Simple Notes on COMP2711H

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1 Logic and Proof

1.1 Propositions

Definition

(1) Proposition

(2) Predicate

Operations on Propositions

(1) Negation

(2) Conjunction

(3) Disjunction

(4) Exclusive-or

(5) Conditional statement

Propositional Formulars

(1) Tautology

(2) Logically equivalence

(3) Counterposition

(4) Converse

(5) Common paris of logically equivalent propositional formulars: identity laws, domination laws, communicative laws, distributive laws, De Morgan's laws...

Comments

(1) You need to fully understand the truth table of $p \to q$.

(2) You need to be aware of the difference between "if" and "only if" as well as the diffrence between "necessary" and "sufficient".

(3) You need to be familiar with common pairs of propositional formulars that are logically equivalent.

1.2 Predicates and Quantifiers

(1) Universal quantifier

(2) Exsitential quantifier

(3) Domain of predicate formular

(4) Order of quantifier

(5) Negation of quantifier

(6) Equivalence of predicate formular

Comments

(1) You need to know when you can change the order of two quantifiers without chaning the meaning of predicate and when you can't.

(2) You need to be familiar with common pairs of equivalent predicate formulars

(3) You need to comprehend why $\forall$ is not distributive over $\vee$ and why $\exists$ is not distributive over $\wedge$.

1.3 Proof

Definition

(1) Proof

Inference Rules

(1) Modus Ponens: $p \wedge (p \rightarrow q) \rightarrow q$

(2) Hypothetical syllogism: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

(3) Addition: $p \rightarrow p \vee q$

(4) Resolution: $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

(5) Universal instantiation

(6) Universal generalization

Proof Techniques

(1) Direct proof

(2) Proof by contradiction

(3) Proof by case

Comments

(1) I highly recommend section 1.5 - 1.10 in the MIT lecture notes. Those sections provide standard templates for various kinds of proofs. Konwing those templates, you can organize your proof in a more logical way and can get across you ideas clearly.

2 Combinatorics

(1) The product rule

(2) The sum rule

(3) Tree diagram

(4) Pigoenhole principle

(5) k-permutation

(6) k-combination

(7) Binomial theorem

(8) Trinomial theorem

(9) Pascal identity and Pascal triangle

(10) Counting things in two ways

(11) Inclusion-exclusion principle

(12) Distribute distinguishable items into distingduishable boxes

3 Probability

3.1 Basics

(1) Sample space

(2) Event

Comments

(1) Set theory and discrete probability theory are closely related. Sometimes it helps to use set theory to interpret theorems in probability theory.

3.2 Probability Theory

(1) Probability distribution

(2) Conditional probability

(3) Independece of events

(4) Bernoulli trials and binomial distribution

(5) Random variables

(6) Bayes' theorem

Comments

(1) The effect of the condition in conditional expectation and conditional probability is the following

  (i) It confines the samle space Ω to the space where that condition holds.

  (ii) Then it scales up the probability of each outcome in the new sample space to make their sum equal to 1.

3.3 Random Variables and Expectation

(1) Expected Value

(2) Linearity of Expectation

(3) Geometric Distribution

(4) Independent Random Variables

(5) Variance

Comments

(1) Although simple, linearity of expectation can significantly facilitate the computation of expectation of a complex random variable.

(2) Geometric random variable is memoryless.

4 Number Theory

4.1 Modular Arithmetic

(1) Euclids Division Theorem

(2) Mod

(3) Congruent

(4) Addition and multiplication mod n operations

(5) Commutative, associative, distributive law

4.2 Greatest Common Divisor

(1) Euclids GCD Algorithm

(2) GCD, linear combination, and mulplicative inverse.

(3) Chinese reminder theorem

(4) System of Linear Congrunce

(5) Fermat's little theorem

(6) Euler's totient function