COMP 2711 Discrete Mathematical Tools for CS
Spring Semester, 2016
Written Assignment # 9
Distributed: 29 April 2016 – Due: 4pm, 6 May 2016

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address, and (iv) your tutorial section. Your work should be submitted to the collection bin outside Room 4210 (Lift 21).

Problem 1: Prove that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$.

Problem 2: Prove that every integer greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
(Hint: first prove the three base cases *of* $n = 8, 9, 10$ *and then prove the inductive step assuming that* $n > 10$.*)*

Problem 3: Consider the recurrence $M(n) = 2M(n-1)+2$, with base case of $M(1) = 1$.
(a) State the solution to this recurrence
(you may use Theorem 4.1 in the book).
(b) Use induction to prove that this solution is correct.

Problem 4: Consider a function $T(n)$ defined on integers n that are powers of 2. Suppose

$$T(1) = 1, \quad T(n) = 3T(n/2) + n^2.$$

Iterate the recurrence or use a recursion tree to find a closed-form expression for $T(n)$. Simplify the closed-form expression using the big Θ notation.

Problem 5: Challenge Problem: Leaving Dot-town
Every person living in Dot-town has a red or blue dot on his forehead but doesn't know the color of his own dot. Every day the people gather in the town square to talk with each other. If anyone ever figures out the color of his own dot he must leave town before the next gathering. People never leave Dot-town unless they figure out their own dot color. One day, a stranger comes to town and casually mentions that at least one person in town has a blue dot on their forehead.

1. Prove that, eventually, every person must leave town.
2. How long does it take before everyone has left town?