COMP 272: Theory of Computation
Fall 2011 Midterm Exam

1. Print your name and student ID at the top of every page (in case the staple falls out!).
2. This is an open-book, open-notes, open-brain exam.
3. Time limit: 120 minutes.
4. You should answer all the questions on the exam. At least you should read all the questions—they are not ordered by their difficulty!
5. When asked to construct an automaton, you can use either a state diagram (recommended) or the formal mathematical definition.
6. You can write on the back of the paper if you run out of space. Please let us know if you need more scratch paper.
7. Relax and breathe, it's just a midterm.

1. (10 pts) Construct a DFA that accepts the language $L = \{w \mid w \in \{0,1\}^*, w$ has an even number of 0's and an odd number of 1's$\}$.

2. (10 pts) Give a context-free grammar that generates the languages $L = \{w \mid w = xy,$ where $x, y \in \{0,1\}^+, |x| = |y|, x \neq y^R\}$.

3. (8 pts) Are the following sets countable or uncountable? You do not need to prove your answers. Let $\mathbb{N}$ be the set of natural numbers, and $\Sigma = \{0, 1\}$.

   (a) $\{(x, y) \mid x, y \in \mathbb{N}, x \neq y\}$.

   (b) Any infinite language over Σ.

   (c) The set of all languages over Σ.

   (d) The set of all finite languages over Σ.

4. (12 pts) In class we proved that context-free languages are not closed under complementation by resorting to intersection. Here is a concrete counter-example. Fix the alphabet $\Sigma = \{a, b, c\}$, and let $L = \{a^n b^n c^n : n \geq 0\}$. We know in class that L is not context-free. Show that the complement of L is context-free. [Hint: Express $\overline{L}$ as the union of three languages, one regular, the other two context-free.]

5. (20pts) Prove that the language $\{w \in \{a, b\}^* : w \text{ has twice as many } b\text{'s as } a\text{'s}\}$ is not regular.

6. (20pts) Let L be a regular language. Prove that $\{w^R \mid w \in L\}$ is also a regular language.

7. (20 pts) Let $M = (K, \Sigma, \Gamma, \Delta, s, F)$ be a pushdown automaton. In class, the language accepted by M is defined to be the set of strings on which there is a computation sequence such that when the entire input string is read, M is at a final state *and* the stack is empty. Here we show that the "stack being empty" condition is dispensable. Define

$$L_F(M) = \{w \in \Sigma^* : (s, w, e) \vdash^* (f, e, \beta) \text{ for some } f \in F, \beta \in \Gamma^*\}.$$

(a) Show that there is a pushdown automaton M' such that $L(M') = L_F(M)$.

(b) Show that there is a pushdown automaton M'' such that $L_F(M'') = L(M)$.