COMP 2711 Discrete Mathematical Tools for Computer Science
2016 Spring Semester – Assignment # 3

Distributed: 26 February 2016 – Due: 4pm, 09 March 2016

Solutions

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address, (iv) your lecture section and (v) your tutorial section. Your work should be submitted to the collection bin outside Room 4210 (Lift 21).

Problem 1: Six schools are going to send their basketball teams to a tournament at which each team must play each other team exactly once. How many games are required?

SOLUTION: Number the six teams 1–6.
Team 1 must play all five others.
Team 2 will play one game with Team 1 (already counted) but must play in four more games with Teams 3, 4, 5, and 6.
Team 3 will play games with Teams 1 and 2 (already counted) and must still play in three more games with Teams 4, 5 and 6.
Team 4 will play games with Teams 1,2 and 3 (already counted) but must still play in two more games with Teams 5 and 6.
Team 5 will play games with Teams 1,2,3 and 4 (already counted) but must still play one more game with Team 6.
All of Team 6's games have already been counted.
Thus, there are $5 + 4 + 3 + 2 + 1 = 15$ games.

Alternatively, there are six teams, each of which must play in five games, giving us 30 ordered pairs of two teams. However, each game is counted twice as an ordered pair, so there are $30/2 = 15$ games.

Alternatively, since we choose two teams for each game, we have $\binom{6}{2} = 6 \cdot 5/2 = 15$ games.

Problem 2: In how many ways can a nine-person club select a president and a secretary-treasurer from among its members?

SOLUTION: There are 9 candidates for the president position; after choosing the president 8 candidates remain for the secretary position. So, by the product principle, there are $9 \cdot 8 = 72$ ways to select them.

Problem 3: In how many ways can a nine-person club select a two-person executive committee from among its members?

SOLUTION: This is the number of ways to choose a two-element subset from an nine-element set which is $\binom{9}{2} = 9 \cdot 8/2 = 36$.

Problem 4: In how many ways can a nine-person club select a president and a two-person executive advisory board from among its members (assuming that the president is not on the advisory board)?

SOLUTION: There are 9 candidates for the president position. After choosing the president there are eight persons remaining so the number of ways to choose the two-person executive advisory board is $\binom{8}{2} = 28$. From the product principle, there are then $9 \cdot \binom{8}{2} = 252$ ways.

Or, first select a two-person board ($\binom{9}{2} = 36$ ways) then a president from the remaining 7 people (7 ways). This gives the same answer $\binom{9}{2} \cdot 7 = 252$.

Problem 5: Using the formula for $\binom{n}{2}$ it is straightforward to show that

$$n\binom{n-1}{2} = \binom{n}{2}(n-2)$$

However, this proof simply uses blind substitution and simplification. Find a more conceptual explanation of why this formula is true. (Hint: Think in terms of officers and committees in a club.)

SOLUTION: This formula counts the number of ways to choose a president and a two-person executive advisory board (not including the president) from a club of n people. The left side chooses the president first, then the board. The right side chooses the board first, then the president.

Problem 6: The local ice cream shop sells eleven different flavors of ice cream. How many different two-scoop cones are there? (The two scoops might be of the same flavor or of two different flavors. Following your mother's rule that it all goes to the same stomach, a cone with a vanilla scoop on top of a chocolate scoop is considered the same as a cone with chocolate on top of vanilla.)

SOLUTION: By the product rule, there are $11 \cdot 10 = 110$ ways to choose two-scoop cones with two different flavors. However, according to your mother's rule, the order of scoops doesn't matter. Because each two-scoop cone can be ordered in two different ways (e.g., chocolate over vanilla and vanilla over chocolate), we have $110/2 = 55$ ways of choosing two-scoop cones with different flavors.

Alternatively, the number of ways to choose two flavors from 11 is $\binom{11}{2} = 55$.

Finally, there are an additional eleven cones with the same flavor for both scoops, giving $55 + 11 = 66$ possible cones.

Problem 7: Suppose you decide to disagree with your mother in Problem 6 – the order of the scoops does matter. How many different possible two-scoop cones are there?

SOLUTION: Because order does matter, we have, from the product principle, $11 \cdot 10 = 110$ ways to choose ice cream cones with two distinct flavors, plus eleven more with the same flavor for both scoops, giving 121 choices.

Alternatively, we have 11 choices for the first scoop and, after choosing the first scoop, 11 choices again for the second scoop. Therefore, by the product principle, we have $11 \cdot 11 = 121$ possible choices.