COMP2711H Tutorial 3

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1 The Pigeonhole Principle

Exercise 1.1. Let $x_1, \ldots, x_{n+1}$ be $n + 1$ positive integers not exceeding $2n$. Prove that $\exists i, \exists j \neq i, x_i$ divides x_j.

Comments. Although the pigeonhole principle is simple, its application can be hard. The difficulty often lies in the construction of pigeonholes.

2 Warm Up

Exercise 2.1. A basketball team has 12 players. How many ways can you choose 2 guards, 2 forwards, and 1 center from those 12 players?

3 Arrangement in Cycle

Exercise 3.1. How many ways are there to seat n people in a round table? (two arrangement are considered identitcal if every person has the same right neighbor)

4 Conditional Arrangement

Exercise 4.1. How many ways are there to seat n couples in a line such that each couples sits together?

Exercise 4.2. Solve the following qustions.

(a) How many five-digit based ten numbers are there?

(b) How many five-digit numbers have no consecutive digits equal?

(c) How many five-digit numbers have at least one pair of equal consecutive digits?

5 Combination with Repetition

Exercise 5.1. I have n types of balls. How many ways can I select k balls if repetition is allowed?

Exercise 5.2. How many nonnegetive integer solutions do $x_1 + \ldots + x_n = k$ have?

References

[1] HKUST. *COMP2711 Lecture Notes.*

[2] V. Koltun. *Discrete Structures Lecture Notes*, chapter 9. 2008.