

COMP 3721: Theory of Computation
Written Assignment 2
Assigned: March 11 Due: March 20

Question 1

Prove that the following languages are not regular. [Hint: use the Pumping theorem and/or the closure properties of regular languages):

- a) $\{01^n 01^n : n \geq 0\}$
- b) $\{w \in \{0, 1\}^* : w = x^2 \text{ for some } x \in \{0, 1\}^*\}$
- c) $\{w \in \{0, 1\}^* : w = xx^R \text{ for some } x \in \{0, 1\}^*\}$

Question 2

A palindrome is a string that is equal to its own reversal (e.g., *aababaa*).

- a) Give a context-free grammar for the language of all strings over $\{a, b\}$ that are palindrome.
- b) Give a context-free grammar for the language of all strings over $\{a, b\}$ that are not palindromes.

Note that this doesn't mean that context free languages are closed under complementation!

Question 3

Let M_1 and M_2 be pushdown automata. Construct a pushdown automaton that accepts $L(M_1)L(M_2)$. Give a state diagram as well as a 6-tuple for your automaton.

Question 4

Consider a variant of pushdown automata (PA) in which the number of symbols that can be pushed onto the stack is bounded by some constant. We define the class of CONSTANT-STACK languages to be those languages L such that there exists a PA M and a constant k , such that M accepts L , and in addition, for every string w accepted by M , there is at least one accepting computation sequence during which the stack never contains more than k symbols at any time during the computation. Note that the value of the constant k may differ for different CONSTANT-STACK languages. For example, for every positive integer i , the language $L_i = \{w c w^R : w \in \{a, b\}^* \text{ and } |w| = i\}$ is a CONSTANT-STACK language, because there is a PA that accepts L_i , with at most i symbols in the stack at any time.

- (a) Briefly explain why $\text{REGULAR} \subseteq \text{CONSTANT-STACK}$.
- (b) Briefly explain why $\text{CONSTANT-STACK} \subseteq \text{CONTEXT-FREE}$.
- (c) Since we know that the set REGULAR is properly contained in the set CONTEXT-FREE, at least one of the containments in (a) and (b) must be proper. Decide which one of the following holds. Explain your answer on an intuitive level.
 - (i) $\text{REGULAR} \subset \text{CONSTANT-STACK}$ or
 $\text{REGULAR} = \text{CONSTANT-STACK}$

(ii) $\text{CONSTANT-STACK} \subset \text{CONTEXT-FREE}$ or
 $\text{CONSTANT-STACK} = \text{CONTEXT-FREE}$