COMP 2711H Discrete Mathematical Tools for Computer Science
2014 Fall Semester
Homework 2
Handed out: Sep 26
Due: Oct 3

Problem 1. In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

(a) the bride must be in the picture?

(b) both the bride and groom must be in the picture?

(c) exactly one of the bride and the groom is in the picture?

Problem 2. How many bit strings of length seven either begin with two 0s or end with three 1s?

Problem 3.

(a) Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

(b) Is the conclusion in part (a) true if six integers are selected rather than seven?

Problem 4. For all parts of this problem, assume that any two people are either friends or enemies.

(a) Show that in any group of six people, there are either three mutual friends or three mutual enemies.

(b) Show that in any group of five people, there are not necessarily three mutual friends or three mutual enemies.

(c) Show that in any group of ten people, there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends. (*Hint:* To prove the first claim in part (c), focus on a particular member of the group. Show that this member has either four friends or six enemies in the group. Then apply the result of part (a) suitably.)

(d) Show that in any group of 20 people, there are either four mutual friends or four mutual enemies.

Problem 5. Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Problem 6. Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where k is an integer.

Problem 7. Prove that

$$C(n+r+1,r) = \sum_{k=1}^{r} C(n+k,k),$$

whenever n and r are positive integers,

(a) using a combinatorial argument.

(b) using Pascal's identity.

Problem 8. Give a combinatorial proof that $\sum_{k=1}^{n} kC(n,k) = n2^{n-1}$. (*Hint:* Count in two ways the number of ways to select a committee and to then select a leader of the committee.)

Problem 9. Give a combinatorial proof of the following fact: Any set has the same number of subsets with an odd number of elements as it does subsets with an even number of elements.

Problem 10. How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 13,$$

where x_1, x_2, and x_3 are nonnegative integers?

Problem 11. How many terms are there in the expansion of $(x+y+z)^{100}$?

Problem 12. How many different strings can be made from the letters in *ORONO*, using some or all of the letters?

Problem 13. A professor packs her collection of 40 issues of a mathematics journal in four boxes with 10 issues per box. How many ways can she distribute the journals if

(a) each box is numbered, so that they are distinguishable?

(b) the boxes are identical, so that they cannot be distinguished?

Problem 14. How many ways are there to travel in $xyzw$ space from the origin $(0,0,0,0)$ to the point $(4,3,5,4)$ by taking steps one unit in the positive x, positive y, positive z, or positive w direction? (Moving in the negative x, y, z, or w direction is prohibited, so that no backtracking is allowed.)

Problem 15. A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen? (*Hint:* Represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of five bars and seven stars so that no two bars are adjacent.)

Problem 16. Show that in any set of $n+1$ positive integers not exceeding $2n$ there must be two that are relatively prime.

Problem 17. How many bit strings of length n, where $n \geq 4$, contain exactly two occurrences of 01.

Problem 18. Suppose that p and q are distinct primes. Use the principle of inclusion-exclusion to find $\phi(pq)$, the number of integers not exceeding pq that are relatively prime to pq.

Problem 19. How many onto functions are there from a set with seven elements to one with five elements?

Problem 20. Find the number of primes less than 300 using the principle of inclusion-exclusion.