COMP3211 Final Exam
Dec 19, 2011

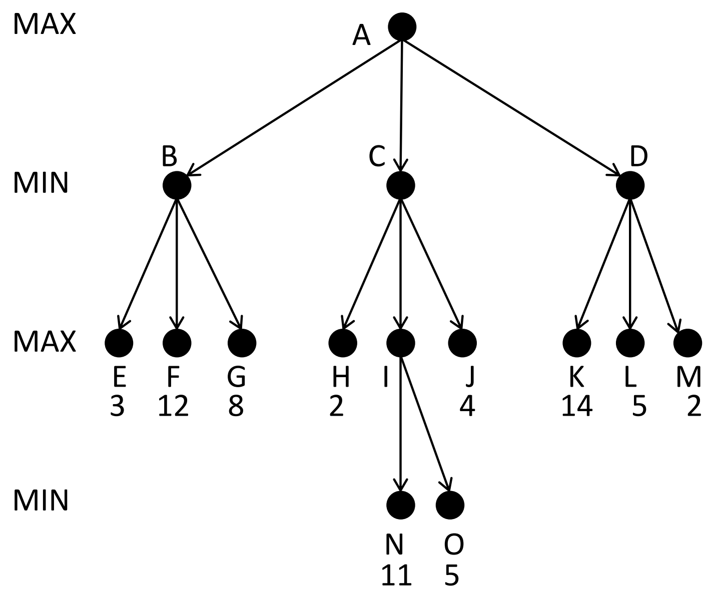
INSTRUCTIONS:

- **This is a closed book exam.**
- **There are total six questions.**
- **You have two and a half hours to complete it.**
- **Do not write your answers in any other space except the ones given.**
- **You can use back pages and the last blank page as your scrap paper.**
- **Use pen only. No pencils, please!**
- **Take out your student ID card and put it at the top left corner of your desk.**

Your student ID number:

Your name:

1. **(14pts) Alpha-Beta Pruning**
   A. Perform minimax with perfect decision on the tree given in the following figure (the leaves are terminal nodes, and the numbers next to them are their utility values).
   B. Perform a left-to-right alpha-beta prune on the tree and list the nodes pruned.
   C. Perform a right-to-left alpha-beta prune on the tree and list the nodes pruned.



A. Minimax:

A	B	C	D	E
3	3	2	2	11

B. Left to right, nodes pruned:

I (hence N and O), J

C. Right to left, nodes pruned:

N

Marking scheme:

4pts for A, 4pts for B and 5pts for C

2. **(16pts) Convert the following sentences into CNF. Notice that negation has the highest precedence, followed by conjunction and disjunction, then implication, and then equivalence. Notice also that the conjunction and disjunction operators are left associative, but implication and equivalence are right associative.**

A. $(p \supset q) \supset r \vee \neg(s \supset q)$.

$\{ (p, r, s), (r, \neg q) \}$

B. $[(p \equiv q) \supset s \supset \neg q] \supset p \supset q \supset p$.

True

C. $\forall x[\forall y(P(x) \supset Q(x, y)) \supset \exists y, z(P(y) \wedge Q(y, z))]$

$$\{ \big(P(x), P(f_1(x))\big), \big(P(x), Q(f_1(x), f_2(x))\big),$$
$$\big(\neg Q(x, f_3(x)), P(f_1(x))\big), \big(\neg Q(x, f_3(x)), Q(f_1(x), f_2(x))\big) \}$$

D. $\exists x F(x) \wedge \forall x G(x) \supset \neg[\forall x F(x) \vee \exists x \neg G(x)]$

$\{(\neg \mathrm{F}(\mathrm{x}), \neg \mathrm{G}(sk_1), \neg \mathrm{F}(sk_2)), (\neg \mathrm{F}(\mathrm{x}), G(u), \neg \mathrm{G}(sk_1))\}$ or

$\{(\neg \mathrm{G}(sk_1), \neg \mathrm{F}(sk_2)), (\neg \mathrm{F}(\mathrm{x}), G(u), \neg \mathrm{G}(sk_1))\}$

Marking scheme:

1. **4pts for each**
2. **Wrong skolemization, 2pts off for each**
3. **Wrong distribution of ∨ over ∧, 1 point off for each**
4. **DNF instead of CNF, 1 point off for each**
5. **For Part D, if $(\neg \mathbf{F}(\mathbf{x}), \neg \mathbf{G}(\boldsymbol{sk_1}), \neg \mathbf{F}(\boldsymbol{sk_2}))$ is written as $(\neg \mathbf{F}(\mathbf{x}), \neg \mathbf{G}(\boldsymbol{sk_1}))$, 1pt off**
6. **For Part B, wrong association, 3pts off**

3. (15pts) Rule Learning with GSCA

Consider a soccer team that decides whether to hire someone as a Goal Keeper based on the following features:

- HS: high speed, true if the applicant has a top speed of 10m/s.
- LT: long throw, true if the applicant has a large range on long throws.
- PS: penalty saver, true if the applicant saves penalties very well.
- QL: quick launch, true if the applicant can reach his top speed quickly.
- FA: frequent attack, true if the applicant comes out to attack frequently.

Use GSCA to learn a set of rules from these examples about when to hire an applicant based on the values of the given attributes.

Please use the heuristic in the lecture notes: it selects a feature p that will yield the highest ratio n+/n, where n is the number of instances covered, and n+ the number of positive instances covered when p is added to the condition of the new rule.

FA	LT	PS	QL	HS	Hire
0	1	1	0	1	0
0	0	0	1	1	1
1	1	0	1	0	0
1	0	1	0	0	1
1	0	1	0	0	1
1	1	0	0	1	0
0	0	0	1	1	1
0	0	0	1	1	1
1	1	1	0	0	1
0	0	0	0	1	0
1	0	1	0	0	1

Marking scheme:

1. **7pts for each iteration and 1 extra point if everything's correct**
2. **Correct result with errors in GSCA procedure, 1-4pts off**
3. **Correct result without GSCA procedure, 5pts off**

Iteration 1:
$\Gamma = true, \tau = \Gamma \supset Hire, \Sigma_{cur} = \Sigma$

FA	LT	PS	QL	HS	Γ
4/6	1/4	4/5	3/4	3/6	PS
4/4	1/2	-	0/0	0/1	$PS \wedge FA$

$\pi = \{PS \wedge FA \supset Hire\}, \Sigma_{cur} = \{e_1, e_2, e_3, e_6, e_7, e_8, e_{10},\}$

Iteration 2:
$\Gamma = true, \tau = \Gamma \supset Hire, \Sigma_{cur} = \{e_1, e_2, e_3, e_6, e_7, e_8, e_{10}\}$

FA	LT	PS	QL	HS	Γ
0/2	0/3	0/1	3/4	3/6	QL
0/0	0/0	0/0	-	3/3	$QL \wedge HS$

$\pi = \{PS \wedge FA \supset Hire, QL \wedge HS \supset Hire\}, \Sigma_{cur} = \{e_1, e_3, e_6, e_{10}\}$

$PS \wedge FA \supset Hire$

$QL \wedge HS \supset Hire$

4. (15pts) Knowledge Representation and reasoning

Consider the following English sentences:

All people who are rich and smart are happy.
All people who read are smart.
John reads and is rich.
David is rich but does not read.
Jack is smart but is not rich.
Happy people have exciting lives.

Marking scheme:

1. **5pts for each**
2. **For Part C, if it's not proved by resolution, 2pts off**

A. Express the statements above as first-order sentences using the following predicates (assuming all individuals are people):
rich(x), smart(x), happy(x), read(x), exciting-life(x)

$\forall x\ (rich(x) \wedge smart(x) \supset happy(x))$
$\forall x\ (read(x) \supset smart(x))$
$read(John) \wedge rich(John)$
$rich(David) \wedge \neg read(David)$
$smart(Jack) \wedge \neg rich(Jack)$
$\forall x\ (happy(x) \supset exciting\_life(x))$

B. Convert each sentence in your answers to part A into CNF.

$\{\, (\neg rich(x), \neg smart(x), happy(x)) \,\}$
$\{\, (\neg read(x), smart(x)) \,\}$
$\{\, read(John), rich(John) \,\}$
$\{\, rich(David), \neg read(David) \,\}$
$\{\, smart(Jack), \neg rich(Jack) \,\}$
$\{\, \left(\neg happy(x), exciting\_life(x)\right) \,\}$

C. Use resolution with refutation to show that there exists someone whose life is exciting. Note that you need to represent the query in first-order logic.

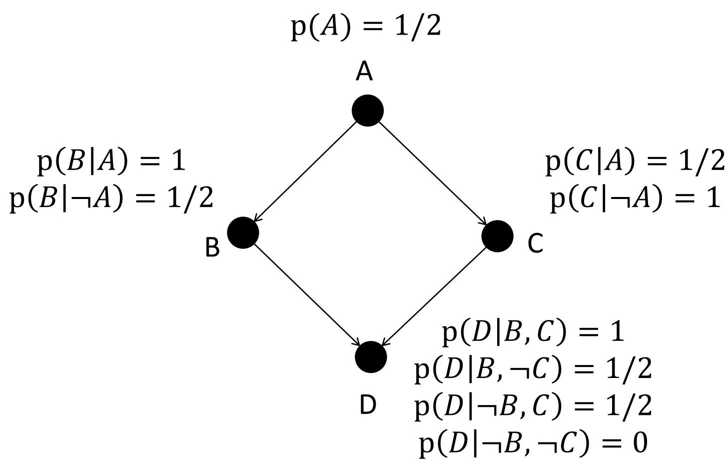
Negation of query: $\neg\exists x(exciting\_life(x))$ i.e. $\neg exciting\_life(x)$

$$
\begin{tikzcd}
\neg exciting\_life(x) \arrow[d] & \left(\neg happy(x), exciting\_life(x)\right) \arrow[dl] \\
\neg happy(x) \arrow[d] & (\neg rich(x), \neg smart(x), happy(x)) \arrow[dl] \\
(\neg rich(x), \neg smart(x)) \arrow[d] & (\neg read(x), smart(x)) \arrow[dl] \\
(\neg read(x), \neg rich(x)) \arrow[d, "Sub(x/John)"] & read(John) \arrow[dl] \\
\neg rich(John) \arrow[d] & rich(John) \arrow[dl] \\
[\,]
\end{tikzcd}
$$

5. (20pts) Uncertainty

An admissions committee for a college is trying to determine the probability that an admitted candidate is really qualified. The relevant probabilities are given in the Bayes network shown here. Calculate $p(A|D)$.

A = applicant is qualified
B = applicant has high GPA
C = applicant has excellent recommendations
D = applicant is admitted
Calculate $p(A|D)$.



$$
\begin{aligned}
P(A,D) &= P(D|B,C)P(B|A)P(C|A)P(A) \\
&\quad +P(D|B,\neg C)P(B|A)P(\neg C|A)P(A) \\
&\quad +P(D|\neg B,C)P(\neg B|A)P(C|A)P(A) \\
&\quad +P(D|\neg B,\neg C)P(\neg B|A)P(\neg C|A)P(A) \\
&= 3/8 \\
P(D) &= P(D|B,C)P(B|A)P(C|A)P(A) \\
&\quad +P(D|B,\neg C)P(B|A)P(\neg C|A)P(A) \\
&\quad +P(D|\neg B,C)P(\neg B|A)P(C|A)P(A) \\
&\quad +P(D|\neg B,\neg C)P(\neg B|A)P(\neg C|A)P(A) \\
&\quad +P(D|B,C)P(B|\neg A)P(C|\neg A)P(\neg A) \\
&\quad +P(D|B,\neg C)P(B|\neg A)P(\neg C|\neg A)P(\neg A) \\
&\quad +P(D|\neg B,C)P(\neg B|A)P(C|\neg A)P(\neg A) \\
&\quad +P(D|\neg B,\neg C)P(\neg B|\neg A)P(\neg C|\neg A)P(\neg A) \\
&= 3/4 \\
P(A|D) &= P(A,D)/P(D) \\
&= 1/2
\end{aligned}
$$

Marking scheme:

1. **8pts for P(A,D), 8pts for P(D) and 4pts for P(A|D)**
2. **Correct equations with incorrect algebra, 2pts off**

6. (20pts) Answer the questions below

A. Indicate which of the following Boolean functions of three input variables can be realized by a TLU. You do not need to give the TLUs.

1. x_1
2. $x_1 x_2 x_3$
3. $x_1 + x_2 + x_3$
4. $x_1 x_2 x_3 + \overline{x_1 x_2 x_3}$
5. 1

1. Yes
2. Yes
3. Yes
4. No
5. Yes

Marking scheme:

1pts for each

B. Formalize the 4-queen problem into an assignment problem (constraint satisfying problem). You need to define the variables, domain and constraints. You do not need to solve it.

Variables:

x_1, x_2, x_3, x_4, where x_i is the column of the queen placed in the i-th row

Domain:

{1,2,3,4}

Constraints:

$i \neq j \supset \; x_i \neq x_j$

$|i - j| \neq |x_i - x_j|$

Marking scheme:

2pts for Variables and domain and 3pts for constraints

C. Prove that the two equations below are equivalent.
$P(A|B,C) = P(A|C)$ and $P(B|A,C) = P(B|C)$

$$\begin{aligned} P(A|B,C) &= \frac{P(A,B,C)}{P(B,C)} \\ &= \frac{P(B|A,C)P(A,C)}{P(B|C)P(C)} \\ &= \frac{P(B|A,C)P(A|C)P(C)}{P(B|C)P(C)} \\ &= \frac{P(B|A,C)P(A|C)}{P(B|C)} \end{aligned}$$

So, P(A|B,C)=P(A|C) and P(B|A,C)=P(B|C) are equivalent.

Marking scheme:
No partial credits

D. Given n variables, how many non-equivalent Boolean formulas there are? Your answer should be a formula of n.

2^{2^n}.
Two Boolean formulas A and B are non-equivalent iff
the set of all models of A is different from the set of all models of B.
As there are totally 2^n different models, there are 2^{2^n} different sets of models, and 2^{2^n} non-equivalent Boolean formulas.

Marking scheme:
No partial credits

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