COMP 2711 Discrete Mathematical Tools for Computer Science
2016 Spring Semester – Assignment # 2
Distributed: 19 February 2016 – Due: 4pm, 26 February 2016

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address, (iv) your lecture section and (v) your tutorial section. Your work should be submitted before 4PM of the due date in the collection bin outside Room 4210 (Lift 21).

Problem 1. Let $P(x, y)$ be a propositional function. Show that the implication $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$ is a tautology.

Problem 2. In Problem 5 of Homework Assignment 1, does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion? Justify your answer briefly.

Problem 3: (a) Construct a contrapositive proof that for all real numbers x, if $x^2 - 2x \neq 3$ then $x \neq 3$.
(b) Construct a proof by contradiction that for all real numbers x, if $x^2 - 2x \neq 3$ then $x \neq 3$.

Problem 4. Prove that $\sqrt{20}$ is irrational.

Problem 5. Is the following reasoning for finding the solutions of the equation $\sqrt{x+3} = 3 - x$ correct?

(1) $\sqrt{x+3} = 3 - x$: Given
(2) $x + 3 = 9 - 6x + x^2$: Square both sides
(3) $x^2 - 7x + 6 = 0$: Re-arrange the terms.
(4) $(x - 1)(x - 6) = 0$: Factor the right hand side.
(5) So, $x = 1$ or $x = 6$.

Problem 6. (Challenge Problem) Three friends Tom, Jerry and Spike play a game with a host. The host has 3 black hats and 2 white hats on his hand initially. He turns off the light, puts one hat on each player, and hides the other two hats. Then he turns on the light and asks each player to guess what color hat he is wearing.

The three players stand on one line and face one direction, so that Tom can see the hats of both Jerry and Spike, Jerry can see only the hat of Spike, and Spike cannot see anyone's hat.

Tom speaks first and says "I don't know". Then Jerry says "I don't know either". After that, Spike says "I know".

What color hat is Spike wearing? How does he know?