Homework 7 – Due 5/1

Option Valuation with Stochastic Volatility

In this project you will value call options via Monte Carlo simulation using a GARCH stochastic volatility model. Take initial stock price $S_0 = 100$, continuously-compounding risk-free rate $r = 5\%$ per year, time to expiration $T = 0.5$ years, and use the GARCH(1,1) model to generate the stock price paths. The starter code `StoVol.cpp` will generate illustrative paths for you. Note that in this code the long-run volatility (σ_{LT}) is 30% annually and the stochastic volatility (σ_0) starts off at 35% annually — use these values in your simulations, along with values of α, β, and γ as specified below. Price each option below with an error tolerance of 0.01 (one penny) with 95% tolerance. Use antithetic variance reduction to improve run-time.

1. Use the model with $\alpha = 0.01$, $\beta = 0.10$, and $\gamma = 0.89$ to value a call option with strike $K = 0$. The payoff at time T is $\max(S_T - 0, 0) = S_T$, so this "option" replicates the payoff of the stock at time T. Its value today should therefore be $S_0 = 100.00$ (± 0.01). If it is not, there's a mistake somewhere in your code.

• In the three problems below, value call options with strikes of 60 to 180 in steps of 10. The program `ImpliedVol.cpp` computes a call option's implied volatility in the Black-Scholes (constant volatility) framework. Once you have computed the value of these thirteen call options, calculate their B-S implied (constant) volatility. Use the 95% confidence intervals for the option prices to calculate 95% confidence intervals for the implied volatilities. Plot the results with the option's strike on the horizontal axis and the implied volatility on the vertical axis. Show the implied volatility confidence intervals as short vertical lines. (This is illustrated in `ImpliedVol.pdf`.) Sanity check: the B-S implied volatilities should all be between 30 and 40. Use this as the vertical scale. I have included `ImpliedVol.tex` if you wish to plot using TeX.

2. Do this for $\alpha = \beta = 0.0$ and $\gamma = 1.0$. Describe qualitatively what the daily volatility process looks like. Explain why you get the results you do.

3. Do this for $\alpha = 0.01$, $\beta = 0.0$ and $\gamma = 0.99$. Describe qualitatively what the daily volatility process looks like. Explain why you get the results you do.

4. Do this for $\alpha = 0.01$, $\beta = 0.10$, and $\gamma = 0.89$. How is the resulting volatility process qualitatively different from those in 2 and 3? What real-world feature of your implied volatility graph does this process yield?