

COMP2711H Tutorial 8

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1 Extended Euclidean Algorithm

Example 1.1. Calculate the greatest common divisor d of $a = 2445$ and $b = 652$. Which integers s and t make $as + bt = d$?

| i | r_i | q_i | s_i | t_i |
|-----|-------|-------|-------|-------|
| 0 | 2445 | | 1 | 0 |
| 1 | 652 | 3 | 0 | 1 |
| 2 | 489 | 1 | 1 | -3 |
| 3 | 163 | 3 | -1 | 4 |
| 4 | 0 | | | |

so we have that $163 = (-1)2554 + (4)652$.

$$r_{i+1} = r_{i-1} - r_i q_i$$

$$r_i = as_i + bt_i$$

$$s_{i+1} = s_{i-1} - q_i s_i$$

$$t_{i+1} = t_{i-1} - q_i t_i$$

2 Fermat's Little Theorem

Exercise 1. What is the value of $9^{794} \bmod 73$?

Exercise 2. What is the value of $34^{70} \bmod 73$?

Exercise 3. Prove that $(2^{70} + 3^{70}) \bmod 13 = 0$.

Reference

1. <http://www.oxfordmathcenter.com/drupal7/node/204>
2. <http://people.brandeis.edu/~jbellaic/nt/ex2sol.pdf>
3. http://db.math.ust.hk/notes_download/elementary/number/ne_N1.pdf