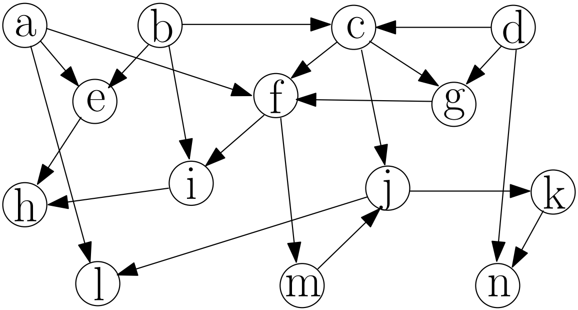
COMP3711: Design and Analysis of Algorithms

Tutorial 11

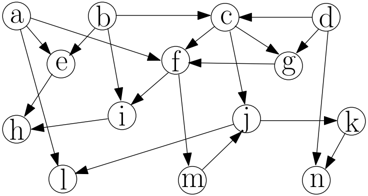
HKUST

Question 1

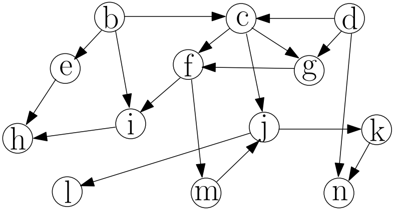
Show the topological ordering of the following graph.



Solution 3



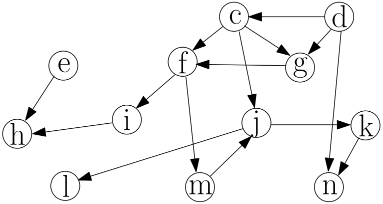
$Q = \{a, b, d\}$



$Q = \{b, d\}$

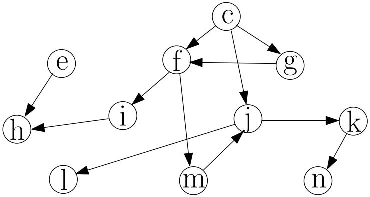
Output: a

Solution 3



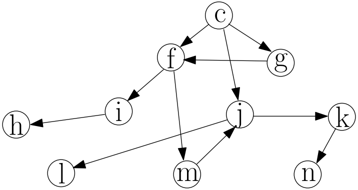
$Q = \{d, e\}$

Output: a, b



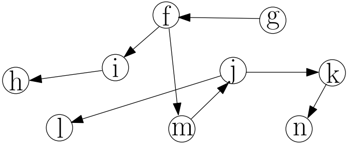
$Q = \{e, c\}$

Output: a, b, d



$Q = \{c\}$

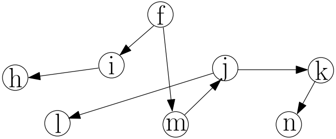
Output: a, b, d, e



$Q = \{g\}$

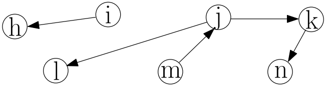
Output: a, b, d, e, c

Solution 3



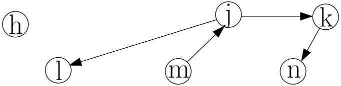
$Q = \{f\}$

Output: a, b, d, e, c, g



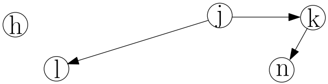
$Q = \{i, m\}$

Output: a, b, d, e, c, g, f



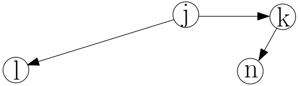
$Q = \{m, h\}$

Output: a, b, d, e, c, g, f, i



$Q = \{h, j\}$

Output: $a, b, d, e, c, g, f, i, m$



$Q = \{j\}$

Output: $a, b, d, e, c, g, f, i, m, h$

$Q = \{k, l\}$

Output: $a, b, d, e, c, g, f, i, m, h, j$

Solution 3

$Q = \{l, n\}$

Output: $a, b, d, e, c, g, f, i, m, h, j, k$

l n

$Q = \{n\}$

Output: $a, b, d, e, c, g, f, i, m, h, j, k, l$

n

$Q = \{\}$

Output: $a, b, d, e, c, g, f, i, m, h, j, k, l, n$

Question 2

Given an undirected weighted graph $G = (V, E)$ with non-negative distinct edge weight and an MST T of it. (a) Replace the weight of each edge w by w^2. Is T still an MST for the new graph? (b) Next we consider a shortest path $u \to v$ in the original graph. Is this path still a shortest path from u to v in the new graph? If yes, prove so; if not, give a counter example.

Solution 2

(a) Yes, the MST remains unchanged when the weights are changing from w to w^2. There are two ways to prove it:

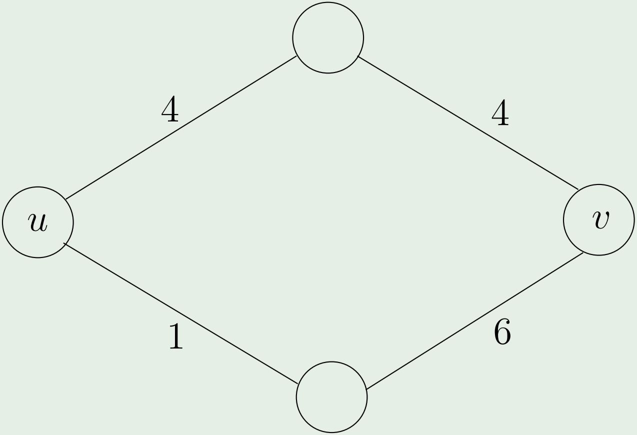
(1) We observe that Prim's algorithm only compares the weights of edges, and changing the weights from w to w^2 does not change the result of any comparison. Thus Prim's algorithm's output will remain the same. From the correctness of Prim's algorithm, we conclude that T is still the MST of the new graph.

(2) We can mimic the proof on the uniqueness of MST. Consider any edge e of T. Removing e from T breaks it into S and $V - S$. e is the min cost edge crossing the cut in the original graph, so it must still be the min cost edge crossing the cut in the modified graph. Apply the cut lemma on S, and we know that any MST in the new graph must contain e. This argument holds for every edge e of T, thus T remains the MST of the new graph.

Solution 2

(b) No, it may not be a shortest path in the new graph.

Example



Question 3

Let G be a connected undirected graph with distinct weights on the edges, and let e be an edge of G. Suppose e is the largest-weight edge in some cycle of G. Show that e cannot be in the MST of G.

Solution 3

Let T be the MST of G and suppose T contains $e = (u, v)$. Removing e from T breaks it into S and $V - S$. Consider the cycle that has e as the largest-weight edge. This cycle starts from u, goes to v, and takes another path to go back to u. This path must cross this cut somewhere via an edge e' with $w(e') < w(e)$. Now we add e' to T and this turns it back to a spanning tree T'. We see that T' has a smaller weight than T, which contradicts with the early assumption that T is the MST. Thus T cannot contain e.