COMP 2711 Discrete Mathematical Tools for CS
Written Assignment # 3
Distributed: 26 September 2014 – Due: 03 Octorber 2014

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions should be submitted before 5PM of the due date, in the collection bin near Room 4210A (Lift 21).

Problem 1: The eight kings and queens are removed from a deck of cards, and then two of these cards are selected (from the eight). What is the probability that the king or queen of spades is among the cards selected?

Problem 2: Calculate

$$\sum_{\substack{i_1,i_2,i_3:\\ 1\le i_1<i_2<i_3\le 5}} i_1 \cdot i_2 \cdot i_3$$

Problem 3 In this problem, a *black card* is a spade or a club.
Remove one card from an ordinary deck of cards. What is the probability that it is an ace, a diamond, or black? Use the inclusion-exclusion formula to solve this problem.

Problem 4: In this exercise you will solve the following problem:
If you roll eight dice, what is the probability that each of the numbers 1 through 6 appears on top at least once?

For $1 \le i \le 6$, let E_i be the event that number i doesn't show up on any of the dice.

(a) Write a formula for $P(E_i)$.

(b) Let $k \leq 6$ and $1 \leq i_1 < i_2 < \cdots < i_k \leq 6$.
Write a formula for $P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k})$.

(c) Now use the inclusion-exclusion formula to write a formula for $P(E_1 \cup E_2 \cup \cdots \cup E_6)$.
This is the probability that *some* number doesn't appear when you roll eight die.
Your formula should use the summation sign, powers and binomial coeficients.

(d) Using the solution to (c), write down the probability that each of the numbers 1 through 6 appears on top at least once (a solution in the form of a sum is fine; it is not necessary to actually calculate the value of the sum).

Problem 5: In this exercise you will solve the following problem:
If you are hashing n keys into a hash table with k locations (buckets), what is the probability that every location gets at least one key? This probability can be expressed as a formula using the summation ($\sum$) symbol.

Hint: To solve this problem let E_i be the event that bucket E_i is empty. Then $E_1 \cup E_2 \cup \cdots \cup E_k$ is the event that at least one bucket is empty.

Let X be the event that every bucket gets at least one key*. Then X is the complement of $E_1 \cup E_2 \cup \cdots \cup E_k$ and the problem is asking you to find*

$$P(X) = 1 - P(E_1 \cup E_2 \cup \cdots \cup E_k).$$

You can now use the inclusion-exclusion formula to find $P(E_1 \cup E_2 \cup \cdots \cup E_k)$.

Problem 6: Six married couples (i.e., 12 people) sit down at random in a row of 12 seats. That is, each one of the 12! different ways of seating the people is equally likely to occur.

We say that a couple *sits together* if the husband and wife in that couple sit next to each other.

In the following, you may express your answers using the summation ($\sum$) sign, binomial coefficients $\left(\binom{n}{m}\right)$, factorials ($n!$) and exponentials ($c^k$). Actual numerical solutions are not necessary.

(a) Consider two specific couples. The first couple c_1 is Peter and Mary and the second couple c_2 is John and Helen. Since Peter and John are friends, they want to sit next to each other. What is the probability that each of these two couples (i.e., c_1 and c_2) sits together, and Peter and John sit next to each other?

(b) What is the probability that every couple sits together?

(c) What is the probability that no couple sits together?