COMP2611: Computer Organization

Tutorial 1: Programs and Numbers

Overview

- You will learn the following in this tutorial:
  - the compilation process of computer programs into machine instructions.
  - the conversion between binary, decimal and hexadecimal numbers.
  - the computer numerical unit prefix.

Programs and Numbers

Computer programs

- the compilation process

Number bases

- the introduction of different bases

Conversion between binary and decimal

- conversion methods and exercises

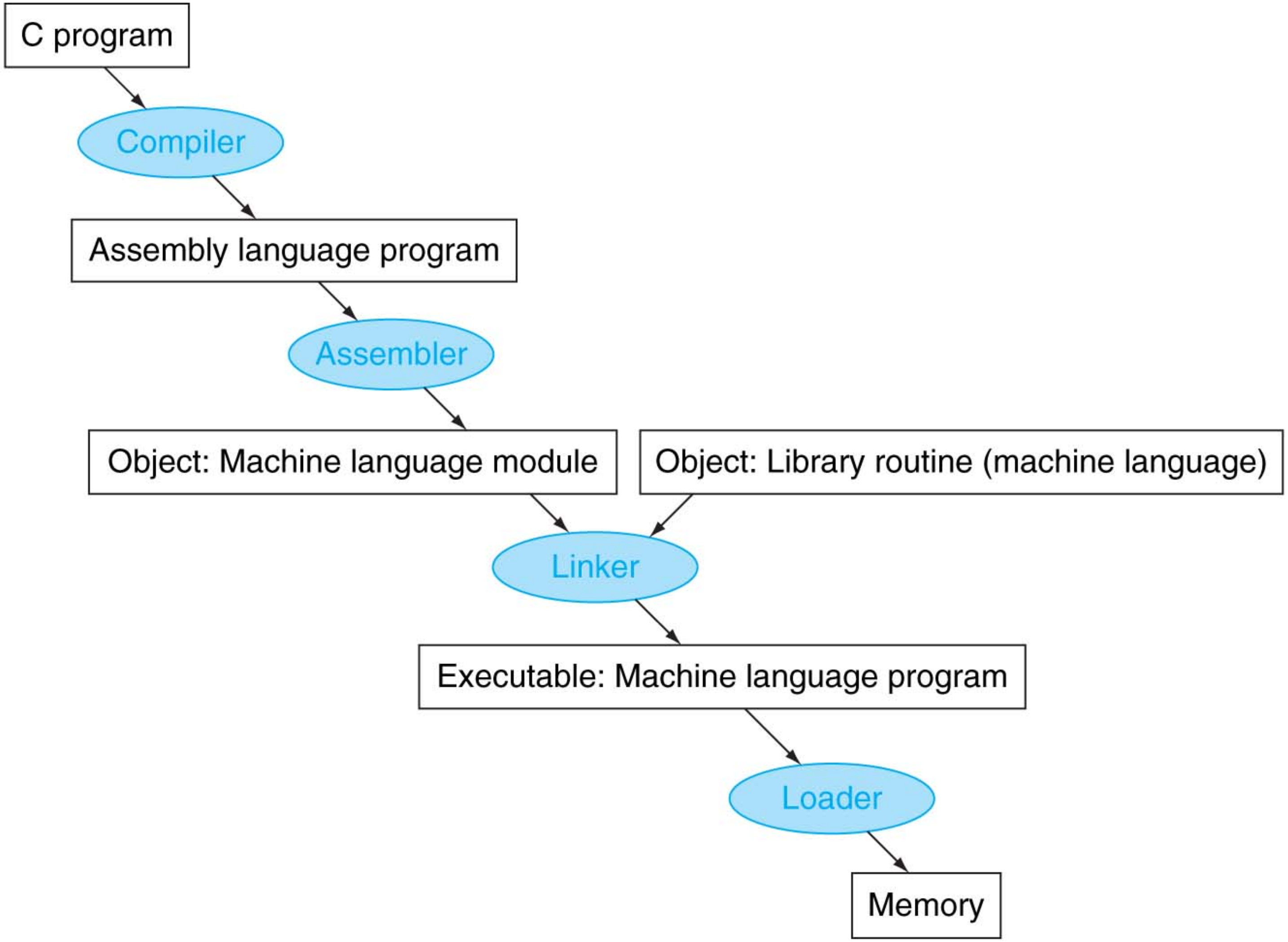
Conversion between binary and hexadecimal

- conversion methods and exercises

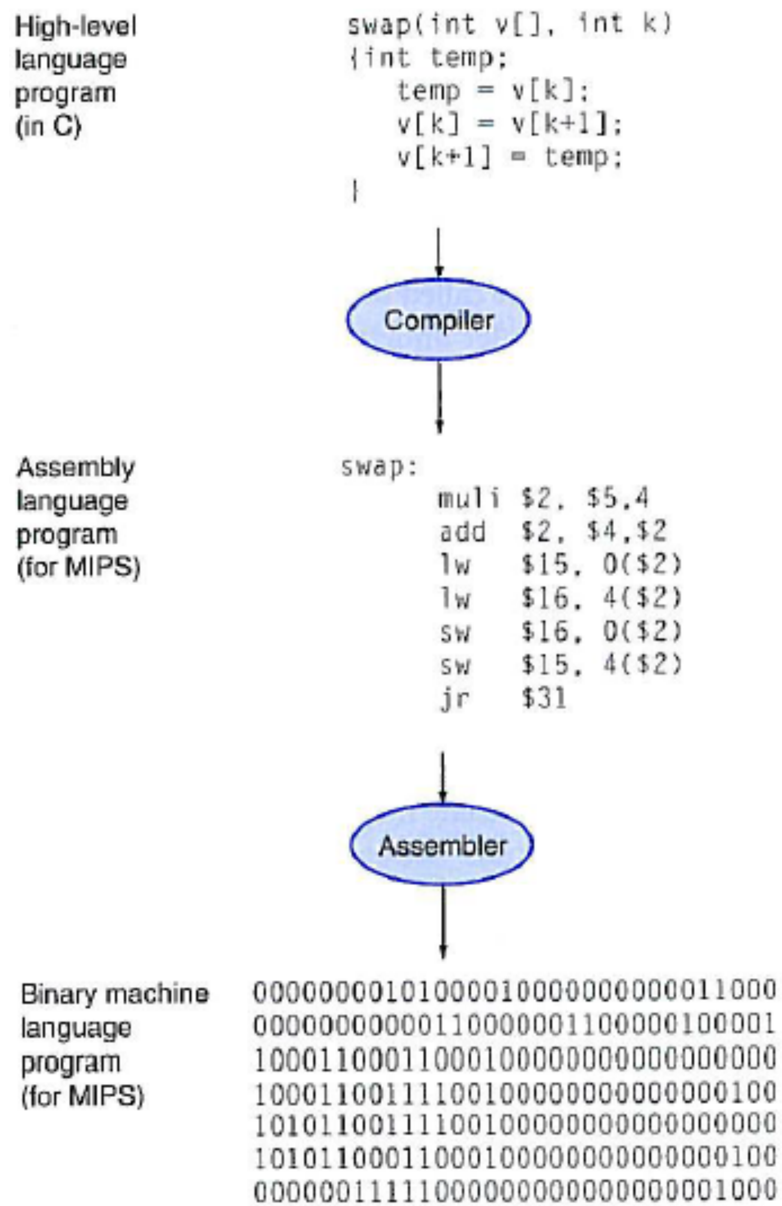
Computer numerical unit prefix

Exercises

Computer programs



Programs for MIPS processor



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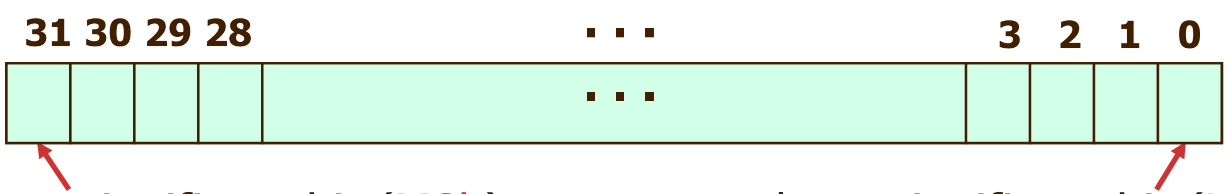
Exercises

Base, Representation and Value

- Numbers can be represented in any base
  - Human: decimal (base 10, has 10 digits 0,1,…,9);
  - Computer: binary (base 2, has 2 digits, 0,1)
- Positional Notation: value of the `i`th digit `d` is `d x Base`^{i}
  - $1001_2 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)_{10} = 9_{10}$

- Bits are grouped and numbered 0, 1, 2, 3 … from right to the left:
  - Byte: a group of 8 bits
  - Word: a group of 32 or 64 bit



most significant bit (MSb) least significant bit (LSb)

- Value of the 32-bit binary numbers =

$$(b_{31} \times 2^{31}) + (b_{30} \times 2^{30}) + \ \ldots + (b_1 \times 2^1) + (b_0 \times 2^0)$$

Hexadecimal Representation

- Hexadecimal (base 16) numbers are commonly used
- To avoid reading and writing long binary numbers

Conversion to hexadecimal

- Since base 16 is a power of 2, we can simply convert by replacing each group of four bits by a single hexadecimal digit, and vice versa

Example of hexadecimal-to-binary conversion:

- 0_{hex} - 9_{hex} for 0000_2 - 1001_2
- a_{hex} - f_{hex} for 1010_2 – 1111_2
- i.e. 0000 1010 0000 0101 0000 1100 0000 0110_2

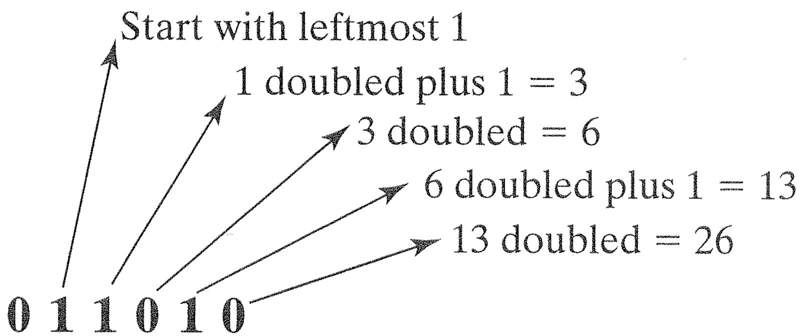
  = 0 a 0 5 0 c 0 6_{hex}

  = 0x0a050c06 # 0x to indicate it is a hexadecimal

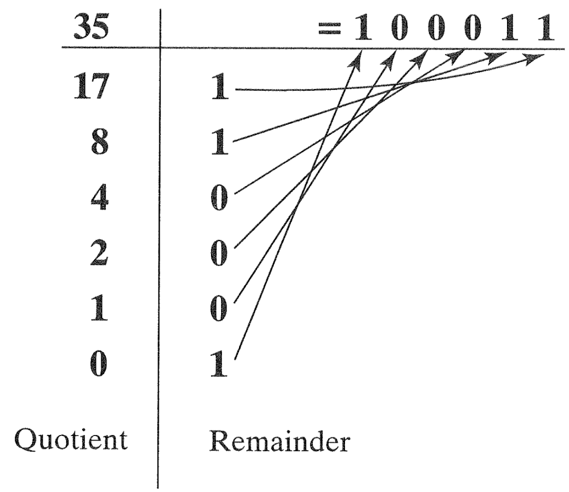
  = 168102918_{10}

- Binary to decimal:
  - Double and add method:
    - For each position *i* starting at the leftmost 1,
      - Double the sum, add bit *i* to the sum

    Ex: 011010



- Decimal to binary:
  - Successive divisions by 2:
    - Inverse of double and add
    - In each step the remainder of the division is the next bit of the sequence



- Remark:
  - useful powers of 2 to memorize

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
1	2	4	8	16	32	64	128	256	512	1024

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Exercises

Converting positive decimal integer to binary

- We keep on dividing the decimal integer by 2 until the quotient is 0. The remainder at each step corresponds to a digit of the integer in base 2, from the Least Significant Digit (LSD) to the Most Significant Digit (MSD).

Exercises

Question 1: Convert $37_{(10)}$ to the binary format.

Converting positive binary integer to decimal

- The value represented by the i-th bit d of a positive binary integer is in fact d x 2^i. Note that the Least Significant bit is the 0-th bit.
- Take the integer $ABCD_{(2)}$ as an example, it effectively corresponds to:

$$ABCD_{(2)} = (A \times 2^3) + (B \times 2^2) + (C \times 2^1) + (D \times 2^0)$$

2's power	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
value	2	4	8	16	32	64	128	256	512	1024

Question 1: Convert the positive integer $10\ 1001_{(2)}$ to the decimal format.

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Hexadecimal format

- The binary number system is verbose in that even small numerical values could require long strings of bits to represent.
- Hexadecimal number system is a number system that has a base of 16 (instead of 2).
- Under the hexadecimal system, there are 16 possible values for each digit, as shown in the table.

Decimal value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexa-decimal digit	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Converting binary number to hexadecimal

- We just group the binary number into groups of 4 bits, and then each group represents one digit of the corresponding hexadecimal number.
- The conversion can be made immediately by eye inspection.

Question 1: Convert 0011 $0010_{(2)}$ to the hexadecimal format.

Converting hexadecimal number to binary

- We just expand each digit of the hexadecimal number into 4 bits, and then the resulting bits from all the digits represent the corresponding binary number.
- Again, the conversion can be made immediately by eye inspection.

Exercises

Question 1: Convert $A7_{(16)}$ to the binary format.

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Computer numerical unit prefix

- For representing a number of bits or bytes in computers, the unit prefix Kilo is often used to represent 2^{10} (equal to 1024) not 1000.

- For examples, 1 kilobytes = 1024 bytes and 1 kilobits = 1024 bits.

- Similarly, we have the following table for the common prefixes used for bytes and bits:

Unit prefix	Value
Kilo	2^{10} (or 1024)
Mega	2^{20} (or 2^{10} Kilo)
Giga	2^{30} (or 2^{10} Mega)
Tera	2^{40} (or 2^{10} Giga)

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Question 1: Convert $176_{(10)}$ to the binary format.

Question 2: Convert the positive integer 11 0100 1001$_{(2)}$ to the decimal format.

Exercises

Question 3: Convert 1010 0011 1001 0111 0100$_{(2)}$ to the hexadecimal format.

Exercises

Question 4: Convert $B12A3F01_{(16)}$ to the binary format.

Conclusion

- You have learnt:
  - the compilation process of computer programs into machine instructions.
  - the conversion between binary, decimal and hexadecimal numbers.
  - the computer numerical unit prefix.