First-Order Logic

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First-Order Logic: Introduction

First-Order Logic

First-order logic is a formal logical system which is more expressive than propositional logic. According to FOL, the world consists of objects and the facts in the world are simply properties about or relations among these objects.

- Syntax
- Semantics

See the lecture nodes.

Representing Knowledge with FOL

Examples:

- Not all birds fly.
- Not all objects that fly are birds.
- For every noisy boy, there is always a girl who can shut him up.
- There is a girl who can shut up all the noisy boys.

Representing Knowledge with FOL

Examples:

- Not all birds fly.
  $\neg(\forall x\, Bird(x) \supset Flies(x))$
- Not all objects that fly are birds.
  $\neg(\forall x\, Flies(x) \supset Bird(x))$
- For every noisy boy, there is always a girl who can shut him up.
  $\forall x\, [(Boy(x) \wedge Noisy(x)) \supset \exists y(Girl(y) \wedge ShutUp(y, x))]$
- There is a girl who can shut up all the noisy boys.
  $\exists y\, [Girl(y) \wedge \forall x\, (Boy(x) \wedge Noisy(x) \supset ShutUp(y, x))]$

Conversion to CNF

1. Eliminate implication and equivalence
2. Move $\neg$ inwards
3. Standardize variables
4. Skolemize
5. Move quantifiers to the front
6. Drop quantifiers
7. Distribute $\wedge$ over $\vee$
8. Flatten nested conjunctions and disjunctions
9. Remove conjunctions to get a set of clauses

Exercises

1. Convert the following to CNF:

$$\forall y\, [\forall x\, (P(x) \supset Q(x,y)) \supset (\exists y\, P(y) \wedge \exists z\, Q(y,z))]$$

Solution:

$\forall y\, [\neg\forall x\, (\neg P(x) \vee Q(x,y)) \vee (\exists y\, P(y) \wedge \exists z\, Q(y,z))]$	Eliminate $\supset$
$\forall y\, [\exists x\, (P(x) \wedge \neg Q(x,y)) \vee (\exists y\, P(y) \wedge \exists z\, Q(y,z))]$	$\neg$ inward
$\forall y\, [\exists x\, (P(x) \wedge \neg Q(x,y)) \vee (\exists u\, P(u) \wedge \exists z\, Q(y,z))]$	unique name
$\forall y\, [(P(f_1(y)) \wedge \neg Q(f_1(y),y)) \vee (P(f_2(y)) \wedge Q(y,f_3(y)))]$	skolemize
$(P(f_1(y)) \wedge \neg Q(f_1(y),y)) \vee (P(f_2(y)) \wedge Q(y,f_3(y)))$	drop $\forall$
$(P(f_1(y)) \vee P(f_2(y))) \wedge (P(f_1(y)) \vee Q(y,f_3(y))) \wedge$ $(\neg Q(f_1(y),y) \vee P(f_2(y))) \wedge (\neg Q(f_1(y),y) \vee Q(y,f_3(y)))$	distribute

Exercises

2. Prove that the following is a tautology.

$$\exists x \, \neg F(x) \wedge \forall x \, G(x) \equiv \neg(\forall x \, F(x) \vee \exists x \, \neg G(x))$$

3. Formalize the following statements with FOL and prove the conclusion with resolution.

KB:

Everyone is either diligent or gifted. All diligent ones succeed. But not everyone succeeds.

Conclusion:

Someone is gifted.