

COMP 3711 Design and Analysis of Algorithms
Spring 2015
Assignment 3

1. Let's consider a long river, along which n houses are scattered. You can think of this river as an axis, and the houses are given by their coordinates on this axis in a sorted order. Your company wants to place cell phone base stations at certain points along the river, so that every house is within 4 kilometers of one of the base stations. Give an $O(n)$ -time algorithm that minimizes the number of base stations used, and show that it indeed yields the optimal solution.
2. Give an $O(n^2)$ -time dynamic programming algorithm to find the longest monotonically increasing subsequence of a sequence of n numbers (i.e, each successive number in the subsequence is greater than or equal to its predecessor). For example, if the input sequence is $\langle 5, 24, 8, 17, 12, 45 \rangle$, the output should be either $\langle 5, 8, 12, 45 \rangle$ or $\langle 5, 8, 17, 45 \rangle$.
3. Given a set $S = \{a_1, a_2, \dots, a_n\}$ of n positive integers and a positive integer W , design an $O(nW)$ -time dynamic programming algorithm to decide whether there is a subset $K \subseteq S$ such that the sum of all numbers in K is W . For example, if $S = \{1, 4, 7, 3, 5\}$ and $W = 11$, then the answer is “yes” since K can be $\{4, 7\}$. (Your algorithm doesn't have to output the subset K .) If no such subset exists, your algorithm should return “no”.
4. A subsequence is *symmetric* if it is the same whether read from left to right or right to left. For instance, the sequence

A C G T G T C A A A A T C G

contains many symmetric subsequences, including A C G C A and A A A A (on the other hand, the subsequence A C T is not symmetric). Design an dynamic programming algorithm that takes a sequence $x[1..n]$ and returns the longest symmetric subsequence. Its running time should be $O(n^2)$.

Note that a “smart” solution to this problem (actually given by some students in previous years) is to simply find the longest common subsequence of the string and its reverse. This is unfortunately not correct. For example, for the string ACBAC, the longest common subsequence of ACBAC and its reverse CABCA is ABC, but it is not symmetric.