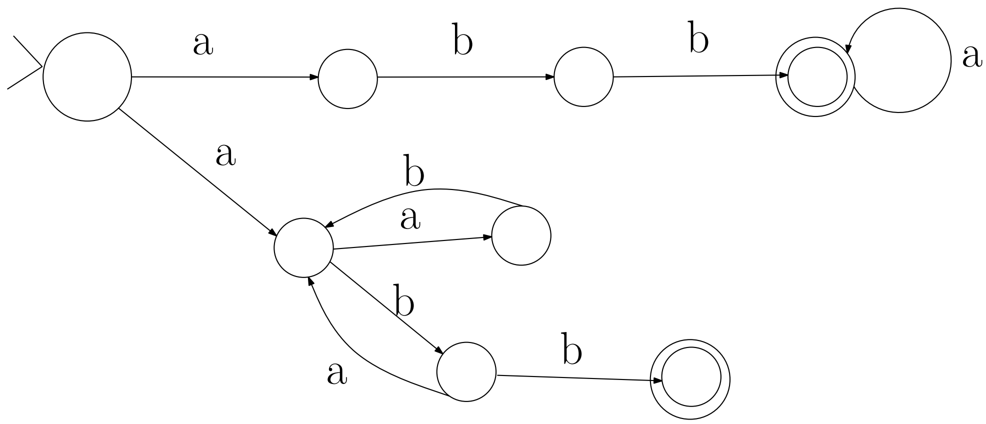
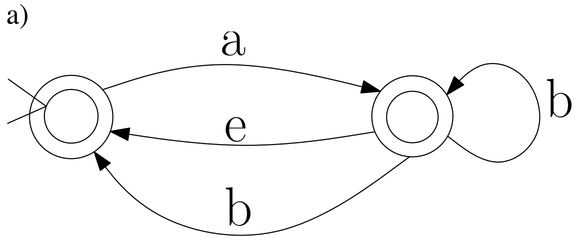
COMP3721 Homework 1

1. Proof 1: We count all A's such that the largest number in A is 0, then all A's such that the largest number in A is 1, $\ldots$, all A's such that the largest number in A is k, $\ldots$. For any k, there are finite number of such A's. We will also reach any A such that $|A|$ is finite since the largest number in any such A is finite.

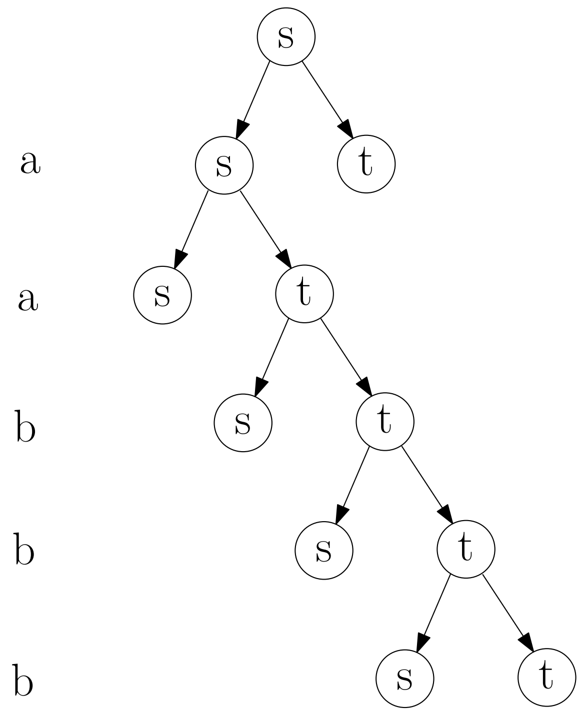
   Proof 2 (sketch): We know the Cartesian product of two countable sets A and B is countable. By induction we can easily prove that $\mathcal{N}^k$ is countable for any fixed integer k. The set $\{A|A \subseteq \mathcal{N}$, A is of size $k\}$ is a subset of $\mathcal{N}^k$, so it is countable for any integer k. Further by using the fact that the union of countably infinitely many countable sets is countable, we complete the proof.

2. a) $(ab^*a \bigcup b)^*ab^*$. b) $(a \bigcup b)^*ab(a \bigcup b)^*$. c) b^*a^*.

3. 

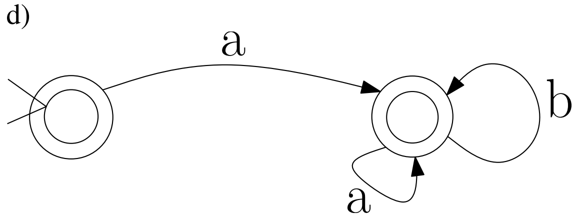
4. a)


   b)



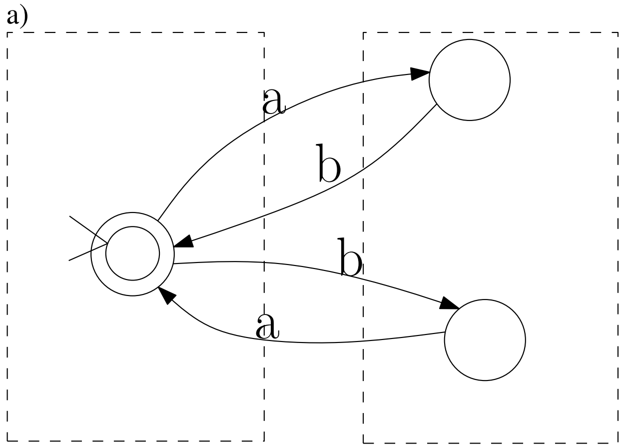
c) $(ab^*)^*$

d)



5. The language of M' is $\{w | w \text{ is a prefix of } w', \text{ where } w' \in L(M)\}$.

6. a)



States for the first tape States for the second tape

b) i) $M = (K_1, K_2, \Sigma, \delta, S, F)$, $F \subseteq K_1 \bigcup K_2$, $\delta : K_1 \bigcup K_2 \times \Sigma \rightarrow K_1 \bigcup K_2$.

ii) $(q, w_1, w_2), q \in K_1 \bigcup K_2, w_1, w_2 \in \Sigma^*$.

iii) $(q, w_1, w_2) \vdash (q', w_1', w_2')$. If $q \in K_1$, $w_1 = aw_1'$ s.t $\delta(q, a) = q'$ and $w_2' = w_2$. If $q \in K_2, w_2 = aw_2'$ s.t. $\delta(q, a) = q'$ and $w_1' = w_1$.

iv) $(s, w_1, w_2) \vdash^* (q, e, e)$, s.t. $q \in F$.

v) $L(M) = \{(w_1, w_2) \mid (s, w_1, w_2) \vdash^* (q, e, e)$, s.t. $q \in F\}$.