

**COMP 2711 Discrete Mathematical Tools for CS**  
**Written Assignment # 6**  
**Distributed: 06 April 2016 – Due: 4:00pm, 13 April 2016**

Your solutions should contain (i) your name, (ii) your student ID #, (iii) your email address, (iv) your lecture section and (v) your tutorial section. Your work should be submitted to the collection bin outside Room 4210 (Lift 21).

**Problem 1:** If a student knows 75% of the material in a course, and if a 100-question multiple-choice test with five choices per question covers the material in a balanced way, what is the student's probability of getting a right answer to a question, given that the student guesses at the answer to each question whose answer he does not know?

**Problem 2:** Suppose a student who knows 60% of the material covered in a chapter of a textbook is going to take a five-question objective (each answer is either right or wrong, not multiple choice or true-false) quiz. Let  $X$  be the random variable that gives the number of questions the student answers correctly for each quiz in the sample space of all quizzes the instructor could construct.

- (a) What is the expected value of the random variable  $X - 3$ ?
- (b) What is the expected value of  $(X - 3)^2$ ?
- (c) What is the variance of  $X$ ?

**Problem 3:** Show that if  $X$  and  $Y$  are independent and  $b$  and  $c$  are constant, then  $X - b$  and  $Y - c$  are independent.

**Problem 4:** (a) Roll a fair die and let  $X$  be the number of dots showing on top. What are  $E(X)$  and  $Var(X)$ ?

(b) What are  $E(2X)$  and  $Var(2X)$ ?

(c) Now roll another die and let  $Y$  be the number of dots showing. What are  $E(X + Y)$  and  $Var(X + Y)$ ?

**Problem 5:** Flip four fair coins. let  $X$  be the number of heads showing. Now flip four  $\frac{1}{3}$ -biased coins (that is, they have  $P(H) = \frac{1}{3}$ ) and let  $Y$  be the number of heads showing.

- (a) What is  $E(X + Y)$ ?
- (b) What is  $Var(X + Y)$ ?

**Problem 6:** A standard *deck* contains 52 cards, 4 each of **2,3,4,5,6,7,8,9,10,J,Q,K,A**. Now start the following process. Pick a random card from the deck, show it, and then return it to the deck. Continue repeating this process, stopping when each type of card, **2,3,4,5,6,7,8,9,10,J,Q,K,A**, has been seen at least once. What is the expected number of cards that you will have drawn?

**Problem 7: (Challenge)** There are  $n \geq 1$  points randomly placed on the circumference of a circle. What is the probability that all  $n$  points lie along a semicircular arc?

For example, the 3 points in the left figure below lie along a semicircular arc but those in the right figure do not.

