COMP 2711 Discrete Mathematical Tools for CS
2014 Fall Semester – Written Assignment # 9
For self-practice. No need to hand in.
Solution Keys

At the top of your solution, please write your (i) name, (ii) student ID #, (iii) email address and (iv) tutorial section.

Some Notes:

- Please write clearly and briefly. For all questions you should also provide a short explanation as to *how* you derived the solution. That is, if the solution is 20, you shouldn't just write down 20. You need to explain *why* it's 20.
- Please follow the guidelines on doing your own work and avoiding plagiarism given on the class home page. Don't forget to *acknowledge* individuals who assisted you, or sources where you found solutions.
- Some of these problems are taken (some modified) from the textbook.
- Please make a *copy* of your assignment before submitting it. If we can't find your paper in the submission pile, we will ask you to resubmit the copy.
- Your solutions should be submitted before 5PM, in the collection bin in front of Room 4213A.

Problem 1: Consider a function $T(n)$ defined on integers n that are powers of 2. Suppose

$$T(1) = 1, \quad T(n) = 3T(n/2) + n^2.$$

Iterate the recurrence or use a recursion tree to find a closed-form expression for $T(n)$. Simplify the closed-form expression using the big Θ notation.

Answer: Iterating the recurrence, we get:

$$\begin{aligned} T(n) &= T(2^j) \\ &= 3T(2^{j-1}) + 2^{2j} \\ &= 3(3T(2^{j-2}) + 2^{2(j-1)}) + 2^{2j} \\ &= 3^2 T(2^{j-2}) + \frac{3}{4}2^{2j} + 2^{2j} \\ &= 3^2(3T(2^{j-3}) + 2^{2(j-2)}) + \frac{3}{4}2^{2j} + 2^{2j} \end{aligned}$$

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\begin{aligned}
&= 3^3 T(2^{j-3}) + (\frac{3}{4})^2 2^{2j} + \frac{3}{4}2^{2j} + 2^{2j} \\
&\quad \vdots \\
&= 3^j T(1) + (\frac{3}{4})^{j-1} 2^{2j} + \ldots + \frac{3}{4}2^{2j} + 2^{2j} \\
&= 3^j + 2^{2j}\frac{1-(3/4)^j}{1-3/4} \\
&= 3^j + 4 \cdot 2^{2j} - 4 \cdot 3^j = 4 \cdot 2^{2j} - 3 \cdot 3^j \\
&= 4n^2 - 3n^{log_2 3} \\
&= \Theta(n^2).
\end{aligned}
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