COMP3721 Homework 2

1. a) Suppose L is regular. Let n be the pump length, choose $w = 01^n01^n$. By P.T. w can be written as xyz, such that $|xy| \leq n, y \neq e$ and $xy^iz \in L$ for all $i \geq 0$. y cannot contain only 0, otherwise there will be at least two consecutive 0's in xy^2z. Then we have $xy^0z \notin L$, contradicting the P.T. So L is not regular.

   b) $L \bigcap 01^*01^* = 01^n01^n$. If L is regular, then 01^n01^n will be regular, too, which is false. So L is not regular.

   c) Suppose L is regular. Let n be the pump length, choose $w = 1^n001^n$. By P.T., w can be written as xyz such that $|xy| \leq n$ and $y \neq e$. Thus y consists of only $1's$, then we have $xy^0z \notin L$, contradicting the P.T. So L is not regular.

2. a)$S \rightarrow e$, $S \rightarrow a$, $S \rightarrow b$, $S \rightarrow aSa$, $S \rightarrow bSb$.

   b)$S \rightarrow aSa$, $S \rightarrow bSb$, $S \rightarrow A$, $A \rightarrow bBa$, $A \rightarrow aBb$, $B \rightarrow e$, $B \rightarrow aB$, $B \rightarrow bB$

3. Let $M_1 = \{K_1, \Sigma_1, \Gamma_1, \Delta_1, s_1, F_1\}$, $M_2 = \{K_2, \Sigma_2, \Gamma_2, \Delta_2, s_2, F_2\}$. $M = \{K, \Sigma, \Gamma, \Delta, s, F\}$, where

   $K = K_1 \bigcup K_2 \bigcup \{s\}$
   $\Sigma = \Sigma_1 \bigcup \Sigma_2$
   $\Gamma = \Gamma_1 \bigcup \Gamma_2 \bigcup \{\alpha\}$
   $s = s$
   $F = F_2$
   $\Delta = \Delta_1 \bigcup \Delta_2 \bigcup \{((s, e, e)(s_1, \alpha))\} \bigcup \{((p, e, \alpha)(s_2, e))|p \in F_1\}$

   State diagram omitted.

4. a) DFA is CONSTANT-STACK PA with $k = 0$.

   b) CONSTANT-STACK PA is a special PA.

   c) REGULAR = CONSTANT-STACK

   Intuitively CONSTANT-STACK is not more powerful than DFA, since we can use a constant number of states to simulate the stack with constant size.

   CONSTANT-STACK $\subset$ CONTEXT-FREE

   since we know that there are some languages, e.g., $\{0^n1^n \mid n \geq 0\}$, that are context-free but not regular.