Sample Solution for Assignment 2

COMP3211

Problem 1 (10%)

The unique Nash equilibrium of this game would be **(Pol:expand, Fed:contract)**, i.e.(3,3) in the payoff matrix.

Problem 2 (10%)

(1)

If (A, A) is a Nash equilibrium, where payoffs are $(a, -a)$, then we have

$$\begin{cases} a \geq c \\ -a \geq -b \end{cases} \implies \quad c \leq a \leq b$$

(2)

Similarly, we have

$$\begin{cases} d \geq b \\ -d \geq -c \end{cases} \implies \quad b \leq d \leq c$$

(3)

If both (A, A) and (B, B) are Nash equilibria, then we have

$$\begin{cases} c \leq a \leq b \\ b \leq d \leq c \end{cases} \implies \quad a = b = c = d$$

Problem 3 (10%)

Formulate this auction as a game in normal form:

- A set of agents $N = \{1, 2\}$;
- The same set of actions for each agent $A_1 = A_2 = \{1, 2, 3, 4, 5, 6\}$;
- Utility functions

$$u_i(x_1, x_2) = \begin{cases} 6 - x_i & \text{if agent } i \text{ wins the auction} \\ 0 & \text{otherwise} \end{cases}$$

Generate the payoff matrix as follows to find the Nash equilibria:

	1	2	3	4	5	6
1	$2.5, 2.5$	$0, 4$	$0, 3$	$0, 2$	$0, 1$	$0, 0$
2	$4, 0$	$2, 2$	$0, 3$	$0, 2$	$0, 1$	$0, 0$
3	$3, 0$	$3, 0$	$1.5, 1.5$	$0, 2$	$0, 1$	$0, 0$
4	$2, 0$	$2, 0$	$2, 0$	$1, 1$	$0, 1$	$0, 0$
5	$1, 0$	$1, 0$	$1, 0$	$1, 0$	$0.5, 0.5$	$0, 0$
6	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, 0$

From the matrix, we can see that the Nash equilibria are $(4, 4), (5, 5)$ and $(6, 6)$.

Problem 4 (10%)

The TSP can be formulated as a search problem by specifying the followings:

- **States**: a set of nodes that have been visited once (the starting node might be visited twice).
- **Initial State**: the salesman is at the starting point, and no other node has been visited.
- **Operators**: the salesman moves from the current node to an adjacent unvisited node.
- **Goal test**: the salesman is at the starting point and all other nodes have been visited exactly once.

Problem 5 (10%)

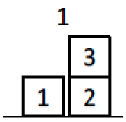
One of the possible ways to formulate *constructing crossword puzzle* as a search problem is to specify:

- **States**: any partially filled grid of rows and columns.
- **Initial State**: any state can be an initial state.
- **Operators**: change the current state by altering one of the squares: remove its current letter, change it to a new letter, or add a new letter.
- **Goal test**: a state is a goal state if it can be extended to a complete grid using the words from the given dictionary.

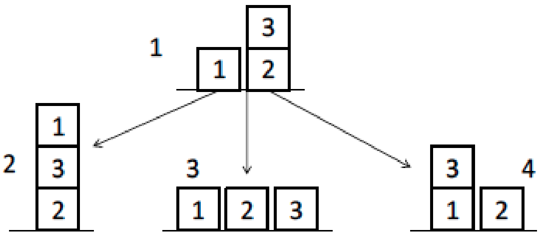
Problem 6 (20%)

(1)

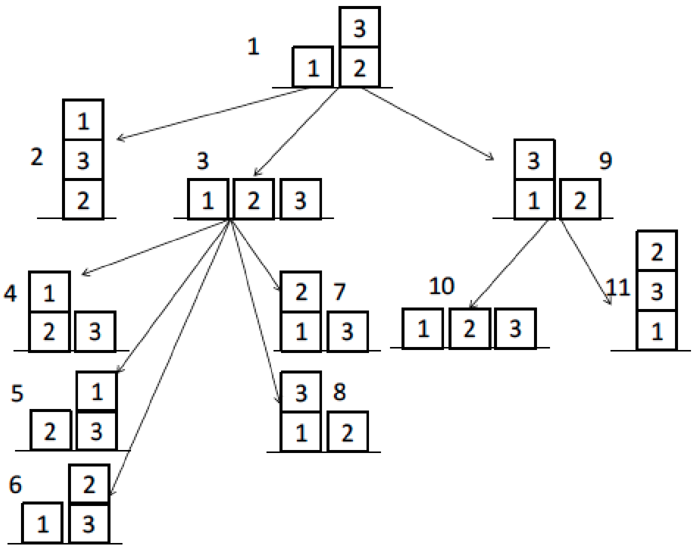
depth 0:



depth 1:



depth 2:



(2)

A possible heuristic function:

$$h(s) = \sum_{i=1}^{3} t(B_i, s)$$

where s is a state and

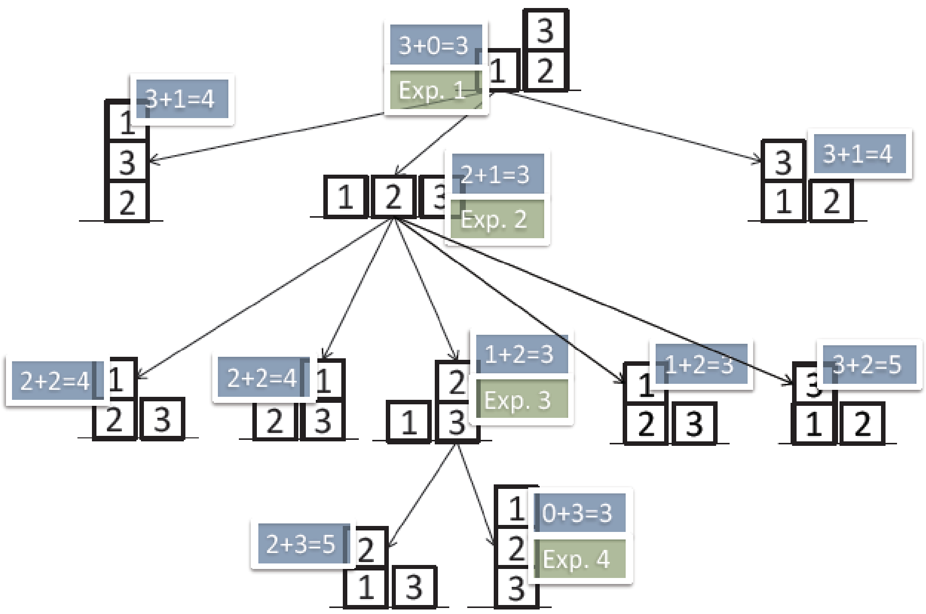
$$t(B,s) = \begin{cases} 0 & \text{if } ontable(B,g) \cdot ontable(B,s) + \exists B' \cdot on(B,B',g) \cdot on(B,B',s) \\ 1 & \text{otherwise} \end{cases}$$

where g is the goal state.

The above heuristic function h represents **the number of blocks not on top of the block or table as stated in the goal state**. Since at least one move must be made to move a misplaced block to the top of the right block, the above heuristic function always underestimates the number of moves required to reach the goal state. In other words, it is admissible.

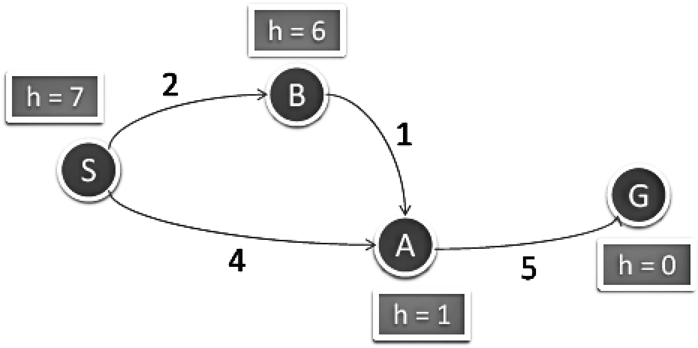
(3)

The search tree is drawn as below:

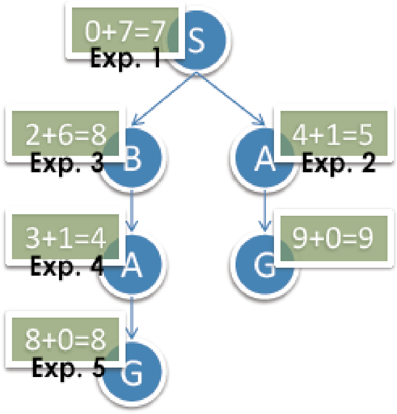


Problem 7 (20%)

The search problem can be illustrated by the graph below:



(1) A^* by tree



(2) A^* by graph

Only the final graph is shown below: