COMP 2711H Discrete Mathematical Tools for Computer Science
2014 Fall Semester
Homework 1
Handed out: Sep 19
Due: Sep 26

Problem 1. Let p, q, and r be the following propositions "you get an A on the final exam", "you do every exercise in this book", and "you get an A in this class", respectively. Write the following propositions using p,q, and r and logical connectives.

(a) You get an A in this class, but you do not do every exercise in this book.

(b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Problem 2. State the converse and contrapositive of each of the following implications.

(a) If it snows tonight, then I will stay at home.

(b) I go to the beach whenever it is a sunny summer day.

(c) When I stay up late, it is necessary that I sleep until noon.

Problem 3. Determine whether the following compound propositions are tautologies. If yes, present two proofs of this fact (one, using truth tables, and the other without truth tables). If it is not a tautology, provide an assignment of truth values that makes the proposition false.

(a) $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

(b) $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$

Problem 4. Prove that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent. Present two different proofs of this fact, only one of which should use truth tables.

Problem 5. Show that $(\forall x P(x)) \vee (\forall x Q(x))$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

Problem 6. Let $P(x, y)$ be a propositional function. Show that the implication $\exists x \forall y P(x, y) \rightarrow \forall y \exists x P(x, y)$ is a tautology.

Problem 7. Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a duck", 'x is one of my poultry", x is an officer", and "x is willing to waltz", respectively. Express each of the following statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

(a) No ducks are willing to waltz.

(b) No officers ever decline to waltz.

(c) All my poultry are ducks.

(d) My poultry are not officers.

(e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion? Justify your answer briefly.

Problem 8. Prove that $\sqrt{20}$ is irrational.

Problem 9. Prove that the square of an integer not divisible by 5 leaves a remainder of 1 or 4 when divided by 5. (*Hint:* Use a proof by cases, where the cases correspond to the possible remainders for the integer when it is divided by 5.)

Problem 10. Prove or disprove that given a positive integer n, there are n consecutive odd positive integers that are primes.

Problem 11. Prove that there is no largest prime number.