LECTURE 6:CENTRALITY MEASURES

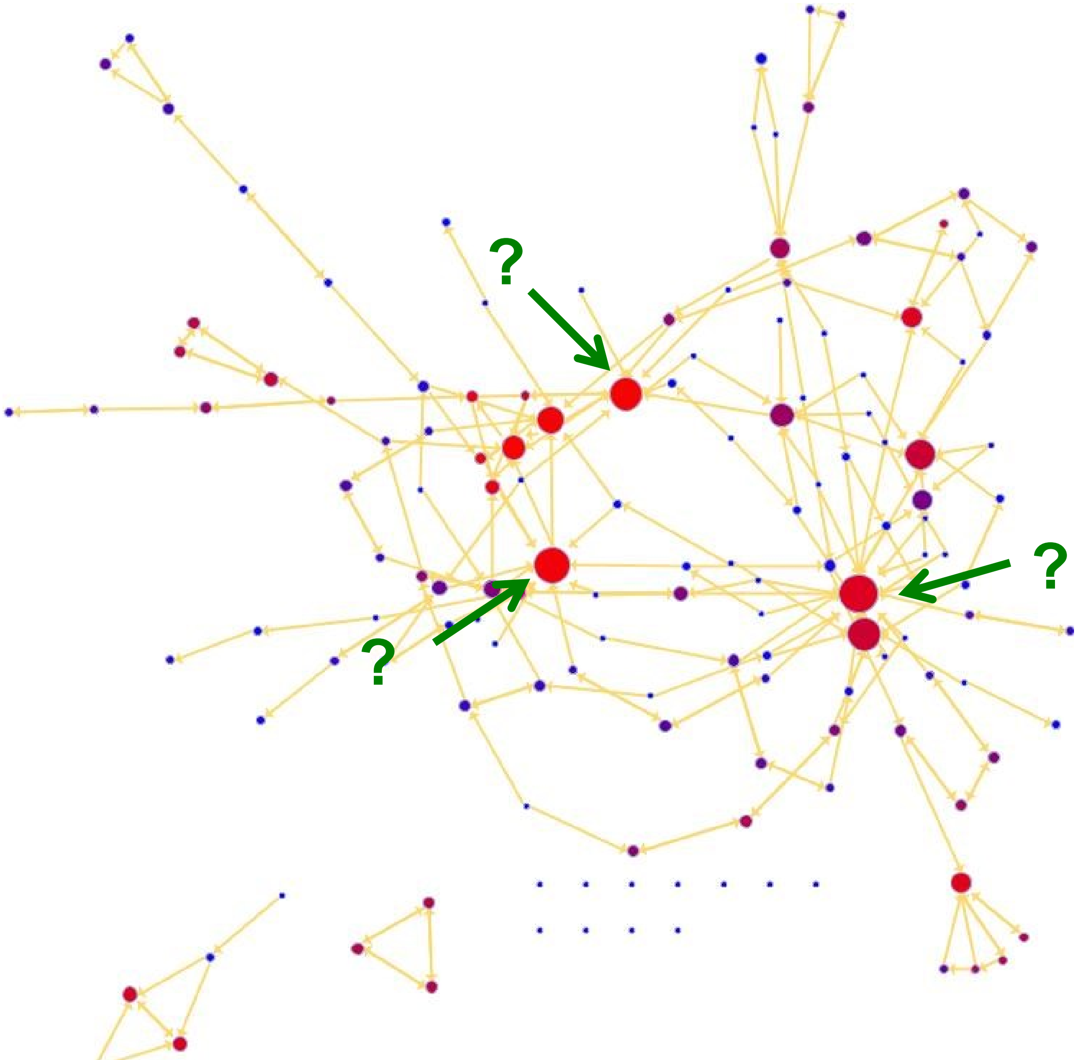
COMP4641: Social Information Network Analysis and Engineering

Wednesday March 4th 2015

In This Lecture

- We will introduce the concept of centrality and the various measures which have been associated to this concept.

Characterizing networks: Who is most central?



Network Centrality

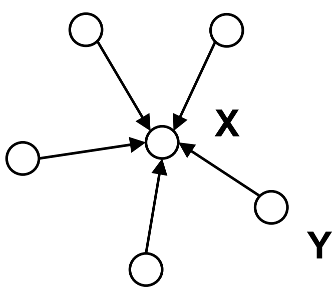
- Which nodes are most ‘central’?
- Definition of ‘central’ varies by context/purpose
- Local measure:
  - degree
- Relative to rest of network:
  - closeness, betweenness, eigenvector (Bonacich power centrality), Katz, PageRank, …
- How evenly is centrality distributed among nodes?
  - Centralization, hubs and autthorities, …

Centrality

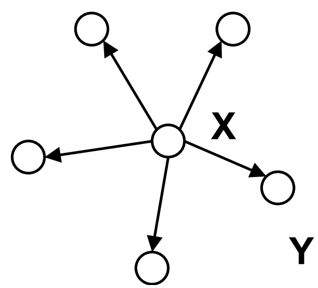
- Finding out which is the most central node is important:
  - It could help disseminating information in the network faster
  - It could help stopping epidemics
  - It could help protecting the network from breaking

Centrality: visually

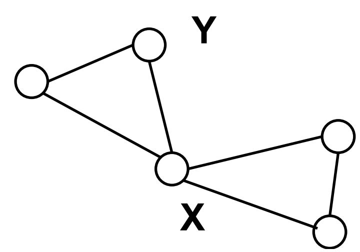
- Centrality can have various meanings:



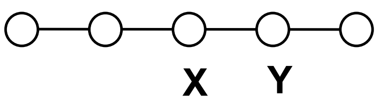
indegree



outdegree

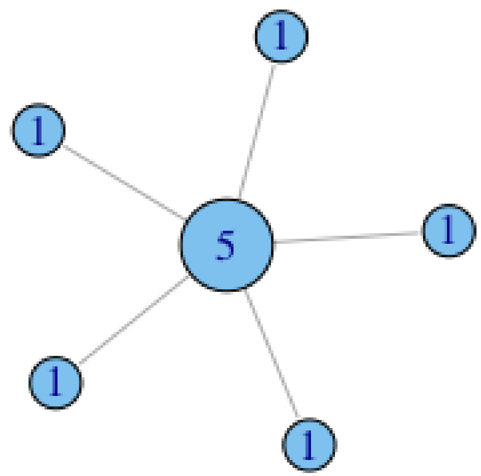


betweenness



closeness

Degree Centrality

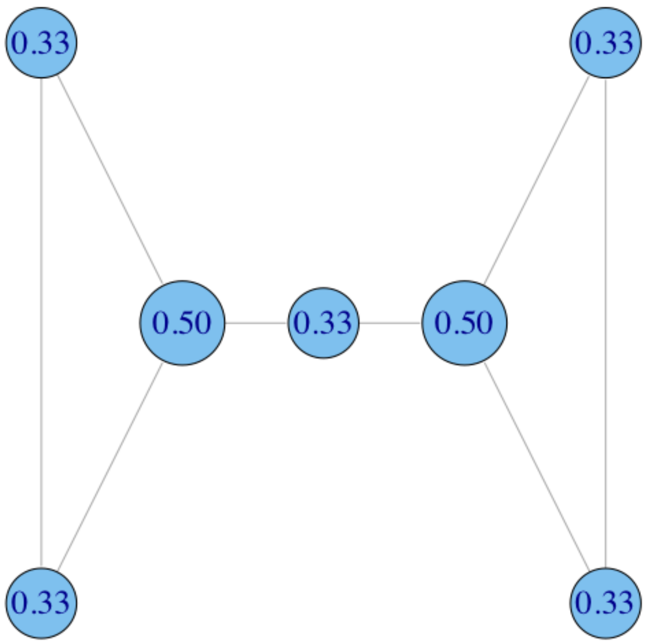


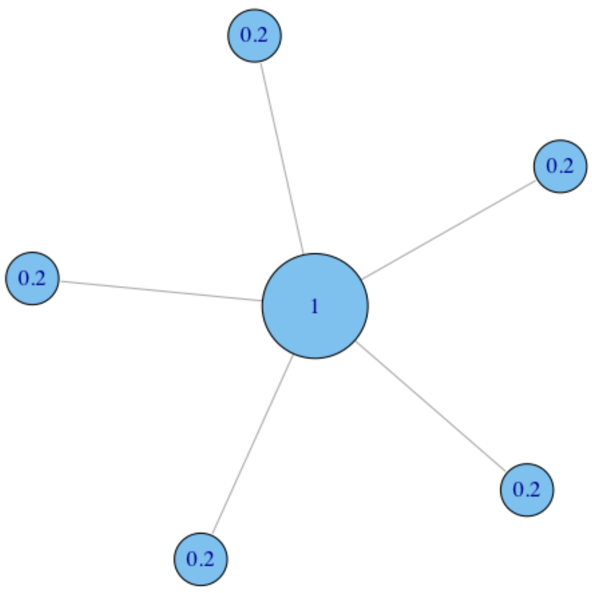
When is the number of connections the best centrality measure?

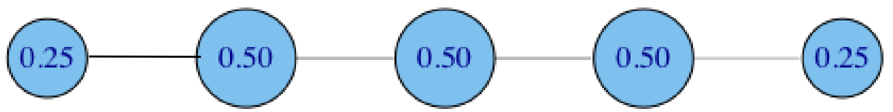
- people who will do favors to you
- people you can talk to / have a beer with

Normalization

- Divide for the max number of nodes (N-1)







Freeman's Network Centrality

- How do we calculate the value of centrality of the network
  - To check how much variation there is among the nodes (heterogeneity?)

Max value of Degree Centrality in the Network

Max value of the above: when network is a star: 1 node has C=N-1 and all others (N-1) have 1.

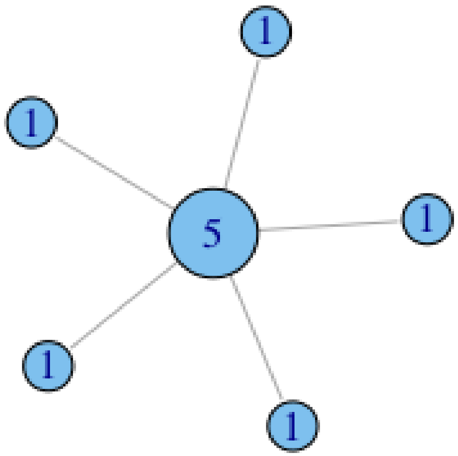
$$C_D = \frac{\sum_{i=1}^{g}\left[C_D(n^*) - C_D(i)\right]}{\left[(N-1)(N-2)\right]}$$

Freeman Network Centrality Explained

- Explanation of the denominator:
- In the star topology one node has degree N-1 and all other nodes have degree of 1:

$$0+((n-1)-1)*(n-1)=(n-2)*(n-1)$$

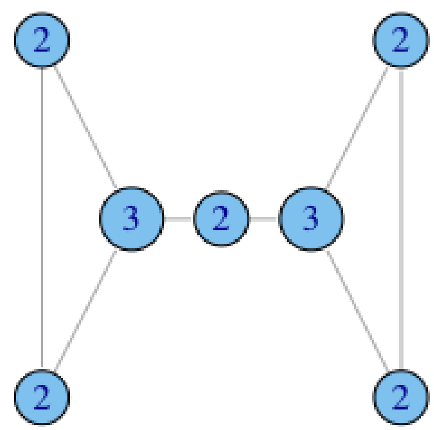
Freeman's Network Centrality



C_D = 1.0

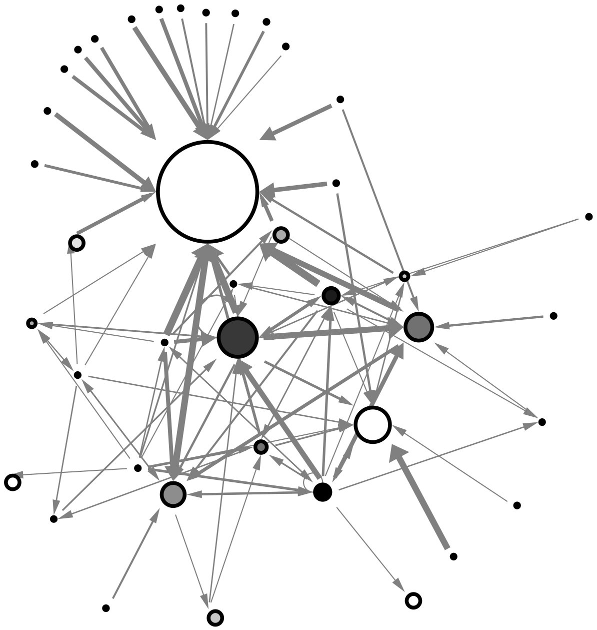
C_D = 0.167

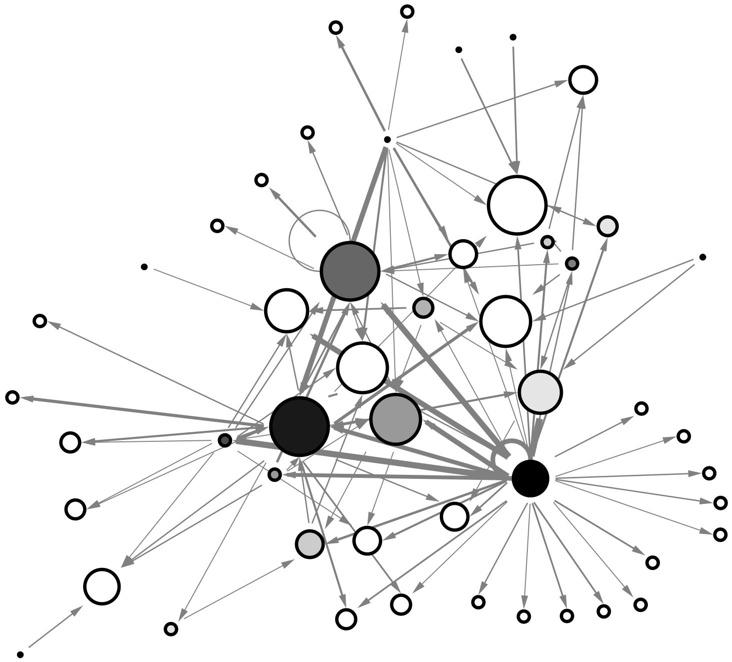
(4+4+4+4+4)/5*4
(1+0+0+0+1)/4*3=1/6
(1+1+0+1+0+1+1)/6*5=5/30



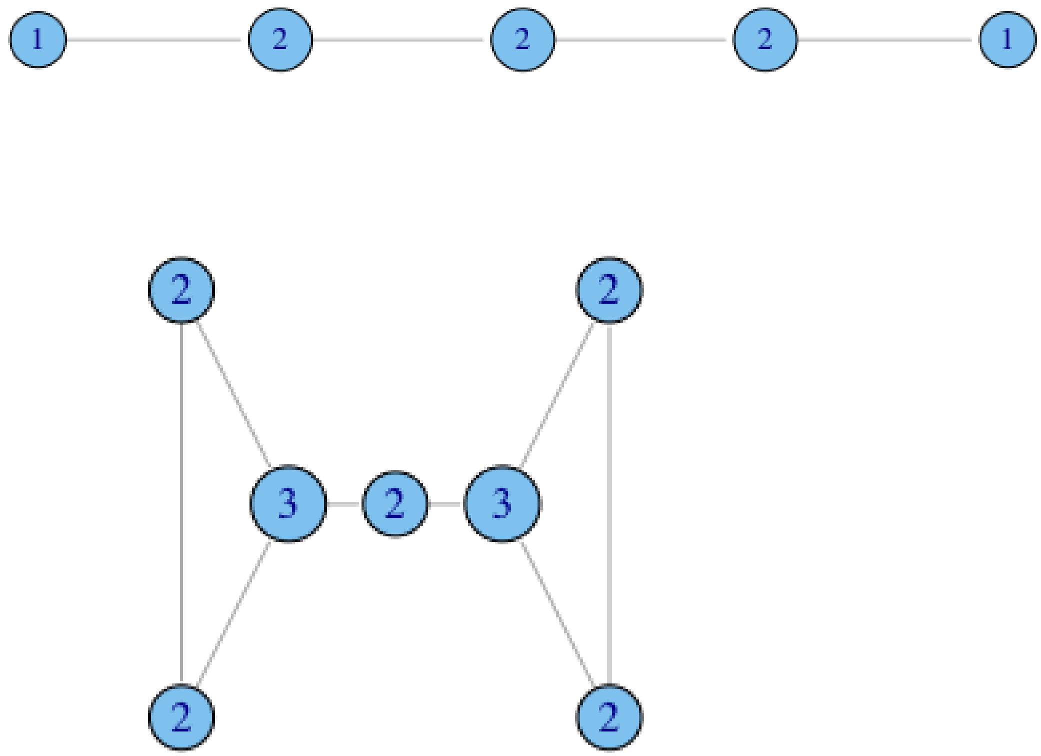
C_D = 0.167

Examples: Financial Networks





When is Degree Centrality not so good?

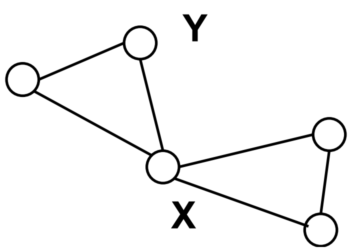


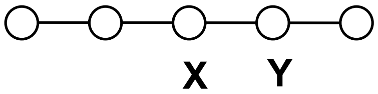
When is Degree Centrality not so good (2)?

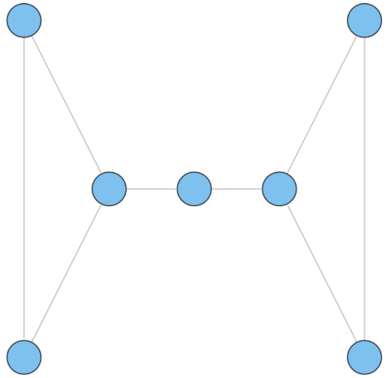
- Ability to broker between groups
- Likelihood that information originating anywhere in the network reaches you…

Betweenness Centrality

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- who has higher betweenness, X or Y?







Betweenness (Formally)

$$C_B(i) = \sum_{j \neq k} g_{jk}(i) / g_{jk}$$

Where g_{jk} (i)= the number of shortest paths connecting *jk* passing through i

g_{jk} = total number of shortest paths

Usually normalized by:

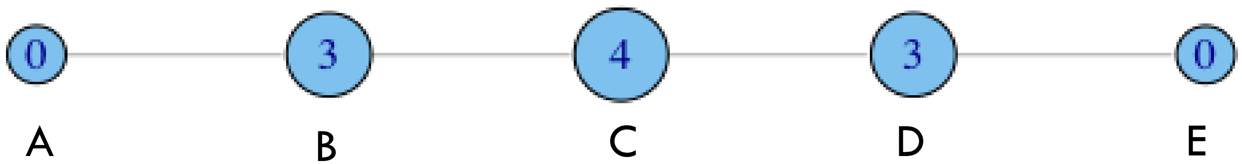
$$C'_B(i) = C_B(i) \boxed{/[(n-1)(n-2)/2]}$$

number of pairs of vertices excluding the vertex itself

Betweenness: Example

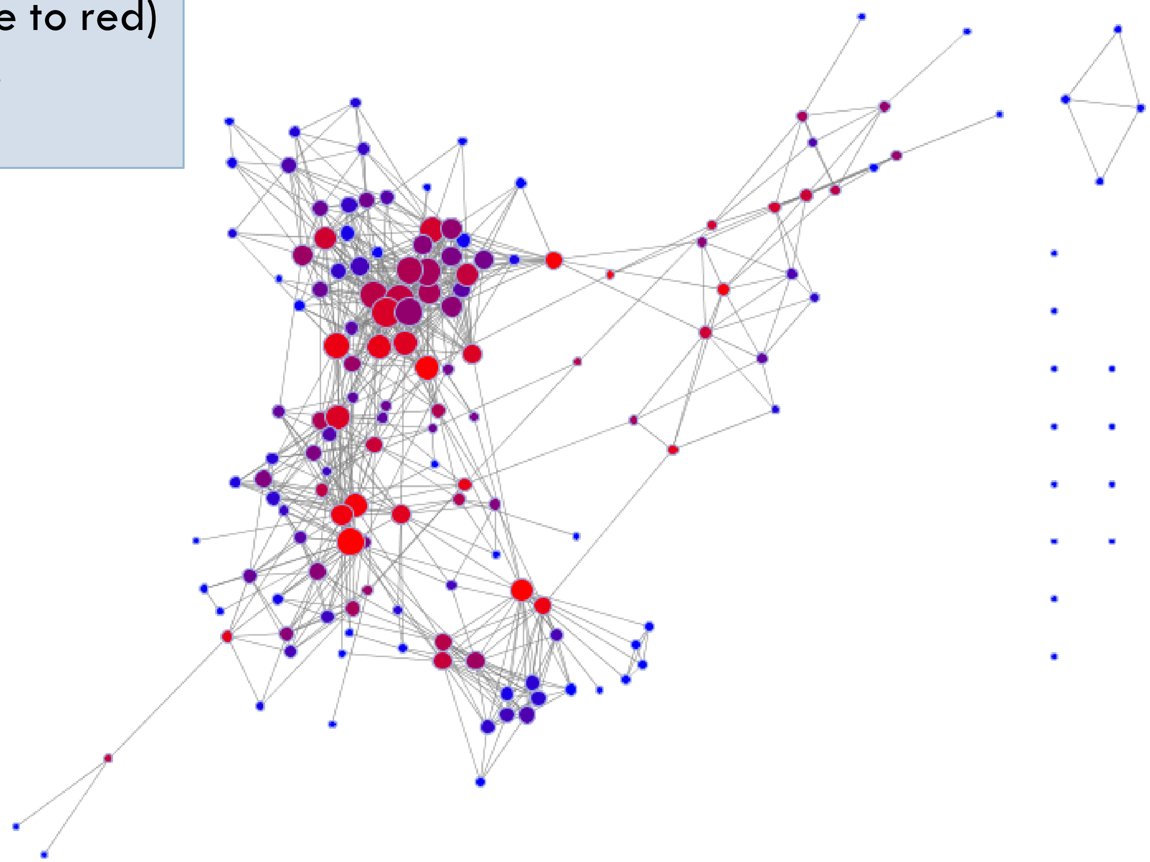
- A lies between no two other vertices
- B lies between A and 3 other vertices: C, D, and E
- C lies between 4 pairs of vertices (A,D),(A,E),(B,D),(B,E)

- note that there are no alternative paths for these pairs to take, so C gets full credit



Facebook Example (Adamic)

Color(from blue to red) is betweenness
Size is degree.



Closeness Centrality

- What if it is not so important to have many direct friends?
- Or be “between” others

- But one still wants to be in the “middle” of things, not too far from the center

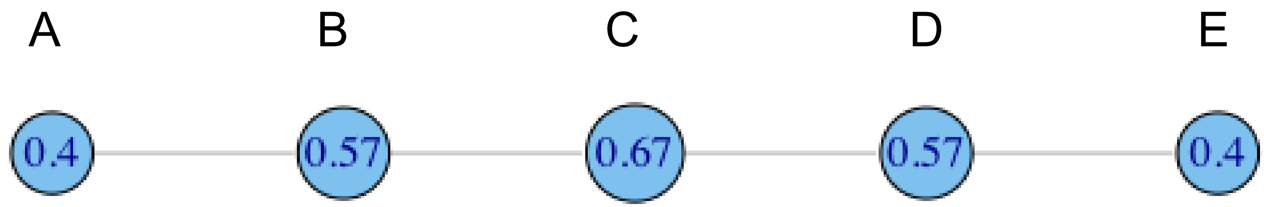
Closeness Centrality (Formally)

- Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

$$C_c(i) = \left[\sum_{j=1}^{N} d(i,j) \right]^{-1}$$

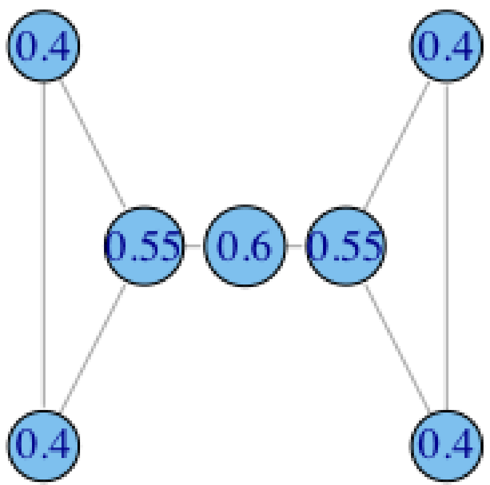
$$C'_c(i) = \left[\sum_{j=1}^{N} d(i,j) / (N-1) \right]^{-1}$$

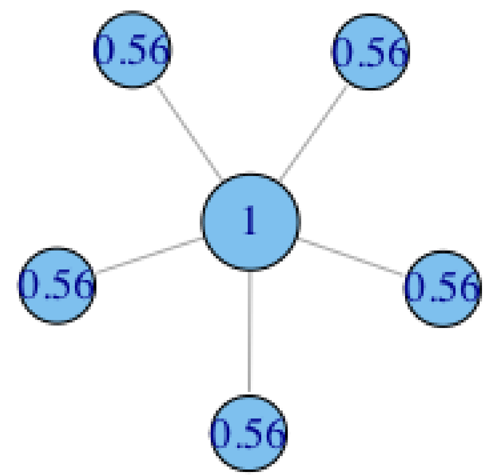
Closeness: Example



$$C_c'(A) = \left[\frac{\sum_{j=1}^{N} d(A, j)}{N-1} \right]^{-1} = \left[\frac{1+2+3+4}{4} \right]^{-1} = \left[\frac{10}{4} \right]^{-1} = 0.4$$

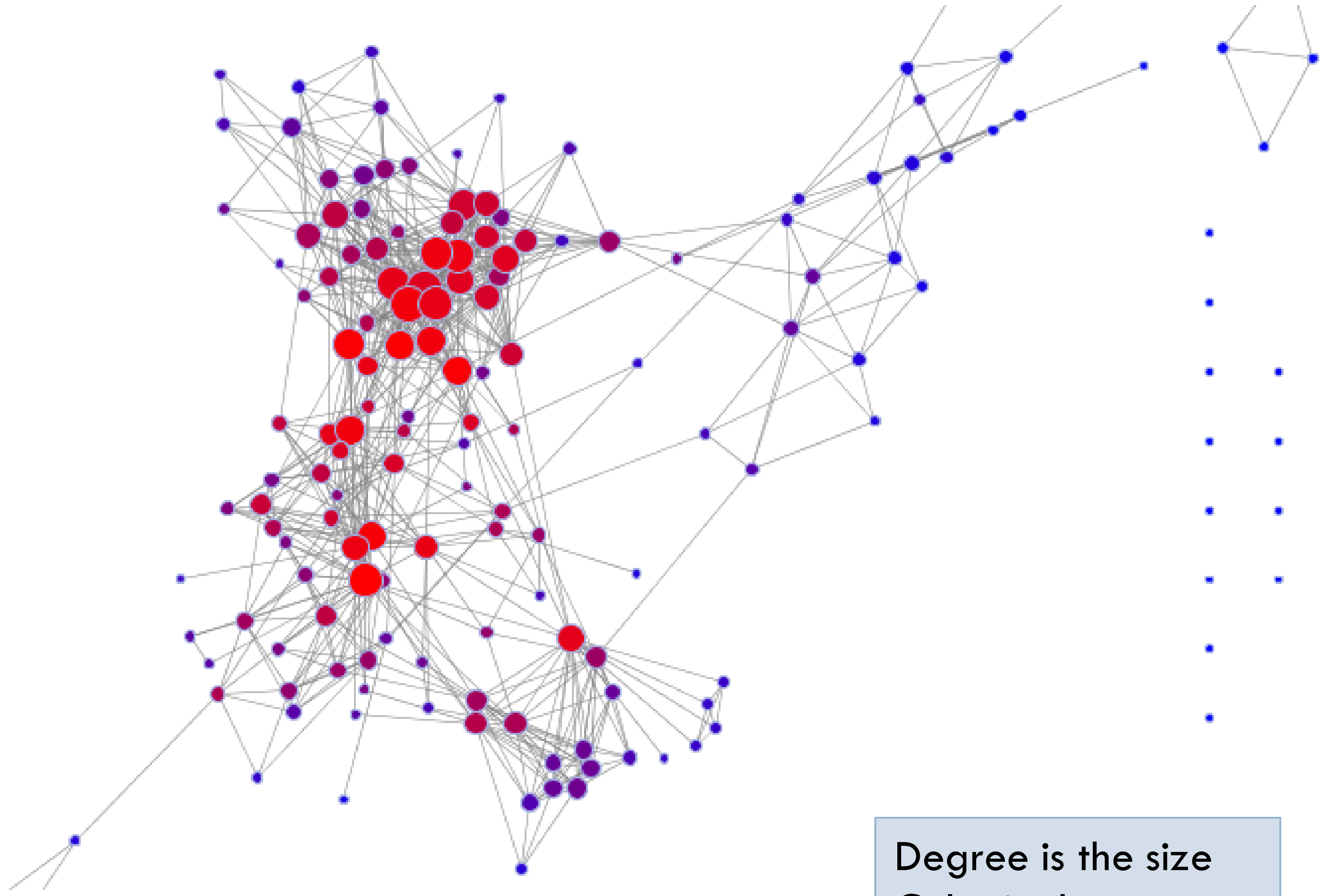
Examples





((1+1+2+3+4+4)/6)^-1=6/15=0.4

Example: Facebook (Adamic)



Degree is the size
Color is closeness

Centrality Comparison

Comparing across these 3 centrality values

- Generally, the 3 centrality types will be positively correlated
- When they are not (low) correlated, it probably tells you something interesting about the network.

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important important/active alters		Probably multiple paths in the network, ego is near many people, but so are many others
High Betweenness	Ego's few ties are crucial for network flow	Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.	

Eigenvector Centrality

- Degree Centrality depends on having many connections: but what if these connections are pretty isolated?
- A central node should be one connected to powerful nodes

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t$$

Neighbourhood of v

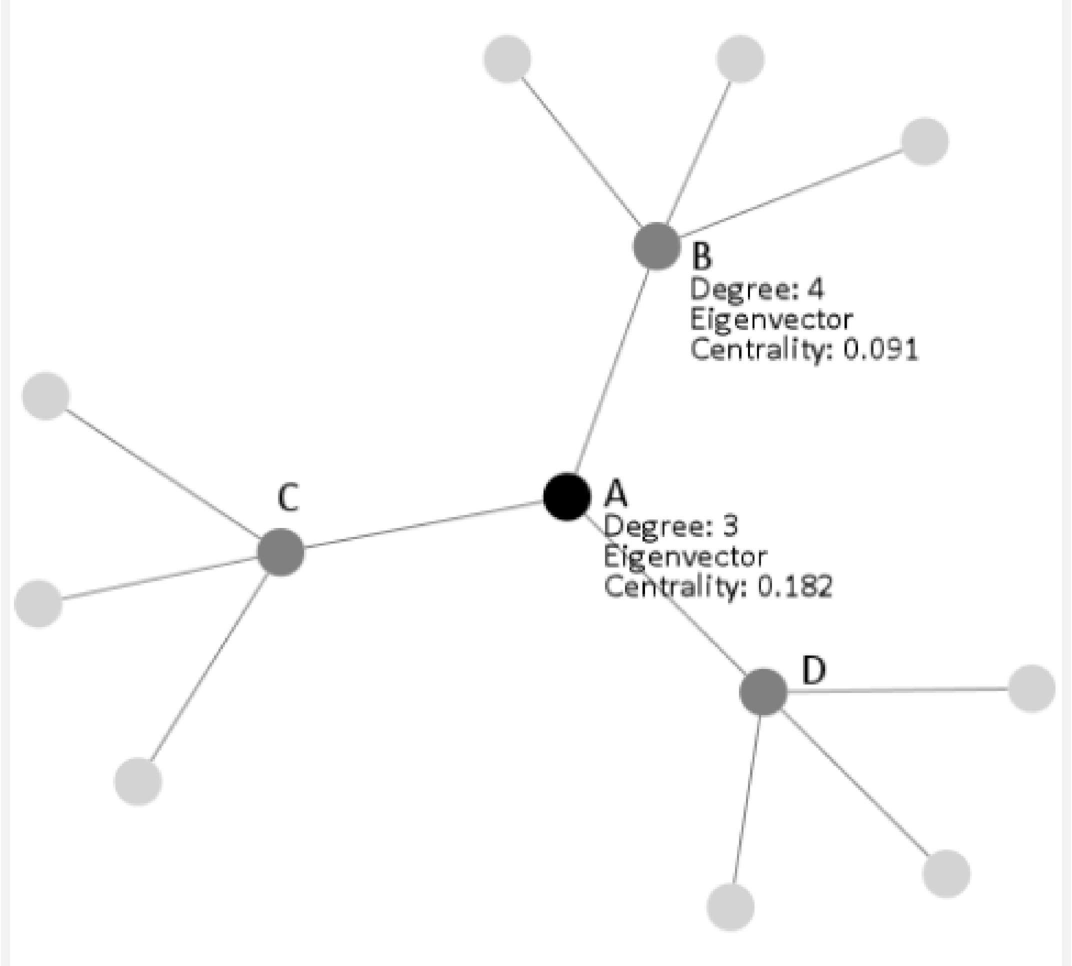
$$\mathbf{Ax} = \lambda \mathbf{x}$$

Adjacency Matrix of the graph

Eigenvector Centrality Algorithm

- 1. Start by assigning centrality score of 1 to all nodes ($v_i = 1$ for all i in the network)
- 2. Recompute scores of each node as weighted sum of centralities of all nodes in a node's neighborhood: $v_i = \sum_{j \in N} a_{ij} * v_j$
- 3. Normalize v by dividing each value by the largest value
- 4. Repeat steps 2 and 3 until values of v stop changing.

Example

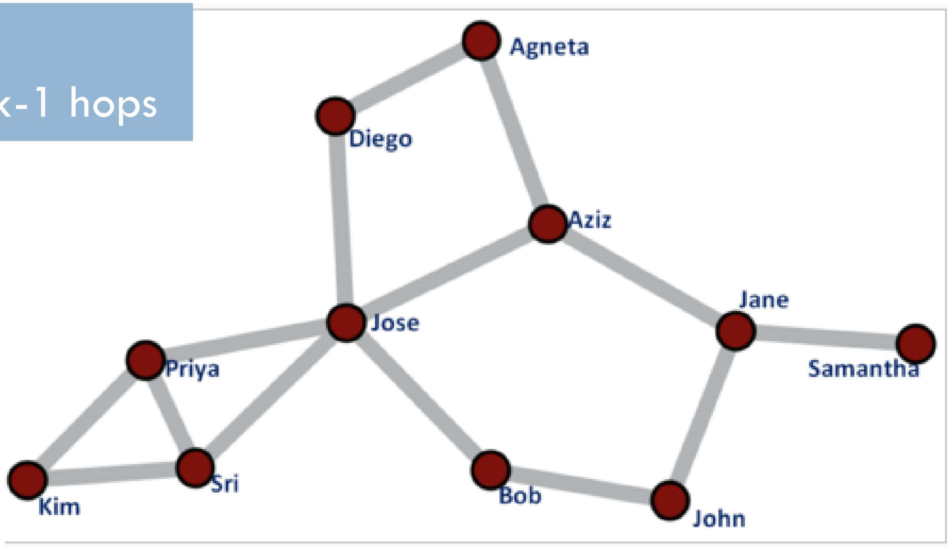

B
Degree: 4
Eigenvector
Centrality: 0.091
C
A
Degree: 3
Eigenvector
Centrality: 0.182
D

Katz Centrality

- Closeness counts the number of shortest paths, but one could count the **number of paths.**

$$C_{\mathrm{Katz}}(i) = \sum_{k=1}^{\infty} \sum_{j=1}^{n} \alpha^k (A^k)_{ji}$$

Alpha is an attenuation factor
A^k (ij) indicates if ij are connected by k-1 hops



Prestige in Directed Social Networks

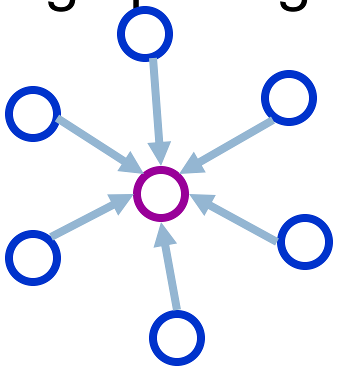
- When 'prestige' may be the right word
  - admiration
  - influence
  - gift-giving
  - trust

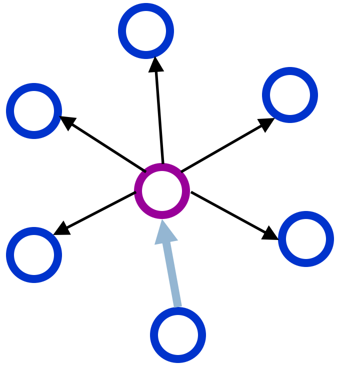
- Directionality especially important in instances where ties may not be reciprocated (e.g. dining partners choice network)

- When 'prestige' may not be the right word
  - gives advice to (can reverse direction)
  - gives orders to (- " -)
  - lends money to (- " -)
  - dislikes
  - distrusts

Extensions of undirected degree centrality - prestige

- Degree centrality
  - Indegree centrality
    - a paper that is cited by many others has high prestige
    - a person nominated by many others for a reward has high prestige

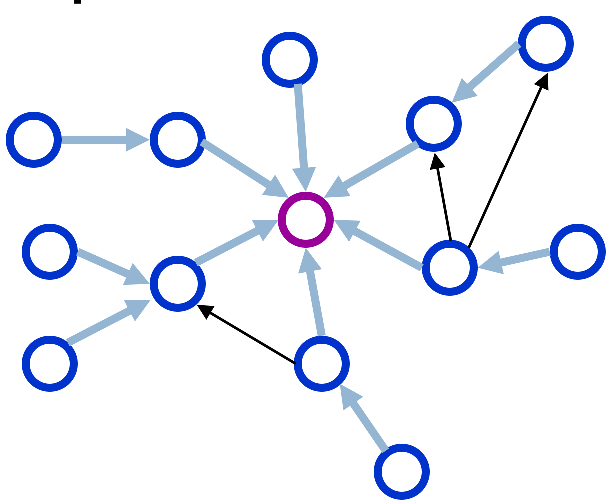




Extensions of Undirected Closeness Centrality

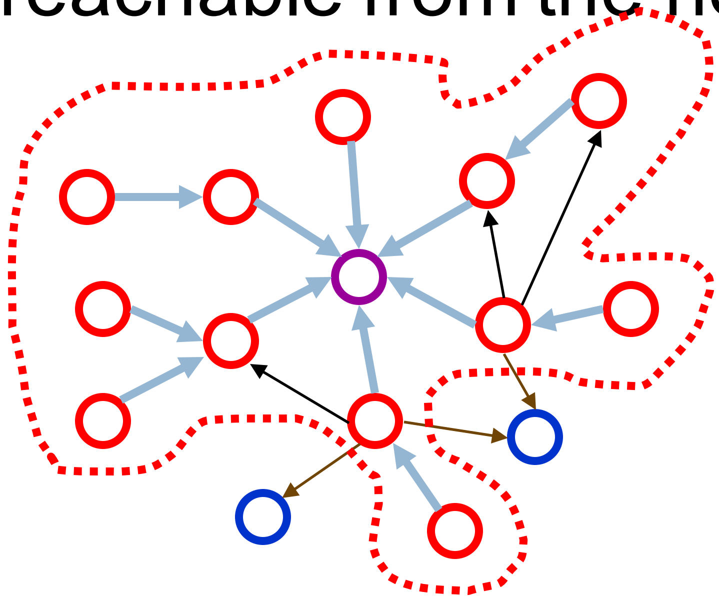
- Closeness centrality usually implies
  - all paths should lead to you
  - paths should lead from you to everywhere else

- Usually consider only vertices from which the node *i* in question can be reached



Influence Range

- The influence range of *i* is the set of vertices who are reachable from the node *i*



Extending Betweenness Centrality to Directed Networks

- We now consider the fraction of all directed paths between any two vertices that pass through a node

betweenness of vertex i

paths between j and k that pass through i

$$C_B(i) = \sum_{j,k} g_{jk}(i) / g_{jk}$$

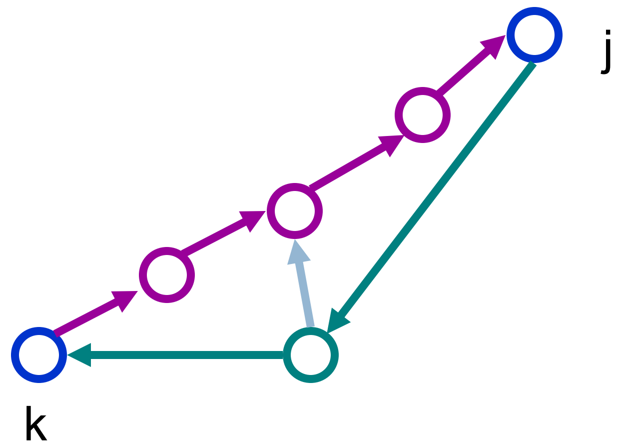
all paths between j and k

- Only modification: when normalizing, we have (N-1)*(N-2) instead of (N-1)*(N-2)/2, because we have twice as many ordered pairs as unordered pairs

$$C'_B(i) = C_B(i) / [(N-1)(N-2)]$$

Directed geodesics

- A node does not necessarily lie on a geodesic from *j* to *k* if it lies on a geodesic from *k* to *j*



World City System

TABLE 3
RANKING OF CITIES ON MEASURES OF POWER AND PRESTIGE

Rank	Outdegree	Value	Closeness	Value	Betweenness	Value	Indegree	Value
1	Tokyo	3,639	Paris	55.51	Paris	25.65	New York	1,425
2	New York	2,601	Tokyo	53.59	Tokyo	15.04	London	1,086
3	Paris	2,535	London	53.47	Düsseldorf	13.61	Paris	944
4	London	1,955	New York	52.87	London	13.31	Tokyo	762
5	Düsseldorf	1,278	San Francisco	51.47	New York	10.01	Los Angeles	538
6	Amsterdam	897	Düsseldorf	50.90	San Francisco	7.29	Chicago	477
7	Zurich	893	Amsterdam	50.84	Munich	4.89	Brussels	452
8	Munich	881	Munich	50.05	Oslo	4.60	Amsterdam	435
9	Osaka	787	Chicago	49.55	Vevey	4.46	Singapore	434
10	San Francisco	755	Stockholm	49.43	Zurich	4.32	Hong Kong	424
11	Frankfurt	515	Toronto	49.06	Beijing	4.23	Toronto	412
12	Vevey	491	Zurich	48.97	Atlanta	4.22	Madrid	338
13	Chicago	455	Los Angeles	48.62	Amsterdam	4.09	Philadelphia	334
14	Stockholm	427	Madrid	48.58	Stockholm	3.99	Milan	322
15	Dallas	413	Dallas	48.46	Osaka	3.98	San Francisco	321
16	Detroit	359	Houston	48.38	Saint Louis	2.95	Mexico City	280
17	Utrecht	336	Detroit	48.28	Detroit	2.71	Sydney	262
18	Toronto	324	Singapore	48.26	Melbourne	2.61	São Paulo	260
19	Saint Louis	315	Brussels	48.19	Dallas	2.49	Dallas	252
20	Basel	304	Seoul	48.18	Omaha	2.33	Munich	250
21	Philadelphia	299	Osaka	48.15	Chicago	2.32	Detroit	243
22	Atlanta	285	Atlanta	48.08	Basel	2.19	Houston	235
23	Oslo	283	Saint Louis	48.02	Philadelphia	1.98	Washington	227
24	Beijing	273	Mexico City	47.87	Turin	1.72	Atlanta	224
25	Hamilton	250	Milan	47.76	Houston	1.69	Bangkok	212
26	Omaha	245	Hong Kong	47.72	Ludwigshafen	1.65	Stockholm	194

Centrality Other Options

There are other options, usually based on generalizing some aspect of those above:

- *Random Walk Betweenness* (Mark Newman). Looks at the number of times you would expect node I to be on the path between k and j if information traveled a 'random walk' through the network.
- *Peer Influence* based measures (Friedkin and others). Based on the assumed network autocorrelation model of peer influence. In practice it's a variant of the eigenvector centrality measures.
- *Subgraph centrality*. Counts the number of cliques of size 2, 3, 4, … n-1 that each node belongs to. Reduces to (another) function of the eigenvalues. Very similar to influence & information centrality, but does distinguish some unique positions.
- *Fragmentation centrality* – Part of Borgatti's Key Player idea, where nodes are central if they can easily break up a network.
- Moody & White's *Embeddedness* measure is technically a group-level index, but captures the extent to which a given set of nodes are nested inside a network
- *Removal Centrality* – effect on the rest of the (graph for any given statistic) with the removal of a given node. Really gets at the system-contribution of a particular actor.