COMP 3721: Theory of Computation
Written Assignment 1
Assigned: February 25 Due: March 11

Question 1

Prove that the set $\{A \mid A \subseteq \mathcal{N}, A \text{ is finite}\}$ is countable.

Question 2

Write regular expressions representing the following languages ($\Sigma = \{a, b\}$):

a) All strings in Σ^* with an odd number of a's.

b) All strings in Σ^* that have ab as a substring.

c) All strings in Σ^* that do not have ab as a substring.

Question 3

Draw an NFA that accepts the language $abba^* \cup a(ab \cup ba)^*bb$ over the alphabet $\{a, b\}$.

Question 4

Given a NFA $M = (\{s, t\}, \{a, b\}, \Delta, s, \{s, t\})$ where $\Delta = \{(s, a, t), (t, b, t), (t, e, s), (t, b, s)\}$,

a) represent M by a state transition diagram;

b) draw the computation tree for input $aabbb$ (is this string accepted by M?);

c) give a regular expression for $L(M)$;

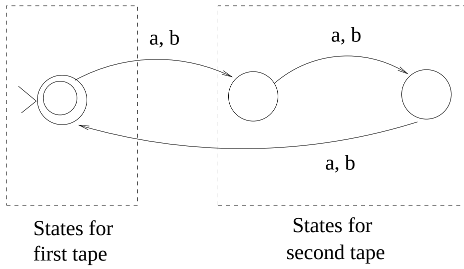
d) Construct a DFA that is equivalent to M.

Question 5

Let $M = (K, \Sigma, \delta, s, F)$ be a DFA. Let $M' = (K, \Sigma, \delta, s, R)$, where R is the set of states that can reach some state in F, i.e., R consists of every $q \in K$ such that there is a path (of any length, including zero) in the state diagram from q to some state in F. What is the relationship between the language recognized by M' and the language recognized by M? Prove your answer.

Question 6

A deterministic 2-tape finite automaton is a device like a deterministic finite automaton, but it accepts ordered pairs of strings $(w_1, w_2) \in \Sigma^* \times \Sigma^*$ where w_1 is on the first tape and w_2 is on the second tape. There are two sets of states: K_1 and K_2. If the machine is in a state of K_1, it reads a symbol from the first tape, evaluates the transition function, and moves to the next state according to the transition function; if the machine is in a state of K_2, it reads a symbol from the second type, and moves to the next state according to the transition function. The machine accepts the input pair of strings if it is in an accepting state after both strings have been entirely read. For example, the automaton shown below accepts all pairs of strings $(w_1, w_2) \in \{a, b\}^* \times \{a, b\}^*$ such that $|w_2| = 2|w_1|$.



a) Draw the state diagram for a deterministic 2-tape finite automaton that accepts the following: All pairs of strings (w_1, w_2) in $\{a, b\}^* \times \{a, b\}^*$ such that $|w_1| = |w_2|$ and $w_1(i) \neq w_2(i)$ for all i.

b) Formally define (namely, in mathematical languages)

   i) a deterministic 2-tape finite automaton;

   ii) the notion of a configuration for such an automaton;

   iii) the yields in one step relation $\vdash$ between configurations;

   iv) the notion that such an automaton accepts an ordered pair of strings;

   v) the notion that such an automaton accepts a set of ordered pairs of strings.