Advanced Graph Algorithms

Spanning Trees, Shortest Paths

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http://softuni.bg

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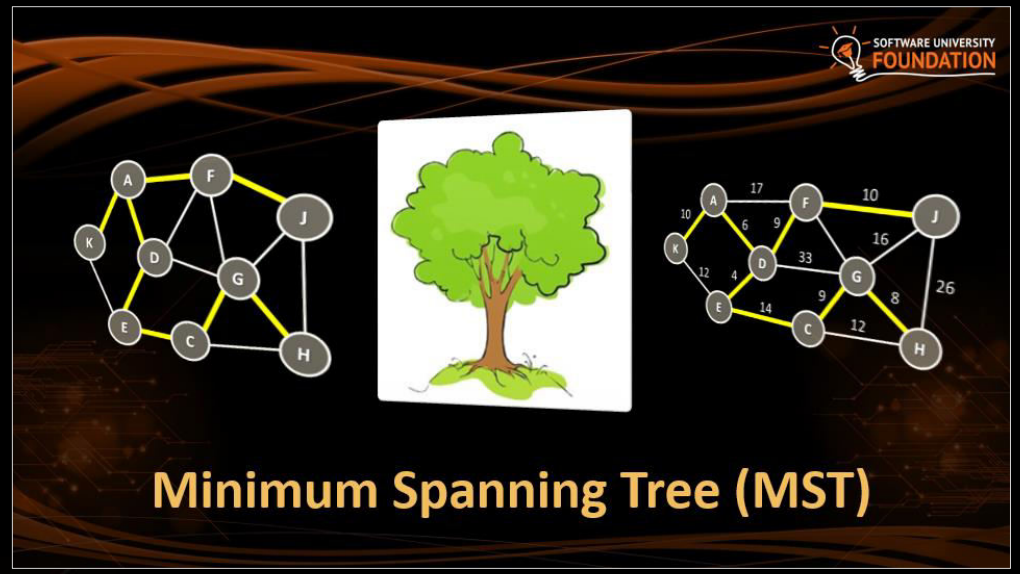

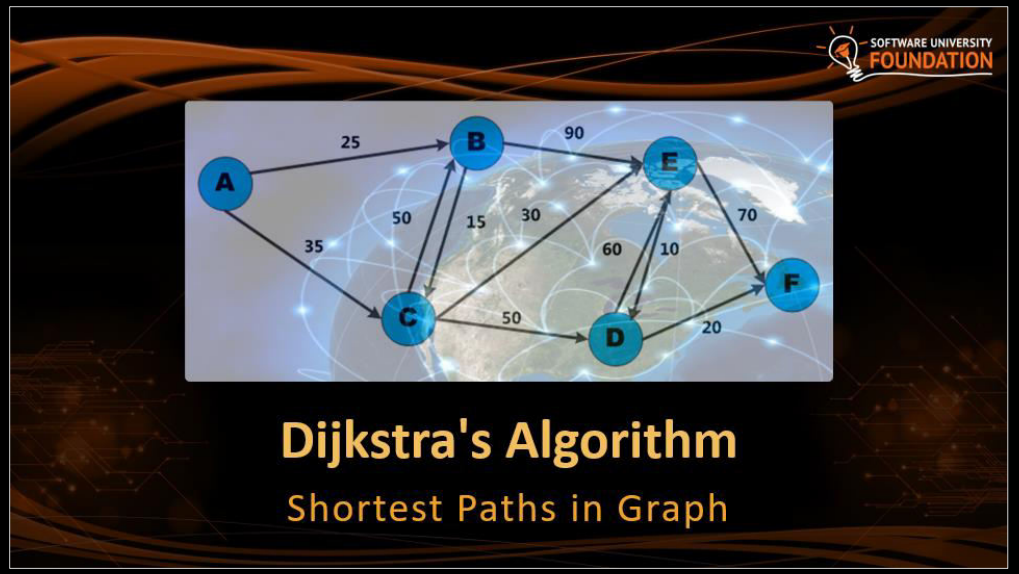

Have a Question?

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sli.do

#DsAlgo

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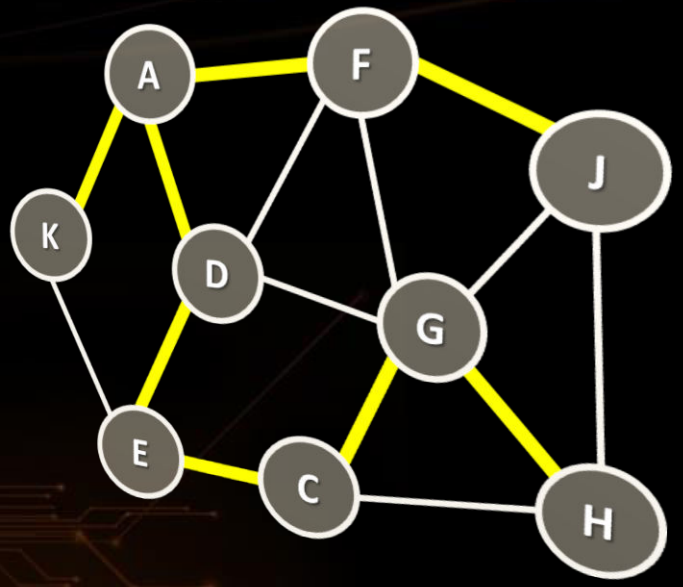
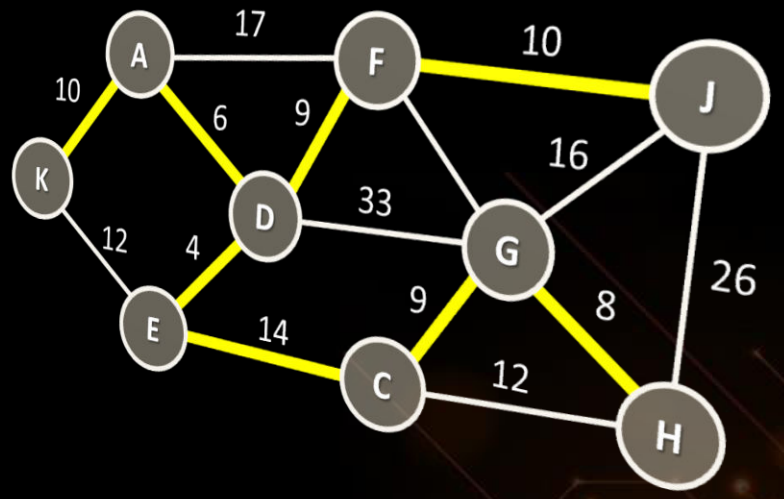
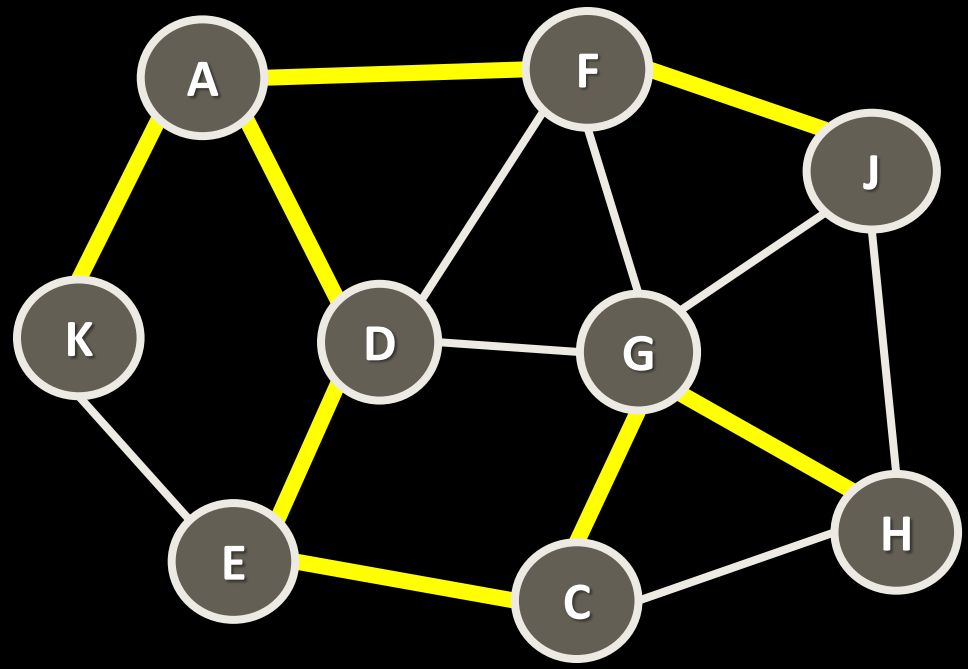




Minimum Spanning Tree (MST)

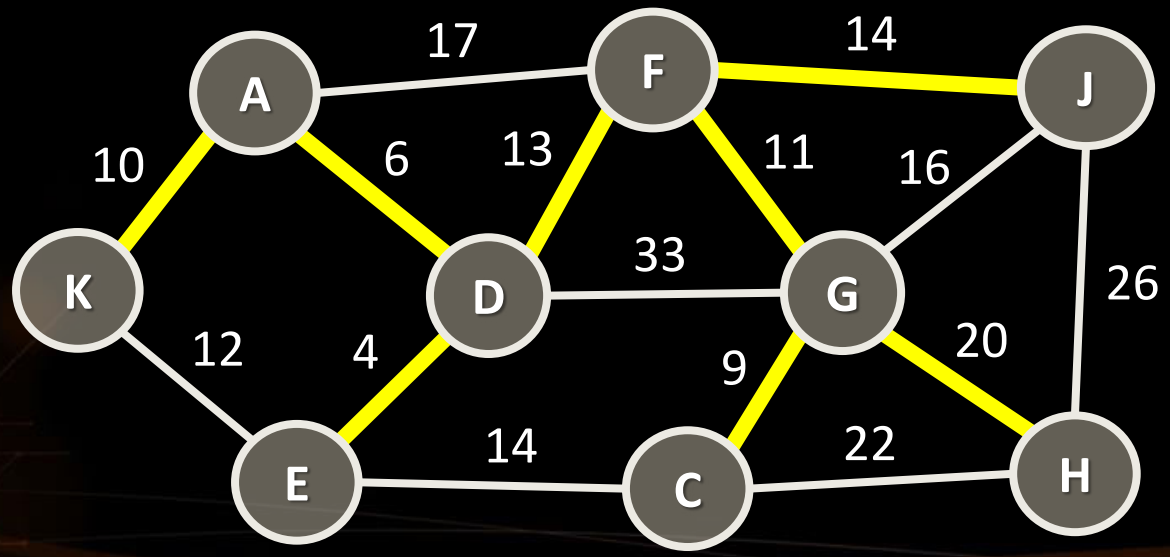
Spanning Tree

- Spanning tree
  - Subgraph without cycles (tree)
  - Connects all vertices together
- All connected graphs have a spanning tree
- All undirected graphs have spanning forest



Minimum Spanning Tree (MST)

- Minimum spanning tree (MST)
  - Weight <= weight(all other spanning trees)
- First used in electrical networks
  - Minimal cost of wiring



Minimum Spanning Forest (MSF)

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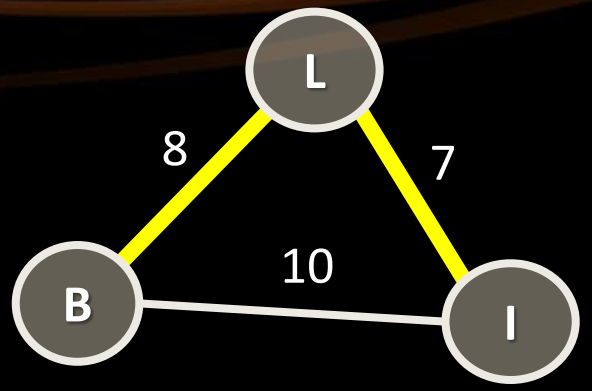
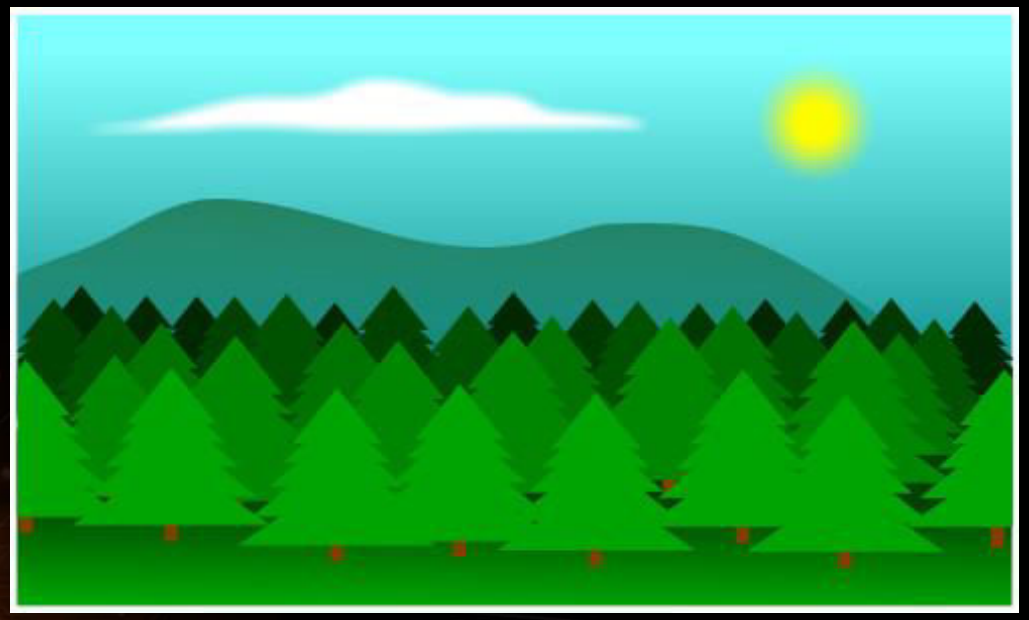
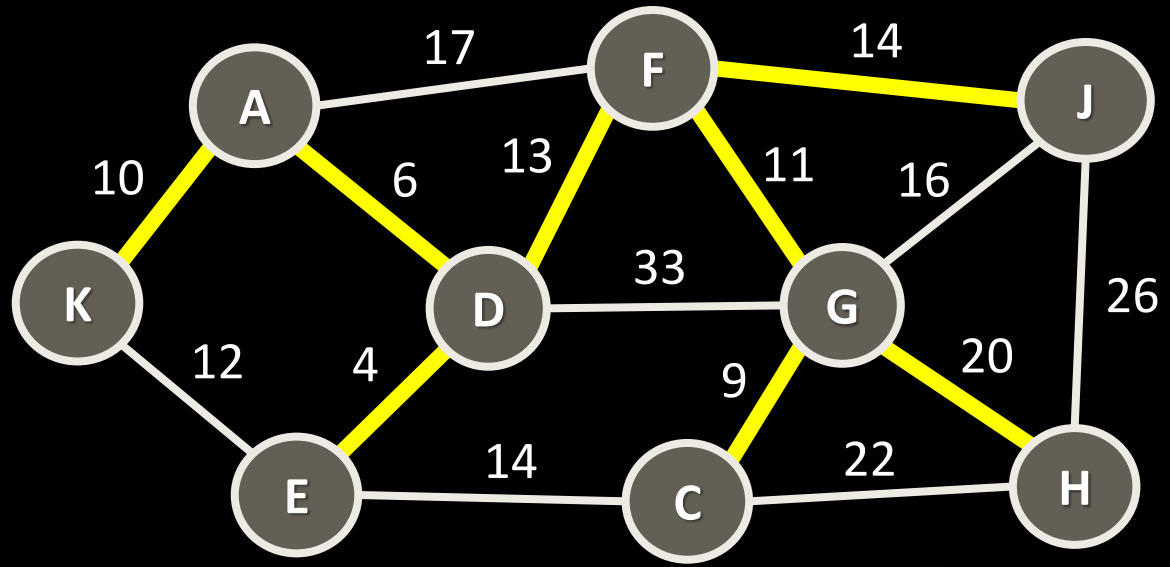
- Minimum spanning forest
- Set of all minimum spanning trees (when the graph is not connected)







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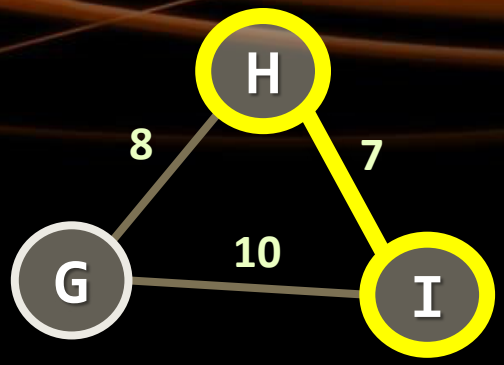
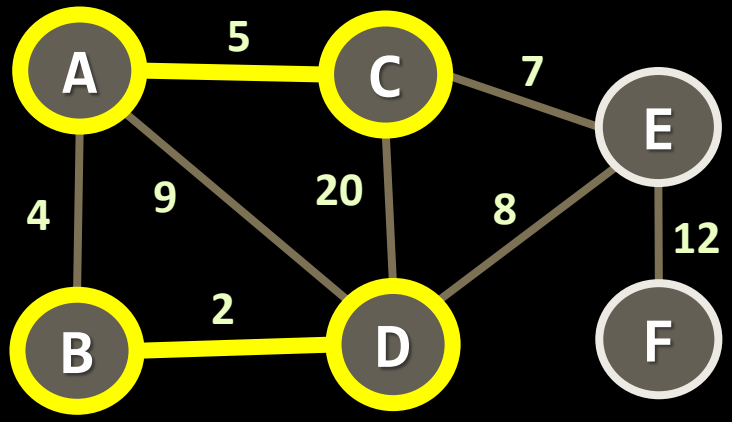
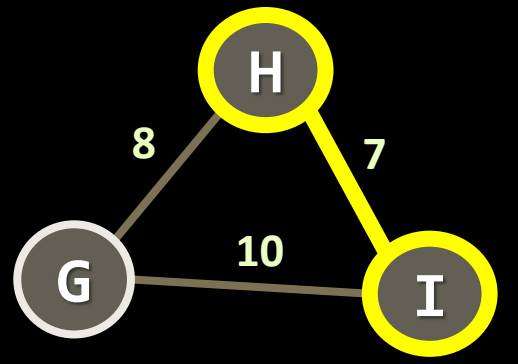


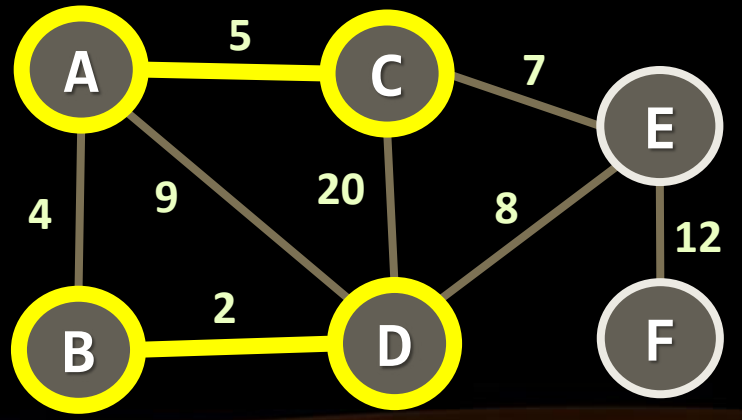


Kruskal's Algorithm

Kruskal's Algorithm

- Create a forest **F** holding all graph vertices and no edges
- Create a set **S** holding all edges in the graph
- While **S** is non-empty
  - Remove the edge ***e*** with min weight from **S**
  - If ***e*** connects two different trees
    - Add ***e*** to the forest
    - Join these two trees into a single tree
- The graph may not be connected





Kruskal's Algorithm – Step #1

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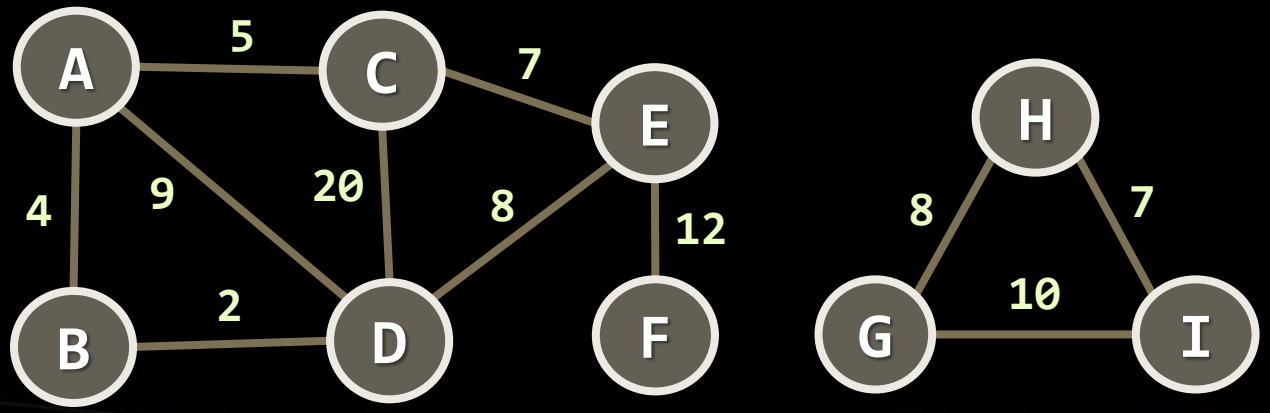
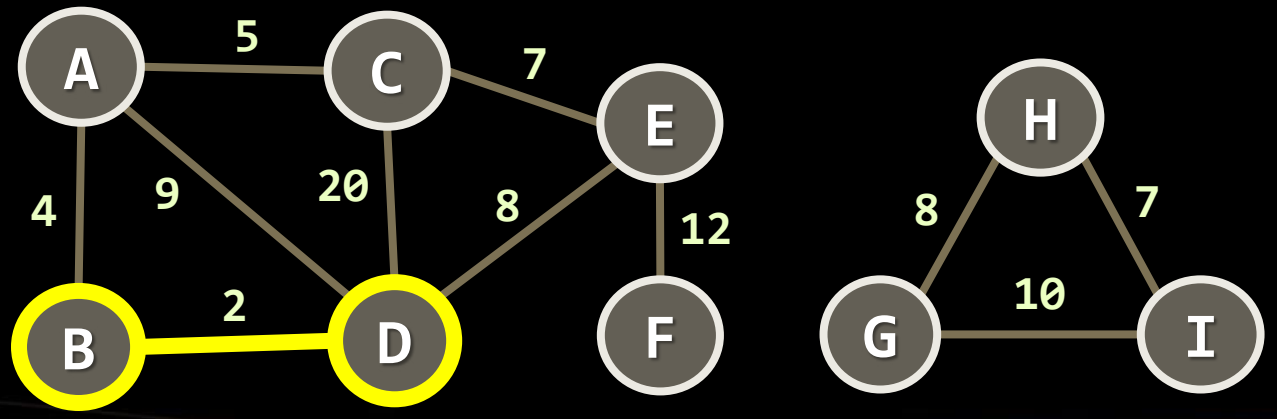
- Start from forest holding all vertices and no edges
- **S** = all edges, ordered by weight
- **F** = { }
- **S** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



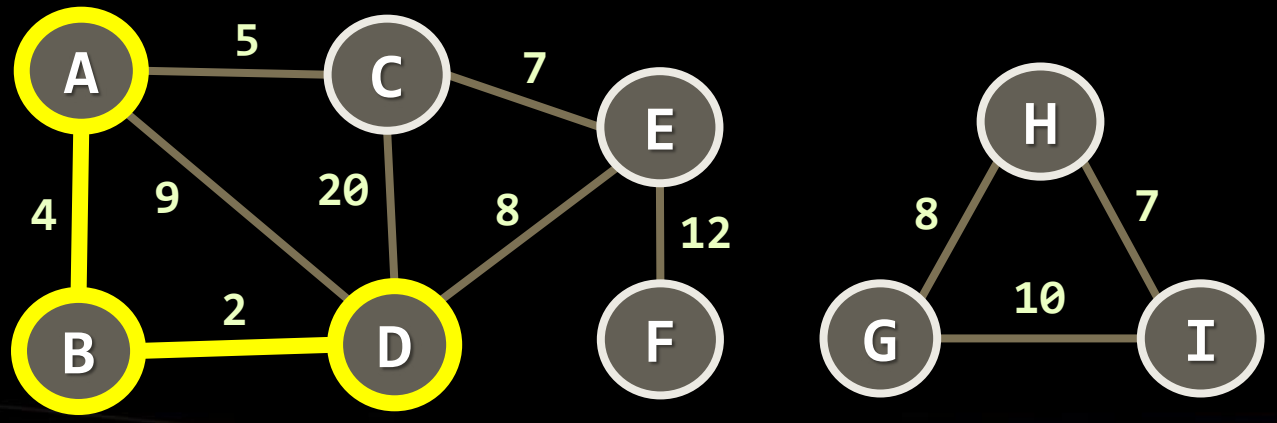
Kruskal's Algorithm – Step #2

- Take the smallest edge **BD** = 2
  - The edge **BD** connects different trees → add it to the forest
- **F** = {**BD**=2}
- **S** = {**AB**=4, **AC**=5, **CE**=7, **HI**=7, **DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



Kruskal's Algorithm – Step #3

- Take the smallest edge **AB** = 4
  - The edge **AB** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4}
- **S** = {**AC**=5, **CE**=7, **HI**=7, **DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



Kruskal's Algorithm – Step #4

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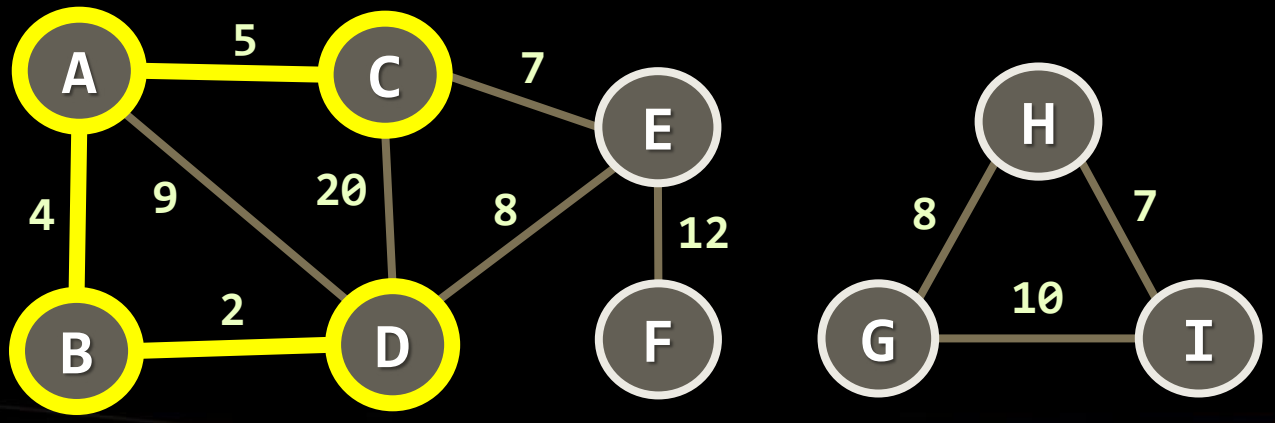
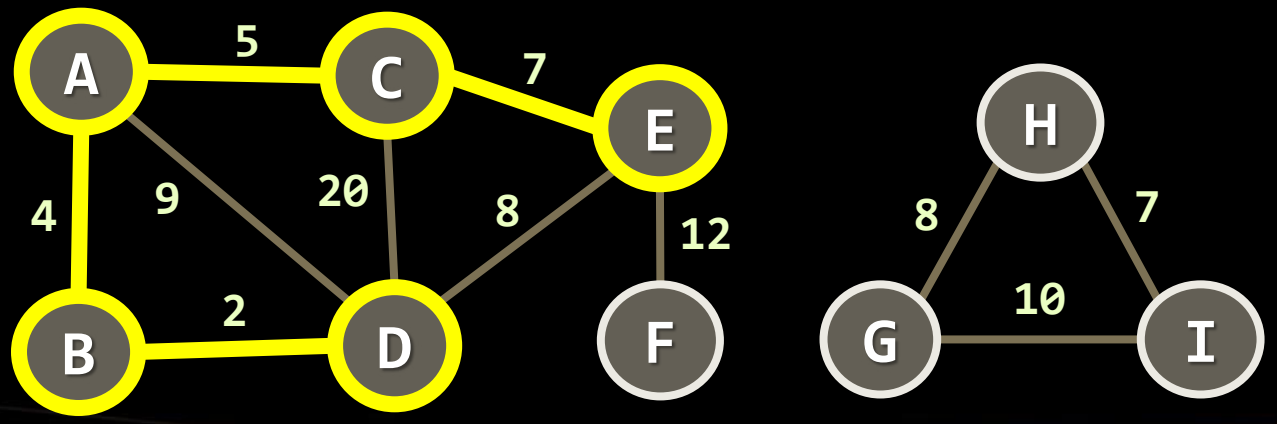
- Take the smallest edge **AC** = 5
  - The edge **AC** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4, **AC**=5}
- **S** = {**CE**=7, **HI**=7, **DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



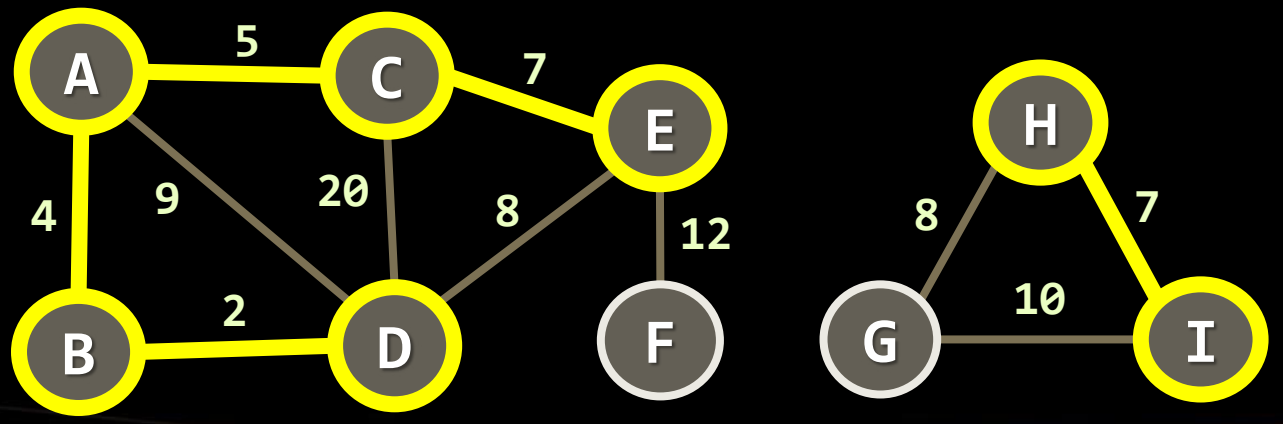
Kruskal's Algorithm – Step #5

- Take the smallest edge **CE** = 7
  - The edge **CE** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7}
- **S** = {**HI**=7, **DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



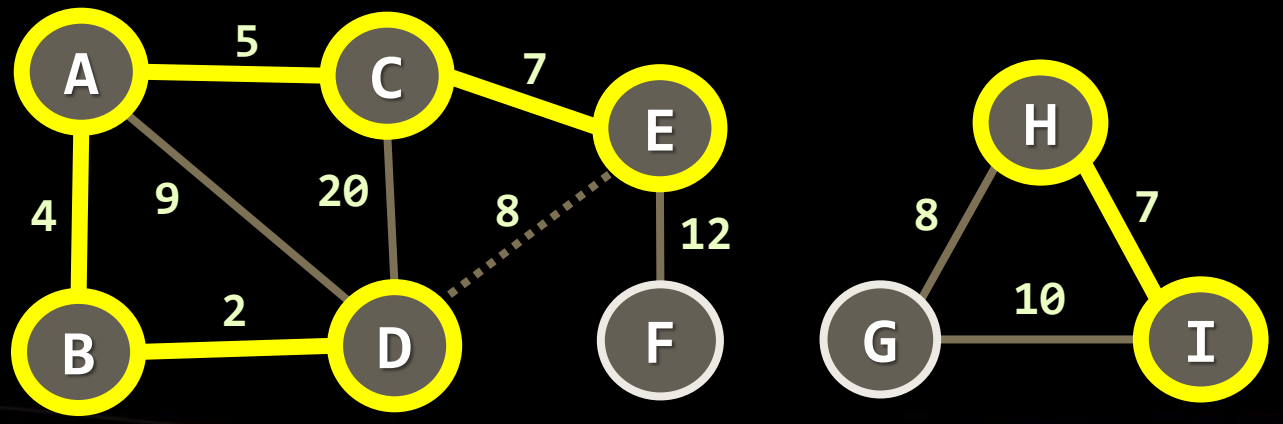
Kruskal's Algorithm – Step #6

- Take the smallest edge **HI** = 7
  - The edge **CE** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7}
- **S** = {**DE**=8, **GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



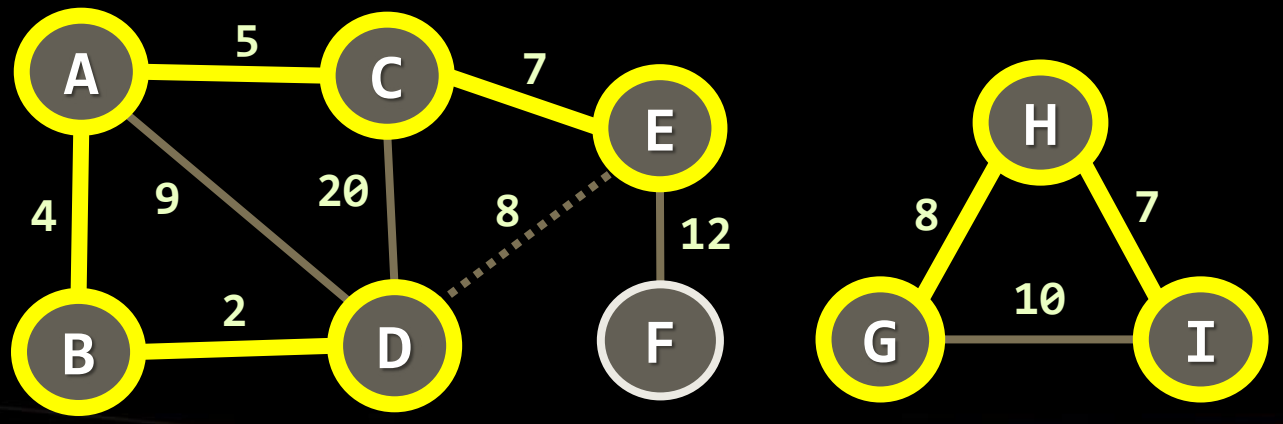
Kruskal's Algorithm – Step #7

- Take the smallest edge **DE** = 8
  - The edge **DE** causes a cycle (connects the same tree) → skip it
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7}
- **S** = {**GH**=8, **AD**=9, **GI**=10, **EF**=12, **CD**=20}



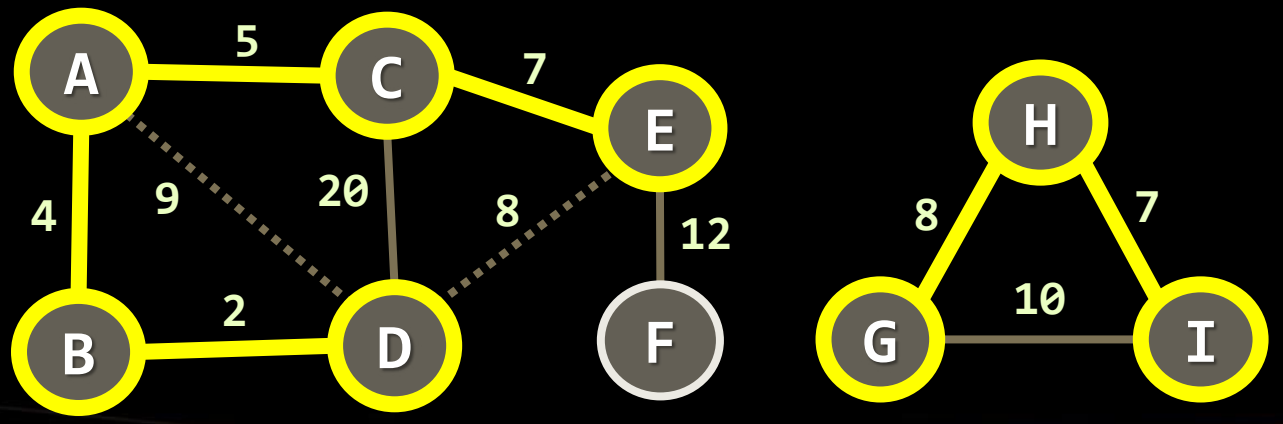
Kruskal's Algorithm – Step #8

- Take the smallest edge **GH** = 8
  - The edge **GH** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **GH**=7}
- **S** = {**AD**=9, **GI**=10, **EF**=12, **CD**=20}



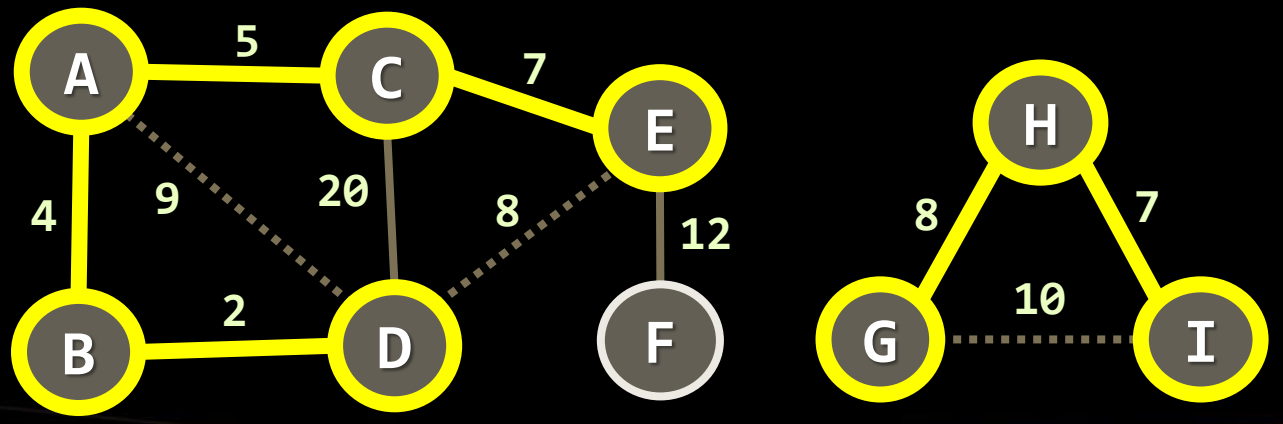
Kruskal's Algorithm – Step #9

- Take the smallest edge **AD** = 9
  - The edge **AD** causes a cycle (connects the same tree) → skip it
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **GH**=7}
- **S** = {**GI**=10, **EF**=12, **CD**=20}



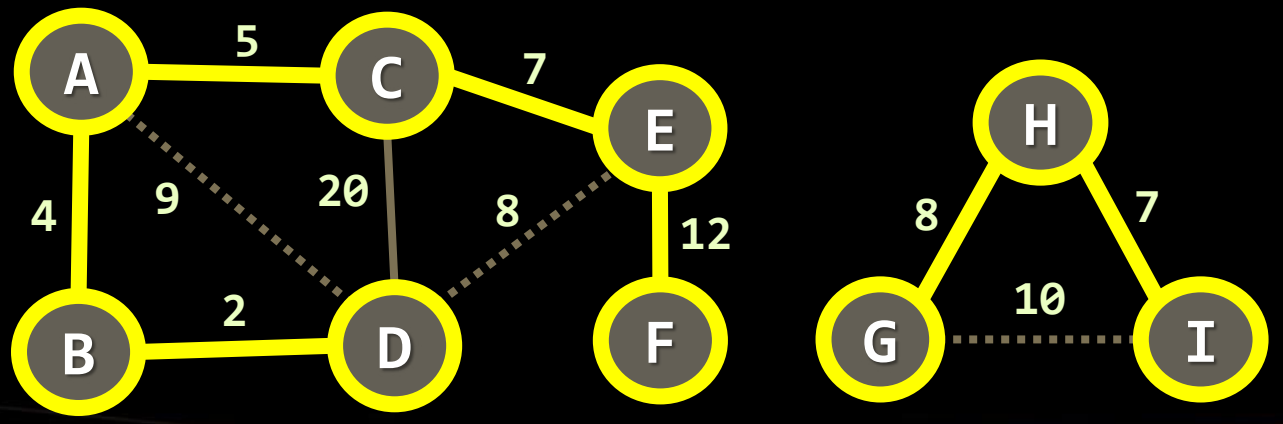
Kruskal's Algorithm – Step #10

- Take the smallest edge **GI** = 10
  - The edge **GI** causes a cycle (connects the same tree) → skip it
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **GH**=7}
- **S** = {**EF**=12, **CD**=20}



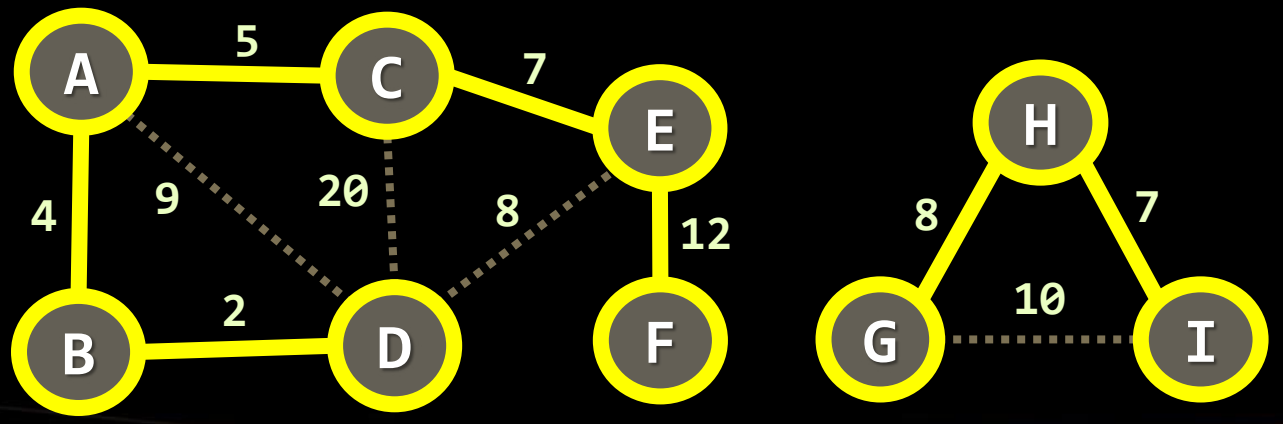
Kruskal's Algorithm – Step #11

- Take the smallest edge **EF** = 12
  - The edge **EF** connects different trees → add it to the forest
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **GH**=7, **EF**=12}
- **S** = {**CD**=20}



Kruskal's Algorithm – Step #12

- Take the smallest edge **CD** = 20
  - The edge **CD** causes a cycle (connects the same tree) → skip it
- **F** = {**BD**=2, **AB**=4, **AC**=5, **CE**=7, **HI**=7, **GH**=7, **EF**=12}
- **S** = { } → stop the algorithm



Kruskal's Algorithm – Pseudo Code

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```
foreach v ∈ graph edges
  parent[v] = v

foreach edge {u, v} ordered by weight(u, v)
  var rootU = FindRoot(u)
  var rootV = FindRoot[v]
  if rootU ≠ rootV
    print edge {u, v}
    parent[rootU] = rootV

FindRoot(node)
  while (parent[node] ≠ node)
    node = parent[node]
  return node
```

Time complexity: $O(|E| * \log^* |E|)$

See http://visualgo.net/ufds.html

Disjoint Sets Optimization

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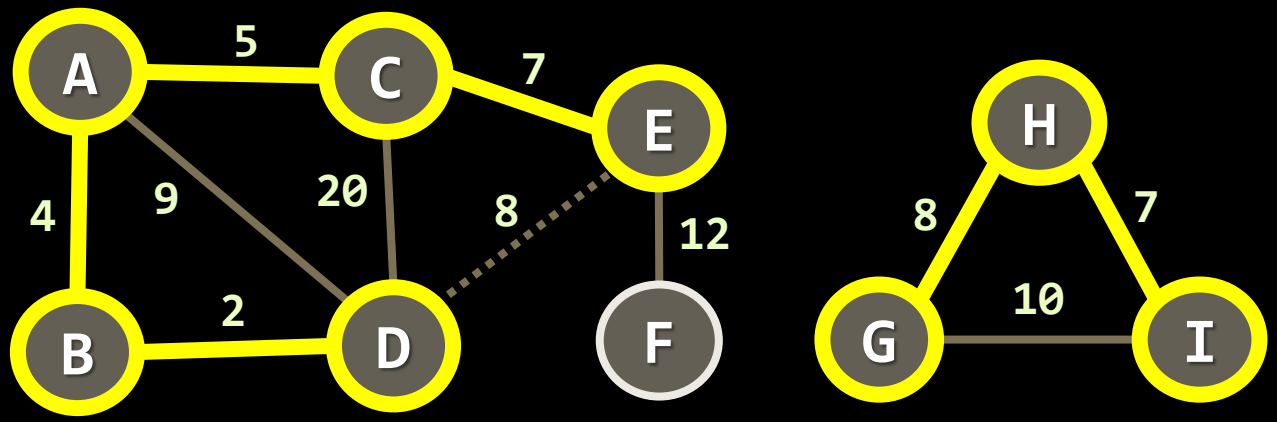
```
FindRoot(node)
  var root = node
  while (parent[root] ≠ root)
    root = parent[root]

  // Optimize the path to root
  // Attach each path node directly to the root
  while (node != root)
    var oldParent = parent[node]
    parent[node] = root
    node = oldParent

  return root
```



Kruskal's Algorithm

Live Demo

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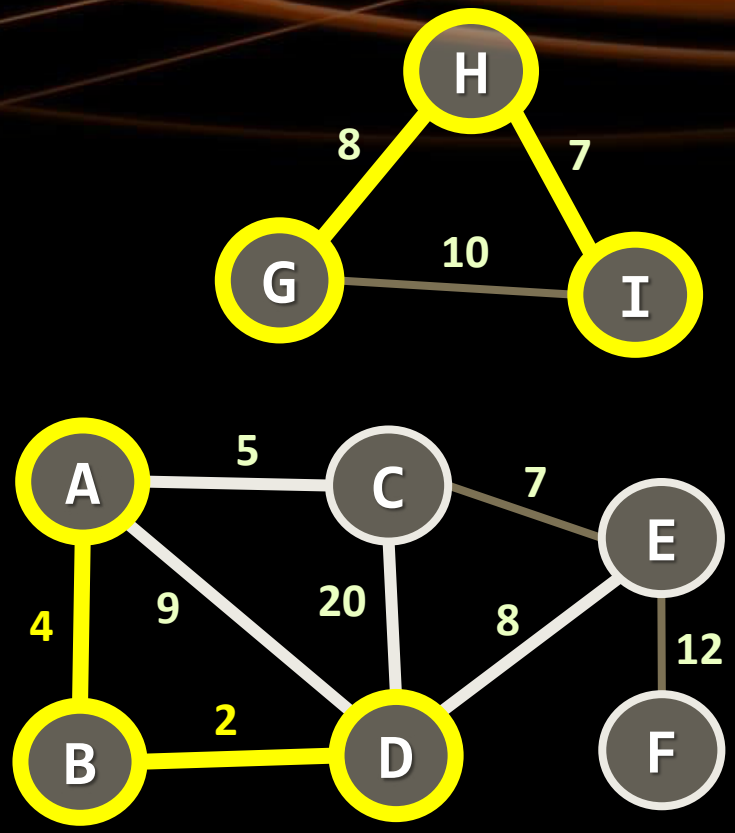


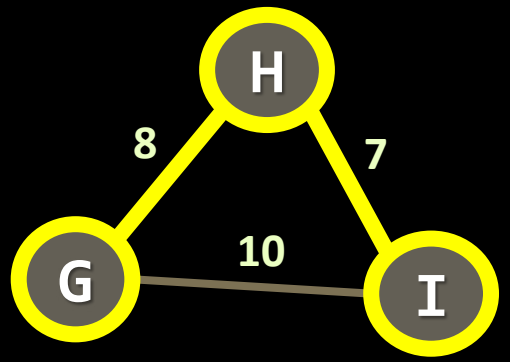
Prim's Algorithm

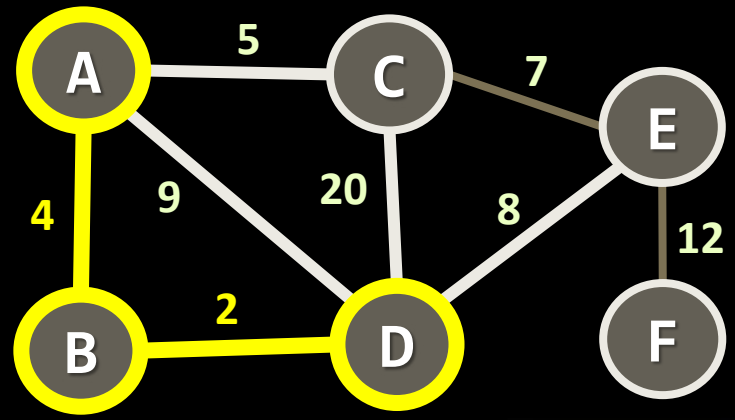
Prim's Algorithm

- Given a graph **G(V, E)** find the minimum spanning forest **T(V', E')**

1. Attach to the tree **T** the starting node
2. While smallest edge exists
   - Attach to **T** the smallest possible edge from **G** without creating a cycle in **T**
     - Use the smallest edge **(u, v)**, such that **u** ∈ **T** and **v** ∉ **T**

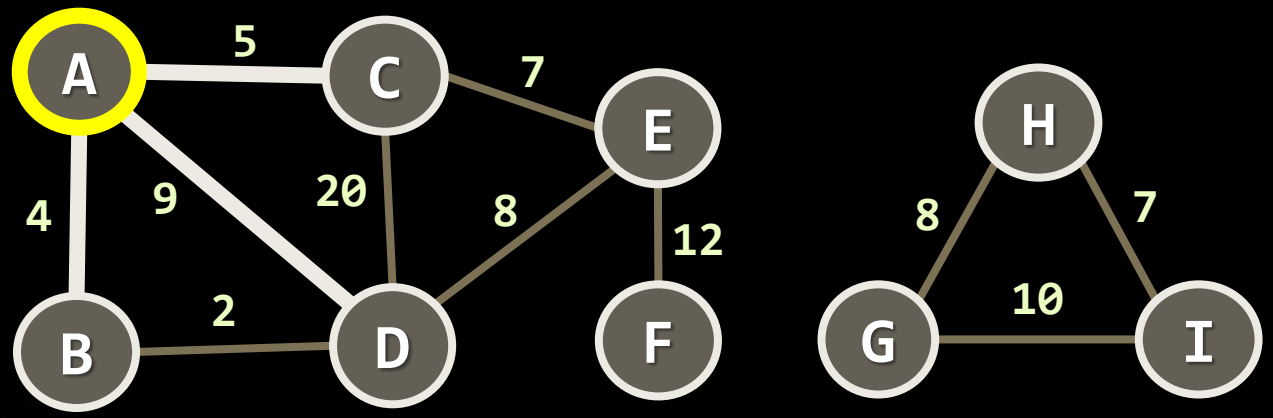
- Start the Prim's algorithm many times from all nodes from **G** still not in **T**





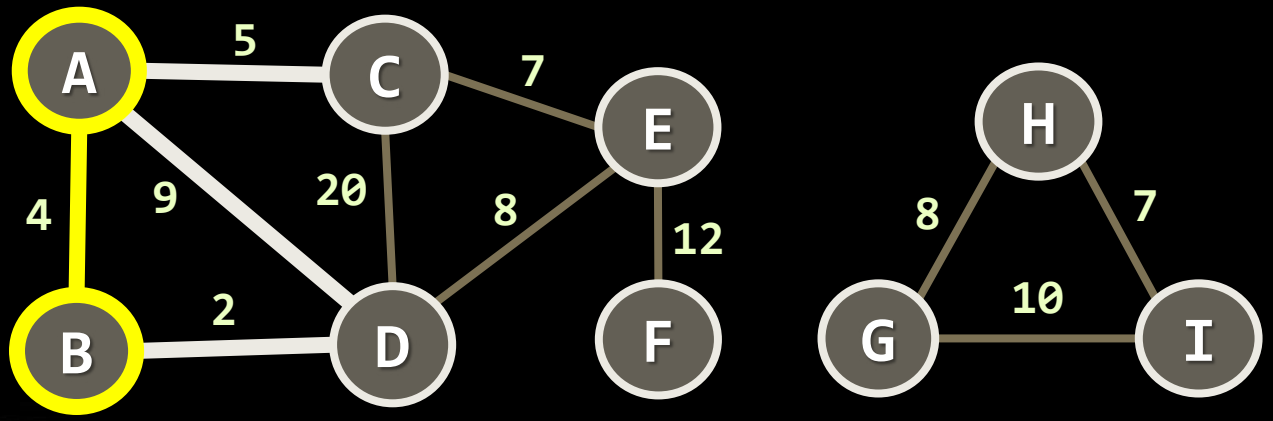
Prim's Algorithm – Step #1

- Start from the initial node **A**
- Enqueue all edges from **A** to other graph nodes: **AB**, **AC**, **AD**
- Spanning tree = {**A**}
- Priority queue = {**AB** = 4}, {**AC** = 5}, {**AD** = 9}



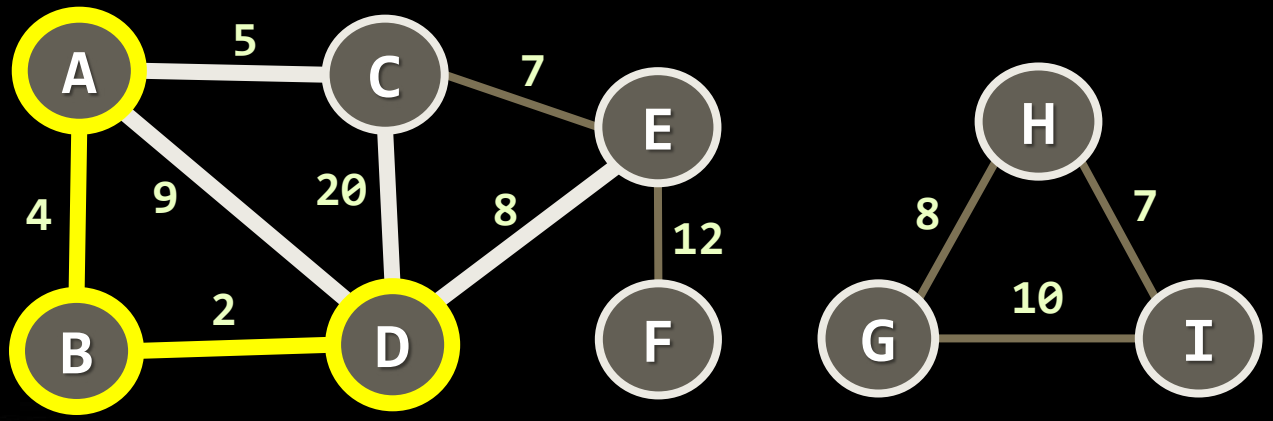
Prim's Algorithm – Step #2

- Dequeue the shortest edge {**AB** = 4} and add it to the tree
- Enqueue all edges from **B** to other graph nodes: **BD**
- Spanning tree = {**AB** = 4}
- Priority queue = {**BD** = 2}, {**AC** = 5}, {**AD** = 9}



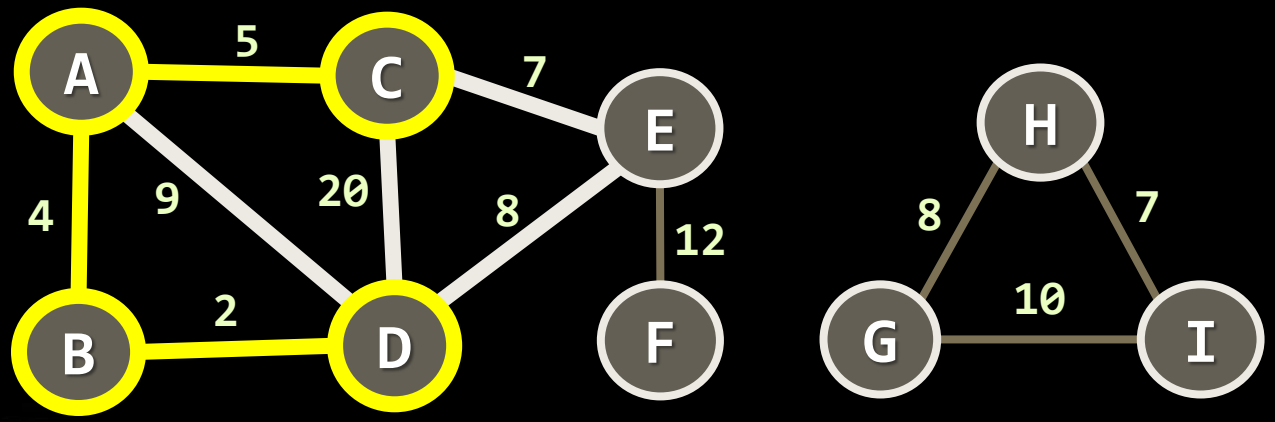
Prim's Algorithm – Step #3

- Dequeue the shortest edge {**BD** = 2} and add it to the tree
- Enqueue all edges from **D** to other graph nodes: **DC**, **DE**
- Spanning tree = {**AB** = 4}, {**BD** = 2}
- Priority queue = {**AC** = 5}, {**DE** = 8}, {**AD** = 9}, {**CD** = 20}



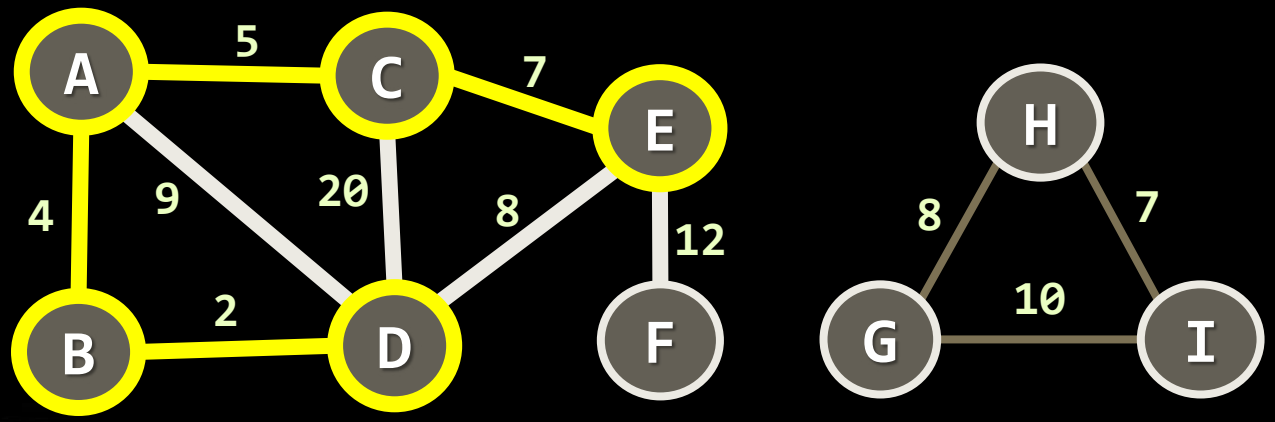
Prim's Algorithm – Step #4

- Dequeue the shortest edge {**AC** = 5} and add it to the tree
- Enqueue all edges from **C** to other graph nodes: **CE**
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}
- Priority queue = {**CE** = 7}, {**DE** = 8}, {**AD** = 9}, {**CD** = 20}



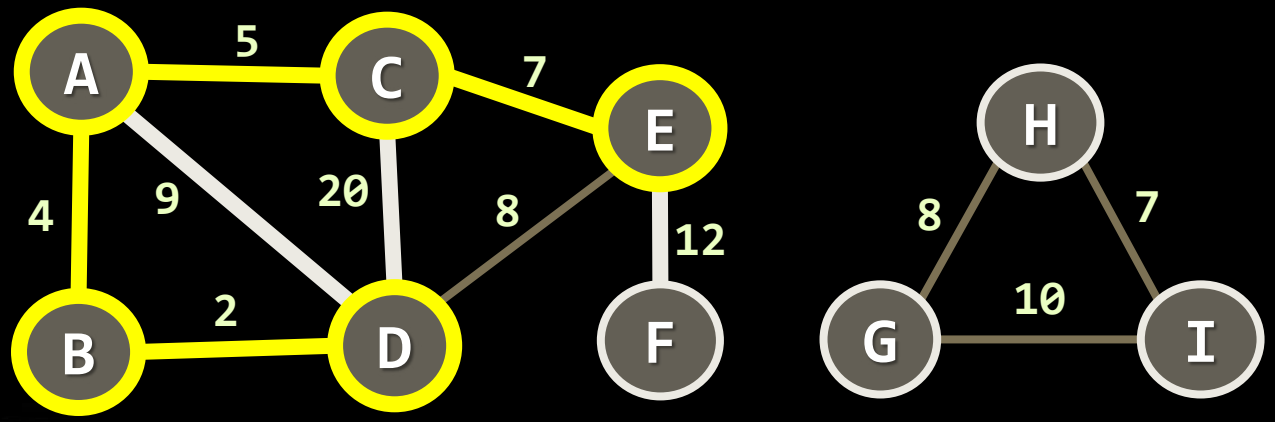
Prim's Algorithm – Step #5

- Dequeue the shortest edge {**CE** = 7} and add it to the tree
- Enqueue all edges from **E** to other graph nodes: **EF**
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}, {**CE** = 7}
- Priority queue = {**DE** = 8}, {**AD** = 9}, {**EF** = 12}, {**CD** = 20}



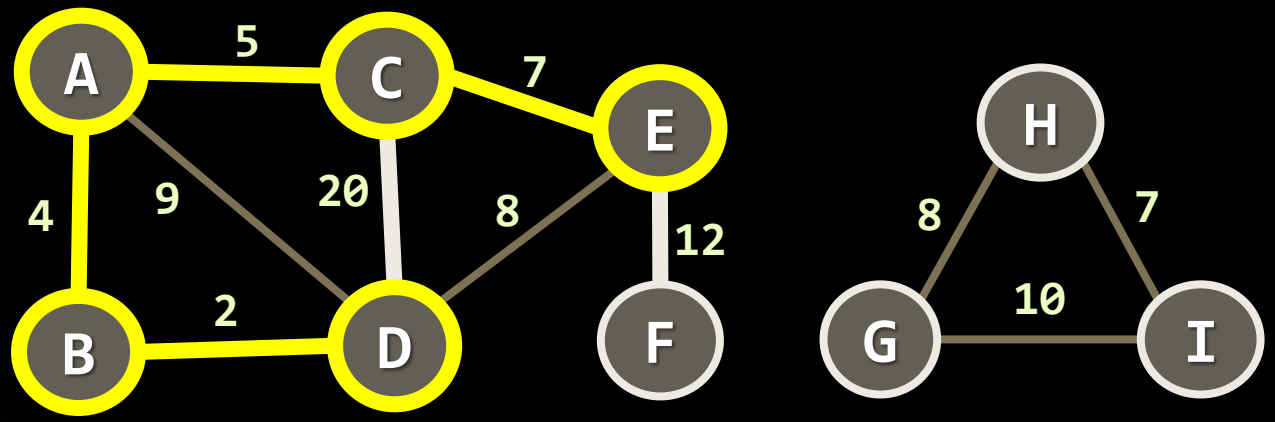
Prim's Algorithm – Step #6

- Dequeue the shortest edge {**DE** = 8}
  - It would create a loop in the spanning tree → skip it
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}, {**CE** = 7}
- Priority queue = {**AD** = 9}, {**EF** = 12}, {**CD** = 20}



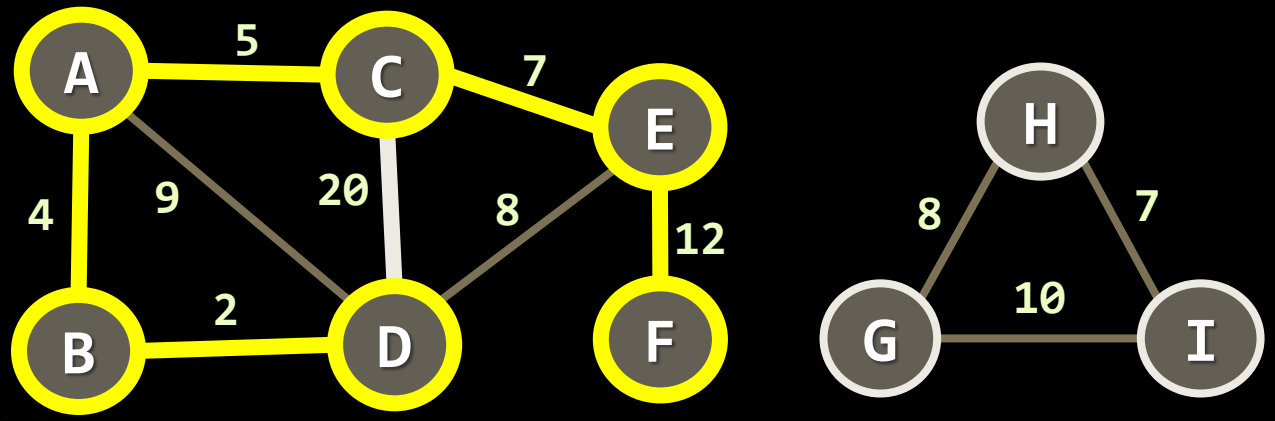
Prim's Algorithm – Step #7

- Dequeue the shortest edge {**AD** = 9}
  - It would create a loop in the spanning tree → skip it
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}, {**CE** = 7}
- Priority queue = {**EF** = 12}, {**CD** = 20}



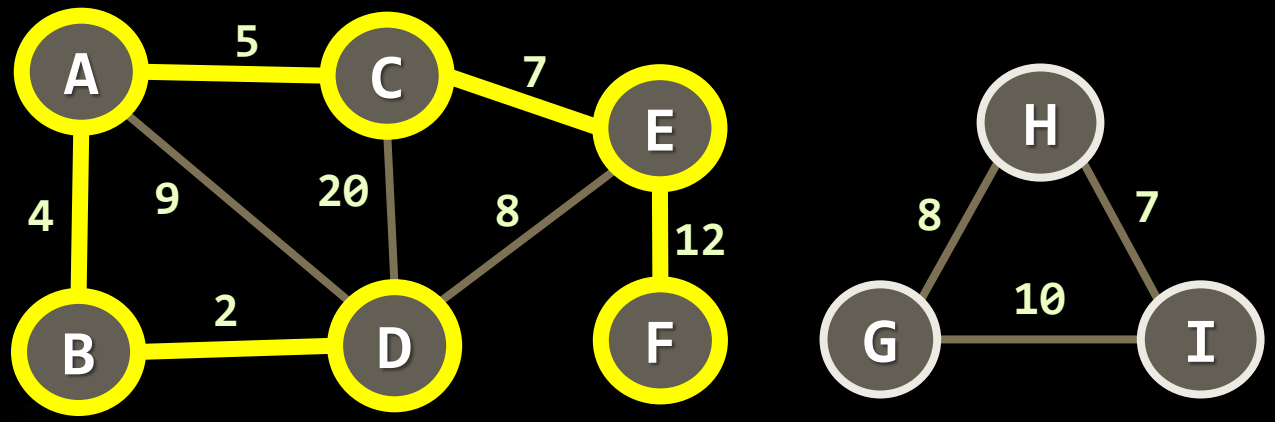
Prim's Algorithm – Step #8

- Dequeue the shortest edge {**EF** = 12} and add it to the tree
- Enqueue all edges from **F** to other graph nodes: no such edges
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}, {**CE** = 7}, {**EF** = 12}
- Priority queue = {**CD** = 20}



Prim's Algorithm – Step #9

- Dequeue the shortest edge {**CD** = 20}
  - It would create a loop in the spanning tree → skip it
- Spanning tree = {**AB** = 4}, {**BD** = 2}, {**AC** = 5}, {**CE** = 7}, {**EF** = 12}
- Priority queue = { } → stop the algorithm



Prim's Algorithm – Step #10

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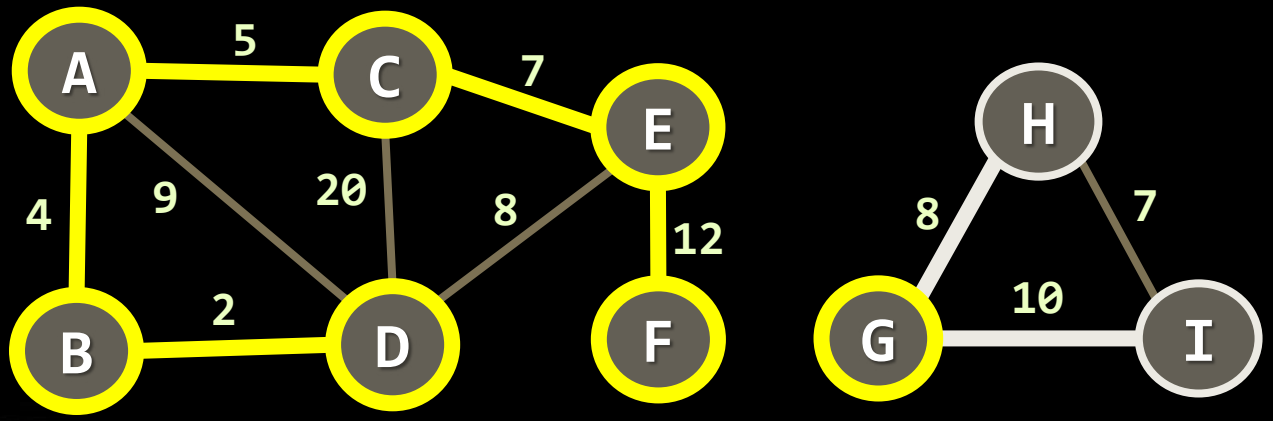
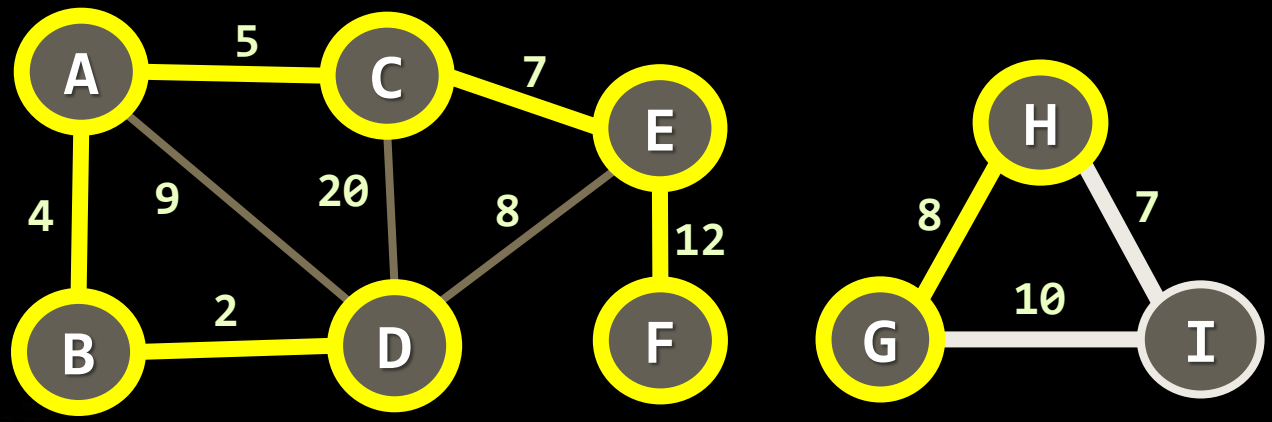
- Start from the initial node **G**
- Enqueue all edges from **G** to other graph nodes: **GH**, **GI**
- Spanning tree = {**AB**, **BD**, **AC**, **CE**, **EF**}, {**G**}
- Priority queue = {**GH** = 8}, {**GI** = 10}



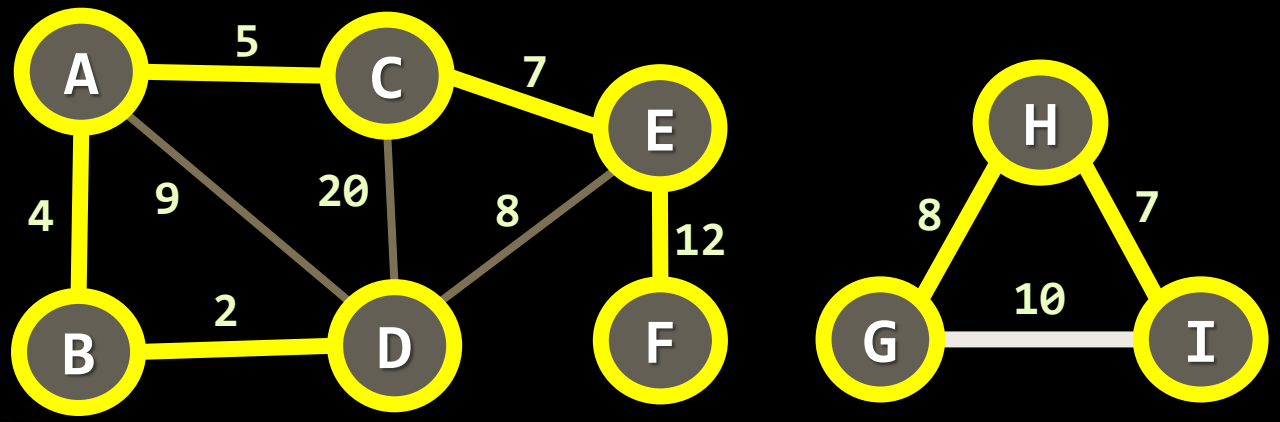
Prim's Algorithm – Step #11

- Dequeue the shortest edge {**GH** = 8} and add it to the tree
- Enqueue all edges from **H** to other graph nodes: **HI**
- Spanning tree = {**AB**, **BD**, **AC**, **CE**, **EF**}, {**GH** = 8}
- Priority queue = {**HI** = 7}, {**GI** = 10}



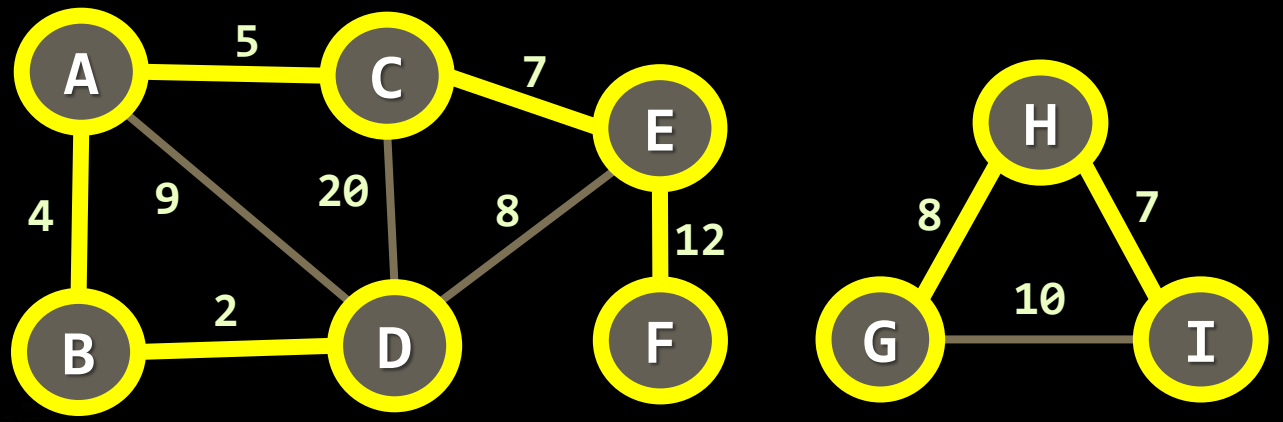
Prim's Algorithm – Step #12

- Dequeue the shortest edge {**HI** = 7} and add it to the tree
- Enqueue all edges from **I** to other graph nodes: no such edges
- Spanning tree = {**AB**, **BD**, **AC**, **CE**, **EF**}, {**GH** = 8}, {**HI** = 7}
- Priority queue = {**GI** = 10}



Prim's Algorithm – Step #13

- Dequeue the shortest edge {**GI** = 7} and add it to the tree
  - It would create a loop in the spanning tree → skip it
- Spanning tree = {**AB**, **BD**, **AC**, **CE**, **EF**}, {**GH** = 8}, {**HI** = 7}
- Priority queue = { } → stop the algorithm



Prim's Algorithm (with Priority Queue)

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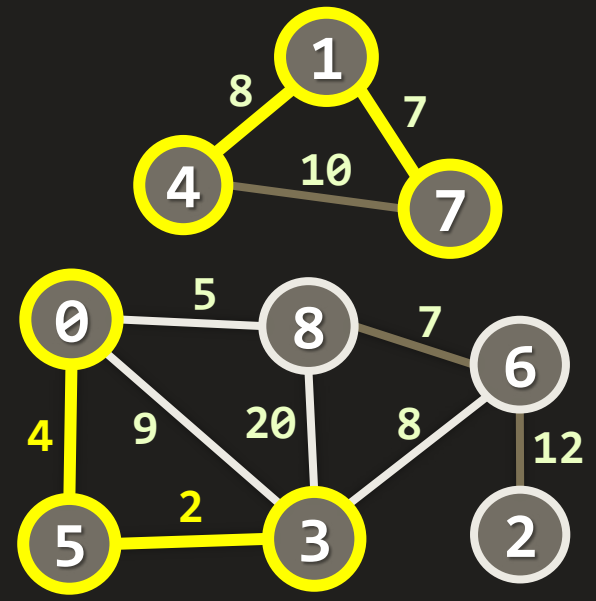
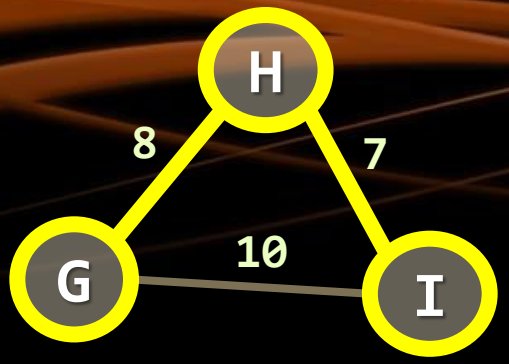
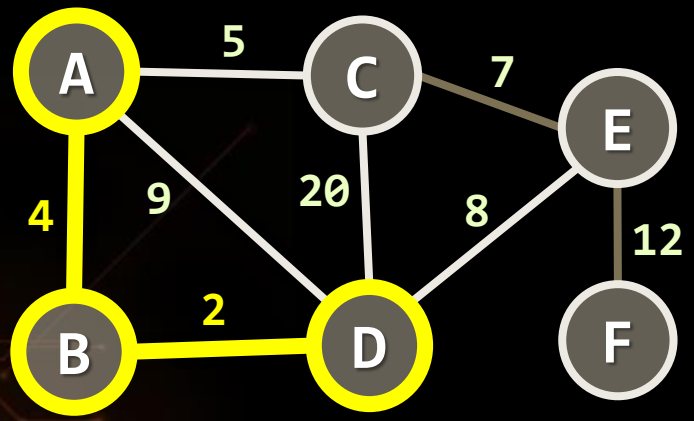
```
var spanningTreeNodes = ∅
foreach (v ∈ graphVertices)
  if (v ∉ spanningTreeNodes)
    Prim(v)

Prim(startNode)
  spanningTreeNodes ← startNode
  var priorityQueue = ∅
  priorityQueue ← childEdges(startNode)
  while (priorityQueue is not empty)
    smallestEdge = priorityQueue.ExtractMin()
    if (smallestEdge connects tree node with non-tree node)
      print smallestEdge
      spanningTreeNodes ← smallestEdge.nonTreeNode
      priorityQueue ← childEdges(smallestEdge.nonTreeNode)
```

Time complexity: $O(|E| * \log |E|)$








```
Minimum spanning forest weight: 45
(C A) -> 5
(B A) -> 4
(D B) -> 2
(C E) -> 7
(E F) -> 12
(I H) -> 7
(G H) -> 8
```

Prim's Algorithm with Priority Queue

Live Demo

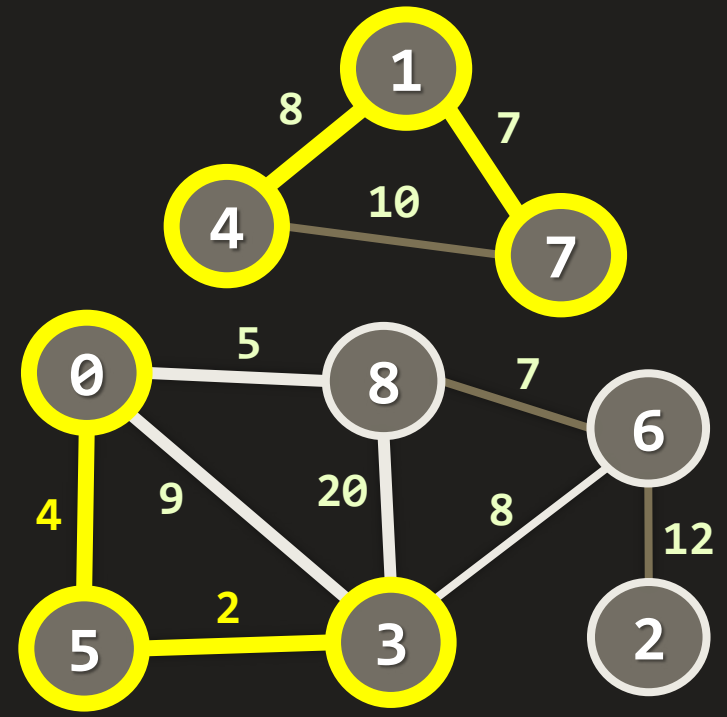
Prim's Algorithm – Pseudo Code

```
var usedNodes = ∅;
for (v = 0…n-1)
  if (not usedNodes[v])
    Prim(v);

Prim(startNode)
  usedNodes ← startNode
  var edgeNode[0 … n-1]
  var nearest[0 … n-1]

  // Initially compute nearest[0…n-1]
  for (i = 0 … n-1)
    nearest[i] = weight[startNode → i]
      or infinity when no edge exists {startNode → i}
    edgeNode[i] = startNode
```

Time complexity: $O(|V|^2)$



Prim's Algorithm – Pseudo Code (2)

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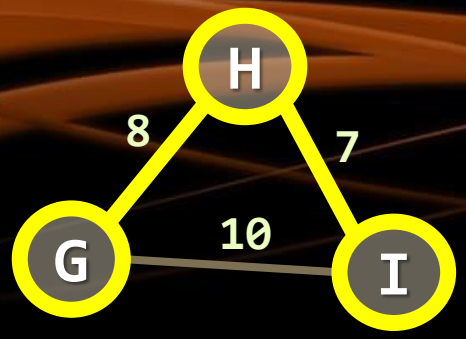
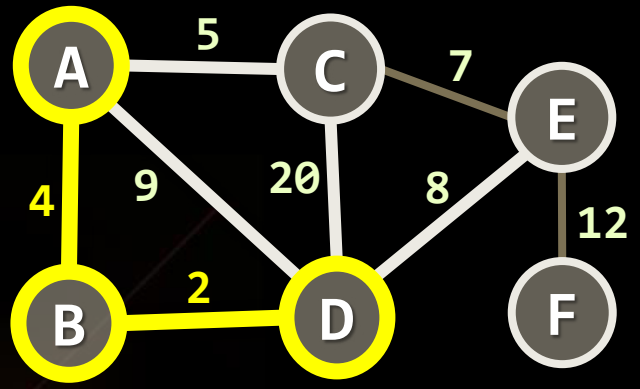
```
while (true)
  nearestNode = node i such that
    i ∉ usedNodes and nearest[i] is the minimum possible
  if (nearestNode does not exists) → return

  usedNodes ← nearestNode
  print edge { edgeNode[nearestNode] → nearestNode }

  // Update nearest[] and edgeNode[] through nearestNode
  for each i ∉ [0 … n-1] and i ∉ usedNodes
    if edge exists { nearestNode → i }
      if weight[nearestNode → i] < nearest[i]
        nearest[i] = weight[nearestNode → i]
        edgeNode[i] = nearestNode;
end while
```






```
Minimum spanning forest weight: 45
(0 5) -> 4
(5 3) -> 2
(0 8) -> 5
(8 6) -> 7
(6 2) -> 12
(1 7) -> 7
(1 4) -> 8
```

Prim's Algorithm with Adjacency Matrix

Live Demo

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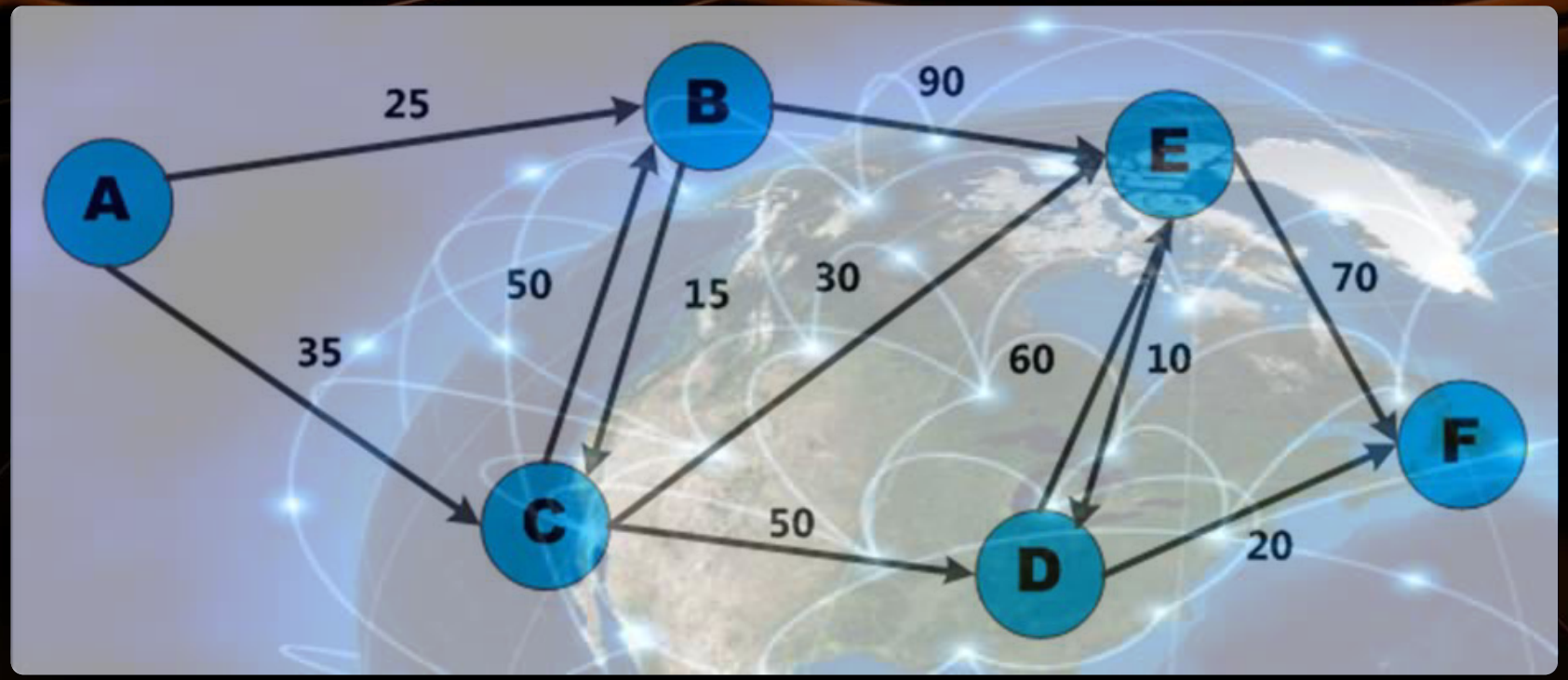
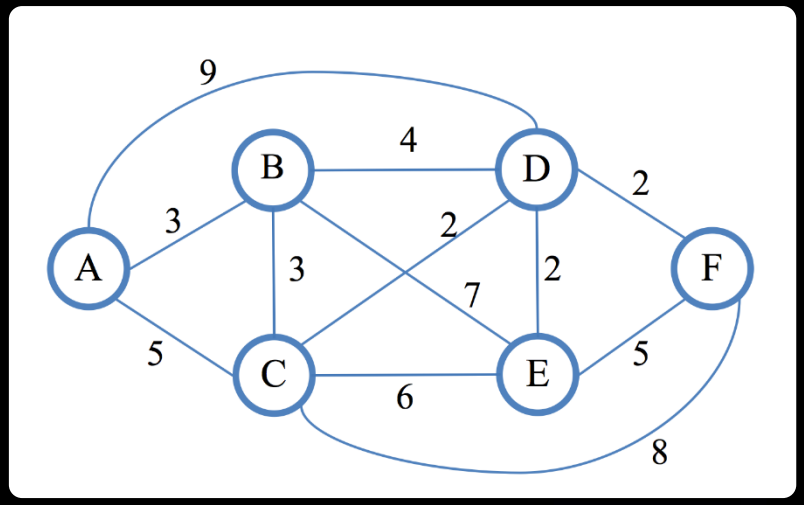


Dijkstra's Algorithm

Shortest Paths in Graph

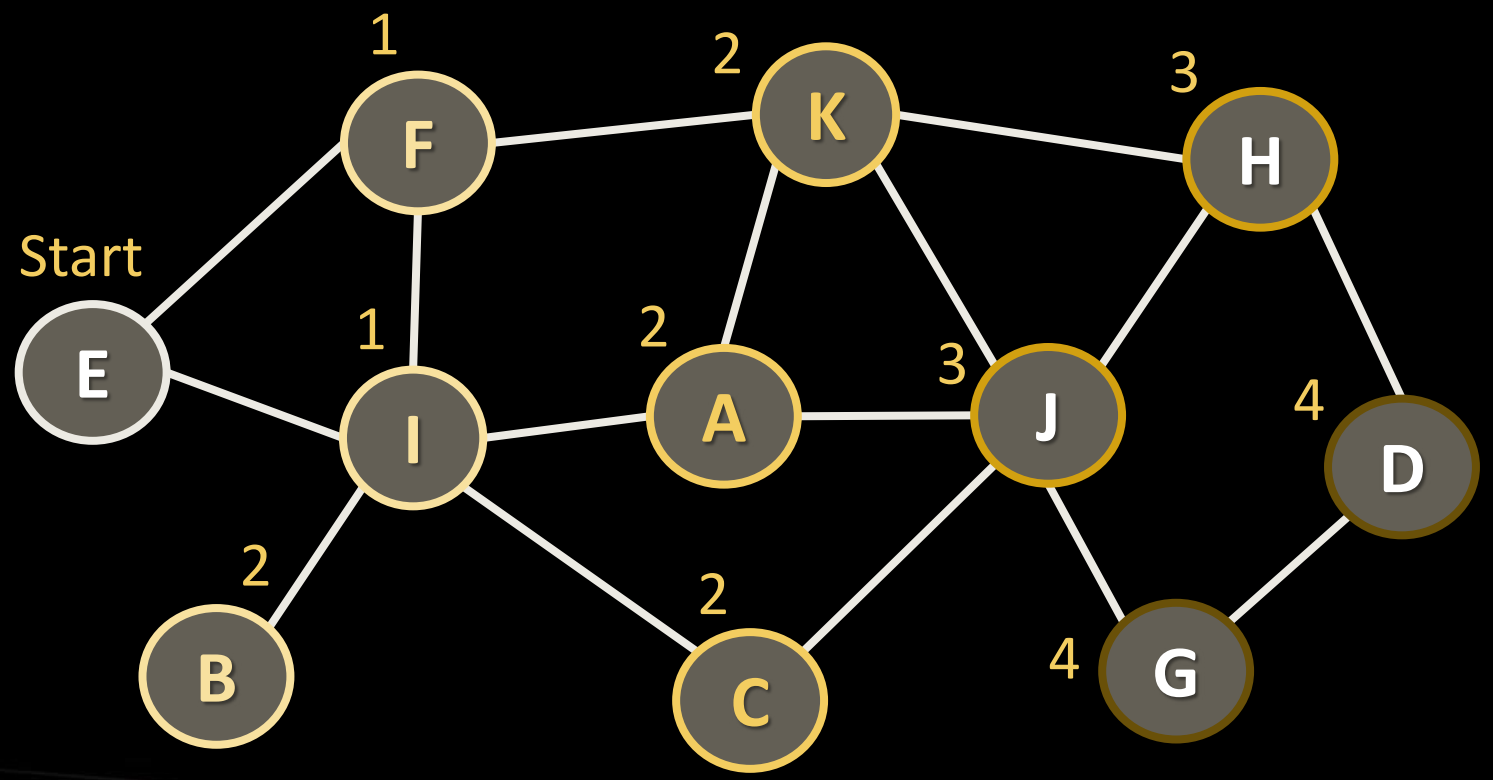
Dijkstra's Algorithm

- Dijkstra's algorithm finds the shortest path from given vertex to all other vertices in a directed / undirected weighted graph
  - First described by Edsger W. Dijkstra in 1956
- Assumptions
  - Weights on edges are non-negative
  - Edges can be directed or not
  - Weights do not have to be distances
  - Shortest path is not necessarily unique
  - Not all edges need to be reachable



Shortest Path in Unweighted Graph

- In unweighted graphs finding the shortest path can be done with BFS (all edges have the same weight):



Weighted Shortest Paths with BFS

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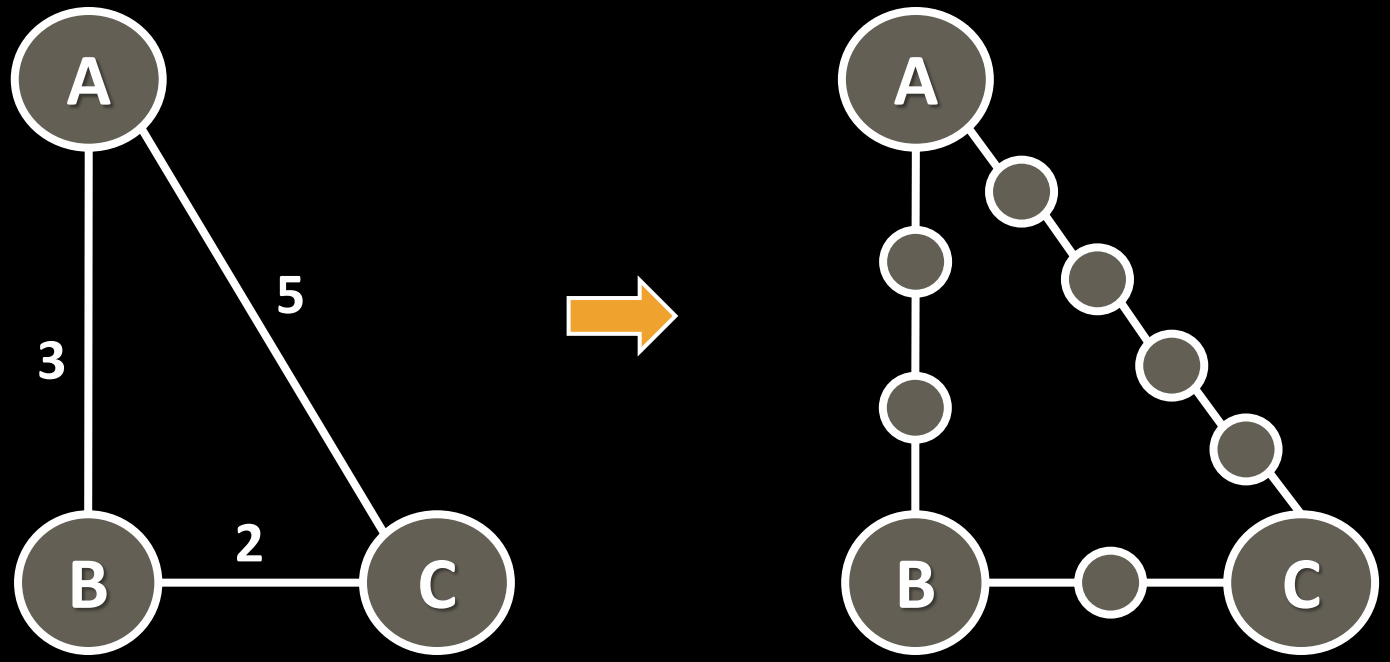
- In weighted graphs
  - Break the edges into sub-vertices and use BFS



  - * Too much memory usage even for smaller graphs!

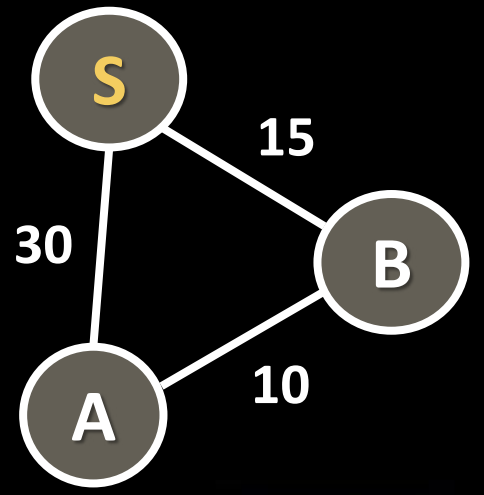
Dijkstra's Algorithm

- Dijskstra's algorithm is similar to BFS and to Prim's algorithms
- Use a priority queue instead of queue
  - Keep the shortest distances so far
- Steps in Dijkstra's algorithm:

v	A	B
d[*v*]	30	15

v	A	B
d[*v*]	25	15

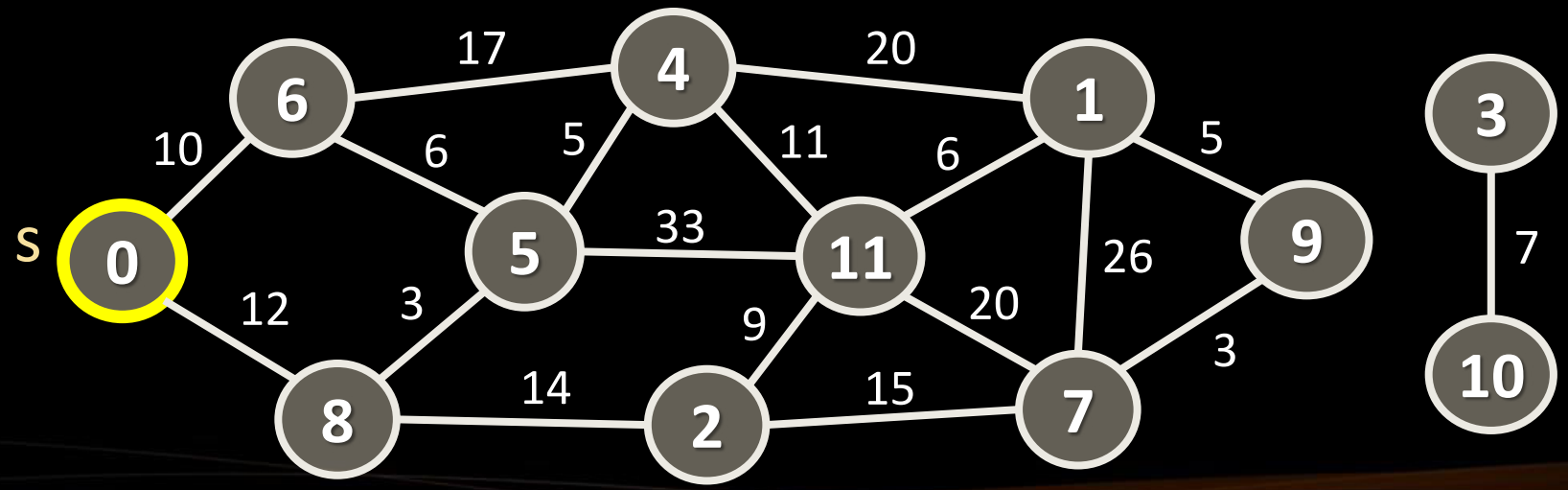
Initially calculate all direct distances d[] from **S**
Enqueue that start node **S**
While (queue not empty)
    Dequeue the nearest vertex **B**
    Enqueue all unvisited child nodes of **B**
    For each edge {B → A}, improve d[A] through **B**:
        d[S → A] = min(d[S → A], d[S → B] + weight[B → A])



Dijkstra's Algorithm: Step #1

- Initialize all distances d[] from s: d[0…n-1] = ∞; d[s] = 0
- Enqueue the start node (0)

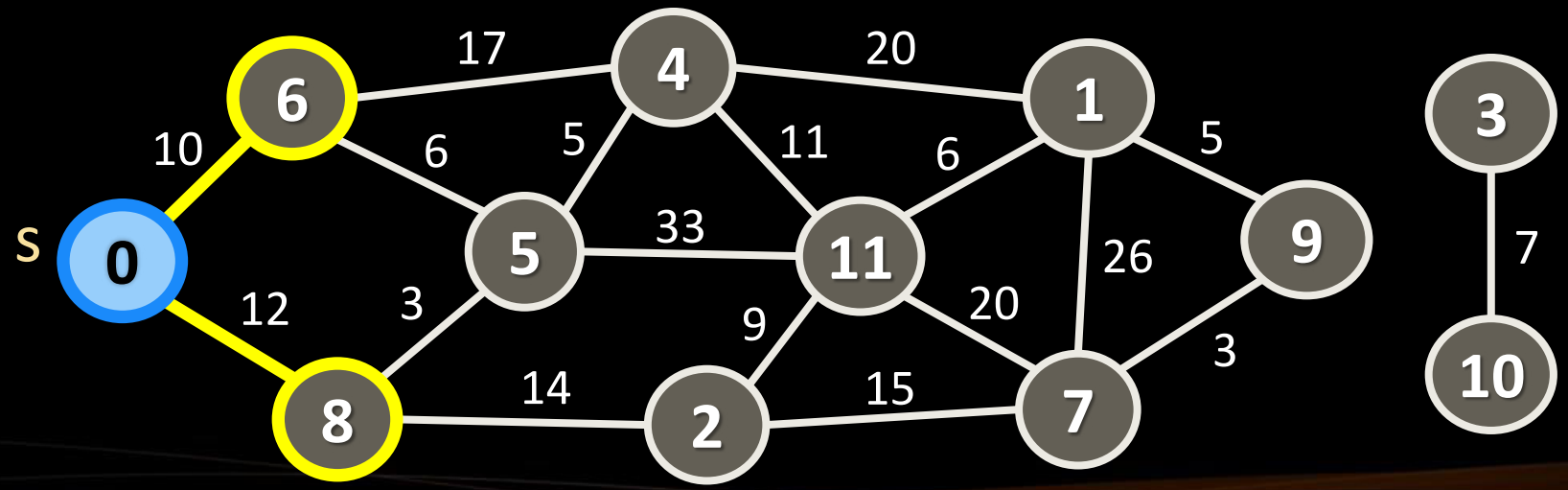
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	-	-	-	-	-	-	-	-	-	-	-
prev[*v*]	-	-	-	-	-	-	-	-	-	-	-	-



Dijkstra's Algorithm: Step #2

- Dequeue the nearest vertex (0) and enqueue unvisited children: 6, 8
- Improve min distances through child edges of 0: {0 → 6}, {0 → 8}

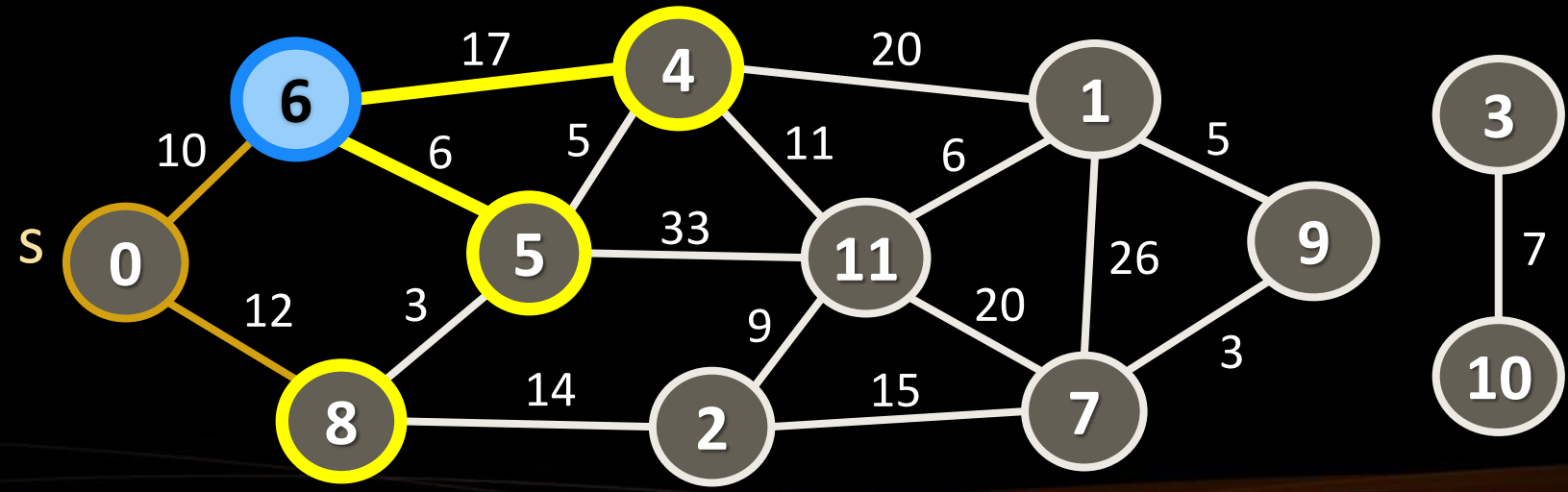
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	-	-	-	-	-	**10**	-	**12**	-	-	-
prev[*v*]	-	-	-	-	-	-	**0**	-	**0**	-	-	-



Dijkstra's Algorithm: Step #3

- Dequeue the nearest vertex (6) and enqueue unvisited children: 4, 5
- Improve min distances through child edges of 6: {6 → 4}, {6 → 5}

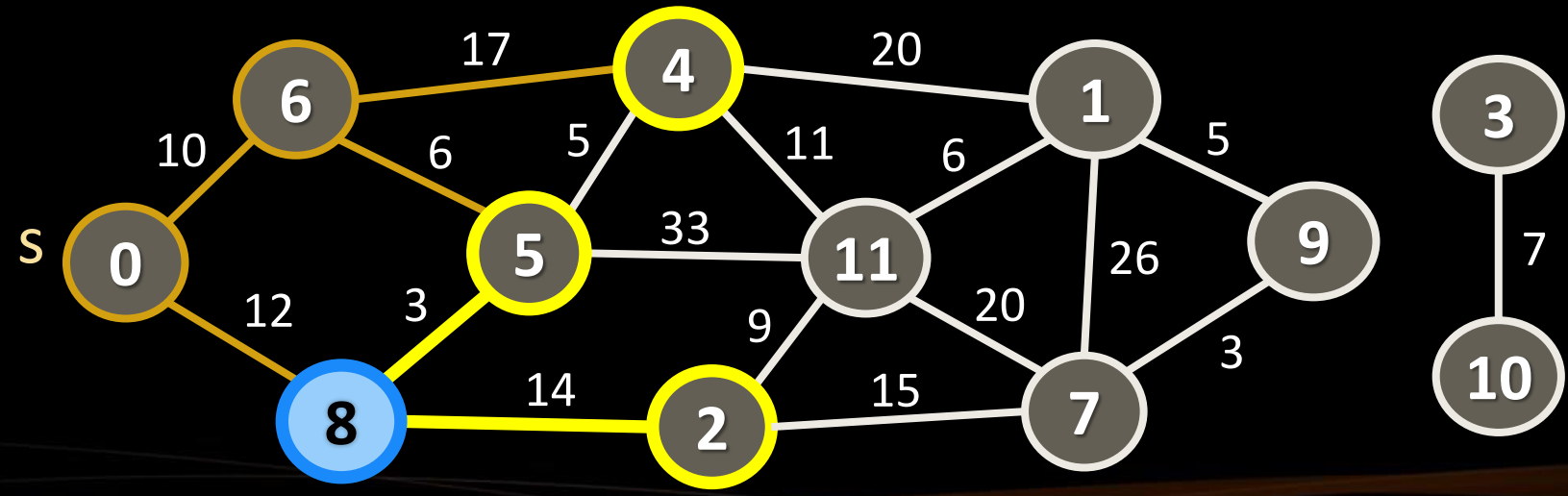
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	-	-	-	27	16	10	-	12	-	-	-
prev[*v*]	-	-	-	-	6	6	0	-	0	-	-	-



Dijkstra's Algorithm: Step #4

- Dequeue the nearest vertex (8) and enqueue unvisited children: 2
- Improve min distances through child edges of 8: {8 → 2}, {8 → 5}

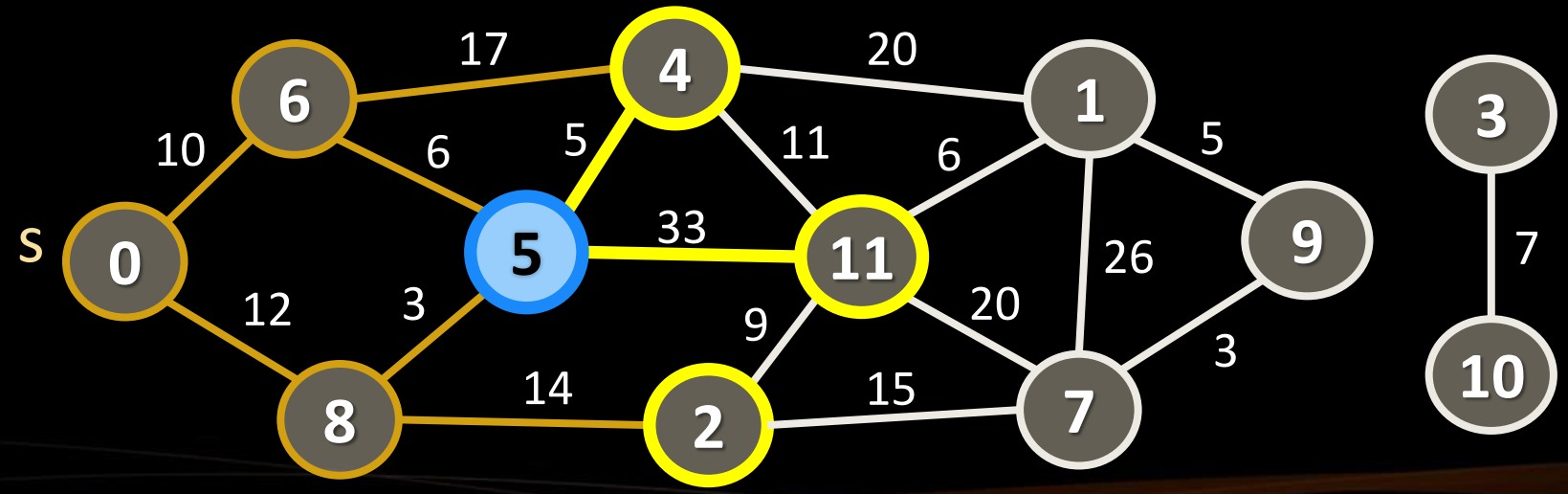
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	-	**26**	-	27	**15**	10	-	12	-	-	-
prev[*v*]	-	-	**8**	-	6	**8**	0	-	0	-	-	-



Dijkstra's Algorithm: Step #5

- Dequeue the nearest vertex (5) and enqueue unvisited children: 11
- Improve min distances through child edges of 5: {5 → 4}, {5 → 11}

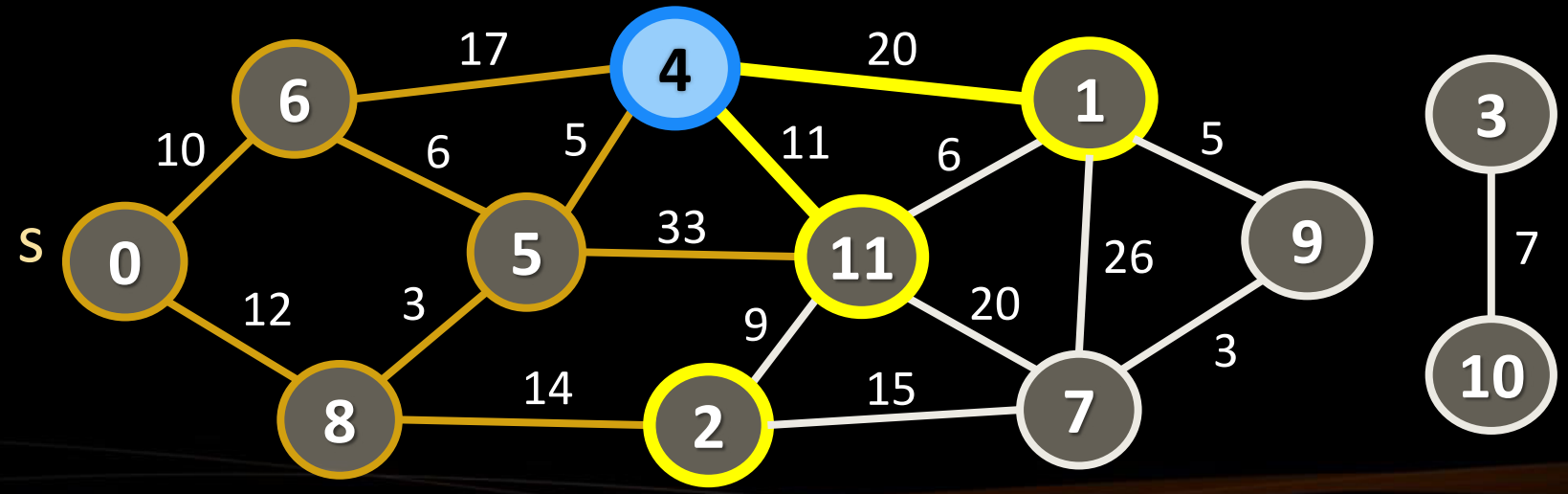
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	-	26	-	**20**	15	10	-	12	-	-	**48**
prev[*v*]	-	-	8	-	**5**	8	0	-	0	-	-	**5**



Dijkstra's Algorithm: Step #6

- Dequeue the nearest vertex (4) and enqueue unvisited children: 1
- Improve min distances through child edges of 4: {4 → 1}, {4 → 11}

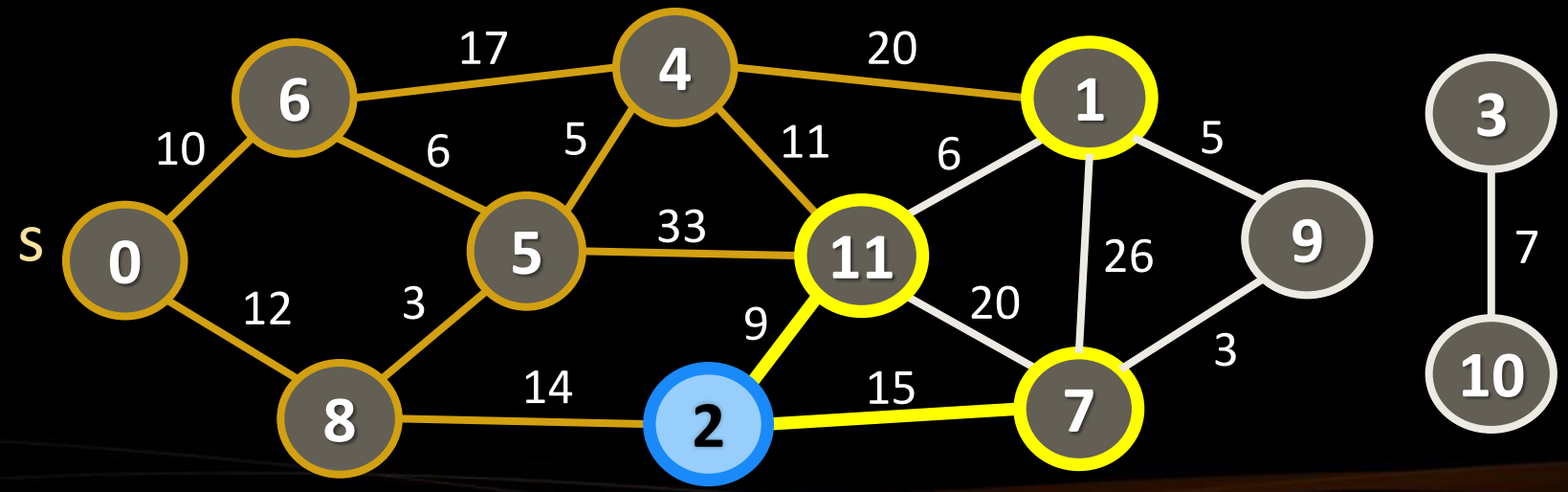
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	**40**	26	-	20	15	10	-	12	-	-	**31**
prev[*v*]	-	**4**	8	-	5	8	0	-	0	-	-	**4**



Dijkstra's Algorithm: Step #7

- Dequeue the nearest vertex (2) and enqueue unvisited children: 7
- Improve min distances through child edges of 2: {2 → 7}, {2 → 11}

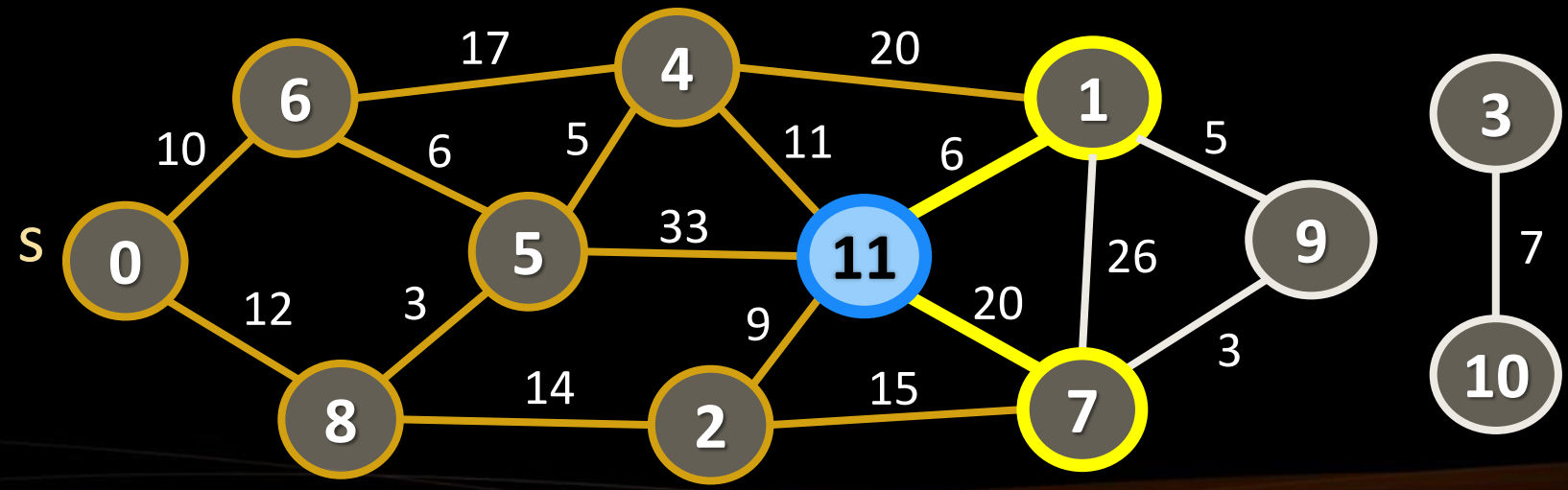
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	40	26	-	20	15	10	**41**	12	-	-	**31**
prev[*v*]	-	4	8	-	5	8	0	**2**	0	-	-	**4**



Dijkstra's Algorithm: Step #8

- Dequeue the nearest vertex (11) and enqueue unvisited children: none
- Improve min distances through child edges of 11: {11 → 1}, {11 → 7}

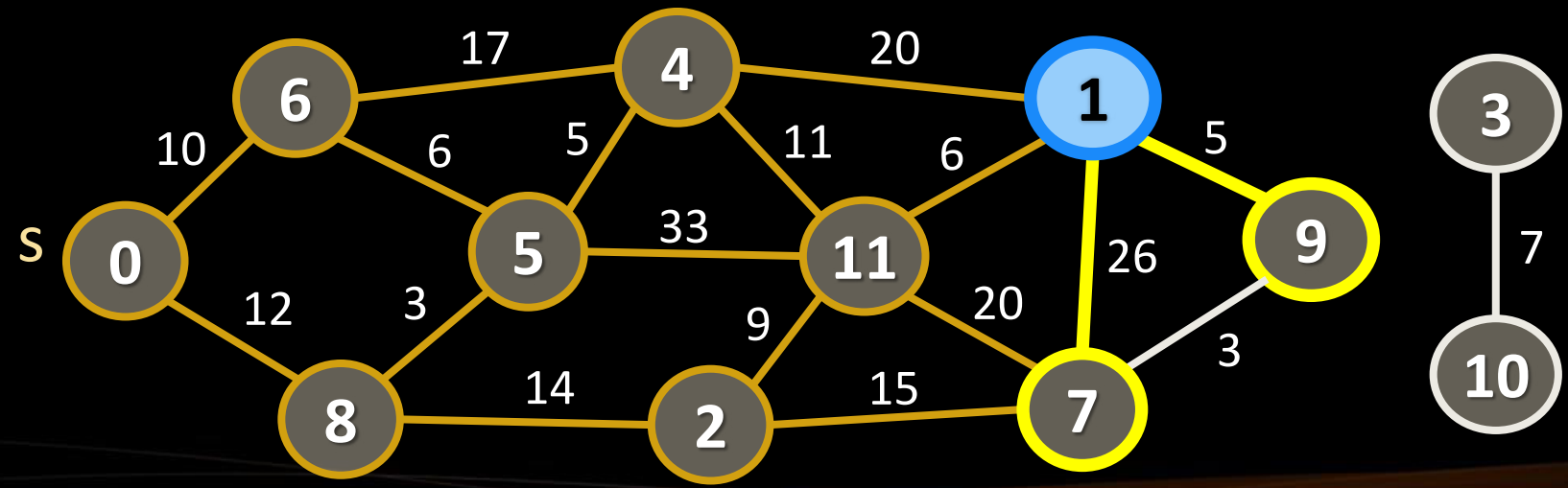
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	**37**	26	-	20	15	10	**41**	12	-	-	31
prev[*v*]	-	**11**	8	-	5	8	0	**2**	0	-	-	4



Dijkstra's Algorithm: Step #9

- Dequeue the nearest vertex (1) and enqueue unvisited children: 9
- Improve min distances through child edges of 1: {1 → 7}, {1 → 9}

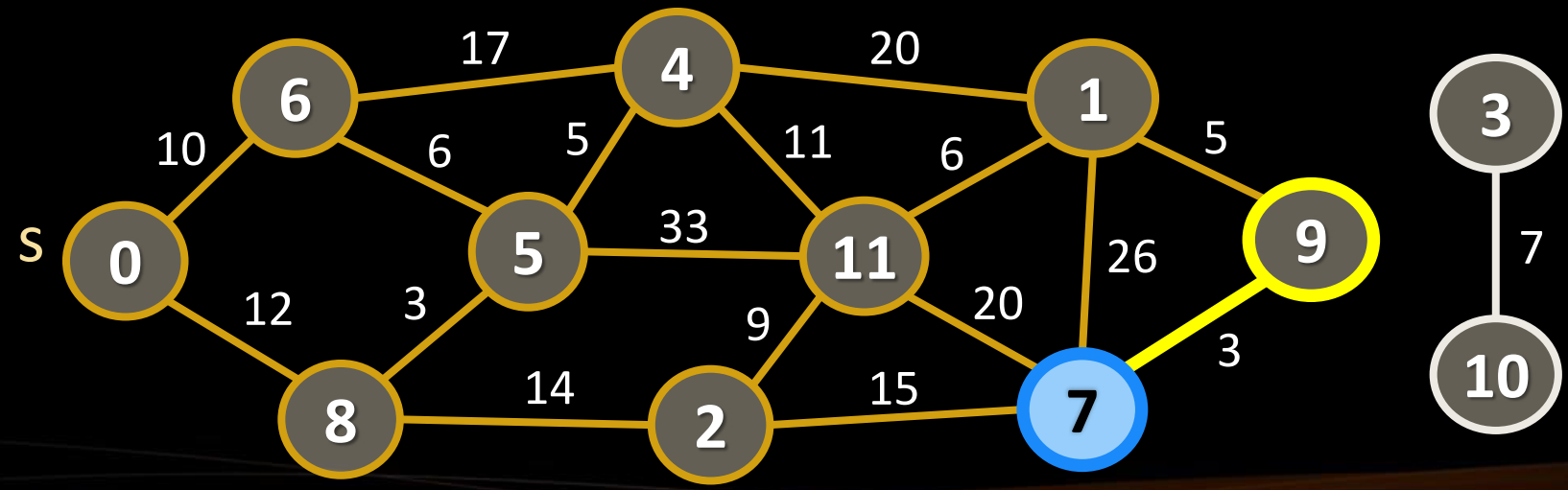
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	37	26	-	20	15	10	**41**	12	**42**	-	31
prev[*v*]	-	11	8	-	5	8	0	**2**	0	**1**	-	4



Dijkstra's Algorithm: Step #10

- Dequeue the nearest vertex (7) and enqueue unvisited children: none
- Improve min distances through child edges of 7: {7 → 9}

v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	37	26	-	20	15	10	41	12	**42**	-	31
prev[*v*]	-	11	8	-	5	8	0	2	0	**1**	-	4



Dijkstra's Algorithm: Step #11

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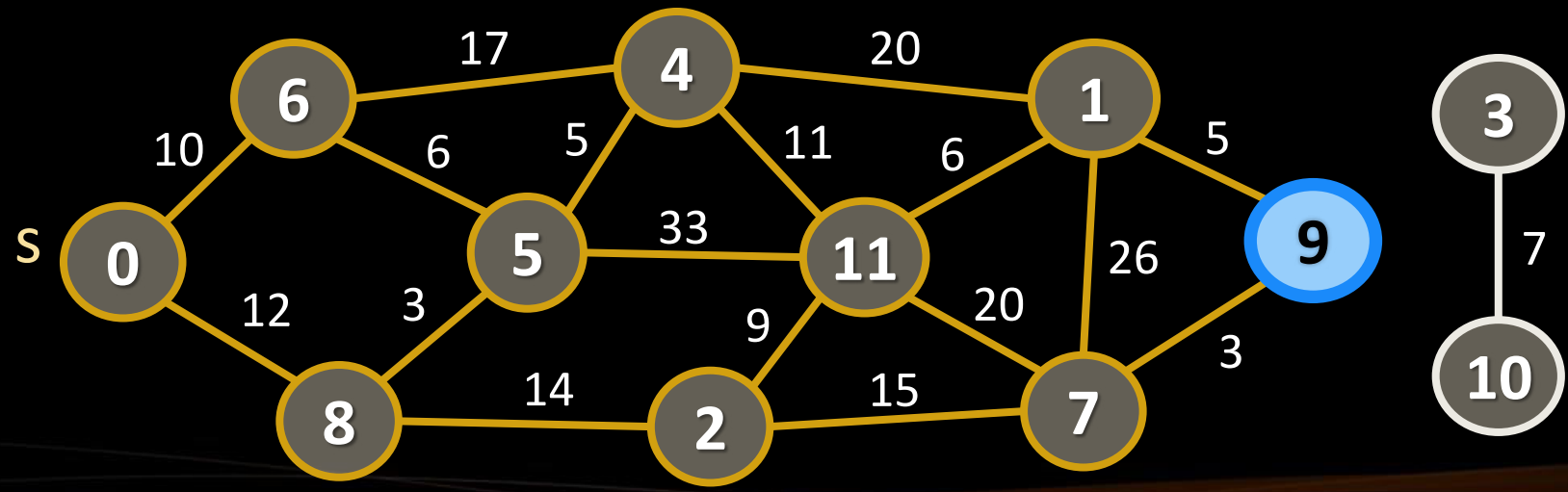
- Dequeue the nearest vertex (9) and enqueue unvisited children: none
- Improve min distances through child edges of 9: none

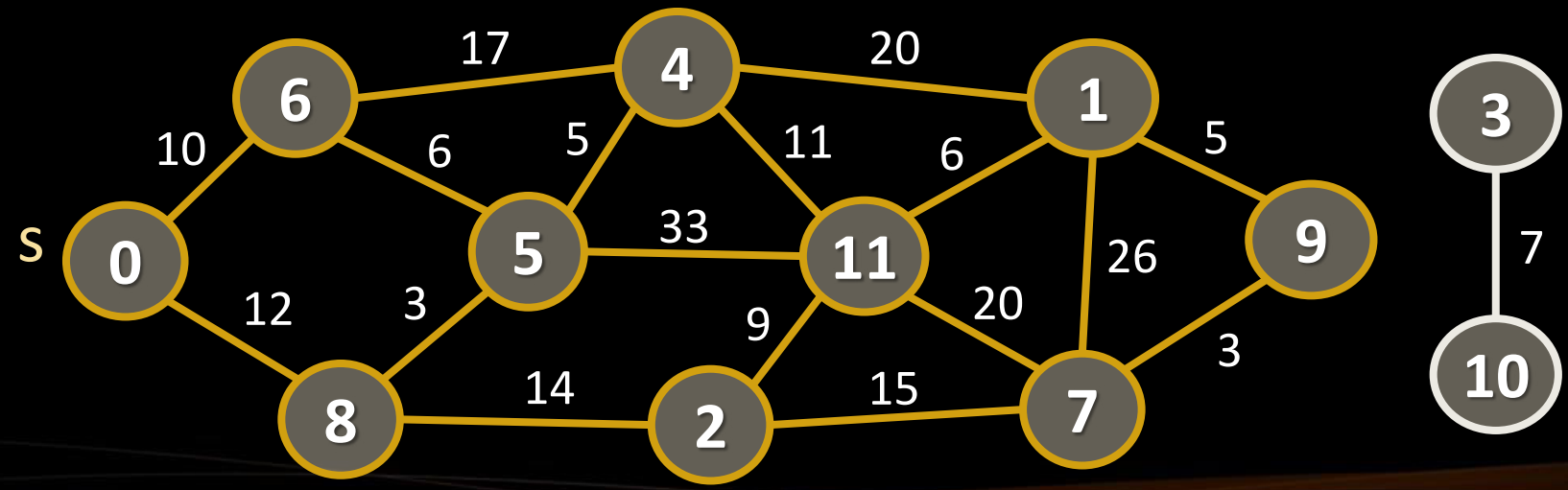
v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	37	26	-	20	15	10	41	12	**42**	-	31
prev[*v*]	-	11	8	-	5	8	0	2	0	**1**	-	4



Dijkstra's Algorithm: Step #12

- The queue is empty → Dijkstra's algorithm is completed
- d[*v*] hold shortest distances; prev[*v*] holds shortest paths tree edges

v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	37	26	-	20	15	10	41	12	**42**	-	31
prev[*v*]	-	11	8	-	5	8	0	2	0	**1**	-	4



Dijkstra's Algorithm: Step #13

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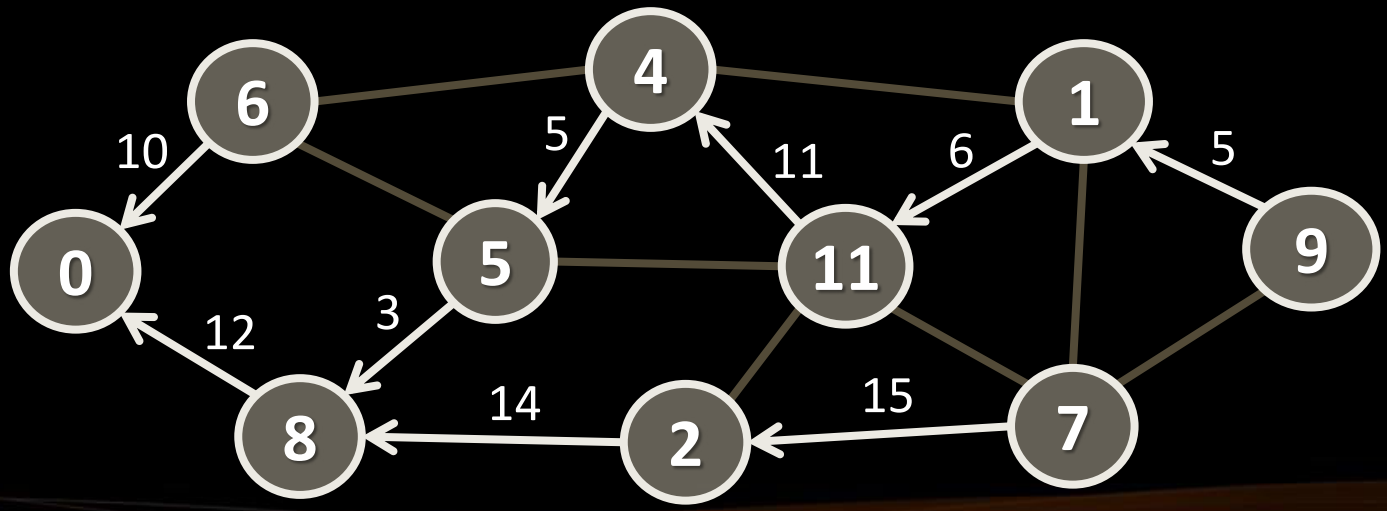
- The output is the shortest paths tree from the starting node to all others
- Reconstruct the path destination to source using **`prev[v]`**

v	0	1	2	3	4	5	6	7	8	9	10	11
d[*v*]	0	37	26	-	20	15	10	41	12	**42**	-	31
prev[*v*]	-	11	8	-	5	8	0	2	0	**1**	-	4

- **`prev[v]`** holds the shortest paths tree edges
- Path[9 → 0] = {9 → 1 → 11 → 4 → 5 → 6 → 0}



Dijkstra's Algorithm – Pseudo Code

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```
d[0…n-1] = INFINITY; d[startNode] = 0
Q = priority queue holding nodes ordered by distance d[]
startNode → Q
while (Q is not empty)
  minNode = dequeue the smallest node from Q
  if (d[minNode] == INFINITY) break;
  foreach (child c of minNode)
    if (c is unvisited) c → Q
    newDistance = d[minNode] + distance {minNode → c}
    if (newDistance < d[c])
      d[c] = newDistance;
      reorder Q;
}
```

Dijkstra's Algorithm – More Details

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- Modifications
  - Implementation with array, priority queue
  - Having a target node + stop when it is found
  - Saving the shortest paths tree (`prev[v]`)
- Complexity depends on the implementation
  - Typical implementation (with array): $O(|V|^2)$
  - With priority queue: $O((|V|+|E|)*\log(|V|))$
- Applications – maps, GPS, networks, air travel, etc.

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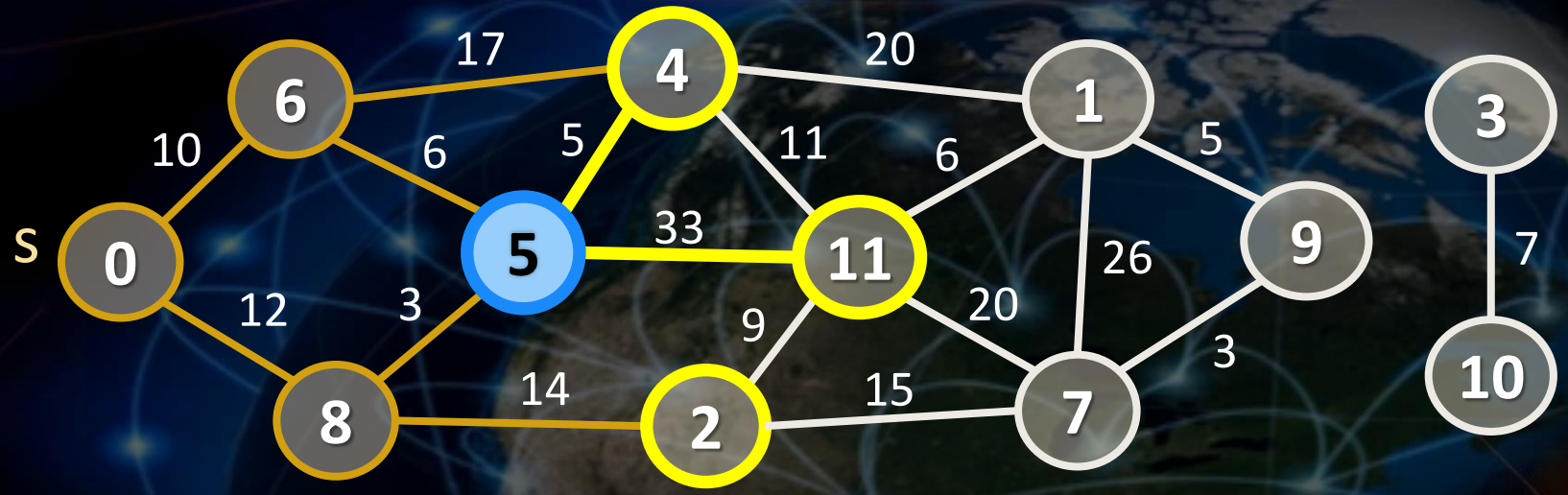

Dijkstra's Algorithm

Live Demo

Bellman-Ford

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- The Bellman-Ford algorithm finds the shortest paths from a single source to all other vertices
  - Unlike Dijkstra's algorithm, Bellman-Ford works with negative weight edges
    - Can detect negative cycles and report them
  - Runs in time $O(|V||E|)$
  - Uses dynamic programming

Floyd-Warshall

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- The Floyd-Warshall algorithm computes the shortest paths between all pairs of vertices
  - Edge weights can be positive or negative
  - Can be extended to detect negative weight cycles
  - Has running time of $O(|V|^3)$
    - Performs better than running Bellman-Ford for each vertex
  - Also uses dynamic programming

Summary

1. Minimum spanning tree (MST)
   - Solved by Prim's and Kruskal's algorithms
2. Shortest paths in a graph:
   - Dijkstra's algorithm – finds shortest path from a single source
     - Does not work with negative weight edges
   - Bellman-Ford – finds shortest path from a single source
     - Works with negative weights and reports negative-weight cycles
   - Floyd Warshall – finds shortest paths between all vertice pairs

Advanced Graph Algorithms

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Questions?

https://softuni.bg/opencourses/algorithms

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