Pruning and CSP

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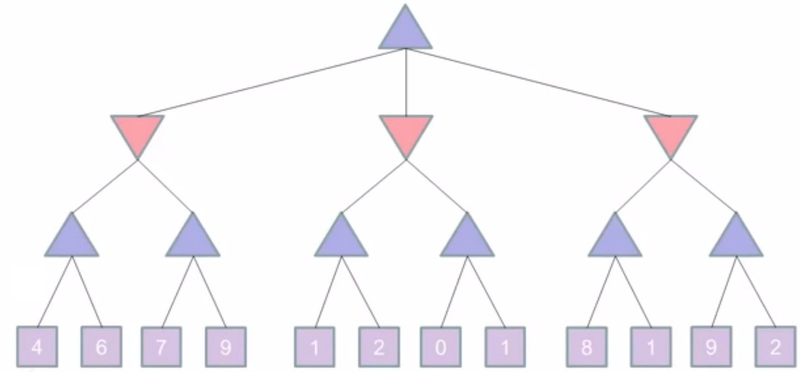
Informal Description

To evaluate a MAX node in a game tree (we shall call the value assigned to a MAX node its alpha value, and that to a MIN node its beta value):

1. Expand the node depth-first until a node where the cutoff test is reached.
2. Evaluate the cutoff node.
3. Update the values of all the nodes that have so far been expanded according to Minimax algorithm, and apply the following pruning strategy:
    - prune all children of any MIN node whose beta value is $\leq$ the alpha value of any of its MAX ancestors.
    - prune all children of any MAX node whose alpha value is $\geq$ the beta value of any of its MIN ancestors.
4. Backtrack to a node that has not been pruned, and go back to step 1. If there are no such node to backtrack to, then return with the value assigned to the original node.

Exercise 1 - Minimax search with alpha-beta pruning



α = best already explored option along path to the root for MAX;
β = best already explored option along path to the root for MIN.

Constraint Satisfaction Problem(CSP)

Problem definition:

- A finite set of variables and their domains.
- A finite set of conditions on these variables.
- A solution is an assignment to these variables that satisfies all the conditions.

Simple problems that can be modeled as a CSP:

- Eight queens puzzle
- Map coloring problem
- Sudoku, Futoshiki, Kakuro (Cross Sums), Numbrix, Hidato and many other logic puzzles

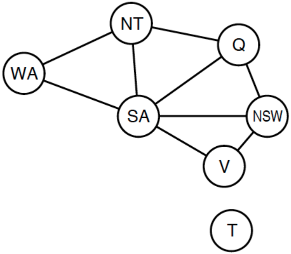
Example: Map Coloring

Color a map so that no adjacent parts have the same color

What are the variables, domains and constraints?

Example: Map Coloring Cont’d

A **constraint graph** is often used to do constraint propagation: nodes are variables, arcs show constraints.

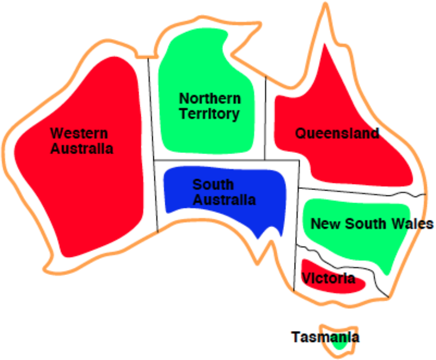


- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $D_i = \{red, green, blue\}$
- Constraints: adjacent regions must have different colors e.g. $WA \neq NT$ (if the language allows this), or $(WA, NT) \in \{(red, green), (red, blue), (green, red), (green, blue), \ldots\}$

Example: Map Coloring Cont'd

Solutions are assignments satisfying all constraints.

e.g., $\{WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green\}$



Exercise 2

Given the following cryptarithmetic problem:

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   S E N D
+  M O R E
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 M O N E Y
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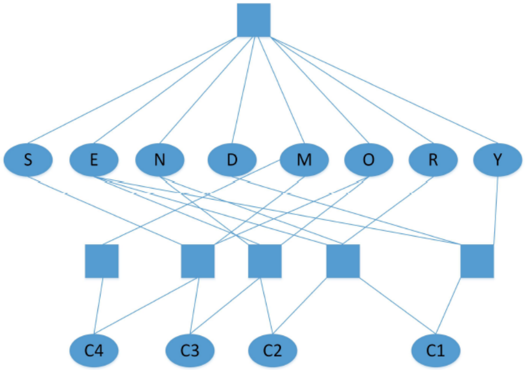
CRYPTARITHM: an arithmetic problem in which letters have been substituted for numbers and which is solved by finding all possible pairings of digits with letters that produce a numerically correct answer.

Call the rightmost column the first column (i.e., the column for D+E). Denote the carry into the second/third/fourth column by C_1, C_2 and C_3, respectively.

1. Define the variables, corresponding domains and constraints.
2. Show the constraint graph.
3. Given $R = 8, M = 1$, what's the remaining possible values of the letters and carry bits after first iteration of *forward checking*?

Exercise 2 - Solution

1. Variables: $D, E, M, N, O, R, S, Y, C_1, C_2, C_3$
   Domains: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
   Constraints: e.g. $(Y + 10C_1 = D + E), (D \neq E), (M \neq 0, S \neq 0)$
2. Constraint graph:



Exercise 2 - Solution Cont'd

forward checking: whenever a variable X is assigned, keep track of remaining legal values for unassigned variables.

3. Given $R = 8, M = 1$, we must then have $O = 0$ and $S = 9$ after forward checking (because of the constraint $S + 1 + C_3 = 10 + O$), and the remaining possible values of the letters and carry bits are:

D	0,2,3,4,5,6,7,9
E	0,2,3,4,5,6,7,9
M	1
N	0,2,3,4,5,6,7,9
O	0
R	8
S	9
Y	0,2,3,4,5,6,7,9
C_1	0,1
C_2	0,1
C_3	0,1

D	2,3,4,5,6,7,8
E	2,3,4,5,6,7,8
M	1
N	2,3,4,5,6,7,8
O	0
R	8
S	9
Y	2,3,4,5,6,7,8
C_1	0,1
C_2	0,1
C_3	0