COMP3711: Design and Analysis of Algorithms

Tutorial 6

HKUST

Question 1

Let X be a random variable that is equal to the number of heads in two flips of a fair coin. What is $E[X^2]$? What is $E^2[X]$?

Question 2

Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $O(n \log n)$?

Question 3

Show the steps of inserting 2,1,4,5,9,3,6,7 into an initially empty AVL tree.

Question 4

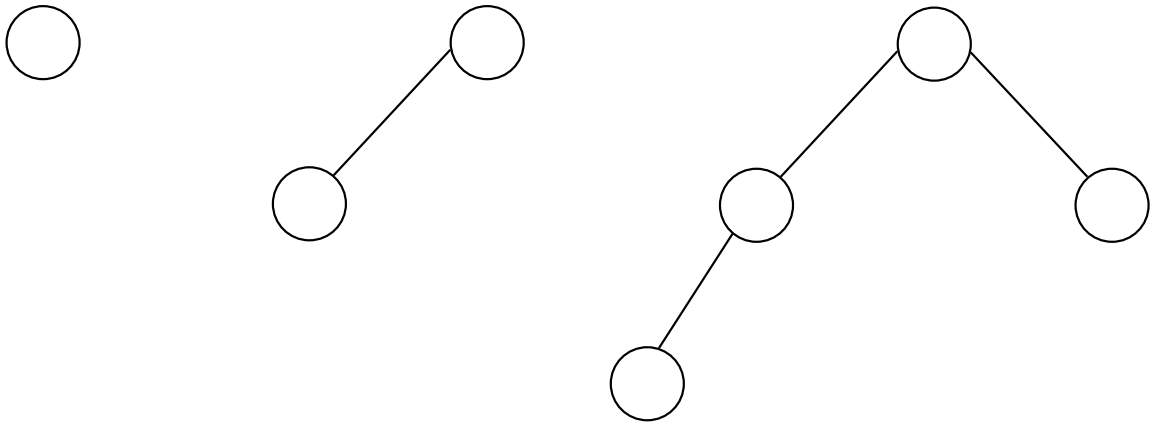
The AVL tree maintains its $O(\log n)$ height by balancing the heights of every two siblings. It is also possible to do so by balancing the weights. More precisely, the *weight* of a node u, denoted as $w(u)$, is the number of nodes in the subtree below u (including u). The weight of an empty tree is 0. We use u_L and u_R to denote the weight of u's left and right child, respectively. A node u is said to be *weight-balanced* if

$$\frac{1}{2} \leq \frac{w(u_L)+1}{w(u_R)+1} \leq 2.$$

A binary tree is weight-balanced if all of its nodes are weight-balanced.

Question 4

(a) The following figures show the smallest weight-balanced trees of height $0, 1$, and 2, respectively. Please draw the smallest weight-balanced trees of height 3 and 4.



$n_0 = 1$ $n_1 = 2$ $n_2 = 4$

(b) Show that the height of a weight-balanced binary tree with n nodes is $O(\log n)$.