

2.1 Order the following functions by growth rate: N , \sqrt{N} , $N^{1.5}$, N^2 , $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, 2^N , $2^{N/2}$, 37 , $N^2 \log N$, N^3 . Indicate which functions grow at the same rate.

Ans $2/N$, 37 , \sqrt{N} , N , $N \log \log N$, $N \log N$, $N \log(N^2)$, $N \log^2 N$, $N^{1.5}$, N^2 , $N^2 \log N$, N^3 , $2^{N/2}$, 2^N .
 $N \log N$ and $N \log(N^2)$ grow at the same rate.

2.2 Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$. Which of the following are true?

- (a) $T_1(N) + T_2(N) = O(f(N))$
- (b) $T_1(N) - T_2(N) = o(f(N))$
- (c) $\frac{T_1(N)}{T_2(N)} = O(1)$
- (d) $T_1(N) = O(T_2(N))$

Ans (a) True.

(b) False. A counterexample is $T_1(N) = 2N$, $T_2(N) = N$, and $f(N) = N$.

(c) False. A counterexample is $T_1(N) = N^2$, $T_2(N) = N$, and $f(N) = N^2$.

(d) False. The same counterexample as in part (c) applies.

2.4 Prove that for any constant k , $\log^k N = o(N)$.

Ans Clearly, $\log^{k_1} N = o(\log^{k_2} N)$ if $k_1 < k_2$, so we need to worry only about positive integers. The claim is clearly true for $k = 0$ and $k = 1$. Suppose it is true for $k < i$. Then, by L'Hôpital's rule,

$$\lim_{N \rightarrow \infty} \frac{\log^i N}{N} = \lim_{N \rightarrow \infty} \frac{\log^{i-1} N}{N}$$

The second limit is zero by the inductive hypothesis, proving the claim.

2.10 Determine, for the typical algorithms that you use to perform calculations by hand, the running time to do the following:

- (a) Add two N -digit integers.
- (b) Multiply two N -digit integers.
- (c) Divide two N -digit integers.

Ans

- (a) $O(N)$
- (b) $O(N^2)$
- (c) The answer depends on how many digits past the decimal point are computed. Each digit costs $O(N)$.

3.37 Suppose that a singly linked list is implemented with both a header and a tail node. Describe constant-time algorithms to

- (a) Insert item x before position p (given by an iterator).
- (b) Remove the item stored at position p (given by an iterator).

Ans

- (a) Add a copy of the node in position p after position p ; then change the value stored in position p to x .
- (b) Set $p.data = p.next.data$ and set $p.next = p.next.next$. Note that the tail node guarantees that there is always a next node.