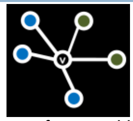
# LECTURE 9:CONTAGION AND VIRAL MARKET



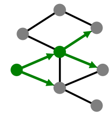
## Models of Cascading Behavior



- **Last time:**
  **Decision Based Models**
  - Utility based
  - Deterministic
  - "Node" centric: A node observes decisions of its neighbors and makes its own decision
  - Require us to know too much about the data




- **Today: Probabilistic Models**
  - Let's you do things by observing data
  - We loose "why people do things"



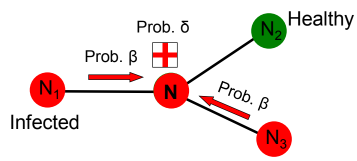
# CLASSICAL MODELS OF DISEASE SPREADING

## Spreading Models of Viruses



**Virus Propagation: 2 Parameters:**

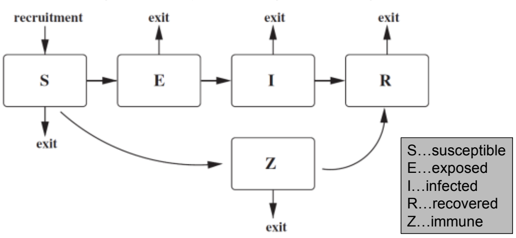
- **(Virus) birth rate β:**
  - probability than an infected neighbor attacks
- **(Virus) death rate δ:**
  - probability that an infected node heals




## More Generally: S+E+I+R Models



- **General scheme for epidemic models:**
  - **Each node can go through phases:**
    - Transition probs. are governed by the model parameters




## SIR Model

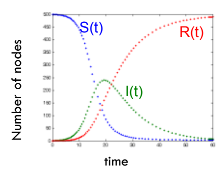


- **SIR model:** Node goes through phases


  - Models chickenpox or plague:
    - Once you heal, you can never get infected again
- **Assuming perfect mixing (**the network is a complete graph**) the model dynamics is:**

$$\frac{dS}{dt} = -\beta SI \qquad \frac{dR}{dt} = \delta I$$

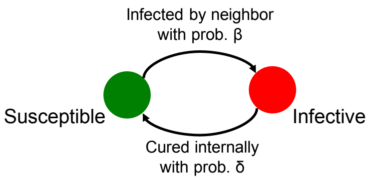
$$\frac{dI}{dt} = \beta SI - \delta I$$



## SIS Model

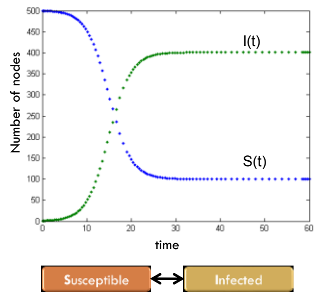
7

- **Susceptible-Infective-Susceptible (SIS) model**
- Cured nodes immediately become susceptible
- **Virus "strength": s = β / δ**
- **Node state transition diagram:**




## SIS Model

8




- **Models flu:**
  - Susceptible node becomes infected
  - The node then heals and become susceptible again
- **Assuming perfect mixing (complete graph):**

$$\frac{dS}{dt} = -\beta SI + \delta I$$

$$\frac{dI}{dt} = \beta SI - \delta I$$

## Question: Epidemic threshold *t*

9

- **SIS Model:**
  **Epidemic threshold of an arbitrary graph G is τ, such that:**
  - **If virus strength $s = \beta / \delta < \tau$ the epidemic can not happen (it eventually dies out)**
- **Given a graph what is its epidemic threshold?**

## Epidemic Threshold in SIS Model

10

- **We have no epidemic if:**

(Virus) Death rate

Epidemic threshold

$$\boxed{\beta/\delta < \tau = 1/\,\lambda_{1,A}}$$

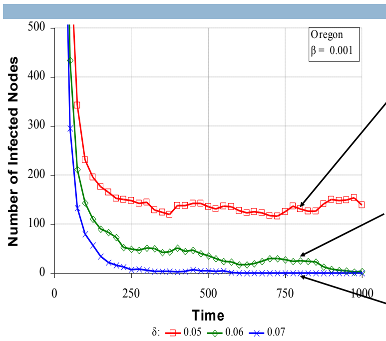
(Virus) Birth rate

largest eigenvalue of adj. matrix ***A***

► ***$\lambda_{1,A}$ alone captures the property of the graph!***

## Experiments (AS graph)

11




10,900 nodes and 31,180 edges

**β/δ > τ (above threshold)**

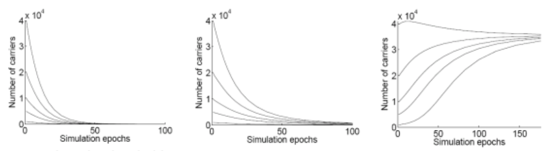
**β/δ = τ (at the threshold)**

**β/δ < τ (below threshold)**

## Experiments

12

- **Does it matter how many people are initially infected?**




(a) Below the threshold, s=0.912

(b) At the threshold, s=1.003

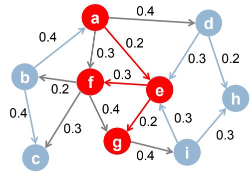
(c) Above the threshold, s=1.1

# MODELS OF INFORMATION SPREAD

## Independent Cascade Model

14

- Initially some nodes S are active
- Each edge $(u,v)$ has probability (weight) $p_{uv}$

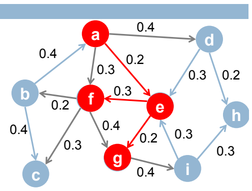



- When node v becomes active:
  - It activates each out-neighbor $v$ with prob. $p_{uv}$
- Activations spread through the network

## Independent Cascade Modal

15

- **Independent cascade model is simple but requires many parameters!**
  - Estimating them from data is very hard [Goyal et al. 2010]

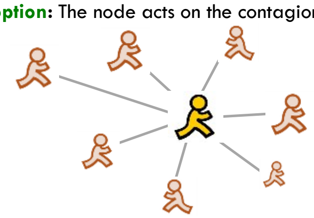



- **Solution:** Make all edges have the same weight (which brings us back to the SIR model)
  - Simple, but too simple
- **Can we do something better?**

## Exposures and Adoptions

16

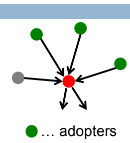
- **From exposures to adoptions**
  - **Exposure:** Node's neighbor exposes the node to the contagion
  - **Adoption:** The node acts on the contagion



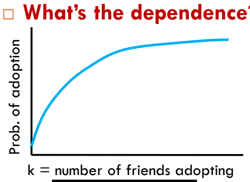
## Exposure Curves

17

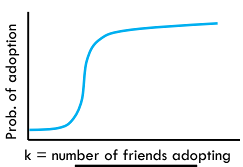
- **Exposure curve:**
  - **Probability of adopting new behavior depends on the number of friends who have already adopted**




- **What's the dependence?**




Diminishing returns:
Viruses, Information

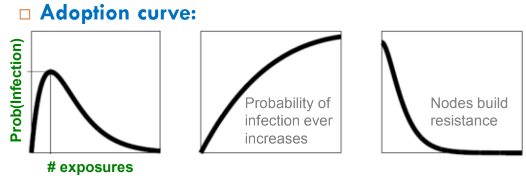



Critical mass:
Decision making

## Exposure Curves

18

- **From exposures to adoptions**
  - **Exposure:** Node's neighbor exposes the node to information
  - **Adoption:** The node acts on the information
- **Adoption curve:**



## Example Application

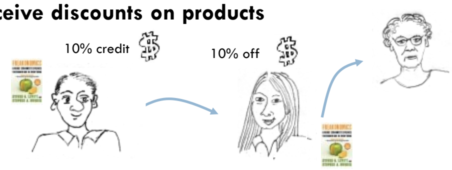
19

- **Marketing agency** would like you to adopt/buy product *X*
- They estimate the adoption curve
- **Should they expose you to *X* three times?**
- **Or, is it better to expose you *X*, then *Y* and then *X* again?**


## Diffusion in Viral Marketing

20

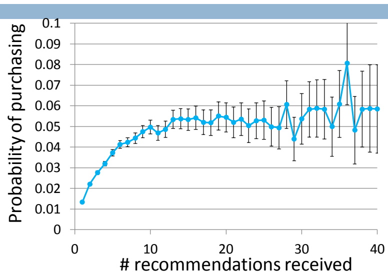
- **Senders and followers of recommendations receive discounts on products**




- **Data: Incentivized Viral Marketing program**
  - 16 million recommendations
  - 4 million people, 500k products
  - [Leskovec-Adamic-Huberman, 2007]

## Exposure Curve: Validation

21



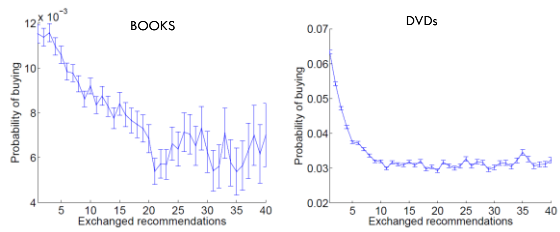

Books

DVD recommendations
(8.2 million observations)

## More Subtle Features

22

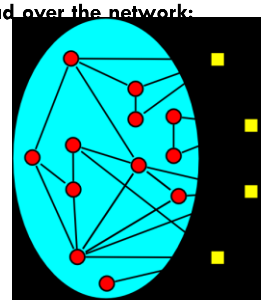
- **What is the effectiveness of subsequent recommendations?**




## Exposure Curve: LiveJournal

23

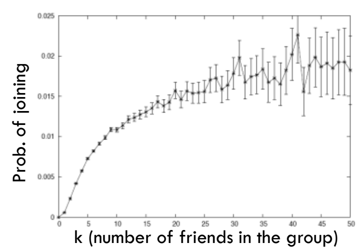
- **Group memberships spread over the network:**
  - Red circles represent existing group members
  - Yellow squares may join
- **Question:**
  - How does prob. of joining a group depend on the number of friends already in the group?



## Exposure Curve: LiveJournal

24

- **LiveJournal group membership**



## What are We Really Measuring?

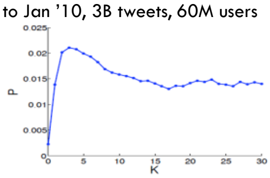
25

- For viral marketing:
  - We see that node v receiving the *i*-th recommendation and then purchased the product
- For groups:
  - At time t we see the behavior of node v's friends
- Good questions:
  - When did v become aware of recommendations or friends' behavior?
  - When did it translate into a decision by v to act?
  - How long after this decision did v act?

## Exposure Curve: Information

26

- Twitter [Romero et al. '11]
  - Aug '09 to Jan '10, 3B tweets, 60M users

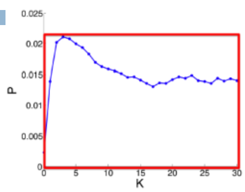


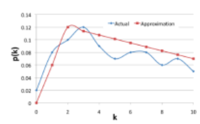

  - Avg. exposure curve for the top 500 hashtags
  - What are the most important aspects of the shape of exposure curves?
  - Curve reaches peak fast, decreases after!

## Modeling the Shape of the Curve

27

- **Persistence of *P*** is the ratio of the area under the curve *P* and the area of the rectangle of length *max(P)*, width *max(D(P))*
  - *D(P)* is the domain of *P*
- **Persistence measures the decay of exposure curves**
- **Stickiness of *P*** is *max(P)*.
- **Stickiness is the probability of usage at the most effective exposure**



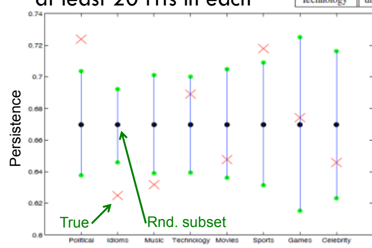




## Exposure Curve: Persistence

28

- Manually identify 8 broad categories with at least 20 HTs in each

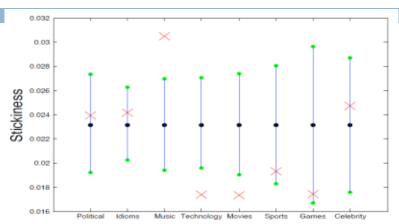
| Category | Examples |
|---|---|
| Celebrity | mj, brazilwantsjb, regis, iwantpeterfacinelli |
| Music | thisiswar, mj, musicmonday, pandora |
| Games | mafiawars, spymaster, mw2, zyngapirates |
| Political | tcot, glennbeck, obama, hcr |
| Idiom | cantlivewithout, dontyouhate, musicmonday |
| Sports | golf, yankees, nhl, cricket |
| Movies/TV | lost, glennbeck, bones, newmoon |
| Technology | digg, iphone, jquery, photoshop |




- Idioms and Music have lower persistence than that of a random subset of hashtags of the same size
- Politics and Sports have higher persistence than that of a random subset of hashtags of the same size

## Exposure Curve: Stickiness

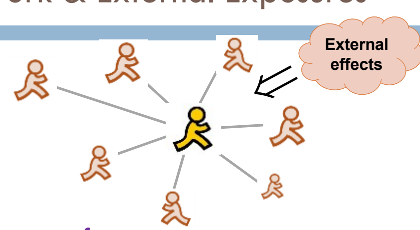
29




- Technology and Movies have lower stickiness than that of a random subset of hashtags
- Music has higher stickiness than that of a random subset of hashtags (of the same size)

## Network & External Exposures

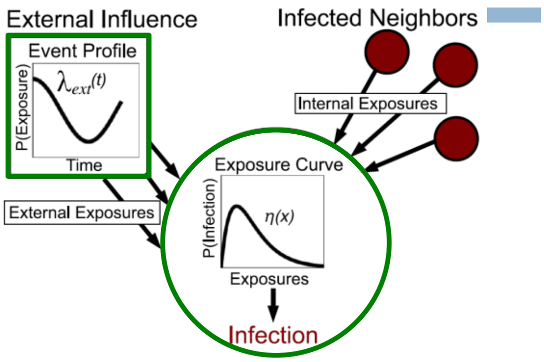
30




- Two sources of exposures [*Myers et al., KDD, 2012*]
  - Exposures from the network
  - External exposures

## Putting it all together

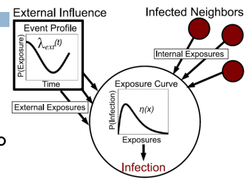
31

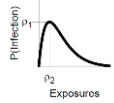



## Model Inference Task

32

- **Given:**
  - Network $G$
  - A set of node adoption times $(u, t)$ single piece of info
- **Goal: Infer**
  - **External event profile:**
    $\lambda_{ext}(t)$ … # external exposures over time
  - **Adoption curve:**

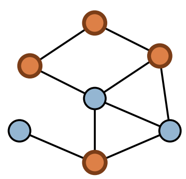



## Experiment

33

- **In social networks people post links to interesting articles**
  - You hear about an article from a friend
  - You read the article and then post it
- **Data from Twitter**
  - *Complete* data from Jan 2011: 3 billion tweets
  - Trace the emergence of URLs
    - Label each URL by its topic



## Results: Different Topics

34

- **Adoption of URLs across Twitter:**

| | max P(k) | k at max P(k) | Duration (hours) | % Ext. Exposures |
|---|---|---|---|---|
| Politics (25) | 0.0007 +/- 0.0001 | 4.59 +/- 0.76 | 51.24 +/- 16.66 | 47.38 +/- 6.12 |
| World (824) | 0.0013 +/- 0.0000 | 2.97 +/- 0.10 | 43.54 +/- 2.94 | 26.07 +/- 1.19 |
| Entertain. (117) | 0.0015 +/- 0.0002 | 3.52 +/- 0.28 | 89.89 +/- 16.13 | 17.87 +/- 2.51 |
| Sports (24) | 0.0010 +/- 0.0003 | 4.76 +/- 0.83 | 87.85 +/- 38.03 | 43.88 +/- 6.97 |
| Health (81) | 0.0016 +/- 0.0002 | 3.25 +/- 0.30 | 100.09 +/- 17.57 | 18.81 +/- 3.33 |
| Tech. (226) | 0.0013 +/- 0.0001 | 3.00 +/- 0.16 | 83.05 +/- 8.73 | 18.36 +/- 1.80 |
| Business (298) | 0.0015 +/- 0.0001 | 3.18 +/- 0.16 | 49.61 +/- 5.14 | 22.27 +/- 1.79 |
| Science (106) | 0.0012 +/- 0.0002 | 4.06 +/- 0.30 | 135.28 +/- 16.19 | 20.53 +/- 2.78 |
| Travel (16) | 0.0005 +/- 0.0001 | 2.33 +/- 0.29 | 151.73 +/- 39.70 | 39.99 +/- 6.60 |
| Art (32) | 0.0006 +/- 0.0001 | 5.26 +/- 0.66 | 188.55 +/- 48.17 | 27.54 +/- 5.30 |
| Edu. (31) | 0.0009 +/- 0.0001 | 3.77 +/- 0.51 | 130.53 +/- 38.63 | 21.45 +/- 6.40 |

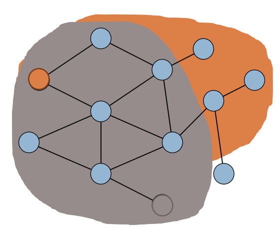
- More in *Myers et al., KDD, 2012*

# MODELING INTERACTIONS BETWEEN CONTAGIONS

## Interactions

36

- So far we considered pieces of information as **independently** propagating
- **Do pieces of information interact?**
  - Does being exposed to blue change the probability of talking about red?



## Modeling Interactions

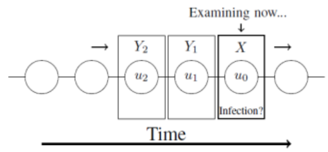
37

- **Goal: Model interaction between many pieces of information**
  - **Some pieces of information may help each other in adoption**
  - **Other may compete for attention**

## Modeling Interactions

38

- **You are reading posts on Twitter:**
  - You examine posts one by one
  - Currently you are examining *X*
  - How does your probability of reposting *X* depend on what you have seen in the past?




P(post X | exposed to X, $Y_1$, $Y_2$, $Y_3$) = ?

## Dataset: Twitter

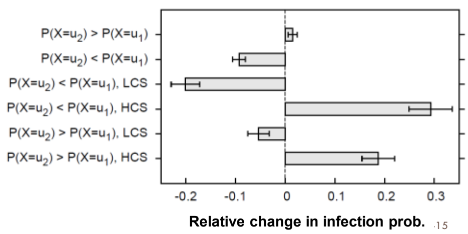
39

- **Data from Twitter**
  - *Complete* data from Jan 2011: 3 billion tweets
  - All URLs tweeted by at least 50 users: 191k
- **Task:**
  Predict whether a user will post URL X
  - Train on 90% of the data, test on 10%
- **Baselines:**
  - **Infection Probability (IP):** $P(X = u_i | Y_k = u_j) = = P(X = u_i)$
  - **IP + Node bias (NB):** $= P(X = u_i) + \gamma_n$
  - **Exposure curve (EC):** $= P(X \mid \#\text{ times exposed to } X)$

## How to Tweets Interact?

40

- **How *P(post $u_2$ | exp. $u_1$)* changes if …**
  - $u_2$ and $u_1$ are similar/different in the content?
  - $u_1$ is highly viral?




**Observations:**
- If $u_1$ is not viral, this boost $u_2$
- If $u_1$ is highly viral, this kills $u_2$

**BUT:**
Only if $u_1$ and $u_2$ are of low content similarity (LCS) else, $u_1$ helps $u_2$

15