COMP 2711 Discrete Mathematical Tools for CS
Fall 2014 – Written Assignment # 2
Distributed: Sept 18, 2014 – Due: Sept 25, 2014

Your solutions should contain (i) your name, (ii) your student ID #, (ii) your email address and (iv) your tutorial section.

Problem 1: Consider the sets $S_4 = \{a, b, c, d\}$ and $S_5 = \{1, 2, 3, 4, 5\}$?
(a) How many functions are there from the set S_4 to S_5?
(b) How many *one-to-one* functions are there from the set S_4 to S_5?
(c) How many *onto* functions are there from the set S_4 to S_5?
(d) How many *bijections* are there from the set S_4 to S_5?
(e) How many functions are there from the set S_5 to S_4?
(f) How many *one-to-one* functions are there from the set S_5 to S_4?
(g) How many *onto* functions are there from the set S_5 to S_4?
(h) How many *bijections* are there from the set S_5 to S_4?
(i) How many functions are there from the set S_4 to S_4?
(j) How many *one-to-one* functions are there from the set S_4 to S_4?
(k) How many *onto* functions are there from the set S_4 to S_4?
(l) How many *permutations* are there from the set S_4 to S_4?

Problem 2: A base ten number is a string of five digits, where the digits are from the set $\{0, 1, \ldots, 9\}$ but the first digit cannot be 0 (so 52375 is a valid number but 02323 and 2323 are not).
(a) How many five-digit base ten numbers are there?
(b) How many five-digit numbers have no two consecutive digits equal?
(c) How many have at least one pair of consecutive digits equal?

Problem 3: Suppose you are organizing a panel discussion on allowing karoke on campus. Participants will sit behind a long table in the order in which you list them. You must choose 4 administrators from a group of 10 and 5 students from a group of 15.
(a) If the administrators must sit together in a group and the students must sit together in a group, in how many ways can you choose and list the 9 people?
(b) If you must alternate students and administrators, in how many ways can you choose and list them?

Problem 4: A by-election is being held to fill 5 seats in a council that have recently become vacant. A total of 9 candidates have registered to run in the election. The candidates can be divided into 3 groups based on their political affiliations with: 4 in Group A, 3 in Group B, and 2 in Group C.

(a) In how many ways can a voter choose 5 candidates?

(b) In how many ways can a voter choose 5 candidates such that at least 3 candidates from Group A are chosen?

(c) In how many ways can a voter choose 5 candidates such that at least 1 candidate from each group is chosen?

Problem 5: In class we stated that
each row of Pascal's triangle first increases and then decreases.
In this question you will prove this statement.

(a) Using the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ prove that

$$\text{if } 0 < k \leq n/2 \text{ then } \binom{n}{k-1} < \binom{n}{k}.$$

(b) Using part (a) and the fact that $\binom{n}{k} = \binom{n}{n-k}$ prove that
each row of Pascal's triangle first increases and then decreases.

Problem 6: (a) If you have thirteen distinct chairs to paint, in how many ways can you paint seven of them orange and six of them red?

(b) Now, how many ways can you paint four of them green, three of them blue, and six of them red?

Problem 7: Give two proofs that

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$$

Your first proof should be purely algebraic, i.e., just plug in the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and show that the left side equals the right side. Your second proof should be combinatorial, i.e., it should show that the left and right sides are just two different ways to count the same thing.

Problem 8: Suppose there are 20 balls and 3 boxes. The three boxes are labeled with 'A', 'B' and 'C' respectively. In how many different ways can we distribute the balls into the boxes under each of the following conditions?

(a) The twenty balls are labeled with integers $1, 2, \ldots, 20$ respectively so that each ball is distinct and is different from other balls.

(b) **(Challenge)** All the balls are identical and hence are indistinguishable from each other.