

**COMP 3721: Theory of Computation**  
**Written Assignment 3**  
**Assigned: April 10      Due: April 22**

**Question 1**

Construct a one-tape Turing machine to perform the following task:

- If the input string is empty or just a symbol, halt; otherwise, transform the input string  $\sigma_1\sigma_2\cdots\sigma_n$  into  $\sigma_1\#\sigma_2\#\cdots\#\sigma_n$ , then halt.

You may assume that the input string consists only of  $a$ 's and  $b$ 's, and that initially the tape head is immediately to the right of  $\triangleright$ . The tape head should return to the initial position when the machine halts.

Note: You need to draw the TM by combining the simpler machines (e.g., symbol-writing and head-moving machines), as described in class.

**Question 2**

Prove or disprove:

- a) If  $L_1$  is regular and  $L_2$  is recursively enumerable (i.e., Turing-acceptable), then  $L_1 - L_2$  is regular.
- b) If  $L_1$  is recursively enumerable and  $L_2$  is context-free, then  $L_1 \cap L_2$  is recursively enumerable.

**Question 3**

Show that a language  $L$  is recursive if and only if there is a Turing machine  $M$  that enumerates the strings of  $L$  in a non-decreasing order of their lengths.

**Question 4**

Are the following problems solvable? Justify your answer.

- a) Given a deterministic Turing machine  $M$ , does there exist a string  $w$  such that  $M$  accepts  $w$  in less than 10 steps?
- b) Given a deterministic Turing machine  $M$ , does there exist a string  $w$  such that  $M$  accepts  $w$  in at least 10 steps?