COMP 2711 Discrete Mathematical Tools for CS
Spring 2016 – Written Assignment # 4
Distributed: March 9, 2016 – Due: March 16, 2016

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address, (iv) your lecture section and (v) your tutorial section. Your work should be submitted before 4PM of the due date in the collection bin outside Room 4210 (Lift 21). Make sure you submit into the box labeled with the correct lecture section.

For all questions you should provide a short explanation as to how you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain why it's 20.

Problem 1: Consider the sets $S_4 = \{a, b, c, d\}$ and $S_5 = \{1, 2, 3, 4, 5\}$?
(a) How many functions are there from the set S_4 to S_5?
(b) How many *one-to-one* functions are there from the set S_4 to S_5?
(c) How many *onto* functions are there from the set S_4 to S_5?
(d) How many *bijections* are there from the set S_4 to S_5?
(e) How many functions are there from the set S_5 to S_4?
(f) How many *one-to-one* functions are there from the set S_5 to S_4?
(g) How many *onto* functions are there from the set S_5 to S_4?
(h) How many *bijections* are there from the set S_5 to S_4?
(i) How many functions are there from the set S_4 to S_4?
(j) How many *one-to-one* functions are there from the set S_4 to S_4?
(k) How many *onto* functions are there from the set S_4 to S_4?
(l) How many *permutations* are there from the set S_4 to S_4?

Problem 2: A base ten number is a string of five digits, where the digits are from the set $\{0, 1, \ldots, 9\}$ but the first digit cannot be 0 (so 52375 is a valid number but 02323 and 2323 are not).
(a) How many five-digit base ten numbers are there?
(b) How many five-digit numbers have no two consecutive digits equal?
(c) How many have at least one pair of consecutive digits equal?

Problem 3: Suppose you are organizing a panel discussion on allowing karoke on campus. Participants will sit behind a long table in the order in which you list them. You must choose 4 administrators from a group of 10 and 5 students from a group of 15.

(a) If the administrators must sit together in a group and the students must sit together in a group, in how many ways can you choose and list the 9 people?

(b) If you must alternate students and administrators, in how many ways can you choose and list them?

Problem 4: In class we stated that
each row of Pascal's triangle first increases and then decreases.
In this question you will prove this statement.

(a) Using the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ prove that

$$\text{if } 0 < k \le n/2 \text{ then } \binom{n}{k-1} < \binom{n}{k}.$$

(b) Using part (a) and the fact that $\binom{n}{k} = \binom{n}{n-k}$ prove that
each row of Pascal's triangle first increases and then decreases.

Problem 5: Give two proofs that

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

Your first proof should be purely algebraic, i.e., just plug in the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and show that the left side equals the right side. Your second proof should be combinatorial, i.e., it should show that the left and right sides are just two different ways to count the same thing.

Problem 6: (a) If you have thirteen distinct chairs to paint, in how many ways can you paint seven of them orange and six of them red?

(b) Now, how many ways can you paint four of them green, three of them blue, and six of them red?

Problem 7: Show that for any $n + 1$ distinct integers, we can pick two of them such that their difference is divisible by n. (Hint: Use the Pigeonhole Principle on the remainders of the integers when divided by n).

Problem 8: **(Challenge)** Suppose there are 20 balls and 3 boxes. The three boxes are labeled with 'A', 'B' and 'C' respectively. In how many different ways can we distribute the balls into the boxes under each of the following conditions?

(a) The twenty balls are labeled with integers $1, 2, \ldots, 20$ respectively so that each ball is distinct and is different from other balls.

(b) All the balls are identical and hence are indistinguishable from each other.