COMP2711H Tutorial 1

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1 Conditional Statement

The trafic rules:

(1) If R: the signal is red, then S: you must stop. $R \to S$.

(2) If G: the signal is green, then M: you must keep moving. $G \to M$

(3) If Y: the signal is yellow, then $S \vee M$: you either stop or keep on moving. $Y \to S \vee M$

How to say that someone obeys the traffic rules in logical language?

$$(R \to S) \wedge (G \to M) \wedge (Y \to S \vee M) = T$$

At a particular moment when the signal is yellow, a man stops. Does he violate the rule? What's the truth value of the above formular? What happens if we force $F \to T$ to be false?

Both R and Y may lead to S, so if a man stops when the signal is not red, we can't say that he violates rule(1). Hences, even when R is false and S is true, $R \to S$ is still true.

2 Tautology

A propositional formular is a tautology, if it is always true, regardless of the truth vale of any of its propositonal variables.

Exercise 2.1. Prove the following propositional formulars are tautologies.

(1) $p \vee \neg p$

(2) $p \wedge (p \to q) \to q$

(3) $(p \to q) \wedge \neg q \to \neg p$

(4) $(p \to q) \wedge (q \to r) \to (p \to r)$

3 Equivalence of Propositional Formulars

Exercise 3.1. Use truth table to prove the following logical equivalence.

1.

$$\begin{aligned} p \to q &\equiv \neg p \vee q \\ p \to q &\equiv \neg q \to \neg p \end{aligned}$$

2. Distributive laws

$$\begin{aligned} p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \\ p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r) \end{aligned}$$

3. DeMorgan's laws

$$\begin{aligned}\neg(p \wedge q) &\equiv \neg p \vee \neg q \\ \neg(p \vee q) &\equiv \neg p \wedge \neg q\end{aligned}$$

4. Absorption laws

$$\begin{aligned}p \vee (p \wedge q) &\equiv p \\ p \wedge (p \vee q) &\equiv p\end{aligned}$$

5. Double negation laws

$$\neg(\neg p) \equiv p$$

6. Commutative laws

$$\begin{aligned}p \vee q &\equiv q \vee p \\ p \wedge q &\equiv q \wedge p\end{aligned}$$

7. Associative laws

$$\begin{aligned}(p \vee q) \vee r &\equiv p \vee (q \vee r) \\ (p \wedge q) \wedge r &\equiv p \wedge (q \wedge r)\end{aligned}$$

When the formular contains n variables, the number of lines of the truth table is 2^n! It is time-consuming to list the truth table when n is large.

4 Substitution and Interchange

Substitution

Exercise 4.1. Prove that $s \vee t \vee (q \wedge r) \equiv (s \vee t \vee q) \wedge (s \vee t \vee r)$. (Hint: let $q = s \vee t$.)

Observation 4.1. Let f be a tautology. Let g be a propositional formular. If we replace all occurrence of a variable in f by g, then the resulting propositional formular is a tautology.

Exercise 4.2. Prove that $s \vee t \vee (q \wedge r \wedge k) \equiv (s \vee t \vee q) \wedge (s \vee t \vee r) \wedge (s \vee t \vee k)$.

Interchange

Exercise 4.3. Prove that $p \wedge (q \rightarrow r) \equiv (p \wedge \neg q) \vee (p \wedge r)$. (Hint: you may use the fact that $q \rightarrow r \equiv \neg q \vee r$.)

Observation 4.2. Let f be tautology. Let g be a subformular in f. Let h be a propositonal formular that is logically equivalnet to g. Then if we replace g in f by h, then the resulting formular is a tautology.

References

[1] V. Koltun. *Discrete Structures Lecture Notes*, chapter 8. 2008.

[2] E. Lehman, T. Leighton, and A. R. Meyer. *Mathematics for computer science*, chapter 3. 2010.

[3] Wikibooks. *Formal Logic in Wikibooks*, chapter Sentential Logic.