TLUs

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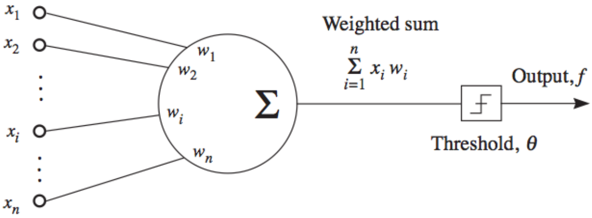
TLUs

- A type of circuits of particular interest in AI is **threshold logic unit** (TLU), also called **perceptron**:



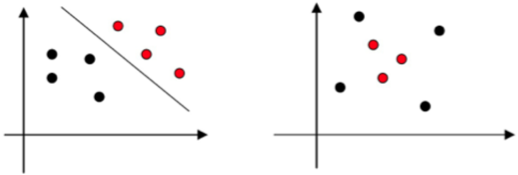
$$f = 1 \text{ if } \sum_{i=1}^{n} x_i w_i \geq \theta$$
$$= 0 \text{ otherwise}$$

Linearly Separable

Definition: Linearly Separable

Given 2 point sets (classes) A and B, if there exists a hyperplane H that can separate A and B, then A,B are linearly separable.

In n-dimension space, a **hyperplane** H is defined by a linear function $a_1x_1 + a_2x_2 + \ldots + a_nx_n + a_0 = 0$, where at least one of a_i $(i = 1, 2, \ldots, n)$ is non-zero.



Relation between TLU and LS

- A function can be represented by a single TLU iff the function is Linearly Separable (two sets of points A and B are divided by function values, 0 or 1)

Question

How to prove the statement above?

- TLU $\Rightarrow$ LS
- LS $\Rightarrow$ TLU

Proof

TLU $\Rightarrow$ LS

If a n-parameter function f can be represented by a TLU T_0:

- weight vector: $W =< w_1, w_2, \ldots, w_n >$
- threshold: θ.

By the definition of TLU, for any sensory vector
$v =< v_1, v_2, \ldots, v_n >$:

- if the value $f(v) = 1$, then $\sum_{i=1}^{n} v_i w_i \geq \theta$;
- if the value $f(v) = 0$ then $\sum_{i=1}^{n} v_i w_i < \theta$.

Thus, we can get a hyperplane H: $w_1 x_1 + w_2 x_2 + \ldots + w_n x_n - \theta = 0$.
H separates the sets of points into two sets according to the function value.

Proof Cont'd

LS $\Rightarrow$ TLU

If a n-parameter function f can be divided into two sets of points by a hyperplane H, according to the function values:

- H: $a_1x_1 + a_2x_2 + \ldots + a_nx_n + a_0 = 0$

There are two possibilities:

- **P1**
  - for all sensory vector $v = < v_1, v_2, \ldots, v_n >$ s.t. $f(v) = 1$, locate above H, so $\sum_{i=1}^{n} a_iv_i + a_0 > 0$;
  - for all sensory vector v s.t. $f(v) = 0$, locate below H, so $\sum_{i=1}^{n} a_iv_i + a_0 < 0$;
- **P2**
  - for all sensory vector v s.t. $f(v) = 1$, $\sum_{i=1}^{n} a_iv_i + a_0 < 0$;
  - for all sensory vector v s.t. $f(v) = 0$, $\sum_{i=1}^{n} a_iv_i + a_0 > 0$;

Proof Cont'd

Then construct a TLU T_0 as:

- if **P1**:
  - the weight vector: $\mathcal{W} = < a_1, a_2, \ldots, a_n >$
  - the threshold: $\theta = -a_0$
- if **P2**:
  - the weight vector: $\mathcal{W} = < -a_1, -a_2, \ldots, -a_n >$
  - the threshold: $\theta = a_0$

Exercises

Exercises on the tutorial page.