2.32745e-04
@id[current]	1.004395e-03
@r1[i]	2.219473e-04
@r2[i]	2.327451e-04
@r3[i]	-1.07978e-05
@r4[i]	-1.19077e-03
@r5[i]	-1.23714e-03
@r6[i]	9.688185e-04
@r7[i]	9.688185e-04
v(1)	5.179800e+00
v(2)	4.949130e+00
v(3)	4.481100e+00
v(4)	4.981968e+00
v(5)	8.811951e+00
v(6)	-1.95815e+00
v(7)	-2.94854e+00
v(8)	-1.95815e+00

Table 3: A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

4 Conclusion

In the end, one can say that the objectives defined for this laboratory session were met. Through the theoretical analysis, all node potentials and mesh currents were determined, with the results proving to be consistent and coherent. Meanwhile, the circuit simulation returned values for the node potentials and some branch currents, all of which matched those determined previously. This was to be expected as the circuit is very straightforward, being composed entirely of time-invariant linear components.