LECTURE 8:NETWORK EFFECTS AND CASCADING BEHAVIOR

COMP4641: Social Information Network Analysis and Engineering

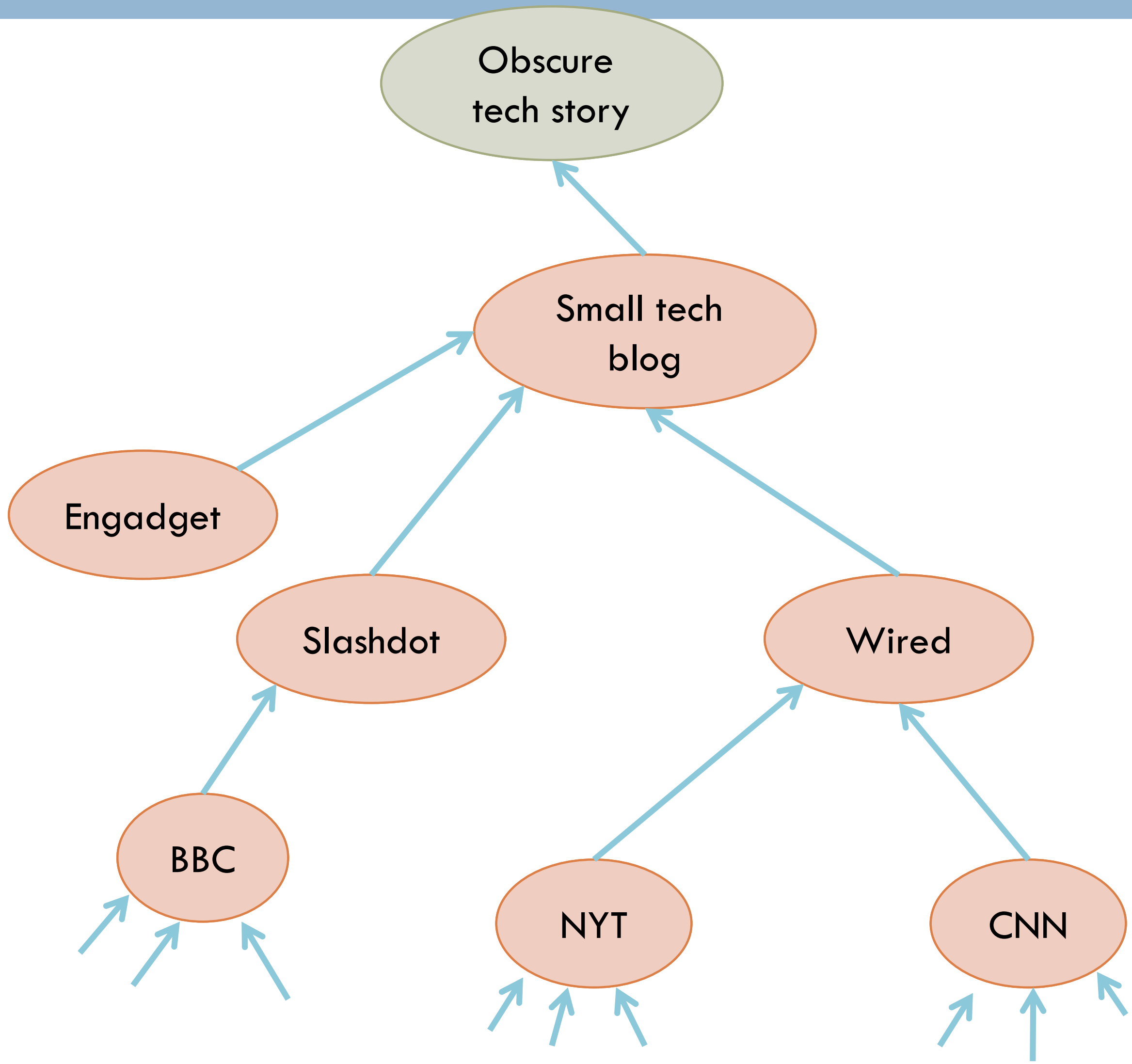
Wednesday March 11th 2015

Spreading Through Networks

- **Spreading through networks:**
  - Cascading behavior
  - Diffusion of innovations
  - Network effects
  - Epidemics
- **Behaviors that cascade from node to node like an epidemic**

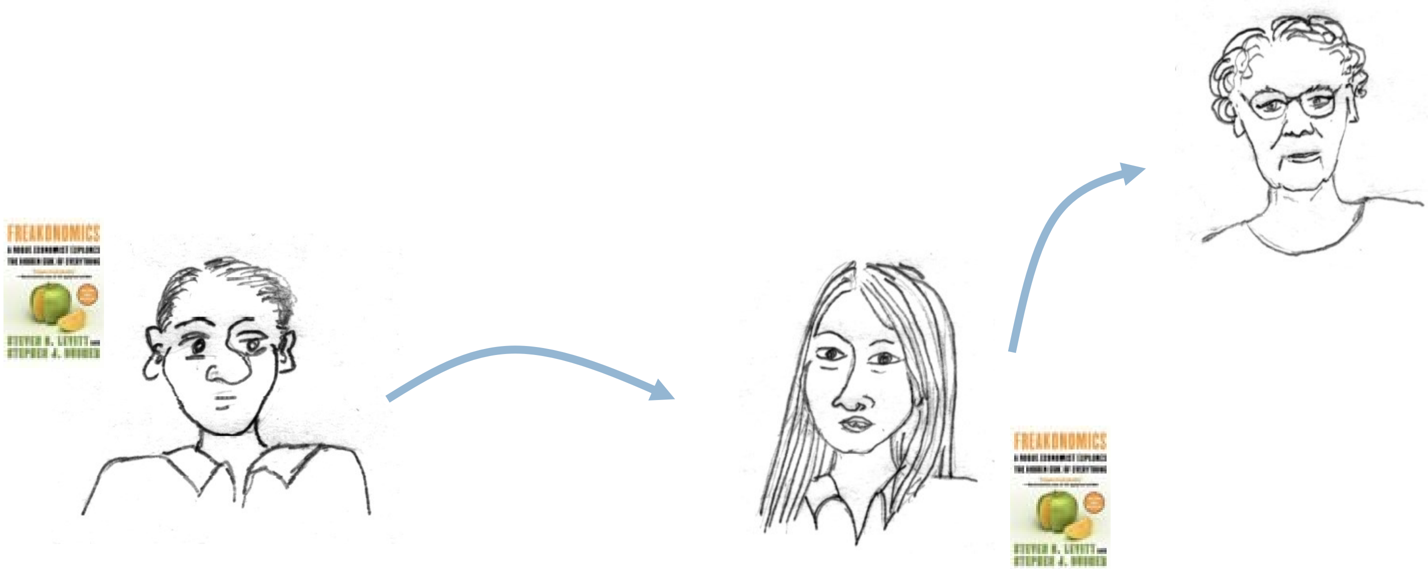
- **Examples:**
  - **Biological:**
    - Diseases via contagion
  - **Technological:**
    - Cascading failures
    - Spread of information
  - **Social:**
    - Rumors, news, new technology
    - Viral marketing

Information Diffusion

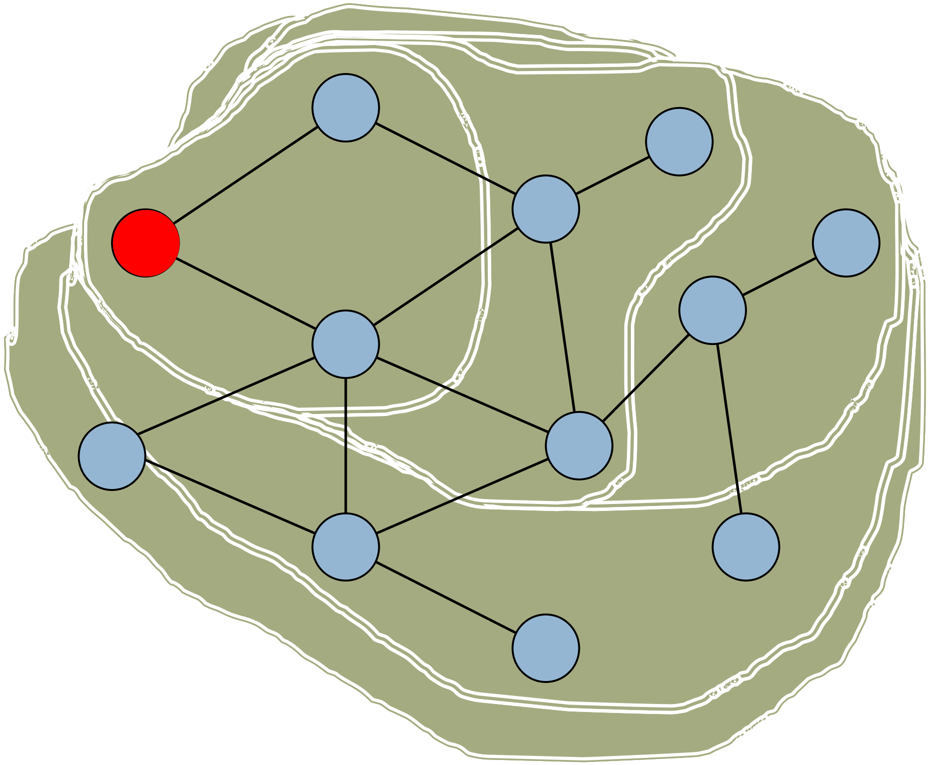


Diffusion in Viral Marketing

- **Product adoption:**
  - **Senders and followers of recommendations**

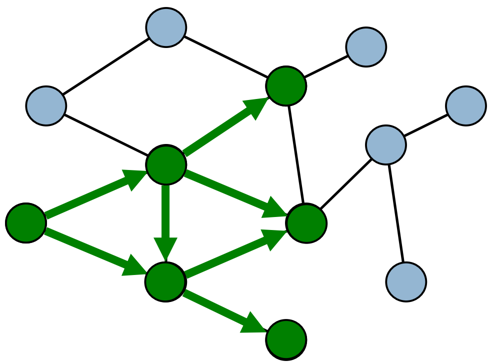


Spread of Diseases

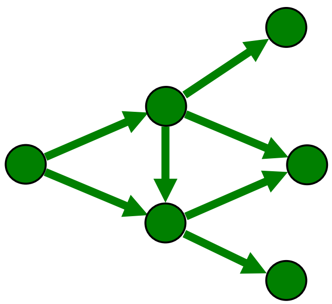


Network Cascades

- **Contagion that spreads over the edges of the network**
- It creates a propagation tree, i.e., **cascade**



Network



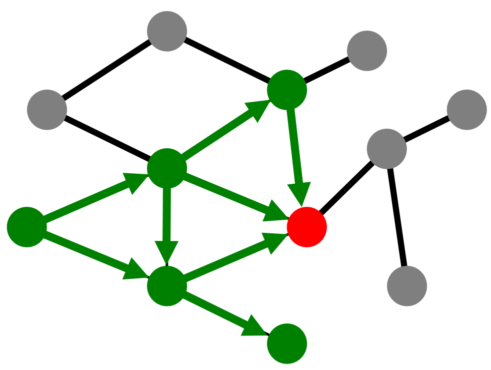
Cascade
(propagation graph)

Terminology:

- Stuff that spreads: Contagion
- "Infection" event: Adoption, infection, activation
- We have: Infected/active nodes, adoptors

How to Model Diffusion?

- **Probabilistic models:**
  - Models of influence or disease spreading
    - An infected node tries to "push" the contagion to an uninfected node
  - **Example:**
    - You "catch" a disease with some prob. from each active neighbor in the network

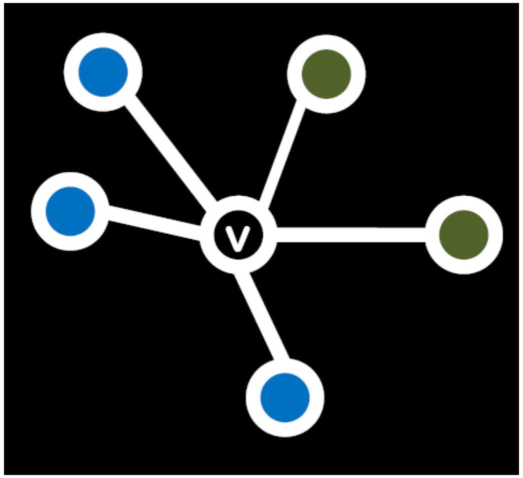


- **Decision based models (today!):**
  - Models of product adoption, decision making
    - A node observes decisions of its neighbors and makes its own decision
  - **Example:**
    - You join demonstrations if *k* of your friends do so too

DECISION BASED MODEL OF DIFFUSION

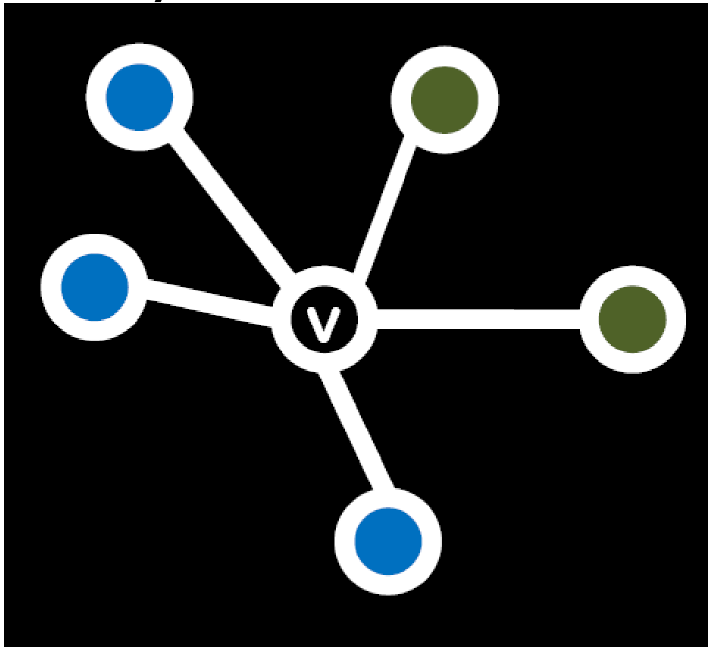
Decision Based Models

- **Two ingredients:**
  - **Payoffs:**
    - Utility of making a particular choice
  - **Signals:**
    - Public information:
      - What your network neighbors have done
    - (Sometimes also) Private information:
      - Something you know
      - Your belief
- **Now you want to make the optimal decision**



Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** "behavior", **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you



Local view of the network of node v

Example: BlueRay vs. HD DVD

Blu-ray Disc

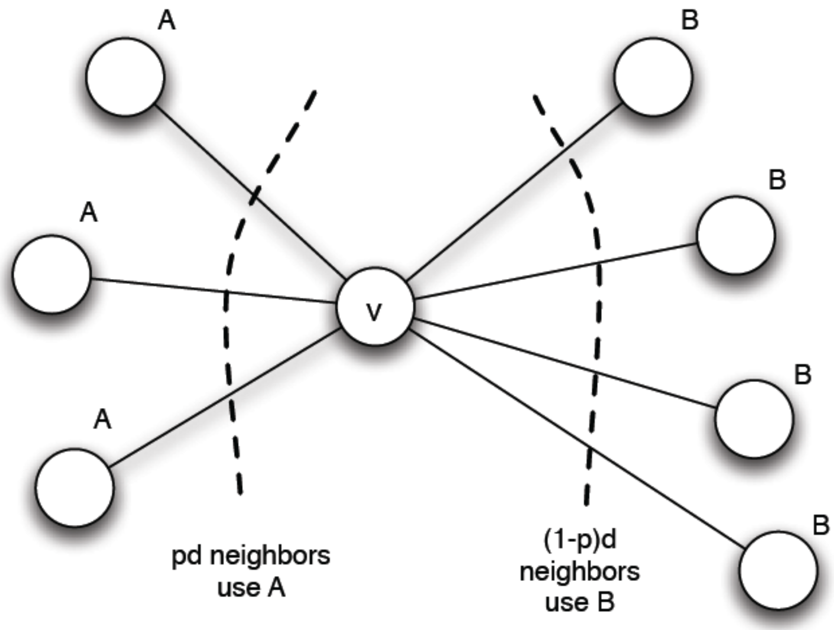
HD DVD™

The Model for Two Nodes

- ***Payoff matrix:***
  - If both v and w adopt behavior A, they each get payoff $a > 0$
  - If v and w adopt behavior B, they reach get payoff $b > 0$
  - If v and w adopt the opposite behaviors, they each get 0

- **In some large network:**
  - Each node v is playing a copy of the game with each of its neighbors
  - **Payoff**: sum of node payoffs per game

Calculation of Node v



Threshold:
v choses *A* if

$$p > q = \frac{b}{a+b}$$

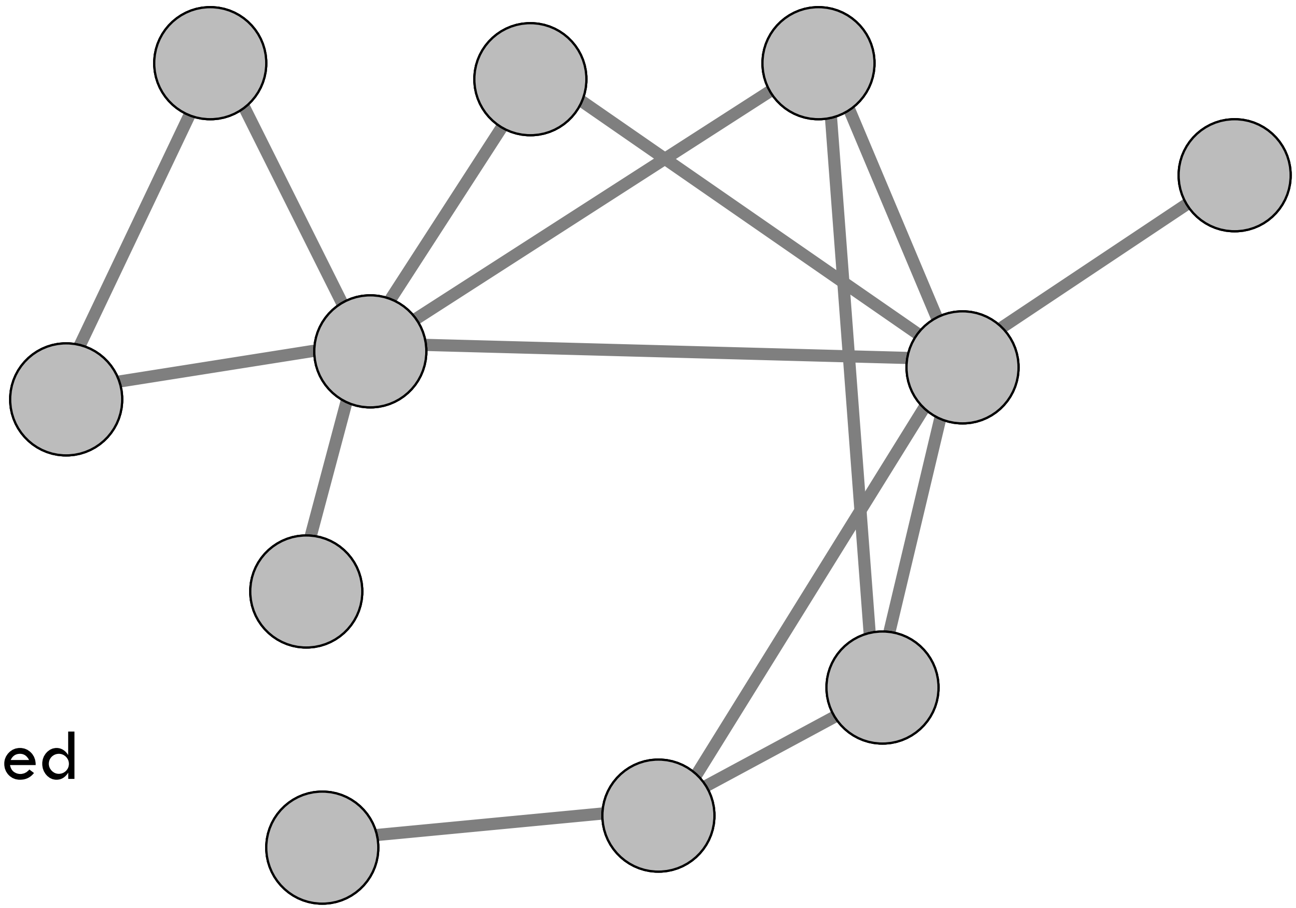
- Let v have *d* neighbors
- Assume fraction *p* of *v*'s neighbors adopt *A*
  - $Payoff_v$ $= a \cdot p \cdot d$ if *v* chooses A
    $= b \cdot (1-p) \cdot d$ if *v* chooses B
- **Thus: v chooses A if:** $a \cdot p \cdot d > b \cdot (1-p) \cdot d$

Example Scenario

- **Scenario:**
  Graph where everyone starts with *B*.
  Small set *S* of early adopters of *A*
  - Hard-wire *S* – they keep using *A* no matter what payoffs tell them to do
- Assume payoffs are set in such a way that nodes say:
  If more than 50% of my friends take A
  I'll also take A
  (this means: $a = b - \varepsilon$ and $q > 1/2$)

Example Scenario

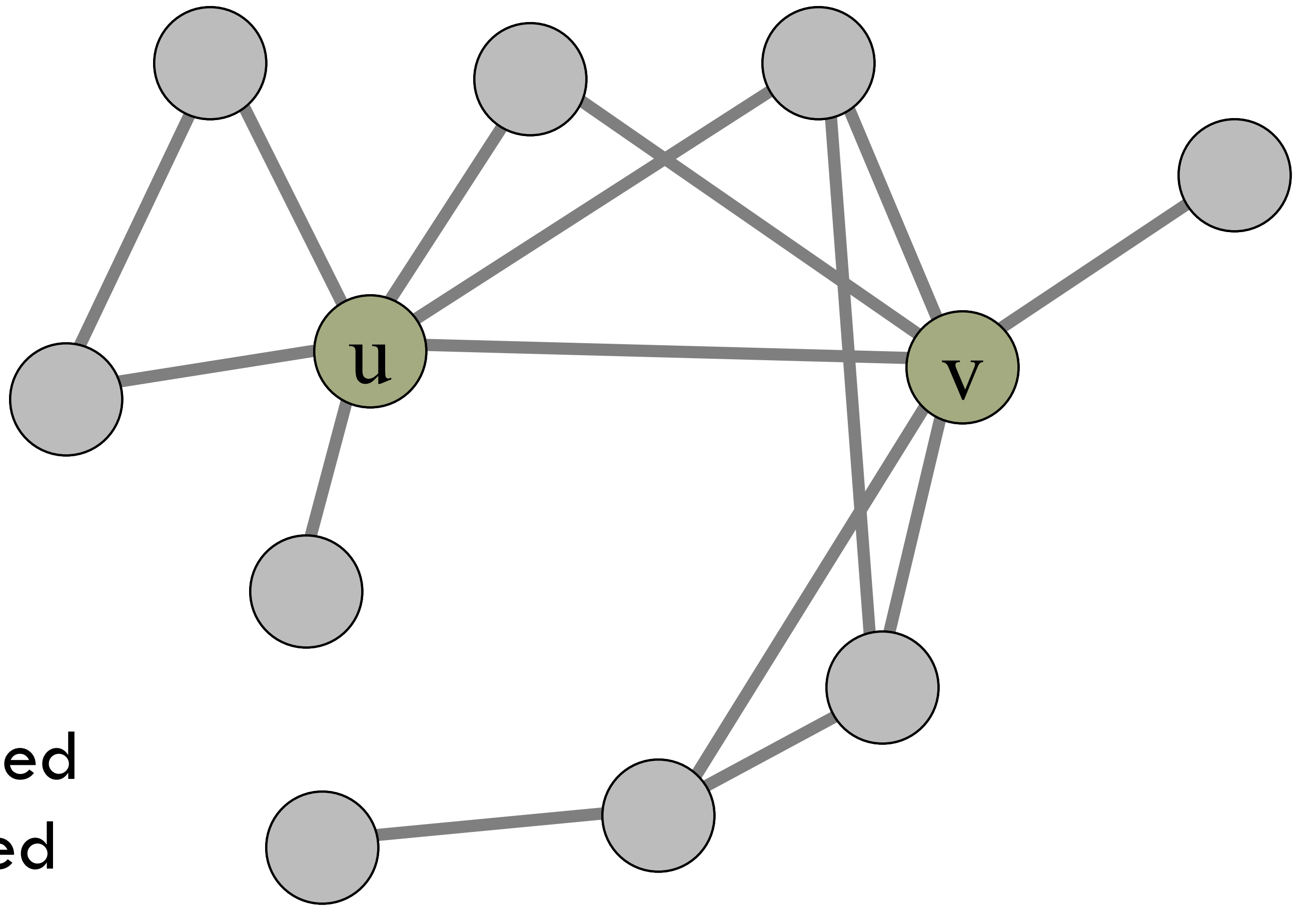
$$S = \{u, v\}$$



If more than 50% of my friends are red I'll be red

Example Scenario

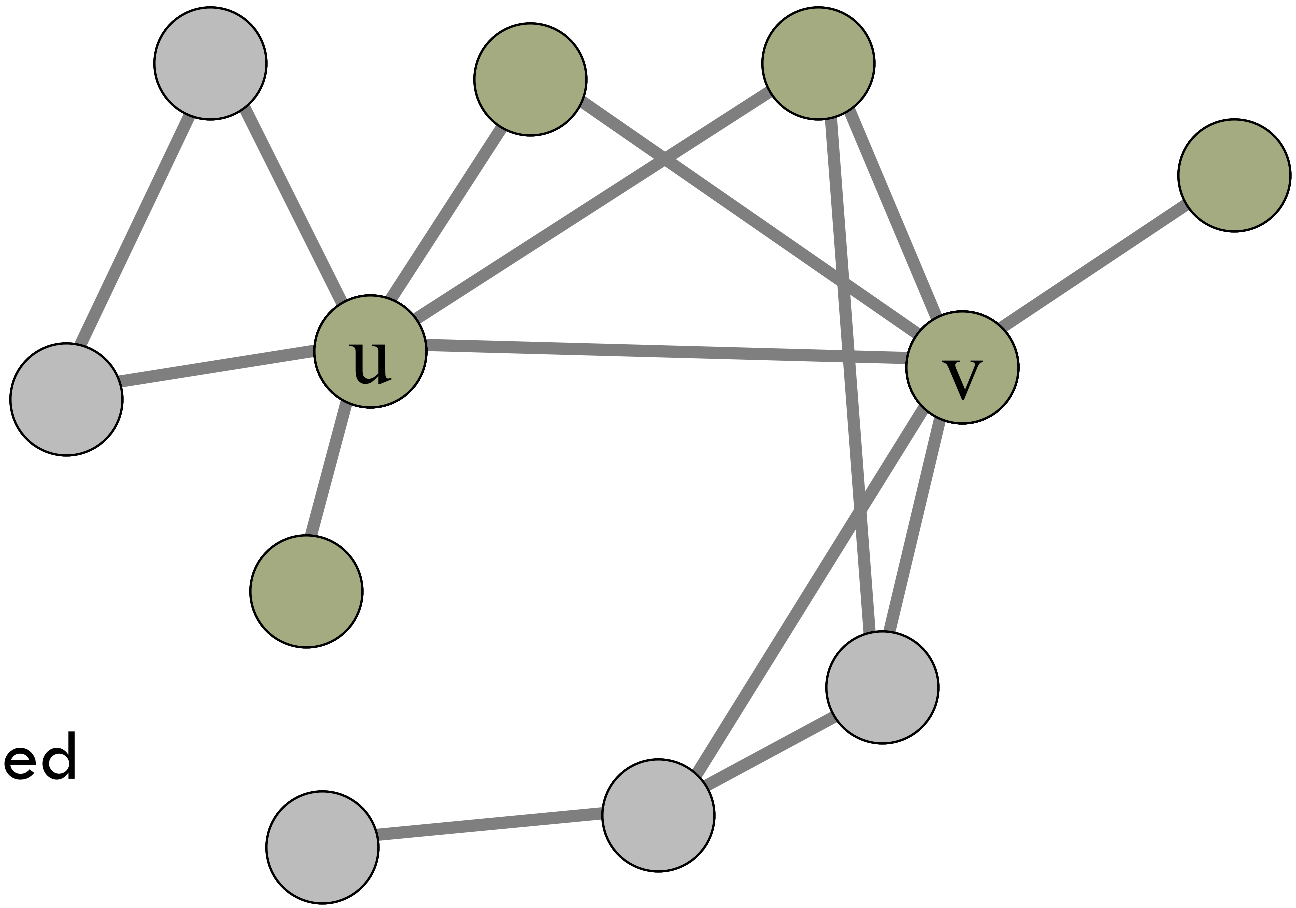
$S = \{u, v\}$



If more than 50% of my friends are red I'll be also red

Example Scenario

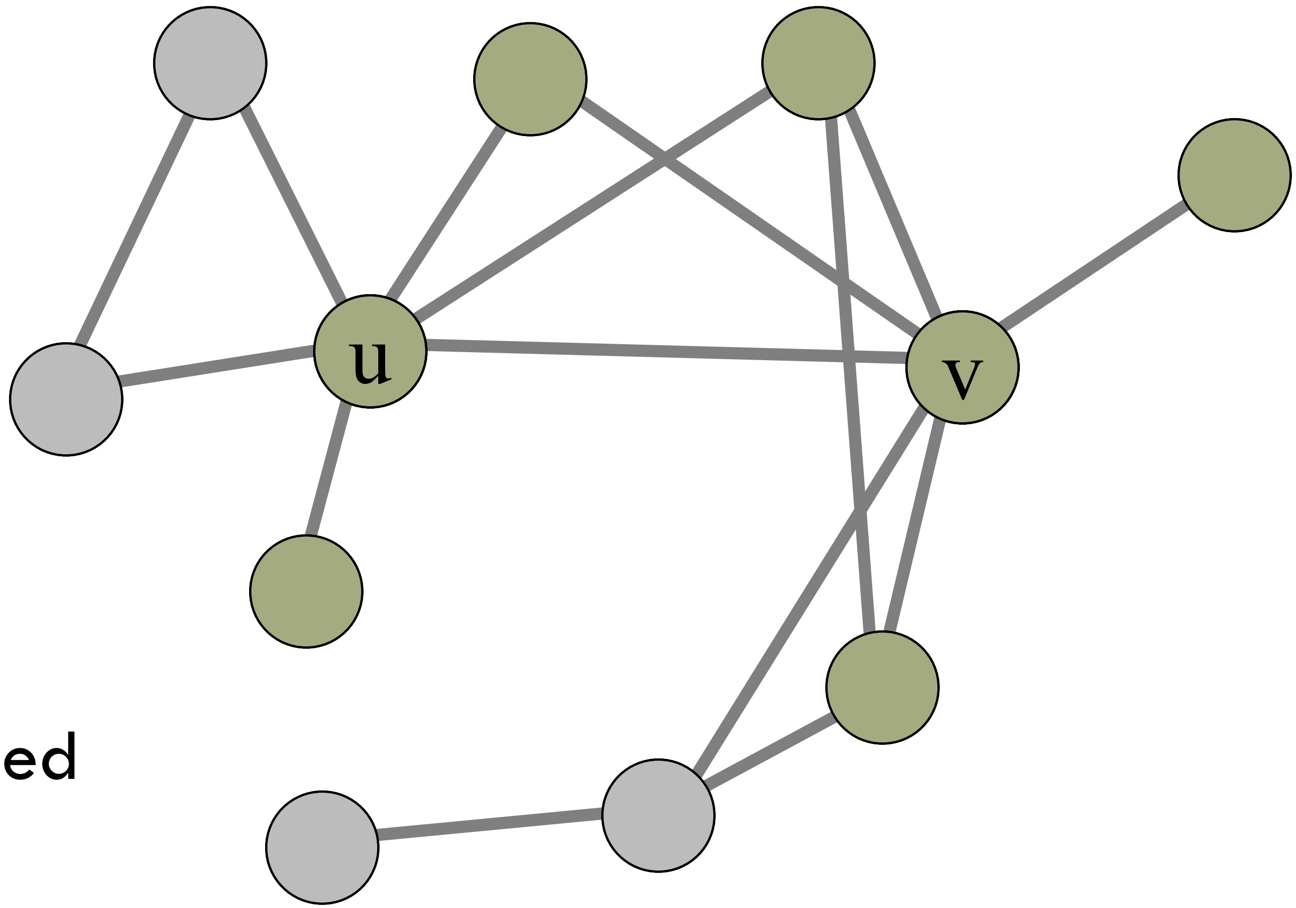
$S = \{u, v\}$



If more than 50% of my friends are red I'll be red

Example Scenario

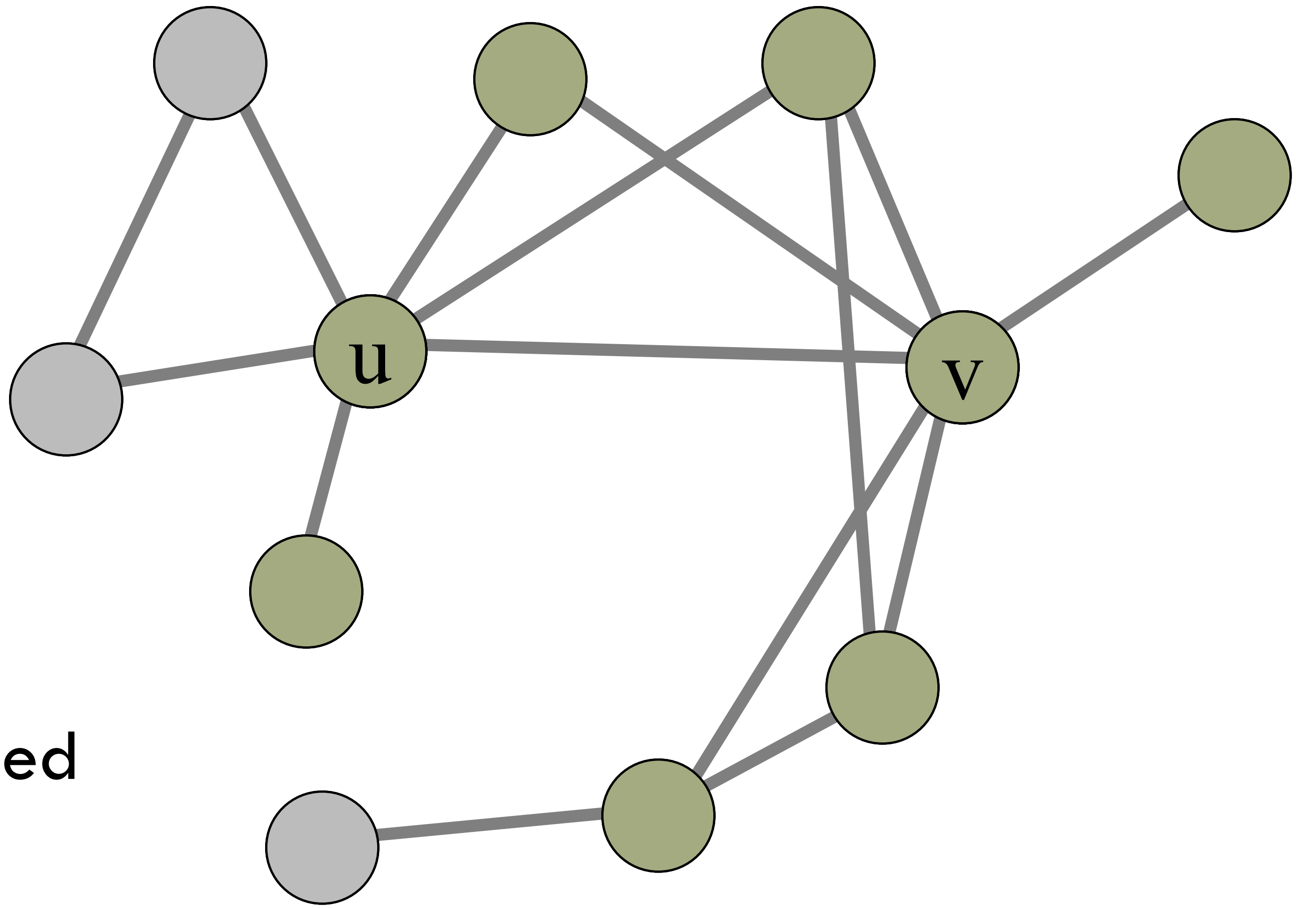
$S = \{u, v\}$



If more than 50% of my friends are red I'll be red

Example Scenario

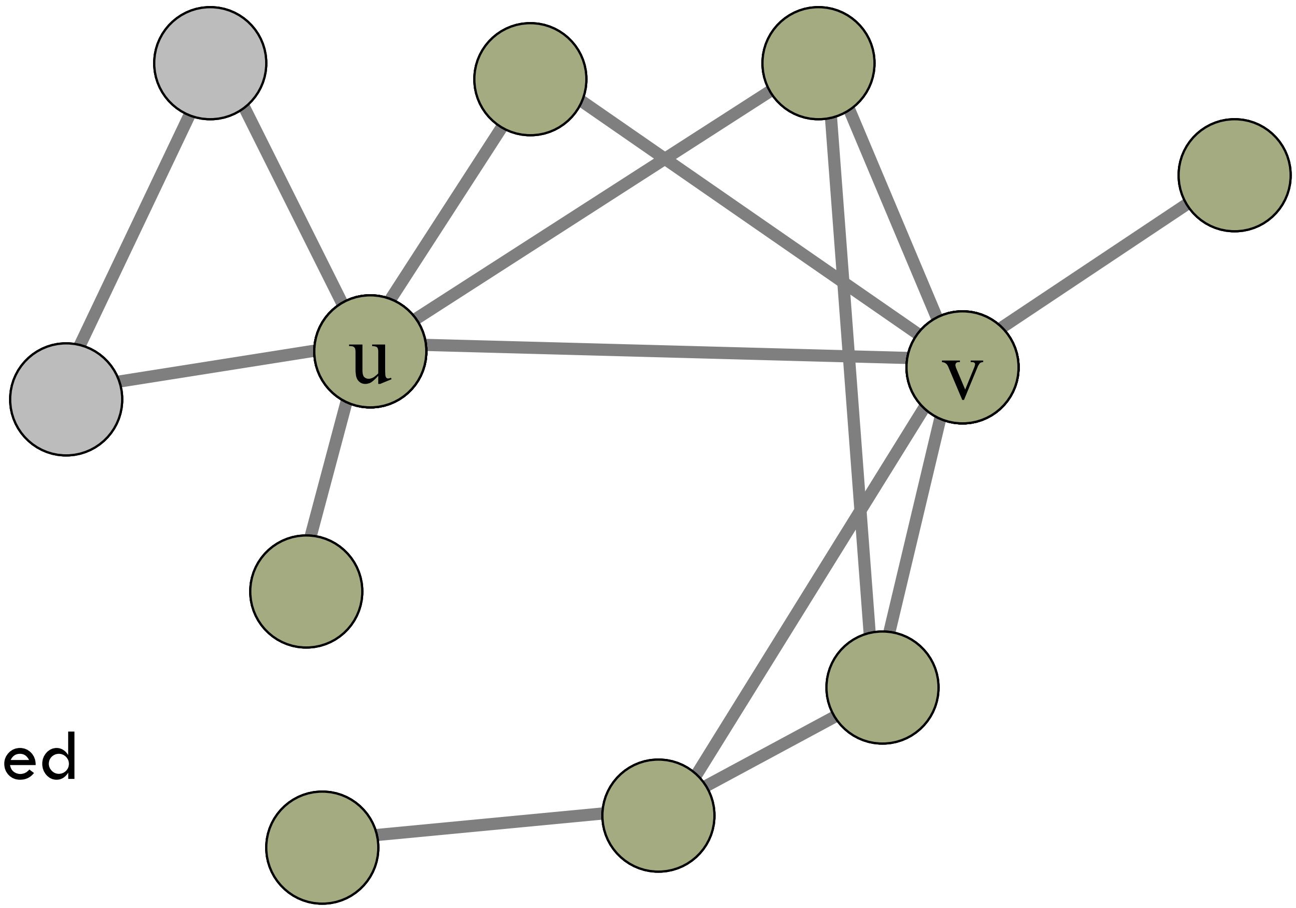
$S = \{u, v\}$



If more than 50% of my friends are red I'll be red

Example Scenario

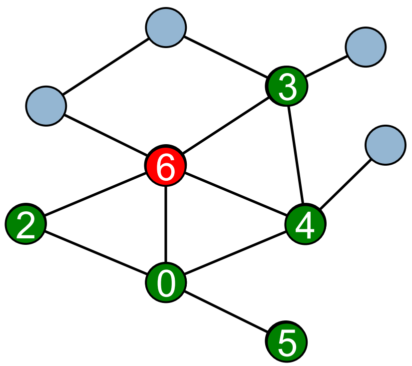
$$S = \{u, v\}$$



If more than 50% of my friends are red I'll be red

Monotonic Spreading

- **Observation: Use of A spreads monotonically**
  (Nodes only switch B→A, but never back to B)
- **Why?** Proof sketch:
  - **Nodes keep switching from B to A:** *B→A*
  - Now, suppose some node switched back from *A→B*, consider the **first** node ***u*** to do so (say at time *t*)
  - Earlier at some time *t'* (*t'*<*t*) the same node ***u*** switched *B→A*
  - So at time *t'* ***u*** was above threshold for *A*
  - But up to time *t* no node switched back to B, so node ***u*** could only had more neighbors who used *A* at time *t* compared to *t'*.
    There was no reason for *u* to switch.
    !! Contradiction !!



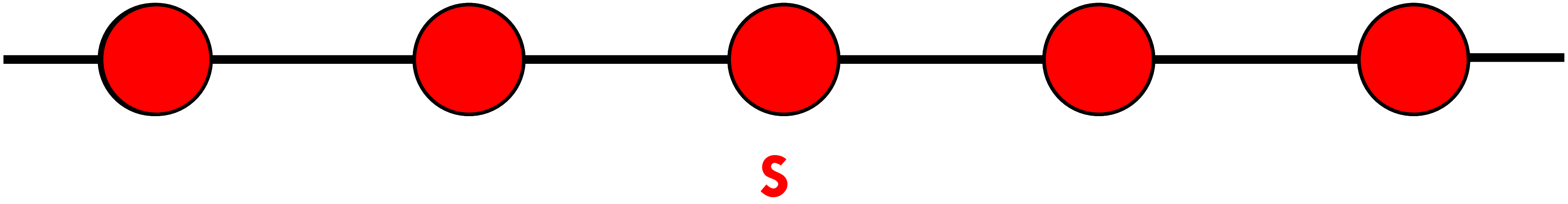
Infinite Graphs

v choses *A* if $p>q$

$$q = \frac{b}{a+b}$$

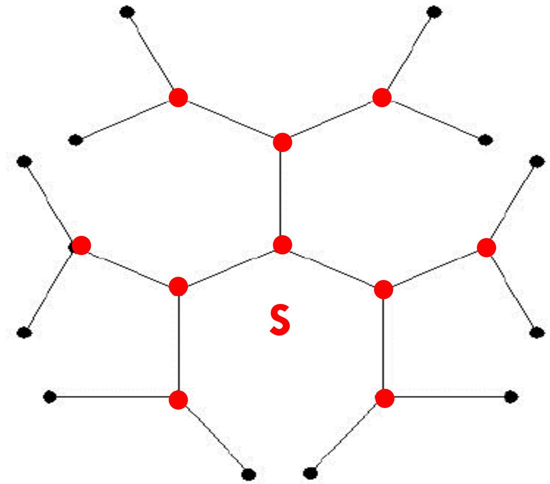
- **Consider infinite graph G**
  - (but each node has finite number of neighbors!)
- We say that a finite set ***S*** causes a cascade in G with threshold *q* if, when *S* adopts *A*, eventually every node adopts *A*
- Example: **Path**

If q<1/2 then cascade occurs



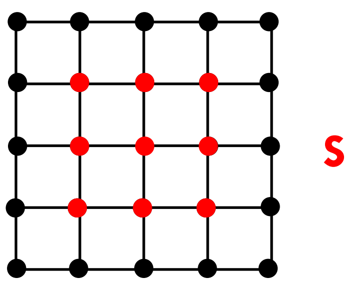
Infinite Graphs

- Infinite Tree:



If q<1/3 then cascade occurs

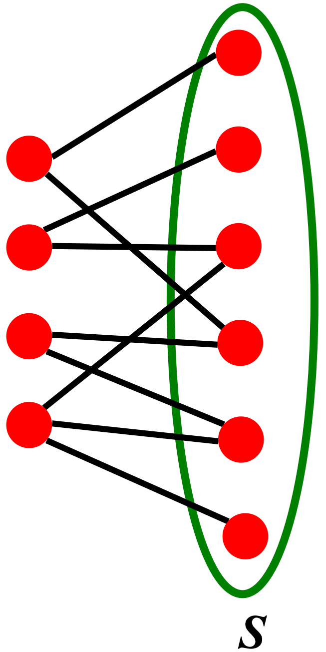
- Infinite Grid:



If q<1/4 then cascade occurs

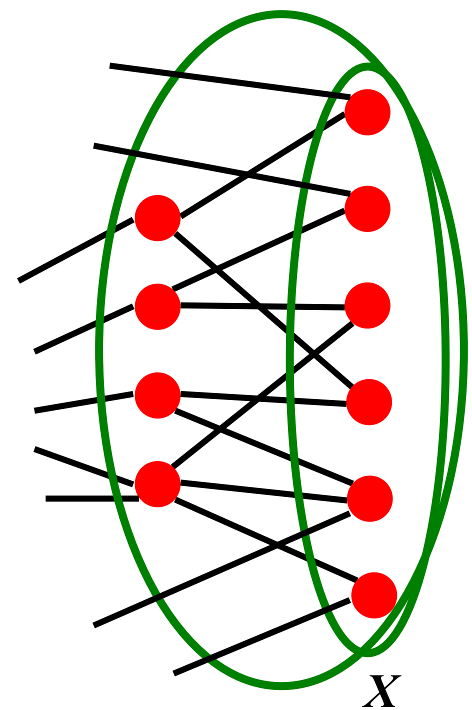
Cascade Capacity

- Def:
  - The **cascade capacity** of a graph G is the **largest q** for which some **finite set S** can cause a cascade
- Fact:
  - There is no G where cascade capacity **> ½**
- **Proof idea:**
  - Suppose such G exists: q>½, finite S causes cascade
  - **Show contradiction:** Argue that nodes stop switching after a finite # of steps



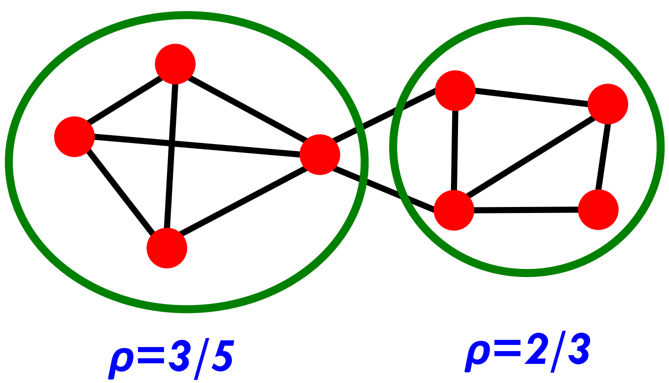
Cascade Capacity

- **Fact:** There is no G where cascade capacity > ½
- **Proof sketch:**
  - Suppose such G exists: q>½, finite S causes cascade
  - **Contradiction:** Switching stops after a finite # of steps
    - Define "potential energy"
    - Argue that it starts finite (non-negative) and strictly decreases at every step
  - "Energy": = $|d^{out}(X)|$
    - $|d^{out}(X)|$:= # of outgoing edges of active set X
    - The only nodes that switch have a strict majority of its neighbors in S
    - $|d^{out}(X)|$ strictly decreases
    - It can do so only a finite number of steps



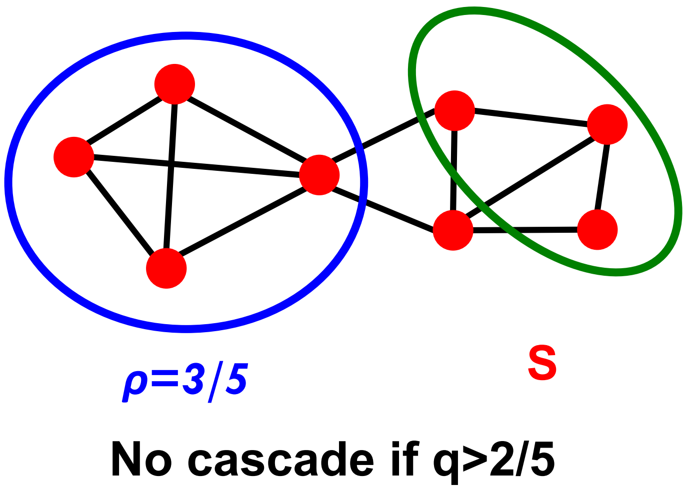
Stopping Cascades

- **What prevents cascades from spreading?**
- Def: **Cluster of density ρ** is a **set of nodes C** where each node in the set has at least ρ fraction of edges in C.



Stopping Cascades

- **Let S be an initial set of adopters of A**
- All nodes apply threshold q to decide whether to switch to A
- **Two facts:**
  - 1) If $G \setminus S$ contains a cluster of density $>(1\text{-}q)$ then S can not cause a cascade
  - 2) If S fails to create a cascade, then there is a cluster of density $>(1\text{-}q)$ in $G \setminus S$



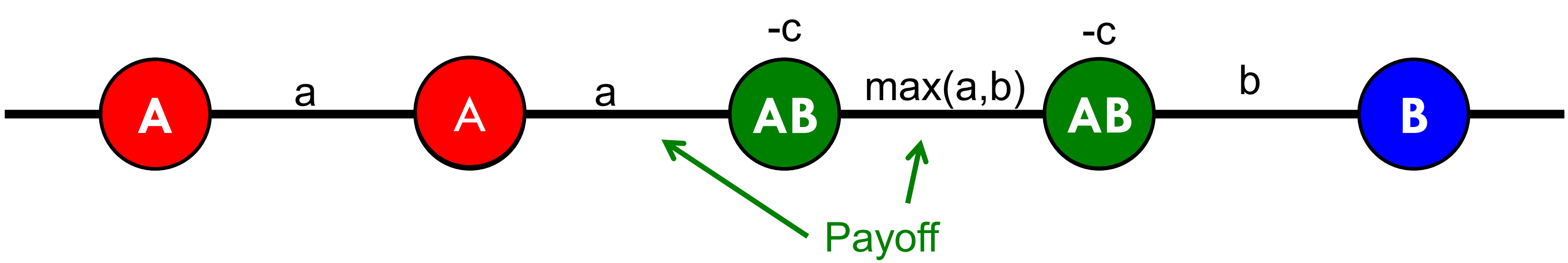
EXTENDING THE MODEL: ALLOW PEOPLE TO ADOPT A AND B

Cascades & Compatibility

- **So far:**
  - Behaviors *A* and *B* compete
  - Can only get utility from neighbors of same behavior: *A-A* get a, *B-B* get *b*, *A-B* get *0*
- **Let's add an extra strategy "*A-B*"**
  - *AB-A*: gets *a*
  - *AB-B*: gets *b*
  - *AB-AB*: gets max(*a*, *b*)
  - Also: Some cost c for the effort of maintaining both strategies (summed over all interactions)

Cascades & Compatibility: Model

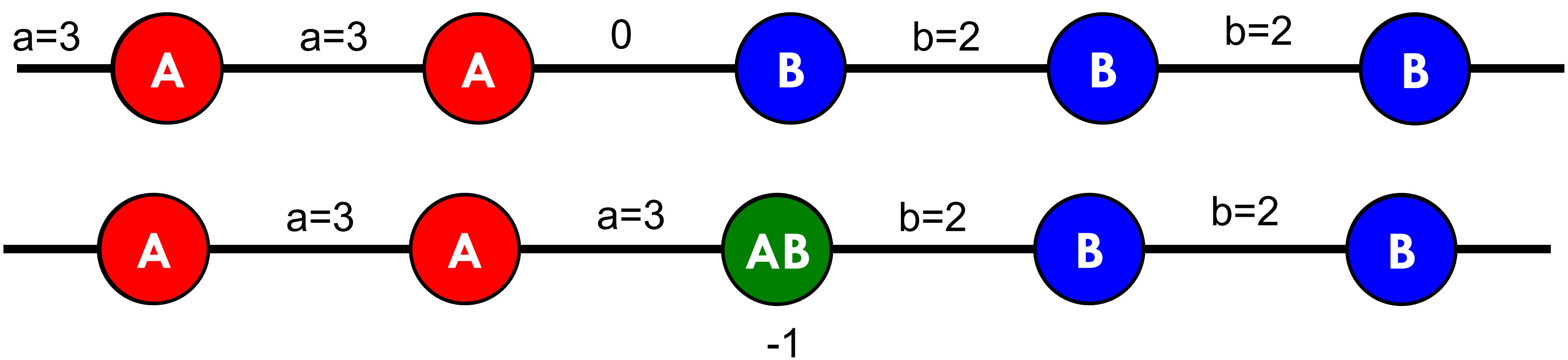
- Every node in an infinite network starts with *B*
- Then a finite set *S* initially adopts *A*
- Run the model for $t=1,2,3,\ldots$
  - Each node selects behavior that will optimize payoff (given what its neighbors did in at time $t-1$)



- **How will nodes switch from *B* to *A* or *AB*?**

Example: Path Graph

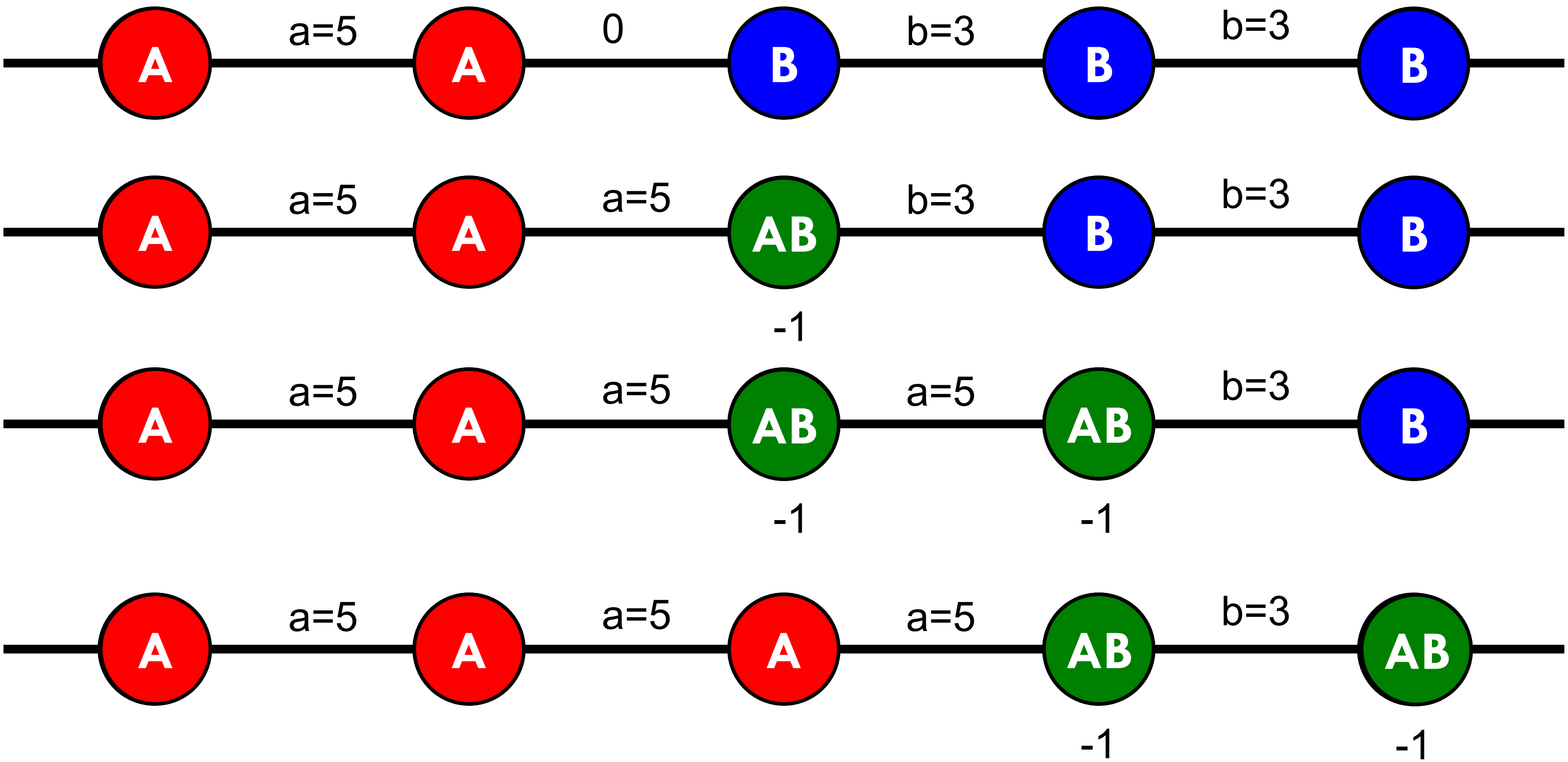
- **Path graph:** Start with all Bs, **a > b** (A is better)
- **One node switches to A – what happens?**
  - **With just A, B:** A spreads if $a > b$
  - **With A, B, AB:** **Does A spread?**
- **Assume a=3, b=2, c=1:**



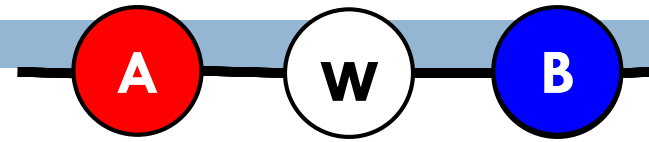
Cascade stops

Example

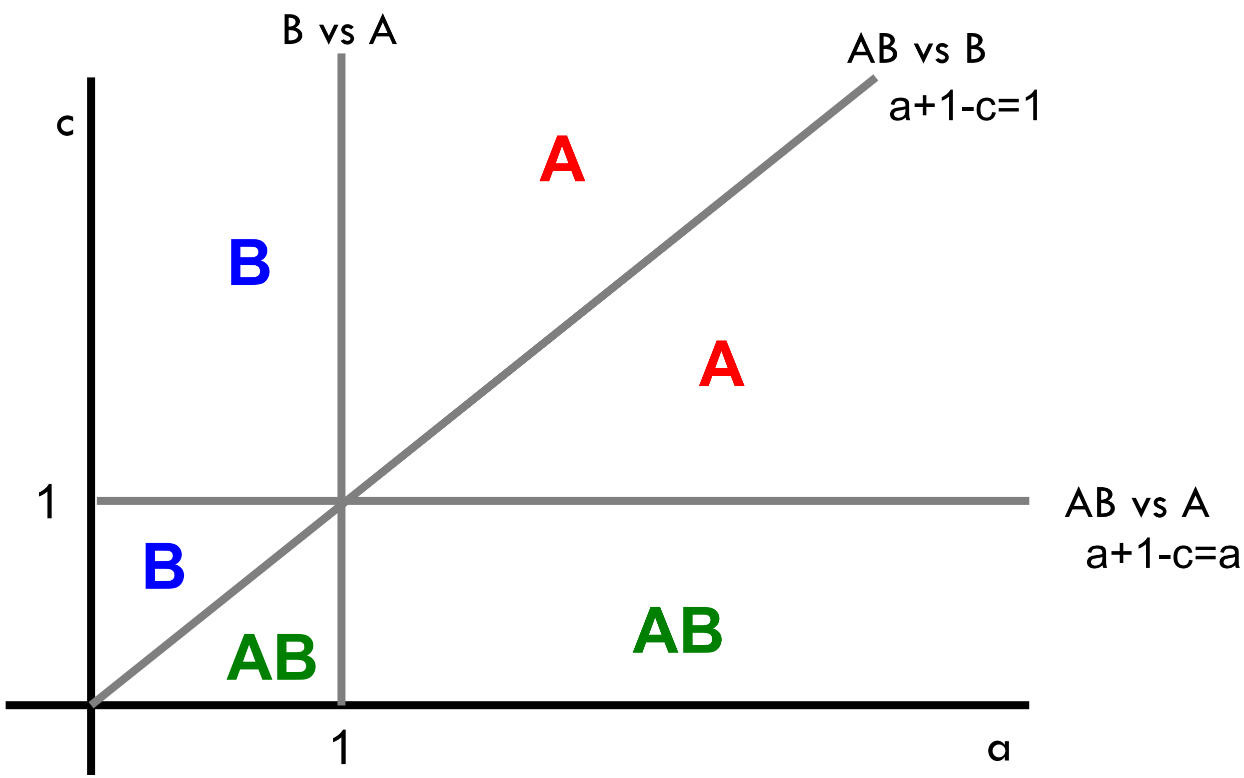
- Let a=5, b=3, c=1



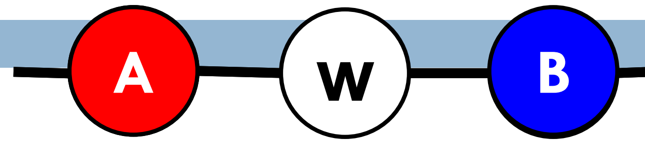
For what pairs (c,a) does A spread?



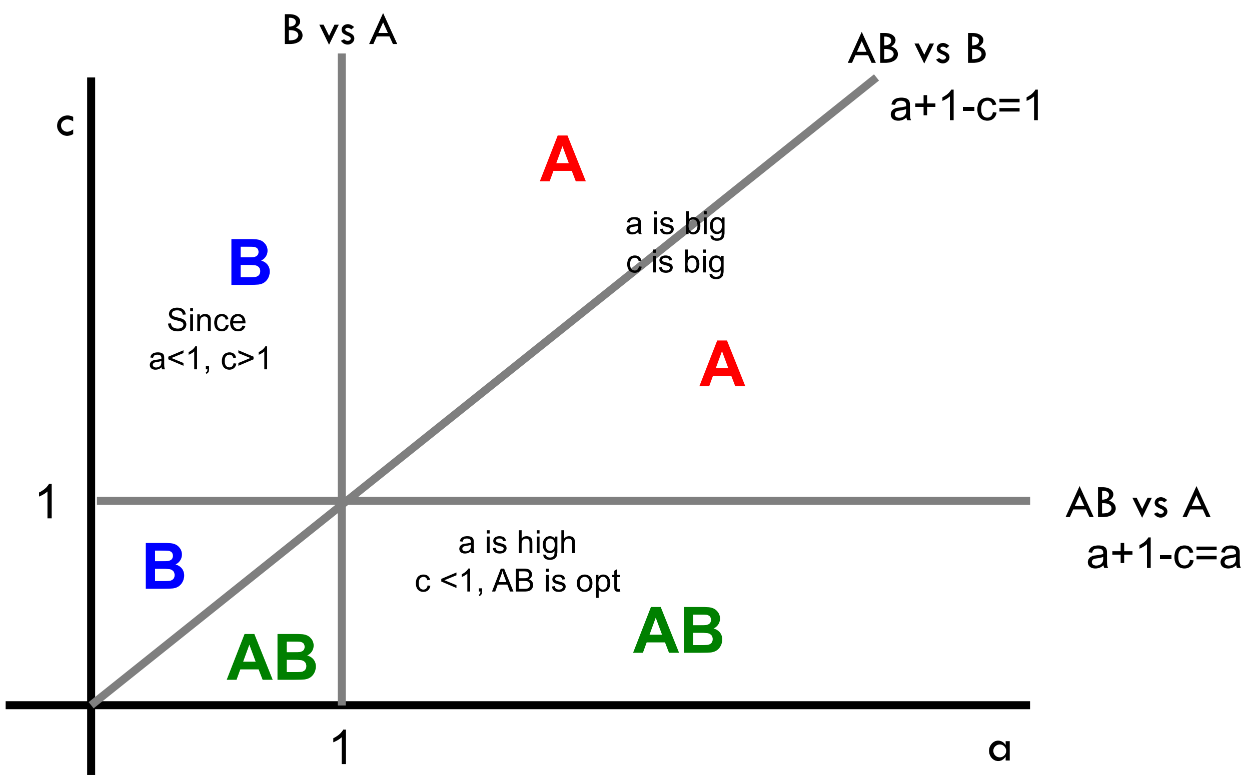
- **Infinite path, start with all Bs**
- **Payoffs for w:** A:a, B:1, AB:a+1-c
- What does node w in **A-w-B** do?



For what pairs (c,a) does A spread?

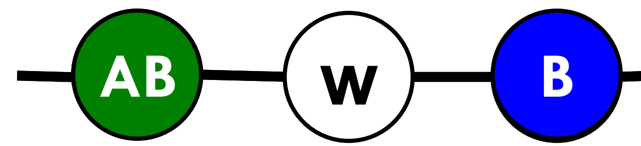


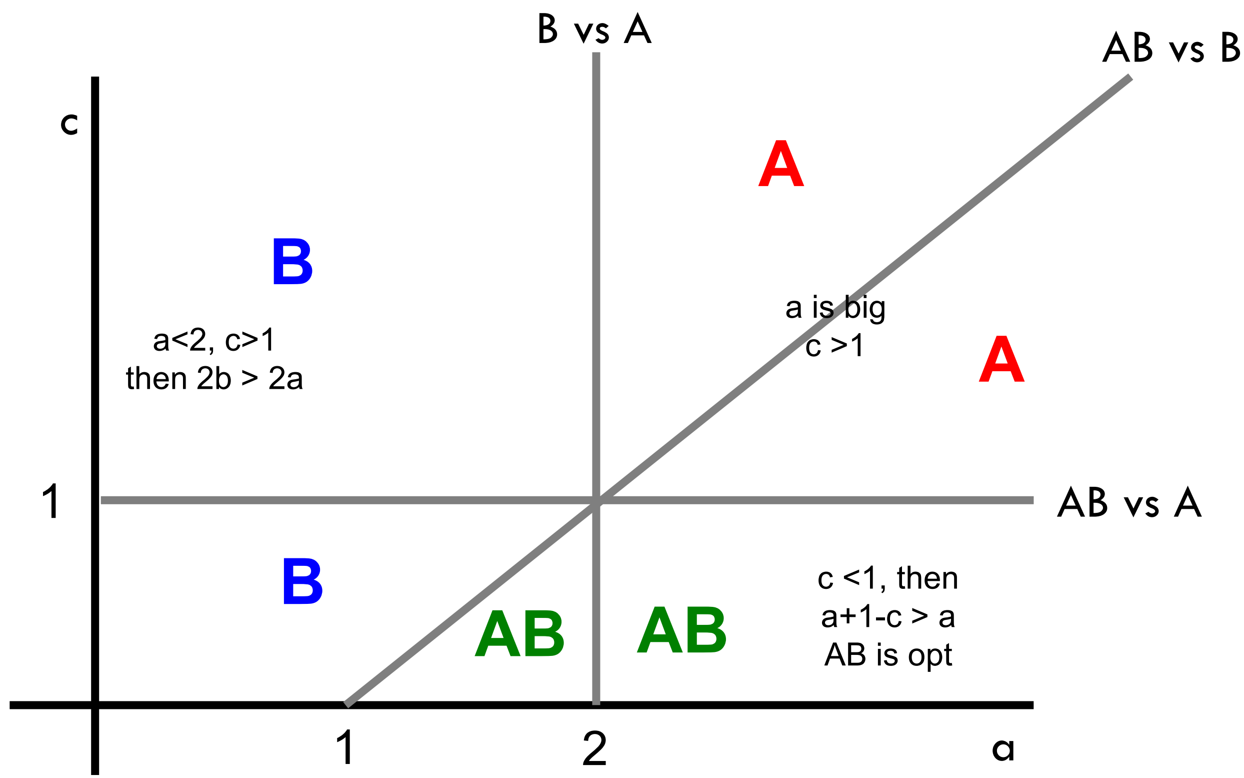
- **Infinite path, start with all Bs**
- **Payoffs for w:** A:a, B:1, AB:a+1-c
- What does node w in **A-w-B** do?



For what pairs (c,a) does A spread?

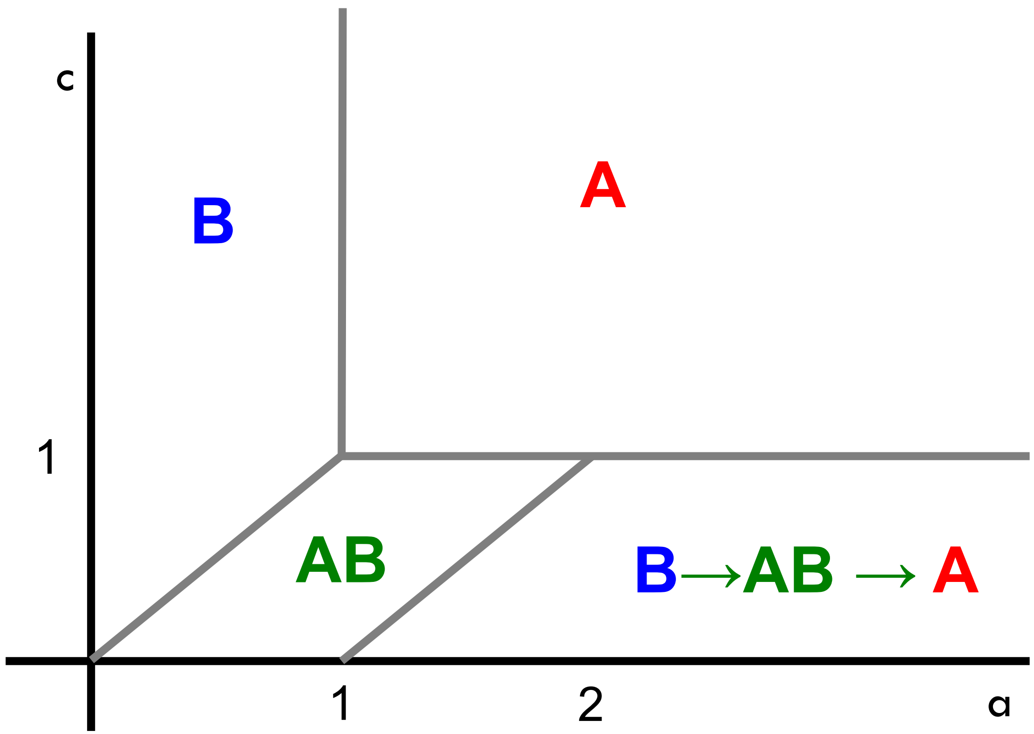
- **Same reward structure as before but now payoffs for *w* change:** A:a, B:1+1, AB:a+1-c
- **Notice: Now also AB spreads**
- What does node w in **AB-w-B** do?





For what pairs (c,a) does A spread?

- Joining the two pictures:



Lesson

- **You manufacture default B and new/better A comes along:**
  - **Infiltration:** If B is too compatible then people will take on both and then drop the worse one (B)
  - **Direct conquest:** If A makes itself not compatible – people on the border must choose. They pick the better one (A)
  - **Buffer zone:** If you choose an optimal level then you keep a static "buffer" between A and B