# COMP 3721 Design and Analysis of Algorithms
# Assignment 4
# For self-practice. No need to submit your answers.

1. **Multiple Choice**

   For each of the following statements, indicate whether it is (a) true, (b) false, or (c) unknown based on our current scientific knowledge. *For any statements that you mark "false", please give a brief explanation why it is wrong.* You do not need to justify the "true" and "unknown" answers.

   1.1 $\mathcal{P} \subseteq \mathcal{NP}$.

   1.2 $\mathcal{P} = \mathcal{NP}$.

   1.3 If we can solve any $\mathcal{NP}$-complete problem in polynomial time, we can solve all the $\mathcal{NP}$-hard problems in polynomial time.

   1.4 If it is not possible to solve SAT in polynomial time, then it is not possible to solve DVC (decision version of vertex cover) in polynomial time.

2. Define *coNP* to be the following class of languages $\{\bar{L}\colon\ L \in NP\ \}$. Show that if $NP \neq coNP$, then $P \neq NP$.

3. Show that the Decision Knapsack problem is NP-complete. You may use the fact that the Set Partition problem is NP-complete.

   Here are the definition of the problems:

   Knapsack: there is a knapsack of capacity $W$ (a positive integer) and $n$ object with weight $w_1 \dots w_n$ and value $v_1 \dots v_n$, where $w_i$ and $v_i$ are positive integers. Given $k$ is there a subset of the objects that fits in the knapsack and has total value at least $k$? (Note: This problem is sometimes called the 0-1 Knapsack problem because one is allowed to take either 0 or 1 copy of any object. )

   Set Partition: Given a finite set $S$ of positive integers, can the set $S$ be partitioned into two subsets $A$ and $\bar{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \bar{A}} x$?

4. A *dominating set* for a directed graph $G = (V, E)$ is a subset $D \subseteq V$ such that every vertex $u \in V - D$ is pointed to from at least one vertex in $D$ by some edge. For example, the shaded vertices in the graph below form a dominating set of size 3. The decision dominating set problem (DDS) is the decision problem where we are given a directed graph $G$ and an integer $k$, and the goal is to decide whether $G$ contains a dominating set of size $k$. Prove that DDS $\in \mathcal{NPC}$ by reducing from *Set Cover* problem. Recall that the Set Cover problem is defined as follows: Given a finite set $X$ and a collection of sets $F = \{S_1, \dots, S_n\}$ whose elements are chosen from $X$, and given an integer $k$, determine if there exist $k$ sets from $F$ such that their union covers all elements in $X$. [Hint: Construct one vertex for each element in $X$ and one vertex for each set $S_i$ in $F$, and build appropriate edges among them.]