COMP2711H Tutorial 9

Yuchen Mao

Department of Computer Science and Engineering,
Hong Kong University of Science and Technology

1 Markov's Inequility

Theorem 1.1. Let X be a nonnegative random variable. Then, for any $a > 0$,

$$Pr(X \geq a) \leq \frac{E[X]}{a}$$

2 Chebyshev's Inequility

Theorem 2.1. Let X be a random variable. Then, for any $a > 0$,

$$Pr(|X - E[X]| \geq a) \leq \frac{Var[X]}{a^2}$$

3 Moment Generting Function

Definition 3.1. The moment generating function of a random variable X is

$$M_x(t) = E[e^{tX}]$$

$M_x(t)$ captures all of the moments of X.

Theorem 3.1. Let X be a random variable with moment generating function $M_X(t)$. Under the assumption that exchanging the expectation and differentiation operands is legitimate, for all $n > 1$ we have

$$E[X^n] = M_X^{(n)}(0)$$

4 Chernoff Bound

Theorem 4.1. For any $t > 0$,

$$Pr(X \geq a) = Pr(e^{tX} \geq e^{ta}) \leq \frac{E[e^{tX}]}{e^{ta}}$$

Exercise 1. Let X be a binomial random variable with parameter n and p. Bound $Pr(X \geq (1+\delta)np)$ by Markov's Inequility, Chebyshev's Inequility and Chernoff bound.