COMP 2711 Discrete Mathematical Tools for CS
2014 Fall Semester – Solution to Written Assignment # 8
Distributed: 20 Nov 2014 – Due: 28 Nov 2014

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions can be submitted before 5PM of the due date, in the collection bin in front of Room 4213A.

Problem 1: Prove that

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$.

Problem 2: Prove that every integer greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
(Hint: first prove the three base cases *of* $n = 8, 9, 10$ *and then prove the inductive step assuming that* $n > 10$*.)*

Problem 3: Consider the recurrence $M(n) = 2M(n-1)+2$, with base case of $M(1) = 1$.
(a) State the solution to this recurrence
(you may use Theorem 4.1 in the book).
(b) Use induction to prove that this solution is correct.

Problem 4: Challenge Problem: Leaving Dot-town
Every person living in Dot-town has a red or blue dot on his forehead but doesn't know the color of his own dot. Every day the people gather in the town square to talk with each other. If anyone ever figures out the

color of his own dot he must leave town before the next gathering. People never leave Dot-town unless they figure out their own dot color. One day, a stranger comes to town and casually mentions that at least one person in town has a blue dot on their forehead.

1. Prove that, eventually, every person must leave town.
2. How long does it take before everyone has left town?