

Logic Examples I

Problem 1. Is the following reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1} = x$ correct?

- (1) $\sqrt{2x^2 - 1} = x$: Given
- (2) $2x^2 - 1 = x^2$: Square both sides
- (3) $X^2 = 1$
- (4) So, $x = 1$ or $x = -1$.

Solution: The reasoning is incorrect. We have

$$(2) \leftrightarrow (3) \leftrightarrow (4).$$

So, both $x = 1$ and $x = -1$ are solutions for (2). However, we don't have

$$(1) \leftrightarrow (2).$$

Instead, we only have

$$(1) \rightarrow (2).$$

Therefore, we cannot conclude that a value of x that satisfies (2) also satisfies (1).

Problem 2. A, B, C are three persons, one of whom is a knight, one a knave, and one a spy.

The knight always tells the truth, the knave always lies, and the spy can either lie or tell the truth.

A says: "C is a knave"

B says: "A is a knight"

C says: "I am the spy".

Who is the knight, who is the knave, and who is the spy?

Solution:

Let us first figure out who is the knight. First of all, C cannot be the knight. If C were, he would be telling the truth, and in which case, he would be a spy (Modus Ponens), which is a contradiction. (Proof by contradiction)

Second, B cannot be the knight either. If B were the knight, he would be telling the truth, and in which case, A is also a knight (Modus Ponens). This is a contradiction because there is only one knight. (Proof by contradiction)

So, A must be the knight.

Because A is the knight, he tells the truth. Hence, C is a knave (Modus Ponens).

Finally, B must be the spy. If B were not a spy, none of the three persons is a spy, which is a contradiction. (Proof by contradiction).