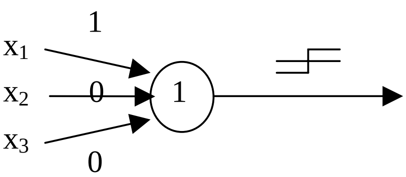
4. (Exercises 2.3) Indicate which of the following Boolean functions of 3 input variables can be realized by a single threshold element with weighted connections to the input.

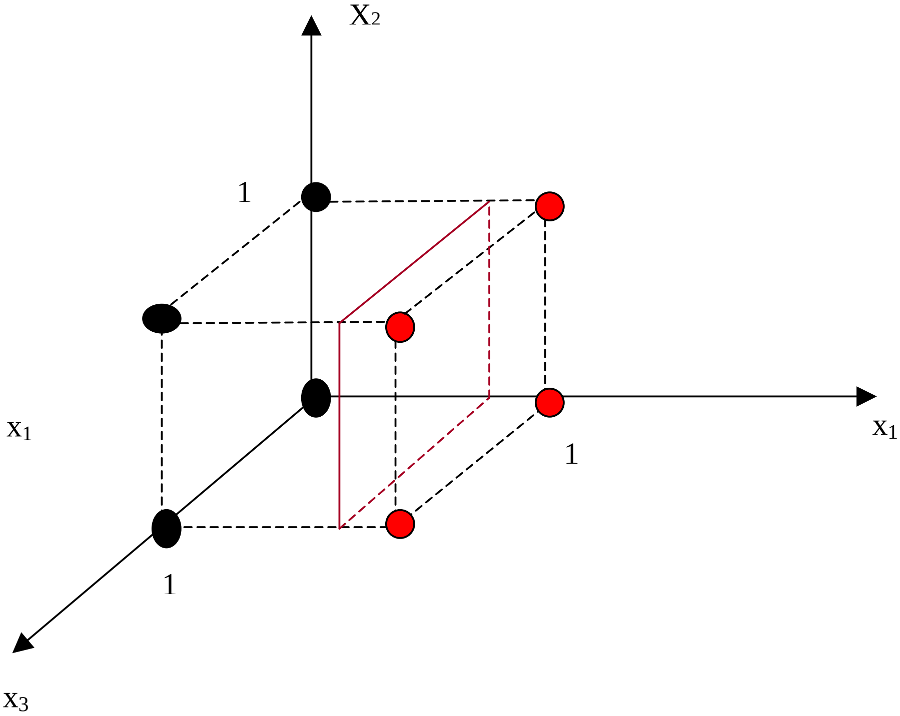
(1). x_1
(2). $x_1x_2x_3$
(3). x_1+x_2
(4). $(x_1x_2x_3) + (\bar{x}_1\bar{x}_2\bar{x}_3)$
(5). 1

Solution:

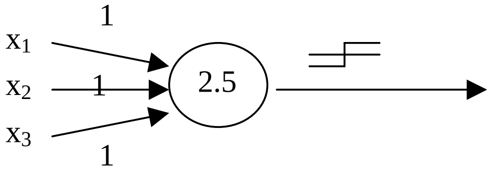
(1) yes.



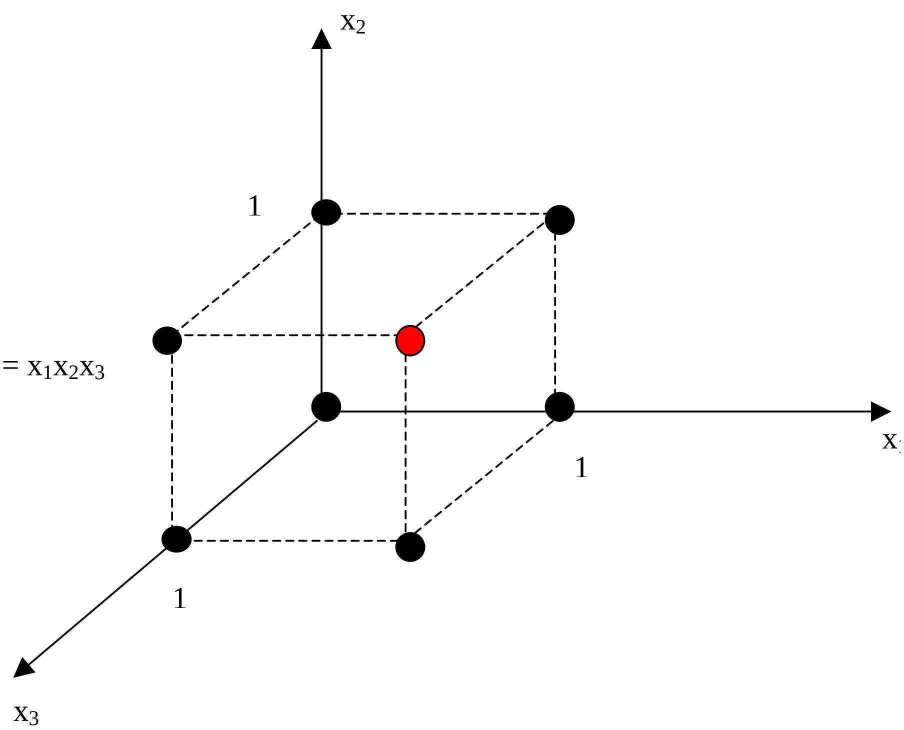
f = x_1



(2). Yes



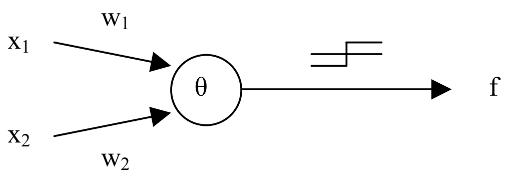
f = $x_1x_2x_3$



(3). x_1+x_2 (**Yes**.)
(4). $(x_1x_2x_3) + (\bar{x}_1\bar{x}_2\bar{x}_3)$ (**No**.)
(5). 1 (**Yes**.)

5. Learning of A Single TLU

- TLU:



- Relation function:

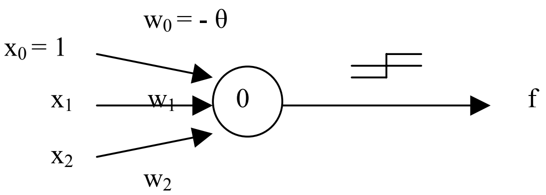
$$f = 1 \quad \text{if} \quad (w_1x_1 + w_2x_2 - \theta) \geq 0$$
$$= 0 \quad \text{otherwise}$$

- Training set:

	x_1	x_2	d
e_1	5	1	0
e_2	2	1	0
e_3	1	1	1
e_4	3	3	1
e_5	4	2	0
e_6	2	3	1

- To learn: w_1, w_2, θ

Solution 1:
(1)



$$f = 1 \quad \text{if} \quad (w0x_0 + w_1x_1 + w_2x_2) \geq 0$$
$$= 0 \quad \text{otherwise}$$

(2)

$W^{new} = W^{old} + c\ (\ d - f\ )\ X$

(3) Take c = 1 and the initial weight values to be 0.

Iteration		W^{old} =(w_0, w_1, w_2)	X =(x_0, x_1, x_2)	d	Σ	f	d= f ?	W^{new} =(w_0, w_1, w_2)
1	e_1	(0, 0, 0)	(1, 5, 1)	0	0	1	no	(-1, -5, -1)
2	e_2	(-1, -5, -1)	(1, 2, 1)	0	-12	0	yes	(-1, -5, -1)
3	e_3	(-1, -5, -1)	(1, 1, 1)	1	-7	0	no	(0, -4, 0)
4	e_4	(0, -4, 0)	(1, 3, 3)	1	-12	0	no	(1, -1, 3)
5	e_5	(1, -1, 3)	(1, 4, 2)	0	3	1	no	(0, -5, 1)
6	e_6	(0, -5, 1)	(1, 2, 3)	1	-7	0	no	(1, -3, 4)
7	e_1	(1, -3, 4)	(1, 5, 1)	0	-10	0	yes	(1, -3, 4)
8	e_2	(1, -3, 4)	(1, 2, 1)	0	-1	0	yes	(1, -3, 4)
9	e_3	(1, -3, 4)	(1, 1, 1)	1	2	1	yes	(1, -3, 4)
10	e_4	(1, -3, 4)	(1, 3, 3)	1	4	1	yes	(1, -3, 4)
11	e_5	(1, -3, 4)	(1, 4, 2)	0	-3	0	yes	(1, -3, 4)
12	e_6	(1, -3, 4)	(1, 2, 3)	1	7	1	yes	(1, -3, 4)

(4). Stable. So we have :

$w_0 = 1$;
$w_1 = -3$;
$w_2 = 4$;

Since $w_0 = -\theta$, We got $\theta = -1$. The boundary line is: $-3x_1 + 4x_2 + 1 = 0$.

Solution 2:

(1)(2) are same with above.

(3) Take c=0.5 and the initial weight values: $w_0 = 0$, $w_1 = 1$,$w_2 = 1$

Iteration		W^{old} =(w_0, w_1, w_2)	X =(x_0, x_1, x_2)	d	Σ	f	d= f ?	W^{new} =(w_0, w_1, w_2)
1	E_1	(0, 1, 1)	(1, 5, 1)	0	6	1	no	(-0.5, -1.5, 0.5)
2	E_2	(-0.5, -1.5, 0.5)	(1, 2, 1)	0	-3	0	yes	(-0.5, -1.5, 0.5)
3	E_3	(-0.5, -1.5, 0.5)	(1, 1, 1)	1	-1.5	0	no	(0, -1, 1)
4	E_4	(0, -1, 1)	(1, 3, 3)	1	0	1	yes	(0, -1, 1)
5	E_5	(0, -1, 1)	(1, 4, 2)	0	-2	0	yes	(0, -1, 1)
6	E_6	(0, -1, 1)	(1, 2, 3)	1	1	1	yes	(0, -1, 1)
7	E_1	(0, -1, 1)	(1, 5, 1)	0	-4	0	yes	(0, -1, 1)
8	e_2	(0, -1, 1)	(1, 2, 1)	0	-1	0	yes	(0, -1, 1)
9	e_3	(0, -1, 1)	(1, 1, 1)	1	0	1	yes	(0, -1, 1)

Stable. So we have another set of weights:

$w_0 = 0$,

$w_1 = -1$,

$w_2 = 1$

then θ = 0 and the boundary is: $-x_1 + x_2 = 0$

- .If a training set of examples is linerly separable, then applying the preceptron weight updating rule can always converge to some solution (i.e. , a set of weights) in a *finite numb*er of steps for any initial choice of weights.
- The exact number of steps needed depends on:
  - ✓ Initial weight values
  - ✓ Learning rate in weight updating rule
  - ✓ Order of presentation of traing examples