An Introduction to Probability

First Probability Ideas and First Steps in R

Martin Summer

10 January, 2025

First Probability Ideas and First Steps in R

Introduction

- Finance inherently involves **uncertainty** and **risk**, making probability essential.
- Finance focuses on allocating and pricing money over time:
  - Saving defers consumption for future purchases.
  - Borrowing enables investment today with future repayment.
- The future is uncertain and unpredictable, but **probability theory** provides tools to:
  - Quantify and analyze uncertainty.
  - Manage financial risks effectively.

Why Study Probability?

- Probability is fundamental to managing uncertainty in Finance.
- Originated as a mathematical theory in the **16th and 17th century**:
  - Influenced by gambling debates.
  - Scholars like **Cardano**, **Pascal**, **Fermat**, and **Bernoulli** contributed.
- Humans have long been aware of chance (e.g., gambling, goddess of chance), but only later sought to **measure uncertainty mathematically**.

First Steps in Probability

A Simple Starting Point

- We'll begin with a **classical probability model**: Tossing a coin.
- This example helps us:
  - Understand fundamental concepts in probability.
  - Explore basic R programming.

Tossing a Fair Coin: First Probability Ideas

- A **fair coin** has an equal chance of showing Heads or Tails.
- When tossed, the outcome is uncertain: Heads or Tails.
- Visualizing the process:

Figure 1: Figure 1: Tossing a coin

How Coin Tossing Matters for Finance

Simulating a coin toss introduces key concepts:

- Equal probabilities for outcomes.
- Link to Finance: Imagine a stock investment with a 50% chance of gain or loss.
- Builds intuition for probability and computational Finance.

The Role and Nature of Idealizations

- **Idealizations** are standard in probability and essential in Finance.
- Example: Stock price movement as a conceptual random experiment.
  - Price cannot fall below 0.
  - Hard to define the highest possible price.

Sample Space of Stock Prices

- Common assumption: Sample space is the entire interval of non-negative real numbers:
  - $S = [0, \infty)$.

Why Use Idealizations?

- Many models in Finance assume **arbitrarily high prices**:
  - Prices can rise without bound, but with **arbitrarily small probabilities** as they increase.
- Practical reasoning:
  - Imposing an upper bound x may seem reasonable, but it introduces awkward assumptions:
    - Why can't the price be just a cent higher than x?
  - Simplicity and convenience outweigh strict realism.

Key Takeaway

- Idealizations simplify models without significant practical harm, enabling tractable analysis in Finance.

Classical Probability: Measuring Uncertainty

- **Probability** quantifies how likely an event is to occur.
- Measurement idea:
  - Similar to measuring length: Choose a standard and count.
  - Measure probability by counting equally probable cases.

Formula for Classical Probability

$$P(A) = \frac{\text{Number of cases where } A \text{ occurs}}{\text{Total number of cases}}$$

- Example: Tossing a fair coin
  - Heads: $P(\text{Heads}) = \frac{1}{2}$ (1 favorable case out of 2 total cases).

Properties of Classical Probability

1. **Probability is never negative**: $P(A) \geq 0$
2. **If an event A occurs in all cases**: $P(A) = 1$
3. **For mutually exclusive events A, B**:
   $P(A \text{ or } B) = P(A) + P(B)$

Example for Rule 3

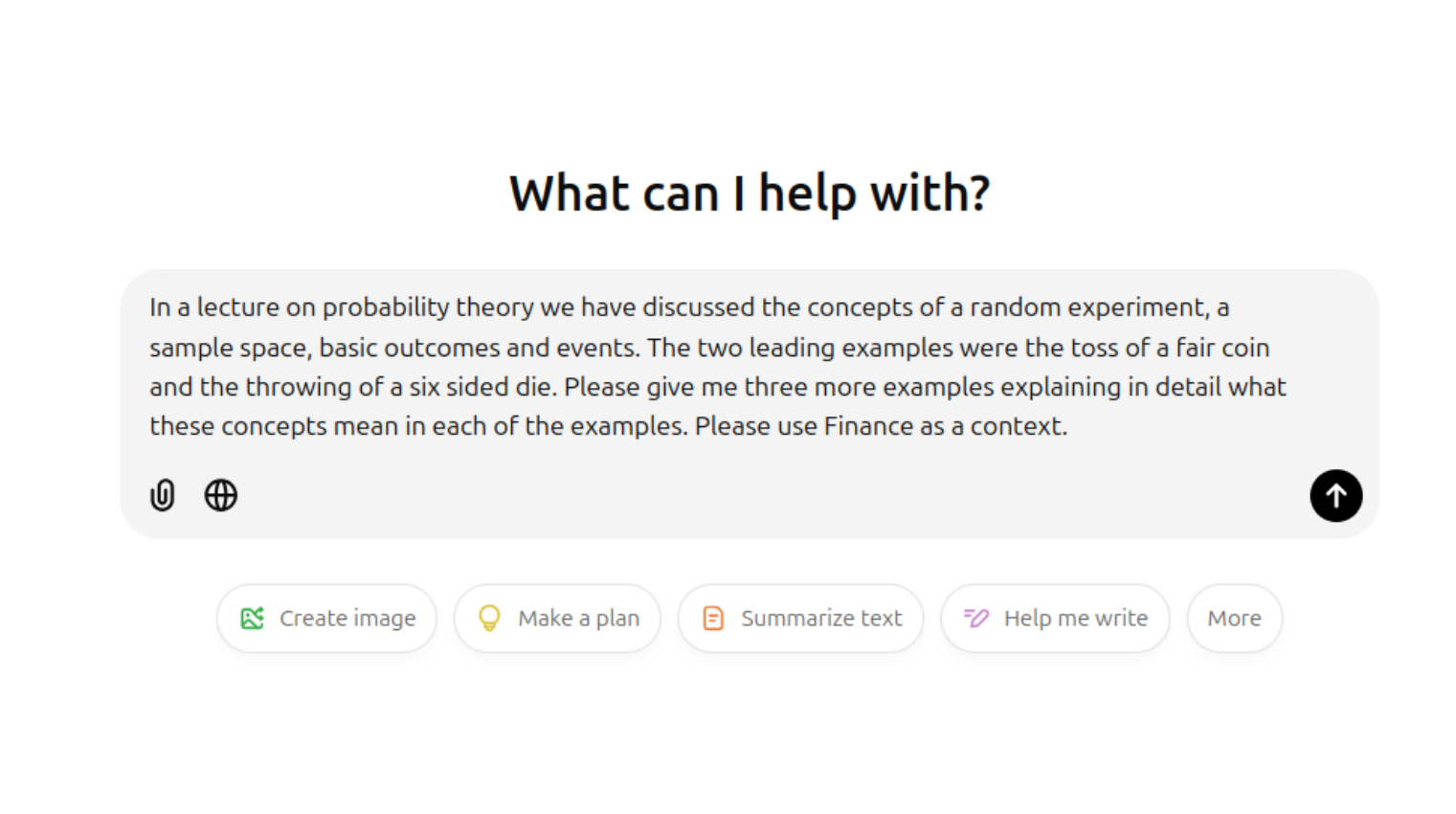
- Drawing a King (A) or a Queen (B) from a deck of 52 cards:
  - $P(A) = \frac{4}{52}$, $P(B) = \frac{4}{52}$.
  - Mutually exclusive events: A and B cannot occur together.
  - $P(A \text{ or } B) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$.

Complementary Events

- Probability of an event **not** occurring: $P(\text{not } A) = 1 - P(A)$

Leveraging ChatGPT for Learning

- Enhance understanding with **large language models (LLMs)** like ChatGPT.
- Example prompt for ChatGPT:
  - Explore random experiments, sample spaces, basic outcomes, and events in Finance.



Transition to R: Making It Tangible

- To apply these ideas practically, we turn to R.
- Coin tossing example revisited:
  - Learn basic R concepts.
  - Simulate probability experiments programmatically.

Tossing a Coin on the Computer: First Steps in R

The R User Interface

- RStudio allows us to interact with the computer and run R commands.
- Launch RStudio to see a screen similar to this:

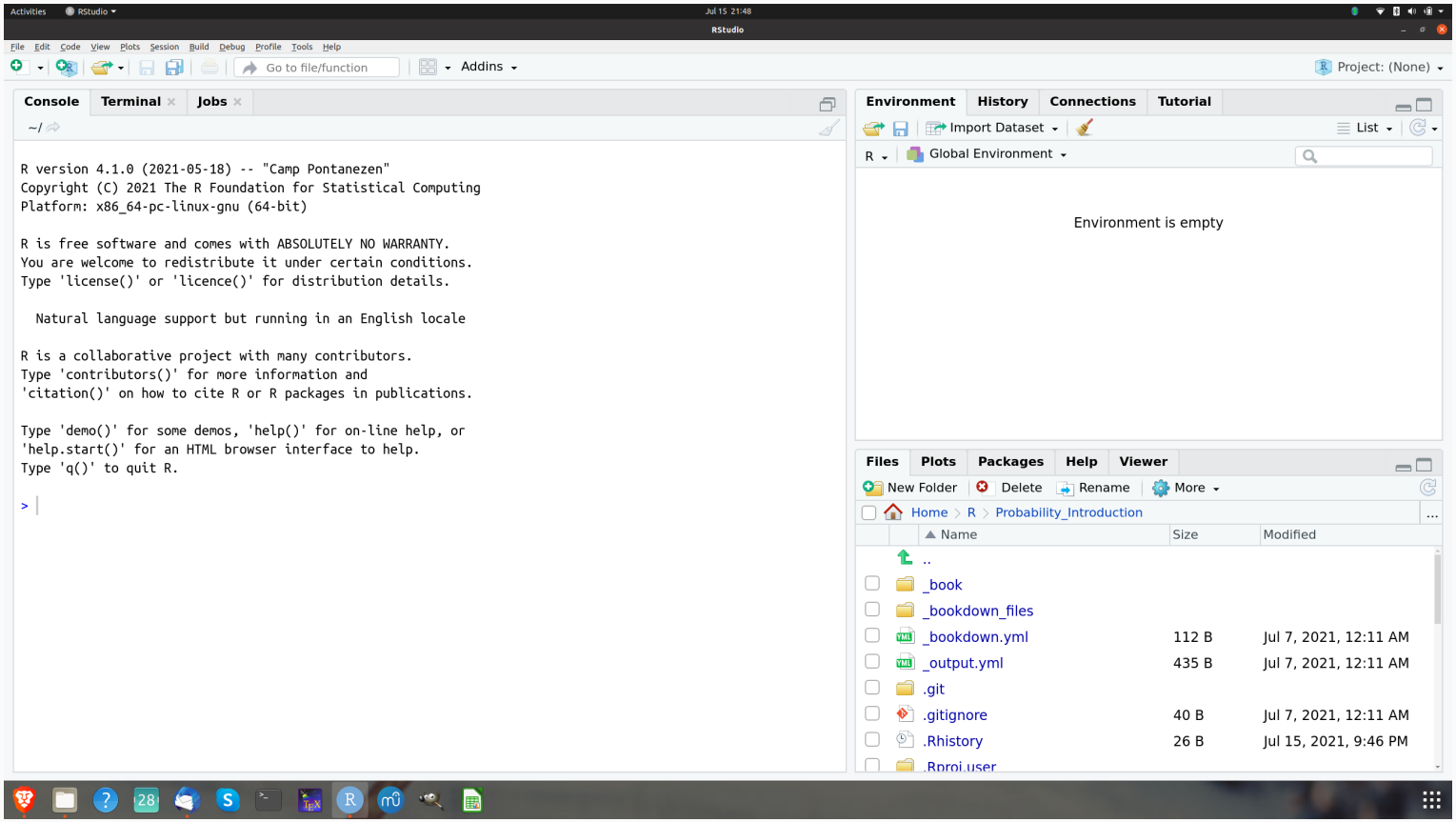


Figure 3: The RStudio startup screen