COMP 2711 Discrete Mathematical Tools for CS
Written Assignment # 4
Distributed: 8 October 2014 – Due: 14 October 2014

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions should be submitted before 5PM of the due date, in the collection bin near Room 4210A (Lift 21).

Problem 1: If a student knows 75% of the material in a course, and if a 100-question multiple-choice test with five choices per question covers the material in a balanced way, what is the student's probability of getting a right answer to a question, given that the student guesses at the answer to each question whose answer he does not know?

Problem 2: Suppose a student who knows 60% of the material covered in a chapter of a textbook is going to take a five-question objective (each answer is either right or wrong, not multiple choice or true-false) quiz. Let X be the random variable that gives the number of questions the student answers correctly for each quiz in the sample space of all quizzes the instructor could construct.
(a) What is the expected value of the random variable $X - 3$?
(b) What is the expected value of $(X - 3)^2$?
(c) What is the variance of X?

Problem 3: Show that if X and Y are independent and b and c are constant, then $X - b$ and $Y - c$ are independent.

Problem 4: (a) Roll a fair die and let X be the number of dots showing on top. What are $E(X)$ and $Var(X)$?
(b)What are $E(2X)$ and $Var(2X)$?
(c) Now roll another die and let Y be the number of dots showing. What are $E(X+Y)$ and $Var(X+Y)$?

Problem 5: Flip four fair coins. let X be the number of heads showing. Now flip four $\frac{1}{3}$-biased coins (that is, they have $P(H) = \frac{1}{3}$) and let Y be the number of heads showing.
(a) What is $E(X+Y)$?
(b) What is $Var(X+Y)$?

Problem 6: A standard *deck* contains 52 cards, 4 each of **2,3,4,5,6,7,8,9,J,Q,K,A**. Now start the following process. Pick a random card from the deck, show it, and then return it to the deck. Continue repeating this process, stopping when each type of card, **2,3,4,5,6,7,8,9,J,Q,K,A**, has been seen at least once. What is the expected number of cards that you will have drawn?

Problem 7: (**Challenge**) There are $n \geq 1$ points randomly placed on the circumference of a circle. What is the probability that all n points lie along a semicircular arc?

For example, the 3 points in the left figure below lie along a semicircular arc but those in the right figure do not.