$$H(u,v) = a + b H_{hp}$$

512×512 $\quad$ 512×512

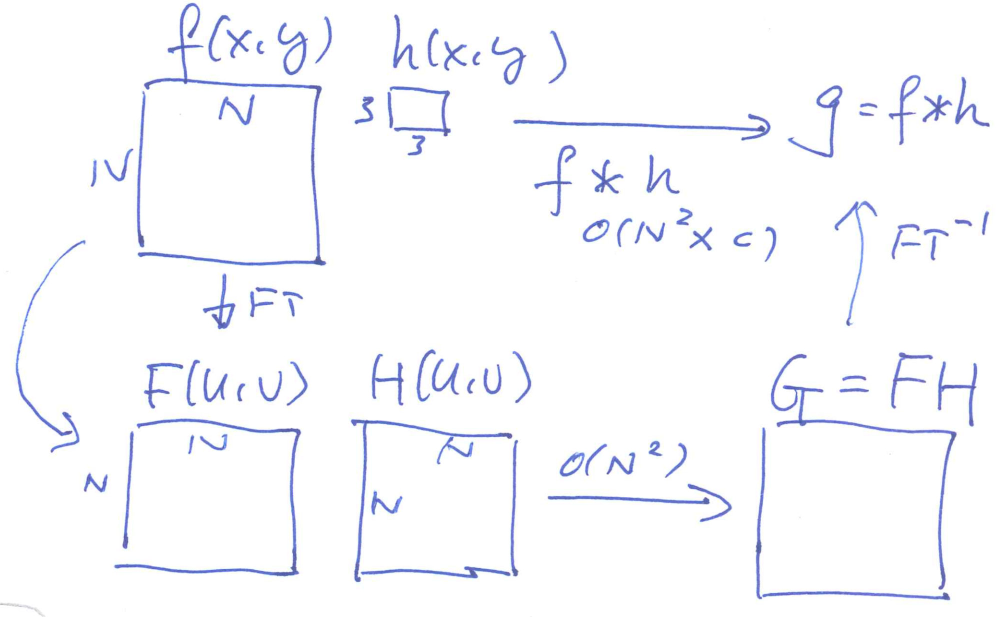
$$\underline{G(u,v)} = F(u,v)\, H(u,v)$$

$$G = F(u,v)\, a + b F(u,v) H_{hp}(u,v)$$

$$\Downarrow FT^{-1}$$

filtered output

$$g(x,y) = \underline{a f(x,y)} + b\, \boxed{f(x,y) * h_{hp}(x,y)}$$



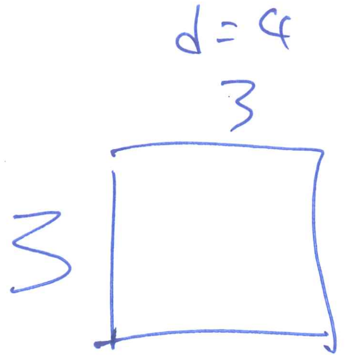
$$H(u,v) = a + b H_{hp}(u,v)$$

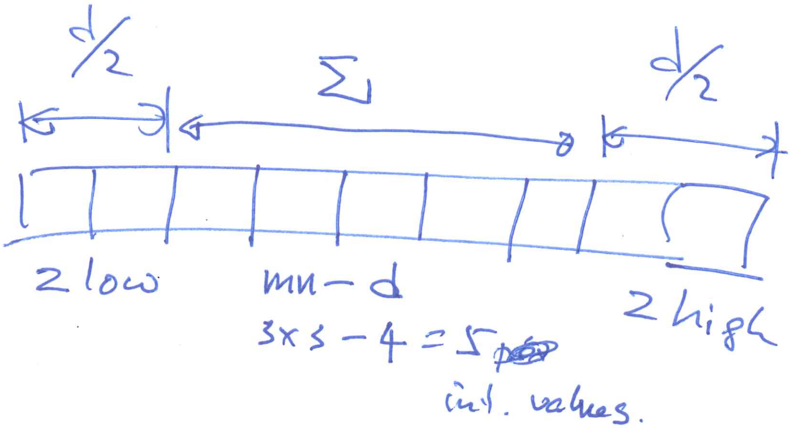
$$G(u,v) = F(u,v) H(u,v)$$

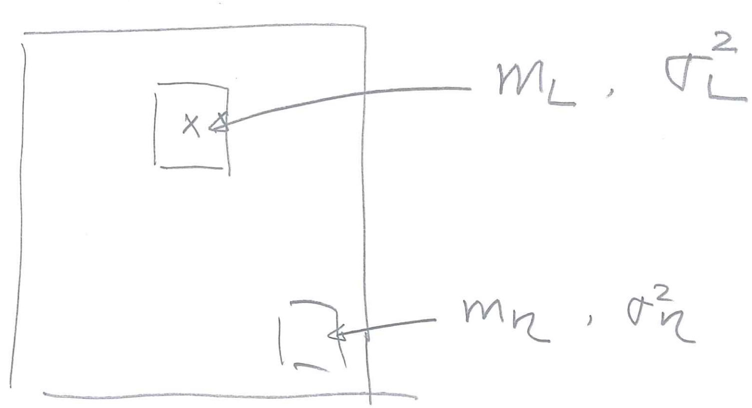
$$= aF(u,v) + b F(u,v) H_{hp}(u,v)$$

$$\Downarrow FT^{-1}$$

$$g(x,y) = a \underbrace{f(x,y)} + b \underbrace{f * h_{hp}(x,y)}$$


d = 4
3
3

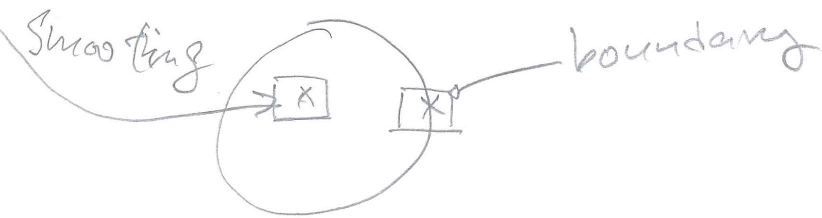

d/2
Σ
d/2
2 low
mn − d
3×3 − 4 = 5
int. values.
2 high



$$\text{If } \sigma_\eta^2 = 0 \rightarrow \hat{f}(x,y) = g(x,y)$$

$$\text{If } \sigma_\eta^2 = \sigma_L^2 \rightarrow \hat{f}(x,y) = g(x,y) - (1)[g(x,y) - m_L]$$
$$= m_L$$

$$\text{If } \sigma_\eta^2 \ll \sigma_L^2 \rightarrow \hat{f}(x,y) = g(x,y) - \cancelto{0}{\frac{\sigma_\eta^2}{\sigma_L^2}}(g - m_L)$$
$$= g(x,y)$$



31		
		39

Z_{med}

Z_{min} , Z_{max}

Z_{max}

Z_{med}

Z_{min}

$A1 = Z_{med} - Z_{min} > 0$ → not pepper noise

$A_2 = Z_{med} - Z_{max} < 0$ → not salt noise

otherwise

0	0	100
200	98	0
0	80	0

$Z_{min} = 0$

0 0 0 0 0 80 ...

$Z_{med} = 0$

if $A1 > 0$ & $A2 < 0$

then ⇒ level B

level B

Z_{max}

Z_{xy}

Z_{min}

$B1 = Z_{xy} - Z_{min} > 0$

$B2 = Z_{xy} - Z_{max} < 0$

if $B1 > 0$ & $B2 < 0$

then output Z_{xy}

else output Z_{med}

25	18	33
27	30	39
13	24	17

18 25 33

27 30 39

13 17 24

18 25 27 30 33 39

$$\begin{tikzcd} \overset{f}{(x_1, y_1)} \arrow[r, bend left] & \overset{g}{(x_1', y_1')} \end{tikzcd}$$

$$x_1' = c_1 x_1 + c_2 y_1 + c_3 x_1 y_1 + c_4$$

$$y_1' = c_5 x_1 + c_6 y_1 + c_7 x_1 y_1 + c_8$$

Complementary information for spatial Transformations

How to find $c_1, c_2, c_3, \ldots c_8$?

If 4 pairs of $(x, y) \Leftrightarrow (x', y')$ are given.

e.g.

$$\left.\begin{array}{l} (x_1, y_1) \Leftrightarrow (x'_1, y'_1) \\ (x_2, y_2) \Leftrightarrow (x'_2, y'_2) \\ (x_3, y_3) \Leftrightarrow (x'_3, y'_3) \\ (x_4, y_4) \Leftrightarrow (x'_4, y'_4) \end{array}\right\} \text{4 pairs}$$

then 8 equations can be written.

$$\left.\begin{array}{l} x'_1 = c_1 x_1 + c_2 y_1 + c_3 x_1 y_1 + c_4 \\ y'_1 = c_5 x_1 + c_6 y_1 + c_7 x_1 y_1 + c_8 \end{array}\right\} \text{2 functions from the 1st pair}$$

$$\vdots$$

$$x'_4 = c_1 x_4 + c_2 y_4 + c_3 x_4 y_4 + c_4$$

$$y'_4 = c_5 x_4 + c_6 y_4 + c_7 x_4 y_4 + c_8$$

g: (known) 2×4 (unknown) f: 4×4 (known)

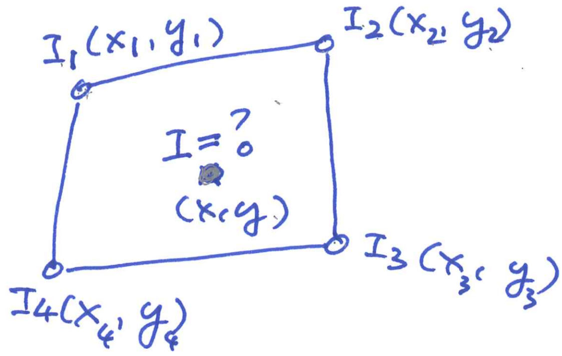
$$\begin{bmatrix} x'_1 & x'_2 & x'_3 & x'_4 \\ y'_1 & y'_2 & y'_3 & y'_4 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ c_5 & c_6 & c_7 & c_8 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ x_1 y_1 & x_2 y_2 & x_3 y_3 & x_4 y_4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ c_5 & c_6 & c_7 & c_8 \end{bmatrix} = \begin{bmatrix} x'_1 & x'_2 & x'_3 & x'_4 \\ y'_1 & y'_2 & y'_3 & y'_4 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ x_1 y_1 & x_2 y_2 & x_3 y_3 & x_4 y_4 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}$$

Complementary information for Grey-level Assignments

Bilinear interpolation

How to find I, given that I_1, I_2, I_3 and I_4 are known?



$$I_1(x_1, y_1) = a x_1 + b y_1 + c x_1 y_1 + d$$

$$I_2(x_2, y_2) = a x_2 + b y_2 + c x_2 y_2 + d$$

$$I_3(x_3, y_3) = a x_3 + b y_3 + c x_3 y_3 + d$$

$$I_4(x_4, y_4) = a x_4 + b y_4 + c x_4 y_4 + d$$

$$\underbrace{[I_1 \quad I_2 \quad I_3 \quad I_4]}_{1\times 4,\ \text{known}} = \underbrace{[a \quad b \quad c \quad d]}_{1\times 4\ \text{unknown}} \overbrace{\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ x_1 y_1 & x_2 y_2 & x_3 y_3 & x_4 y_4 \\ 1 & 1 & 1 & 1 \end{bmatrix}}^{4\times 4\ \text{known}}$$

$$[a \quad b \quad c \quad d] = [I_1 \quad I_2 \quad I_3 \quad I_4] \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ x_1 y_1 & x_2 y_2 & x_3 y_3 & x_4 y_4 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1}$$

$\therefore$ a, b, c & d are found and

$$I(x, y) = a x + b y + c x y + d$$