COMP 3721: Theory of Computation
Spring 2013 Midterm Exam

1. Print your name and student ID at the top of every page (in case the staple falls out!).
2. This is an open-book, open-notes exam.
3. Time limit: 80 minutes.
4. When asked to describe a DFA, NFA, or pushdown automaton, you can use either the state diagram (the preferred method) or the formal definition.
5. You can write on the back of the paper if you run out of space. Please let us know if you need more scratch paper.

1. (5 pts) Let $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Design a DFA or an NFA that only accepts one string which is your student ID.

2. (20 pts) Let $\Sigma = \{0, 1\}$. For any natural number n, let $b(n) \in \Sigma^*$ be the binary representation of n. For example, $b(0) = 0, b(11) = 1011$. Define the language

$$L = \{b(n) : 0 \leq n \leq 11\}.$$

(a) Write a regular expression to represent L.

(b) Design an NFA that accepts L.

(c) Design a DFA that accepts L.

3. (35 pts) Denote by $\mathsf{Regular}$ the set of all regular languages and CFL the set of all context-free languages. For each of the following statements, decide if it is right or wrong. For a wrong statement, please give a counter example (you can use any alphabet Σ).

   (a) If $A \in \mathsf{Regular}$ and $B \in \mathsf{Regular}$, then $A \ominus B \in \mathsf{Regular}$. ($\ominus$ is the *symmetric difference* of two sets, defined as $A \ominus B = (A - B) \cup (B - A)$)

   (b) If $A \in \mathsf{Regular}$ and $B \notin \mathsf{Regular}$, then $A \cap B \notin \mathsf{Regular}$.

   (c) If $A \notin \mathsf{Regular}$ and $B \notin \mathsf{Regular}$, then $A \cup B \notin \mathsf{Regular}$.

   (d) If $A \in \mathsf{Regular}$, $B \notin \mathsf{Regular}$, and $A \cap B = \emptyset$, then $A \cup B \notin \mathsf{Regular}$.

   (e) If $A \in \mathsf{CFL}$ and $B \in \mathsf{Regular}$, then $A \cap B \in \mathsf{Regular}$.

   (f) If $A \in \mathsf{CFL}$ and $B \in \mathsf{CFL}$, then $A \cup B \in \mathsf{CFL}$.

   (g) If $A \in \mathsf{CFL} - \mathsf{Regular}$ and $B \in \mathsf{CFL} - \mathsf{Regular}$, then $A \cap B \notin \mathsf{Regular}$.

4. (20 pts) Let $\Sigma = \{0, 1\}$. For any string $x \in \Sigma^*$, denote by $\overline{x}$ the string obtained by flipping every symbol of x, e.g., $\overline{00110} = 11001$. Let $L = \{x\overline{x} : x \in \Sigma^*\}$. Prove that L is not regular.

5. (20 pts) Prove that any regular language is also a context-free language, by showing that for any regular expression α, there exists a context-free grammar G such that $L(\alpha) = L(G)$. Give a direct proof without using the equivalence between regular expressions and DFA/NFA, or the equilavence between context-free grammars and pushdown automata.