Introduction to Uplift Modeling

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Motivation

How can we optimally select customers to be treated by marketing incentives?

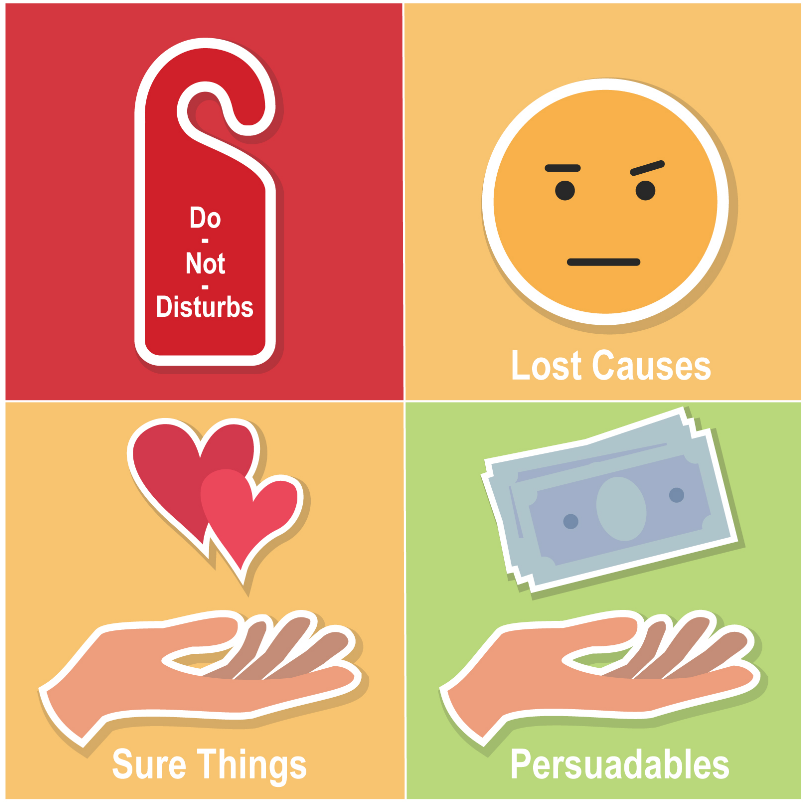


Image taken from https://www.uplift-modeling.com/en/latest/user_guide/introduction/clients.html

We can not send and not send incentives to the same customers at the same time

What is Uplift Modeling?

From Gutierrez, P., & Gérardy, J. Y. (2017). "Causal Inference and Uplift Modelling: A Review of the Literature"

> - Uplift modeling refers to the set of techniques used to model the incremental impact of an action or treatment on a customer outcome.
> - Uplift modeling is therefore both a Causal Inference problem and a Machine Learning one.

Conditional Average Treatment Effect

- Let Y_i^1 denote person i's outcome when it receives the treatment and Y_i^0 when it does not receive the treatment.
- We are interested in understanding the *causal effect* $Y_i^1 - Y_i^0$ and the *conditional average treatment effect* $CATE = E[Y_i^1|X_i] - E[Y_i^0|X_i]$, where X_i is a feature vector of the i-th person.
- **However, we can not observe them!** 🙁

Taken from Gutierrez, P., & Gérardy, J. Y. (2017). *"Causal Inference and Uplift Modelling: A Review of the Literature"*

Uplift

Let W_i is a binary variable indicating whether person i received the treatment, so that

$$Y_i^{obs} = Y_i^1 W_i + (1 - W_i) Y_i^0$$

Unconfoundedness Assumption

If we **assume** that the treatment assignment W_i is independent of Y_i^1 and Y_i^0 conditional on X_i, then we can estimate the $CATE$ from observational data by computing the empirical counterpart:

$$\textbf{uplift} = \widehat{CATE} = E[Y_i | X_i, W_i = 1] - E[Y_i | X_i, W_i = 0]$$

Taken from Gutierrez, P., & Gérardy, J. Y. (2017). *"Causal Inference and Uplift Modelling: A Review of the Literature"*

Estimating Uplift

- Meta algorithms
- Direct measurements (trees)

Some python implementations

- `causalml`
- `EconML`
- `scikit-uplift`

S-Learner

Step 1: Training

$$\underbrace{\begin{pmatrix} x_{11} & \cdots & x_{1k} & w_1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{11} & \cdots & x_{nk} & w_n \end{pmatrix}}_{X \bigoplus W} \xrightarrow{\mu} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

Step 2: Uplift Prediction

$$\widehat{\textbf{uplift}} = \hat{\mu} \begin{pmatrix} x_{11} & \cdots & x_{1k} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{11} & \cdots & x_{mk} & 1 \end{pmatrix} - \hat{\mu} \begin{pmatrix} x_{11} & \cdots & x_{1k} & 0 \\ \vdots & \ddots & \vdots & \vdots \\ x_{11} & \cdots & x_{mk} & 0 \end{pmatrix}$$

Taken from https://www.uplift-modeling.com/en/latest/user_guide/models/index.html

T-Learner

Step 1: Training

$$\underbrace{\begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_C k} \end{pmatrix}}_{X|_{\text{control}}} \xrightarrow{\mu_C} \begin{pmatrix} y_1 \\ \vdots \\ y_{n_C} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_T k} \end{pmatrix}}_{X|_{\text{treatment}}} \xrightarrow{\mu_T} \begin{pmatrix} y_1 \\ \vdots \\ y_{n_T} \end{pmatrix}$$

Taken from https://causalml.readthedocs.io/en/latest/methodology.html#meta-learner-algorithms

T-Learner

Step 2: Uplift Prediction

$$\widehat{\textbf{uplift}} = \hat{\mu}_T \begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{mk} \end{pmatrix} - \hat{\mu}_C \begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{mk} \end{pmatrix}$$

Taken from https://causalml.readthedocs.io/en/latest/methodology.html#meta-learner-algorithms

X-Learner

Step 1: Training: Same as T-Learner

Step 2: Compute imputed treatment effects

$$\tilde{D}^T := \begin{pmatrix} y_1 \\ \vdots \\ y_{n_T} \end{pmatrix} - \hat{\mu}_C \begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_T k} \end{pmatrix}$$

$$\tilde{D}^C := \hat{\mu}_T \begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_C k} \end{pmatrix} - \begin{pmatrix} y_1 \\ \vdots \\ y_{n_C} \end{pmatrix}$$

Taken from https://causalml.readthedocs.io/en/latest/methodology.html#meta-learner-algorithms

X-Learner

Step 3: Train with different targets

$$\underbrace{\begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_C k} \end{pmatrix}}_{X|_{\text{control}}} \xrightarrow{\tau_C} \begin{pmatrix} \tilde{D}_1^C \\ \vdots \\ \tilde{D}_{n_T}^C \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} x_{11} & \cdots & x_{1k} \\ \vdots & \ddots & \vdots \\ x_{11} & \cdots & x_{n_C k} \end{pmatrix}}_{X|_{\text{treatment}}} \xrightarrow{\tau_T} \begin{pmatrix} \tilde{D}_1^T \\ \vdots \\ \tilde{D}_{n_T}^T \end{pmatrix}$$

Taken from https://causalml.readthedocs.io/en/latest/methodology.html#meta-learner-algorithms

X-Learner

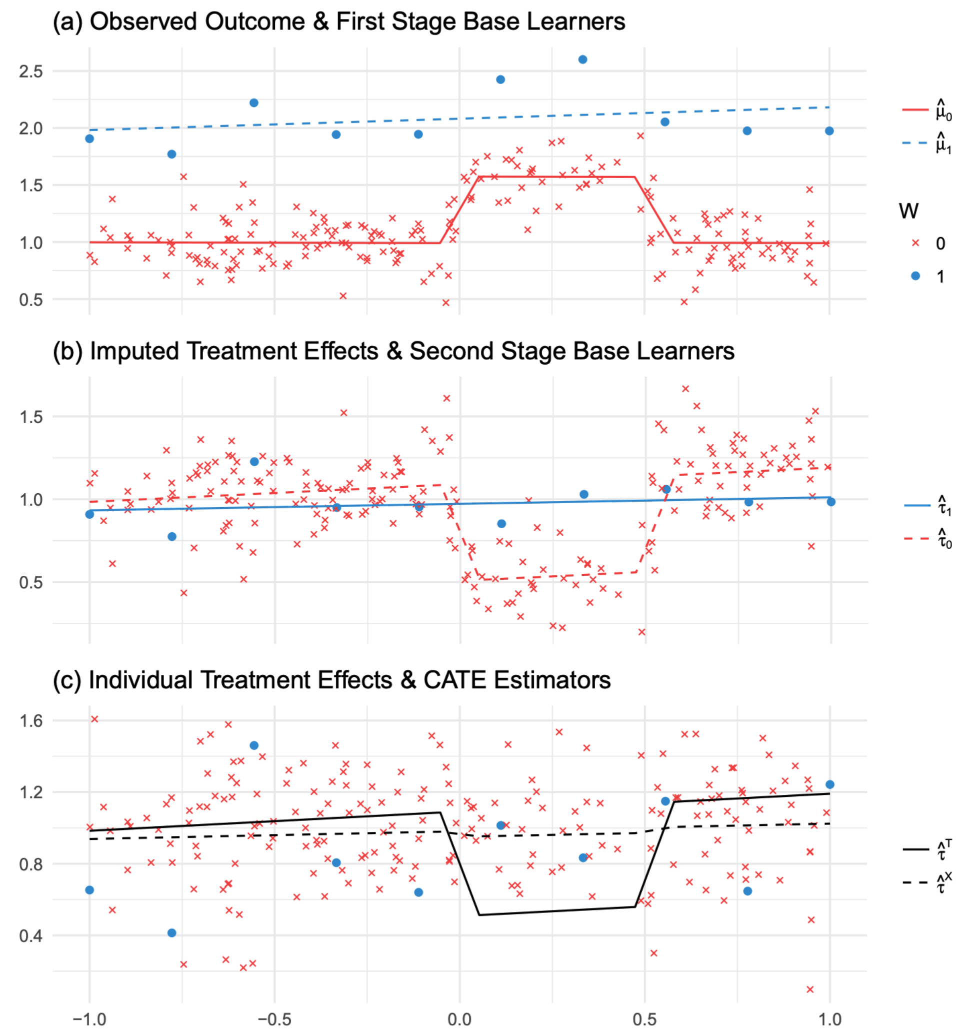
Step 4: Uplift Prediction

$$\widehat{\textbf{uplift}} = g(x)\hat{\tau}_C(x) + (1 - g(x))\hat{\tau}_T(x)$$

where $g(x) \in [0, 1]$ is a weight function.

Taken from https://causalml.readthedocs.io/en/latest/methodology.html#meta-learner-algorithms

Intuition behind the X-Learner



Taken from Sören, R, et.al. (2019) *"Meta-learners for Estimating Heterogeneous Treatment Effects using Machine Learning"*

Python code: Example

```
from causalml.inference.meta import BaseTClassifier
from sklearn.ensemble import HistGradientBoostingClassifier

# define ml model
learner = HistGradientBoostingClassifier()

# set meta-model
t_learner = BaseTClassifier(learner=learner)

# compute ate
t_ate_lwr, t_ate, t_ate_upr = t_learner.estimate_ate(X=x, treatment=w, y=y)

# predict treatment effects
t_learnet.predict(X=x)

# access ml models
t_learner.models_c[1]
t_learner.models_t[1]
```

Uplift Metrics: Cumulative gain chart

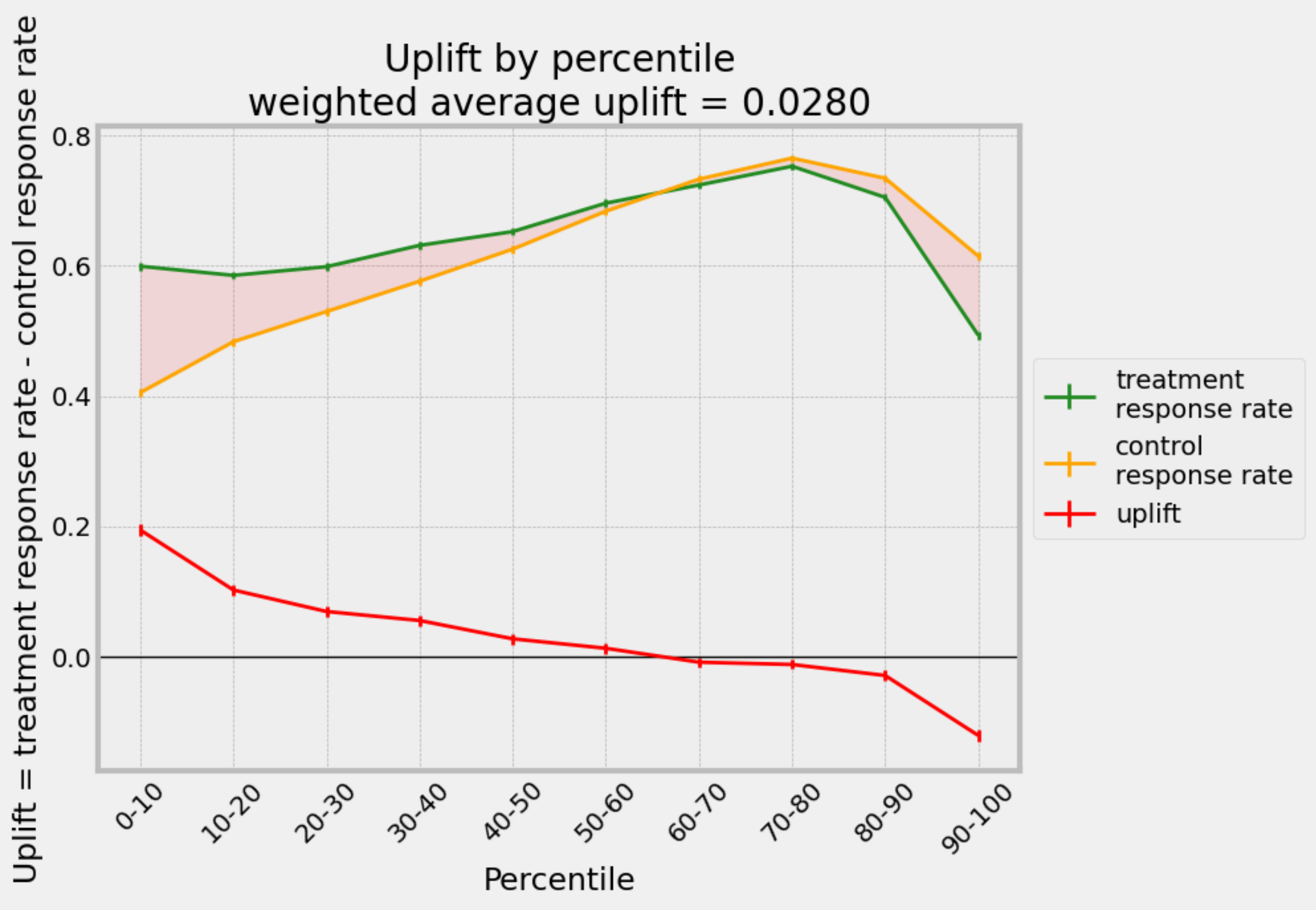
Predict uplift for both treated and control observations and compute the average prediction per decile (bins) in both groups. Then, the difference between those averages is taken for each decile.

$$\left(\frac{Y^T}{N^T} - \frac{Y^C}{N^C}\right)(N^T + N^C)$$

- Y^T/Y^C: sum of the treated / control individual outcomes in the bin.
- N^T/N^C: number of treated / control observations in the bin.

Taken from Gutierrez, P., & Gérardy, J. Y. (2017). *"Causal Inference and Uplift Modelling: A Review of the Literature"*

Uplift Metrics: Cumulative gain chart



Plot function from `scikit-uplift`

Uplift Metrics: Uplift Curve

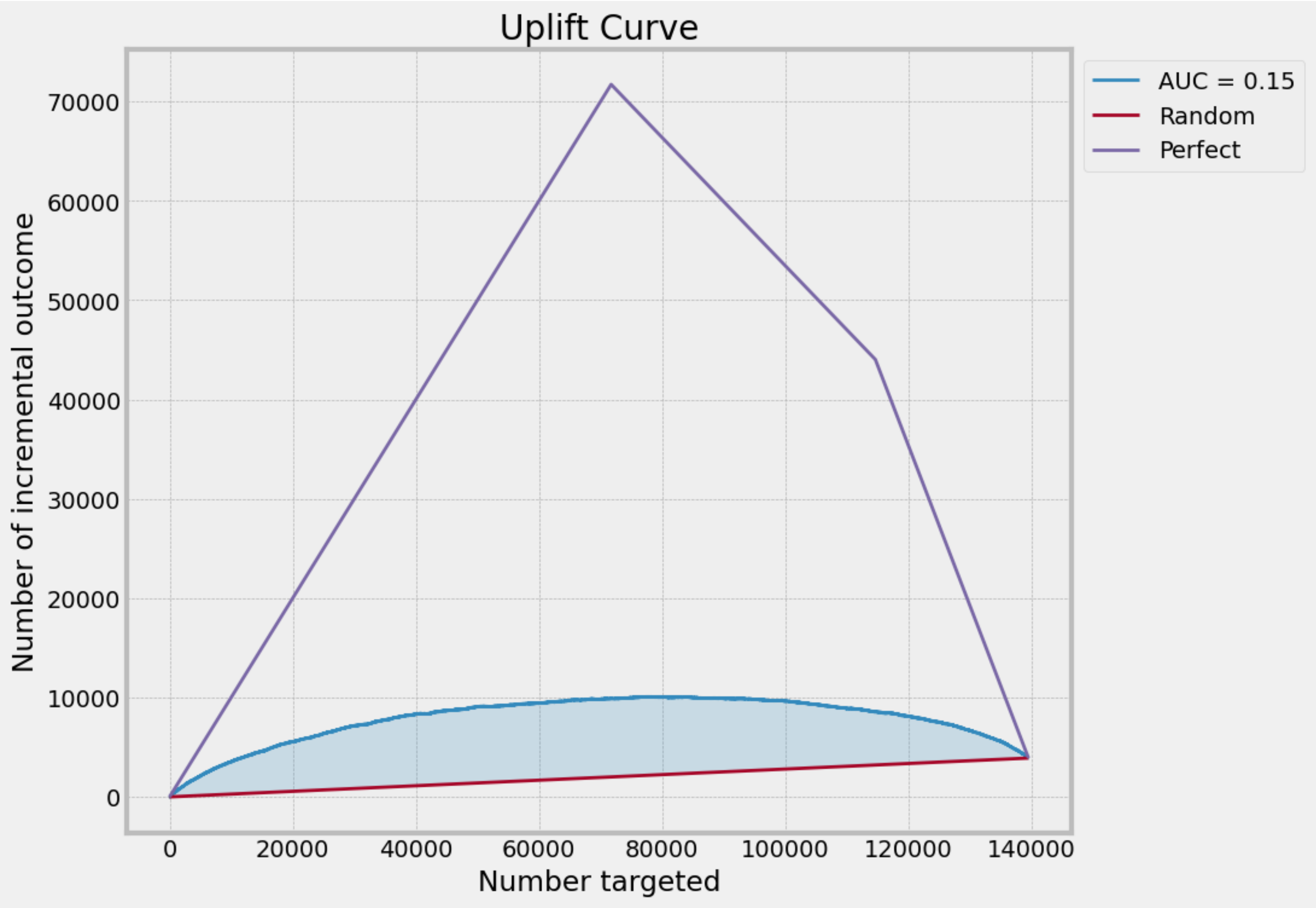
We can generalize the cumulative gain chart for each observation of the test set:

$$f(t) = \left(\frac{Y_t^T}{N_t^T} - \frac{Y_t^C}{N_t^C}\right)(N_t^T + N_t^C)$$

where the t subscript indicates that the quantity is calculated for the first t observations, sorted by inferred uplift value.

Taken from Gutierrez, P., & Gérardy, J. Y. (2017). *"Causal Inference and Uplift Modelling: A Review of the Literature"*

Uplift Metrics: Uplift Curve



Plot function from `scikit-uplift`

Perfect Uplift Curve

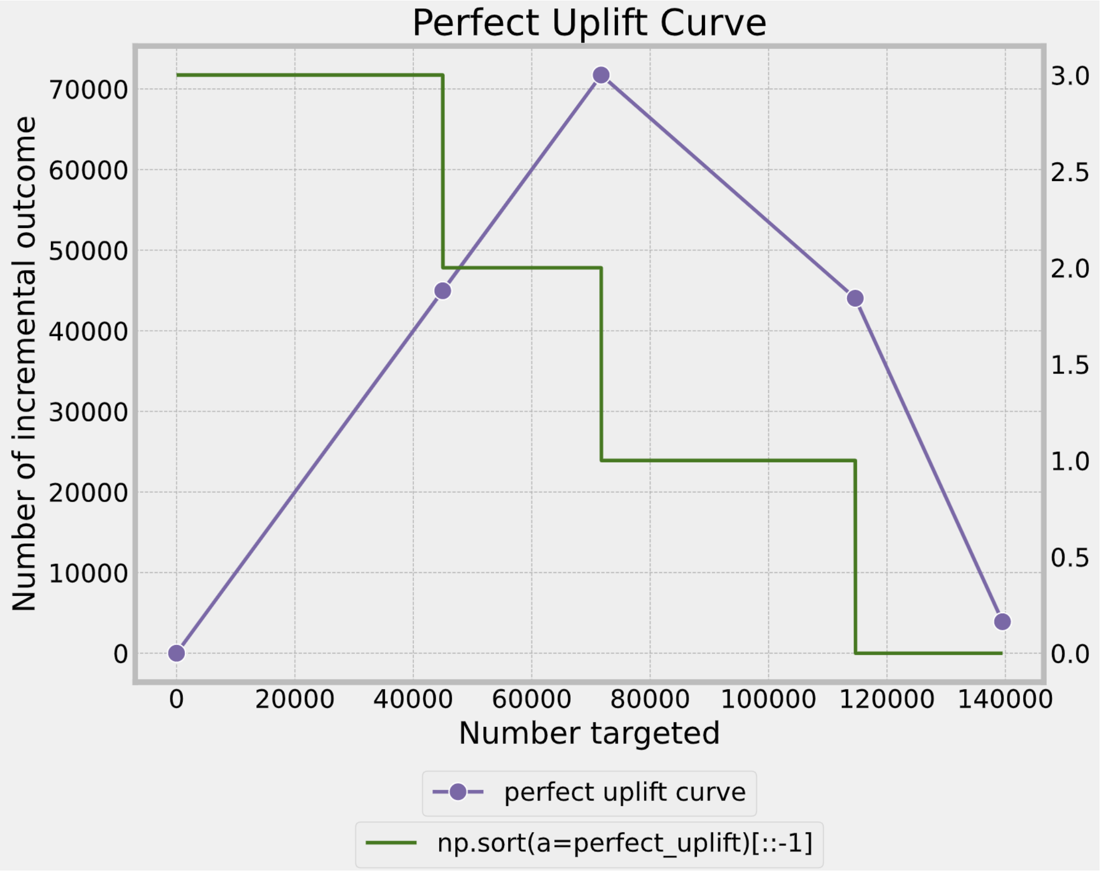
> A perfect model assigns higher scores to all treated individuals
> with positive outcomes than any individuals with negative outcomes.

```
cr_num = np.sum((y_true == 1) & (treatment == 0))  # Control Responders
tn_num = np.sum((y_true == 0) & (treatment == 1))  # Treated Non-Responders

summand = y_true if cr_num > tn_num else treatment
perfect_uplift = 2 * (y_true == treatment) + summand
```

Taken from Diemert, Eustache, et.al. (2020) *"A Large Scale Benchmark for Uplift Modeling"*

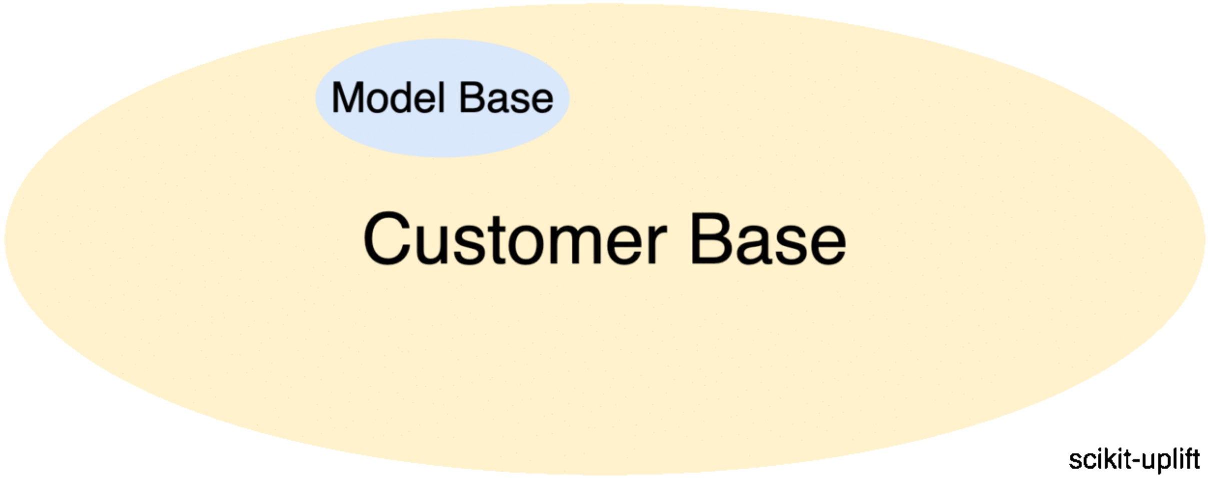
Perfect Uplift Curve

```
from sklift.metrics import uplift_curve

a, b = uplift_curve(y_true=y_true uplift=perfect_uplift, treatment=treatment)
```



Data Collection

1. Randomly subset the customer base to create a representative model base.



Taken from https://www.uplift-modeling.com/en/latest/user_guide/introduction/data_collection.html

Demo

Notebook Link

See https://juanitorduz.github.io/uplift/

References:

- Diemert, Eustache, et.al. (2020) *"A Large Scale Benchmark for Uplift Modeling"*
- Gutierrez, P., & Gérardy, J. Y. (2017). *"Causal Inference and Uplift Modelling: A Review of the Literature"*
- Karlsson, H. (2019) *"Uplift Modeling: Identifying Optimal Treatment Group Allocation and Whom to Contact to Maximize Return on Investment"*
- Sören, R, et.al. (2019) *"Meta-learners for Estimating Heterogeneous Treatment Effects using Machine Learning"*

Thank you!

More Info: juanitorduz.github.io/