

# **An Introduction to Probability**

With Applications to Computational Finance using R

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## **The Motto of this Course**

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# The Motto of this Course

*“What I hear, I forget; What I see, I remember; What I do,  
I understand”*

*Confucius, 551-479 BC*

**What is this lecture about?**

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- These lecture notes support the course **An Introduction to Probability - with Applications to Computational Finance using R**.
- The course introduces essential probability concepts that every finance practitioner must understand using R within the context of computational finance.
- **Why Probability?**
  - Because Finance is about making decisions under uncertainty.
  - Probability is the most powerful conceptual tool we have at hand to think about uncertainty and decision making under uncertainty in a systematic and rational way.
- **Why R ?**
  - Because it is a language which is well adapted to probability and hands on coding of applied probability problems.
  - The computer can make abstract problems tangible.

# Course Objectives

- **Understand the fundamentals of probability:** concepts, rules, and theorems.
- **Learn R programming for probability simulations:**
  - Generating random variables.
  - Visualizing probabilities.
  - Solving real-world problems computationally.
- **Apply probability concepts to finance:** Use cases in risk management, pricing, and investment decisions.

# Building Abstract Concepts

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# Conceptual Experiments

- How can we build abstract concepts like **probability**, **random phenomena**, and **chance** by our own hands?
- Probability gains practical value through real or conceptual experiments, such as:
  - Future changes in a **stock price index**.
  - The future value of a **portfolio of securities**.
  - The chance that a **creditor cannot repay a loan**.



# Simulations with Computers

- Computers allow us to **simulate** a wide variety of random phenomena:
  - Models of random **fluctuations in asset prices**.
  - Models of **financial risks**.
  - Exploring **future scenarios** through simulation.
- The arrival of computers has been a **revolution** in the mathematics of probability:
  - Enables both theoretical reflections and practical simulations.
  - Abstract concepts can be built **hands-on** using computational tools.

## **R Example for Simulation**

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## Simulating a Stock Price in R

```
# Simulate daily returns for a stock

# Ensure reproducibility
set.seed(123)

# Number of days
n <- 100

# Normal distribution
daily_returns <- rnorm(n, mean = 0.001, sd = 0.02)

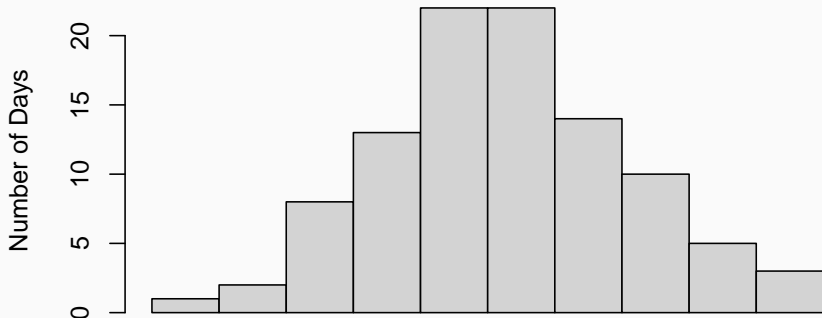
# Compute cumulative returns, starting price = 100
price <- cumprod(1 + daily_returns) * 100
price[1:10]
```

```
[1] 98.97905 98.62237 101.79547 102.04081 102.40670 106
[8] 104.50237 103.17132 102.35490
```

## Visualize the distribution of daily returns:

```
hist(daily_returns,  
     main = "Distribution of daily stock returns",  
     xlab = "Returns",  
     ylab = "Number of Days")
```

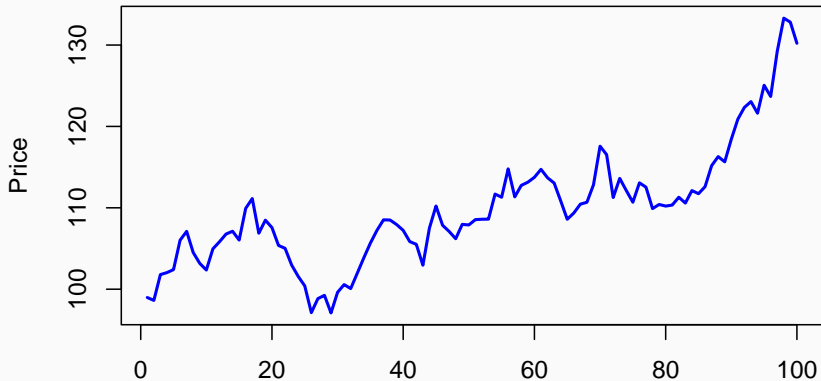
**Distribution of daily stock returns**



## Visualize the Price Dynamics

```
# Visualize the stock price
plot(price, type = "l", col = "blue", lwd = 2,
      main = "Simulated Stock Price",
      xlab = "Days", ylab = "Price")
```

**Simulated Stock Price**



## Insights from the Example

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# Key Takeaways

- **Highlights:**
  - How randomness influences stock price movements.
  - The usefulness of R for simulations and visualization.
- **Critical Reflection:**
  - The real world is **different** from theoretical and simulated worlds.
  - The real world is **richer and more complex** than both theory and simulation.
  - Features in the real world may be **absent** in theory or simulations but are crucial for explanations.

# The Interdependence of Theory, Simulation, and Reality

- Theoretical concepts, simulations, and applications are **interdependent**.
- Developing a sense of their differences and connections is a core goal of this course.



# **Building Virtual Objects and Simulations in R**

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# The Power of Simulations

- Simulations allow us to:
  - **Construct and manipulate** virtual objects.
  - Explore and analyze random phenomena.
  - Understand complex systems through hands-on experimentation.

# Why R and RStudio?

- **R Programming Language:**

- Well-suited for probability and hands-on coding of applied problems.
- Widely used in data analysis, statistics, and data science.

- **RStudio IDE:**

- Provides an integrated environment for coding, visualization, and simulation.
- A popular tool in both industry and academia.

# Leveraging AI Tools in Learning

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# The Role of Large Language Models (LLMs)

- Since this course was first taught in 2021, **AI tools** such as large language models (LLMs) have emerged.
- Examples include **ChatGPT**, **Claude**, and **Gemini**.
- These tools create new opportunities for learning and reinforcing concepts:
  - Generate examples.
  - Explain difficult topics.
  - Debug R code.
  - Translate code into familiar languages.

# Integrating AI into This Course

- Throughout the course, I will show you how to:
  - Use ChatGPT to **enhance your learning experience**.
  - Leverage AI for **interactive and engaging studies**. # Using ChatGPT for Learning

# Benefits of ChatGPT in This Course

- ChatGPT offers capabilities to support learning, such as:
  - Clarifying concepts.
  - Generating examples.
  - Debugging R code.
  - Practicing exercises.
  - Simulating discussions.
  - Learning best practices in R programming.
- **Example Use Cases:**
  - **Clarify Concepts:**
    - > “Explain the concept of a probability distribution with an example.”
  - **Generate Examples:**
    - > “Can you give me an example of a random variable and how it applies in finance?”

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# Practical Examples of ChatGPT Use

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## More Use Cases

- **Debug R Code:**

Paste your code and ask for help: > “Why does this R code not run, and how can I fix it?”

- **Practice Exercises:**

Generate exercises to reinforce learning: > “Create three exercises to practice calculating probabilities for dice rolls.”

- **Simulate Discussions:**

Test your understanding with interactive discussions: > “I think the variance of a constant is zero. Am I correct? Explain why or why not.”

- **Learn R Best Practices:**

Ask for coding tips: > “What are the best practices for writing clean and efficient R code?”

## Important Considerations

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# Verifying AI Outputs

- While ChatGPT is powerful, always **verify outputs**, especially for:
  - Complex calculations.
  - Detailed explanations.
  - Programming suggestions.
- Use additional tools for cross-checking:
  - **Wolfram|Alpha:**  
A computational engine available for free:  
<https://www.wolframalpha.com/>

# Downloading and Installing R

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# Steps to Get R

- R is an open-source project maintained by an international team of developers.
- The software is available at:
  - Comprehensive R Archive Network (CRAN)

## Download Instructions

1. Visit the “Download and install R” section at CRAN.
2. Select the link corresponding to your operating system (Windows, Mac, or Linux).
3. Choose **precompiled binaries** for an easier installation.
  - Optionally, build R from source if you have the tools and expertise.
4. Install either the 32-bit or 64-bit version:
  - **64-bit versions** handle larger files and datasets more efficiently.

# Downloading and Installing RStudio

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# Why Use RStudio?

- **RStudio**: A user-friendly application that simplifies R coding.
- Provides a consistent interface across all operating systems.

## Steps to Install RStudio

1. Visit: <https://www.rstudio.com/products/rstudio/>
2. Select **RStudio Desktop** and follow the download instructions.
  - RStudio Desktop is free to use.
3. Ensure you have a version of R installed before using RStudio.

- If R and RStudio are successfully installed, you're ready to begin.
- Throughout this course:
  - I will demonstrate code using **RStudio**.
  - Other options (e.g., Jupyter Notebooks) are available for advanced setups.

# Using R with Jupyter Notebooks

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- R can also be used in **Jupyter Notebooks**.
- For those interested:
  - Ask ChatGPT for step-by-step instructions: > “Please give me a step-by-step instruction on how to set up R in Jupyter Notebooks.”
- Note: This setup will not be covered in this course, but you’re welcome to explore it further.

## Signing Up for ChatGPT

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## Steps to Create a Free Account

1. Visit: <https://chat.openai.com>
2. Click “Create a free account.”
  - Sign up using an email address, Google, or Microsoft account.
3. Verify your email and complete the registration process.
4. Log in to start using ChatGPT for learning and exploration.

## Free vs. Paid Versions

- The **free version** (GPT-3.5) is sufficient for most learning needs.
- Paid versions (e.g., GPT-4.0) offer enhanced capabilities but require a subscription.
- Explore the free version to understand its potential.

# Prerequisites

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# An Intuitive Introduction

- This course is an **elementary introduction** to probability and R.
- **No prior knowledge** of probability or R is required.
- You can rely on these **lecture notes** and slides without needing additional textbooks.

## Designed for All Levels

- For newcomers:
  - Learn essential concepts from scratch.
  - Develop curiosity and excitement about the field.
- For experienced learners:
  - Gain new perspectives and deepen understanding of familiar concepts.



# Course Structure

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# Parallel Development of Concepts

- Probability and R concepts are developed **together**.
- Build knowledge by **constructing and experimenting** with concepts on the computer.
- Reinforces:
  - Probability understanding.
  - R programming skills.

# Main Chapters and Projects

- The course is divided into **five chapters**:
  - Each corresponds to one double lecture.
- After each lecture:
  - Work on a **project** to practice and deepen understanding.
  - Projects include discussions and worked solutions.

## Lecture Highlights

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# Lecture 1: Foundations

- Explore the historical context of probability.
- Learn basic concepts using a **coin-toss game**:
  - Application: Modelling security prices.
- Address a real-world problem:
  - Understand coincidences and cryptographic safety (e.g., hash functions in Bitcoin).
- **Project 1**: Design secure transaction identifiers for a digital payment system.



## Lecture 2: Frequencies and Data Manipulation

- Discuss probability and frequency relationships.
- Explore **Benford's Law** for detecting anomalies in data.
- Enhance R skills with data manipulation and structures.
- **Project 2:** Analyze and manipulate data for probabilistic insights.

## Lecture Highlights (Continued)

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## Lecture 3: Conditional Probability

- Understand **dependence** and its role in finance.
- Learn probability updates with new data.
- Applications:
  - Risk management.
  - Investment decisions.
- **Project 3:** Apply conditional probability to real-world financial scenarios.

## Lecture 4: Random Variables and Asset Dynamics

- Explore:
  - Expected value.
  - Variance.
  - Covariance and correlation.
- Understand **binomial lattice models** for asset price dynamics.
- Develop R skills for programming control structures.
- **Project 4:** Simulate asset dynamics using probabilistic models.

## Lecture 5: Continuous Random Variables

- Study the **normal distribution**:
  - Its power and limitations in finance.
- Optimize R code for performance and efficiency.
- **Project 5**: Analyze continuous random phenomena and risks in finance.

# Acknowledgements

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# Special Thanks

- **Branko Urošević:**
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- **Past Students:**
  - For their feedback and enthusiasm, which improved these notes.





## References

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# Key Resources

## 1. Probability:

- William Feller: *An Introduction to Probability Theory and Its Applications*.
- Karl Schmedder: *An Intuitive Introduction to Probability* (Coursera).

## 2. R Programming:

- Garrett Grolemund: *Hands-On Programming with R*.

## 3. Finance:

- David Luenberger: *Investment Science*.

## 4. History and Philosophy:

- Persi Diaconis and Brian Skyrms: *10 Great Ideas About Chance*.
- These notes combine well-known ideas with new approaches to teaching probability and R.