Greedy Algorithms

Brute-Force, Heuristic Algorithms

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Table of Contents

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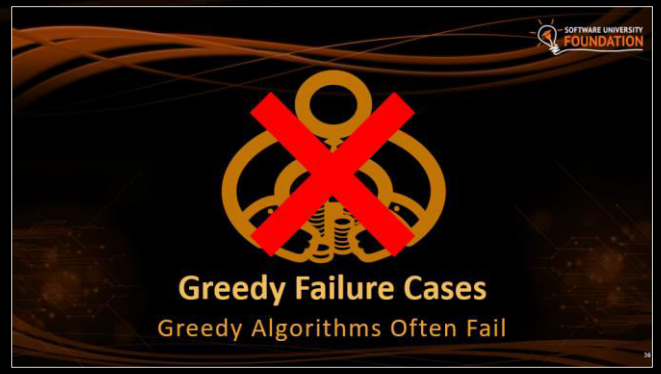

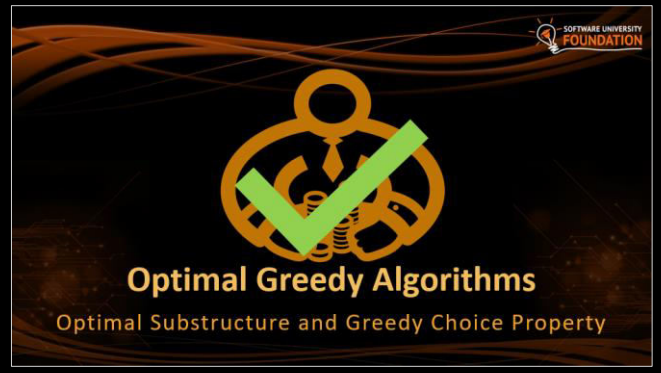

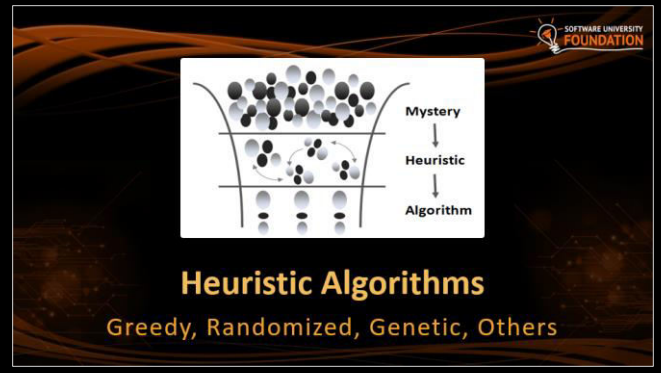

Have a Question?

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sli.do

#DsAlgo

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Brute-Force Algorithms

Trying All Solutions

Brute-Force Algorithms

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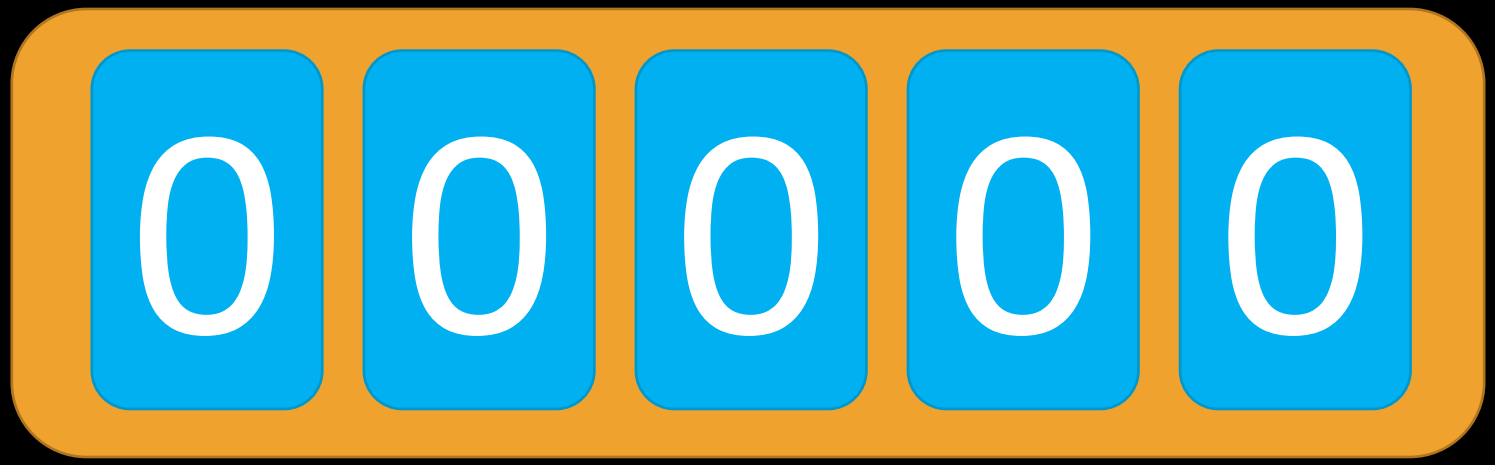
- Trying all possible combinations
- Picking the best solution
- Usually slow and inefficient



Brute-Force Algorithms

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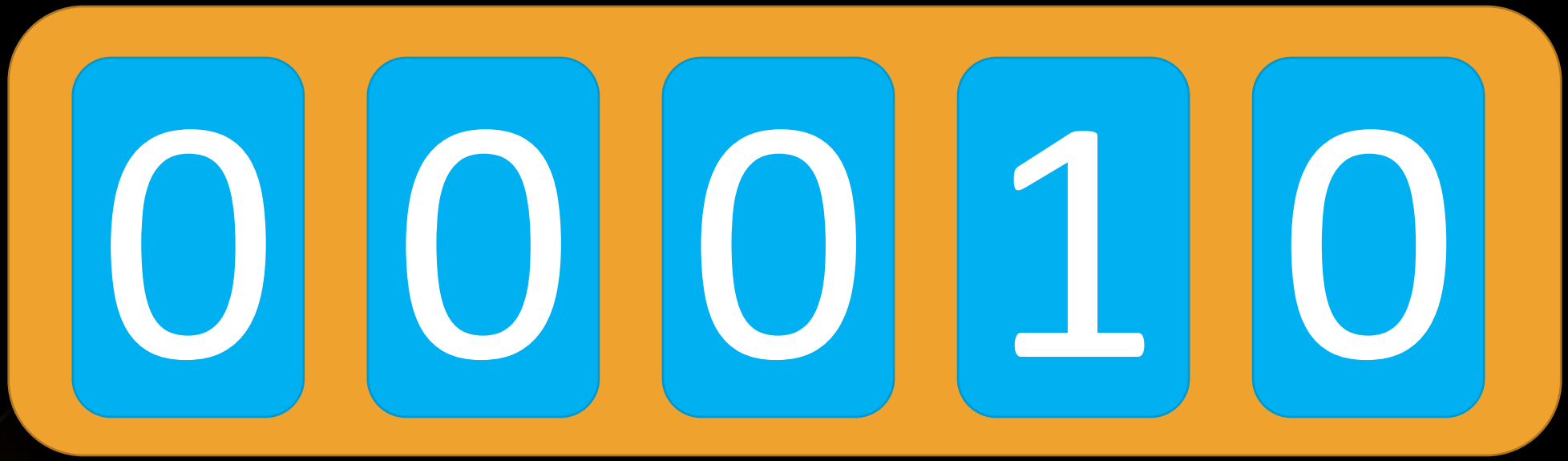

Brute-Force Algorithms

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Brute-Force Algorithms

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10 x 10 x 10 x 10 x 10 = 100,000 combinations

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Greedy Algorithms

Picking Locally Optimal Solution

Optimization Problems

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- In computer science, an optimization problem is the problem of finding the best solution from all feasible solutions
  - Finding the optimal among many candidates
- Examples of optimization problems:
  - Find the shortest path from Sofia to Varna on the map
  - Find the maximum increasing subsequence in integer sequence
  - Traveling salesman problem (minimal Hamilton cycle)
    - Find the shortest route that visits each city and returns to the origin city

Greedy Algorithms

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- Greedy algorithms are a category of algorithms
  - For solving optimization problems
  - Usually more efficient than the other algorithms
    - But can produce non-optimal (incorrect) result
- Greedy algorithms pick the best local solution
  - Pick the optimum from their current position & point of view
- Greedy algorithms assume always choosing a local optimum leads to the global optimum
  - This is sometimes correct, but sometimes it is not

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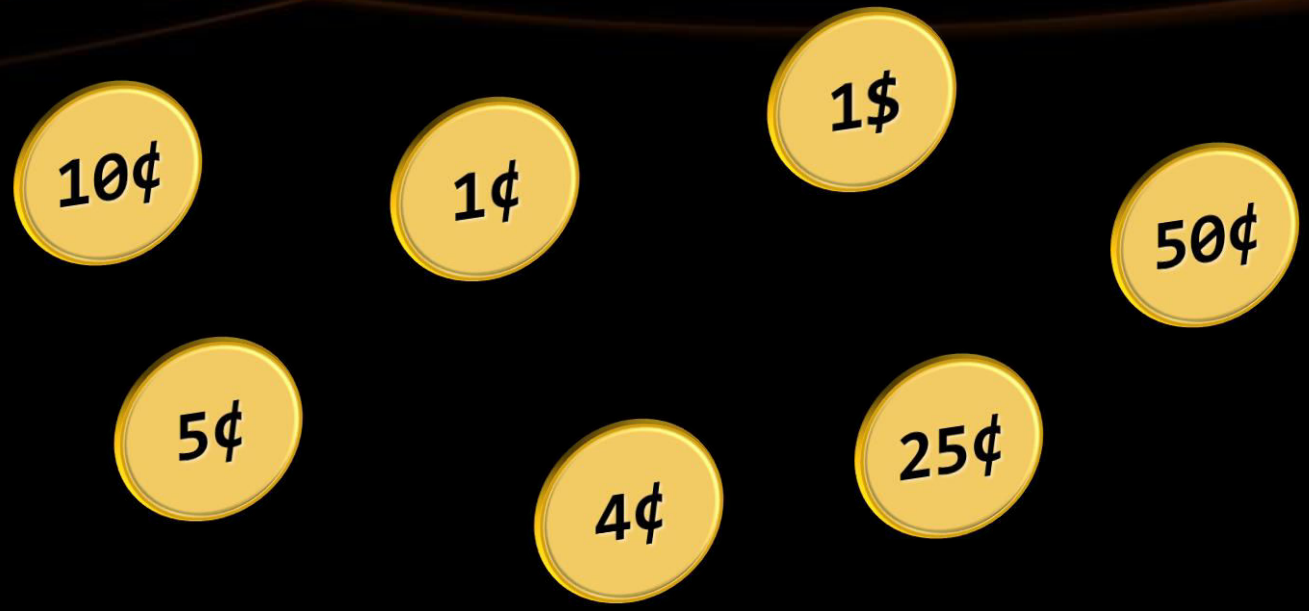

Sum of Coins

Problem

Greedy Algorithms: Sum of Coins

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- Consider the US currency coins:
  - { 0.01, 0.02, 0.05, 0.10}
- Problem "Sum of Coins":
  - Gather a sum of money, using the least possible number of coins
- Greedy algorithm for "Sum of Coins":
  - Take the largest coin while possible
  - Then take the second largest
  - Etc.

Sum of Coins Visualization

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Target: 18

Actual: 0

Sum of Coins Visualization

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Target: 18

Actual: 10

Sum of Coins Visualization

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Target: 18

Actual: 15

Sum of Coins Visualization

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Target: 18

Actual: 17

Sum of Coins Visualization

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Target: 18

Actual: 18

Sum of Coins – Solution

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```
int finalSum = 18;
int currentSum = 0;
int[] coins = { 10, 10, 5, 5, 2, 2, 1, 1 };
Queue<int> resultCoins = new Queue<int>();

// Next Slide

Console.WriteLine("Sum not found");
```

Sum of Coins – Solution(2)

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```
for (int i = 0; i < coins.Length; i++)
{
  if (currentSum + coins[i] > finalSum) continue;

  currentSum += coins[i];
  resultCoins.Enqueue(coins[i]);

  if (currentSum == finalSum)
    // Sum Found
}
```

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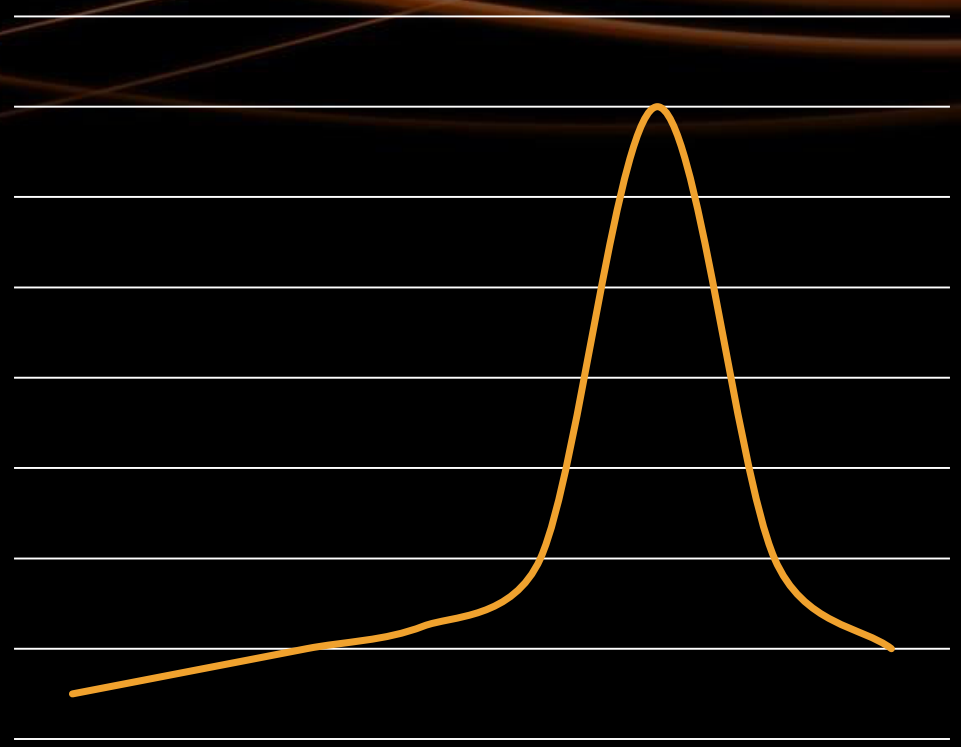
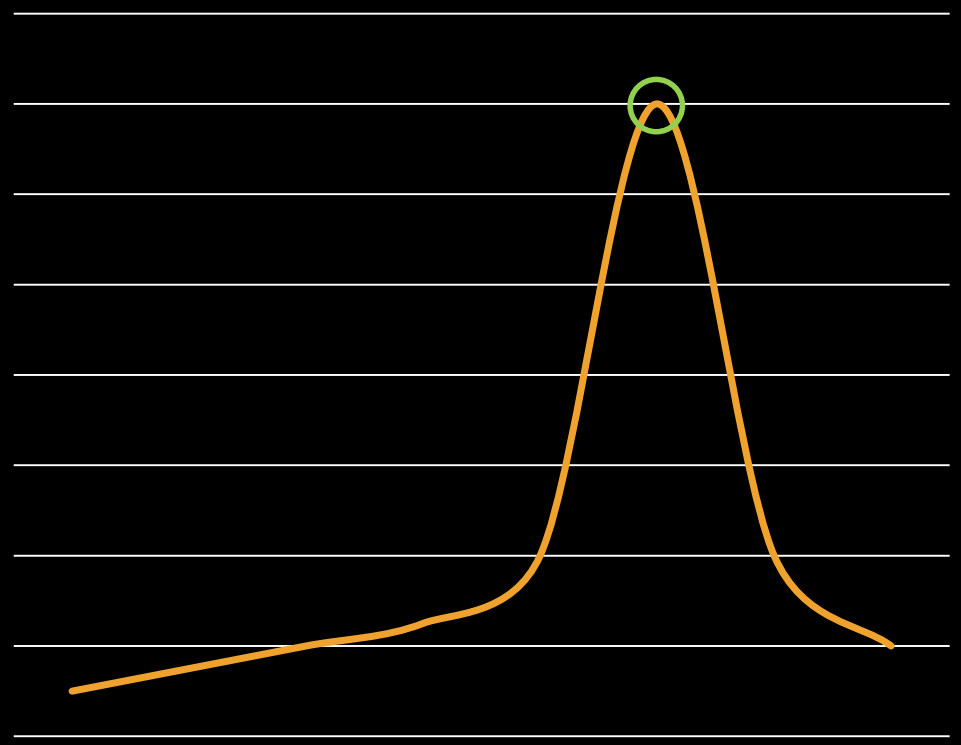


Peak Point

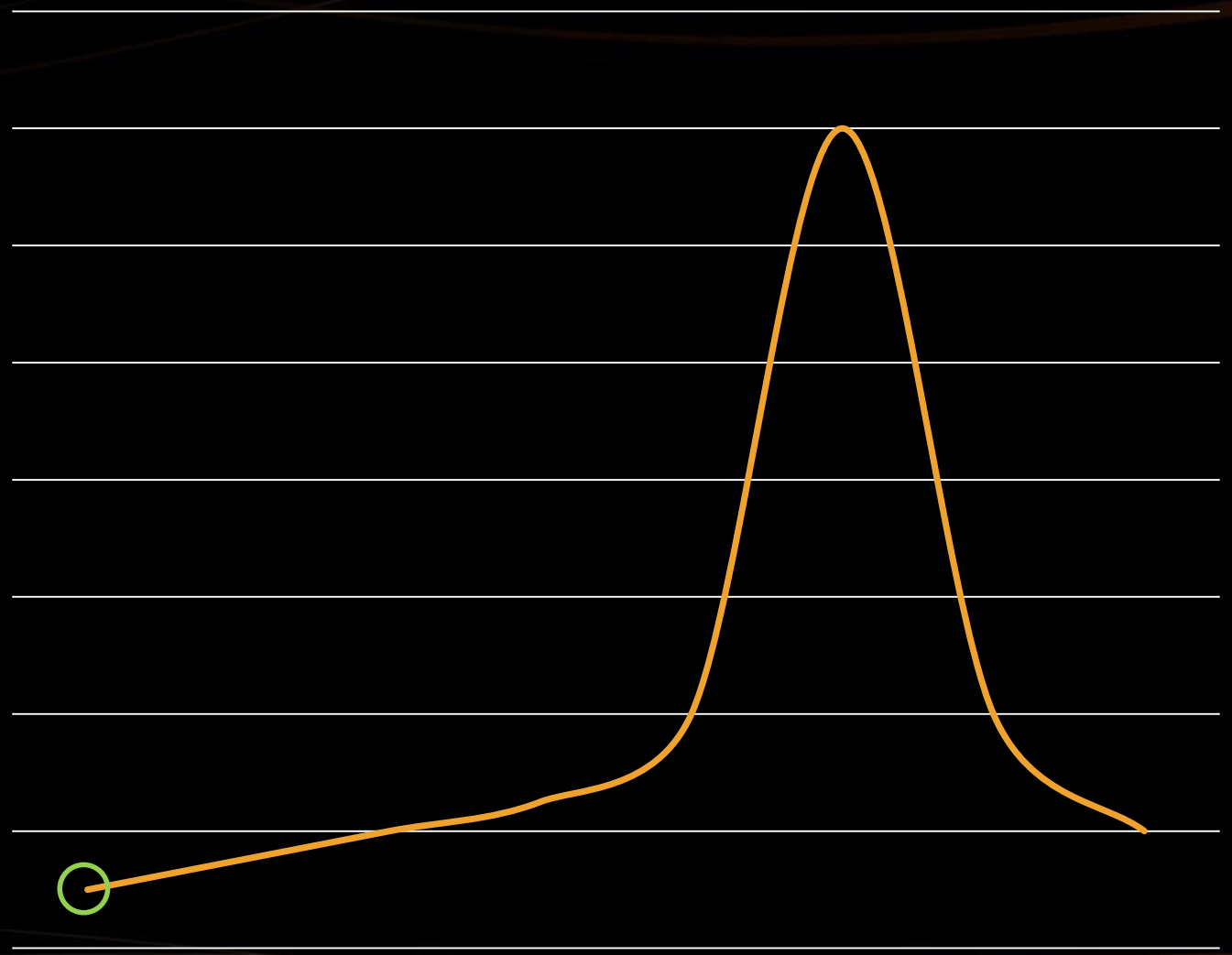
Problem

Greedy Algorithms: Peak Point

- Find the peak point on a curve



Peak Point Visualization



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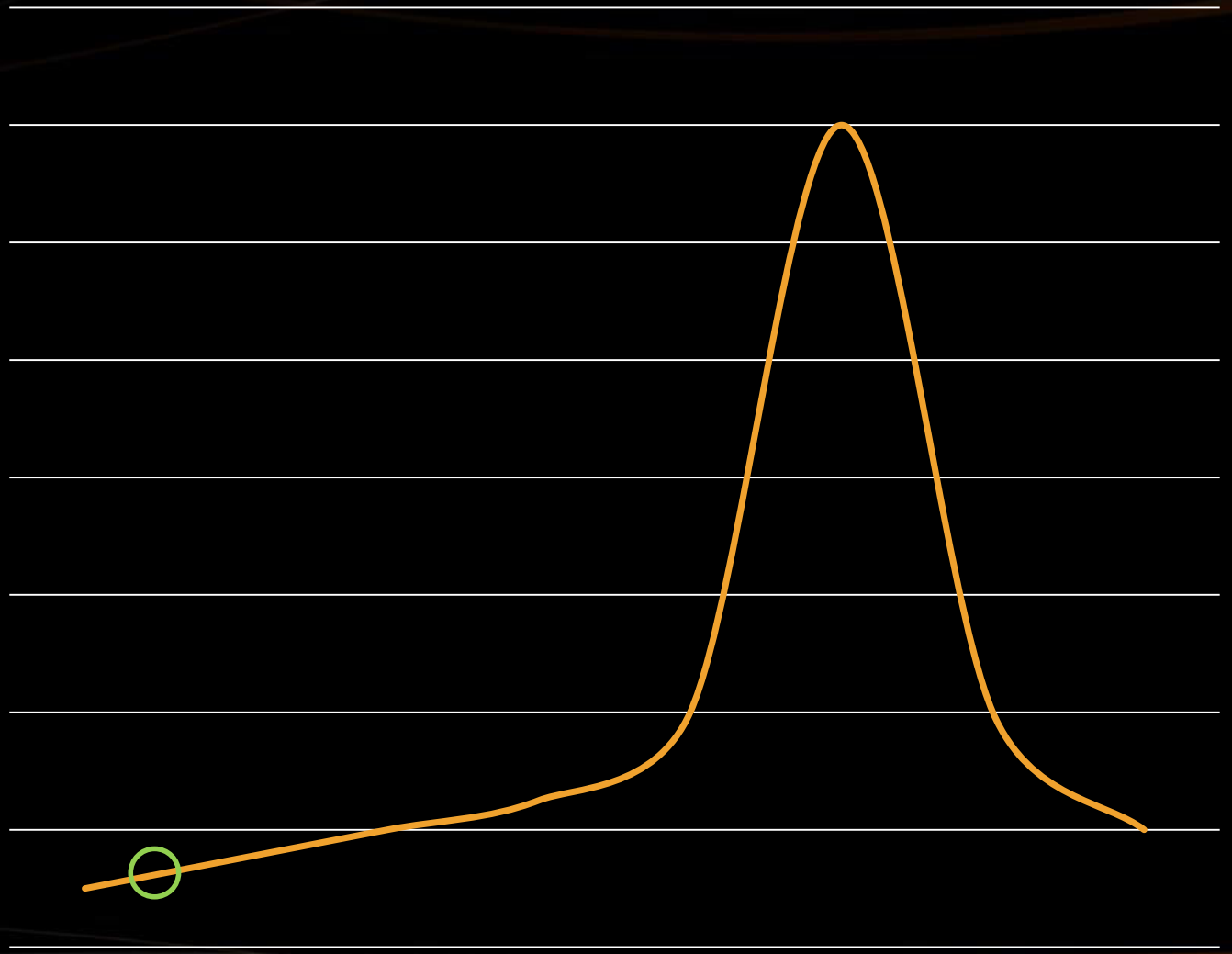

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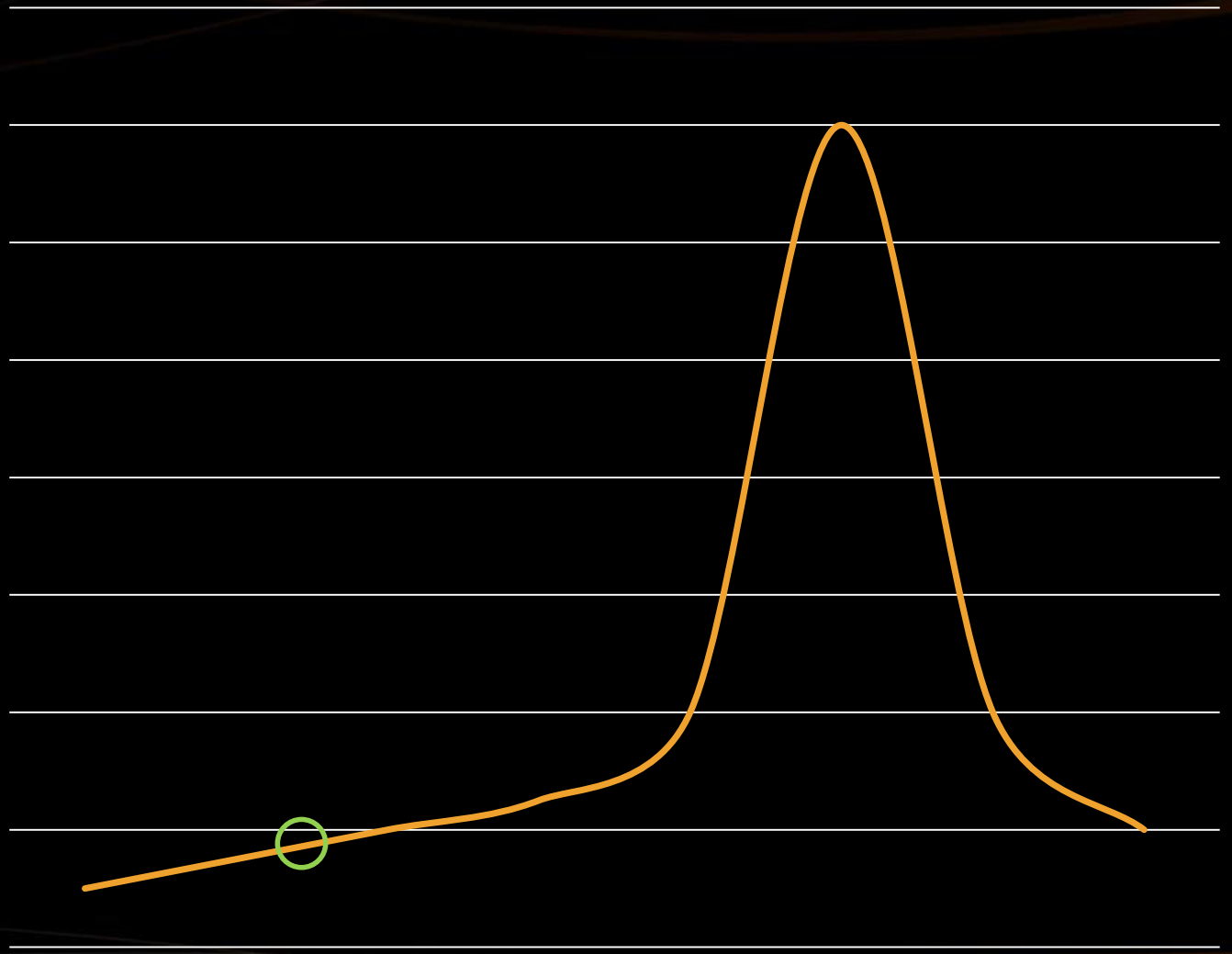

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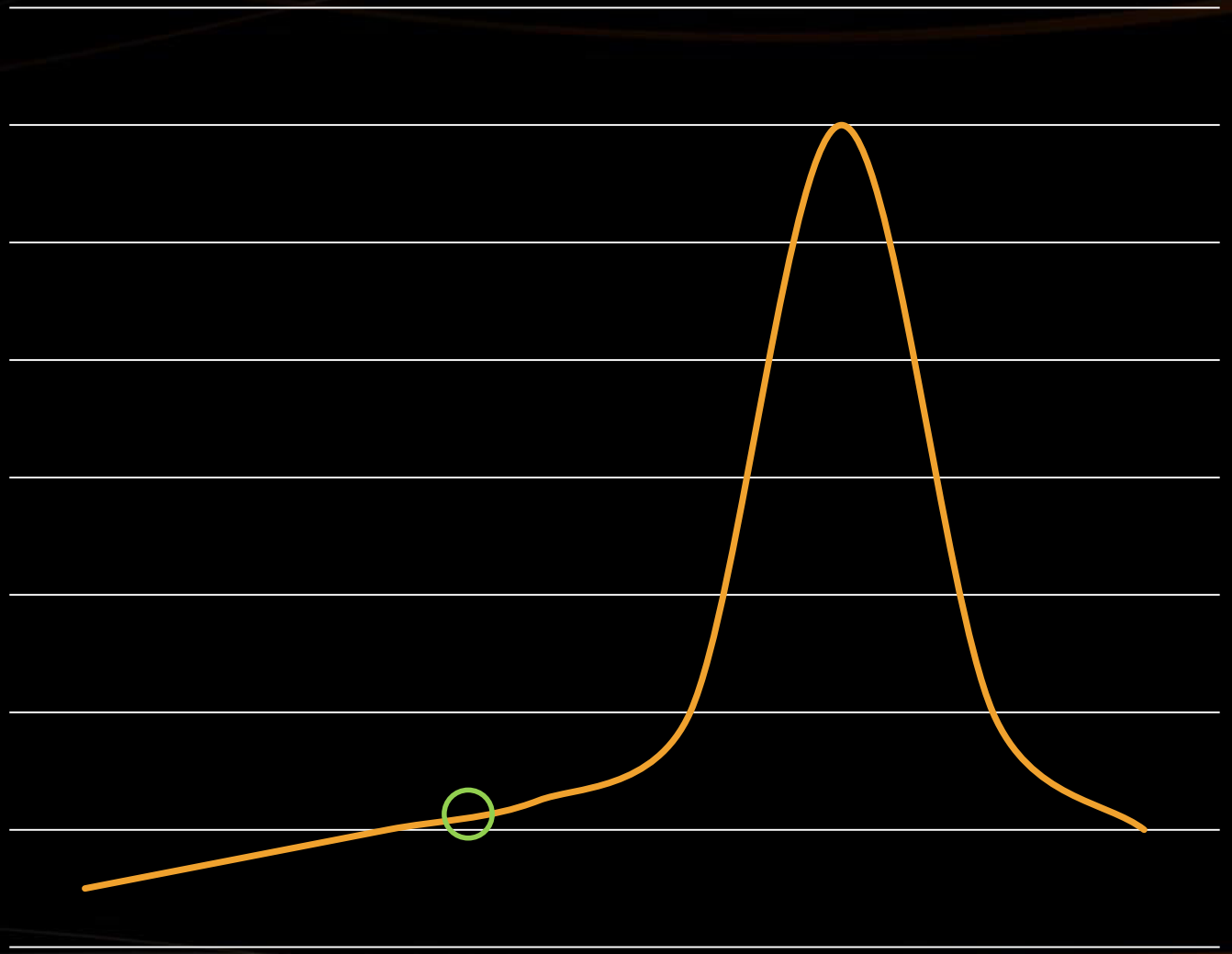

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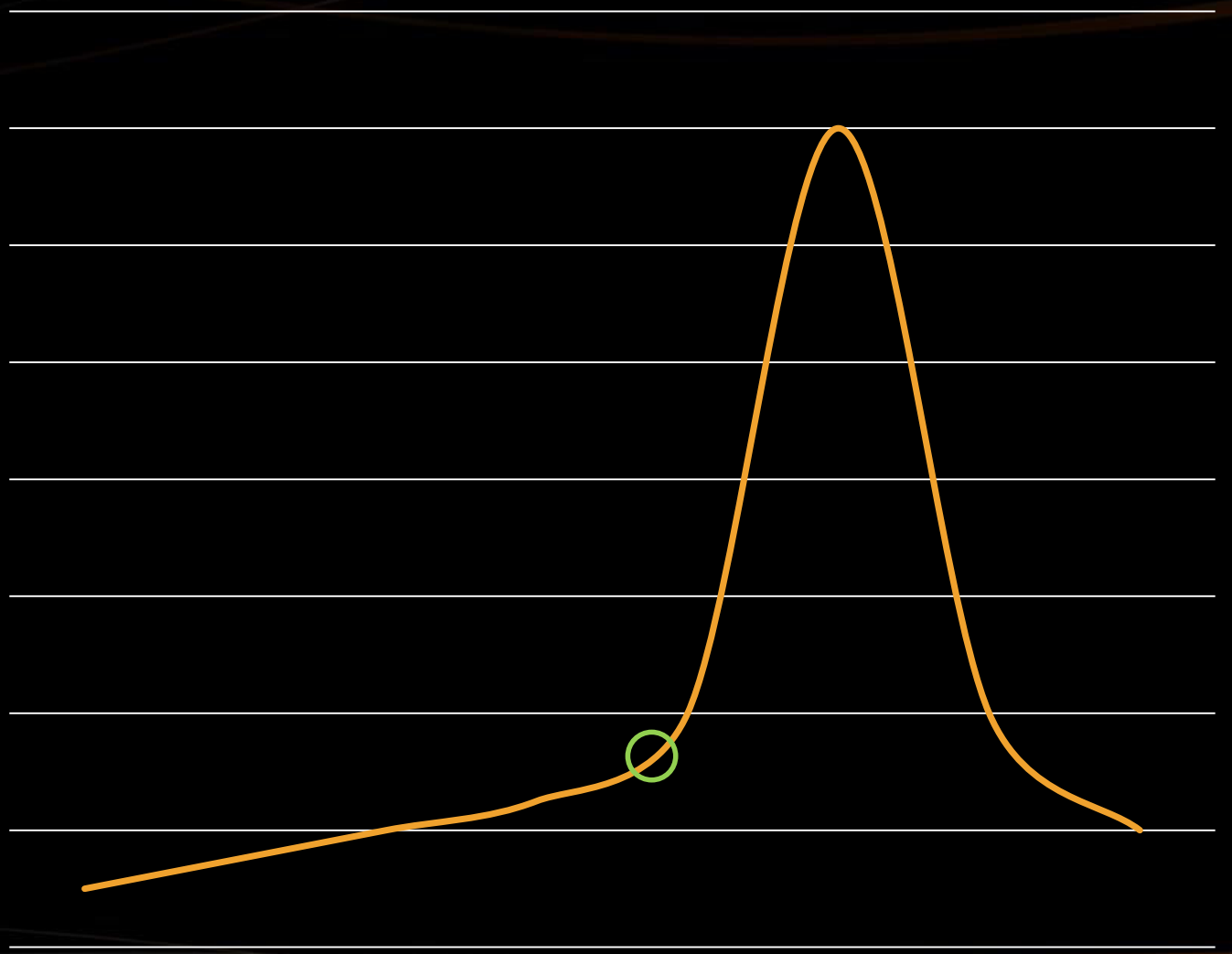

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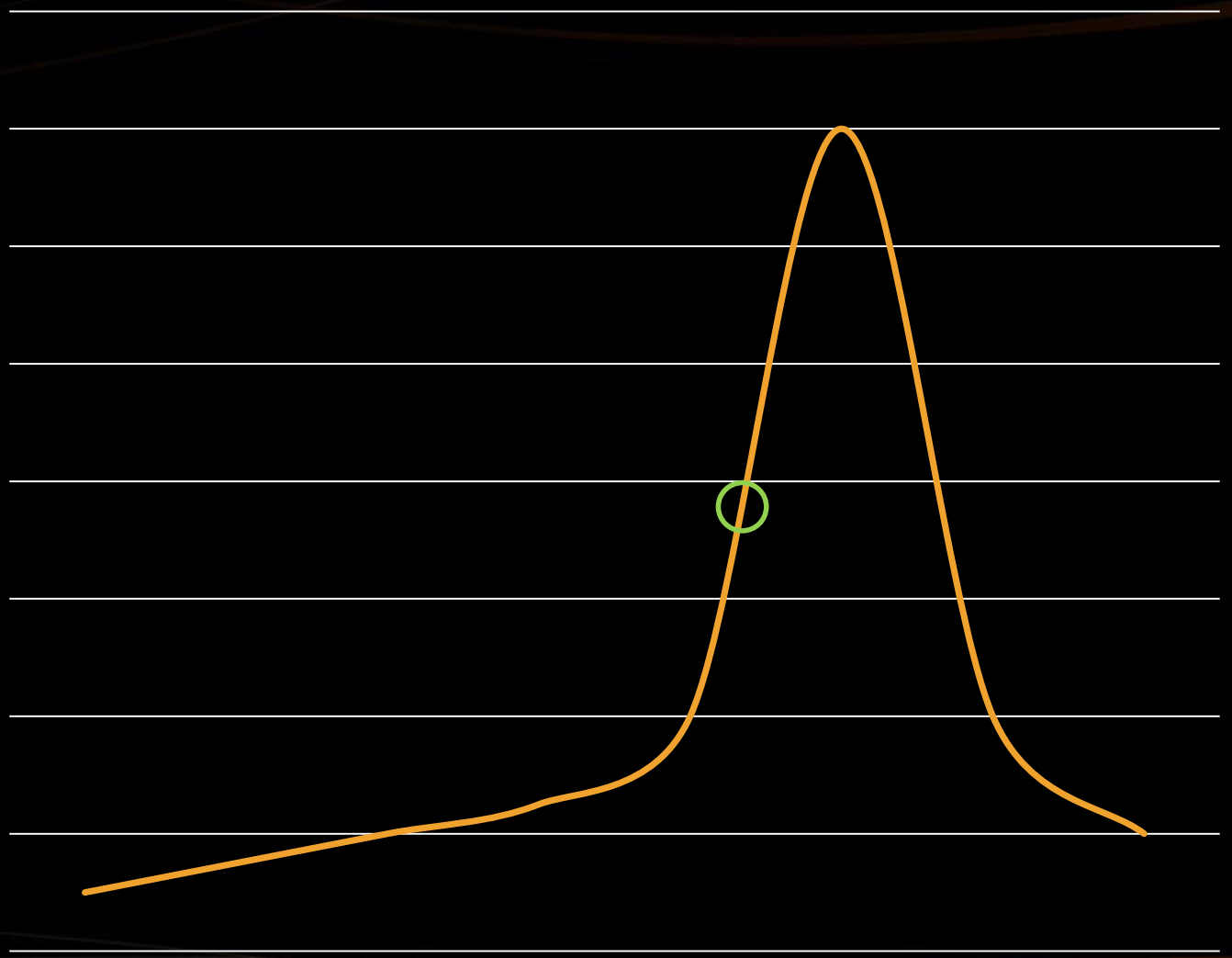

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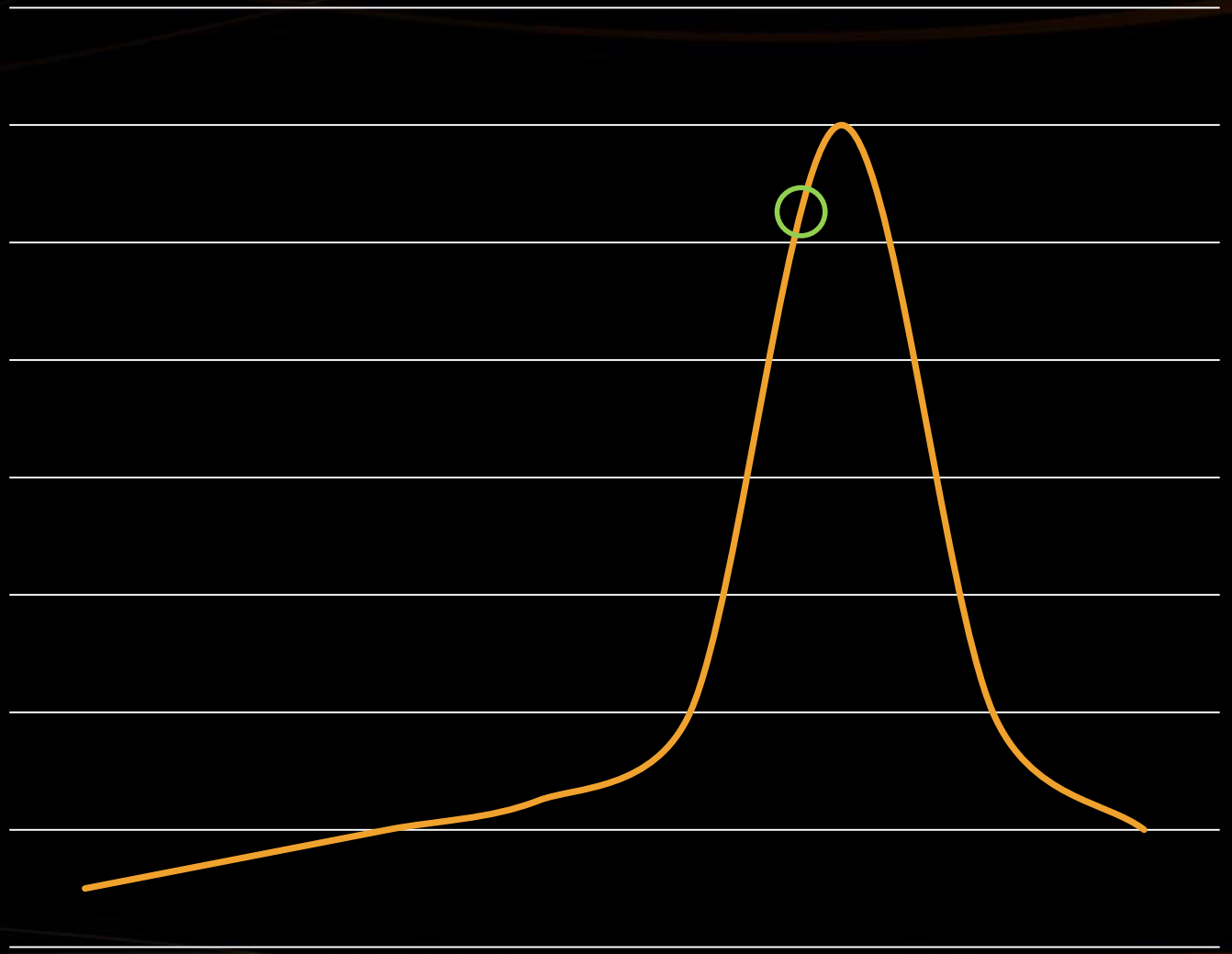

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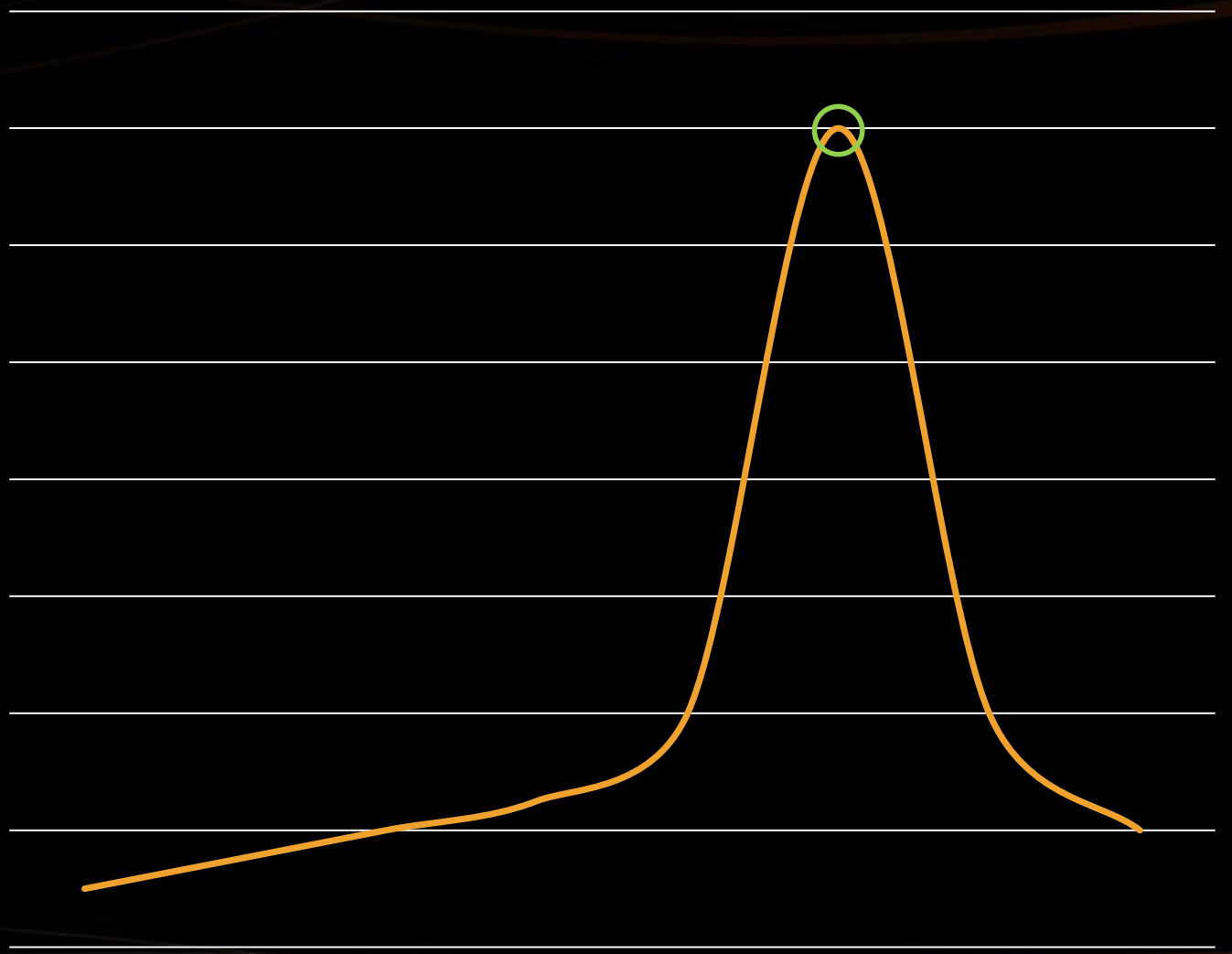

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Greedy Algorithm for Sum of Coins

Live Coding Exercise (Lab)

Greedy Failure Cases

Greedy Algorithms Often Fail

Sum of Coins Failure

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Target: 18

Actual: 0

Sum of Coins Failure

Target: 18

Actual: 10

Sum of Coins Failure

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Target: 18

Actual: 15

Sum of Coins Failure

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Target: 18

Actual: 16

Sum of Coins Failure

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Target: 18

Actual: 17

Sum of Coins Failure

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Target: 18

Actual: 18

Sum of Coins Failure

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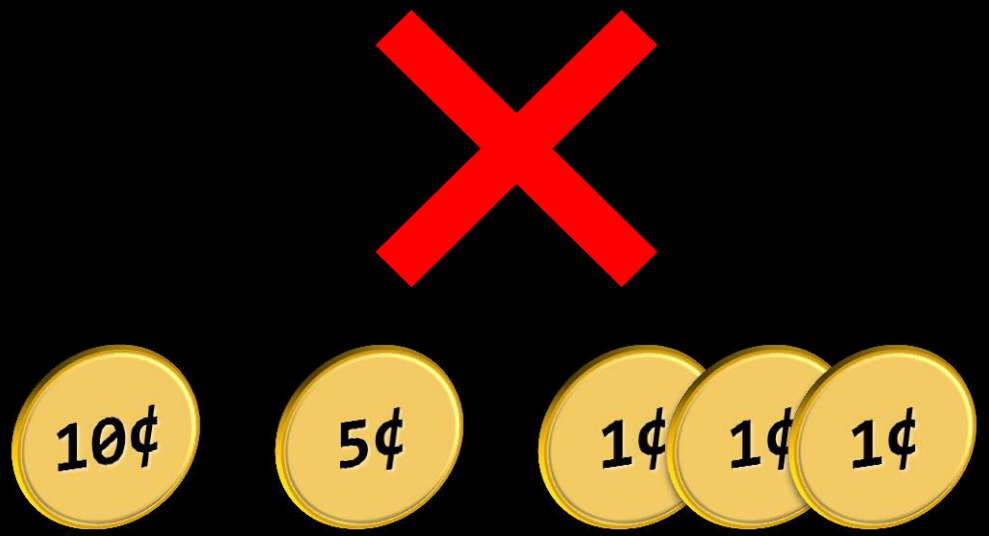
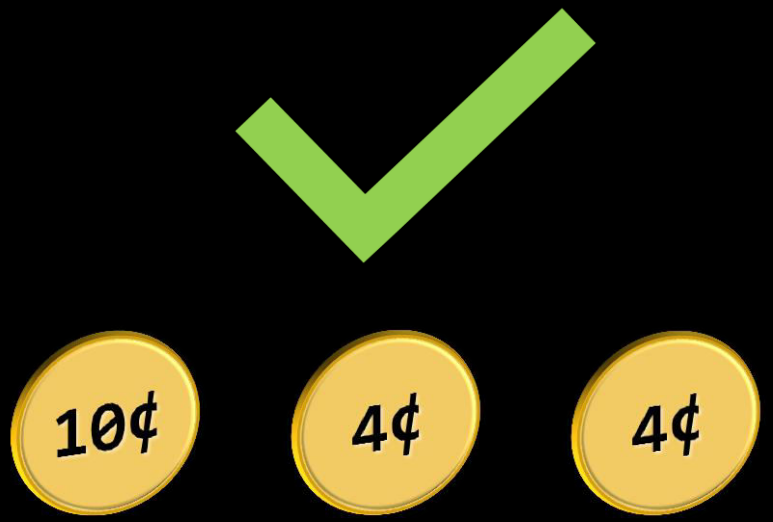
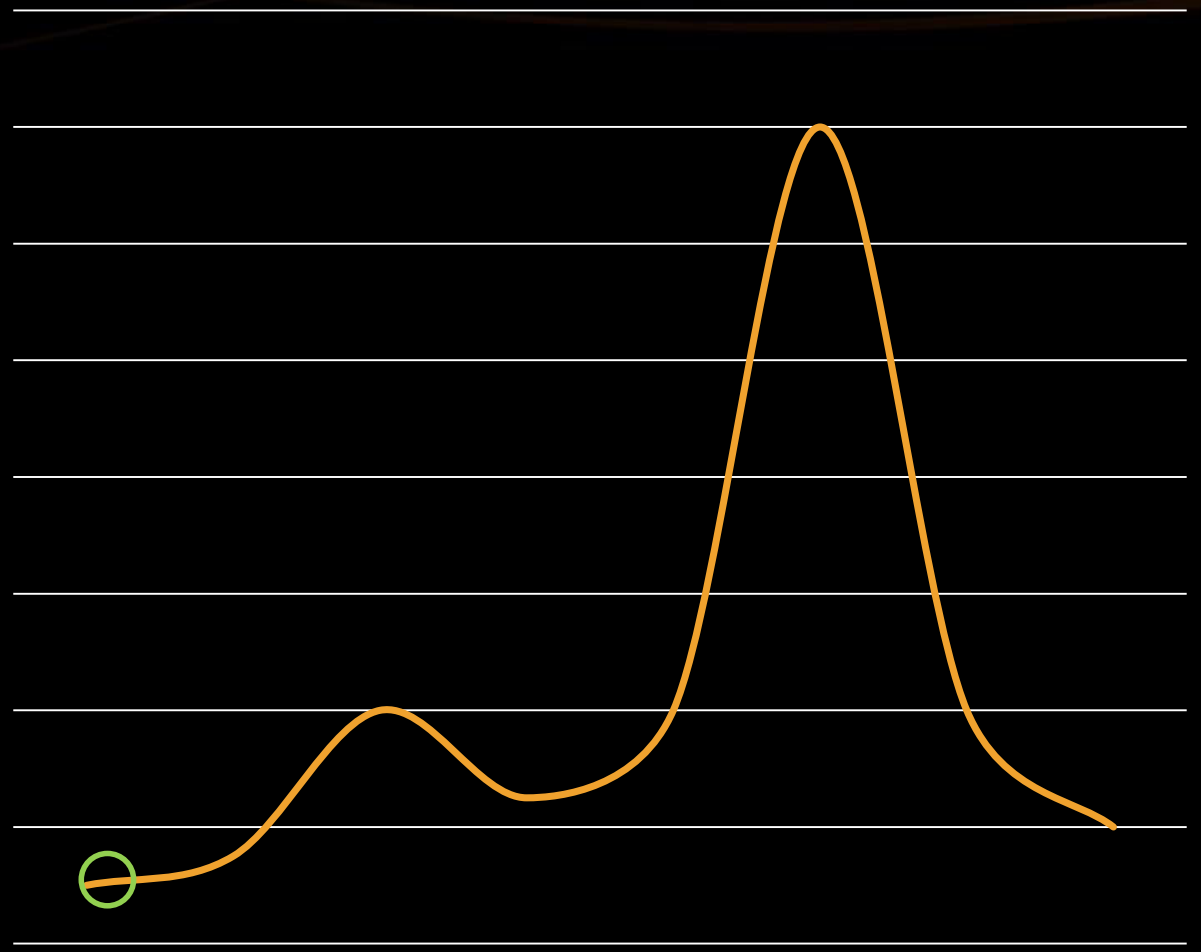
Target: 18





Peak Point Failure



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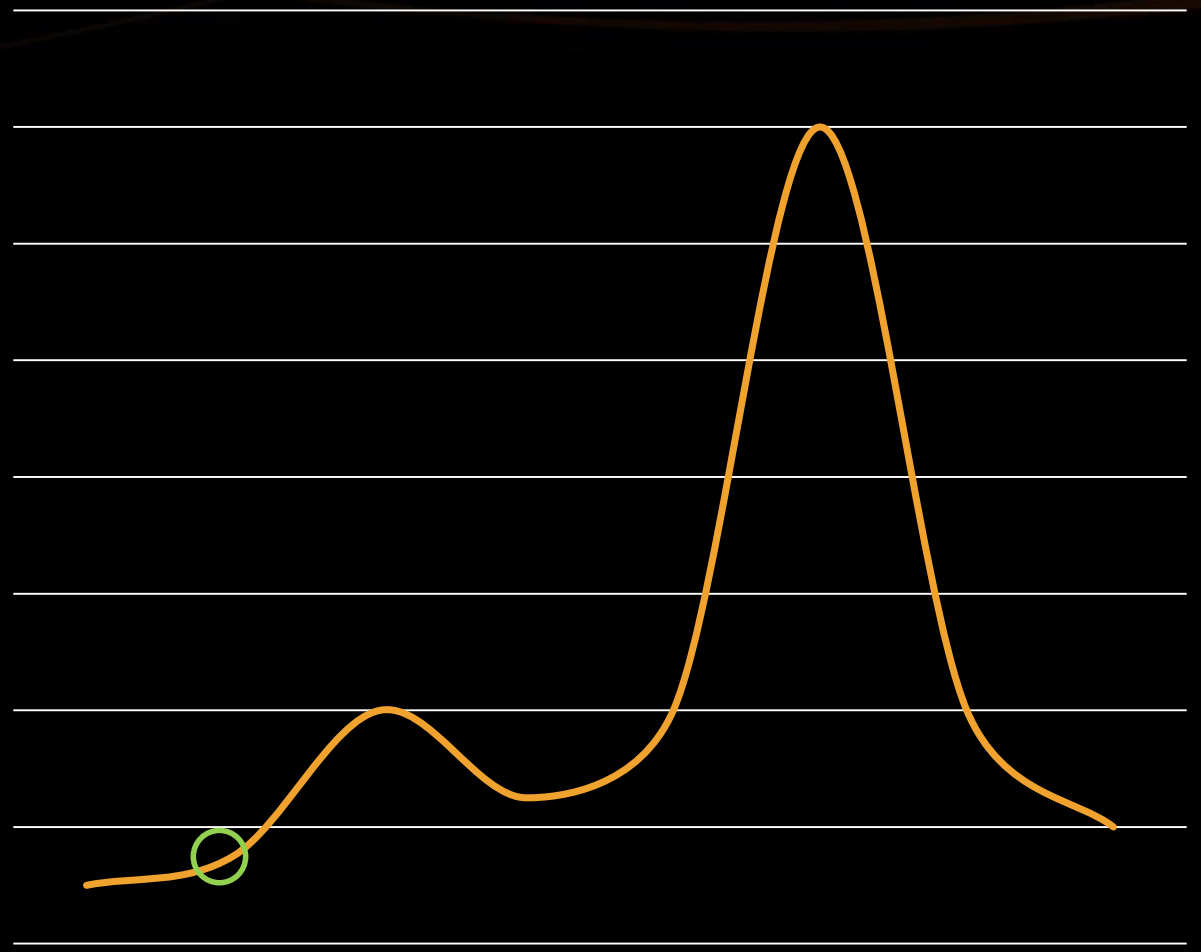

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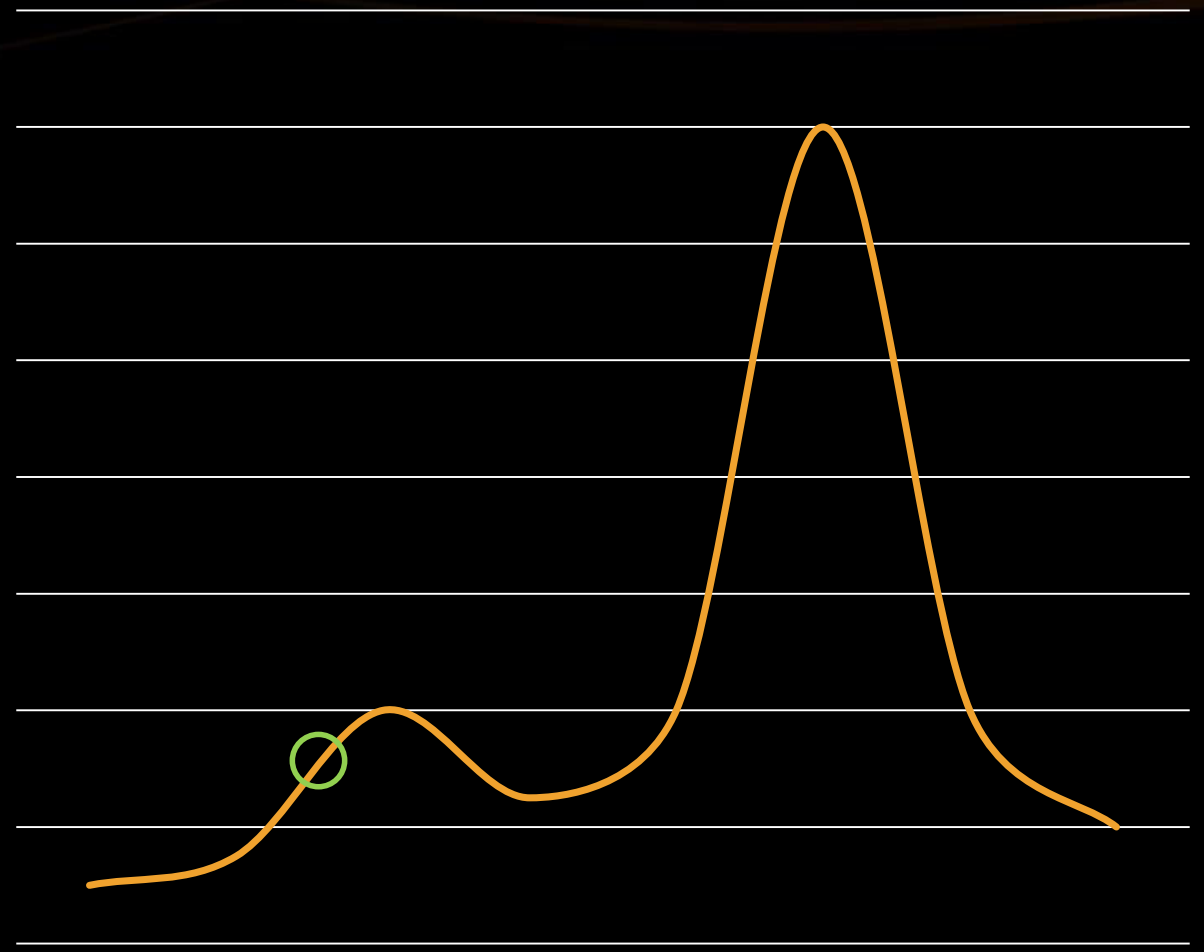

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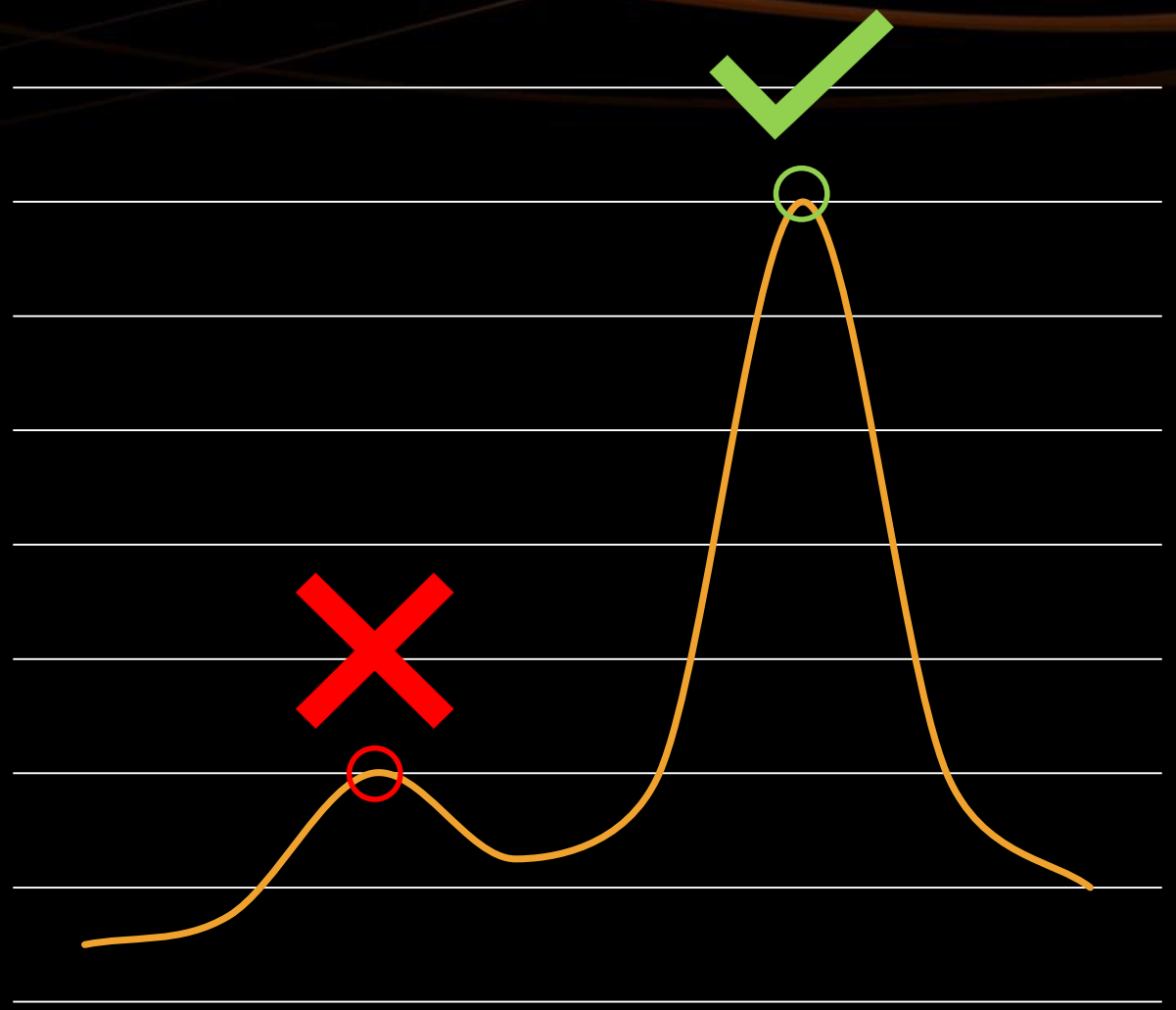
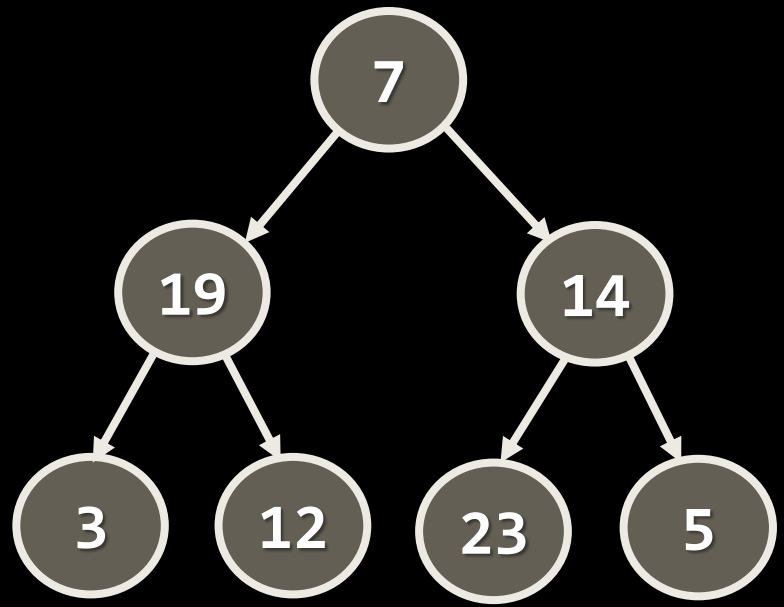


Max Sum Path in Binary Tree

- Largest sum path in a tree (starting at root)
  - Greedy strategy: choose largest next node



Max Sum in Binary Tree

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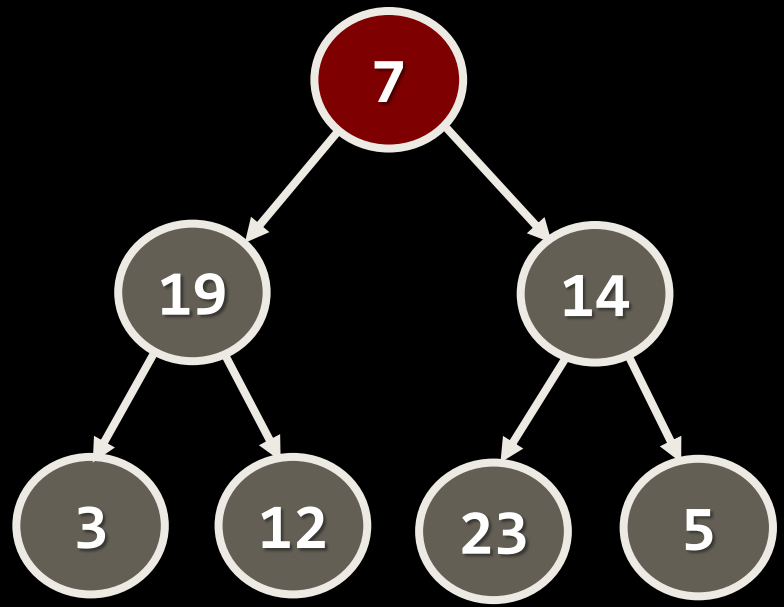
Sum: 7



Max Sum in Binary Tree

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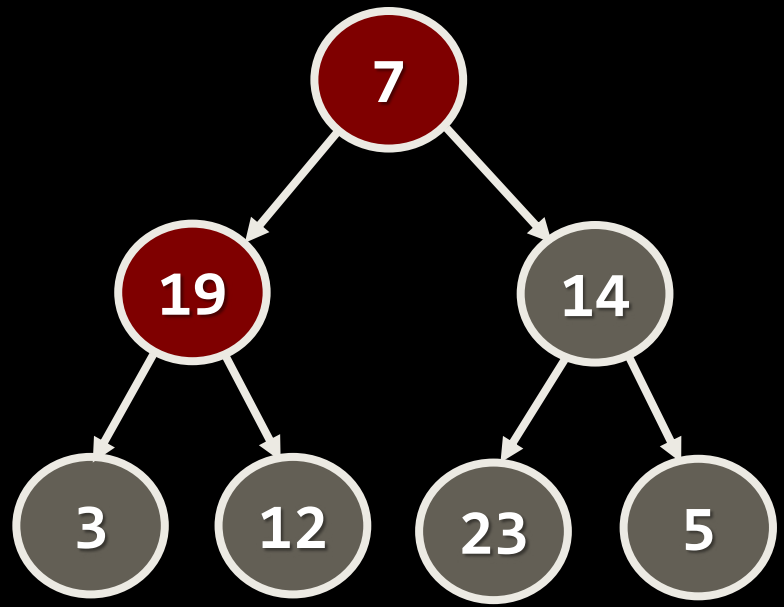
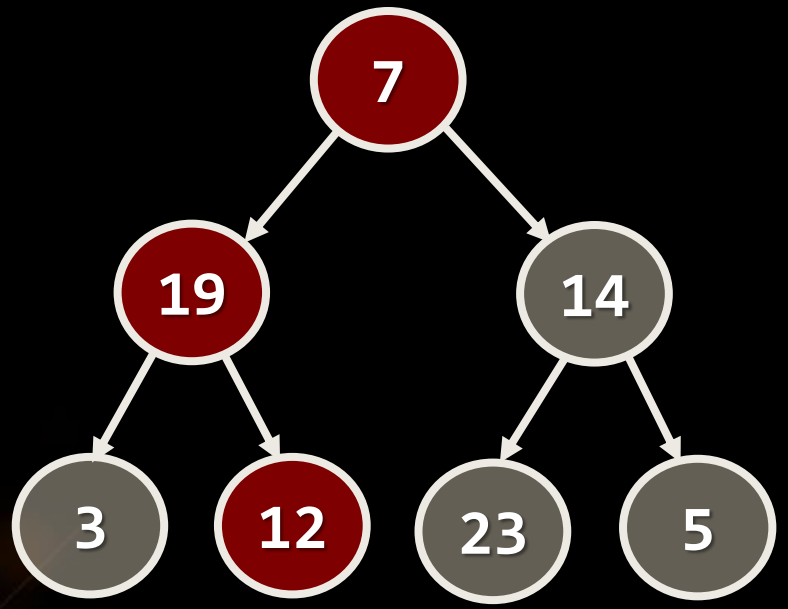
Sum: 26

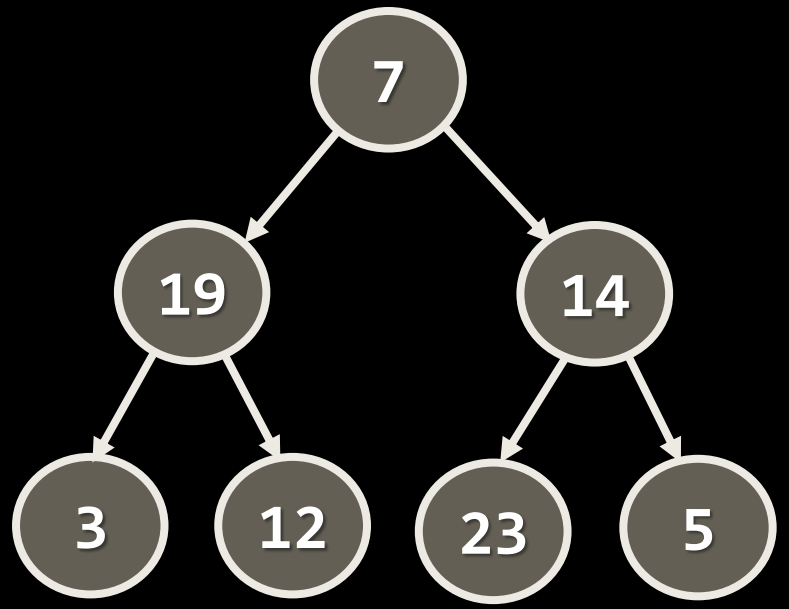


Max Sum in Binary Tree

Sum: 38
Greedy

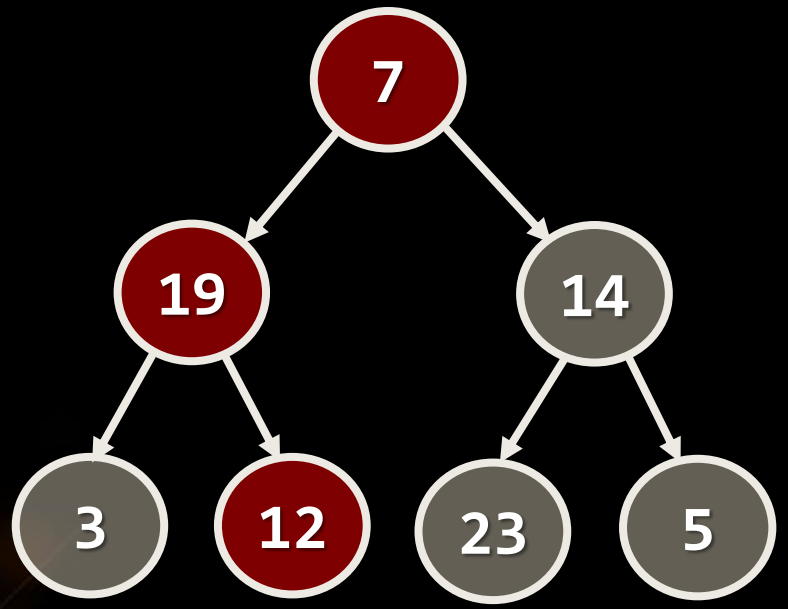


Sum: 0
Actual

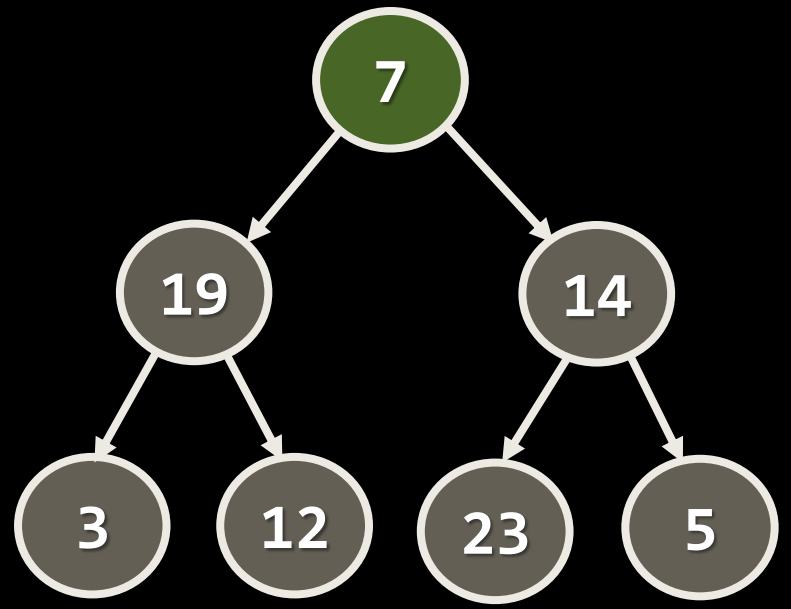


Max Sum in Binary Tree

Sum: 38
Greedy



Sum: 7
Actual



Max Sum in Binary Tree

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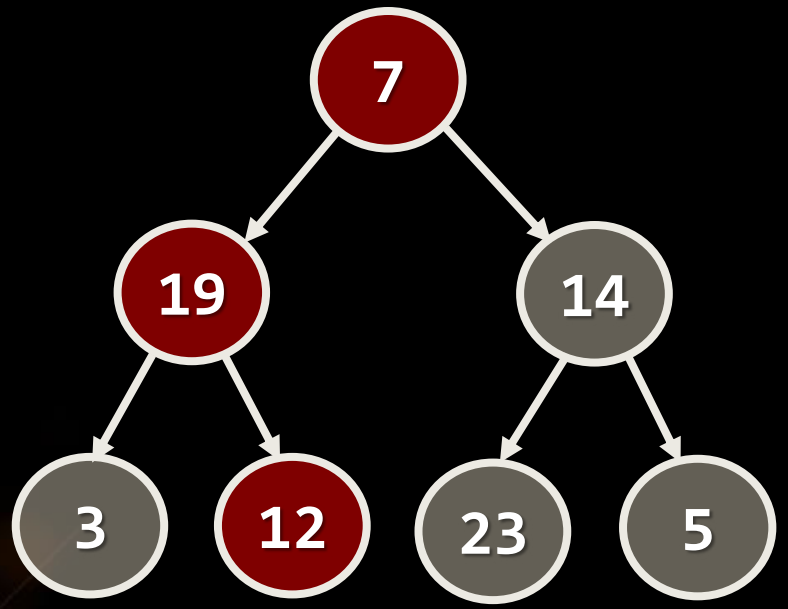
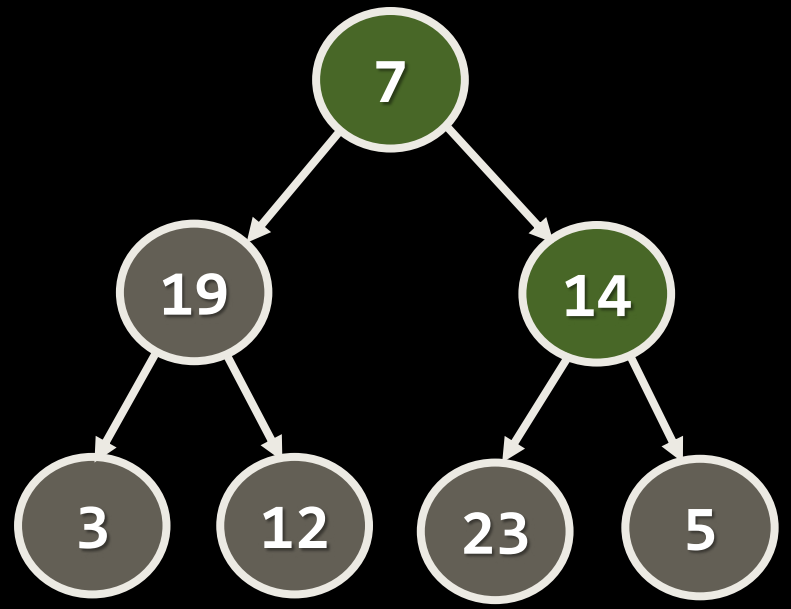
Sum: 38
Greedy

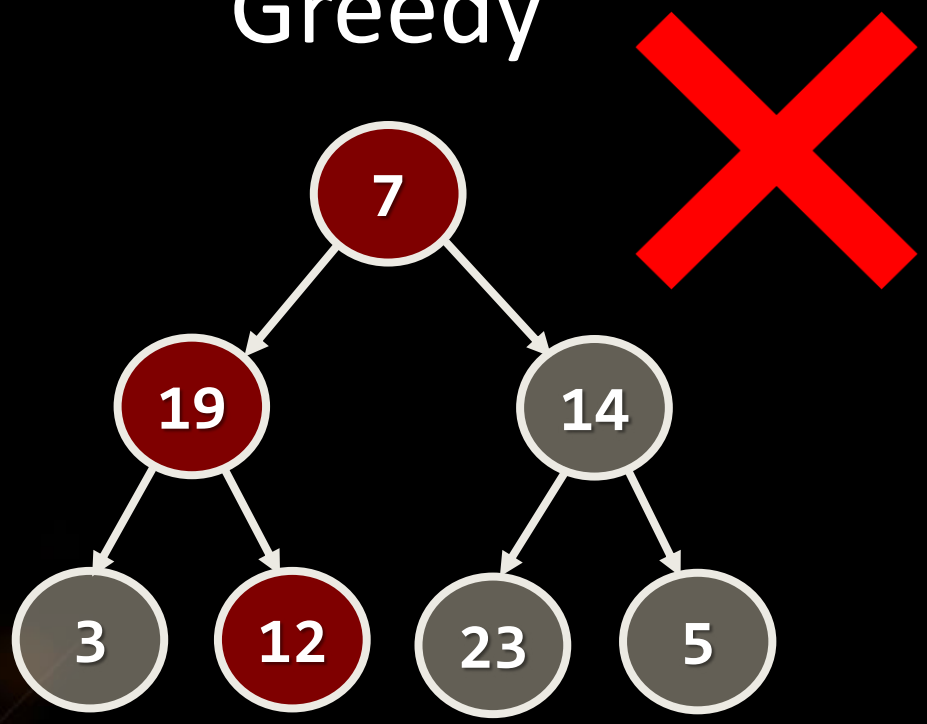


Sum: 21
Actual

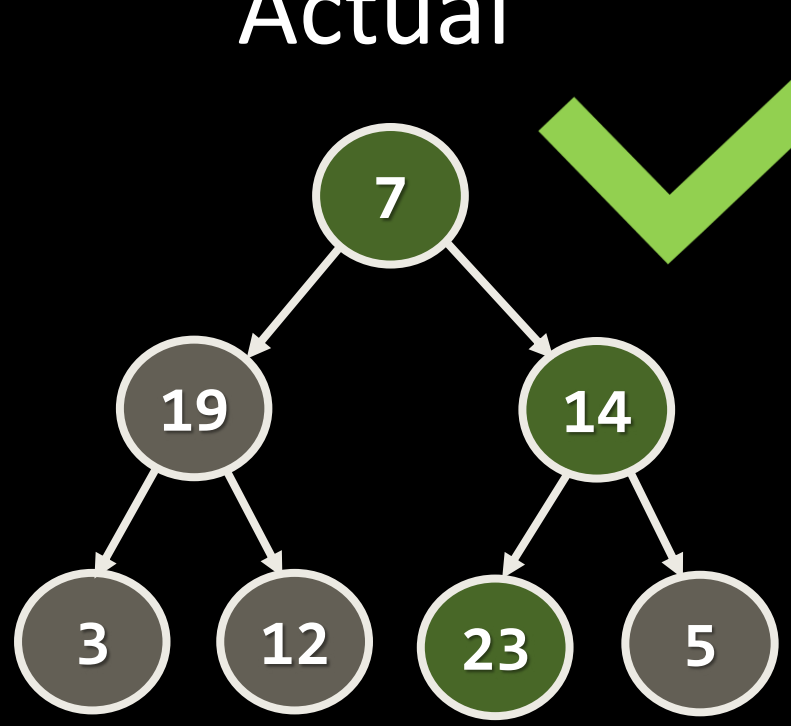


Max Sum in Binary Tree

Sum: 38
Greedy



Sum: 44
Actual



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Optimal Greedy Algorithms

Optimal Substructure and Greedy Choice Property

Optimal Greedy Algorithms

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- Suitable problems for greedy algorithms have these properties:
  - Greedy choice property
  - Optimal substructure
- Any problem having the above properties is guaranteed to have an optimal greedy solution

Greedy Choice Property

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- Greedy choice property
  - A global optimal solution can be obtained by greedily selecting a locally optimal choice
  - Sub-problems that arise are solved by consequent greedy choices
    - Enforced by optimal substructure

Optimal Substructure Property

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- Optimal substructure property
  - After each greedy choice the problem remains an optimization problem of the same form as the original problem
  - An optimal global solution contains the optimal solutions of all its sub-problems

Greedy Algorithms: Example

- The "Max Coins" game
  - You are given a set of coins
  - You play against another player, alternating turns
  - Per each turn, you can take up to three coins
  - Your goal is to have as many coins as possible at the end

Max Coins – Greedy Algorithm

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- A simple greedy strategy exists for the "Max Coins" game

```
At each turn take the maximum number of coins
```

- Always choose the local maximum (at each step)
  - You don't consider what the other player does
  - You don't consider your actions' consequences
- The greedy algorithm works optimally here
  - It takes as many coins as possible

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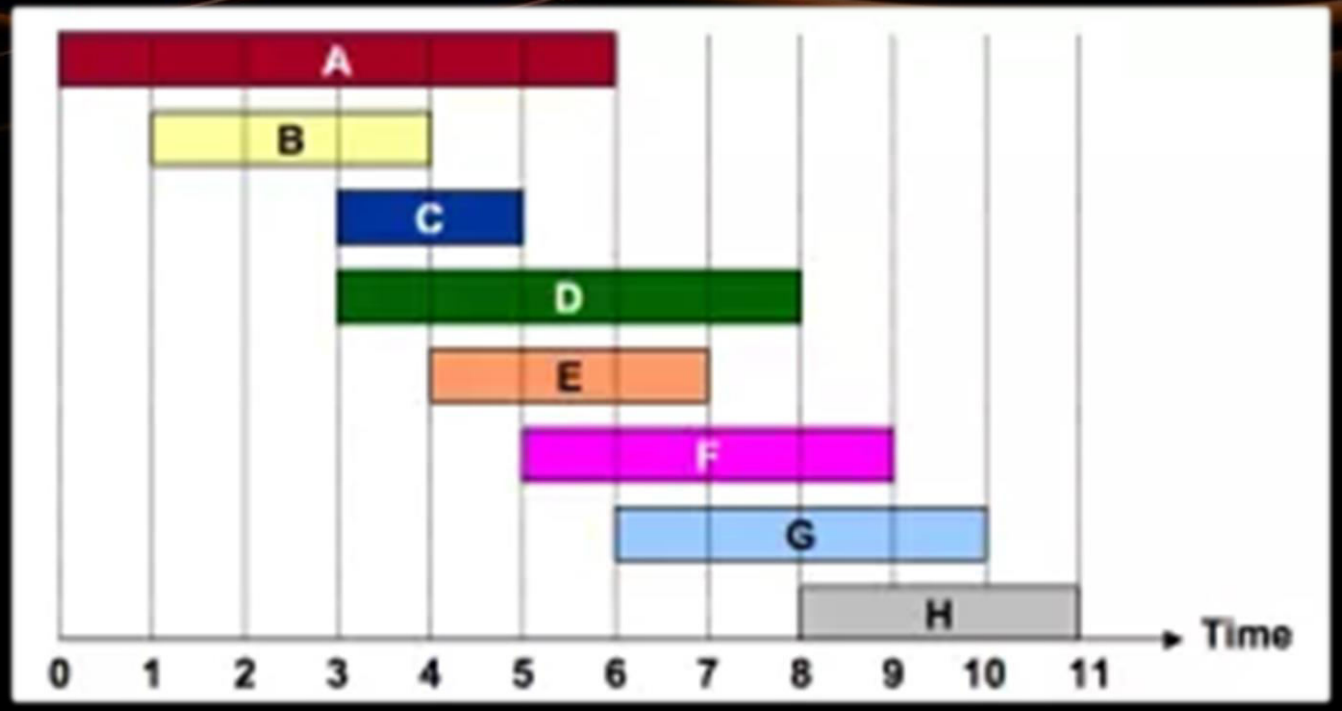

Activity Selection Problem

The Greedy Algorithm and Why It Is Optimal

Activity Selection Problem

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- The Activity Selection Problem
  (a.k.a. Conference Scheduling Problem)
  - Given a set of activities $S = \{a_1, a_2, \ldots, a_n\}$
    - Each having a start & finish time: $a_i = \{s_i, f_i\}$
  - Activities are "compatible" if they don't overlap
    - i.e. their intervals do not intersect
  - What is the maximum-size subset of compatible activities?
    - i.e. which is the largest list of compatible activities that can be scheduled?

Activity Selection Problem – Example

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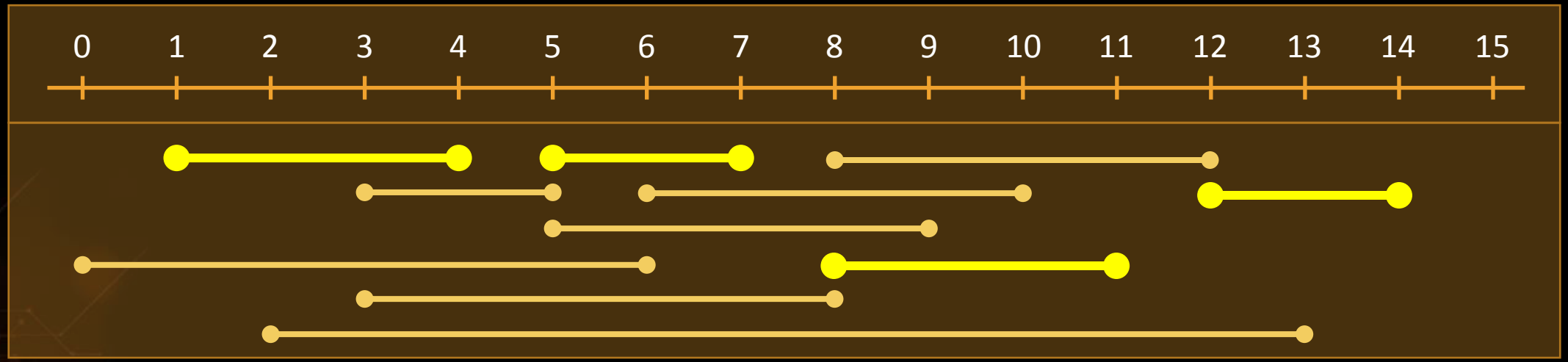
- Activity selection problem example:

Index	1	2	3	4	5	6	7	8	9	10	11
Start (s_i)	1	3	0	5	3	5	6	8	8	2	12
Finish (f_i)	4	5	6	7	8	9	10	11	12	13	14



- Several optimal solutions: $\{a_1, a_4, a_8, a_{11}\}$, $\{a_2, a_4, a_9, a_{11}\}$, ...

Activity Selection Problem: Greedy

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- A greedy algorithm for the Activity Selection Problem:

```
While (S in not empty)
  Select from S an activity A with the earliest finish
  Remove A + all activities from S conflicting with A
```

- Greedy characteristics of the above algorithm
  - Taking the earliest finish activity gives more time for all others
  - We pick an activity preserving the "maximum remaining time"

Activity Selection Problem: Implementation

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- Single scan (linear) implementation of the greedy algorithm for the Activity Selection Problem:

```
Sort the activities from S by their finish time
last = first activity from S
for (a in S)
  if (a.Start >= last.Finish)
    // a does not conflict with last
    print a
    last = a
```

Proving Greedy Optimality

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- To prove the discussed greedy is optimal we:
  - Need to prove the problem has a greedy choice property
  - Need to prove the problem has optimal substructure
- Greedy properties are satisfied:
  - A solution starting with a greedy choice exists: the earliest finish
  - Optimal substructure exists:
    - If we remove the activity A with the earliest finish time activity
    - We reduce the problem to the same problem of smaller size (without activities in S, which intersect the activity A)

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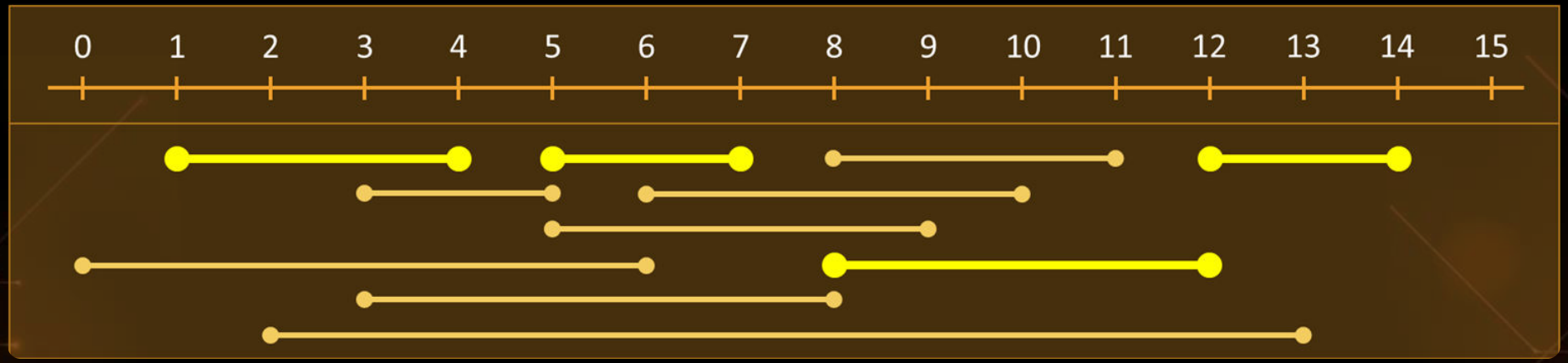
Index	1	2	3	4	5	6	7	8	9	10	11
Start (s_i)	1	3	0	5	3	5	6	8	8	2	12
Finish (f_i)	4	5	6	7	8	9	10	11	12	13	14



Activity Selection Problem

Live Demo

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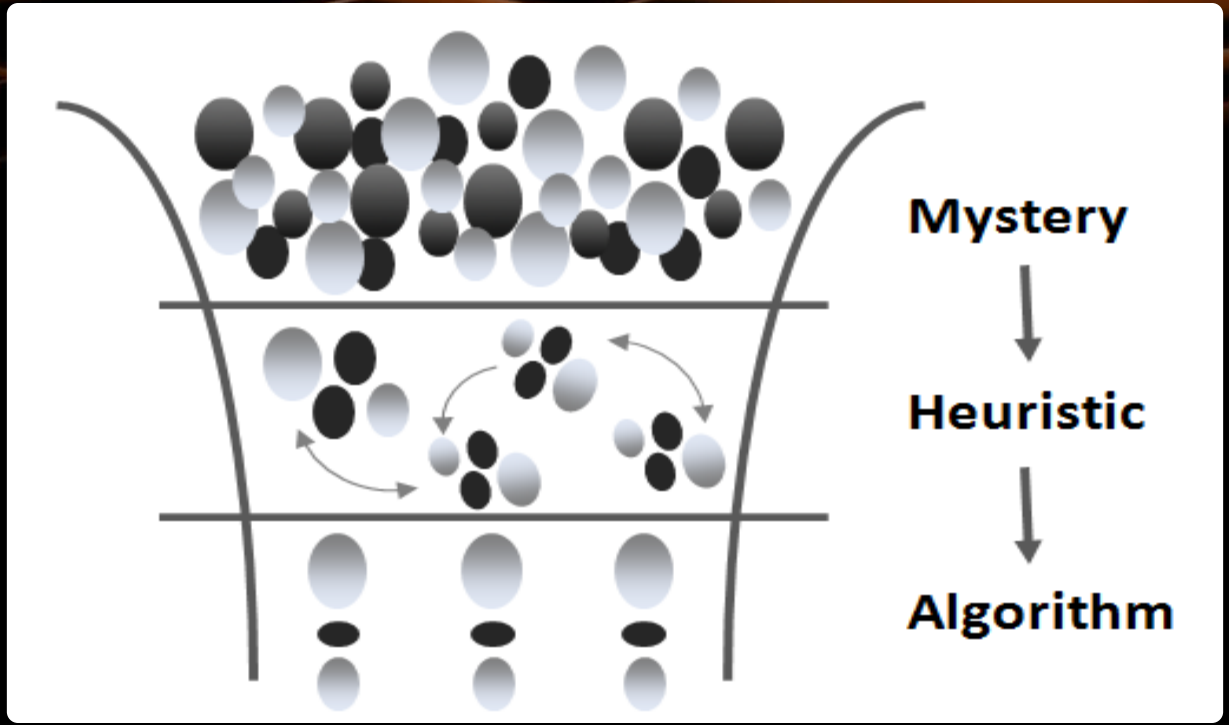

Heuristic Algorithms

Greedy, Genetic, Branch and Bound

Heuristic Algorithms

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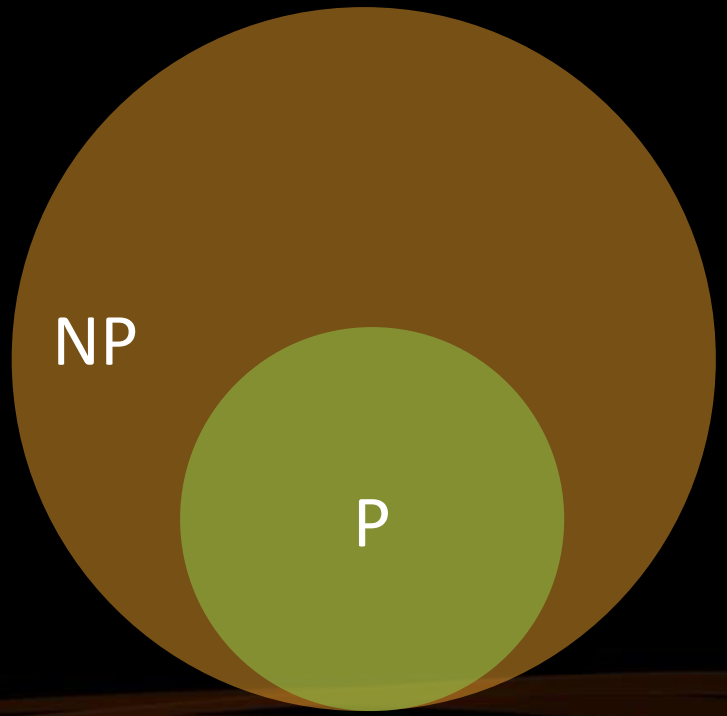
- Some problems are hard to be solved (no fast algorithm exists)
- Heuristic algorithms solve hard problems approximately
  - They find a good solution, but cannot guarantee it is optimal
- Examples of heuristic algorithms:
  - Greedy algorithms – make a greedy choice at each step
  - Genetic algorithms – based on inheritance, mutation, selection, …
  - Branch and bounds – optimized backtracking with cut-offs
  - Others…

P vs NP

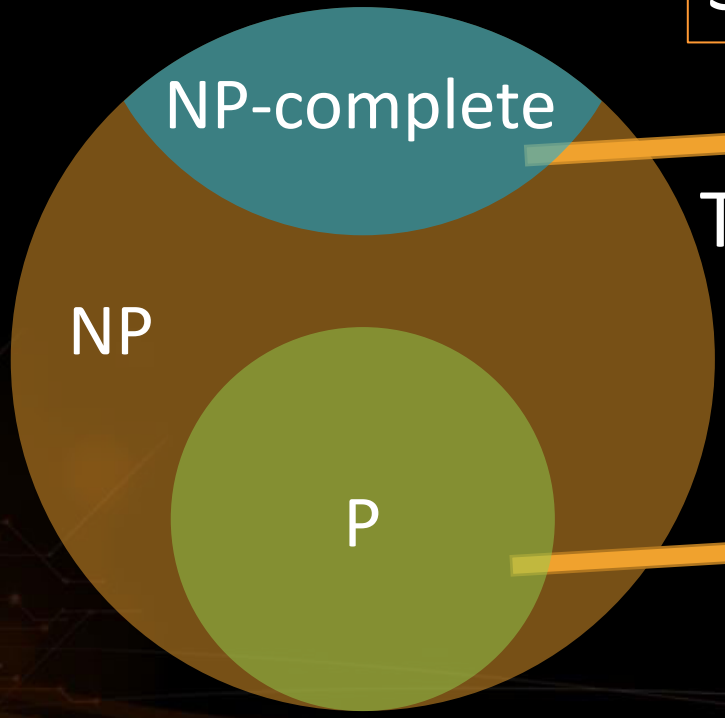
- P - Category in which the algorithms are solvable in polynomial time
- Polynomial -> $5N^5 + 3N^4 + N$ is $O(N^5)$
- Not a polynomial -> 2^N

P vs NP

- NP - Category in which the algorithms are verifiable in polynomial time.
- Decision problems
- Example:
  - Does a route shorter than L exist?
  - Yes/No



P vs NP



Subset sum

Knapsack
(Can a value >= V be achieved?)

Set-Cover problem

The hardest in NP

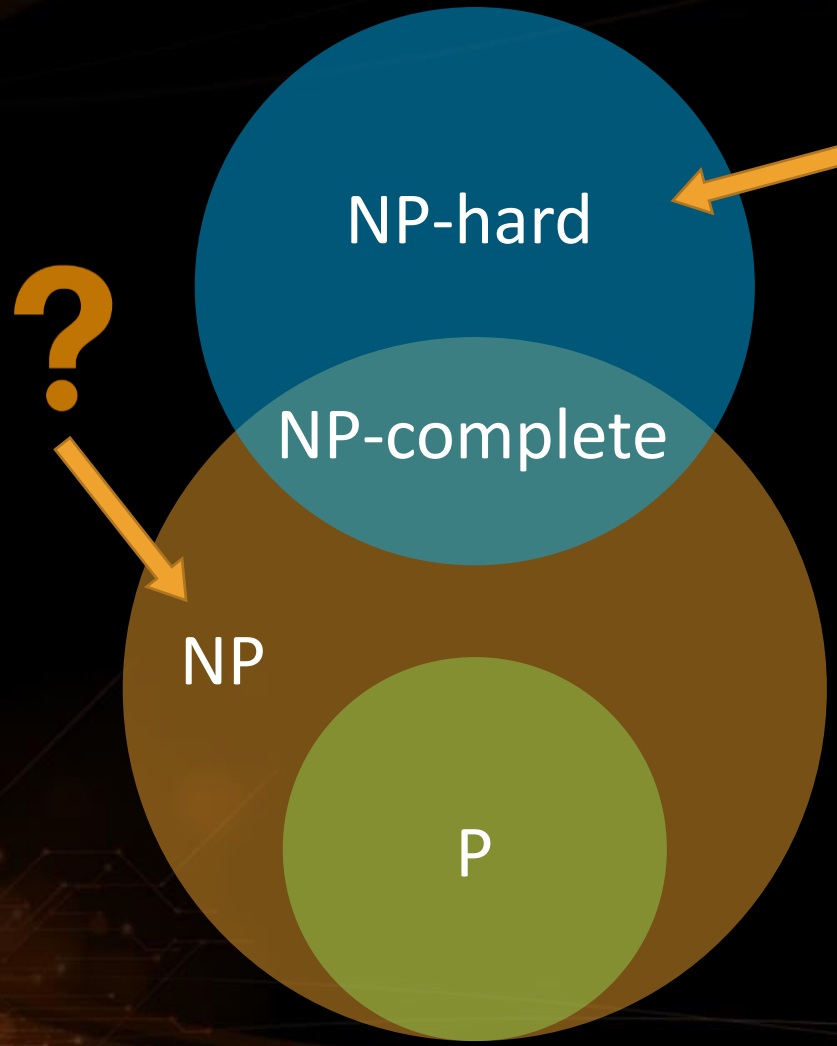
Finding shortest route

Sorting

Look-ups in hash tables

Traversing lists

P vs NP



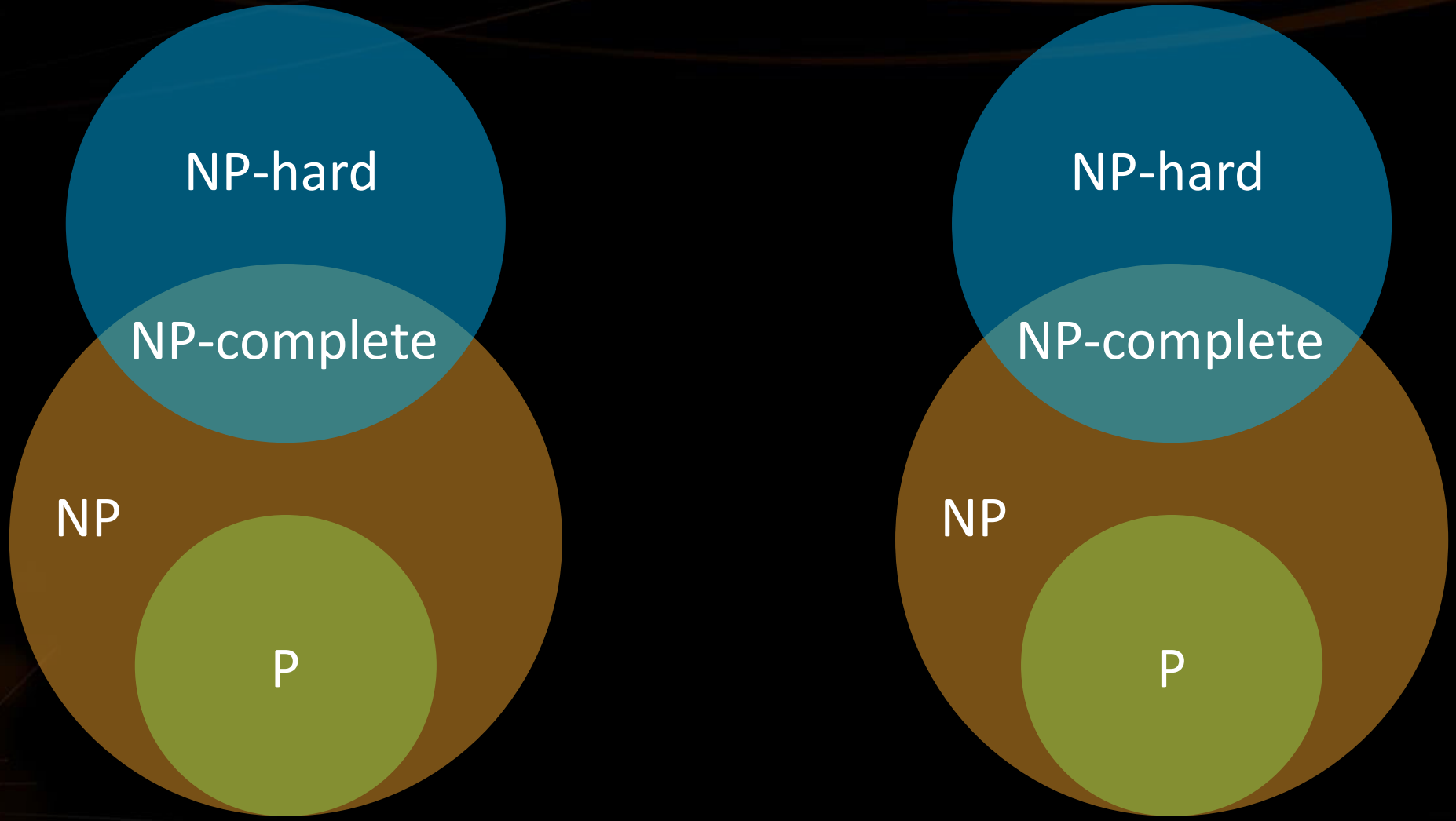
At least as hard
as NP-complete

Knapsack (optimization)
(maximize V)

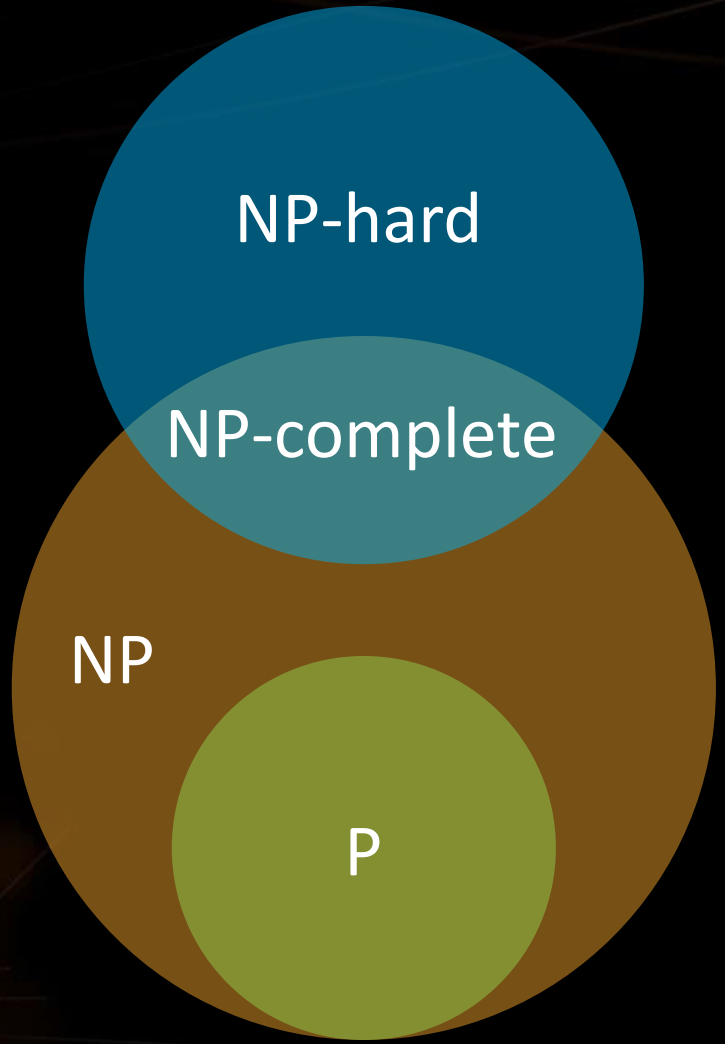
Halting problem

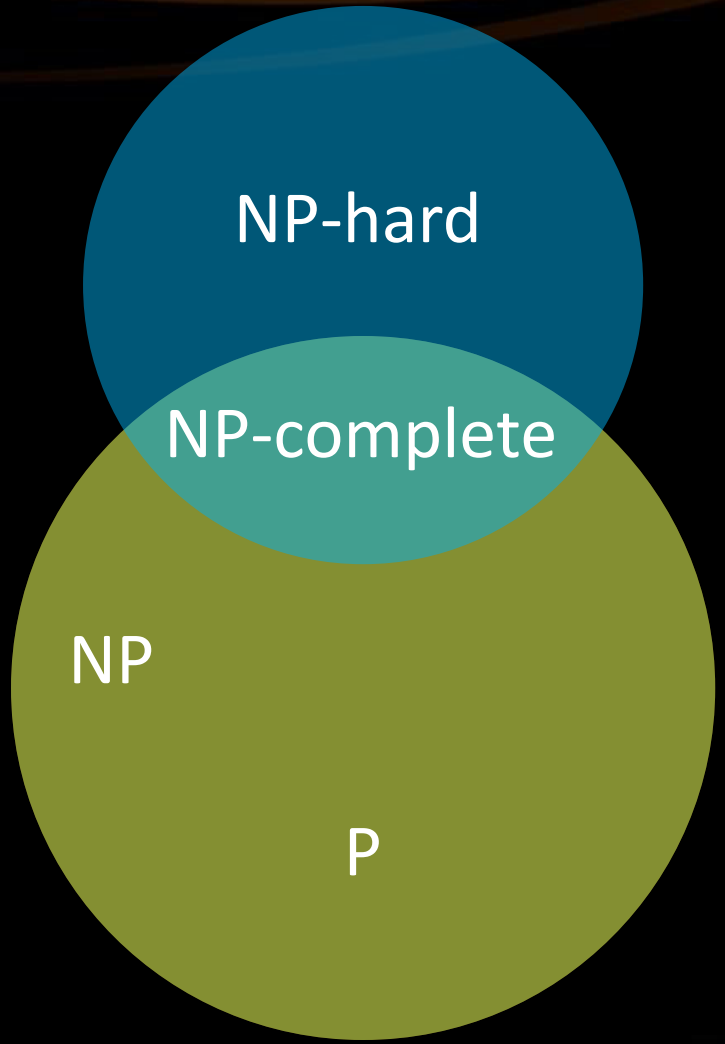
Traveling salesman

P vs NP

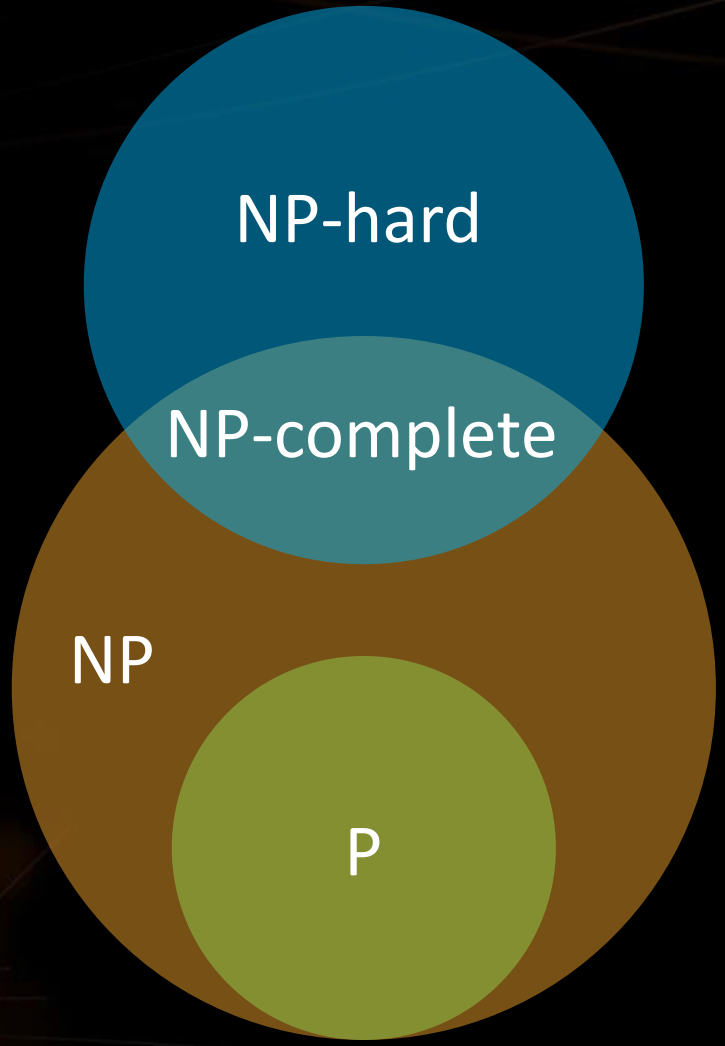


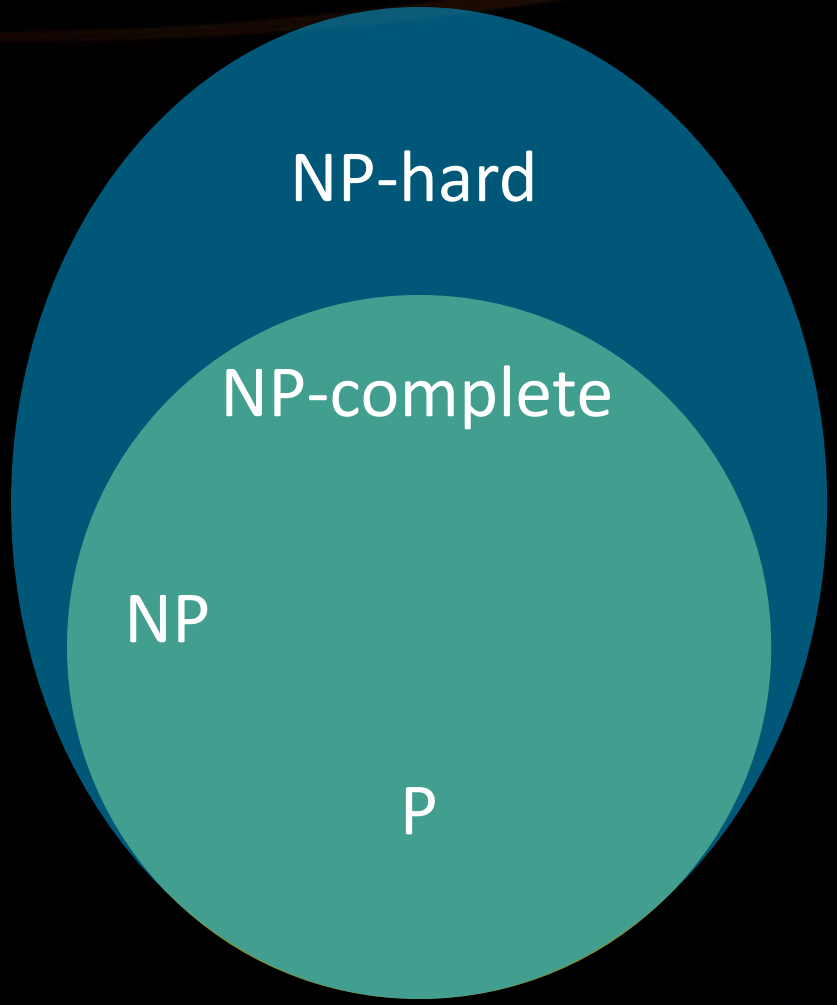
P vs NP





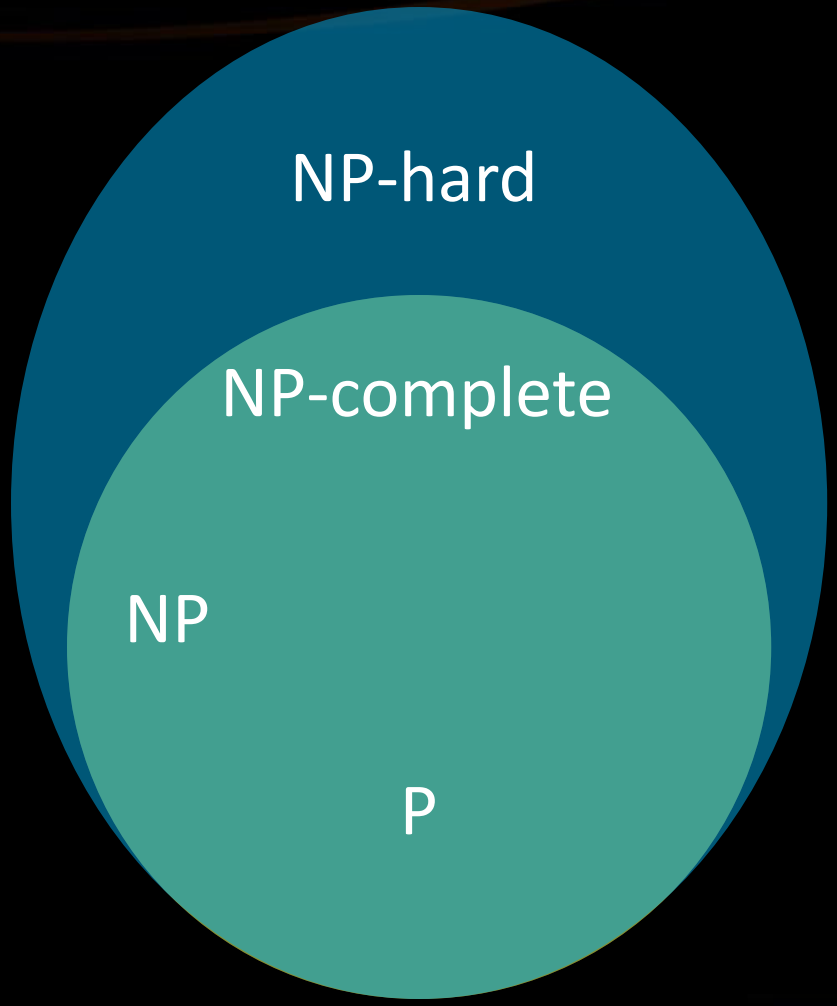
P vs NP





P vs NP

- Millennium Problem
  - Prize of $1,000,000
  - Prove either P=NP or P!=NP
  - Currently > 100 attempts



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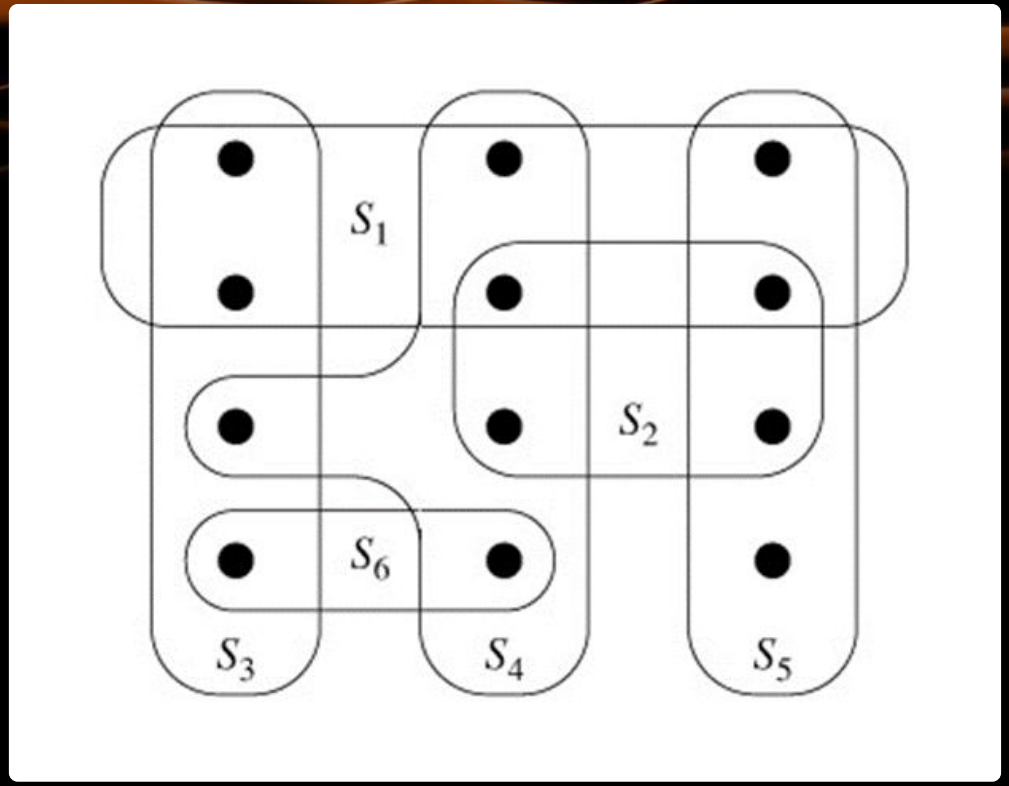

The Set Cover Problem

Using Greedy for Approximation

Set Cover Problem

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- The Set Cover Problem (SCP) is such a problem
- SCP formulation:
  - Given a set U={1,2,…,m} called "the Universe"
  - And a set S={{…}, {…}, {…}, …} of n sets whose union = U
  - Find the smallest subset of S, the union of which = U (if it exists)
- Example of SCP:
  - U = {1, 2, 3, 4, 5, 6}
  - S = { {1, 3}, {1}, {2, 4}, {2, 5}, {2, 6}, {2, 3, 6}, {4, 6}, {4, 5, 6}, {6} }

Set Cover Problem

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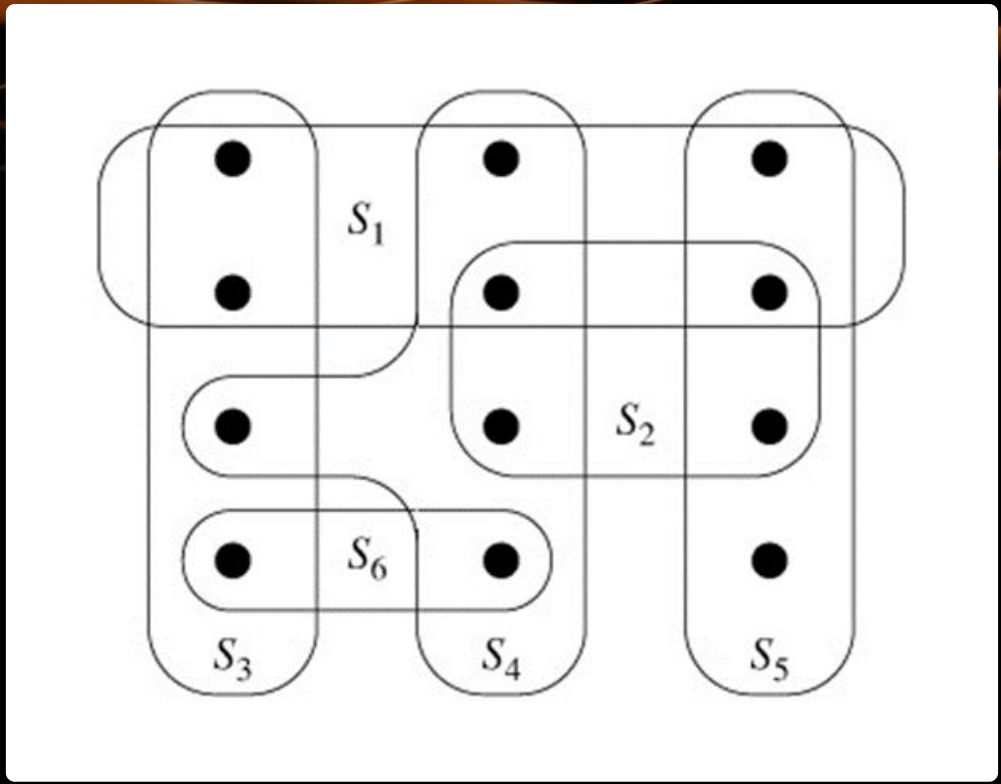
- The SCP turns out very complex!
  - The optimal solution is NP-complete
  - Infeasible for real-world calculations (unless P = NP)
- Good solutions can be achieved through a greedy approach:
  - At each step pick the set containing the largest number of uncovered elements, e. g.
    - U = {1, 2, 3, 4, 5, 6}
    - S = { {1, 3}, {1}, {2, 4}, {2, 5}, {2, 6}, {2, 3, 6}, {4, 6}, {4, 5, 6}, {6} }



The Set Cover Problem

Live Coding Exercise (Lab)

References & Further Reading

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- Recommended further reading on greedy algorithms:
  - Lecture @ Boston University (Shang-Hua Teng)
    - www.cs.bu.edu/~steng/teaching/Fall2003/lectures/lecture7.ppt
  - Lecture @ University of Pennsylvania
    - www.cis.upenn.edu/~matuszek/cit594-2005/Lectures/36-greedy.ppt
  - Book on Algorithms @ Berkeley University
    - www.cs.berkeley.edu/~vazirani/algorithms/chap5.pdf
  - Book "Programming = ++ Algorithms;" – Chapter 9
    - www.programirane.org

Summary

- Greedy algorithms make local optimal decision at each step and expect to achieve the global optimal solution
- Greedy algorithms give optimal solution for problems holding:
  - Greedy choice & optimal substructure
- Heuristic algorithms give approximation
  - Not always the optimal solution

Greedy Algorithms

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Questions?

https://softuni.bg/opencourses/algorithms

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