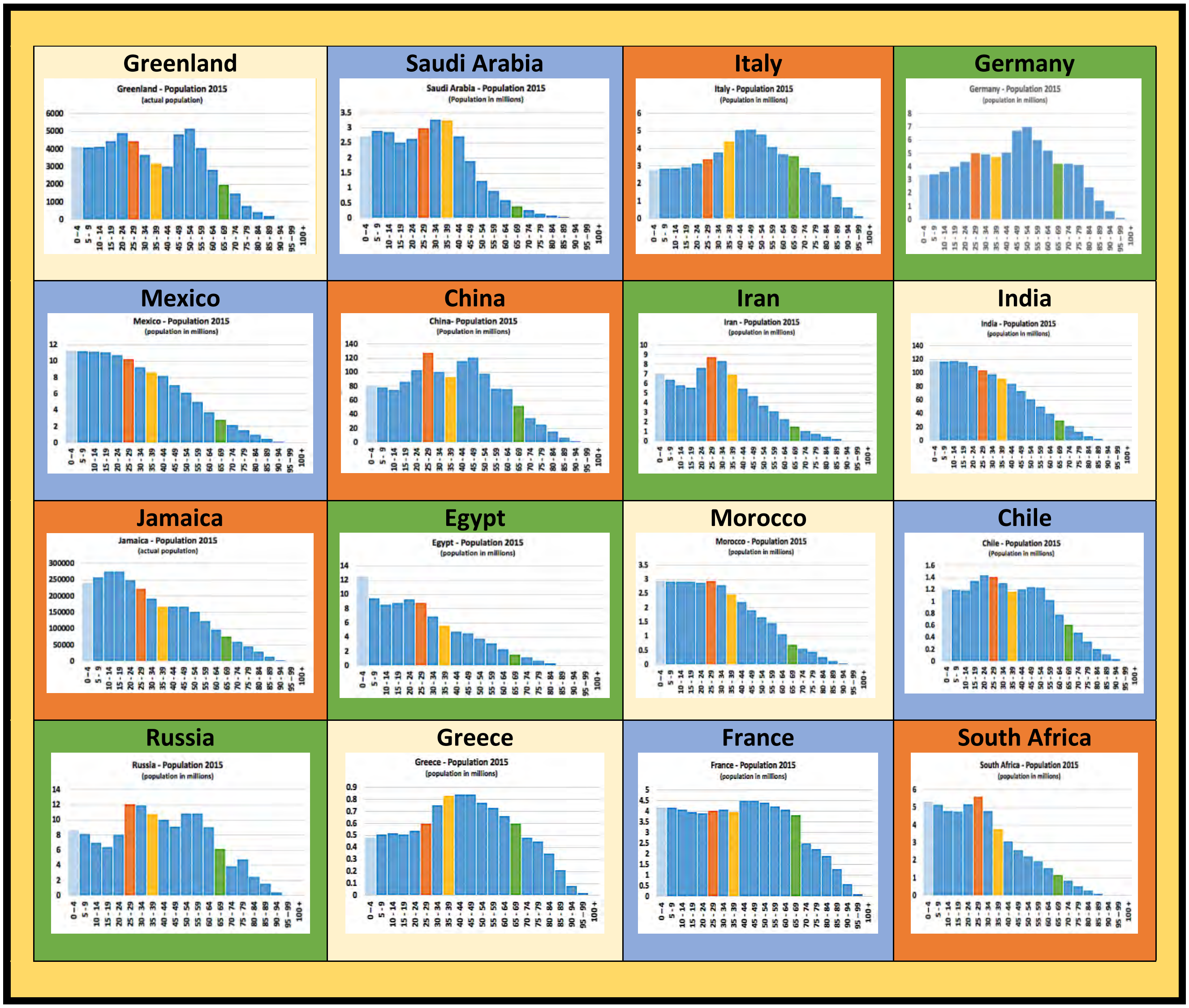
People Count!

(and their data stories)

Henry Kranendonk



Henry's Quilt of Countries

Data Stories

Modeling with Data

Teaching Notes

People Count!
and the stories of ...

Kristin
Generation X

Abbey
The Millennial

Adeline
Generation ?

* Henry's Quilt

Parents
Baby Boomers

And generations yet to be named

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The population data used in this module were extracted from the **International Data Base** (**IDB**) website https://www.census.gov/programs-surveys/international-programs/about/idb.html. This website is housed and maintained by the United States Census Bureau. Data used in this module were obtained from the **IDB** in 2018 - 2020. Population estimates used in the module may be different than what is posted on the website due to periodic updates and revisions. The author is indebted to the United States Census Bureau for providing this valuable data and allowing it to be used in this module.

Acknowledgements

Let me start by thanking my former students at Rufus King High School in Milwaukee, Wisconsin who piloted the first several lessons of this module. Students in my International Baccalaureate (IB) Math Studies courses as well as students in my computer science courses were involved in working through problems in the early stages of developing several lessons.

I thank Gail Burrill for introducing me to population pyramid graphs and for her encouragement and support through the years. I thank David Ebert, a friend and fellow mathematics teacher, for his feedback and writing the **Foreword**. I also thank the early reviewers, namely Donald Cramer, Kevin McLeod, Mary Mooney, Michael Steele, Melissa Manley, and Steven Gorski. I especially thank Johnny Lott for reviewing the first draft and encouraging me to continue writing this module at a time I was not sure if it would have any value to students and teachers.

I thank my family for putting up with me while trying to weave this material together. In particular, I thank my wife Jean Kranendonk. I also thank my daughters Laura DeCicco, Kristin Moala, and Abbey Kranendonk, and my grandchildren Adeline, Paul, and Dominic DeCicco for allowing me to use their names as part of the data stories.

I would especially like to thank my daughter Kristin Moala for creating the illustrations that are included in several of the lessons and the data stories that include her name, and to my son-in-law Peter Moala for editing an early edition.

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Finally, allow me to dedicate this module to the memory of my infant son, Jeffrey Scott Kranendonk. “There was too little time.”

Thanks again to everyone involved in this project.
Henry Kranendonk
2020

Foreword

I first met Henry Kranendonk at a workshop nearly twenty years ago. I was a young early career teacher, and Henry was a veteran mathematics teacher. That workshop was my first introduction to the population pyramid graphs that are featured in ***People Count***. I was, and continue to be, astounded by the beautiful simplicity of the graphs, and how such a simple graph can tell such a complex story. Henry introduced me to a new way of thinking about the visualization and analysis of data.

The National Council of Teachers of Mathematics' book, ***Catalyzing Change in High School Mathematics***, states that high school mathematics empowers students to expand professional opportunity, understand and critique the world, and experience wonder, joy, and beauty. Most high school mathematics and statistics textbooks and curricula do a good job expanding our students' professional opportunities, but fall short in bringing our students to understand and critique the world. Very few materials lead our students to experience wonder, joy, and beauty. ***People Count*** fills that void by guiding our students to an understanding of world population issues through the use of beautiful population pyramid graphs.

Too often, unfortunately, what is presented as "mathematical modeling" in many textbooks and curricula is either low on actual mathematics, or low on real-world connections. ***People Count*** is a great example of real, important data analysis that involves a high amount of mathematical rigor. Throughout the book, the knowledge and skills required by students builds on their experience in each successive unit, and each unit is another piece in the story of real people identified through their stories. The narratives present the mathematics as a student-centered story to be explored rather than a teacher-centered set of problems to be completed.

The book you have in front of you is the culmination of a career dedicated to sharing the simple complexity of population pyramid graphs with hundreds of students and teachers. It is a valuable addition to the field, and offers a compelling example of the investigative process of statistical problem solving called for in the American Statistical Association's ***Guidelines for Assessment and Instruction in Statistics Education*** (2007) or **GAISE**. Thank you, Henry, for bringing this material to hundreds of your students; and thank you, through this book, for sharing this material with thousands of students across the world.

David Ebert
High School Mathematics Teacher
Member, NCTM Board of Directors, 2016-2020

People Count!
(and their data stories)

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Teacher Edition 1

Overview of Module

People Count! (and their data stories) is a collection of 16 lessons for high school or college students, along with data stories and a quilt of countries. This module evolved from several projects originating in the author's computer science and International Baccalaureate (IB) Math Studies classes at Rufus King High School in Milwaukee, Wisconsin. Working with real data, even if the data is not initially of high interest to students, brings out a response that captures students' attention. The initial lessons were expanded based on students' input that ultimately resulted in this module.

People Count! is designed for students enrolled in most high school mathematics classes. Pre-requisite skills are essentially skills learned in a first year algebra course. This module also involves several Microsoft Excel files to complement several lessons and align with the modeling objectives described in this Overview. It is not necessary, however, for students to have access to the files to complete most of the lessons. (It might also be possible to use the files with other spreadsheet applications.) The lessons and files support the main objective of this module, namely modeling with data. Students develop, analyze, and redesign population projection models using past and present population totals by age groups to estimate future population estimates of various countries.

Included in this **Overview** is a general description of the lessons and ideas for using the lessons in a high school or first year mathematics/statistics course. In addition, an explanation of a **Modeling Continuum** is provided to guide the implementation of each lesson. References to the **Continuum** are also included in the **Teaching Notes** for each lesson. Connections to related state mathematics standards and the **Common Core State Standards for Mathematics** (**CCSSM**) are also identified in the section ***Supporting Standards for Mathematics***.

The data used throughout this module were obtained from the International Data Base (IDB) that is a resource of the United States Census Bureau's website (https://www.census.gov/programs-surveys/international-programs/about/idb.html). The incredible depth of the data available has several opportunities for additional projects. Efforts were made to use this data as intended, however, there were times that some liberties were taken. Mostly noticeably, the population totals for a given year are interpreted in this module as the count of people *at the beginning of the stated year*. This interpretation allows a more workable way for students to derive the year of birth of a selected person from a country's population. (The year of birth is important in the data stories highlighted in the lessons.) As reported by the International Data Base, the counts are actually mid-year counts or estimates. This discrepancy (and a few others that are pointed out when using the data) do not result in misleading results. The author and students remain indebted to the US Census Bureau for providing this data.

Ideas for Using this Module

Several ideas emerged as ways to use this module with students in high school mathematics courses.

(1) Stand-alone resource for a unit that involves the study of modeling.
Several courses in high school mathematics directly address modeling. This module provides an opportunity to study modeling within a two to three-week time period. At the time of the writing of this module, mathematics courses that stated modeling as an objective within each course include: Algebra, Algebra 2, Advanced Mathematics, Finite Mathematics, Precalculus, Statistics, and IB Math Studies, as well as several integrated mathematics series.

(2) Supplementary resource that is aligned with the goals of a mathematics course.
Lessons are aligned to several states' mathematics standards for high school students. Selected lessons from the module could be used to complement the textbooks used in these courses. Included in this **Overview** is a section entitled **Implementation Guidelines** that provides a description of each lesson to coordinate with the goals of a specific mathematics course.

(3) Development of student-centered projects.
A specific implementation of this module with students included the following: after a brief introduction of a selected lesson at the beginning of the week, students worked independently or in small groups to complete the lesson and to discuss their answers to the problems at the end of the week. The lessons served the purpose of developing student-centered projects. In addition, the **Teaching Notes** include possible extensions for several lessons that could also be completed as independent projects. Discussions were often led by students in groups as they highlighted the key problems using posters, PowerPoint, or other slide presentation software. See the **Implementation Guidelines** for a description of the lessons that would provide the basis for independent work. The number of lessons involved if implemented in this format will vary.

(4) Intergenerational resource for courses involving parents, guardians, and students.
High schools are frequently looking for ways to involve parents and guardians in the study of mathematics. Several schools have started monthly Parent Evenings in which a specific "mini-course" in mathematics is offered. A mini-course using selected lessons would be of interest to both parents and students. Lessons would be completed and discussed at the Parent Evenings mini-course. The **Implementation Guidelines** identify lessons that provide the best opportunity for an intergenerational study of mathematics.

(5) Summer-school offering
Summer or interim courses are often designed to provide additional opportunities for students to develop their understanding of mathematics. A four or five-week summer course using this material would provide an alternative curriculum for courses offered during the summer or during interim breaks.

(6) Development in a Computer Science course
Several of the lessons could be developed in a computer science course. One particular implementation of this module directed students to develop the recursive models as a programming assignment. The Microsoft Excel files were provided as examples of what the computer programs were expected to accomplish. Modifications of the lessons will be needed to complete this connection.

All of the above suggestions are based on the expectation that students are provided opportunities to complete the lessons in small groups Several lessons have components that involve some tedious calculations. Providing opportunities for students to participate in small groups makes these lessons more workable. Composition of groups should consider identifying students with a range of dispositions and abilities in mathematics.

Managing Real-World Mathematics Problems

The challenge with real-world mathematics problems is that they are about the real world. The range of topics that come under the umbrella of real-world problems is wide. Topics involving climate change, energy, animal proliferation or extinction, space travel are just a few topics that require mathematics to understand the problems connected to them. Sometimes these topics, however, result in passions that are not easily expressed in group discussions nor are they easy for you to referee. This challenge is also the case with the **People Count** stories. Immigration, death, birth rates and family planning are topics important in making sense of the mathematics introduced in the models, but these topics may also result in challenges for a meaningful discussion as students struggle to express their interpretation of the problems.

Teenagers are emerging adults. Sometime the lack of maturity is in full display. They do not always listen to each other, they do not always treat each other fairly or with sensitivity, and they do not always allow for other opinions or ideas to be part of their thinking. Then at other times (and often within a short period of time from their less than mature actions), they listen, they balance ideas in their thinking, and they work with each other and consider opposing ideas in forming their own ideas.

A premise in this module is that teenagers should be involved in real-world problems even if there are some possible challenges. Saying that, there is a need for you as their teacher to be prepared to filter the discussions and to weigh to what extent the discussions around immigration or family planning will be received by teenagers or their families that is productive. Consideration of other members of the class must be taken into account. For example, will a personal opinion on immigration from one student make another student uncomfortable or even scared?

This module has been designed to assist you regarding these discussions in at least two ways. First, if you think some of the questions might single out certain students in a way that is uncomfortable for you or the class, focus the discussion on the characters in the **Kristin's**

Stories that are used to introduce the issues mentioned. Rather than open up the discussion in a way that you are not sure where it is headed, focus on the characters they read about in the stories – for example, Raphine who immigrates to the United States and then emigrates, Adeline who is born during one of the stories, Kristin's mother and Hana's grandmother and mother who are senior citizens. Each character is represented as "where do they count" in the graphs and data sets, but they are also characters for discussing the issues of immigration, birth rates, and death. Feel free to use their names and circumstances to further discuss the issues that are asked in the problems.

A second way to allow students to connect with the material and to minimize the possible distraction from the objectives of the lessons is to suggest students write their own data stories previously described in this **Overview of Module**. This approach was used with students in several classes that piloted this module. Their personal stories provided an opportunity to personally connect with these sensitive topics in a positive and more private way.

There are a few problems that have an "*" identification. This is a gentle reminder that this problem might require a little more attention if used in any group discussion. Yet, the implications of these topics are critical for teenagers to address as they move closer into the decision-making age groups of this country. Part of learning to be a problem-solver is using mathematics to understand, and maybe solve, real-world problems.

The Modeling Continuum

High school and college teachers of mathematics are often asked, "What are the important areas of mathematics students should learn?" Generally, the discussion follows with an identification of specific skills or specific standards. This module addresses several of these skills and standards, but the primary answer to that question would be to describe a **process** of solving problems. This module is not solely about developing specific skills or standards; this module is about skills or standards working together to answer important questions. The process in which these components work together is defined as **modeling**.

In a broader sense, modeling is developed in this module in the same way a room in a house might be remodeled or upgraded (for example, a kitchen). The process starts by putting together an initial blueprint, or a sketch, or a "to do" list. Tools and materials are assembled and then used to complete the construction. This module highlights in the students' text a different example that has a similar process - making a quilt. The process starts by organizing tools and materials (needles, cloth, patterns). A sketch or pattern is often designed as a blueprint to complete the next steps. These initial tasks are then followed by assembling and building the quilt. Similar to remodeling a room, it is the final process of building a product that makes each an example of modeling. The tools in this module (i.e., proportions, ratios, median centers, mean centers, sequences) are used to build or rebuild population projection models. If a student does not have the right tools, or if a student does not know what tools are needed, then the process starts with figuring out the tools and the materials. As students grow more comfortable with understanding the tools, the more they are able to participate in one of the most important areas of mathematics - namely modeling.

The interplay between skills and a process, however, is complex. Sometimes it is difficult to see where learning a specific skill and developing a process are different (and maybe they are not different). This module provides a guide in sorting through students' work that involves the interplay between skills used in this module and the process to build a population projection model. This guide is called the ***Modeling Continuum***. It has four specific levels that are used to classify problems and analyze students' work. Although the first two levels are not in themselves modeling, they describe the necessary first steps. The last two levels put the skills together to solve special problems and provide evidence of modeling.

Why is important to have a working definition of modeling? Several formal definitions of modeling were considered for this module. In the end, it was difficult to find one that would address the specific process used to solve the problems in this module. Modeling is a major component of several past and current mathematics standards with varying definitions and examples of implementation. Modeling is a process that is difficult to define, although teachers generally know when students are demonstrating its role in learning mathematics. The **Modeling Continuum** designed in this module is intended to provide both teachers and students descriptions of a specific process needed to answer the specific questions in this module.

The **Modeling Continuum** has two basic questions that form umbrellas over the continuum levels. The first umbrella question is simply, "What is ….?" Questions that either directly use that lead or can be reworded to ask "what is" questions are Levels 1 and 2 of the continuum. What is the count of males in the United States in 2015? What is the percent of males in 2015? The first question simply requires students to find and state the information from a table or graph that is part of a lesson. Using a graph to answer that question may involve an approximation, but it is nonetheless directly obtained from a given data set. This process represents a Level 1 of the Continuum. The second question adds one more step, namely, students find the count of males from a table and then derive what percent this count represents of the total population. This question relies on a student's understanding of a proportion (a tool) and how to convert it to a percent. This summary represents an extension of the first level and is classified as a Level 2 of the Continuum.

In a similar way, the umbrella with the lead "What if…?" represents the classification of questions at Levels 3 or 4 of the continuum. What if there are 3 million more males within the next 5 yeas? What proportion of the resulting population are males? What if there are more people moving into our country than moving out? Will the population grow as a result of more people moving into the country? What if we calculate the population factors for Kenya in the same we calculated them for the United States? What do the population factors tell us about Kenya? Each "What if …?" question is followed by a question that requires an interpretation of the answer. Level 3 is a more general interpretation of these questions. It uses the tools of population factors (as defined in this module) based on proportions or percent. Level 4 requires a more robust remake of the projection model, asking students to recreate the conditions summarized in the model. Level 4, as expected, evolves as the lessons progress. The lessons lay the ground work for remodeling the projection models and applying the parameters that were involved in their initial design. The problems often rework the model for use with a country highlighted in the quilt (see cover) after completing the process for the United States. The following summary of the **Modeling Continuum** is used to guide your understanding of the process developed by students in this module to solve problems:

The Modeling Continuum

"What is …?"		"What if …?"	
Level 1	**Level 2**	**Level 3**	**Level 4**
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

The lessons summarized in the **Teaching Notes** include suggested levels of the **Modeling Continuum** for each of the problems. The suggestions are not to be interpreted as absolute – with minor tweaking, some of the problems might be classified at a higher level. With assistance provided by the instructor or other students, some problems might be classified at a lower level. The primary role that the Continuum plays is to provide opportunities ***for students*** to self-evaluate their work. It is suggested in several Units 1, 2, and 3 lessons to assign students to complete the **Exit Summary** (**Handout 13**) of their work. If this handout is used for a specific lesson, direct students to select one or two of the problems completed and to identify a description of one of the continuum levels from the **Exit Summary** or **Handout 13** for each problem that they think describes the way they solved the problem. Students proceed to write a brief summary of the process or method they used to solve each problem and why they selected the continuum level and description. An example of a completed **Exit Summary** from a student is provided below. Share this example with students before they attempt their first **Exit Summary**.

People Count! (and their data stories)

Exit Summary

Name: Stephanie
Lesson Number: 1

At your teacher's discretion, identify problems or questions in this lesson that you answered by using one or more of the levels of the Modeling Continuum. Within the column of the level or levels you identified, explain the steps you used to answer the questions or problems.

The Modeling Continuum

Level 1	Level 2	Level 3	Level 4
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

Level 1	Level 2	Level 3	Level 4
Problem: 3	**Problem:**	**Problem: 14**	**Problem:**
I used the population pyramid graph to answer this question. The longest bar on the male side of the graph was for the 20 – 24 years old males.		This problem was difficult. I knew all of the people 15 – 19 years old would be part of my answer. My question was how to include in my answer the 13 and 14 years old. What if 2/5 of the 10 – 14 years old age group are 13 and 14? So, I took 2/5 of the total count in that age group and added that value to the total of the 15 – 19 years old.	

Supporting Standards of Mathematics

Identifying important mathematics students need to learn is a major challenge for teachers of mathematics to sort out. Most states created standards for mathematics that carefully identify what expectations are important and at what grade levels they should be addressed for students in kindergarten through grade 8. Although standards differ from state to state, overall they address similar expectations and grade level expectations.

The **Common Core State Standards for Mathematics**, or **CCSSM**, is used in this **Overview of Module** to provide a general blueprint of the important mathematics for high school students. Most state standards provide a similar delineation of important mathematics for high school students. Also similar to the **CCSSM**, most state standards for high school students indicate what students should achieve upon the completion of high school and not grade level expectations.

The CCSSM standards supported by this module are outlined below by identifying the important mathematics addressed in each unit. Particularly note the connections in the table to the ***Standards for Mathematical Practice***. The strength of this module is the development of the lessons addressing the ***Standards for Mathematical Practice***.

	Mathematics Standards for High School	**Standards for Mathematical Practice (MP)**
Unit 1: **A Country's Shape**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	**MP-1: Make sense of problems and persevere in solving them.**

Unit 2: **Looking Back**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **Conceptual Category: Modeling**	**MP-1: Make sense of problems and persevere in solving them.** **MP-2: Reason abstractly and quantitatively.** **MP-3: Construct viable arguments and critique the reason of others.**
Unit 3: **Looking Forward**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 6a,6c):*** Represent data on two quantitative variables on a scatter plot and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. c. Fit a linear function for a scatter plot that suggests a linear association.	**MP-1: Make sense of problems and persevere in solving them.** **MP-3: Construct viable arguments and critique the reason of others.** **MP-4: Model with Mathematics** Students apply their new mathematical understanding to real-world problems. They also discover mathematics through experimentation and by examining patters in data from real-world contexts.

	Statistics and Probability ***Making Inferences and Justifying Conclusions (S-IC 2):*** Decide if a specific model is consistent with results from a given data-generating process. **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 1):*** Represent data with plots on the real number line (dot plots, histograms, and box plots). **Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 2):*** Use statistics to approximate the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **Conceptual Category: Modeling**	
Unit 4: **"What if …?"**	**Statistics and Probability** ***Interpreting Categorical and Quantitative Data (S-ID 6a):*** Fit a function to the data. **Statistics and Probability** ***Making Inferences and Justifying Conclusions (S-IC 2):*** Decide if a specific model is consistent with results from a given data-generating process. **Conceptual Category: Modeling**	**MP-1: Make sense of problems and persevere in solving them.** **MP-3: Construct viable arguments and critique the reason of others.** **MP-4: Model with Mathematics** Students apply their new mathematical understanding to real-world problems. They also discover mathematics through experimentation and by examining patters in data from real-world contexts.

Writing a Data Story

As your students begin reading **People Count**, they will be introduced to three people – Kristin from the United States, Raphine from Kenya, and Hana from Japan by their ***data stories***. (Interestingly, their names were on the list of the most popular names in their respective countries when they were born.) Each character's age (and the year of their birth) and her or his location on a population pyramid graph are shared in data stories students read in the lessons.

Data stories for this module are short, essentially factual summaries of a person based on age and his or her connection to a larger population. The characters in this module are reflections of people important in understanding a country – they are part of what makes the study of a country's population interesting, and also important.

If students have problems reading the narratives either because of a lack of interest or skills, please read the stories together as a class. They are short and focused on the study of a population. They are also intended to personalize the problems presented in the lesson. The problems are not just about the count of people (or the percent of people) older than 36 years old – they are problems about the count (or percent) older than Kristin and Raphine and Hana and why the percent of older people is important in Kristin's, or Raphine's, or Hana's life. It is also about why Kristin or Raphine or Hana is important in their respective countries. Let the data explain their placement in the population through the stories.

This module provides an opportunity for students to write their own data stories. Consider assisting students to write a story as they progress through the lessons. In particular, this module provides an opportunity for students to communicate with a person of a different generation who Is important in understanding their country.

Consider asking students in what age groups of a population pyramid graph their parents or guardians, or grandparents, or other relatives would be counted, and why a person in that age group is important to study? If students identify a person, work with them (possibly as a class project) in developing 5 to 10 questions to summarize in a short data story. Questions such as: What is your favorite movie? What is your favorite (or least favorite) subject in school? What were the main problems in the country when you were a teenager (assuming the person is older than a teenager)? How did you communicate (e.g., a phone, or a letter) with other people when you were a teenager? Were you an immigrant? Direct students to summarize a person's past by identifying a pyramid graph that summaries the year they were born using the International Data Base (IDB) site that is referenced in the module, along with researching the primary issues in the news when this person was born.

Consider assembling the data stories to share with the class, and also as part of a larger group setting (for example, a parent's night or a newsletter sent home with students). This might also be a project to conduct with another class that also looks at intergenerational topics. The **Teaching Notes** for several of the lessons include ideas that could be considered in a data story.

Implementation Guidelines

The following section summarizes the topics addressed in each unit. This section is followed by a general description of each lesson, a general description of how to implement each lesson, and an estimate of the class time to complete each lesson.

It is assumed that students will have access to a complete copy of each lesson as well as any handouts identified in the **Teaching Notes** and the student's lesson. Copies of the lessons could be printed or electronic. Handouts generally should be provided in print form. Handouts are completed as part of the problems in a lesson and then used in other lessons to interpret and expand students' understanding of the data.

Unit 1: A Country's Shape
This unit defines shape based on the population distribution as displayed by population pyramid graphs and histograms. Specific age groups are analyzed to highlight significant features of each country's population. Summaries include the age groups representing the greatest percent of the population, a ratio of old-to-young, and general descriptions based on the defined shape of a country. The implications of these summaries are also developed in the problems.
Points to consider: In order to develop an understanding of shape as defined in the module, completing Lessons 1 to 3, or Lessons 1 to 4 as a unit is recommended. Lesson 5 is optional, although follow-up work based on the data identified in Lesson 5 is referenced in later units. Consider implementing Lesson 1 for all students as a whole-group discussion, and then assigning Lessons 2 and 3 to small groups of students. If this suggestion is implemented, conduct a whole group discussion of students' work led by the Kenya groups and a whole group discussion led by the Japan groups. Consider directing each group to summarize their work by presenting to class the main results they learned about their country using a PowerPoint or posters. Also consider the optional **Case Study** projects. See Henry's Quilt, **Case Study Project: *Developing a Country Poster,*** after completing Lessons 1 to 3. See Henry's Quilt, **Case Study Project: *Deriving a Country's Centers and Spread,*** after completing Lessons 1 to 4.

	Description	Implementation Design	Anticipated instructional time
Lesson 1: The United States – A Lower Middle-Layered Country	This lesson introduces students to a population pyramid graph of the United States. Using the 2015 pyramid graph, or a histogram of the counts in 5-year age groups or a table prepared from data provided by the United States Census Bureau, students discover the age summaries of a lower middle-layered country. Lesson could stand alone, although comparisons to Kenya and Japan provide a deeper understanding of definition of the shape of a country.	• Provide students: **Handout 1: *United States - 2015*** •Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 1.	One 50-minute class period
Lesson 2: Kenya – A Bottom Layered Country	The 2015 pyramid graph of Kenya, along with a histogram of the counts in 5-year age groups and a table prepared from data provided by the United States Census Bureau, are used to discover the age summaries of Kenya. Lesson could stand alone, although comparisons to United States and Japan provide a deeper understanding of shape.	• Provide students: **Handout 2: *Kenya - 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 2.	One 50-minute class period
Lesson 3: Japan – An Upper Middle-Layered Country	The 2015 pyramid graph of Japan, along with a histogram of the counts in 5-year age groups and a table prepared from data provided by the United	• Provide students: **Handout 3: *Japan - 2015*** • Set up lesson to be completed individually or in small groups.	One 50-minute class period

	States Census Bureau, are used to discover the age summaries of Japan. Lesson could stand alone, although comparisons to United States and Kenya provide a deeper understanding of shape.	• Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 3.	
Lesson 4: **The Center and Spread of a Country's Shape**	This lesson organizes the 2015 United States population data into tables that students use to calculate the mean age and the median age. Tables are also provided for students to derive an estimate of the spread of ages anchored by the median.	• Set up lesson to be completed Individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 4. • Consider organizing small groups of students to also calculate the mean age, median age, and spread for Kenya and Japan. The templates needed are included in the lesson. Each group is assigned a country and reports to the whole class the results of their work.	One 50-minute class period (An additional 50-minute class period is needed if students include Kenya and Japan.)
Lesson 5: **My Country**	The lesson directs students to create a population distribution by age groups for their own country (referenced in later lessons as ***My Country***). The details students should consider are provided in the lesson. An alternative set-up is to direct students to work with a data set representing a top layered country that is provided in the **Teaching Notes**.	• This lesson could be completed as an independent project. If this suggestion if followed, provide time for students to individually present the pyramid graphs and histograms for the population distributions they created. • The data included in the **Teaching Notes** provide an example of a Top Layered Country that could be used by the whole class. This data set is also referenced in later	One 50-minute class period (if completed during class).

	This lesson is considered optional. It can be used as a way to determine students understanding of the previous 4 lessons, as well as a sense of their disposition in working with this data.	lessons to indicate the unusual features of a top layered country. • If the data provided in the **Teaching Notes** are used, conduct whole class discussions at periodic points in the lesson.	

Unit 2: Looking Back

A country's shape is a result of many factors that emerge by studying the past and present population pyramid graphs and histograms. This unit develops a timeline that analyzes both past distributions and the resulting connection to 2015. In Lesson 6, the timeline identifies the emergence of two significant subgroups (although subjective descriptions are used to define these subgroups) that result in the unique shape of the United States – namely, the Baby Boom Generation and the Millennial Generation. Events that resulted in a change in the count of people over time are identified in the lessons and traced through several decades.

Points to consider:
The primary focus in this unit is developed in Lesson 6. It is important for several reasons that are outlined in the **Teaching Notes**. It is the first lesson that begins to imply what are factors that result in a change of the population over time. It also uses the graphs to indicate why these factors need to be considered in analyzing a country's current distribution.

Lesson 6 builds a foundation for the goal of the next unit, n1amely, to derive population estimates of the future. Lesson 7 continues to connect Kenya and Japan in this study as a comparison of different shapes to the shape of the United States. Lesson 7 is an optional lesson, although it also links Kenya and Japan's current shape to their past.

	Description	Implementation Design	Anticipated instructional time
Lesson 6: Looking Back at the Shapes of the United States	This lesson looks back at the population distributions of the United States using population pyramid graphs and histograms. Links to the past are used to identify the emergence of descriptions of generations (i.e., Baby Boomers or the Millennial Generation).	•Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 6.	One 50-minute class period.

	Lesson could stand alone. The connection to Kenya and Japan adds to an understanding of past distributions to the current shape (as defined in this module) of a country.		
Lesson 7: Looking Back at the Shapes of Kenya and Japan and My Country	This lesson links the past population distributions of Kenya and Japan to their current distributions. This lesson is considered optional.	•Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 7.	One to two 50-minute class periods.

Unit 3: Looking Forward
This unit builds on the analysis of past distributions in deriving estimates of future counts. Students are challenged in each lesson to explain the factors behind each new iteration of a population projection model. The unit begins by developing an arithmetic sequence based on the 2010 and 2015 population totals. Lesson 8 summarizes this sequence with a linear model that is used to estimate the future population totals. A geometric series is used to derive an exponential model in Lesson 9. Each of these lessons, however, do not obtain estimates for the age groups. Although possible to derive age group estimates using these models, the age group analysis begins in Lesson 10 with the special recursive model designed for this module. Lesson 10 introduces an important feature of the recursive model that is identified as **Population Factors**. Applying these factors to build estimates of future age groups is done in Lesson 11. Lesson 12 completes the recursive model by developing **Foundation Factors** that fill in the "holes" left by the 0 – 4 years old age group in each iteration. As the holes are derived, the resulting projections based on population factors are completed. Lessons 13 and 14 build off of these estimates to summarize the shapes and projected changes in the population of the United States, Kenya, and Japan.
Points to consider: Lessons 8 and 9 provide students the more familiar models they might have previously studied. The development of a linear model and an exponential model are often used to start a study of modeling. Lesson 10 is an important lesson to launch an understanding of a specific iterative or recursive model. Important features of this model are evident by studying specific

age groups over time, namely the impact of immigration, emigration, and death in population projections. Lessons 10 to 12 should be completed as a unit. Lessons 13 and 14 are optional but add to students' understanding of the population changes over time. In particular, Lessons 13 and 14 compare the linear, the exponential, and the recursive models.

Also consider the optional Case Study projects. See Henry's Quilt, **Case Study Project: *Interpreting Population and Foundation Layer Factors,*** **Case Study Project: *Developing Projection Model,*** and **Case Study Project: *A Country Quilt Scavenger Hunt*** after completing lessons 8 to 12. A description of each case study is provided in the introduction to the Case Student Projects.

	Description	**Implementation Design**	**Anticipated instructional time**
Lesson 8: Looking Forward with an Arithmetic Sequence and a Linear Model	This lesson uses the 2010 and 2015 population totals of the United States, Kenya, and Japan to estimate the population of each country in the future by deriving and applying a linear model. The linear model is also used to derive estimates of past years, comparing these estimates to the actual population summaries.	• Lesson could be assigned as an independent project and discussed with all students after they complete the problems. Lesson could also be organized as a class project in which students work independently or in small groups and discuss as a class their answers or responses. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 8.	Two 50-minute class periods.
Lesson 9: Looking Forward with a Geometric Sequence and an Exponential Model	This lesson uses the 2010 and 2015 population totals of the United States, Kenya, and Japan to estimate the population in the future. The exponential model is also used to derive estimates of past years, comparing these estimates to the actual population summaries.	• Lesson could be assigned as an independent project and discussed with all students after they completed the problems. Lesson could also be organized as a class project in which students work independently or in small groups and discuss as a class their answers or responses. • Conduct whole class discussions at periodic points in the lesson – read	One 50-minute class period.

		suggestions in **Teaching Notes** for Lesson 9.	
Lesson 10: Looking Forward with a Recursive Model	The recursive model designed in this module emphasizes the changes in 5-year age groups as a reading of what happened during the last 5 years. Was the count in those age groups changed primarily by immigration, by emigration, or by death? (The impact of births in the country is addressed in Lesson 12.) These changes are examined by forming ratios of ***connected age groups*** that are then represented by decimal values called the **Population Factors.** This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.	• Provide students **Handout 1: *United States – 2015*** and **Handout 4: *United States Connected Age Groups*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 10. • Highlight during the discussion with students the problems identified in the Assessment section of the **Teaching Notes**, possibly placing selected student responses on a poster for referencing in the next several lessons.	One 50-minute class period.
Lesson 11: The Recursive Model and Falling Dominos	The population factors are applied to obtain future population estimates. The problems in this lesson direct students to summarize these changes over time; for example, if the changes noted in the 2010 to the 2015 connected age groups continue, what is the effect on future counts? A hole is left in the population summaries, however, that is filled in the next lesson.	• Provide students **Handout 5: *Looking Forward for the United States (Student Edition)*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 11.	One 50-minute class period.

	This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.		
Lesson 12: Completing the Recursive Model with the Foundation Layer	This lesson fills in the holes left in the lesson. A **Foundation Factor** is derived based on the 2015 population. If this factor based on the 0 – 4 years old age group (the only age group that counts the births) stays the same over time, what are the estimates of the future counts in this age group? Filling in this hole will then generate estimates for connected age groups (or, the effect of the falling dominos). Each iteration completes an estimate of the 0 – 4 years old age group that ultimately fills in all holes summarized in the handout. This lesson is an important lesson for students to complete in order to accomplish the overall goals of this module.	• Provide students **Handout 6: *United States 2010 - 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 12.	One 50-minute class period.
Lesson 13: "The More Things Change, the More Things Stay the Same"	This lesson analyzes the recursive model developed in Lessons 10 to 12 in more detail. Over time, the results of applying the population factors derived in Lesson 10 are a growing population and a leveling off	• Provide students: **Handout 6: *United States 2010 – 2015*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points	One 50-minute class period.

	of several age groups. Students examine the subtle changes in specific age groups that result in the leveling off of the population. Analyzing changes over time is important, especially when several iterations of this particular model are completed. This lesson is considered optional.	in the lesson – read suggestions in **Teaching Notes** for Lesson 13.	
Lesson 14: Kenya, Japan, and the United States – Summing it Up	Unit 1 launched the importance of examining the populations of the United States, Kenya, and Japan. Unit 2 examined the past populations primarily using graphs. Unit 3 implemented various models to estimate population counts in the future. This concluding lesson of Unit 3 organizes the results of the various models and summaries the different counts for the 3 countries. This lesson provides students an important wrap-up of what has been developed in the module. This lesson builds on the previous lessons from Unit 3 and provides students a better understanding of what the purpose of a model. In particular, students are directed to understand the importance of identifying the	• Provide students the following handouts: **Handout 6: *The United States 2010 – 2015,*** **Handout 7*: Kenya 2010 – 2050,* and** **Handout 8*: Japan 2010 – 2050*** • Set up lesson to be completed individually or in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in **Teaching Notes** for Lesson 14.	One 50-minute class period.

	assumptions and design of a model when interpreting its results.		

Unit 4: "What if …?"

The ultimate goal of any model is to provide possible answers to "What if …?" questions. A working model is now in place. What if the population factors and foundation factors noted in the set-up of the recursive model continue? What if they change? What if fewer counts of people move into a country? What if economic conditions require more workers? What if a downturn in the economy results in people having fewer children? These, and several similar questions, are explored in this unit. Students are expected to apply the tools they learned to rebuild the recursive model to answer these types of questions. Students are also expected to be able to articulate what changes they propose to the original model, and why they propose those changes. The two lessons designed in this unit, and the concluding wrap up section, are putting students in control of the model (and the highest level of the **Modeling Continuum**).

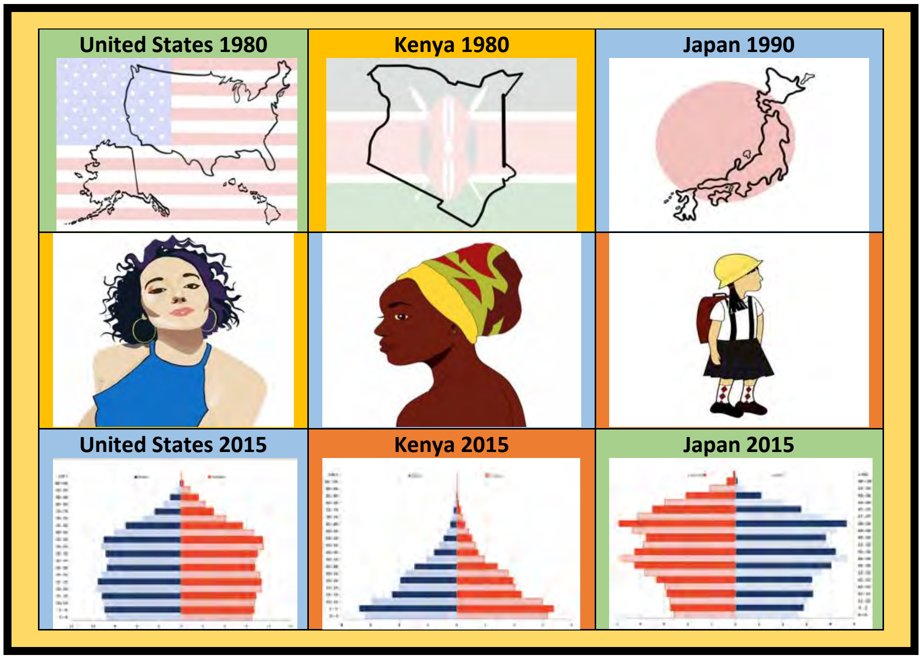
Points to consider:
Students complete Lesson 15 with access to the Excel projection files provided with this module. Handouts are provided for students to identify specific factors (or tools) of the recursive model they would alter to address a "What if…?" scenario. They are also expected to explain their decisions based on the impact of what factors they would alter. Lesson 16 is similar in design. Students again rework the recursive model with the goal of matching their results to the projections summarized by the United States Census Bureau. Lesson 16 involves a more precise analysis of the impact of the proposed changes to the model.

	Description	**Implementation Design**	**Anticipated instructional time**
Lesson 15: "What if …?" Scenarios	Students select a "What if…? scenario from several presented in the lesson. They complete a plan that indicates how they would alter the recursive model based on the selected scenario. In their plan, students indicate which population factors they would alter and what values they would assign to those factors. They also indicate if they would alter the	• Provide students the handouts indicated in the opening section of the lesson. • Provide students access to the projection files indicated in the opening section of the lesson. • Consider organizing students in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in the **Teaching Notes** for Lesson 15.	Two 50-minute class periods. The extension in time is based on students have access to the projection files.

	foundation factors and what values they would assign.		
Lesson 16: The US Census Models and the Recursive Model	Students are presented the 2050 population projections from the Census Bureau. They compare the Census Bureau's result to the results from the 2050 recursive model. They complete a plan that indicates how they would alter the recursive model that would result in a better match to the United States Census projections. In their plan, students indicate the impact of changing specific factors over time. Students also indicate if they would alter the foundation factors and what values they would assign to the foundation factors. Students are again expected to indicate the impact of their changes in the foundation factors over time. This lesson is considered optional.	• Provide students the handouts indicated in the opening section of the lesson. • Provide students access to the projection files indicated in the opening section of the lesson. • Consider organizing students in small groups. • Conduct whole class discussions at periodic points in the lesson – read suggestions in the **Teaching Notes** for Lesson 16.	One to two 50-minute class periods.
Wrap-up of the People Count Stories	Three projects are described in this wrap-up section. Each project involves identifying key components of the recursive model and interpreting their impact on the model.	• Read with students the selected project or projects. • Direct students to complete the project or projects independently or in small groups. • Provide time for students to report their answers to the whole class.	Work is primarily done outside of class. Schedule time for students to summarize their work to

	Students summarize their work by answering questions that require an understanding of the design of the recursive model.	• Other suggestions are provided in the **Teaching Notes**.	the entire class.

Teaching Notes
Unit 1
A Country's Shape



Introduction

The United States, Kenya, and Japan, like all countries, are a collection of people often described by age, sex, race, education, and a number of other factors. Are these descriptions important? In particular, does the description of a country by age groups help us better understand a country? Unit 1 defines the shape of a country based on the counts of people living in the country by age. Age groups are displayed by way of population pyramid graphs and histograms. The graphs, and the summaries derived from them, begin to unpack stories about people living in these countries. Kristin, Raphine, Hana, their sisters and brothers and parents and nieces and nephews are people counted in these graphs. Their stories are not just about them. They are stories about special people who help us understand the United States, Kenya, and Japan.

The graphs are more than just visual summaries of each country. They serve another purpose that is also connected to the stories in this module. Each graph is a point on a country's timeline that tells us something about its past and possible future. From this point, we unpack a part of each country's history as well as speculate about its future. This module uses the count of people by ages to build a quilt that is understood by applying mathematical tools that emerge from a country's past and present shapes. Building this quilt, called a recursive model, starts by counting people. Let us begin the process, after all, "People Count!"

Teaching Notes

Lesson 1
The United States
A Lower Middle-Layered Country

Overview:

This lesson introduces students to a definition of a country's shape using the 2015 population pyramid graph or the 2015 population histogram. Students unpack a definition of a country's shape by answering questions and problems linked to the ***lower middle-layered*** shape of the United States. A similar study of Kenya and Japan follow this lesson that introduce students to different population shapes.

After students complete the lesson, consider assigning an **Exit Summary** using at least two of the problems of this lesson. The **Exit Summary** is **Handout 13** found in the handout section of the teacher's edition. Most of the problems of this lesson address Levels 1 or 2 of the **Modeling Continuum**. A suggested alignment of the problems is provided in the following table. This alignment, however, can be altered based on shared work with other students or special directions provided by you, the teacher. Remind students to think about the descriptors of the **Modeling Continuum** as they complete an **Exit Summary**. An example of a completed summary is included in the **Overview of the Module** section. Consider sharing this example with your students.

A consistent interpretation of these descriptors is initially challenging; however, Lessons 1 to 3 primarily focus on the descriptors in Levels 1 and 2 that are more straight forward. As students continue to work with the **Exit Summary**, the descriptors begin to make more sense, especially as they are introduced to problems that are aligned to Levels 3 and 4. Providing the opportunity for students to summarize their work using the **Exit Summary** is one way to assess a student's thinking and understanding of the problems.

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2, 7 8, 9, 10, 15, 16, 21,22,23	Problems: 3, 4, 5, 6, 11, 12, 13, 14, 15, 18, 19, 24, 25, 26		

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, areas, ratios.
See the connection of these tools to high school standards in the ***Overview of the Module.***

Resources needed for this lesson:

Provide students a printed or electronic copy of the **Introduction to the Module** and the **Introduction** of Unit 1. Students will need a copy of Kristin's data story, **Kristin's Story – Chapter 1**, to start this lesson and a printed or electronic copy of the problems for Lesson 1. **Kristin's Story – Chapter 1** sets the stage for the data stories used throughout the module. Students will also need a copy of **Handout 1: *United States – 2015*** to complete the problems.

Consider providing a globe, a world map, or a geographical app to help students understand the geographical connection of the United States to the other countries they will study in the next lessons. Lesson 2 will involve similar questions and problems using the population data for Kenya. Lesson 3 will involve a similar study using the population data of Japan. Each lesson highlights a different population shape and a different population distribution. Extensions in **Henry's Quilt** are also included at the end of the module that can be used at any time to study the shape and spread for the countries displayed on the the cover of the module. Various maps can be obtained from the website of the United States Geological Survey if maps are not available (https://www.usgs.gov/products/maps/overview).

Launch:

Read and discuss the **Introduction to the Module**. After reading this introduction, ask students what they think this module is about. Also read and discuss the **Introduction** to Unit 1 and again ask students what they Unit 1 is about. Consider recording a few of the students' comments on poster paper. Review and discuss these comments after the unit has been completed.

Provide time for students to read **Kristin's Story, Chapter 1**. After appropriate reading time, ask students to summarize some of the points of the story. For example:

- How old is Kristin at the start of 2015?
- Why did Kristin's mother's comment confuse her?
- Why do you think Kristin did not always like the more popular movies?

Direct students to study the population pyramid graph and the histogram displayed in the lesson and handout. Ask general questions to determine if they understand the information summarized by the graphs. Consider asking questions such as:

- What is represented by the horizontal bars of the pyramid graphs?
- What does the left side of the pyramid graph represent? What does the right side represent?
- What is an estimate of the count of 0 – 4 years old from the histogram?
- How do you think the counts summarized in the pyramid graph were obtained? (This question is also raised in the **Introduction to the Module** section as it desribes the challenges undertaken by the United States Census Bureau.)

When you think students understand the graphs, direct them to work in small groups or individually as they complete the problem section.

Implementation Ideas:

There are several problems that are directed at finding the count of people from the table included on **Handout 1**. Other questions involve approximations that could be answered by the height, length, and area of the graphs (either the population pyramid graph or the histogram). Discuss with students questions that are answered by either the table or the graphs. Also, summarize with students the percent of each layer (for example, the percent of the population in the lower middle-layer or the top layer) and the possible implications that are linked to a country's shape.

Using Kristin as an example, identify where people counted in her age group are summarized in the graphs. Highlight that there is a larger count of people in the older age groups than Kristin, and similarly, a larger count of people is in the younger age groups. This observation will be picked up and enhanced in the next chapters of the data stories as it impacts the generational diversity of the United States.

Consider arranging students in groups of 2 to 3 to answer the problems in the lesson. Several of the problems involve calculations that encourge students to coordinate their work with other students. In addition, the questions that ask students to share an idea are more interesting when discussed within smaller groups before discussed with the entire class.

Student responses or descriptions

Lesson 1 - Problems

Handout needed to complete the following problems: Handout 1: *United States – 2015*

1. What 5-year age group has more people (males and females) than any other age group?
   Based on the table, the 20 – 24 years age group with 22,693,026 people is the largest count of people. Students might also indicate that they used the visual length of the bars from the population pyramid graph or histogram.

2. In what age group was Kristin counted in the 2015 population pyramid graph or histogram? In what age group was Kristin's mother counted? In what age group was Kristin's younger sister counted?
   Kristin was counted in the 35 – 39 years old age group at the start of 2015. Her mother was counted in the 65 – 69 years old age group. Her younger sister was counted in the 25 – 29 years old age group.

3. What 5-year age group of just males has more counts of males than any other age group of males?
   Using the table, the 20 – 24 years age group with 11,644,934 males is the largest age group of males.

4. What 5-year age group of just females has more counts of females than any other age group of females?
   Using the table, the 50 – 54 years age group with 11,348,281 is the largest age group of females.

A population distribution is defined by the following *layers*:

- The ***bottom-layer*** refers to the counts of people in the 0 to 24 years old age groups.
- The ***lower middle-layer*** refers to the counts of people in the 25 to 49 years old age groups.
- The ***upper middle-layer*** refers to the count of people in the 50 to 74 years old age groups.
- The ***top layer*** refers to the count of people in the 75 to 100+ years old age groups.

Based on the summary of layers, a country's shape is defined by the following terms:

- A country that has most of its people in the bottom-layer is identified as **Bottom-Layered Country**.
- A country that has most of its people in the lower middle-layer is identified as a **Lower Middle-Layered Country**.
- A country that has most of its people in the upper middle-layer is identified as an **Upper Middle-Layered Country**.
- A country that has most of its people counted in the top layer is identified as a **Top-Layered Country**.

5. Estimate what age group layer (bottom, lower middle, upper middle, top) you think will have the least number of people in the United States? Did you use the population pyramid graph, the histogram, or the table to make your estimate? Explain how you made your estimate using the graphs or table.
   Using the graphs, the top-layer appears to have the smallest total area covered by the age groups.

6. Estimate what age group layer you think will have the greatest number of people? Did you use the population pyramid graph, the histogram, or the table to make your estimate? Explain how you made your estimate using the graphs or table.
   Answers vary. The increasing/decreasing pattern in the lower middle-layer and also in the upper middle-layer makes it difficult to summarize this question based on the graphs. If the table is used, it would be the lower middle-layered group.

7. Kristin used the table included with Handout 1. She added up the count of people who were 0 – 4 years old, 5 – 9 years old, 10 – 14 years old, 15 – 19 years old, and 20 – 24 years old. The total count she obtained was 104,776,994 people. What is the percent of people 0 to 24 years old based on the estimates summarized by the table? Summarize your answer to the nearest tenth of a percent.
   104,776,944 people/320,896,618 people or approximately 32.7%.

8. In a similar way, what is the percent of people 25 to 49 years old?
   105,289,239 people/320,896,618 people is approximately 32.8%.

9. What is the percent of people 50 to 74 years old?
   90,623,165 people/320,896,618 people is approximately 28.2%.

10. What is the percent of people 75 to 100+ years old?
    20,207,271 people / 320,896,618 people is approximately 6.3%.

11. Identify two different age groups in which the count of people in the first age group is approximately double the count of people in the second age group. (There are several examples to answer this problem.)

    Using the table, the possibilities that could be considered are the 70 – 74 years old is nearly double the 80 – 84 years old age group. Also, the 20 – 24 years old age group is nearly double the 70 – 74 years old. Analyze other possibilities.

12. Kristin's data story indicates that she felt disconnected from the more popular choices of movies and political views. Look at the age group that includes Kristin at the start of 2015. Why might people older or younger than Kristin have different interests than Kristin in movies or political views? Using the population estimates provided in **Handout 1**, why might the entertainment choices or political views of people older or younger than Kristin be reported in the news or social media more often?

    Using the graphs or table, the 35 – 39 years age group is less than the count of people in the age groups just younger than her and is also less than the count of people in the age groups just older than her. Kristin's age group is a type of "valley" with age groups older and younger more dominant in count. Several of the areas of disconnect that Kristin mentioned are influenced by age. People identify with a certain actress or actor by age, or by a certain political leader by age, etc. This results in the more dominant age groups seemingly more targeted by social media, entertain options, and political outreach.

13. For this problem, adjacent age groups are age groups next to each other. For example, 0 – 4 years old is adjacent to 5 – 9 years. In a similar way, the 45 – 49 years old age group is adjacent to the 50 – 54 years old age group. The age group 45 – 49 years old is also adjacent to the 40 – 44 years old age group. Identify two adjacent age groups that have approximately the same count of people in each age group. (There is more than one answer to this problem.)
Answers vary. Visually, age groups 20 – 24 and 25 – 29 years old age groups appear to be nearly the same. Also, 50 – 54 and 55 – 59 years old age groups are nearly the same.

14. Estimate the count of teenagers (13 -19 years old). Explain how you derived your estimate.
Caution: This problem is more rigorous than the previous problems. Consider discussing this problem before students derive an estimate. If necessary, highlight that each group counts 5 distinct ages. Consider the number of 13 and 14 years old to be approximately 2/5 of the count of 10 – 14 years old. Therefore, an estimate of the number of teenagers is:
2/5 x (20,605,579) + 21,084,710 which is approximately 29,326.942 people (to the nearest person), or a general estimate of 29 or 30 million people.

15. What is the count and percent of people who are under 10 years old?
The count of people 0 – 4 years is 19,912,499 people and the count of people 5 – 9 years is 20,481,130.
Total: 40,393,629 people. This is approximately 12.6% of the country's population.

16. What is the count and percent of people who are 65 years old or older?
There are approximately 47,734,292 people 65 years old or older. This is approximately 14.9% of the country's population.

17. Why is it important that the count and percent of people under 10 years old and the count and percent of people 65 years old or older are given special attention when analyzing a country's population?
These age groups include people who are not working (at least fulltime in many cases). People in these groups may need the most care from the country in terms of health care, education, and retirement support.

18. "Old" and "young" are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of "young" as people less than 10 years old, and the definition of "old" as people who are 65 years old or older. What is the ratio of "old" to "young" using the above definitions of young and old? Derive a

decimal from this ratio and interpret it by describing the approximate count of "old people" to the count of one "young person." Express your answer to the nearest whole number.
47,734,292 people to 40,393,629 people, or approximately 1.18 as a decimal. This means a little more that 1 old person to 1 young person to the nearest person, or approximately 1 old person to 1 young person.

19. Kristin's data story indicated that she worked in the health field. Do you think that working in the health field is a major area of employment? Explain your answer by referring to the data.
Yes, the combined percent of young and old as defined above is greater than 27% of the population, or more than one-quarter of the population is at the age that they may require more health care than other age groups.

20. The voting age in the United States is 18 years old or older. Derive an estimate of the number of potential voters in the United States at the start of 2015.

    Consider 18 and 19 to be 2/5 of the age group 15 – 19 years. So, voting ages would be:

    2/5 x (21,084,710) plus the total of the age groups older than the age group of 15 – 19 years old. This resulting estimate is 247,246,584 people. This estimate includes, however, the count of immigrants and other people who may not be eligible to vote. Therefore, this is a high estimate.

21. Identify the age groups in which the count of males is estimated to be more than the count of females.
Age groups 0 – 4, 5- 9, 10, 14, 15 – 19, 20 – 24, 25 – 29, and 30 – 34 years age groups.

22. Identify the age groups in which the count of females is estimated to be more than the count of males.
All age groups from 35 – 39 years to 100+ years.

23. Identify an age group that has approximately the same count of males and females?

    The age group 35 – 39 years has approximately the same count of females and males as noted in the table.

24. Similar to the way you estimated the ratio of old to young people in problem 18, estimate the ratio of females to males for the following age groups. Derive a decimal from the ratio and estimate the number of females to one male in that age group (round your answers to the nearest whole number):
    a. In the age group of 85 – 89 years, there are approximately _____ females to one male.
       2,423,021 females to 1,441,268 males, or 2 females to 1 male.
    b. In the age group of 90 – 94 years, there are approximately _____ females to one male.
       1,263,123 females to 588,497 males, or 2 females to 1 male.
    c. In the age group of 95 – 99 years, there are approximately _____ females to one male.
       367,526 females to 127,836 males, or 3 females to 1 male.
    d. In the age group of 100+ years, there are approximately _____ females to one male.
       62,137 females to 15,105 males, or 4 females to 1 male.

25. Notice the changes in the ratio of females to males that you derived in problem 24 as the age groups grew older. Write a sentence or two that describes what is happening. Why might these changes be important for people interested in the health care of the population?
    As people age, the count of females per male increases. Females live longer than males. Health care that provides for these differences will be critically important.

26. What questions would you like summarized for specific age groups? Answers to your questions would not necessarily be derived by the population graphs or table. Discuss with your class at least one of your questions and why you think the answer to your question is important.
    Questions or summaries will vary. The goal of this problem is for students to reflect that the population of the country consists of age groups that differ in interests, activities, opinions, etc. Some of the suggested questions students list in the table could be further analyzed by actually asking people to respond to the questions (i.e., family members, friends of the family, older or younger friends) and discussing the range of answers collected by the students. Several of these type of questions will discussed in other lessons based on the changes in the country's shape over time.

Age group	Questions you would like summarized for this age group
Example: 0 – 4 years old	How many children 0 – 4 years old are in a pre-school program? (The answer to this question is important in determining whether or not our country has enough trained pre-school teachers.) How many children 0 – 4 years old can count out loud to 20? (The answer to this question would help determine what skills 0 – 4 years old have or have not mastered or what skills can be used to extend learning opportunities.)
15 – 19 years old	
35 – 39 years old	
An age group of your choice:	

Assessment Ideas

Assessment Task:

Consider the following assessment task to determine a student's understanding of the lesson:

Oostburg, a small town in Wisconsin, had a population at the start of 2015 of 1000 people. Ages of the people in the town ranged from 0 years old to 102 years old. Someone described the town as a lower middle-layered town. Sketch a histogram of the 1000 people's ages using the same design presented in this lesson, namely, set-up your histogram with age groups of 0 – 4 years old, 5 – 9 years old, etc.

After you have made your sketch, answer the following:

1. Indicate why your sketch represents a lower middle-layered town.

2. Identify a 5-year age group in your sketch that you think will have the greatest percent of town's population. Explain your reasoning in selecting that age group.

3. What age group do you think will have the least count of people? Explain you reasoning in selecting that age group.

Comments on the Assessment Task:
Evaluate the assessment task based on a student's understanding of an age distribution as presented in this lesson. A sketch that has the greatest count or percent of the people in the age groups of 25 - 49 years old would indicate an understanding of a lower middle-layered town. Any sketch that is similar to the shape of the United States represents a lower middle-layered town. If time is a factor, a general overview of the shape is the most important to evaluate.

Any of the 5-year age groups in the lower middle-layer or 25 – 49 years old would be a reasonable age group to identify as having the greatest count or percent of the population. It is possible, of course, for students to create a histogram that has the greatest count in an age group either below or about the lower middle-layer. Anticipate some students will be creative with their sketches as various arrangements could be possible. The key is that the ***total count*** of the layer defined as the lower middle-layer, or people 25 – 49 years old age groups, has the greatest count of people of the 4 layers. Also anticipate that students would identify the 100+ age group as having the least count due to the aging of the population.

Additional Assessment Ideas:
Several of the problems could be used to determine if students understood the lesson's overview. Particularly, discuss as a whole group problems 7, 8, 9, 10 and what these problems indicate about the shape. Also, consider evaluating problems 12 and 13 to determine if students understood the smaller count of people in Kristin's age group to the greater estimates of the count of people in the age groups above and below.

Discuss as a whole group answers to problems 7, 8, 9, 10 and 26 and what these problems indicate about the shape of the population distribution. Discuss with students why the age groups with the larger estimated counts may impact preferences of entertainment, employment skills, types of technology, etc. For example, why might the target audience of a certain smartphone be 20 to 29 years old and not 35 to 44 years old?

Students will discover in later lessons that the estimated counts of various age groups will vary over time, and result in different shapes of the population pyramid graphs or histograms. The connection of a country's shape to some of the challenges the country faces is an important step in understanding why models that estimate future counts are important. Problem 26 particularly encourages students to extend their thinking about the population by age groups.

Teaching Notes
Lesson 2
Kenya
A Bottom-Layered Country

Overview:

Similar to Lesson 1, the problems in this lesson summarize data from the population pyramid graph, the population histogram, and the table. Students calculate decimals, percent, and ratios of various age groups and connect their calculations to the definition of a country's shape that is also defined in the lesson. The Modeling Continuum provides an overview of the rigor of the problems. In general, the problems are designed to address Levels 1 or 2 of the **Modeling Continuum**. The suggested classification of the problems is provided in the following table to help you assess students' work. Classification of the problems can be altered based on your implementation of the lesson with students. Consider assigning an **Exit Summary** after students complete the problems. If an **Exit Summary** is assigned, remind students to explain their answers using the **Exit Summary's** descriptors of the **Modeling Continuum**.

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2, 13, 15	Problems: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 17, 18, 19		

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, areas, ratios.
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

The data story, **Kristin and Raphine's Story – Chapter 2**, begins the lesson. This story introduces new characters who are important in understanding the population distribution of Kenya. Students need **Handout 2: *Kenya – 2015*** to solve the problems. This lesson would benefit with visuals that were also suggested in Lesson 1, namely a map of the world or a globe or a geographical app.

Launch:

Consider starting this lesson with a discussion of Kenya. Locate it on a map or globe. Highlight the geographical shape of the country, its location on the continent of Africa, and its distance from the United States or Japan (the next country they will study).

Direct students to read **Kristin and Raphine's Story – Chapter 2.** This data story is included in the student edition for Lesson 2. After appropriate reading time, ask students to summarize some of the points of the story. For example:

- How old was Raphine at the start of 2015?
- What was Raphine's job in Kenya?
- Why was Raphine interested in moving for a period of time to the United States?
- How was Kristin going to help Raphine?

Direct students to study the population pyramid graph and the histogram of Kenya. As you did in Lesson 1, ask general questions to determine if students understand the information summarized by the graphs.

- What is an estimate of the count of 0 – 4 years old from the histogram?
- What is an estimate of the count of 35 – 39 years old from the histogram?
- How do you think the counts summarized in the pyramid graph were obtained? This question is far more difficult to answer, however, for Kenya than for the United States. Allow students to suggest possible ideas. Point out, however, that unlike the United States, Kenya and most of the countries of the world do not have a constitution that directs a census every 10 years. The data reported by the International Data Base was obtained by actual counts of people, projection models similar to the models developed in this module, and sampling techniques.

Implementation Ideas:

As suggested in Lesson 1, consider assigning students to small groups. Several of the problems involve calculations that could be more efficiently derived through group work. In addition, the questions that ask students to share an idea are more interesting when discussed within small groups before discussed with the entire class.

The population graphs (both the pyramid graph and the histogram) have very different shapes than the graphs representing the United States. Students are instructed in the lesson to summarize the data using **Handout 2.** Comparing the counts and percent of various age groups can be derived using the table or the lengths of the bars or areas of the histogram . The greater counts of the younger age groups will also be highlighted in Lesson 4 as students estimate a mean age, median age, and spread for Kenya.

Kenya's future shape will be discussed in Units 3 and 4. The projected shape for Kenya in 2050 is different than its current shape and its recent past shapes. Students will continue to work with the population data of Kenya and summarize how the estimated population counts might be revised in the future, resulting in noticeable changes on the pyramid graph. Highlight that the youngest age group in 2015 (the 0 – 4 years old) indicates fewer younger people than the 5 – 9 years old age group. This noticeable decrease will explain some changes in the shape of the country in the latter units of this module.

Student responses or descriptions

Lesson 2 - Problems

Handout needed to complete the following problems: Handout 2: *Kenya – 2015*

1. What 5-year age group has more people counted in it than any other age group?
   The age group 5 - 9 years old has the most people.

2. In what age group was Raphine counted in the 2015 population pyramid graph or histogram? In what age groups were the students in his school counted at the start of 2015?

   The 35 – 39 years old identifies the age group Raphine was counted in the beginning of 2015. (Note, Raphine is approximately the same age as Kristin. This age group is an important marker in looking back and looking forward.) The students in Raphine's school are counted in the 5 – 9 and 10 – 14 years old age groups.

Review again the definitions that were presented in Lesson 1:
A population distribution is defined by the following *layers*:

- The ***bottom-layer*** refers to the counts of people in the 0 to 24 years old age groups.
- The ***lower middle-layer*** refers to the counts of people in the 25 to 49 years old age groups.
- The ***upper middle-layer*** refers to the count of people in the 50 to 74 years old age groups.
- The ***top layer*** refers to the count of people in the 75 to 100+ years old age groups.

Based on the summary of layers, a country's shape is defined by the following terms:

- A country that has most of its people in the bottom-layer is identified as **Bottom-Layered Country**.
- A country that has most of its people in the lower middle-layer is identified as a **Lower Middle-Layered Country**.
- A country that has most of its people in the upper middle-layer is identified as an **Upper Middle-Layered Country**.
- A country that has most of its people counted in the top layer is identified as a **Top-Layered Country**.

3. Estimate what layer (bottom, lower middle, upper middle, top) you think will have the least number of people in Kenya? Explain how you made your estimate.
   Top layer has the smallest area of the graph, representing the older people in the country.

4. Estimate what layer you think will have the greatest number of people? Explain how you made your estimate.
   Bottom layer has the largest area of the graph, representing the youngest people in the country.

Mark on the ***pyramid graph*** and the ***histogram*** a designation indicating where each of the above layers begin and end.

5. Using the table of population counts on **Handout 2**, what is the percent of people 0 to 24 years old?
   Total count of the 0 to 24 years old is 27,655,707 people using the table. This is approximately 60.2% of the Kenyan population.

6. In a similar way, what is the percent of the count of people 25 to 49 years old?
   Total count of the 25 – 49 years old is 13,977,786 people. This is approximately 30.4%.

7. What is the percent of the count of people 50 to 74 years old?
   Total count of the 50 to 74 years old is 3,871,954 people. This is approximately 8.4%.

8. What is the percent of the count of people 75 to 100+ years old?
   Total count of the 75 to 100+ years old is 419,854 people, or approximately 0.9% or approximately 1%.

9. Identify two age groups in which the number of people in one age group is approximately double the count in the other age group.
   Answers vary. Possible selections: the count of 45 – 49 years old is approximately double the 60 – 64 years old. Also, the count of 0 – 4 years old is a little more than double the count of 30 – 34 years old. Other answers should be considered.

10. Raphine's data story indicates that he was unable to accept all of the kids who wanted to attend his school. In what way do the graphs indicate that finding a school for all of the young people might be one of Kenya's challenges?
    There are 12,709,028 people (or approximately 27.7% of the country) who are in the age groups 5 to 9 and 10 to 14 years old. This is a large percent of the country expecting educational services.

11. Identify one of the age groups from the table that has more than 10% of the total population of Kenya.
    Age groups 0 – 4, 5 – 9, 10 – 14 years old each have more than 10% of the total population within the age group.

12. Estimate the count and percent of teenagers (13 to 19 years old). Explain how you derived your estimate. (Estimates will vary.)
    Consider the count of 13- and 14-years old people to be 2/5 of the people counted in the 10 – 14 age group plus all of the people estimated in the 15 -19 years old age group. Therefore, an estimate of the count of teenagers is:
    (2/5) x (5,950,852) + (4,494,168) or approximately 6,874,509 people or approximately 15% of the country's population.

13. What is the count of people who are under 10 years old?

    The estimated count of 0 – 4 years old is 6,376,220 and the estimated count of 5 - 9 years old is 6,758,176. The total is 13,134,396 people.

14. What is the percent of the count of people who are under 10 years old?
    Approximately 28.6%

15. What is the count of people who are 65 years old or older?
    Count of age groups from 65 – 69 to 100+ is 1,309,451 people.

16. What is the percent of the count of people who are 65 years old or older?
    Approximately 2.9%

17. Why is it important that the count and percent of people under 10 years old and 65 years old or older are given special attention?
    The people in these age groups are likely to need the most care.

18. "Old" and "young" are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of "young" as people less than 10 years old, and the definition of "old" as people who are 65 years old or older. What is the ratio of "old" to "young" using the above definitions of young and old? Derive a decimal from this ratio and interpret it by describing the approximate count of "old people" to the count of "young people." Express your answer to the nearest person.
    The "old" population is 1,309,451 people and the "young" population is 13,134,396 people. The ratio is 1,309,451 to 13,134,396. The decimal value is approximately 0.10. To answer the question indicating the number of old to young people, work with students in multiplying 0.10/1 by 10/10. The result is 1 over 10 or 1 old person to 10 young people.

19. If there are approximately 500 students in a typical school for students who are 5 to 14 years old, estimate the number of schools needed to educate the students who are 5 to 14 years old.
    Add the count of the 5- 9 years old and the count of the 10 – 14 years old, or:

    6,758,176 + 5,959,852 = 12,709,028 people. The estimate of the number of schools would be:
    12,709,028 people /500 students per school is approximately 25,418 schools.

Assessment Ideas

Assessment Task:

Consider the following assessment task to evaluate a student's understanding of the lesson. This task also provides an opportunity to evaluate whether or not the goals of this lesson are making sense to students.

Waldo, a small town of 1000 people, is located near a large city. Over the last 10 years, several people moved out of the city to Waldo to raise their children. Waldo has over 200 children in their elementary school and more than 100 students in their middle and high schools. The town is described as having a lower layered population distribution. Sketch a histogram of the people's ages in Waldo using the same design presented in this lesson. After you have made your sketch, answer the following:

1. Indicate why your sketch represents a lower middle-layered town.

2. Identify a 5-year age group in your sketch that you think will have the greatest percent of town's population. Explain your reasoning in selecting that age group.

3. Estimate the percent of students in Waldo's grade school and middle and high school?

Comments on the Assessment Task:
Evaluate students' work based on the definition of a lower layered population distribution. Given the counts of people in school, a sketch of the histogram of Waldo should represent most of the town in the younger age groups, or 0 – 4 years old, 5 – 9 years old, 10 – 14 years old, 15 – 19 years old, and 20 – 24 years old. As a lower layered population distribution, expect students to identify an age group in the lower layer to have the greatest percent of the town's population. Given the estimated counts of children in the elementary school (over 200 people), and the young people in the middle and high schools (over 100 people), students' estimate of the people in the 0 – 24 years old age groups would be at least 30% (or over 300 people in a town of 1000 people.

Additional Assessment Ideas:
Wrap up the discussion of this lesson by asking students or the small groups to identify at least one problem to summarize to the whole class. Ask each group to include in their summary why they think their answer is connected to the lower layer shape of Kenya. If time is limited and organizing a class discussion is not possible, direct students to write a couple of sentences explaining their summaries.

Wrap up the discussion with students by indicating that the age groups 30 – 34 years old and 35 – 39 years old are key groups of people in any country. People in this age group are making decisions about where to live, about whether or not to have children, and about how to make a living. Clearly, these questions need to be addressed by Raphine as he plans his future.

Teaching Notes

Lesson 3
Japan
An Upper Middle-Layered Country

Overview:

Similar to Lessons 1 and 2, the problems in this lesson summarize data from tables or graphs. Students derive and interpret decimals, percent, and ratios. The suggested classification of the problems using the **Modeling Continuum** is included in the following table. Similar to the previous two lessons, the classification of these problems is subject to interpretation by students or teachers. This lesson primarily asks students to summarize data provided in **Handout 3**. Students are asked to compare their summaries in this lesson to similar summaries for the United States and Kenya that they previously derived. In this way, the problems are beginning to move students thinking to higher levels.

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2, 13, 15, 17	Problems: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 20, 21		

Upon completion of the problems, consider directing students or small groups of students to identify one problem to discuss with the whole class, explaining to the class the solution to the problem and the steps used to solve the problem. Also consider assigning students to identify at least 2 problems to summarize on the **Exit Summary** handout. As previously stated, remind students to focus their classification of the selected problems using the descriptors of the ***Modeling Continuum.***

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, areas, ratios.
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

The data story, **Kristin and Hana's Story – Chapter 3**, begins the lesson. This story introduces new characters who are also identified in the problems. Students need **Handout 3: *Kenya – 2015*** to complete the problems. This lesson, as was the case for the previous two lessons, would benefit with visuals that were also suggested in Lesson 1, namely a map of the world, a globe, or a geographical app of the world.

Launch:
Similar to Lessons 1 and 2, start this lesson by finding out what students know about Japan. Where is Japan located on a map or globe? Is it a big country?

Direct students to examine the population pyramid graph and the histogram included on ***Handout 3 – Japan 2015***. The raggedness of this graph is something that stands out as different about Japan's graphs. The special features of Japan's graphs will be discussed in other lessons of this module.

After the initial review of the population graphs, provide time for students to read the data story, **Kristin and Hana's Story – Chapter 3.** Anticipate at least 3 minutes of reading time. After they have read the story, discuss the following questions:

- What age group counted Hana at the beginning of 2015?
- What is one of Hana's main concerns as she looks to the future?
- Does the population pyramid graph or population histogram provide any indication why Hana is concerned about caring for her mother and grandmother?

Identify the uneven population counts noticed in the population pyramid graph and the population histogram. Particularly point out to students the count of people 65 – 69 years old and the count of people 40 - 44 years old age. Highlight that these two age groups have similar counts of people. Also observe that the counts in these age counts are greater than the counts in other age groups of the country. Although this observation is also addressed in the problems, the counts in these age groups contribute to the unique shape of this country's population graphs.

You might consider asking students to explain their thinking about these unusual counts, especially when comparing Japan to Kenya and to the United States. Also, students might ask why is the 0 - 4 years old age group so small. If time is available, encourage students to independently research these questions. To move this lesson along, however, explain that the primary explanations for these counts are a result of changes in the birth rates and events such as wars (both of these factors were a major part of Japan's past).

Implementation Ideas:
Japan's population pyramid graph is quite different than the pyramid graphs of the United States and Kenya. As students work through the problems, consider displaying the 3 population pyramid graphs or the population histograms of the countries side-by-side using the handouts. Compare the different counts for selected age groups. For example, Kenya's pattern is larger counts for younger age groups and smaller counts for older age groups; the United States has a pattern of increasing, decreasing, increasing, and then decreasing counts of the age groups. Japan's pattern has several of the older age groups increasing in counts. The age groups that stand out in Japan's graph should also be compared to the people in the same age groups in Kenya and the United States. Consider comparing the percent of people in the age groups of 65

– 69 years old and 40 – 44 years old for each country. The differences in the percent of people in these age groups is part of the reason Japan's population is summarized as an upper middle-layered country.

As suggested in Lessons 1 and 2, consider arranging your students in small groups. Several of the problems involve calculations that could be more efficiently derived by sharing the work among students. The questions that ask for students to explain their answers are more interesting when discussed within small groups before discussing them with the entire class.

Student responses or descriptions

Lesson 1 - Problems

Handout needed to complete the following problems: Handout 3: *Japan – 2015*

1. What 5-year age group records the greatest count of people?
   Based on the table, the age group 65 – 69 years old have the most count of people.

2. Identify the age group that has the second highest count of people.
   Based on the table, the age group 40 – 44 years old has the second most count of people.

3. Identify the age group of people younger than 80 years old with the least count of people.
   Of the age groups less than 80 years old, the age group 0 – 4 has the least count of people.

4. The story indicated that Hana celebrated her 36th birthday in 2015. Identify on the population graph or the population histogram Hana's age group in 2015.
   a. What is the age group adjacent to Hana's age group that is younger? Is that age group "less than" or "greater than" that the count in Hana's age group?
      Hana is counted in the 35 – 39 years old age group. The count of people 30 to 34 years old is less than the count of 35 to 39 years old by approximately 800,000 people.
   b. What is the age group adjacent to Hana's age group that is older? Is that age group "less than" or "greater than" that the count in Hana's age group?
      The age group just older than Hana's age group is 40 – 44 years old. The count of people 40 to 44 years old is approximately 1.2 million greater than the count of 35 to 39 years old.
   c. Is the above summary of the age groups younger and older than Hana different than the summary of these same age groups in the United States? If yes, describe the difference. (See **Handout 1** to review the count of people in these age groups in the United States.)

In the United States the count of people 30 to 34 years old is greater than the count of 35 to 39 years old. Also, the count of people 40 to 44 years old was slightly less than the count of people 35 to 39 years old. The comparisons are not the same when comparing countries.

5. Estimate what layer (bottom, lower middle, upper middle, top) you think will have the least count of people? Explain how you made your estimate.
   The top layer as it has a sharp narrowing decline in the counts as the age groups get older.

6. Estimate what layer you think will have the greatest count of people? Explain how you made your estimate.
   Simply based on the graphs, it is not clear what layer of the two middle layers (lower middle-layer or upper middle-layer) has the greatest count of people. The sum of the counts is needed to determine which layer has the most count of people.

7. Use **Handout 3** to derive an estimate of the percent of the count of people who are 0 to 24 years old in Japan. (Round your answer to the nearest 10^{th} of a percent.)
   28,918,110/126,919,659 is equal to 0.228 or 22.8%.

8. In a similar way, what is the percent of the count of people 25 to 49 years old?
   40,240,247/126,919,659 is equal to 0.317 or 31.7%.

9. What is the percent of the count of people 50 to 74 years old?
   41,396,600/126,919,659 is approximately equal to 0.326 or 32.6%.

10. What is the percent of the count of people 75 to 100+ years old
    16,364,699/126,919,659 is approximately equal to 0.129 or 12.9%.

11. Based on the above definitions of a country's shape, what is the description of the shape of Japan's population graphs?
    *This country would be classified as an **upper middle-layered country** as the layer representing the count of people 50 – 75 years old has the greatest percent of the population.*

12. Identify two age groups in which the count of people in one age group is approximately double the count in the other age group.
    Answers vary. There are 6,151,388 people who are 15-19 years old. This is nearly double the count of people who are 85 – 89 years old. Also consider that there are

6,260,248 people in the age group 75 – 79 years old. This is nearly double the 3,171,849 people who are 85 – 89 years old.

13. Hana's data story indicates that she was concerned about her parent's future. In what way do the graphs indicate why she might be concerned about their future?

    Hana's parents age group, or age groups 65 – 69 or older, have a large count of people. People in these age groups may require special home or health care services in the future. Hana is unclear if there will be a sufficient number of people or resources to provide that help to a large percent of the country, especially given that the age groups who would be taking care of this group of people are a smaller percent of the country's populationsz.

14. Identify at least one age group that has approximately 1 million more females than males.
    Answers vary. The female count of the 80 – 84 years old age group has close to 1 million more females than males. Also, the 85 – 89 age group has approximately 1 million more females than males. There are other examples for this question.

15. Estimate the count of teenagers (13 to19 years old). Explain how you derived your estimate. (Estimates will vary.)
    Let the count of teenagers 13 and 14 years old be estimated as 2/5 of the people who are 10 – 14 years old. Therefore, an estimate of the count of teenagers is: (2/5) x (5,749,262) + 6,151,388 people or approximately 8,451,093 teenagers.

16. What is the count of people who are under 10 years old?

    0 – 4 years old are 5,272,998 people and 5 – 9 years old are 5,612,088 people for a total of 10,885,086 people.

17. What is the percent of people who are under 10 years old?
    10,885,086/126,919,659 is approximately equal to 0.086 or 8.6%.

18. What is the count of people who are 65 years old or older?
    Using the table, the count of 65 to 100+ years old is 33,750,203 people.

19. What is the percent of the count of people who are 65 years old or older?
    33,750,203/126,919,659 is approximately equal to 0.266 or 26.6%.

20. Why is it important that the count and percent of people under 10 years old and 65 years old or older are given special attention?
    The people in these age groups are likely to need the most care.

21. “Old” and “young” are subjective descriptions that in many cases are defined by several factors other than age (for example, health status, or income status). For this unit, however, consider the definition of “young” as people less than 10 years old, and the definition of “old” as people who are 65 years old or older. What is the ratio of “old” to “young” using the above definitions of young and old? Derive a decimal from this ratio and interpret it by describing the approximate count of “old people” to the count of one “young person.” Express your answer to the nearest person.
    33,750,203 to 10,885,086 is the ratio. As a decimal, this is 3.10. This indicates a slightly more than 3 old people to 1 young person.

22. If there are approximately 500 people in a typical special care facility designed for people 90 years old or older, how many facilities were possibly needed in 2015?
    There are 1,452,801 people 90 – 94 years old plus 441,584 people 95 – 99 years old plus 91,338 people 100+ years old for a total of 1,985,723 people. Divide this total by 500 people per special care facility. The results are approximately 3,971 facilities.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to evaluate a student’s understanding of this lesson. This task also provides an opportunity to evaluate whether or not the problems completed by your students result in an understanding of an upper middle-layered country.

Consider the following scenario. Your teacher described the population of a small town she or he visited as an upper middle-layered town. Answer the following questions:

1. Your teacher decided to have lunch in a restaurant of this upper middle-layered town. What age groups of people would you expect your teacher encountered the most and why?

2. Your teacher heard that the town's leadership team was holding talks about what they plan to do to help their residents. What issues do you think the residents mentioned the most in these talks?

3. If the population pyramid graph of this town looks very similar the population pyramid graph of Japan, would you expect to see people older than 90 years old in this town? Why

4. Do you think this town has a large percent of elementary school students (or students who would be in kindergarten to 5th grade)? Explain your answer.

Comments on the Assessment Task:
This task highlights the distinctive features of a population in which the older age groups have the greater counts of people. If the people eating at a restaurant are similar to the population distribution of Japan, you would expect most of the people in the restaurant are 65 – 69 years old, or 40 – 44 years old. It is anticipated that most students would summarize these people as "old". This summary is very different than what you would expect in Kenya or the United States.

Students might indicate that the most important topic the residents of the town want to discuss is related to the care or life style of older people. Whatever topic students identify, their reasoning should be connected to the distribution of the older age groups.

The counts of people 90 years old or older is noticeable in the population pyramid graph of Japan (approximately 2 million people are 90 years old or older out of approximately 127 million people, or approximately 1.5%). Although a small percent of the total population, people in these age groups are more noticeable in the graphs of an upper middle-layered country than in the graphs of the United States or Kenya.

Students assessment of whether or not the town has a large number of elementary students is subjective (what is large?). What is important is that students would highlight that if it was similar to Japan's shape, you would expect less than 10% of the population in the age groups of 0 – 4 years old and 5 – 9 years old. This is a smaller percent than what they noted for Kenya or the United States or countries with similar shapes to the United States or Kenya.

Additional Assessment Ideas:
After students complete the problems, consider asking students to discuss selected problems from the lesson. Several problems could be identified for discussion that indicate if students understood the lesson, and specifically how Japan is different than the United States or Kenya. Problem 20 returns to the question of what is the ratio of "old" to "young". Review with students that the same ratio represented approximately 1 older person to 10 younger people in Kenya, and approximately 1 older person to 1 younger person in the United States. The ratio in Japan is approximately 3 older people to 1 younger person. This ratio is an important summary of the population of Japan. Also consider using problems 13, 14, and 21 in assessing students understanding of the unique counts of Japan's age groups.

Teaching Notes
Lesson 4
The Center and Spread of a Country's Shape

Overview:

Describing and summarizing data are the primary goals of this lesson. Lessons 1, 2, and 3 introduced students to population data by focusing on the shape of the population pyramid graphs or the histograms. This lesson examines measures of center, specifically the median and mean ages, and the spread of the data as additional tools in understanding the population distributions of the United States, Kenya, and Japan. The centers and the spread work together in providing important summaries of the data. The spread developed in this lesson is a generalization of the Interquartile Range (or IQR) for grouped data and provides a description of how spread out the data is from the median. Lessons in Unit 2 will look back at the population distributions and use the centers and spread to summarize changes in each country's population distribution. Lessons in Unit 3 will analyze projected or future population data sets of each of the countries and will also summarize the changes by discussing the centers and the spread.

The topics addressed in this lesson might be familiar to students based on their previous work involving statistics. A different twist to these familiar statistical topics, however, is that the median age, the spread, and the mean age are derived for grouped data. A list of all of the data of the population is obviously not known and too massive, therefore, a summary of the data by age groups is provided. Deriving centers and spread based on working with grouped data might be new to most students.

A major challenge in completing this lesson is estimating the number of 50-minute class periods needed to complete the entire lesson. If students are familiar with these topics, anticipate 2 class periods. If these topics are relatively new to students, anticipate 3 to 4 class periods. Assigning students to work in small groups (especially when students are completing the tables) is again suggested and may decrease the time needed to complete the lesson. This lesson addresses student's analysis at the higher levels of the **Modeling Continuum**.

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 5, 6,	Problems: 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 17, 18, 19	Problems: 14, 15, 16, 20, 21, 22, 21, 24, 25	Problems: 12

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, mean of a data set, median of a data set, and spread of a data set.
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy, either electronic or printed, of a complete Lesson 4 for each student. This lesson does not require any additional handouts although it is also longer than most lessons. Consider providing students printed copies of the selected tables you direct them to complete. Students can answer several of the remaining problems using an electronic copy of the lesson.

Launch:

Begin this lesson by directing students to study the 1980 population histogram of the United States. After a minute or two of examining the graph, ask students questions similar to the following:

- What age group had the greatest count of people?
- What was the total population of the country in 1980?
- What description of the shape of the country would you estimate for this graph?

Discuss with students that often a population is described by a single value, generally identified as a center of the population. For example, ask students what age they might use to describe a typical person from the United States. Is 80 years old a good description? Why or why not? Is 20 years old a good description of a typical person? And again, why or why not? The focus of this lesson is to derive centers that are often used to describe a collection of data, along with a value called a spread that is also necessary to summarize the data.

After highlighting several of these type of questions, direct students to complete the problems. Monitor their responses. Provide opportunities to discuss with students their answers to the problems. Also provide opportunities to discuss problems as a class, especially if they are not initially clear to students.

Implementation Ideas:

Work through the steps outlined in the lesson to find the mean and median ages for a person in the United States. At the end of the problem set, students are asked to find the mean and median ages for Kenya and Japan. If students have access to scientific or graphing calculators, then organize students in small groups to complete the problems. Assign one group to find the mean and median ages for Kenya and the other group to find the mean and median ages for Japan. The process to derive each of these centers is organized for students in the tables that are part of the lesson.

Another possibility is to move the calculations to a spreadsheet or to a graphing calculator. The list options of calculators similar to the TI-84 allow for multiple calculations to be completed by the calculator. Similarly, multiple calculations can be completed with a spreadsheet. After students complete the calculations (either individually or in small groups), discuss as whole group the estimates of the mean and median ages for each country as directed in the problems.

Student responses or descriptions
Lesson 4 – Problems

1. Answer the following based on the 1980 and the 2015 population histograms.
   a. Do you think the population increased from 1980 to 2015? Explain your answer.
   Anticipate that students will indicate the population increased based on the increased area of the 2015 histogram. Students might quickly add up the values of the 2015 histogram and determine that they exceeded the total value included on 1980 histogram of 227.24 million people.
   b. What two age groups recorded the greatest count of people in 1980? What two age groups recorded the greatest count of people in 2015?
   The greatest counts in 1980 were the 20 – 24 years old age group (approximately 21 million people and the 15 – 19 years old age group (also approximately 21 million people). The greatest counts in 2015 are more challenging as there are several age groups that have similar counts for the first and second greatest count of people. The 20 – 24 years old age groups had the greatest count. The 25 – 29 years old age group had the second greatest count. Accept 50 – 54 years old, 55 – 59 years old, and 30 – 34 years old age groups as estimates of the second greatest count. Each of these age counts appear to have similar heights on the 2015 histogram. A table of the age group counts is needed to distinguish the counts.
   c. Identify the age group where a person 34 years old would be counted on the 1980 population histogram. Also identify the age group where a person 34 years old would be counted on the 2015 population histogram. Do you think the typical age of a person in 2015 is also 34 years old? If not, do you think the typical age will be older or younger? Explain your answer. Several of the problems in this lesson will develop an answer to these questions more precisely.

   The counts of the older age groups have increased. As a result, an estimate of a typical age would identify an age that is older in 2015 than in 1980.

Finding the Median Age of the United States Population in 2015

2. Are the ages of the people summarized in the histograms from 1980 and 2015 in order from youngest to oldest? Explain your answer.
   The ages are essentially in order by age groups. They could be considered a summary of the population in order.

3. Is it possible to determine how many people in the United States were exactly 34 years old? Explain your answer.

   Yes, ***if*** *it were possible obtain the ages of everyone in the United States. The task of finding everyone and counting everyone, however, is not likely. From that standpoint, the answer would be no.*

4. Answer the following questions based on the histogram of the 2015 population provided in this lesson.
   a. Do you think a person who is 20 years old would be an estimate of the median age? Why or why not
      No, as most of the area of the histogram is above (or older) than this estimate. Therefore, there would be more than 50% above this age.
   b. Do you think 40 years would be an estimate of the median age? Why or why not?
      This estimate is difficult to determine based on the area of the histogram. Visually, the areas above and below 40 years appear similar, so this age could be an estimate of the median age.
   c. Do you think 80 years would be an estimate of the median age? Why or why not?
      No, as most of the area of the histogram is below (or younger) than this estimate. Therefore, there would be more than 50% below this age.

Discuss with students as a class the following table. Point out the column headings and the values completed in the table. Particularly discuss the **Cumulative count of people** and how the values in that column are calculated.

United States - 2015

Age group	**Count** of people in each age group (in millions of people to the nearest hundredth)	**Cumulative count** of people (in millions of people to the nearest hundredth)	Proportion of cumulative count of people to the total population of the country (to the near thousandth)	Proportion as a percent (to the nearest tenth of a percent)
0 – 4	19.91	19.91	$\frac{19.91}{320.91} = 0.062$	6.2%
5 - 9	20.48	40.39	$\frac{40.39}{320.91} = 0.126$	12.6%
10 - 14	20.61	61.00	$\frac{61.00}{320.91} = 0.190$	19.0%
15 - 19	21.09	82.09	$\frac{82.09}{320.91} = 0.256$	25.6%
20 - 24	22.69	104.78	$\frac{104.78}{320.91} = 0.327$	32.7%
25 - 29	22.40	*127.18*	$\frac{127.18}{320.91} = 0.396$	*39.6%*
30 - 34	21.62	*148.80*	$\frac{148.80}{320.91} = 0.464$	46.4%
35 - 39	20.31	169.11	$\frac{169.11}{320.91} = 0.527$	52.7%
40 - 44	20.16	*189.27*	$\frac{189.27}{320.91} = 0.590$	*59.0%*

45 - 49	20.80	210.07	$\frac{210.07}{320.91} = 0.655$	65.5%
50 - 54	22.29	*232.36*	$\frac{232.36}{320.91} = 0.724$	72.4%
55 - 59	21.77	254.15	$\frac{254.13}{320.91} = 0.792$	79.2%
60 - 64	19.04	273.17	$\frac{273.17}{320.91} = 0.851$	85.1%
65 - 69	16.05	*289.22*	$\frac{289.22}{320.91} = 0.901$	*90.1%*
70 - 74	11.48	300.70	$\frac{300.70}{320.91} = 0.937$	93.7%
75 - 79	8.12	*308.82*	$\frac{308.82}{320.91} = 0.962$	*96.2%*
80 - 84	5.80	314.62	$\frac{314.62}{320.91} = 0.980$	98.0%
85 - 89	3.86	*318.48*	$\frac{318.48}{320.91} = 0.992$	*99.2%*
90 - 94	1.85	320.33	$\frac{320.33}{320.91} = 0.998$	99.8%
95 – 99	0.50	*320.83*	$\frac{320.83}{320.91} = 0.999$	99.9%
100+	0.08	**320.91**	$\frac{320.91}{320.91} = 1.000$	100.0%
Total	**320.91**			

5. What does each of the columns of the above table summarize? What does the cumulative count summarize? Answer each of the following questions based on the table to indicate your understanding of each of the columns.
   a. How many people are younger than 5 years old?
      19.91 million people
   b. What is the percent of people younger than 5 years old?
      6.2%
   c. How many people are younger than 10 years old?
      40.39 million people
   d. What is the percent of people younger than 10 years old?
      12.6%
   e. What is the percent of people younger than 20 years old?
      25.6%

f. How many people are younger than 50 years old?

210.07 million people

g. What is the percent of people younger than 50 years old?

65.5%

6. There are several blank cells in the table. For each blank cell, complete the expected calculations.

   See the above completed table.

7. What is the first age group that captures at least 50% of the cumulative population?

   The 35 – 39 years captures 52.7% of the cumulative population.

8. Estimate the age group in which the median age is located.

   The estimate of the median age would be within the age group of 35 - 39 years age. This age group captures at least 50% of the population.

9. Is it possible to identify the exact age within the age group identified in problem 8 that would be the median age (or the age where at most 50% of the population would be less than this age)? Explain your answer.

   As we do not know the specific ages (just a grouping of the ages as indicated by age group), we cannot find an exact age.

10. Using the above description of spread and the completed table, answer the following:
    a. What is the first age group that captures 75% of the cumulative population?
    The age group 55 – 59 years captures 79.2% of the cumulative population. It is the first age group that has at least 75% of the cumulative population within that age group.
    b. What is the first age group that captures 25% of the cumulative population?
    The age group 15 – 19 years captures 25.6%. It is the first age group that has at least 25% of the cumulative population within that age group.
    c. Calculate the spread of the population based on the difference in ages of the two age groups in 10(a) and 10(b) using the oldest age from the age group that captures 75% of the cumulative population and the youngest age from the age group that captures 25% of the cumulative population. Approximately what percent of the total population is captured between the two ages used to calculate the spread?
    At least 50% of the cumulative population is captured between 15 – 19 years old and 55-59 years old. The difference of 59 years and 15 years is 44 years. This difference is an estimate of the spread of the population. The actual spread based on the goal of finding where 50% of the population is less than 44 years.

d. Why is the last cell in the column representing the proportion as a percent equal to 100.0%?
The last cell is the percent based on the proportion of the entire count of people or the entire population. Therefore, the resulting proportion is 1.00 and the percent is 100%.

11. Describe the typical person in 2015. How does this person differ from the 1980 typical person?
The median age is estimated to be between 35 – 39 years old, which is slightly higher than the median age reported in 1980 of 34 years old. The spread of approximately 44 years is slightly higher than the reported spread in 1980 which was estimated at 39 years (the difference of 54 years and 15 years). The greater value of the spread, however, is based on the age groups which are overestimating where 50% of the cumulative population would be counted. As a result, the estimate of 44 years is greater than the actual spread; therefore, the spread for 2015 is similar to the spread reported for 1980.

12. Sketch a histogram of a country with the same median age as the United States in 2015 but with a spread that is one-half of the value derived for the United States. Also sketch an approximate box plot of this country using the following grid:
A sketch of the population histogram would be "pushed" more closely toward the estimate of the median age. The age group where at least 75% of the cumulative population is captured would probably be younger than the age group for the United States. Similarly, the age group where at least 25% of the cumulative population is captured would probably be older than the age group for the United States. It is likely there would be a noticeable build-up of the population around the age group containing the median age.

Finding the Mean Age of the United States Population in 2015

Topics for Discussion:

• Mean as a Balance Point

Point out to students that the mean is an important summary of a population. Although often summarized as a simple "average" of data, it is a value that also indicates a **balance point** of the data. If students do not understand the significance of the mean as a balance point, take a few minutes to share an example of this description. Tape coins to a light-weight ruler. Possibly start off by taping a stack of 5 pennies at the end of a 12-inch ruler (position equals 12 inches) and 10 pennies taped together at the other end of the ruler (position equals 0 inches). Place a pencil under the ruler and try to identify the position that balances the ruler. The ruler should approximately balance at the position of 4 inches. Indicate to students that 4 is the mean or: (10 x 0 + 5 x 12)/15. A frequent clarification that might be needed is that the mean for this example is derived by dividing the sum of the positions for ***each*** of the 15 pennies (ten 0's and five 12's) by the total number of pennies (15 pennies).

In addition, point out that the distance of the mean to the position 12 is 8 inches. The sum of the distances from the mean of 4 inches to the 5 pennies taped at the position 12 inches is 5 x 8 inches or 40 inches to the left of the mean. The distance of the mean to the position 0 is 4 inches. The sum of the distances from the mean of 4 inches to the 10 pennies taped at the position 0 is 10 x 4 inches or 40 inches to the right of the mean. The mean is a special summary in which the sum of the distances to the right and the sum of the distances to the left are equal. This is what makes the mean a balance point. (A resource for additional examples and lessons can be found on the ASA website and the **Data-Driven Module Exploring Centers**, or: https://www.amstat.org/asa/files/pdfs/ddmseries/ExploringCenters--TeachersEdition.pdf)

• Mid-Intervals as an Estimate

The table uses the mid-interval age of an age group as an estimate of all of the ages in an age group. This estimate is then used to calculate the sum of all of the ages in that age group. This technique will likely not result in an exact value of the sum of the ages, but given that the exact values are not known, it is considered a reasonable estimate. If further discussion about why this might result in a reasonable estimate, consider providing a specific example. For example, if you have 5 people with the ages of 1, 1, 2, 3, and 4, the sum of the ages is 11 years. If you use the mid-interval value of 0 – 4 years old, or 2 years as estimate of each person's age, the sum of the ages would be 10 years. Although not exact, it is close. There could, however, be examples in which the estimate is not close. For example, what if the ages of the 5 people were 0, 0, 0, 0, 1. The sum of the five ages is 1 which is considerably less than 10 years. The main point to discuss with students is that with a large count of people in each age group, the mid-interval age is a reasonable estimate that is larger than the ages that are smaller and smaller than the ages that are larger, and therefore, it should be a good estimate of an age for deriving the sum of the ages in that age group.

United States – 2015
Results for finding the mean age of the United States

Age group	Mid-interval Age	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	19.91	2 X 19.91 = 39.82
5 - 9	7	20.48	7 X 20.48 = 143.36
10 - 14	12	20.61	12 X 20.61 = 247.32
15 - 19	17	21.09	17 X 21.09 = 358.53
20 - 24	*22*	22.69	*22 X 22.69 = 499.18*
25 - 29	27	22.40	27 X 22.40 = 604.8
30 - 34	32	21.62	32 X 21.62 = 691.84
35 - 39	37	20.31	*37 X 20.31 = 751.47*
40 - 44	42	20.16	42 X 20.16 = 846.72
45 - 49	*47*	20.80	47 X 20.80 = 977.6
50 - 54	52	22.29	52 X 22.29 = 1159.08
55 - 59	57	21.77	57 X 21.77 = 1240.89
60 - 64	62	19.04	*62 X 19.04 = 1180.48*
65 - 69	*67*	16.05	*67 X 16.05 = 1075.35*
70 - 74	*72*	11.48	72 X 11.48 = 826.56
75 - 79	77	8.12	77 X 8.12 = 625.24
80 - 84	82	5.80	82 X 5.80 = 475.6
85 - 89	87	3.86	*87 X 3.86 = 335.82*
90 – 94	*92*	1.85	*92 X 1.85 = 170.2*
95 – 99	97	0.50	97 X 0.50 = 48.5
100+	102	0.08	102 X 0.08 = 8.16
	Total	**320.91**	**12306.52**

13. Explain how the mid-interval ages were determined for each age group.

    The mid-interval ages for each age group can be found by adding the endpoints and dividing by 2. Also, the mid-interval age for an age group of 5 years is the age within the group that has two ages above it and two ages below it.

14. Do you think all of the 19.91 million people who were 0 – 4 years old are 2 years old? Why might, however, the mid-interval age of 2 years be a reasonable estimate of the age of each of the children in the 0 – 4 years old age group?

    Clearly all of the people within the age group are not 2 years old, however, it is a good estimate of the age of each person within that interval as there are likely a similar count of people above 2 years as there are people below 2 years.

15. To determine the mean age, the sum of the ages in each age group is needed. Consider the age group 0 – 4 years old. If 2 years is a good estimate of the age of all of the people in that age group, what does the product of 2 and 19.91 represent?

    As indicated, 2 years is a good estimate of the age of each person within that age group. As a result, to find the sum of all of the ages in that age group, the product of 2 and the count of people in that age group, or 19.91, is a good estimate.

16. In the same way, what does the product of 7 and 20.48 represent?

    The age of 7 years is a good estimate of the age of each person within the age group of 5 – 9 years old. As a result, the sum of all of the ages in that age group could be estimated by the product of 7 and the count of people within the age group.

17. Analyze what is missing in each of the blank cells of the above table. For each blank cell, complete the expected calculations.

    See the completed cells in the table.

18. The last column of the above table represents an estimate of the sum of the ages for each age group. To determine the mean, the sum of all of the ages for 320.91 million people is needed. Based on the estimates recorded for each age group, 12306.52 million is the approximate sum of all of the age groups in that column. Describe the last step needed to calculate an estimate of the mean age of a person in the United States.
    The last step needed is to divide the sum of the all of the ages by the total population of the country, or divide 12306.52 million years by 321.91 million people.

19. What is an estimate of the mean age of a person in the United States?
    Divide the sum of the ages (12306.53 million) by the number of people or 320.91 million. The mean age is approximately 38.35 years old. This estimate is similar to what the Census Bureau estimated as the mean age for 2015.

20. Do you think the estimated mean age is a good description of a typical person in 2015? Explain why or why not.
    Mean age is 38.35 years. This estimate is similar to what the Census Bureau estimates for 2015 and could be used to describe a typical person.

21. The estimate of the mean age in 2015 is greater than the estimate for 1980. What does this indicate about the change in the population during this time?

    The mean age is greater as the count of older people in the 2015 population is greater. Compare the 1980 and 2015 histograms and observe the higher bars (or greater counts) for ages 40 and older.

Tables are provided for calculating the median and mean values for Kenya and Japan. The tables are designed so that the calculations of the mean and median ages are completed in the same way as the calculations for mean and median ages were done for the United States. Consider organizing this part of the lesson in small groups, with each group and each member of the group completing a portion of the calculations. Also consider assigning some groups to derive the median age, the spread, and the mean age for Kenya, and assigning different groups to derive the median age, the spread, and the mean age for Japan.

Kenya 2015

Results for finding the median age of Kenya:

Age group	Count of people (in millions of people)	**Cumulative count** of people (in millions)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	6.38	*6.38*	*0.138*	*13.8%*
5 - 9	6.76	*13.14*	*0.286*	*28.6%*
10 - 14	5.95	*19.09*	*0.416*	*41.6%*
15 - 19	4.49	*23.58*	*0.513*	*51.3%*
20 - 24	4.08	*27.66*	*0.602*	*60.2%*
25 - 29	3.92	*31.58*	*0.686*	*68.6%*
30 - 34	3.60	*35.18*	*0.766*	*76.6%*
35 - 39	2.89	*38.07*	*0.829*	*83.9%*
40 - 44	2.01	*40.08*	*0.873*	*87.3%*
45 - 49	1.55	*41.63*	*0.906*	*90.6%*
50 - 54	1.25	*42.88*	*0.934*	*93.4%*
55 - 59	0.98	*43.86*	*0.955*	*95.5%*
60 - 64	0.75	*44.61*	*0.971*	*97.1%*
65 - 69	0.53	*45.14*	*0.983*	*98.3%*
70 - 74	0.36	*45.50*	*0.991*	*99.1%*
75 - 79	0.23	*45.73*	*0.996*	*99.6%*
80 - 84	0.12	*45.85*	*0.998*	*99.8%*
85 - 89	0.05	*45.90*	*0.999*	*99.9%*
90 - 94	0.01	*45.91*	*1.000*	*100.0%*
95 – 99	0.01	*45.92*	*1.000*	*100.0%*
100+	0.01	*45.93*	*1.000*	*100.0%*

22. Derive an estimate of the median age group for Kenya and a description of the spread based on the definition of spread in this lesson.

The median age would be within the age group of 15 – 19 years as it captures 50% of the cumulative population. The age group 5 - 9 captures 25% of the cumulative population and the age group 30 – 34 captures 75% of the cumulative population. The spread as defined in this lesson would be the difference of 34 years and 5 years, or approximately 29 years. The median age and spread is less than the United States indicating a younger population.

Kenya – 2015
Results for finding the mean age of Kenya:

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	6.38	*12.76*
5 - 9	7	6.76	*47.32*
10 - 14	12	5.95	*71.40*
15 - 19	17	4.49	*76.33*
20 - 24	22	4.08	*89.76*
25 - 29	27	3.92	*105.84*
30 - 34	32	3.60	*115.20*
35 - 39	37	2.89	*106.93*
40 - 44	42	2.01	*84.42*
45 - 49	47	1.55	*72.85*
50 - 54	52	1.25	*65.00*
55 - 59	57	0.98	*55.86*
60 - 64	62	0.75	*46.5*
65 - 69	67	0.53	*35.51*
70 - 74	72	0.36	*25.92*
75 - 79	77	0.23	*17.71*
80 - 84	82	0.12	*9.84*
85 - 89	87	0.05	*4.35*
90 – 94	92	0.01	*0.92*
95 – 99	97	0.01	*0.97*
100+	102	0.01	*1.02*
	Total	45.93	*1046.41*

23. Derive an estimate of the mean age of Kenya. Compare the estimated mean age to the median age. Are they similar? Explain.

The mean age is 1046.91 million years divided by 45.93 million people, or approximately 22.78 years. The mean age is pulled by the older age groups resulting in a higher estimate of the age of a typical person than the median age. Remind students that an estimate of the mean age is more impacted by the older age groups. The difference between the estimates for the mean age of 22.78 years and median ages for Kenya is large.

Japan 2015

Results for finding the median age of Japan:

Age group	Count of people (in millions of people)	**Cumulative count** of people (in millions)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4	5.27	*5.27*	*0.042*	*4.2%*
5 - 9	5.61	*10.88*	*0.086*	*8.6%*
10 - 14	5.75	*16.63*	*0.131*	*13.1%*
15 - 19	6.15	*22.78*	*0.179*	*17.9%*
20 - 24	6.13	*28.91*	*0.228*	*22.8%*
25 - 29	6.54	*35.45*	*0.279*	*27.9%*
30 - 34	7.47	*42.92*	*0.338*	*33.8%*
35 - 39	8.27	*51.19*	*0.403*	*40.3%*
40 - 44	9.50	*60.69*	*0.478*	*47.8%*
45 - 49	8.46	*69.15*	*0.545*	*54.5%*
50 - 54	7.82	*76.97*	*0.606*	*60.6%*
55 - 59	7.57	*88.54*	*0.698*	*69.8%*
60 - 64	8.62	*98.46*	*0.776*	*77.6%*
65 - 69	9.57	*102.73*	*0.809*	*80.9%*
70 - 74	7.82	*110.55*	*0.871*	*87.1%*
75 - 79	6.26	*116.81*	*0.920*	*92.0%*
80 - 84	4.95	*121.76*	*0.959*	*95.9%*
85 - 89	3.17	*124.93*	*0.984*	*98.4%*
90 - 94	1.45	*126.38*	*0.996*	*99.6%*
95 – 99	0.44	*126.82*	*0.999*	*99.9%*
100+	0.09	*126.91*	*1.000*	*100.0%*
Total	**126.91**			

24. Derive an estimate of the median age group for Japan and a description of the spread based on the definition of spread in this lesson.

The median age would be within the age group of 45- 49 years as it captures 50%of the cumulative population. The age group 25 -29 captures 25% of the cumulative population and the age group 60 – 64 captures 75% of the cumulative population. The spread as defined in this lesson would be the difference of 64 years and 25 years, or approximately 39 years. The median age is greater than the estimate of the median range for the United States, with a similar spread. This estimate indicates that a typical person in Japan is older than the typical person in the United States.

Japan – 2015
Results for finding the mean age of Japan:

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2	5.27	*10.54*
5 - 9	7	5.61	*39.27*
10 - 14	12	5.75	*69.00*
15 - 19	17	6.15	*104.55*
20 - 24	22	6.13	*134.86*
25 - 29	27	6.54	*176.58*
30 - 34	32	7.47	*239.04*
35 - 39	37	8.27	*305.99*
40 - 44	42	9.50	*399.00*
45 - 49	47	8.46	*397.62*
50 - 54	52	7.82	*406.64*
55 - 59	57	7.57	*431.49*
60 - 64	62	8.62	*534.44*
65 - 69	67	9.57	*641.19*
70 - 74	72	7.82	*563.04*
75 - 79	77	6.26	*482.02*
80 - 84	82	4.95	*405.90*
85 - 89	87	3.17	*275.79*
90 – 94	92	1.45	*133.40*
95 – 99	97	0.44	*42.68*
100+	102	0.09	*9.18*
	Total	**126.91**	*5802.22*

25. Derive an estimate of the mean age of Japan. Compare Japan's estimate of the mean age to the median age. Are they similar? Explain.

    The mean age is 5802.22 millions of years divided by 126.91 millions of people. This is approximately equal to 45.72 years old. This is also within the age group of the median age.

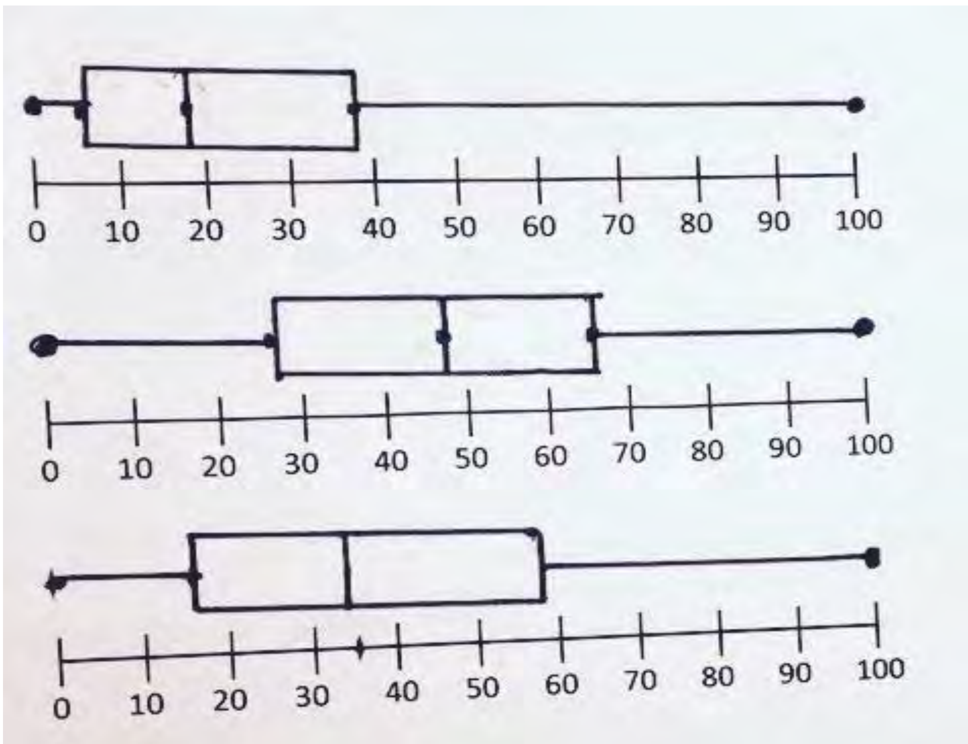
Consider discussing the following with students: The estimated mean and median ages for a typical person in the United States are similar ages. This observation is also true for a typical person in Japan. The median for Kenya, however, is within the age group 15 – 19 years old, and the mean age was estimated to be 22.78 years old. The difference in the estimated values of median and mean for Kenya is one of the larger differences of median and mean ages for countries of the world. Recall that the mean is a balance point (a type of fulcrum). Although there are not many people in the older age groups in Kenya, the older people result in a "pull" on the location of the mean age. The mean and median ages calculated in this lesson for each of the countries are very similar to what is reported by the United States Census Bureau. The Census Bureau emphasizes the median age in its reports as the median is considered a more appropriate estimate for skewed distribution.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to evaluate a student's understanding of the lesson.

The following 3 box plots were sketched by a student completing this lesson. One box plot represents the country of Kenya, one box plot represents the country of Japan, and one box plot represents the country of the United States. Unfortunately, the student forgot to write a label of the country for each box plot. Study the box plots and identify the correct country that is represented by each box plot.



Comments on the Assessment Task:
The top box plot is Kenya. The second box plot is Japan and the bottom box plot is the United States.

Students connect their understanding of the 5-number summaries to the box plots . The box plots also provide visuals that highlight the differences of each country's population distribution and their different shapes. Consider aligning the box plots to the population histograms studied in Lessons 1, 2, and 3.

Highlight a few of the summaries that a comparison of the box plots point out. For example, discuss how more than 50% of the population in Japan is older than 25% of the population in Kenya. Also discuss that Q1 for the United States is approximately equal to the median age of Kenya.

Additional Assessment Ideas:
This lesson has multiple questions and problems that serve as formative assessments. The last set of problems (problems 22 to 25) provide opportunities to assess a student's understanding of centers and spread. Students use the guided work of estimating the centers and spread for the United States to derive centers and spread for Kenya and Japan. These problems also extend a student's thinking to levels 3 and 4 of the **Modeling Continuum**. Consider conducting a whole class discussion on what role students' think the centers and spread play in analyzing what are the challenges a country might face. Are the challenges faced by the 3 countries studied in this module similar or different? Do the data and the estimates of center and spread suggest the challenges faced by these countries?

Teaching Notes
Lesson 5
My Country

In this lesson students design their own population distribution. Directions to students should be minimal. The intent of this lesson is to wrap-up Unit 1 by providing an opportunity to demonstrate the 3rd level of the **Modeling Continuum**. If an example is needed to help students understand the expectations, consider discussing the design and analysis of the data provided for a fictitious country identified as My Country in this lesson.

Launch this lesson by discussing the 3 scenarios described in the student lesson. Encourage students to select one of the scenarios. Students can also create, however, a distribution that does not fit one of these scenarios. For students that select to create their own distribution, ask them to indicate what challenges or opportunities they think their distribution reflects.

Student responses or descriptions
Lesson 5 – Problems

1. After a discussion addressing the scenarios, students may intentionally design a country that is bottom-layered (similar to Kenya), or lower middle-layered (similar to the United States), or upper middle-layered (similar to Japan), or design a country representing a completely different shape.

   Students may simply enter age group estimates without specifically thinking about a shape. For now, that is not a problem, and may even provide some insights when students incorporate their country's population in various projection models to estimate the projected population in 2050. Encourage students, however, to think about selecting one of the scenarios if they have no particular interest in the type of country they are creating.

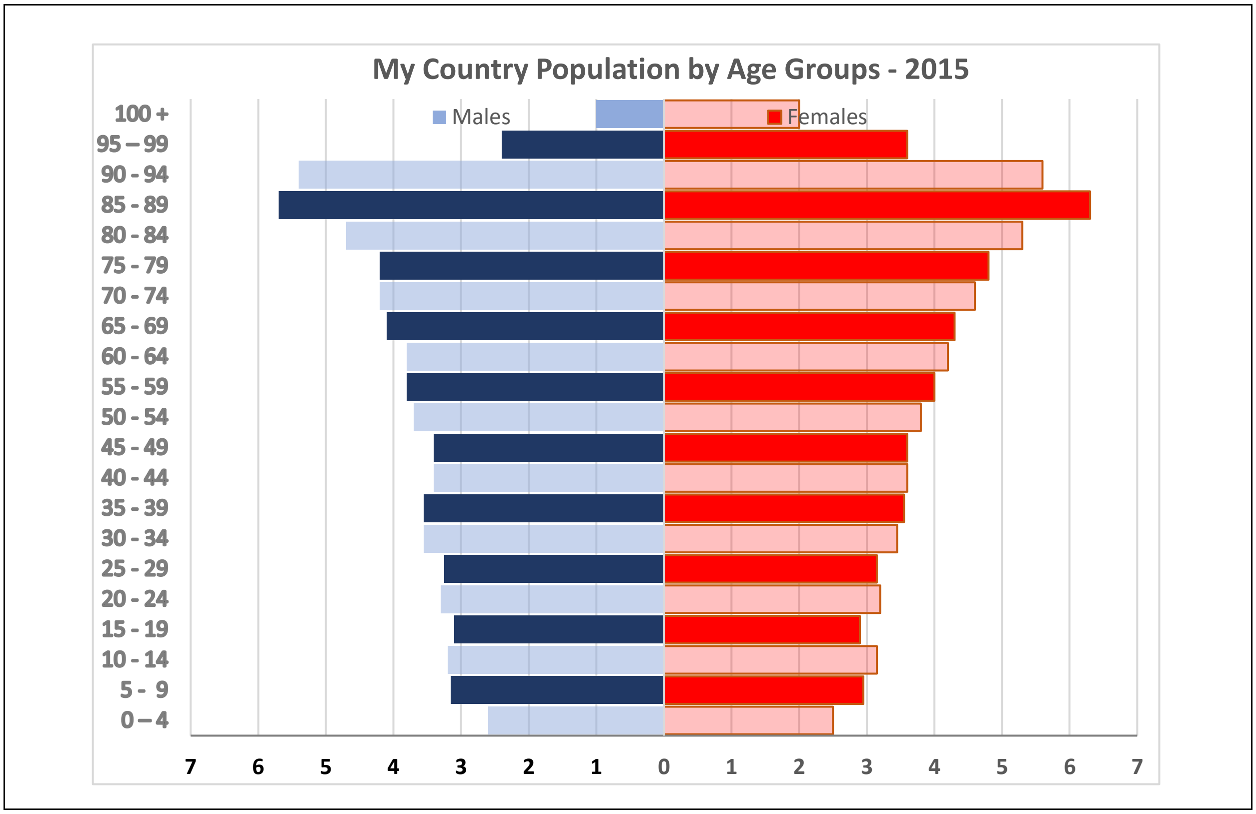
   A few other notes to consider regarding this lesson. As indicated, students may design a country that will pose unusual challenges later in this module. For example, if students create a country with all males or all females, the model does not alter estimates of the future. An improved model would take into account models in which the proportion of males and females is considered. Improving the model to include other factors of change demonstrates students' thinking at Levels 3 and 4 of the **Modeling Continuum**.

For students not confident of the directions in this lesson, share with them the following population distribution. This distribution represents a top-layered country. Provide students a copy of this distribution (printed or electronic or both) as you may want to include this country in the projection models of later lessons. (A copy of this distribution is provided as the last page of this lesson for distribution to students if this example is used.)

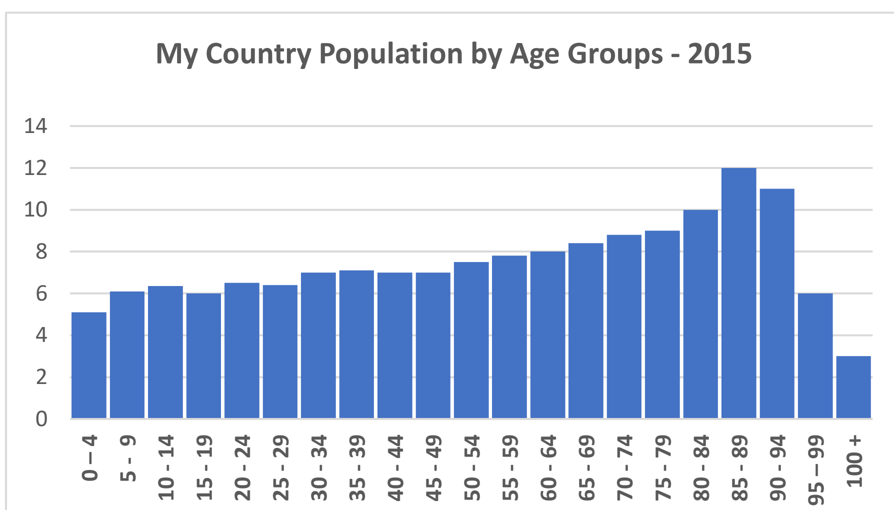
Country: **My Country** in 2015

Age Group	Males	Females	Total
0 - 4	2.60	2.50	5.10
5 - 9	3.15	2.95	6.10
10 - 14	3.20	3.15	6.35
15 - 19	3.10	2.90	6.00
20 - 24	3.30	3.20	6.50
25 - 29	3.25	3.15	6.40
30 - 34	3.55	3.45	7.00
35 - 39	3.55	3.55	7.10
40 - 44	3.40	3.60	7.50
45 - 49	3.40	3.60	7.00
50 – 54	3.70	3.80	7.50
55 - 59	3.80	4.00	7.80
60 - 64	3.80	4.20	8.00
65 - 69	4.10	4.30	8.40
70 - 74	4.20	4.60	8.80
75 - 79	4.20	4.80	9.00
80 - 84	4.70	5.30	10.00
85 - 89	5.70	6.30	12.00
90 - 94	5.40	5.60	11.00
95 – 99	2.40	3.60	6.00
100+	1.00	2.00	3.00
	75.50	78.05	156.55

2. Based on the table of counts in each age group, use the following template to complete a pyramid graph of **My Country**:
   A population pyramid graph of the top-layered country provided above is included as an example of the expectations of this problem.



3. Create a histogram of My Country.
   The following histogram is based on the age groups of the fictitious country provided with this lesson.



Age Groups
Total Population is 156.55 millions of people
(based on the data of a top-layered fictitious country)

4. Determine if your country is bottom-layered, lower middle-layered, upper middle-layered, or top-layered or a combination of these descriptions.
   If the discussion involving the My Country data is conducted, the population pyramid graph and the histogram indicate a top-layered country with approximately 32.6% of the population 75 years old or older.

5. Identify any features of your country you want to watch as you look into the future.
   Consider sharing with students that a major question for a top-layered country is what happens in the future when many of the people are counted in the older age groups. Can a country like this survive? In time, it probably will "fade away".

6. Use the templates provided to calculate estimates of the median age, the spread as defined in Lesson 4, and the mean age for My Country.
   Median age:
   Spread of ages from approximately 25% to 75% of the population:
   Mean age:
   If the My Country example is used, the following summaries describe the population of the My Country example:
   Median age is located in the 45 – 49 years old age count.
   Mean age is approximately 55.33 years (approximately 55 years old)
   Spread: Approximately 50% of the population is between the age groups 20 – 24 years old and 65 – 69 years old resulting in a spread of approximately 49 years.

7. Describe a typical person in your country.
   Students are expected to answer this problem by using the ages derived in problem 6 to describe a person in their country.

As an extension to this lesson, encourage students to research a country using the International Data Base (IDB) available at the United States Census Bureau website (https://www.census.gov/programs-surveys/international-programs/about/idb.html). At this website, navigate through the available options including selecting a country, the type of data summary (population pyramid graph or 5-year age intervals), and the year of interest. Over 200 countries are summarized at this site. Especially interesting shapes can be found for the 2015 population for Saudi Arabia, Cuba, Brazil, Italy, Spain, and Mexico.

Attached are templates for estimating the mean and the median ages of a student's country.

My Country – 2015
Template for finding the estimate of the mean age:

Age group	**Mid-interval Age**	Count of people (in millions of people)	Sum of ages in age group: (Estimated in millions of years)
0 – 4	2		
5 - 9	7		
10 - 14	12		
15 - 19	17		
20 - 24	22		
25 - 29	27		
30 - 34	32		
35 - 39	37		
40 - 44	42		
45 - 49	47		
50 - 54	52		
55 - 59	57		
60 - 64	62		
65 - 69	67		
70 - 74	72		
75 - 79	77		
80 - 84	82		
85 - 89	87		
90 – 94	92		
95 – 99	97		
100+	102		
	Total		

My Country 2015

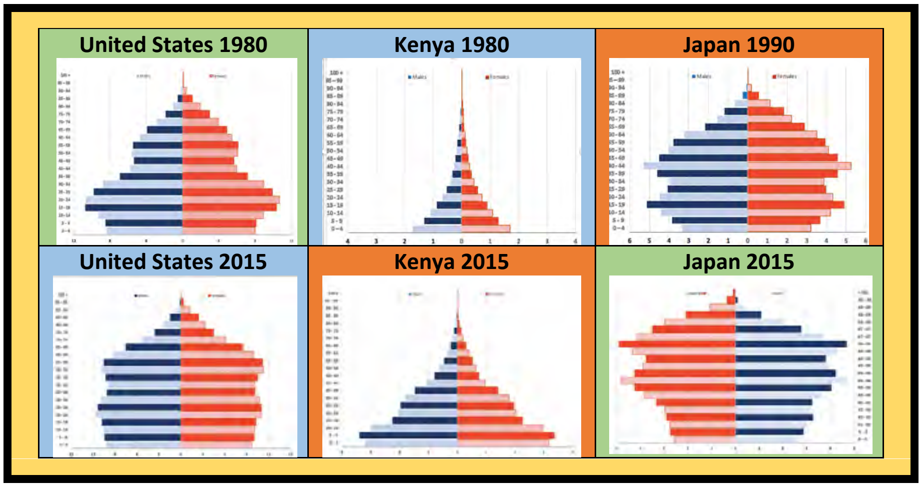
Template for finding the estimate of the median age:

Age group	Count of people (in millions of people)	**Cumulative coun**t of people (in millions)	Proportion of cumulative count of people to total population (to the near thousandth)	Proportion as a percent
0 – 4				
5 - 9				
10 - 14				
15 - 19				
20 - 24				
25 - 29				
30 - 34				
35 - 39				
40 - 44				
45 - 49				
50 - 54				
55 - 59				
60 - 64				
65 - 69				
70 - 74				
75 - 79				
80 - 84				
85 - 89				
90 - 94				
95 – 99				
100+				
Total				

Fictitious Country: **My Country** in 2015

Age Group	Males	Females	Total
0 - 4	2.60	2.50	5.10
5 - 9	3.15	2.95	6.10
10 - 14	3.20	3.15	6.35
15 - 19	3.10	2.90	6.00
20 - 24	3.30	3.20	6.50
25 - 29	3.25	3.15	6.40
30 - 34	3.55	3.45	7.00
35 - 39	3.55	3.55	7.10
40 - 44	3.40	3.60	7.50
45 - 49	3.40	3.60	7.00
50 – 54	3.70	3.80	7.50
55 - 59	3.80	4.00	7.80
60 - 64	3.80	4.20	8.00
65 - 69	4.10	4.30	8.40
70 - 74	4.20	4.60	8.80
75 - 79	4.20	4.80	9.00
80 - 84	4.70	5.30	10.00
85 - 89	5.70	6.30	12.00
90 - 94	5.40	5.60	11.00
95 – 99	2.40	3.60	6.00
100+	1.00	2.00	3.00
	75.50	78.05	156.55

Teaching Notes
Unit 2
Looking Back



Introduction

Kristin, Raphine and Hana are part of the same age group. While Raphine finds most of the people in his country younger and Hana finds most of the people older, Kristin finds most of the people older or younger. Each of them belongs to an age group that plays a major role in telling the stories of their countries. Although each of them was 36 years old at the start of 2015, the population pyramid graphs of their countries tell different stories over time.

The United States, Kenya, and Japan experienced events that resulted in their unique shapes. Natural disasters such as hurricanes, earthquakes, volcanic eruptions, or unusual drought or flood conditions resulted in a loss of life that reshaped each population distribution for decades. Other events, such as major wars, also decreased each country's population and reshaped the population distribution reflected in the population pyramid graphs. In the same way, periods of relative peace and positive economic conditions increase a country's population that is also evident in a country's past and current shape.

"Looking Back" is more than just a reflection of how things have changed. It is identifying those events that resulted in increasing or decreasing the population in ways that are evident in each country's past and current shape. This short unit looks back at each of our countries through the pyramid graphs and the histograms that tell us where the generation that includes Kristin, Raphine, and Hana began or made their impact.

Teaching Notes

Lesson 6

Looking Back at the Shapes of the United States

Overview:

This lesson looks back at the United States population using pyramid graphs and histograms from 1980 to 2015. Many current issues and challenges are a direct result of generational differences regarding lifestyle choices, entertainment choices, use of technology, and financial priorities. The generational labels most commonly used in this lesson (Baby Boom Generation, Generation X, Millennials) are traced back in time using the population graphs. Students interpret census data involving past and current counts to speculate why a generational study is important in understanding the demographic make-up of the United States.

Percentages of the various generations are derived over time that uncover factors that explain a change in a population distribution (for example, immigration and death). As students uncover the factors of change, carefully monitor class discussion. Explaining these factors (particularly those that summarize immigration and death) are mature topics that some students may be uncomfortable discussing in class. They are also topics, however, that result in students discovering mathematical tools that are needed in constructing models that look into the future.

This lesson poses several ideas that complement the goal of looking back by analyzing the count of selected age groups. Consider ways for your students to communicate with people of different generations. One extension that was implemented involved an activity of interviewing parents or grandparents or guardians. Students should provide you details before they conduct the interview so you can give them feedback. Some extensions that were also implemented involved students watching old television shows or reading portions of older novels that were popular in the 1950s or 1960s as ways to understand the lifestyle and values of Baby Boomers. Here again, however, a cautionary note is given. Some old television shows resulted in negative feedback regarding racial and gender issues. For some students and classes this is not a recommended extension as the topics were uncomfortable for students to discuss or share in their class. For other students or classes, however, this added to the value of what students were learning about the background of different generations. This activity also provided an opportunity to discuss with family members and friends important topics that have changed over time. If this extension is conducted, be aware of the topics that might be discussed, and monitor these discussions closely.

An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems:	Problems: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 18	Problems: 12, 13, 14, 15, 16, 19	Problems: 17

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, extracting and interpreting data from graphs
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of the **Introduction** to Unit 2 and a complete Lesson 6 for each student. This lesson does not require any additional handouts.

Launch:

Begin by reading and discussing the **Introduction** to Unit 2. Discuss with students that the statement "The History of America's Future" (also the title of a book by Strauss and Howe that is referenced in the lesson) is a summary of this unit. Mathematics will be used to uncover the past and analyze the future.

After a brief discussion of the introduction, direct students to individually study the 1980 and the 2015 pyramid graphs and histograms that are used to begin the lesson. Ask students to comment on what is different about the shapes of the graphs. Also ask them to think about identifying people counted in 1980 and also counted in 2015. What part of the population of 2015 includes people counted in 1980 and what part of the population in 2015 are new people? The problems in the lesson address these differences in more detail, however, it is likely that an opening discussion will set the stage for understanding the problems that ask students to study the changing counts, shapes, and demographics of the United States over time.

Implementation Ideas:

The differences in the counts of people connected to various age groups are emphasized in the problems, with some age groups having a more dominant count and percent than others. Students connect the counts with generational labels that are frequently used in analyzing the United States population. These labels (specifically the Silent Generation, the Baby Boom Generation, Generation X, and the Millennial Generation) are connected to periods of time in which these generations were born. Encourage students to conduct some research regarding generations. Please monitor their resources. There are significant summaries from polls, surveys, economic data, and health care data that use these labels as a way to explain differences or similarities of the various generations. Also consider assigning students a topic mentioned in the lesson (for example, musical preferences or movies) and research the preferences cited by various age groups or generations.

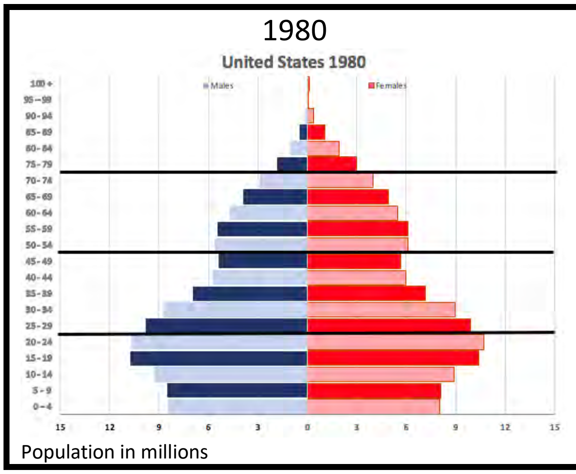
Extension

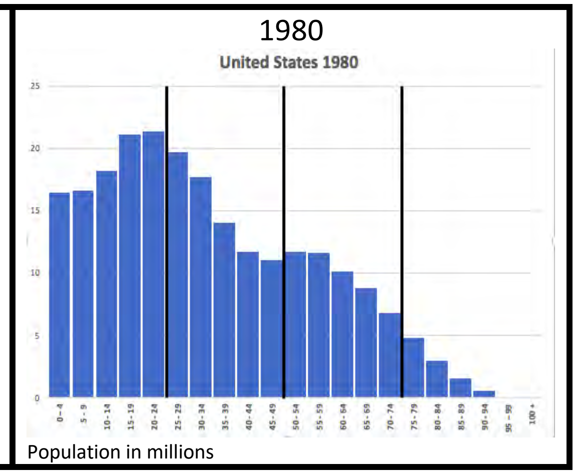
Consider the following extension to the research mentioned above. As a class, design a short survey that students use to collect data from their parents or guardians and other family members of different ages. Questions on the survey might include preferences in music, movies, games or entertainment. In addition, survey questions might ask people to indicate who they consider as the most influential people in the country (or state or city or town), what daily activities are important, or what problems are considered the most critical in the country. Include on the survey a question to determine the age or an age group of the person completing the survey. Organize a short time period in which the surveys are given out so that there is some consistency in the responses collected. Use the data as a way to reflection on what are possible factors that impact people from different generations.

Discuss with students that the results from the surveys, however, do NOT necessarily reflect characteristics of a generation due to challenges of obtaining a representative sample. Students should not report what they compiled from the surveys or interviews as a statistical summary of generations but rather as a type of class project or case study. Students may, however, learn insights that a statistical study would need to consider. Young and emerging adults (or students completing this extension) valued the reflections received from people of other generations.

Student responses or descriptions
Lesson 6 - Problems

1. Lesson 1 defined the age group layers (bottom-layered, lower middle-layered, upper middle-layered, and top-layered) for the 2015 pyramid graph or histogram. Based on the percent of people in each layer, the United States was identified as a lower middle-layered country in 2015. Identify the layers in the 1980 pyramid graph and the 1980 histogram similar to the way they are identified in the 2015 graphs.
Students identify the 0 to 24 years old age groups as the bottom layer, 25 to 49 years old age groups as the lower middle-layer, 50 to 74 years old age groups as the upper middle-layer, and the 75 to 100+ years old age groups as the top layer.

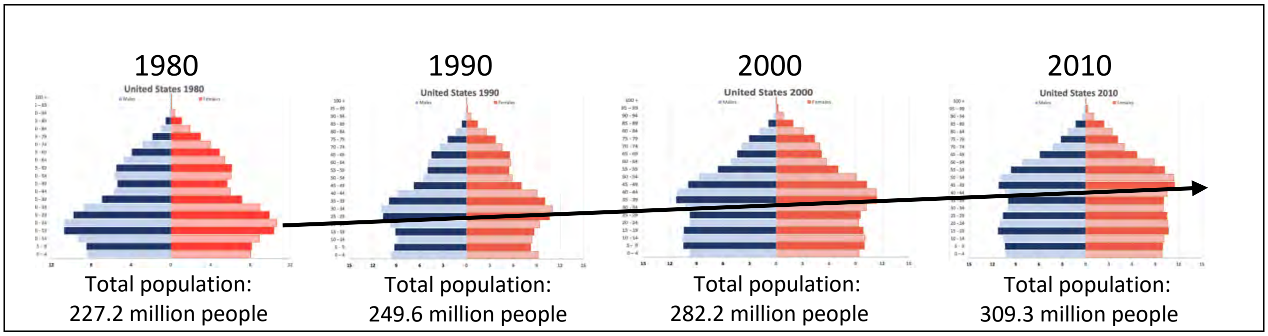


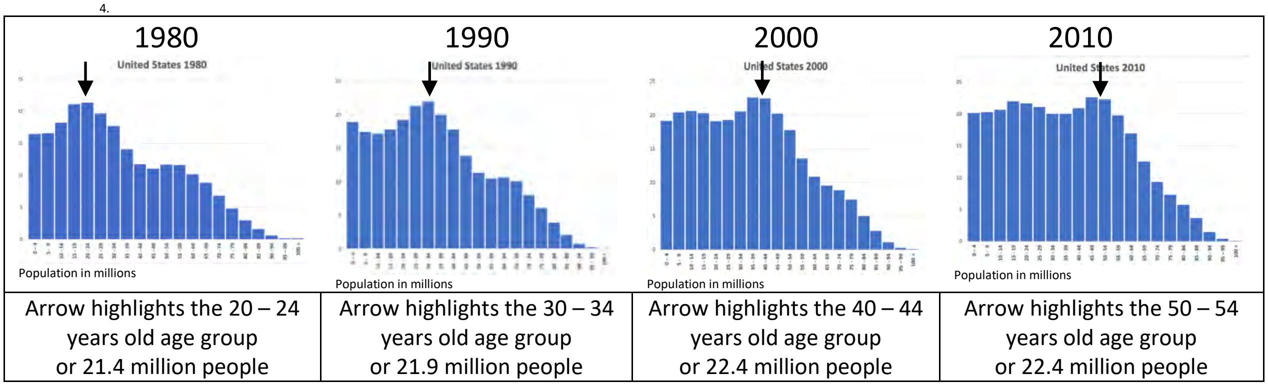


2. Based on the layered marks, what identification would summarize the shape of the country in the year 1980? Which graph, the pyramid graph or the histogram, did you use to make your identification? Explain why you selected the graph you identified.
The 1980 population is bottom-layered. (Allow students to estimate this identification based on the area of the graph that is marked off in that layer. The percent of the population in the bottom layer is 41.3% of the population.)

Either the pyramid graph or the histogram could be identified to answer the question about the change in the shape. The scale for the 1980 and the 2015 pyramid graph are the same, so the change in the area of the graphs and the changes in the length (or counts) of corresponding age groups can be used to compare overall changes between 2015 and 1980. The scales are also the same for the histograms. Most students selected the histogram when field testing this problem, but their reasons were focused more on the appearance and not the scales. This question is more important in the next lesson as the scales of the pyramid graphs are not the same, thus, comparing graphs is more challenging.

3. The change in the shape from 1980 to 2015 is highlighted by observing the aging of certain age groups over two to three decades. The arrows sketched on the following pyramid graphs and histograms trace the aging of people who were 20 – 24 years old at the start of 1980 through three decades. What is significant about this age group in 1980?





The significance is that the age group highlighted in 1980 was the largest age group in the 1980 graphs. As the people in this age group grew older, the shape of the country also changed as the lower middle layer increased in the count of people.

4. Based on the above timeline, the people 20 - 24 years old in 1980 would be counted in the 30 – 34 years old age group of the 1990 Census if they did not move out of the country or die. What is significant about the count of the 30 – 34 years old age group in 1990?
   The significance is again that the count of people in the 30 – 34 years old age group in 1990 was the largest age group in the graphs. Highlight that during the ten years from 1980 to 1990, the people counted as 20 – 24 years old in 1980 and then counted as 30 – 34 years old in 1990 remain as the most dominant age group in the country. Also note that the count of 30 – 34 years old in 1990 is greater than the count of 20 – 24 years old in 1980. More people were added to that group over the ten years. The only way that count could increase is immigration – a factor that will be part of the projection models developed in Unit 3.

5. Observe that the people 20 – 24 years old in 1980 are counted in the 40 – 44 years old age group in the 2000 census if they did not move out of the country or they did not die. Did the count of people in the 40 – 44 years old age group in 2000 increase, decrease, or stay the same when compared to the count of people in the 20 – 24 years old age group in 1980? Explain what happened during the 20 years from 1980 to 2000 that might increase or decrease the count of people 20 – 24 years old in 1980 to the count of people 40 – 44 years old in 2000.
The count of people increased for that 5-year age group of people from 1980 to 2000 by approximately 1 million people. The increase was a result of more people moving into the country than people dying or moving out of the country.

6. Calculate the following based on the above pyramid graphs and histograms highlighted in this lesson:
   a. What percent of the population in 1980 were 20 – 24 years old?
      21.4 / 227.2 = 0.0.94 or 9.4%
   b. What percent of the population in 1990 were 30 – 34 years old?
      21.9 / 249.6 = 0.088 or 8.8%
   c. What percent of the population in 2000 were 40 – 44 years old?
      22.4 / 282.2 = 0.079 or 7.9%
   d. What percent of the population in 2010 were 50 – 54 years old?
      22.4 / 309.3 = 0.072 or 7.2%
   e. The count of people in 2015 who were 55 – 59 years old was 21.8 million people. What percent of the population in 2015 were 55 – 59 years old?
      21.8 / 320.9 – 0.068 or approximately 6.8%

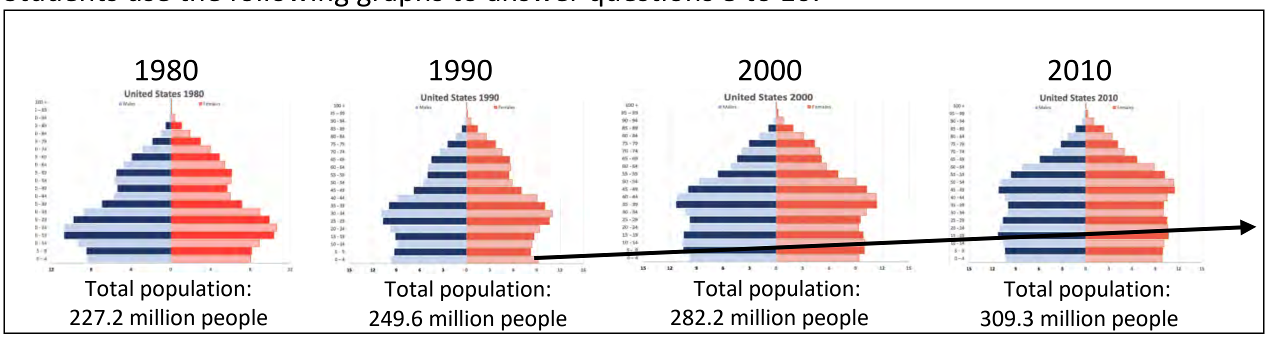
7. Recall that for this module, the count of people in an age group is based on ages ***at the start of the year***. Therefore, a person born in 1943 was counted as a one-year old at the start of the 1945 United States Census count, although this person turned two years old sometime in 1945. If this person continued to live in the United States, he or she was six years at the start of 1950 and turned seven sometime in 1950. Based on that interpretation of the age groups represented in the graphs, answer the following:
   a. What was the age of a person whose birth year is 1943 at the start of 2015?
      There are several ways students discovered how to calculate the age. One method was to subtract the two given years, and then subtract 1 from the answer addressing the fact that the person did not have a birthday in the last year (in this case, 2015). For this question:
      2015 – 1943 = 72. Subtract 1 as this person will turn 72 sometime in 2015, the age would be 71 years at the start of 2015.

   b. What was the age of a person whose birth year is 1960 at the start of 2015?
      2015 – 1960 = 55. Subtract 1 or 54 years.

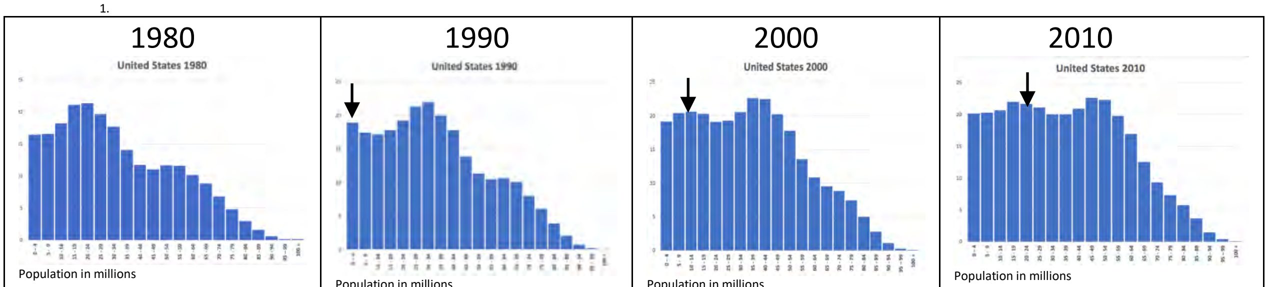
c. What age groups in the 2015 population graphs would include most of the people born in 1943 to 1960?
The oldest age group of 70 – 74 years to the youngest age group of 50 – 54 years old. Students may reason that the youngest group is 55 – 59 as there is only 1 group in the 50 – 54 years old, or the people 54 years old, who would belong to the Baby Boom generation as identified in this lesson. As the years identifying generations are debated among researchers, also allow that answer to identify the aging Baby Boom Generation.

d. What do the population graphs from 1980 to 2010 indicate is happening to the Baby Boom Generation?
The counts and the percent of this group is declining. They still indicate a major count of people, but the percent in these age groups is declining due to death or possibly people leaving the country.

Students use the following graphs to answer questions 8 to 10.



1.



	Arrow highlights the 0 – 4 years old age group or 18.8 million people	Arrow highlights the 10 – 14 years old age group or 20.6 million people	Arrow highlights the 20 – 24 years old age group or 21.7 million people

8. What is significant about the 0 – 4 years old age group in 1990?
The count of 0 – 4 years old is greater than the count of several of the older age groups, specifically the 5 – 9 years old, 10 – 14 years old, and the 15 – 19 years old age group.

9. People 0 – 4 years old at the start of 1990 (or born in 1985 to 1989) would be 20 - 24 years old at the start of 2010 if they remained in the country or did not die. Did the count of people in the 0 – 4 years old age group in 1990 increase, decrease, or stay the same when compared to the count of people in the 20 – 24 years old age group in 2010? Explain what might increase or decrease the count of the people who were 0 – 4 years old in 1990 and then 20 – 24 years old in 2010.

The count of people increased nearly 3 million during that time. This is again explained by immigration. Some people during that time died or moved out of the country (which decreased the count). The overall increase is explained by more people within the ages of this age group moving into the country.

10. Calculate the following based on the pyramid graphs or the histograms:
    a. What percent of the population is 0 – 4 years old in 1990?
    18.8 / 249.6 = 0.075 or 7.5%
    b. What is the percent of the population 10 – 14 years old in 2000?
    20.6 / 282.2 = 0.073 or 7.3%
    c. What is the percent of the population 20 – 24 years old in 2010?
    21.7 / 309.3 = 0.070 or 7%

11. Kristin was directed by the administrators at the health clinic to write a report highlighting the changes of the United States population by ages from 1980 to 2015. Which one of the following summaries would be appropriate for her report based on the descriptions of the Baby Boom Generation and the Millennial Generation in the previous problems? Explain why you think your selection is the most accurate.

    a. The Baby Boom Generation is growing in numbers.

    b. The Millennial Generation is catching up to the Baby Boom Generation.

    c. The Millennial Generation is decreasing in numbers.

    d. The percent of the United States population in 2010 who are considered the Baby Boom Generation and the Millennial Generation is less than most other age groups.

    The answer is b. The Baby Boom Generation is showing a leveling off of the counts during that time. The Millennial Generation is still increasing in counts. The percent of the primary age groups of the Baby Boom Generation is also decreasing (as well as the percent of the primary age groups of the Millennial Generation), but the decreases indicate that in 2015 the percentage of the primary age group of the Baby Boom Generation is less that the primary age group of the Millennial Generation highlighted the histograms and pyramid graphs.

12. Why are these two generations (Baby Boom Generation and the Millennial Generation) important in understanding the United States in the following areas:
    a. entertainment watched on TV or cable or streaming services?
    A major generational divide is frequently reported regarding how people receive their entertainment. At the time of the writing of this module, mostly older people (the Baby Boom Generation) prefer cable and/or regular TV. The younger generations (the Millennials) prefer streaming services. These preferences, however, are constantly changing.

b. musical preferences?

Musical preferences are often summarized as generational preferences (this is an opinion by many and may or may not be true). As a result, the impact of these generations on the musical business is significant.

c. movies?

Similar to the answer to b, the impact of these generations on the movie industry is significant due to the greater count of people in these age groups.

d. health care?

The Baby Boom Generation is reaching the ages in which health care is a major concern. Millennials at this time are likely not as concerned (as research indicates that the majority of the Millennials are still at the age in which they are generally healthy). The tension between these two groups over this issue in 2015 was significant.

13. Let us return to the characters of the data stories in this module. Complete the following table (Kristin has been completed for you):

	Age at start of 2015	Birth year	Age at start of 1980	Generation
Kristin	36	1978	***1***	Generation X
Abbey (Kristin's sister)	26	***1988***	*Was not born yet.*	***Millennial***
Kristin's mother	66	***1948***	***31***	***Baby Boomer***

Observe the answers in the above table.

14. Answer the following:

a. In what age group did Kristin's mother belong in the 1980 population histogram?

Kristin's mother belonged to the 30 – 34 years old age group in 1980.

b. In what age group did Kristin belong in the 1980 population histogram?

Kristin belonged to the 0 – 4 years old age group in 1980.

15. Use the 2015 population histogram to answer the following:

a. Identify the three age groups immediately younger than Kristin's age group in 2015. Are the counts in these age groups greater than or less than the count in Kristin's age group?

The counts in the 2015 population pyramid graph or the 2015 histogram of the 3 age groups immediately younger than Kristin are 30 – 34 years old, 25 – 29 years old, and 20 – 24 years old. Each of these age groups has a greater count than the count in Kristin's age group. The count of 20 – 24 years old is greater than the count of 25 – 29 years old. And, the count of 25 – 29 years old is greater than the count of 30 -34 years old.

b. Identify the three age groups immediately younger than Kristin's mother's age group in 2015. Are the counts in these age groups greater than or less than the count in her mother's age group?
The counts in the 2015 population pyramid graph or the 2015 histogram of the 3 age groups immediately younger than Kristin's mother are 60 – 64 years old, 55 – 59 years old, and 50 – 54 years old. Each of these age groups is also greater than the count in her mother's age group. The count of 50 – 54 years old is greater than the count of people 55 – 59 years old. And, the count of 55 – 59 years old is greater than the count 60 – 64 years old.

16. In what age group was Abbey counted in 2015?. Identify the three age groups immediately younger than Abbey's age group in 2015. Are the counts in these age groups greater than or less than the count in Abbey's age group?
Abbey was counted in the 25 – 29 years old age group in 2015. The age group 20 – 24 years old is greater in count than her age group. However, the counts in the 15 – 29 years old and the 10 – 14 years old are less in count than her age group.

17. Kristin's mother was 30 years old when Kristin was born and 40 years old when Abbey was born. Members of the Baby Boom Generation generally had children at an older age than people in the generations older than the Baby Boom Generation. In what way did the decision of Baby Boomers to have children at an older age possibly impact the counts of people in age groups that included Kristin's age group and 3 age groups immediately younger than Kristin's age group?
The decision to have children at an older age may have contributed to the increase in the counts in the age groups younger than Kristin's age group as her mother represents one of the older members of the Baby Boomers.

18. Determine the percent change in the age groups from 1980 to 2015 by completing the following table (round off the percent increases to the nearest tenth of a percent): *(Consider assigning completion of this table in small groups.)*

Age Group	1980 Population	2015 Population	Percent change from 1980 to 2015
0-4	16,451,184	19,912,499	(19,912,499 – 16,451,184)/16,451,184 0.210 = 21.0%
5-9	16,602,353	20,481,130	(20,481,130 - 16,602,353)/16,602,353 0.234 = 23.4%
10-14	18,236,335	20,605,579	*(20,605,579 - 18,236,335)/18,236,335* *0.130 = 13.0%*
15-19	21,110,940	21,084,710	*(21,084,710 - 21,110,940)/21,110,940* *-0.0012 = - 0.12%*
20-24	21,385,705	22,693,026	(22,693,026 - 21,385,705)/21,385,705 .061 = 6.1%
25-29	19,685,966	22,401,168	*(22,401,168 - 19,685,966)/19,685,966* *0.138 = 13.8%*
30-34	17,742,706	21,617,533	*(21,617,533 - 17,742,706)/17,742,706* *0.218 = 21.8%*
35-39	14,076,734	20,312,646	(20,312,646 - 14,076,734)/14,076,734 0.443 = 44.3%
40-44	11,728,497	20,156,736	*(20,156,736 - 11,728,497)/11,728,497* *0.719 = 71.9%*
45-50	11,048,040	20,801,156	*(20,801,156 - 11,048,040)/11,048,040* *0.883 = 88.3%*
50-54	11,694,715	22,289,734	*(22,289,734 - 11,694,715)/11,694,715* *0.906 = 90.6%*
55-59	11,611,382	21,767,855	(21,767,855 - 11,611,382)/11,611,382 0.875 = 87.5%
60-64	10,142,668	19,038,554	(19,038,554 - 10,142,668)/ 10,142,668 0.877 = 87.7%
65-69	8,809,479	16,049,246	*(16,049,246 - 8,809,479)/8,809,479* *0.822 = 82.2%*
70-74	6,841,235	11,477,776	(11,477,776 - 6,841,235)/6,841,235 0.678 = 67.8%
75-79	4,829,832	8,119,847	*(8,119,847 - 4,829,832)/4,829,832* *0.681 = 68.1%*
80-84	2,955,279	5,798,910	*(5,798,910 - 2,955,279)/2,955,279* *0.962 = 96.2%*
85-89	1,580,234	3,864,289	(3,864,289 - 1,580,234)/1,580,234 1.445 =144.5%
90 - 94	557,241	1,851,620	*(1,851,620 - 557,241)/557,241* *2.3228 = 232.28%*
95 – 99	119,057	495,362	(495,362 - 119,057)/119,057 = 316.07%
100+	15,099	77,242	*(77,242 - 15,099)/15,099* *4.1157 = 411.57%*
Totals	227,224,681	320,896,618	*(320,896,618 - 227,224,681)/ 227,224,681* *0.4122 = 41.22%*

Note: After students complete the blank cells of the table, discuss with them a summary of the percent changes. Students should observe that in most cases there is an increase in the percent change of older age groups. The table indicates how the country grew over this period of time. It also indicates how the country grew older.

19. Identify age groups that you would like to follow as projection estimates are derived in the following lessons. Why are you interested in these age groups?
    This problem is optional. A student might want to continue to watch an age group that is part of one of the identified generation labels (Baby Boomers, Generation X, Millennial) to see how the counts change over time.

Assessment Ideas:

Consider an assessment that has students write a 3 or 4-sentence summary on what happened to an age group from 1980 to 2015. In this summary, students should highlight the changes in the percent of this age group, changes in the count and percent of age groups above and below this age group, and changes in the shape of the country over this period of time. If a student highlights an age group that was dominant in 1980, does this age group remain dominant in 2015? Or, if a student highlights an age group that was not as dominant in count in 1980 or other decades, does its impact change over the decades?

Teaching Notes
Lesson 7
Looking Back at the Shapes of Kenya and Japan and My Country

Overview:

This lesson looks back at the population of Kenya and Japan in a similar way to the study of the United States in Lesson 6. However, the pyramid graphs and the histograms of these countries tell different stories. The problems in the lesson ask questions that link the graphs and the initial summaries derived in Lessons 2 and 3 to past graphs and summaries. This lesson concludes with students observing the factors of immigration, death, and birth rates on the total population and key age groups highlighted in this lesson.

An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 4, 9, 10	Problems: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	Problems: 11, 12, 13, 14, 15	Problems:

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, extracting and interpreting data from graphs. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 7 for each student. This lesson does not require any additional handouts.

Launch:

Begin this lesson with a whole group discussion of the pyramid graphs and the histograms for Kenya and Japan of 2015. Consider questions similar to the following:

- Describe the different shapes, total population, and the counts of selected age groups for Kenya and Japan.
- What is the count of the 0 – 4 years old age group for each country in 2015? What is the percent of the 0 – 4 years old age group in each country?

After an opening discussion of the two countries, direct students to compare Kenya's population in 1980 to the population in 2015 using the pyramid graphs.

- What do the graphs tell us about the overall changes in Kenya's population from 1980 to 2015?
- Did the country grow, and did any age group significantly increase or decrease in count?

In the same way, highlight the population pyramid graphs and histograms of Japan and ask the same questions. Kenya and Japan have different shapes, and different summaries of the count and percent of the highlighted age groups. How do we know if an age group increased in population over time? What features indicate these changes? For example, is there a change in the scale of the graph that indicates the counts in an age group changed, or is there a change in the area of the graphs? After an opening discussion, direct students to complete the problems.

Implementation Ideas:

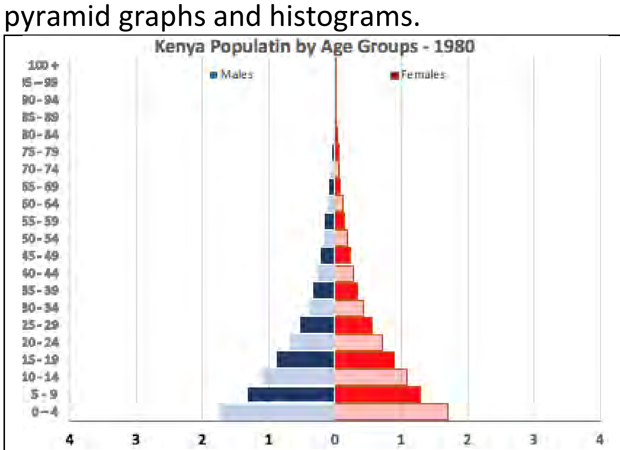
After students complete the problems, discuss selected students' answers as a whole group. Students might be encouraged to research what events happened in Kenya or Japan that would explain the increase or decrease of the count of people over time. This research may result in discussions of other issues (economic conditions, diseases, wars) that you may not wish to discuss as a class, but these factors provide some background to the key events that altered each country's population. The comparison of each country's population to the United States population is an indication why these 3 countries were selected and highlighted in this module. Each country has a unique shape, a changing shape over time, and a different set of challenges over time.

Student responses or descriptions

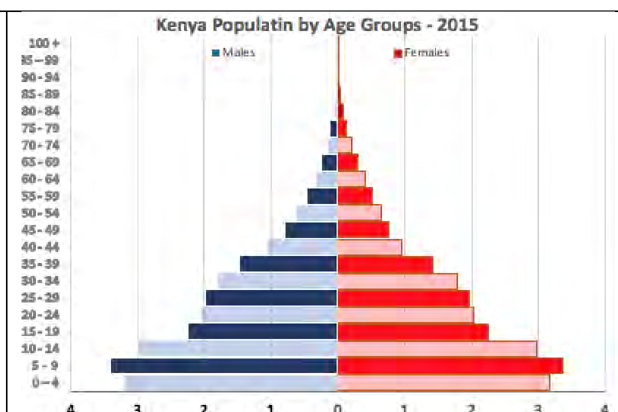
Lesson 7 - Problems

Kenya

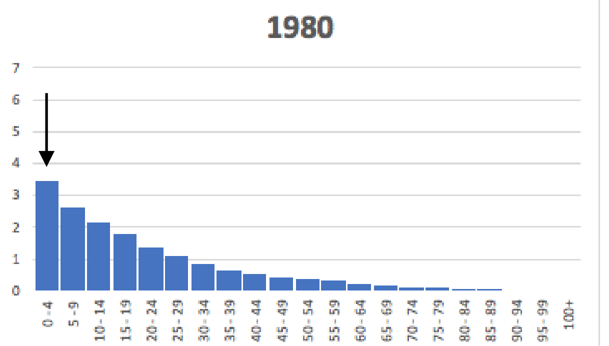
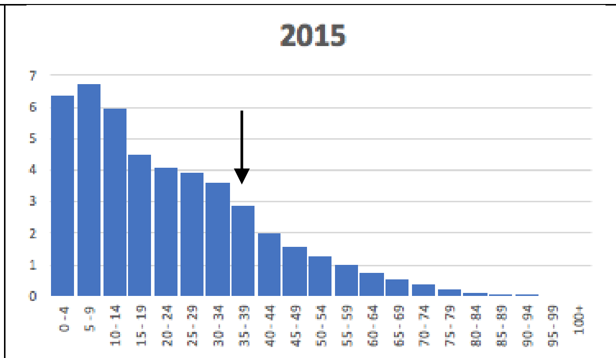
The 1980 population and the 2015 population are summarized by the following population pyramid graphs and histograms.



Total population: 16.33 million people

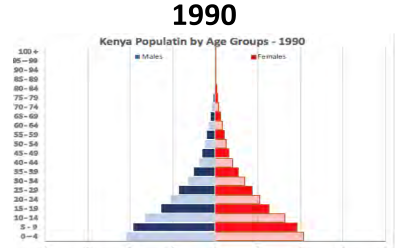
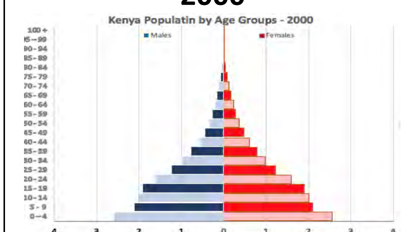
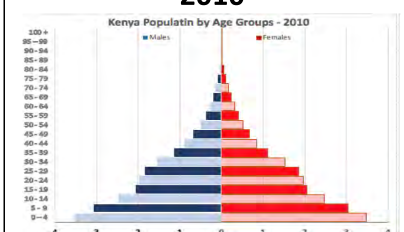


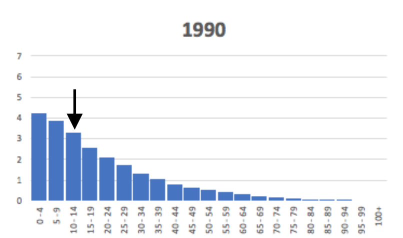
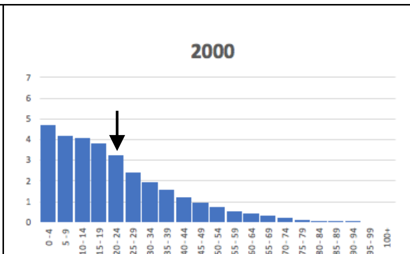
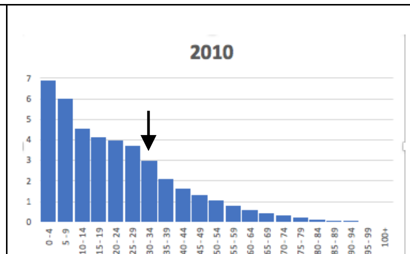
Total population: 45.93 million people

 Population in millions	 Population in millions
Arrow highlights the 0 – 4 years old age group or 3.48 million people	Arrow highlights the 35 – 39 years old age group or 2.89 million people

The population pyramid graphs and histograms of the intervening years of 1990, 2000, and 2010 are also provided. Note the changes in the shape of the population pyramid graphs and histograms.

Journey through the intervening years:

Kenya **1990**	**Kenya** **2000**	**Kenya** **2010**
		
Total population: 23.36 million people	Total population: 30.61 million people	Total population: 40.84 million people

 Population in millions	 Population in millions	 Population in millions
Arrow highlights the 10 – 14 years old age group or 3.28 million people	Arrow highlights the 20 – 24 years old age group or 3.22 million people	Arrow highlights the 30 – 34 years old age group or 3.00 million people

1. The information provided above indicates that the total population of Kenya increased from 16.33 million people in 1980 to 45.93 million people in 2015.
   a. Calculate the percent increase of the population from 1980 to 2015.
      Calculate (45.93 – 16.33)/16.33 which is approximately 1.81 or 181%.

b. Describe how the graphs (either the population pyramid graph or the population histogram) also indicate an increase of the population from 1980 to 2015.
The area of the 2015 histogram is noticeable greater than the 1980 histogram. It is important to indicate that the histograms were used to explain an increase in area as an explanation of an increase in the population. The histograms maintained the same scale for the count of people. The scale for the population pyramid graphs changed, and therefore, the change in area is not as accurate an indication of the change in the population.

2. The 0 – 4 years old in 1980 are highlighted by an arrow in the 1980 histogram. If the people in the 0 – 4 years old age group did not move to another country or did not die in the next 35 years, they were also counted in the 35 – 39 years old age group in 2015. An arrow is used to also identify the 35 -39 years old age group in 2015.
   a. What count and percent of the people in Kenya were 0 – 4 years old at the beginning of 1980?
   The count of 0 – 4 years old is 3.48 millions of people. The percent of 0 – 4 years old is approximately (3.48/16.33) x 100 or 21.3%.

   b. What count and percent of the people in Kenya were 35 – 39 years old at the beginning of 2015?
   The count of 35 – 39 years old is 2.89 millions of people. The percent of 35 – 39 years old is approximately (2.89/45.93) x 100 or 6.3%.

   c. Explain what factors contributed to the decrease in the population of the age group 0 – 4 years old in 1980 to the 35 – 39 years old in 2015 while the total population of Kenya increased.
   The decrease in the count of people from 3.48 millions of people to 2.89 millions of people is primarily explained by people moving out of the country or dying. The increase in the total population is primarily explained by an increase in the number of births.

3. Answer the following questions using the population graphs and the information provided about the total population and the population within highlighted age groups:
   a. In what age group in the 1990 population were people counted who were 0 – 4 years at the start of 1980?
   People counted in the 0 – 4 years old age group in 1980 were 10 – 14 years old in the 1990 population.

   b. What percent of the people in 1990 belonged to the age group identified in 3(a)?
   (3.28 millions of people / 23.36 millions of people) x 100 or approximately 14.0%.

4. Continue to use the population graphs and the information provided to answer the following:
   a. In what age group in the 2000 population were people counted who were 0 – 4 years at the start of 1980?
   People in the 20 – 24 years old age group in 2000 were counted in the 0 – 4 years old age group in 1980.

   b. What percent of the people in Kenya belonged to the age group you identified in 4(a)?
   (3.22 millions of people/30.61 millions of people)/30.61 millions of people or approximately 10.5% of the country's population.

   c. In what age group in the 2010 population were people counted who were 0 – 4 years at the start of 1980?
   People in the 30 – 34 years old age group in 2010 were counted in the 0 – 4 years old age group in 1980.

   d. What percent of the people in Kenya belonged to the age group you identified in (c)?
   (3.00 millions of people /40.84 millions of people) x 100 is approximately 7.3% of the country's population.

5. Summarize the change in the count and percent of the people who were 0 – 4 years old at the start of 1980 to the count and percent of people 35 – 39 years old in 2015.
   The count of people 0 – 4 years old decreased in count for each of the next age groups that counted them. The decrease is primarily explained that as people grew older, they died.

6. Although a decrease in both the count and the percent of people were noted in the 0 – 4 years old age group to the 35 – 39 years old age group, there are other summaries that indicate changes in the population of Kenya from 1980 to 2015.
   a. Complete the following table by calculating the percent of the population in the given year who were 35 – 39 years old. (The calculation for 1980 has been completed as an example. Calculate your answer to the nearest tenth of a percent.)

Year	Count in age group 35 – 39 years old (millions of people)	Count in total population (millions of people)	Percent of 35 – 39 years old in the total population
1980	0.66	16.33	4.0%
1990	1.05	23.36	*4.5%*
2000	1.56	30.62	*5.1%*
2010	2.10	40.83	*5.1%*
2015	2.89	45.93	*6.3%*

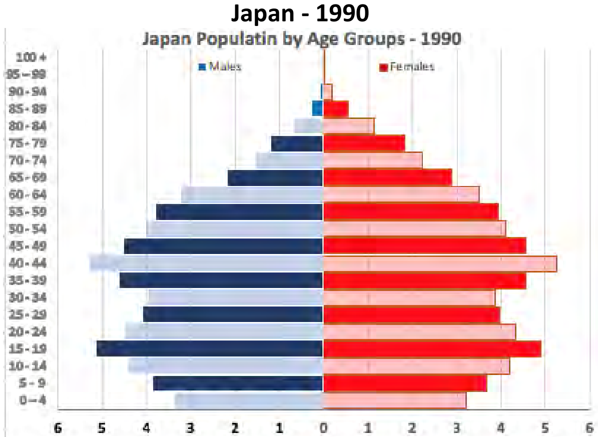
b. Based on the above changes in this age group, and similar changes in several older age groups, describe a change in the median age of people in Kenya from 1980 to 2015.
Older age group represent more of the population in 2015 than in 1980. This greater percent indicates that although people died due to age, the percent of the people who died was less.

c. Why was the change in the median age an encouraging summary for the population of Kenya?
As indicated, more people were living longer and making up a greater percent of the country's population. The changes in Kenya will be picked up in Units 3 and 4 as the impact of these changes are significant.

7. In what way do you think the mean ages has also changed from 1980 to 2015 in Kenya? Explain your answer.
Similar to the change in the median age, the mean age is also greater. The reasons for the change are again that more people in the older age groups are living longer, and the percent (or weight) of the older age groups is more significant in 2015. The calculation of the mean is more impacted by those changes. It is not intended, however, that students would carry out the details of estimating the mean ages as outlined in Lesson 4. A description and a discussion of the mean as a balance point is the focus of the answer to this question.

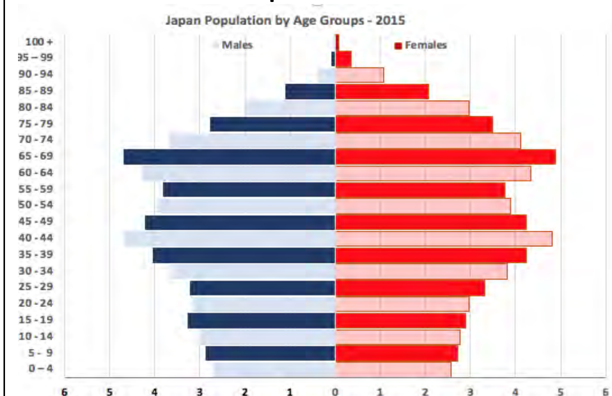
Japan
The 1980 population and the 2015 population are summarized by the following population pyramid graphs and histograms.

Japan - 1990

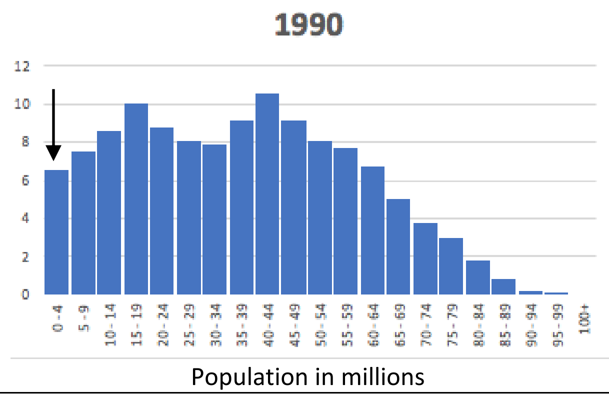


Total population: 123.54 millions of people

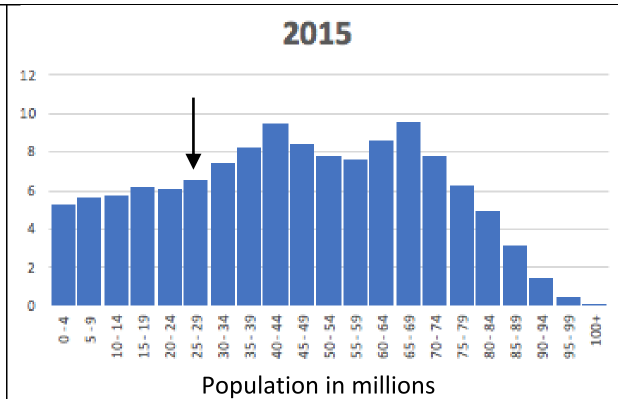
Japan - 2015



Total population: 126.91 millions of people



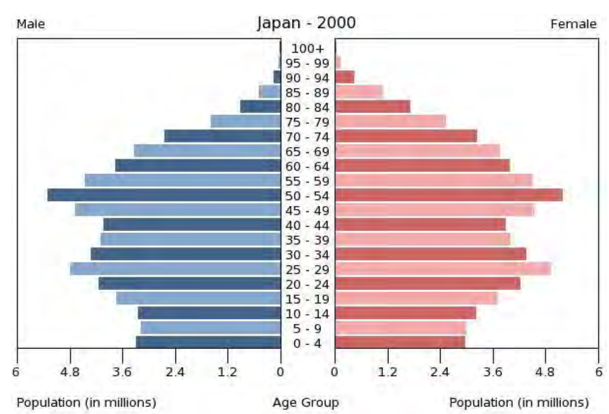
Arrow highlights the 0 – 4 years old age group or 6.57 million people



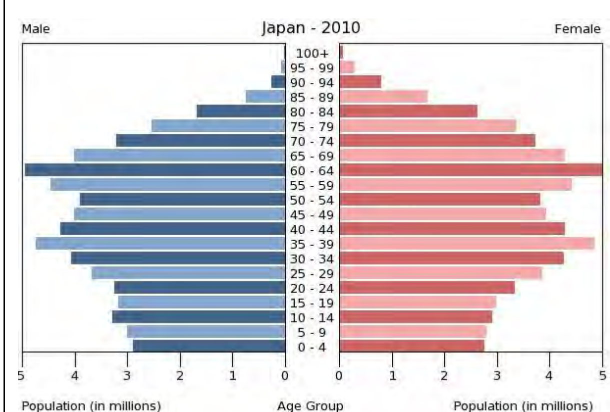
Arrow highlights the 25 – 29 years old age group or 6.54 million people

The population pyramid graphs and histograms of the intervening years of 2000 and 2010 are also provided. Note the changes in the shape of the population pyramid graphs and histograms.

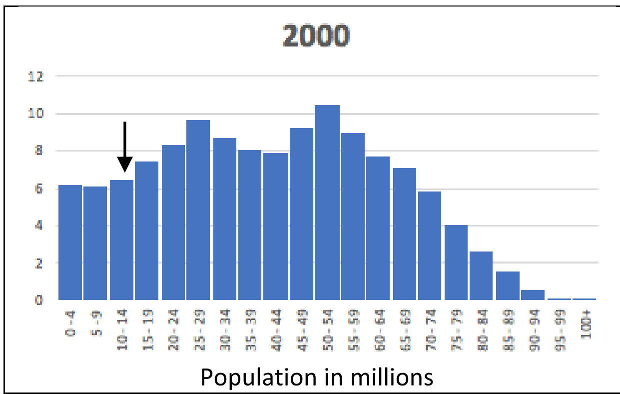
Journey through the intervening years:



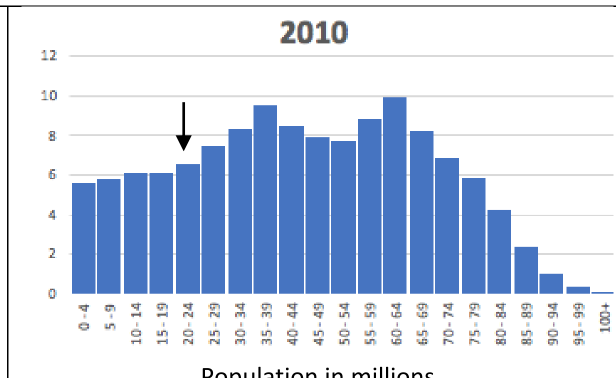
Total population: 126.78 million people



Total population: 127.56 million people



Arrow highlights the 10 – 14 years old age group or 6.42 million people



Arrow highlights the 20 – 24 years old age group or 6.55 million people

8. What do the graphs tell us about the changes in the population of Japan from 1990 to 2015? Identify at least 2 changes highlighted by the graphs.
   The graphs indicate a slight increase in the areas that indicate an increase in the population of the country ***except for the 2010 to 2015 graphs****. The age groups that have the greatest count of people are also clearly aging over the time analyzed in this lesson. Students may see Japan's population similar to the summaries of the United States, specifically related to the Baby Boom and Millennial Generations.*

9. Use the 1990 and 2015 population pyramid graphs or histograms to answer the following questions.
   a. What is the count of people 0 – 4 years old in 1990?
      6.57 million people
   b. What is the percent of people 0 – 4 years old in 1990?
      6.57 / 123.54 is approximately 0.053, or 5.3%.
   c. What is the count of people 25 – 29 years old in 2015?
      6.54 million people
   d. What is the percent of people 25 – 29 years old in 2015?
      6.54 / 126.91 is approximately 0.052, or 5.2%.

10. Use the 2000 and 2010 population pyramid graphs or histograms to answer the following questions?
    a. What is the count of people 10 – 14 years old in 2000?
       6.42 million people
    b. What is the percent of people 10 – 14 years old in 2000?
       6.42 / 126.78 is approximately 0.0.51, or 5.1%.
    c. What is the count of people 20 – 24 years old in 2010?
       6.55 million people
    d. What is the percent of people 20 – 24 years old in 2010?
       6.55 / 127.56 is approximately 0.051, or 5.1%.

11. Based on the counts and percent derived in questions (9.) and (10.), what happened to the people and the country of Japan who were 0 – 4 years old at the start of 1990 that changed the counts and the percent from 1990 to 2015?
    The counts and percent are interesting for Japan, and different than the United States and Kenya. The counts show a slight increase in the counts except from 2010 to 2015. The percent were slightly less or the same. The total population had slight increases, except from 2010 to 2015, in which the total population actually decreased. The aging of the people in Japan results in deaths that resulted in the slightly decreasing counts as people aged. The slight changes in percent, however, indicate that these changes are not major as was the case with Kenya.

12. In what way do you think the median ages changed from 1990 to 2015 in Japan? Explain your answer.
The median ages are increasing over the time highlighted by the histograms. If students add up the age group totals to estimate the age group that captures approximately half of the population, 50% of the population is captured in the 35 – 39 years old age group for 1990, 50% is captured in the 40 – 44 years old age group for 2000 and 2010, and 50% is captured in the 45 – 49 years old age group for 2015. This increase in the median age is a major indicator of the aging population of Japan.

13. In what way do you think the mean ages changed from 1990 to 2015 in Japan? Explain your answer
The mean ages are also increasing. Again, the general shift of the balance point to the older age groups is the main focus of the change in the mean ages.

14. If you knew a person from Japan who was well aware of the history of Japan, what questions would you ask this person based on the above pyramid graphs or histograms?
Questions that would ask what happened just before and after the age groups have the greatest count of people (specifically 15 – 19 years old and 40 – 44 years old in the 1990 graphs). Are the counts connected in the same way to the Baby Boom and Millennial Generations discussed in a summary of the United States population? What events might have caused the decrease in the population in the set of histograms? What challenges are faced in this country with a growing aging population?

15. Complete the following summary table of the United States, Kenya, and Japan:
This problem represents a key summary of the work in this lesson and Lesson 6. Use students' responses as an indication of their understanding of the changes in population due to death, immigration, aging, and increased birth rates. These descriptions will be a major part of the recursive model developed in Unit 3.

Country	Explain the changes of the counts of the age groups over time that were highlighted in this lesson.	Summarize the changes in the overall population count and shape of the countries during the years highlighted in the lesson.
United States	*The counts of the people in the age groups cited increased over the years highlighted by the graphs that are explained by immigration.*	*The country's total population increased over the years highlighted. The increase is again primarily explained by immigration. Also, the mean and the median ages appear to increase, thus slightly changing the shape of the country to reflect an aging population.*
Kenya	*The counts of the people in the age groups cited decrease in both counts and percent. The explanation is*	*The total population of the country increases over the years highlighted. The primary explanation is the high*

	related to the decrease counts from death. The crease in the counts and the increase in the total population of the country (growth due to high birth rates) explains the decrease in the percent of the age groups cited.	*birth rate that impacts the count of people 0 – 4 years old. This increased count is even more important in explaining the increase in the population as the count of older age groups decrease.*
Japan	*The counts and the percent of the people in the age groups cited slightly increased over time except as Japan moved from 2010 to 2015. The decrease in the count of people from 2010 to 2015 is a major description of Japan's aging population.*	*The population slightly increased over the time periods highlighted in the lesson, except for the time period from 2010 to 2015, where the total population of the country slightly decreased. This decrease is a major factor of the unusual population changes for Japan.*

Use the population totals for 2010 and 2015 as the anchor years in observing changes over time. If this module is done when the actual values of the population are known for years 2020 or beyond, consider adjusting the anchor years. Also add 2015 population estimates for the country you envisioned in Lesson 5, or My Country.
(Population expressed in millions of people.)

The following table for Kenya and Japan will be referenced in the lessons that continue analyzing the counts of Kenya, Japan, and the United States. The My Country data are provided as a follow-up for the country designed in Lesson 5. The population for 2010 and the population for 2015 will also be used in the modeling lessons that follow as an example of a top-layered country and possible changes over time a country with that shape might face. You do not have to share data the My Country data with students unless a complement to the other countries would improve the investigations.

	Kenya		Japan		My Country	
Age Group	2010	2015	2010	2015	2010	2015
0 – 4	6.87	6.38	5.63	5.27	*6.20*	*5.10*
5 – 9	6.01	6.76	5.76	5.61	*6.40*	*6.10*
10 - 14	4.55	5.95	6.16	5.75	*6.20*	*6.35*
15 - 19	4.13	4.49	6.13	6.15	*7.00*	*6.00*
20 - 24	3.99	4.08	6.55	6.13	*6.00*	*6.50*
25 - 29	3.70	3.92	7.50	6.54	*7.20*	*6.40*
30 - 34	3.00	3.6	8.30	7.47	*7.15*	*7.00*
35 - 39	2.10	2.89	9.55	8.27	*7.60*	*7.10*
40 - 44	1.63	2.01	8.52	9.50	*7.05*	*7.50*
45 - 49	1.32	1.55	7.91	8.46	*7.60*	*7.00*
50 - 54	1.04	1.25	7.69	7.82	*8.00*	*7.50*
55 - 59	0.81	0.98	8.84	7.57	*8.10*	*7.80*
60 - 64	0.59	0.75	9.92	8.62	*8.50*	*8.00*
65 - 69	0.43	0.53	8.27	9.57	*9.00*	*8.40*
70 - 74	0.31	0.36	6.89	7.82	*9.34*	*8.80*
75 - 79	0.20	0.23	5.86	6.26	*10.20*	*9.00*
80 - 84	0.10	0.12	4.27	4.95	*15.00*	*10.00*
85 - 89	0.04	0.05	2.40	3.17	14.00	12.00
90 - 94	0.01	0.01	1.03	1.45	7.00	11.00
95 - 99	0.01	0.01	0.33	0.44	3.50	6.00
100+	0.00	0.01	0.05	0.09	1.50	3.00
Totals	40.84	45.93	127.56	126.91	163.34	156.55

Assessment Ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of the lesson.

Adeline created a country she named Awesome. She generated counts for a 2010 population distribution and a 2015 population distribution for ages 0 – 100+ years old. When estimating the mean and median ages of Awesome, she found that from 2010 to 2015 the mean age got younger but the median age stayed the same.

a. Is the above possible, or do you think that Adeline made an error in her calculation of the mean and median ages? Explain your answer

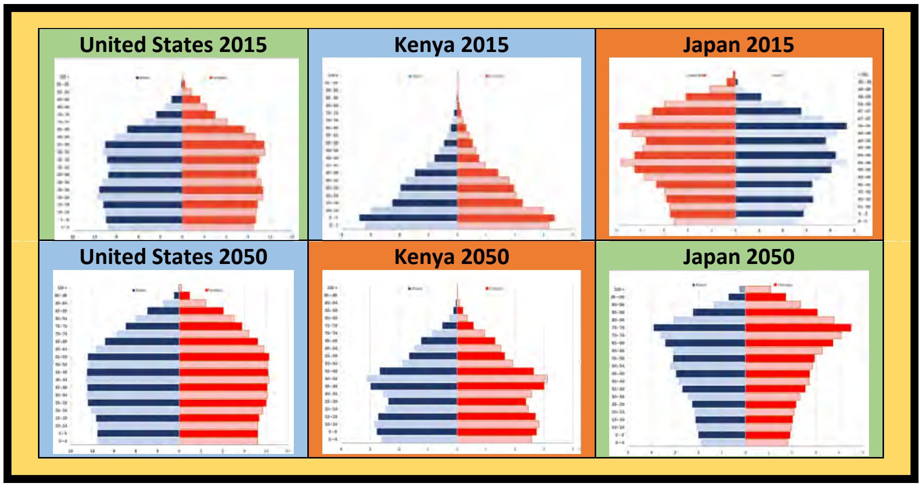
b. Develop sketches of a 2010 histogram and a 2015 histogram that Adeline might have developed.

Comments on the Assessment Task:
There are several possible explanations and sketches that would result in the mean age decreasing and the median age staying the same. To help students provide an explanation of their thinking, direct them to mark on their sketches the location of the median age on both the 2010 and 2015 histograms. The most straight forward explanation would be to add people in the 0 – 4 years old age group of the 2015 histogram (each new person a result of additional births). Then students would also add **the same number of people** to older age groups (counts added by immigration) but the new people would be added to the age groups in the lowest range of the upper 50% of the distribution (need to make sure that the people added result in a younger mean age). The assumption of this approach is that no one has died or left the country during that time. If students add death and immigration to the mix, then a little more adding and subtracting of people and their ages would need to be analyzed.

Additional Assessment Ideas:
Question 14 adds insights into students understanding by assessing what ***questions*** they would raise about the unusual shape of Japan's pyramid graphs to someone who could answer questions about Japan's past. What events happened to explain the large count of people in age groups 40 – 44 years old and 65 – 69 years old in the 2015 graph? Why does Japan's population seem to be decreasing since 2010? Students who understand the objectives of this lesson will probably not be able to answer what specifically happened in the past, but they will be able to suggest ideas that would result in a decline of the population of Japan.

Question 15 provides a good indication of a student's understanding of the lesson as its summaries the focus of this lesson and the previous lessons.

Teaching Notes
Unit 3
Looking Forward



Introduction

"Looking Forward" is a series of lessons to build models and rebuild models that estimate the future counts of people in a country. The models are put together by mathematical tools (common ratios, population factors, foundation factors) that are carefully developed in the lessons. In this unit, assumptions will be made, data will be selected and organized, and procedures or algorithms will be designed to generate outputs that give us a possible glimpse into a country's future. The models speculate about a country's shape for future decades based on well-defined assumptions. Ultimately these forecasts, however, will be evaluated on whether or not the predicted outcomes actually happen. In other words, we need to stick around and wait to see the results of the count of people in a country. The stories continue!

Teaching Notes

Lesson 8

Looking Forward with an Arithmetic Sequence and a Linear Model

Overview:

This lesson is the first in a series of lessons that explore models that estimate a country's population in the future. The problems in this lesson ask students to think about what a population projection is, what is a model that results in a population projection, why is this important, and are these projections reliable. Answers to these problems are examined by developing projection models, analyzing the projections from the models, and evaluating if the models provide accurate estimates of past population distributions. Essentially, answers to these problems are grounded in applying and interpreting mathematical tools.

This lesson begins with a basic linear model using the 2010 and 2015 population totals. The conclusions are limited as this model is not developed to provide projections of the main discussion points of this module, namely the counts in the ***age groups*** that make-up a country's population.

This lesson stretches students' development of the **Modeling Continuum** by moving their thinking to levels 3 and 4. Students are involved in incorporating data into the projection models and making "What if …?"decisions. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1	Problems: 2, 3, 4, 5, 8	Problems: 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21	Problems: 9, 10

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, extracting and interpreting data from graphs, working with coordinate graphs, slopes, and solving and setting up linear equations.
See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of the **Introductio**n to Unit 3 and a complete Lesson 8 for each student. This lesson does not require any additional handouts.

Launch:

Direct students to read the introduction to Unit 3. Consider asking students: "If you and your family remain healthy and well, how old will you be in 2050? How old will your parents or guardians be in 2050? How old will your teacher be in 2050?" Looking forward, or looking into the future, is the goal of this lesson.

Ask students why looking forward is important? Expect mixed responses to that question, including that it is not an important question. Discuss, however, the implications for a country if its population increases by millions of people in the next several decades. What if those increases were not anticipated? Also discuss the implications for a country if its population decreases by millions of people. Emphasize that discussing the future includes them in the count of people, and that they will see their country through the eyes of possibly a different shape as defined in previous lessons. After these opening questions and discussion, direct students to complete the problems.

Implementation Ideas:

This lesson involves some topics that may be familiar to some students and new to others. Topics such as what is an arithmetic sequence, the slope of a linear equation, the y-intercept of a linear equation, and the best-fitting line are topics most students have studied, but they may be topics that not all students understand or readily apply. Modify the expectations based on students' previous understanding of sequences and linear equations if these topics are unfamiliar to students. The depth of explanations of the topics involving slope, y-intercept, best-fitting line, and scatter plot are minimal and may require additional explanations. The problems in this lesson, however, can be used to strengthen an understanding of these topics, as well as provide connections to meaningful applications.

This lesson is the first of several lessons that introduce students to specific ***models*** that look into the future. The projections in this lesson are derived from equations based on population estimates for 2010 and 2015 of the United States, Kenya, and Japan. Discuss with students what might alter some of the estimates that are derived from these models. For example, will the count of people moving into a country each year be the same? Will a country record a similar count of births each year? If those counts are not the same, how will the models for each country be altered? Begin discussing these questions as students work through the problems in Lesson 8 and in the next several lessons.

Student responses or descriptions
Lesson 8 - Problems

Arithmetic Sequence

1. Calculate the difference of the 2015 and 2010 population estimates.
   12 million people

2. Add the above difference to the 2015 population. The result is an estimate for the 2020 population assuming the population increases by the same count from 2015 to 2020 as it did from 2010 to 2015. In the same way, add the difference to the 2020 estimate to obtain the 2025 population estimate. Complete the following table by continuing the process:

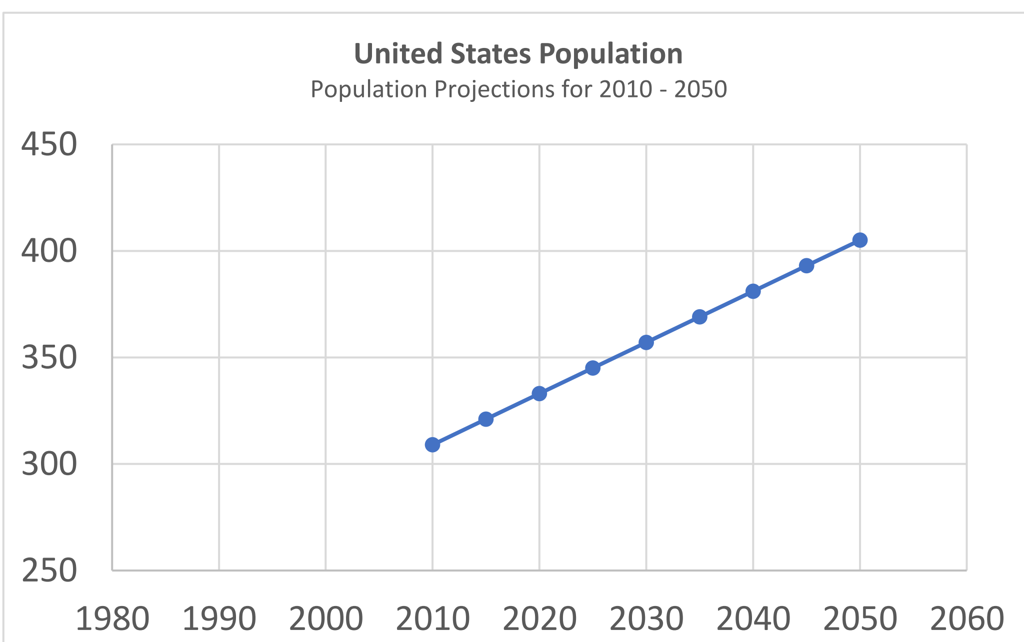
United States

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	309	321 *+12*	*333 +12*	*345 +12*	*357 +12*	*369 +12*	*381 +12*	*393 +12*	*405*

3. The above estimates form a list that is described as a ***finite arithmetic sequence***. Based on this example, how would you explain an arithmetic sequence to someone who was unfamiliar with it?
The constant difference between the numbers that started the sequence is added to each element to derive the next element. The sequence is a list of numbers where each successive term of the sequence is found by adding a constant.

Linear Model

4. Consider the following coordinate grid in which the x-axis represents the year and the y-axis represents the estimated population values. Plot each of the coordinate points of the sequence in problem 3 on this grid or a grid of this type. The first two points are plotted for you.
The following graph connects the points based on the arithmetic sequence.



Population in millions of people

5. Connect the coordinate points you plotted on the graph. What do you notice?
   The points appear to lie on a straight line when connected.

6. There are several procedures designed to derive an equation given two points. Consider the following steps to derive a linear equation by representing the coordinate values (x , y) for 2010 as (x_1 , y_1) and 2015 as (x_2 , y_2).
   a. Calculate the slope or change in population per year (to the nearest tenth) using the two points (2010, 309) and (2015, 321) as represented below:

      Slope = $\frac{y2-y1}{x2-x1} = \frac{Change\ in\ population}{change\ in\ years} = \frac{321-309}{2015-2010} = \frac{12}{5}$ *or 2.4 million people per year.*

      Explain this slope as a change in the population in 1 year.
      The country is projected to grow by a count of 2.4 million people per year.

   b. An equation called the "***point-slope***" equation derives the slope based on the two given points. The slope is then used with one of the two points to create a linear model. For this problem, let x represent the year and y represent the population in millions. Use the slope previously calculated from the two points (2010, 309) and (2015, 321). Use the first point (2010, 309) to complete the equation of the linear model. The steps are outlined below:

      $y - y_1 = \frac{y2-y1}{x2-x1}(x - x_1)$ for (x_1, y_1)

      Use the point (2010, 309) and the value of the slope to complete this linear model:
      y - ____ = $\frac{(y2-y1)}{(x2-x1)}$ (x - ____)

      Using (2010, 309) as the coordinate point and the value of the slope, the equation becomes:
      y – 309 = 2.4(x – 2010)

   c. If an estimate of the population for 2020 is derived from this equation, what is the value of x used in this model ? Derive an estimate of the population for 2020 using the above. Round your estimate of the 2020 population to the nearest millions of people.
      The value of x would be the year 2020, and the value of y would be the projected population estimate for 2020.
      y – 309 = 2.4(2020 – 2010)
      y – 309 = 2.4(10)
      y – 309 = 24
      y = 24 + 309 = 333

The estimated population of the United States in 2020 would be 333 million people.

d. Rework the above equation into a form called the "***slope - y-intercept***" linear equation. This form is also a representation addressed in a study of algebra. This form is often summarized as **y = mx + b** where **m** is the slope previously calculated in the above problem and **b** is the y-intercept, or the value of y when x =0. (There are, however, other representations of the ***slope - y-intercept*** equation. For example, a statistical representation of this equation is often summarized as $y = b_0 + b_1x$ were b_1 represents the slope and b_0 is the y-intercept.)

$y - y_1 = 2.4(x - x_1)$

$y - 309 = 2.4(x - 2010)$

$y - 309 = 2.4x - 4824$

$y = 2.4x - 4824 + 309$

$y = 2.4x - 4515$

7. Answer the following.
   a. Determine the ***units*** to complete the following statement:
      The slope is "__________" per "_______"
      The slope is 2.4 million people per year, or people per year.
   b. What does a positive slope indicate?
      The population is increasing constantly per year.
   c. What would a negative slope indicate?
      The population is decreasing constantly per year.
   d. What does a slope that is equal to 0 indicate?
      The population is neither increasing nor decreasing per year. It stays the same.

8. The United States Census estimates for 1980, 1990, 2000, and 2005 are included in the following table. Use the linear model to derive population estimates for each of these years. Record your estimates in the table.

	1980	1990	2000	2005
US Census estimates	227	250	282	296
Linear model estimates	*237*	*261*	*285*	*297*

9. Compare the estimates derived from the linear model to the counts of people reported by the Census Bureau.
   a. Calculate the difference between the linear model estimate and the Census Bureau for 2005. Do you think the linear model provided a good estimate for 2005? Explain your answer.
      The difference was 1 million people. This is very close.

b. In the same way, calculate the differences between linear model estimates and the estimates reported by the Census Bureau for 2000? 1990? 1980? Do you think the linear model provided a good estimate for those years? Explain your answer.
Direct students to answer the years one at a time. The difference in 2000 is 3 million people. Consider this difference as also a good estimate. The difference in 1990 is 11 million people. This estimate is not as good. The difference in 1980 is 10 million. This estimate is also not as good.

c. Derive the estimates for the years 2010 and 2015 using the linear model. Are the estimates different from the previous estimates? Explain your answer.
$y = 2.4(2010) - 4515$
$y = 309$
$y = 2.4(2015) - 4515$
$y = 321$
The estimates are equal to the Census estimates as each year was used in constructing the model.

10. The Census Bureau estimates that the population of the United States at the time of the signing of the Declaration of Independence in 1776 was approximately 2.5 millions of people. This estimate is not based on any census as the United States Constitution was not completed at that time. What is the linear model's estimate of the population in 1776? Do you think the linear model provided a good estimate for 1776? Explain your answer.
The linear model estimate is: $y = 2.4(1776) - 4515$, or $y = -252.6$
Obviously this estimate is extremely incorrect as a negative population does not make sense.

11. Estimate the following future counts of the United States using the linear model:
    a. 2020 (332)
    $y = 2.4(2020) - 4515 = 333$ millions of people
    b. 2030 (343)
    $y = 2.4(2030) - 4515 = 357$ millions of people
    c. 2050 (397)
    $y = 2.4(2050) - 4515 = 405$ millions of people
    d. 2300 (947)
    $y = 2.4(2300) - 4515 = 1005$ millions of people

12. What years do you think the estimates derived in problem 11 are accurate predictions of the future? What years do you think the estimates derived in problem 11 are not accurate predictions of the future? Why would an estimate from a model like the linear model not be a good estimate?
The best estimates are probably for 2020 and 2030 as they are close to the years used to derive the model. The estimates for 2050 and 2300 are probably not accurate and could be quite different than what will be the actual population.

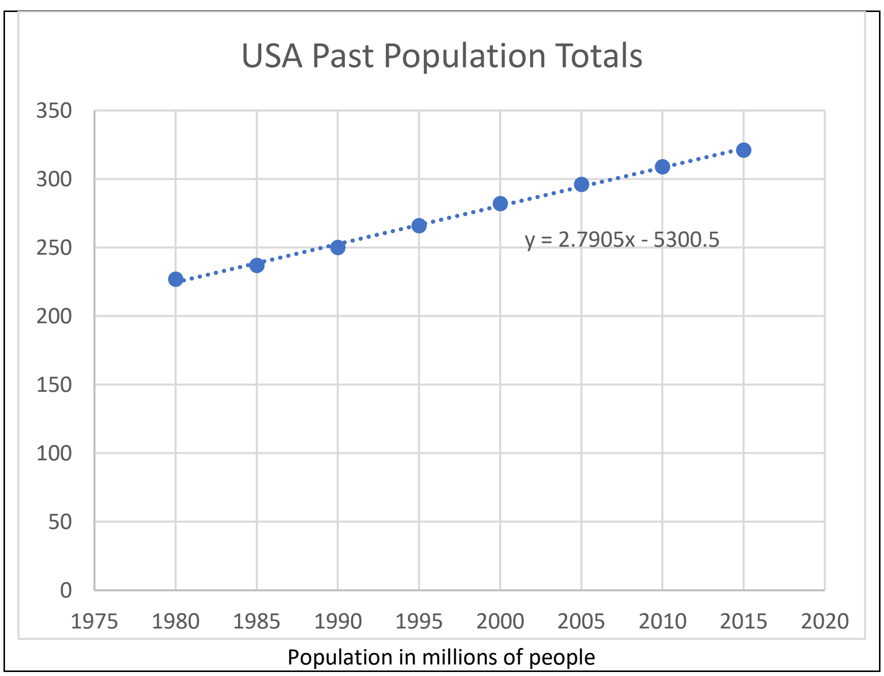
The model begins to fail as changes in a 5-year period will be different than the changes used in deriving the model. Changes due immigration, emigration, births, and deaths are not consistent.

There are many other models that could be developed, including other linear models. Consider the data collected from the Census Bureau of the United States population from 1980 to 2015 and a scatterplot based on this data. Each of these counts are considered the best estimates of the actual population.

Year	**1980**	**1985**	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**
Population (in millions of people)	227	237	250	266	282	296	309	321

13. Sketch a line that you think is a best-fitting line of the scatter plot. Identify any two points on the best-fitting line you drew and derive a linear equation based on the two points you selected using the point-slope summary. (The points you select will probably not be points represented by the scatter plot.) Determine the estimates of the United States population for the following years using the equation of the best-fitting line. Estimate each population to the nearest millions of people:

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates (in millions of people)							



Answers will vary given the line drawn by students. If the line approximates a best-fitting line, work with students to estimate the population estimates using their line. Encourage students to derive the slope, y-intercept, and the resulting equation of the line they drew by estimating the values from two points on the line (and remind them that the points do not necessarily have to be the data points). They will then use the equation to estimate the values for the years in the table.

If technology is available and similar to a graphing calculator with software, the data provided for the scatter plot can be used to derive the linear regression equation of the scatter plot. The linear regression equation or least squares regression equation is approximately equal to the following:

$$y = 2.8x - 5300$$

The following projected estimates are based on the least square regression equation:

Year	***2020***	***2025***	***2030***	***2035***	***2040***	***2045***	***2050***
Population (in millions of people)	*356*	*370*	*384*	*398*	*412*	*426*	*440*

Note that the slope of this line is greater than the slope of the linear model in problem 6, and the population estimates are considerably higher over time based on this model.

14. Compare the estimates derived from the linear model in problem 6 to the estimates derived from the best-fitting line.
The values to complete this table will vary based on the estimate of the best-fitting line equation. The values in the following table were based on the least squares' regression equation.

Year	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population estimates from model in problem 6 (in millions of people)	*333*	*345*	*357*	*369*	*381*	*393*	*405*
Population estimates from best-fitting line (in millions of people)	*356*	*370*	*384*	*398*	*412*	*426*	*440*

Do the models produce similar estimates? Explain.
The models are different, with the best-fitting model projecting higher estimates of the population in the future. Discuss with students that differences this large may result in

varied decisions regarding the building of schools, infrastructure (roads, airports, etc.), financial planning, and many other issues frequently discussed and debated.

15. Complete the following table for Kenya by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Kenya

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	41	46	*51*	*56*	*61*	*66*	*71*	*76*	*81*

16. Derive a linear model for the above population estimates for Kenya in the same way as linear model was developed for the United States in problem 6.
The linear equation based on the above sequence is:
$y = x - 1969$
Note: the change in the population is estimated to be approximately 1 million people per year.

17. Complete the following table for Japan by adding the difference of the 2015 population and the 2010 population to obtain the next population estimates.

Japan

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	128	127	*126*	*125*	*124*	*123*	*122*	*121*	*120*

18. Derive a linear model for the above population estimates for Japan in the same way as linear model was developed for the United States in problem 6.
The linear equation based on the above sequence is:
$y = -0.2x + 530$
Note: the change in the population is estimated to decrease by approximately 200,000 people per year.

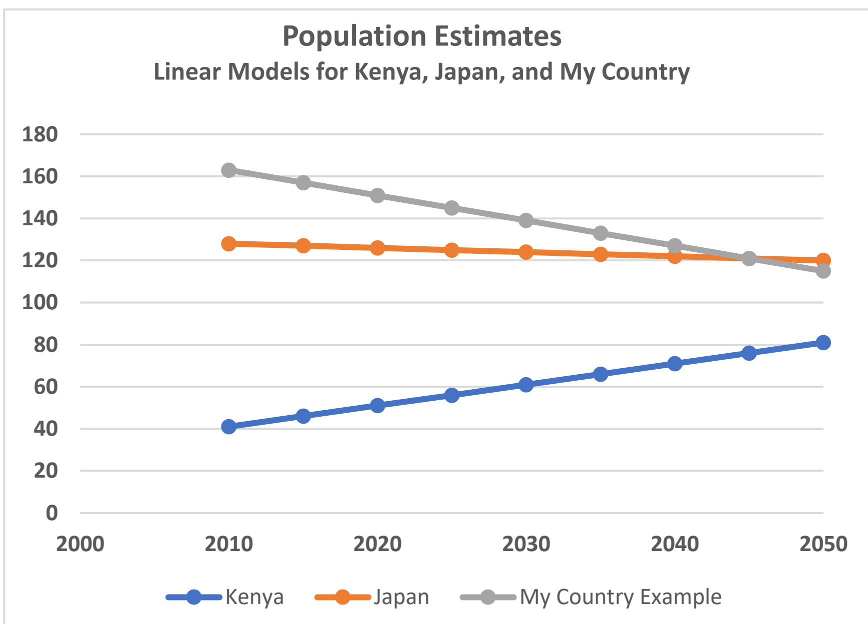
19. Finally, complete a table for the population you created for your own country, or My Country.
The values entered in the table will vary depending on the My Country population totals designed by the students in previous lessons. If students completed their own country data, they should be encouraged to use the data and derive a linear model in the same way as the previous examples. The values entered in the following table were provided in the Teaching Notes to analyze a country whose shape is classified as a top-layered country.

My Country

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
Population (in millions of people)	163	157	*151*	*145*	*139*	*133*	*127*	*121*	*115*

20. Derive a linear model for the population estimates of My Country in the same way as the linear model was developed for the United States.
Answers will vary depending on the students' data. Using the example provided in the above table, the linear equation is:
$y = -1.2x + 2575$

21. Use the following grid to record the projected population values for Kenya, Japan, and My Country based on the linear models developed in this lesson
The following sketch includes the points and line for the example of My Country previously discussed.



Population in millions of people

The linear models developed in this lesson provide estimates of the total population. The models do not, however, provide estimates of the age groups that make-up the population. This challenge must still be addressed.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of the lesson.

Paul used population data of his fictitious country from Lesson 5 and derived a linear model of $y = 2.5x - 2000$, where x represented the year and y represented the count of people for that year in millions of people.

a. If Paul's model is accurate, how many people are added to his country's population each ten years?

b. Paul derived his model using the 2010 and 2015 population totals for his country. Do you think his model will be accurate for predicting the population in 2100? Explain your answer.

c. Paul also was interested in the estimated population of his country in 2005. Do you think his model will be accurate in predicting the population in 2005? Explain you answer.

Comments on the Assessment Task:
The assessment task is a general review of the main goals of this lesson. Paul's country will grow by 2.5 million people per year which is based on an understanding of slope. In ten years, the country will grow by 25 million people (provided the linear model is accurate). Paul would not consider the 2100 estimate as accurate as it is several decades away from the years used to derive the model. However, Paul would consider the estimate for 2005 to be accurate as it is close to the years used to derive the model.

Additional Assessment Ideas:
Ask students what factors would alter the population projections using the linear models as a wrap-up question. What could happen to a country during a 5 or 10-year period that would result in a count of the total population that is different than the count projected by the linear model? This question provides opportunities for summarizing this lesson, and also suggesting where the next several lessons are headed.

Consider directing students complete an **Exit Summary** for this lesson. Discuss with students the levels and in particular, levels 3 and 4 as more questions and problems require use of the tools to make more independent summaries or decisions.

Teaching Notes

Lesson 9

Looking Forward with a Geometric Sequence and an Exponential Model

Overview:

This lesson works with students in deriving a geometric sequence based on the 2010 and 2015 population estimates. From this sequence, an exponential model is presented and used to derive estimates of the count of people in the future. Similar to the last lesson, however, the conclusions are limited as the model is not designed to provide counts in the age groups that make-up a country's population. The role of an exponential projection model is another tool in estimating the total population of a country over time.

The exponential model represents the most robust model of the projection models. This lesson concludes with the observation that if this model were to continue indefinitely, populations of growing countries will ultimately be too large to support the population, and populations of declining countries will ultimately approach 0. This observation continues to provide discussion about other models and their impact over time.

This lesson stretches students' development of the **Modeling Continuum** by moving their thinking to solve problems to levels 3 and 4. Students are involved in incorporating data into the projection models and making "What if …?"decisions. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2	Problems: 3, 4, 12, 15, 16, 19, 20	Problems: 5, 6, 7, 8, 9, 10, 11, 13, 17, 18	Problems: 14, 15

Primary tools students use in this lesson to answer the above problems are:
Arithmetic operations, proportions, percent, extracting and interpreting data from graphs, working with coordinate graphs, and solving and setting up an exponential model or equation. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 9 for each student. This lesson does not require any additional handouts.

Launch:

Launch the lesson by providing time for students to read ***Kristin's Story – Chapter 5***. Discuss some of the details of the doubling results highlighted in the story. This story provides an example of a geometric sequence (and the resulting series) in which the common factor multiplied to derive the next payment for doing her homework is 2. Write out the sequence detailed in the story and discuss how the values cited in the story were derived. Consider providing other examples such as the amount of money earned in a compound savings' account.

Pose a few questions before students start the problems, and then revisit these discussion questions at the end of the lesson as a wrap-up:

- What do you think will happen to a country's population if it increases its population every year for over 200 years?
- Why would a country grow every year? Why would a country not grow every year?
- Is it possible for a country to grow for a period of time and then not grow for a similar period of time? Think of specific examples.

Implementation Ideas:

The problems are designed for students to complete individually or in small groups. They are also designed with some expectation that students have previously worked with an exponential function, although if they did not, the problems allow you to introduce and expand their understanding of an exponential model. Details involved in setting up the exponential model are minimal. Students learn in this lesson the starting value and the common ratio or common factor of an exponential function. The problems are connected to the goals of this module involving the count of people in a country. However, if time is available, financial problems involving compound interest, mortgages, future and present value are related topics that have a foundation in the exponential model.

Provide time for students to complete the problems, with periodic whole class discussions of their answers. The explanations provided in the Teaching Notes include a summary of the answers to several problems or explanations of the problems that students are expected to develop. Several of the problems also provide an opportunity for you to check their understanding of this important model.

Student responses and descriptions
Lesson 9 - Problems
Geometric Sequence

1. Consider the proportion $\frac{321}{309}$ or approximately 1.039 as the decimal representation to the nearest thousandth. What is the result of multiplying this proportion and 309 to the nearest million?
   $\frac{321}{309}$ x 309 million = 321 million

2. For the next estimate of the sequence, multiply 321 million by 1.039. Record your result to the nearest million in the population projection for 2020.
   321 x 1.039 million = 334 million

3. Continue multiplying each estimate by $\frac{321}{309}$ or 1.039 to obtain the next estimate until you reach the estimate for the year 2050. Complete the following table (round off your calculations to the nearest millions of people):

United States

Year	2010	2015	2020	2025	2030	2035	2040	2045	2050
Population (in millions of people)	309	321	*334*	*347*	*361*	*375*	*390*	*405*	*421*
		x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	x $\frac{321}{309}$ or 1.039	

4. The counts in the table form a *geometric sequence*. How would you describe the difference of an arithmetic sequence and a geometric sequence to someone unfamiliar with these sequences?
   *Each next count in the list of a geometric sequence is derived by **multiplying** the previous count by a constant factor . In an arithmetic sequence each count is derived by **adding** (or subtracting) a constant.*

5. Consider a country in which the 2010 population was 309 million people and the 2015 population was also 309 million people. If a geometric sequence is generated to estimate the population in the future, what is the value of the common ratio or constant factor?
   The constant factor would be equal to 1. A factor of 1 indicates the population stays the same for each 5-year period.

6. Consider a country in which the 2010 population was 200 million people and the 2015 population was 100 million people. If a geometric sequence is also generated for this country, what is the value of the common ratio or constant factor? How would you summarize the change in the population for each 5-year period?
   The population factor would be equal to one-half, or 0.5. The population would be cut in half for each 5-year period.

7. By multiplying a population estimate by 1.039, what do you know about the next 5-year population estimate? For example, does the population grow, decline or stay the same? Derive the percent change in the population over each 5-year period as a percent of the population.
   The population will increase for each 5-year period. The percent increase for each 5-year period will be 3.9%. .

Exponential Model

The geometric sequence can be used to derive an exponential model to estimate the population for any given year. Consider the following exponential model derived from the above geometric sequence with modifications due to rounding off key values:

$$y = 309(1.0078)^{x - 2010}$$

8. If the above exponential model is used to estimate the population for a given year, answer the following:
   a. Let x = 2010. What is the value of y?
      The value of y would be 309 to the nearest whole number.
   b. Let x = 2015. What is the value of y?
      The value of y to the nearest whole number is 321 to the nearest whole number.
   c. What does x represent in the exponential model?
      x represents the independent variable and the year when calculating the population.
   d. What does 309 represent in the exponential model?
      309 is the starting value in the geometric series, or the population at the start of the given values.
   e. What does y represent in the exponential model?
      y represents the dependent variable or the value of the population to the nearest whole number.
   f. What does 1.0078 represent?
      1.0078 represents the common factor of the geometric series or the exponential model.

9. Derive an estimate of the population for the start of the 2022 using the exponential model. Also derive an estimate of the population for the start of the year 2008. Would you have been able to derive these estimates using the values in the geometric sequence? Explain your answer?

   For 2022, let x = 2022

   $y = 309(1.0078)^{2022-2010}$ *or* $y = 309(1.0078)^{12}$

   = 339 million people

   For 2008, let x = 2008

   $y = 309(1.0078)^{2008-2010} = 309(1.0078)^{-2} = 304$ *million people*

   The above calculations could be estimated from the sequence by doing a messy extrapolation of the values between 2020 and 2025. It is more straightforward to use the exponential model. Allow students to indicate that they would not have been able to derive these estimates from the geometric sequence.

10. The constant factor for the 5-year estimates of the geometric sequence was 1.039. The exponential model involves a constant factor of 1.0078. How do you think the constant factor of 1.0078 was derived?

    Students could explain that the constant factor is an estimate of a yearly factor and would be 1/5 of the constant factor for a 5-year period, or 1/5 of 0.039. This is approximately 0.0078. An exponential model, however, does not exactly work like this, therefore, another method to solve for r (where r is the yearly constant factor) is indicated below. This method is addressed in an intermediate or advance algebra class.

    $321 = 309r^5 \Rightarrow 321/209 = r^5 \Rightarrow \sqrt[5]{\left(\frac{321}{309}\right)} = r \Rightarrow$ *r is approximately 1.00765.*

    The resulting yearly common factors are approximately the same for each method.

11. Does the exponential model and the geometric sequence derive the same estimates for each 5-year interval? Complete the following table by calculating y from the exponential model. Let y represent an estimate of the population to the nearest millions of people. Recall that 309 million people at the start of 2010 was rounded off to the nearest millions of people from the Census data.

x	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
y	309	*321*	*334*	*348*	*362*	*376*	*391*	*407*	*423*

12. Compare the estimates from the exponential model to the estimates from the geometric sequence. Record population estimates from the model to the nearest million. Complete the following table:

Year	Estimate of population from exponential model	Estimate of population based on the geometric sequence:
2020	*334 million*	*334 million*
2030	*362 million*	*361 million*
2040	*391 million*	*390 million*
2050	*423 million*	*421 million*

13. The exponential model was derived from the first two values of the sequence. Why are the values in the table for problem 12 not exactly the same? Do you think the estimates are close? Explain your answer.
    Differences are due to the round off of the common factor and the population estimates. The estimates are very close.

14. Compare the estimates derived from the exponential model to the estimates reported by the Census Bureau for 1980 and 2000. Is the exponential model close to the actual counts? Summarize your comparisons.
    Population is represented in millions of people:

Year	*Exponential Estimates*	*Population from Census Bureau (best estimate of actual population)*
1980	*245 million*	*227 million*
2000	*286 million*	*282 million*

15. Do you think the estimates using the exponential model for years after 2015 will be accurate population projections for 2020 to 2050? Explain your answer.
    Point out to students that when looking back the estimate for 1980 was noticeably different than the best estimate of the actual population from the Census Bureau, however, the estimate for 2000 was considerably closer to the Census estimate. Answers will vary but discuss with students that population estimates closer to the starting population estimates for 2010 and 2015 are anticipated to be closer to what will be (or what was) the actual population estimates. This was the same observation students made with the arithmetic sequence or the linear model..

Consider the 2010 and 2015 population counts for Kenya and Japan.

16. Complete the following geometric sequence for Kenya by multiplying each estimate by the common factor based on the 2010 and 2015 counts of people:

Kenya

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	41	46 *x 1.12*	*52 x 1.12*	*58 x 1.12*	*65 x 1.12*	*73 x 1.12*	*82 x 1.12*	*92 x 1.12*	*103*

To derive the next count in the list in millions, multiply the previous count by $\frac{46}{41}$ *or approximately 1.12. This factor indicates an increase in the population of approximately 12% for each 5-year period.*

17. Complete the following geometric sequence for Japan:

Japan

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	128	127 *x 0.992*	126 *x 0.992*	125 *x 0.992*	124 *x 0.992*	123 *x 0.992*	122 *x 0.992*	121 *x 0.992*	120

To derive the next count in the list in millions, multiply the count by $\frac{127}{128}$ *or approximately 0.992 This common factor indicates an approximately 0.8% decrease in each 5-year period.*

Note: Discuss with students why the decrease in Japan's population looks consistent (or, a decrease of 1 million people for each 5-year period). It almost looks like a linear model. The constant factor is close to 1. As a result, the decrease is slight for each year, but accumulates to approximately 0.8% for each 5-year period. If, however, we project the populations far into the future, the decreases are more noticeable in comparison to the starting values.

18. In what way is the exponential model for Japan different than the exponential models for the United States and Kenya?

The population of Japan is forecast to decrease. Discuss with students that for a decreasing geometric sequence, the constant factor is equal to (1 – the rate of change per year), as opposed to (1 + the rate of change per year) which is the case for Kenya and the United States. If students have previously worked with compound interest problems, including present value or future value problems , they may have worked with formulas that included (1 + i) or (1 – i) where "i" is the interest rate. The above examples are similar.

19. Finally, complete a geometric sequence for the population data you created for your own country, or the My Country data in Lesson 5.

Students complete the table using the data they created for My Country. The data in the following table is based on the fictitious My Country introduced in the Teachingr Notes of Lesson 5. It continues to provide an example of what happens to a top-layered country. The data and the resulting model are intended to provide an opportunity to discuss how a model affects different types of countries.

My Country

Year	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
Population (in millions of people)	163	157 x 0.963	151 x 0.963	145 x 0.963	139 x 0.963	134 x 0.963	129 x 0.963	124 x 0.963	119

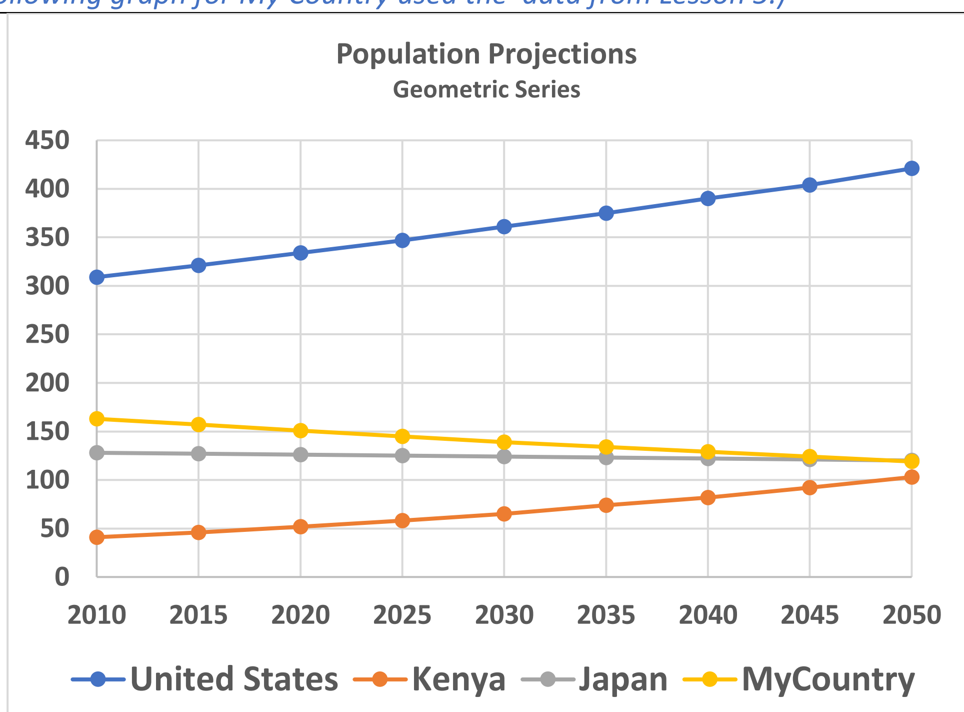
The constant factor used in the example of My Country is $\frac{157}{163}$ or approximately 0.9631 which is approximately a decrease of 0.0368 or 3.68% per 5-year period. A yearly estimate of 0.0074 or a decrease of 0.74% per year is used in the following model.

An exponential mode for the above is:

$$y = 163(1 - 0.0074)^{x-2010} \text{ or } y = 163(0.9926)^{x-2010}$$

20. Use the values you derived for each geometric series of the United States, Kenya, Japan, and My Country data to graph their population projections.

(The following graph for My Country used the data from Lesson 5.)



Population in millions of people
My Country example – students use their own country data

Recap the exponential model by recording the 2050 estimates to the nearest millions of people for each country . Compare these estimates to the United States Census estimates.

Projections of the 2050 population projects:

	Census Bureau model	Linear model	Exponential model
United States	398,328,349	405 million people	*421 million people*
Kenya	70,755,460	81 million people	*103 million people*
Japan	107,209,536	120 million people	*120 million people*

Similar to the linear models developed in Lesson 8, the exponential models derived in this lesson provide estimates of the total population for the United States, or Kenya, or Japan. In this current form, the models do not provide estimates of the age groups that make-up a population.

Assessment ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of the lesson.

Oostburg is a village in Wisconsin that had a population of 1000 people in 2010 and a population of 1050 in 2015. In addition, Milwaukee, Wisconsin had a population of 600,000 in 2010. Milwaukee's population declined 4% from 2010 to 2015. Changes in the population are anticipated to continue for the next several decades for both Oostburg and Milwaukee. If geometric sequences are derived for both Oostburg and Milwaukee to estimate their population counts for 5-year periods, determine the following:

a. What is the common ratio or constant factor for the geometric sequence of Oostburg for a 5-year time frames?

b. What is the common ratio or constant factor for the geometric sequence of Milwaukee for a 5-year time frames?

c. Is it possible that in the future Oostburg might be larger than Milwaukee? Explain your answer.

d. Why is it unlikely that the population changes for Oostburg and Milwaukee will continue to change as they did from 2010 to 2015?

Comments on the Assessment Task:
The assessment task directs students to summarize and interpret the main components of a geometric sequence. First challenge is to derive the growth rate for Oostburg in a 5-year period of time. The change in population was 5%, thus the common ratio or common factor is 1.05. The decline for Milwaukee indicates that the common ratio or constant factor is 1 – 0.04 or 0.96 for a 5-year period. It is important for students to distinguish that the rate of growth is added to 1 when the population grows and subtracted from 1 when the population declines. With Oostburg increasing in count, and Milwaukee decreasing in count, it is possible that their population counts will equal each other provided the common factors do not change over time. However, it is very unlikely for all conditions to remain the same, given that it would take a long time for the population values to reach the point where Oostburg would exceed Milwaukee. Students are expected to interpret when a population increases and when a population decreases, and the unlikely time frame for conditions that change each population to remain constant.

Additional Assessment Ideas:
The lesson concludes with a summary of why linear models or an exponential model would not continue indefinitely. At some point, a country cannot continue to add population counts without affecting its ability to support the population, or a country cannot continue to lose population without going out of existence. As a formative assessment question, ask students to start describing what might happen in the United States that would slow down the counts or even cause a decrease in the population? (For example, economic conditions that result in having a family very expensive.) Also, what might happen in the United States that would accelerate its growth? Record their ideas from small group discussions. Revisit these ideas as the next model (the recursive model) is developed.

Teaching Notes

Lesson 10

Looking Forward with a Recursive Model (Present to Past … to Future)

Overview:

An important tool in designing the ***Recursive Model*** is a population factor. The age group 0 - 4 years old in 2010 is counted to the 5 - 9 years old age group in 2015. The two age groups are defined as connecting age groups. A population factor is calculated by deriving a ratio of connecting age groups and representing that ratio as a decimal. The story behind the population factors is that:

- factors greater than 1 identify connecting age groups in which the change in population during a 5-year period is dominated by immigration,
- factors less than 1 identify connecting age groups in which the change in population during a 5-year period is dominated by death or emigration.
- factors equal to 1 indicate no change in the population of the age groups during a 5-year period.

The problems and questions use the 2010 to 2015 population data to form the ratios from connecting age groups and then summarize what the ratios indicate about the changes in age groups. The subtitle (***Present to Past … to Future***) is based on the connection that the count of people in 2015 (considered the present in this module) is compared to the count of people in 2010 (the past). The ratios are then changed to decimals and percent that will be used to predict the future. Students are continuously asked to explain what these ratios mean about the changes in the count of important age groups.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. Students are asked to interpret the implications of the population factors and what they indicate about the changes in the population of a country. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1,7	Problems: 2, 8, 11, 13, 14	Problems: 3, 4, 5, 6, 9, 10, 12	Problems: 15, 16, 17, 18, 19, 20, 21

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 10 for each student. This lesson requires 2 additional handouts for students to complete the problems, **Handout 1: *United States – 2015*** and **Handout 4: *United States Connected Age Groups***. Provide an electronic or a printed copy of the handouts.

Launch:

Begin this lesson by directing students to read ***Kristin's Story – Chapter*** 6. After they have read the story, ask them questions about the characters in the story. Consider the following questions:

- What age was Kristin in 2010?
- What age was Kristin in 2015?
- Was Raphine counted in the 2010 census of the United States? Why or why not?
- In what age group was Adeline counted at the start of 2015?

Work with students who have problems answering these questions by highlighting the details in the story that answer these questions.

Consider forming small groups to work on these problems as discussions among students might help them understand the key points of this important lesson. If the problems are assigned individually, consider providing more completed answers to **Handout 4**. The design of **Handout 4** is to promote small group work.

Implementation ideas:

The problems are designed to build students' understanding of the lesson's objectives. Note several summaries are provided to organize the objectives in the student lesson. Use these summaries as opportunities to have a brief whole group discussion of the problems. Wrap up this lesson with a discussion of the questions highlighted in the assessment ideas.

Student responses or descriptions

Lesson 10 - Problems

1. Examine again the **Handout** 1: ***United States - 2015***. In what age group was Kristin and Raphine counted in 2015?
   Kristin and Raphine are in the 35 – 39 years old age group in 2015.

2. Would a summary of the United States population in 2010 include Raphine? Explain your answer.
   No. Raphine moved to the United States in 2011, therefore, at the start of 2010, he was not counted in the United States population.

For the following problems, use **Handout 4: *United States Connected Age Groups.***

3. At the start of 2010, there were 20,189,589 people who were 0 to 4 years old. At the start of 2015, there were 20,481,130 people who were 5 to 9 years old. What is the connection of these two age groups?
   The people who were 5 - 9 years old in 2015 are for the most part the people who were 0 – 4 years old in 2010. After the start of 2010, the count of people in the 0 – 4 years old age group changed by people moving into the country or moving out of the country or dying. At the start of 2015, the resulting count of people in the age group 5 – 9 years old suggests what happened to the 2010 age group of 0 – 4 years old. Note, there were more people counted in the age group 5 – 9 years old in 2015 than in the 0 – 4 age group at the start of 2010.

4. At the start of 2010, there were 21,983,206 people 15 to 19 years old. In what age group will these people be counted at the start of 2015? How many people were counted in that age group?
   The group of people in the 15 to 19 years old in 2010 will be in the 20 – 24 years old group in 2015. There were 22,693,026 people in that age group.

5. Adeline was born in 2012. In what age group was Adeline counted in 2010? Explain your answer.
   She was not counted at the start of 2010 as she was not born yet.

6. In what age group would Adeline be counted in 2015?
   Adeline would be counted in the 0 – 4 years old age group in 2015. She would turn 3 years old during that year.

Examine the age groups that are described as connected age groups in ***Handout 4***. This handout indicates a connection of the age group 0 – 4 in 2010 to the age group 5 – 9 in 2015. For the recursive model developed in this module, these age groups are called ***connected age groups***.

7. What is the ratio of the count of people 5 to 9 years old at the start of 2015 to the number of people 0 to 4 years old at the start of 2010?
   The ratio is 20,481,130 people to 20,189,589 people.

8. The ratio is represented by a decimal in column 4, or 1.014 to the nearest thousandth. As stated in problem 7, this decimal is defined as the **population factor** for the connected age groups. This population factor is greater than 1 for this example. What does that tell you about the connected age groups?
   A population factor greater than 1 indicates that the count of people in the 5 – 9 years old age group in 2015 is greater than the count of people in the 0 – 4 years old age group in 2010. The increase is explained by immigration as that is the only way that people in the connected age groups could increase the counts of people.

9. During the 5 years summarized on the table, what is the approximate percent increase of people 5 - 9 years old in 2015 based on the count of people who were 0 – 4 years old in 2010? Is the percent increase also part of the population factor for these connected age groups? Explain.
   First step is to determine the difference of the counts, or:
   20,481,130 – 20,189,589 = 291,541 people.
   Divide the above difference by the people in the 0 – 4 years old age group, or
   291,541 people / 20,189,589 = 0.014 to the nearest thousandth.
   Convert to a percent, or 1.4%
   Note the population factor for these age groups is 1.014. This factor also indicates the count of people in the connected age groups will increase by 1.4%.

10. What is the explanation for the growth in the connected age groups with a population factor greater than 1?

    Connected age groups with a population factor greater than 1 indicate that the increase in the counts were based on immigration or people moving into the country.

11. During the 5 years summarized on the table, what is the approximate percent of change of the count of people 55 – 59 years old in 2015 who were 50 – 54 years old in 2010?
    Count of people 55 – 59 years old in 2015 is 21,767,855, and the count of people 50 – 54 years old in 2010 is 22,353,471. The difference is:

    (21,767,855 – 22,353,471) = - 585618

    The rate of change: -584618/22,353,471 which is approximately -0.026 to the nearest thousandth. The percent change is approximately – 2.6%. Note, the percent change is negative. The population factor would be (1 – 0.026) or 0.974.

12. What is the explanation of the changes in the connected age groups with a population factor less than 1?
    The changes in the count of the connected age groups are a result of people moving out of the country or dying during the 5 years from 2010 to 2015.

Complete the calculations missing in ***Handout 4***. After you have completed the handout, answer the following questions:

To check students' work, see **Handout 4 *(Teacher Edition).***

13. What is noticeable about the population factor for the connected age groups of 40 – 44 years old in 2010 to 45 - 49 years old in 2015 when compared to the population factors for younger connected age groups??

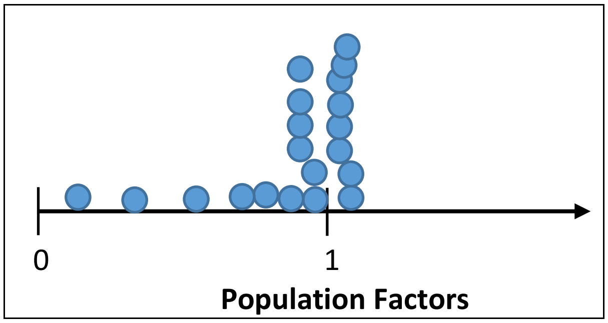
    The population factor of these two connected age groups is less than 1 (the first population factor less than 1).

14. What happens to the population factors for connected age groups that count people 40 years old or older in 2015?

    The population factors for connected age groups in which people are 40 years old or older are close to 1 or less than 1.

Any change in the count of people counted after the start of the year are a result of people moving into this country (immigrating), people leaving this country (emigrating), and dying. The following problems examine the collection of population factors for the United States.

15. Place a dot for each of the population factors derived on **Handout 4** on the above Population Factors number line. (Stack dots if they are close to each other.)



(Essentially look for 8 dots slightly greater than 1, 6 dots slightly less than 1, and 6 dots less than 1 as estimated in the above example.)

16. Why is 1 considered an important value in interpreting a population factor? Explain your answer.
A population factor of 1 indicates the count of the age group did not change during 5-years. This result suggests that no one died, or the number of deaths was equal to the number of people moving into the country. Any factor greater than 1 indicates the population increased. Any factor less than one indicates the population decreased.

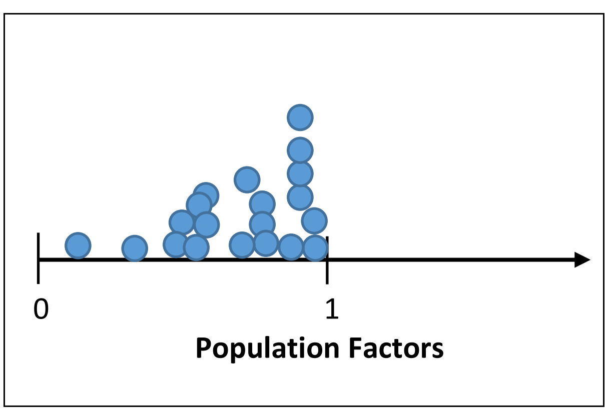
17. Is it possible for the population factor of connecting age groups be equal to 0? Explain your answer.
Yes. If everyone after the start of the year in an age group died or moved out of the country, and no one moved into the country during the 5-year period, the count in the connecting age group would be 0.

18. Changes in the count of people over a 5-year period of connected age groups are explained by **birth**, **death**, **immigration**, and **emigration** (people leaving a country). Use the value of the population factor to Identify what changed the counts in the following Connected Age groups from 2010 to 2015. Identify the most dominant explanation for the changes in the connected age groups in the last column. (The first connecting age group is completed for you.)

Connected Age groups from 2010 to 2015	Population Factor for the connected age groups	What could explain the changes in the count of people in the connected age groups?	What was the dominant explanation of the change in the connected age groups?
0 – 4 to 5 – 9	1.014	Immigration, emigration, deaths	Immigration
20 – 24 to 25 – 29	*1.032*	*Immigration, emigration, deaths*	*immigration*
40 – 44 to 45 – 49	*0.995*	*Immigration, emigration, deaths*	*deaths*
85 – 89 to 90 – 94	*0.509*	*Immigration, emigration, deaths*	*deaths*
95 to 99 to 100+	*0.205*	*Immigration, emigration, deaths*	*deaths*

19. Summarize what a population factor indicates about the connected age groups.
Population factors provide an explanation for the change in counts of connected age groups. If a population factor is less than 1, then the dominate explanation for the change is death or people moving out of the country or both. People could still move into the country, but the count is less than the count of people dying and moving out of the country. If a population factor is greater than 1, then the dominate explanation for the change is immigration. People could still move out of the country or die, but the counts are less than the count of people moving into the country. A population factor equal to 1 indicates that the counts that decrease connected age groups by death and leaving the country equal the count that increases the age group by immigration.

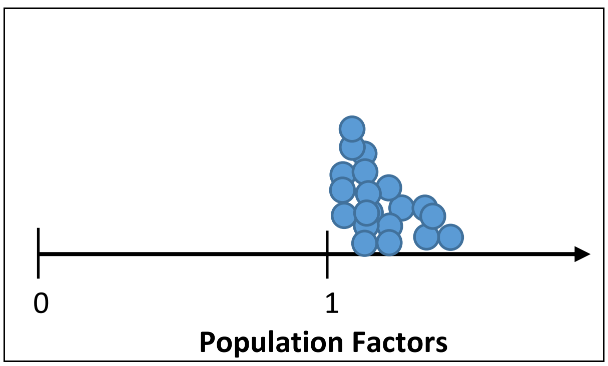
20. Consider the following dot plot of the population factors of a fictitious country:



a. What is the dominant explanation of change in the connecting age groups for a country represented by the above dot plot?
The dominant explanation of change is death or people leaving the country.

b. Do you think it is possible for a country with the above population factors to have an increase in its total population during a 5-year period? Explain.
Yes. Population factors do not provide a summary of births or immigration in the 0 – 4 years old age group during the previous 5 years. There is no connecting age group to the 0 – 4 years old age group. As a result, a large count of births or possibly a large count of children who immigrated into the country could still result in a total population increase.

21. Consider the following dot plot of the population factors for another fictitious country::



a. What is the dominant explanation of change in the connecting age groups for the above dot plot?
The dominant explanation of change is immigration

b. Do you think it is possible for a country with the above population factors to have a decrease in its total population during a 5-year period? Explain.
The above dot plot would suggest a country with all population factors greater than 1 would increase its population. In most cases, this summary is accurate. However, if the country had an unusually large count of older people (for example 90 - 94 years old, 95 – 99 years old, 100+ years old) and an unusually small count of younger people in 2015, the changes in 5 years could decrease. Over a longer period of time, the counts increase. (After students have an understanding of the recursive models as organized in the EXCEL files, you may want to return to this question and allow students to play around with the 2015 counts to see if they can configure a 2015 population that results in a declining population in 2020.)

How can the population factors be used to estimate future counts, or population projections? Applying the above population factors to the actual count of people in 2015 will start the recursive model.

Assessment ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of this lesson.

The city of Awesome had 1000 people who were 0 – 4 years old in 2015. The population factor for the connecting layer of 0 – 4 years old to 5 – 9 years old is 1.05.

a. What is an estimate of the count of people for the 5 – 9 years old in 2020?

b. Explain the reasons behind the change in population for 0 – 4 years old to 5 – 9 years old.

c. The population of 85 – 89 years old in 2010 was 200 people. The population of people who were 90 – 94 years old in 2015 is 300 people. What is the population factor for these connecting layers?

d. What do you know about the changes in the connecting layers in (c)? What reasonable explanation might explain the changes?

Comments on the Assessment Task:

The questions expect students to recognize that a population factor of 1.05 indicates an anticipated increase of 5% of the population in the connecting layer of 5 – 9 years old for 2020, or a 5% increase in the count of 1000 people. An estimate of 5 – 9 years old in 2020 would be 1050 people. The primary reason for the change would be more people moving into the country during the 5-year interval.

The population factor for (c) would be 1.5 as there is a 50% increase in the population during the 5-year period. An explanation might be related to a health issue or a lifestyle issue. For example, this country may have a particularly attractive climate for older people, resulting in older people moving into the country.

Additional Assessment Ideas:

Students' explanations to questions 19 – 21 provide a key insight to their understanding of population factors. What a population factor indicates about a connecting age group is a main component of the recursive model that will be continued in the following lessons.

Do not necessarily expect students to correctly explain whether or not a country will increase or decrease in population over time as that is related to an understanding of another feature of this recursive model, the foundation factor. If students' answers do not address the count of the 0 – 4 years old age group in the future, revisit the questions about when Adeline was counted in this lesson to remind them that this age group impacts the total population differently than the connected age groups. The Foundation Factors will be developed in the next several lessons and will complete the design of the recursive model. Questions 19 – 21 are designed to have students think about all of the age groups, and their possible impact on the country's population.

Consider assigning an **Exit Summary** for this lesson.

Teaching Notes
Lesson 11
The Recursive Model and Falling Dominos

Overview:

This lesson is the first of two lessons that develop the Recursive Model referenced throughout this module. It builds off the ***Population Factors*** derived in the previous lesson. Counts for each of the age groups in 2020 (except the 0 – 4 years old age group) are determined by multiplying the appropriate population factor to the count of the connected age group. This process generates estimates for the 2020 population. These estimates in turn are used to derive the 2025 counts, followed by the 2030 counts, and continued until 2050. It is this process that indicates the recursive or iterative design of this model.

Why is this important? Unlike the linear or exponential models, this model provides students an estimate of the counts of the underlying age groups, and as a result, the new shape of the country after the process is completed. By first looking back, this model also looks forward. This particular model is built on applying the population factors as an indicator of what happened over the last five years that changed the counts of each age group. The model is completed in Lesson 12 with the design of the ***Foundation Factor*** and its role in estimating counts of age groups left blank in this lesson.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 4	Problems: 2, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16	Problems: 10, 12, 17, 18	Problems: The extension

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 11 for each student. This lesson also requires 1 additional handout for students to complete the problems, **Handout 5: *Looking Forward for the United States (Student Edition)*.** Provide an electronic or printed copy of the handout.

Launch:

Consider starting this lesson with actually setting up dominos in such a way that when the first domino falls, the next domino falls, and then the next, until the last domino is down. Several online videos of falling dominos are available to visualize this idea. A similar process is set-up in this lesson involving projecting counts in the future that resemble falling dominos, namely, the interconnection of one age group to another age group, to another, etc.

Review with students before they begin the problems the main characters in the story and their ages. Specifically, have a brief discussion that reminds them that at the start of 2015, Kristin is 36 years old, Adeline was 2 years old, Abbey was 26 years old, and Kristin's mother was 66 years old. The problems in this lesson will continue working with the past and future ages of these characters.

Implementation Ideas:

Similar to the previous lesson, the problems are designed to build students' understanding of the lesson's objectives. Note several summaries are provided to organize the objectives in the student lesson. Use these summaries as opportunities to have a brief whole group discussion of the problems. Wrap up this lesson with a discussion of the questions highlighted in the assessment ideas. The suggested assessment task utilizes an Excel file.

Student responses or descriptions

Lesson 11 - Problems

Recall from the Student Edition the following:

(Population of age group 0 – 4 years old 2010) X *(Population Factor for connected age groups)* =>

(Population of age group 0 – 4 years old 2010) X $\frac{Population\ of\ age\ group\ 5-9\ for\ 2015}{Population\ of\ age\ group\ 0-4\ for\ 2010}$ = *Population of age group 5 – 9 for 2015*

↑

(Population Factor for connected age groups 0 – 4 to 5 – 9 years old)

1. Complete the multiplication indicated in the following table. The first calculation has been completed for the 0 – 4 age group of 2010 to the 5 – 9 age group of 2015 and set-up in the above equation. The population factors listed were derived in the previous lesson.

Population of Age group **2010**	X	Population Factor	=	Population of Connected Age Group **2015**	Census Bureau counts for **2015**
0 – 4 20,189,589	X	1.014	=	**5 – 9** 20,472,243	**5 – 9** 20,481,130
5 – 9 20,331,807	X	1.013	=	**10 – 14** *20,596,120*	**10 – 14** 20,605,579
10 - 14 20,681,214	X	1.020	=	**15 – 19** *21,094,838*	**15 – 19** 21,084,710

2. Why are the answers in problem 1 not equal to the Census Bureau estimates?
   The population factors were rounded off. As a result, any calculations based on these factors will be different from the actual counts of the Census Bureau.

3. Examine the connected age groups of 0 – 4 years old in 2010 to 5 – 9 years old in 2015. A population factor of 1.014 indicates an approximate increase of 1.4% in the 5 – 9 years old age group in 2015 when compared to the 0 – 4 years old age group in 2010.

   a. Using the above summary of the changes in these age groups, is it possible to derive the number of people who moved into the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.
   No. The population factor of 1.014 indicates an overall increase but it does not provide a specific breakdown. From 2010 to 2015, the connecting age groups had more people moving into the country than people moving out or dying, however, the specific counts of people moving in compared to the count of people moving out or dying is not possible to determine from the population factor.
   b. Is it possible to derive the number of people who moved out of the country from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.

   No. A student's answer indicates that the overall change does not reflect a specific breakdown of the change in the count of people.
   c. Is it possible to derive the number of people who died from 2010 to 2015 for the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain why or why not.
   No. A student's answer indicates that the overall change does not reflect a specific breakdown of the change in the count of people.
   d. What was the dominate explanation for the change in the number of people from 2010 to 2015 of the connected age groups of 0 – 4 years old to 5 – 9 years old? Explain your answer.
   The population factor indicates an overall increase in the count. This increase could only occur through immigration. Immigration is the dominate explanation of the changes in the population, however, emigration and dying also changed the count of people in the 5 – 9 years old age group.

4. Population factors listed in the following table were recalculated based on the approximations of millions of people in 2010 and 2015. The 2015 population estimates were also rounded off as indicated. Complete the following table. Several age group projections have been completed. (Note: Some of the posted estimates may differ from your calculations due to the software used to implement the recursive model. The differences are minor. Allow a difference of +/- 0.01 of your answers and the answers posted.)

Age group	Population 2015 (millions to the nearest hundredth)	Population Factor	Population 2020 (millions to the nearest hundredth)
0 – 4	19.91	1.014	
5 - 9	20.48	1.014	20.20
10 - 14	20.61	1.020	20.76
15 - 19	21.09	1.032	21.02
20 - 24	22.69	1.032	21.77
25 - 29	22.40	1.022	*23.42*
30 - 34	21.62	1.012	22.90
35 - 39	20.31	1.004	*21.88*
40 - 44	20.16	0.995	*20.39*
45 - 49	20.80	0.985	20.05
50 - 54	22.29	0.974	20.48
55 - 59	21.77	0.962	*21.71*
60 - 64	19.04	0.945	20.93
65 - 69	16.05	0.917	*17.99*
70 - 74	11.48	0.869	*14.72*
75 - 79	8.12	0.792	*9.98*
80 - 84	5.80	0.670	*6.43*
85 - 89	3.86	0.508	3.89
90 - 94	1.85	0.340	*1.96*
95 – 99	0.50	0.205	*0.63*
100 +	0.08		*0.10*

5. Recall that Kristin was 36 years old at the start of 2015. In what age group would she be counted in 2020?
   Kristin would be counted in the 40 – 44 years old age group in 2020.

6. One **age group** for 2020 (the shaded cell) is left blank by this process. What age group is left blank? Explain why this age group does not have a population factor for this recursive model. (This particular age group is called the **Foundation Layer** and will be the focus of the next lesson.)
   The age group left blank is the 0 – 4 years old age group for 2020. To determine the count of this age group, we need to know how many people were born during the last 5 years (or from 2015 to 2019), along with how many people moved into the country, how many people died, and how many moved out of the country during the last 5 years who were 0 – 4 years old. This group of people were not previously counted. As a result, there is no connecting age group and no population factor can be set-up.

7. One **population factor** also has no entry in the above table. Why is there no population factor entered for this age group?
   There are no population factors for the age group 100+ years old (or people who will turn 105 or older in the next 5 years). There are definitely people older than 105 years old in the country, but the count is so small in comparison to the total population that we generalize the counts in the 100+ age group.

The Falling Dominos

8. Record the missing 2020 projections in the column identified as 2020 in **Handout 5**. Be careful not to provide a projection for the age group 0 - 4 years old in 2020.
   Check results with the Teacher Edition of Handout 5.

9. Use the Population Factors to complete the projections for 2025 from the 2020 projections based on the previous 5-year projections. Several calculations have been completed in the handout. (Remember, round your answers to the nearest hundredth. Each value indicates the count of people in millions.)
   Check results with the Teacher Edition of Handout 5.

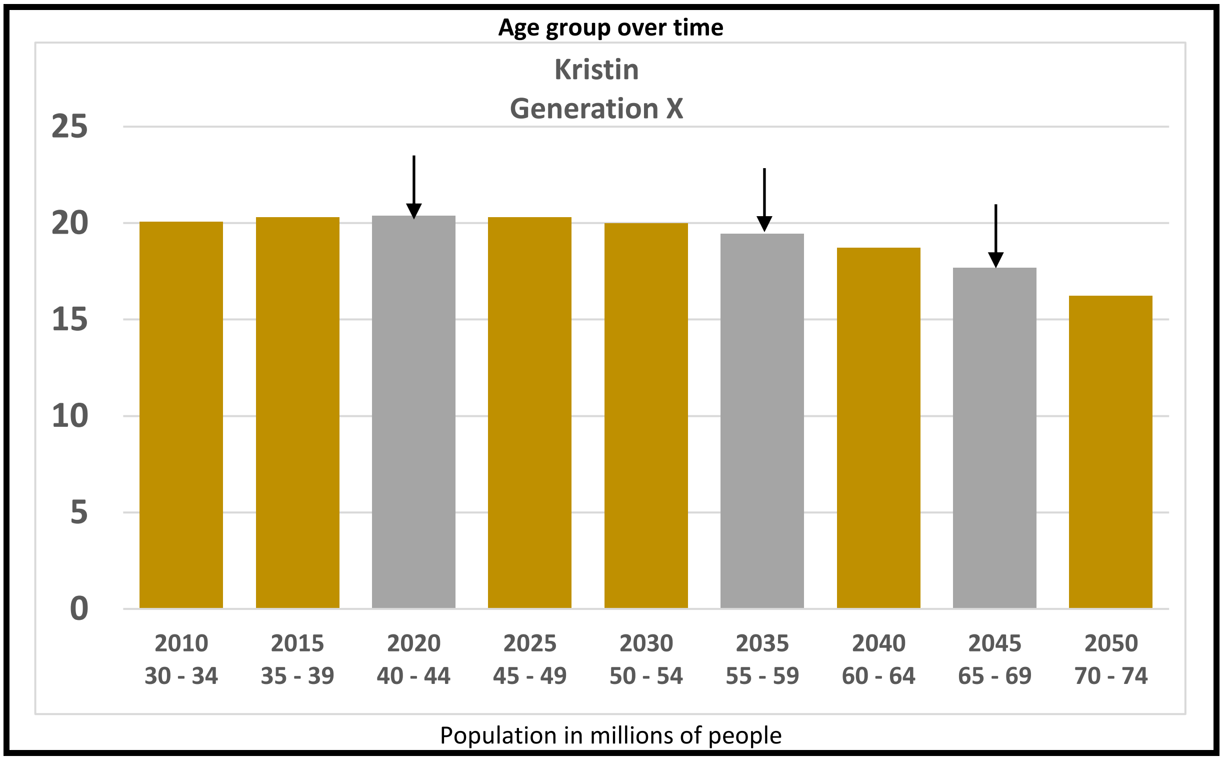
10. In 2020, there was one age group that was left blank. In 2025, there are two age groups that are blank. Why is this second age group blank?

    As there was no estimate of the 0 – 4 years old age group in 2020, there would be no estimate for the 5 – 9 years old age group in 2025. In addition, the count of the 0 – 4 years old age group in 2025 is not connected to any age group, so for the same reasons as indicated previously, that age group also remains blank (for now).

11. Complete the missing calculations in **Handout 5** for the years 2030, 2035, 2040, 2045, and 2050. Again, several calculations have been completed. Use the completed calculations to check your work.
    Check results with the Teacher Edition of Handout 5.

12. Use your calculations from **Handout 5** to complete the following bar graph of Kristin's age groups. Recall that Kristin was 36 years old in 2015.

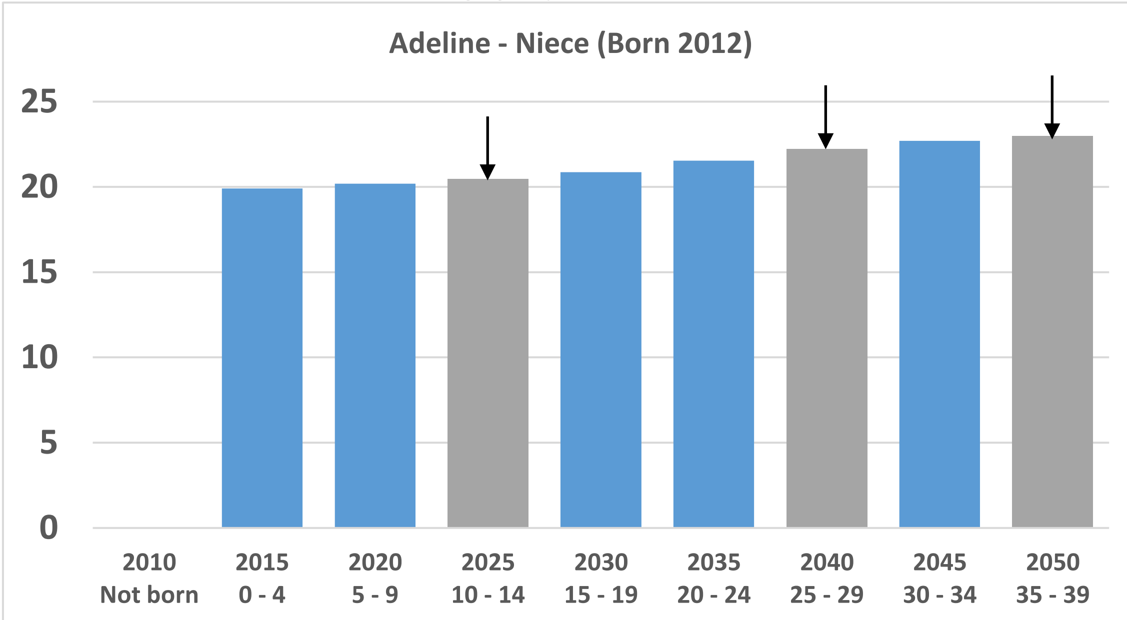
    Missing age groups 40 – 44, 55 - 59, and 65 – 69 years old are included on the following graph:



13. Recall that Kristin's niece Adeline was born in 2012. In what age group is she counted at the start of:
    a. 2015?
       Age group 0 – 4 years old
    b. 2020?
       Age group 5 – 9 years old
    c. 2035?
       Age group 20 – 24 years old
    d. 2050?
       Age group 35 – 39 years old

14. Using **Handout 5,** complete the following bar graph representing Adeline's age groups from 2015 to 2050:
    Missing age groups 10 – 14, 20 – 24, and 30 - 34 years old are included on the following graph:

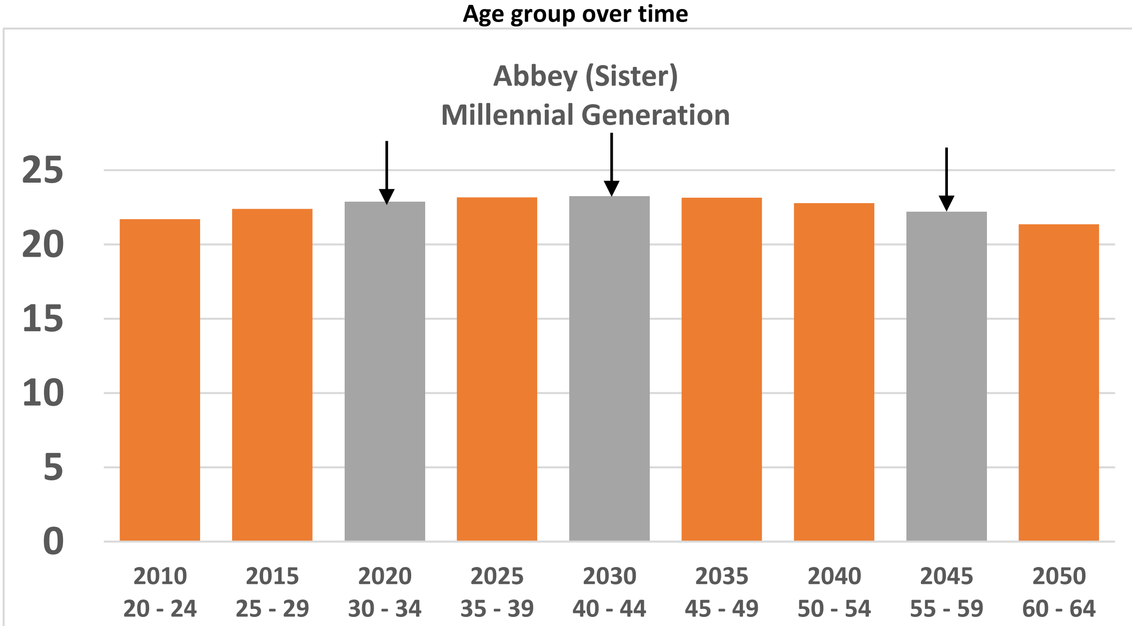
Age group over time



Population in millions of people

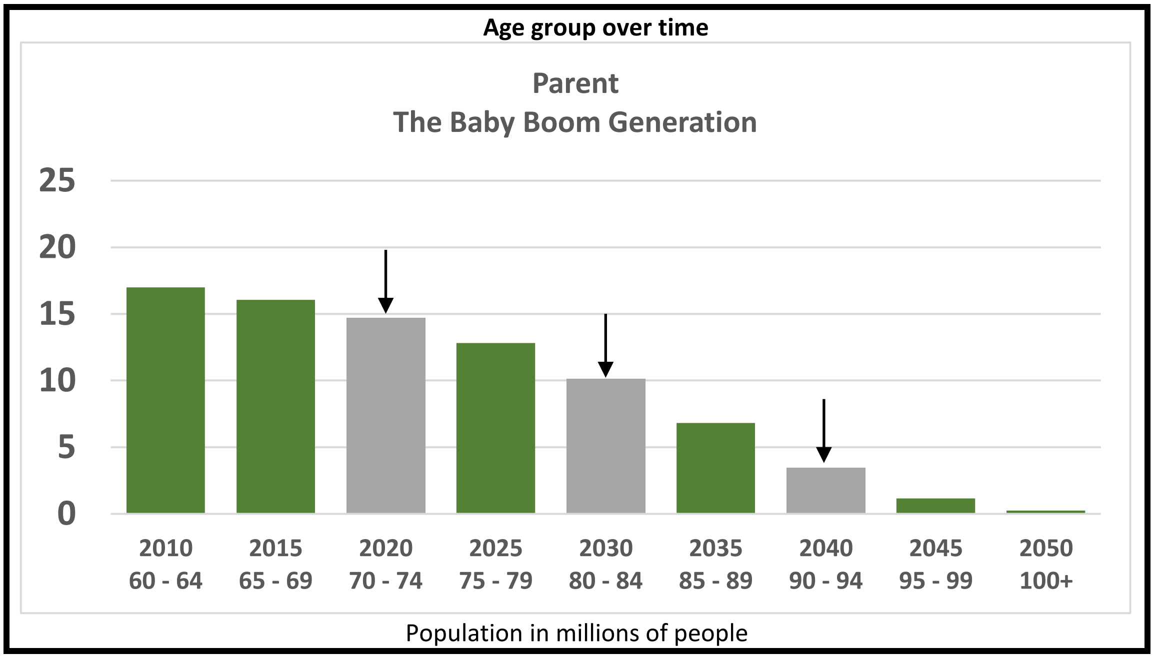
15. Using **Handout 5,** complete the following bar graph representing Kristin's sister Abbey age groups from 2010 to 2050. Recall that Abbey was 26 years old in 2015.
    Missing age groups 30 – 34, 40 – 44, and 55 – 59 years old completed on the following graph:

Age group over time



Population in millions of people

16. Using **Handout 5**, complete the following bar graph representing Kristin's mother's age groups from 2010 to 2015. Recall that Kristin's mother was 66 years old in 2015.
    Missing age groups 40 – 44, 50 – 54, and 60 – 64 years old completed on the following graph:



17. Examine the completed graph for Adeline. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?
    The counts of the age groups that include Adeline increase during each 5-year period. The first count at the start of 2015 was approximately 20 million people. The count at the start of 2050 for the age group 35 – 39 years old is projected to be approximately 23 million people. The primary reason for the increases is fewer deaths and higher counts of immigration.

18. Examine the completed graph for Kristin's parent. Summarize the changes of the count of people in her age group. What is the primary explanation for these changes over time?
    The counts of the age groups that include Kristin increase slightly from the start of 2010 to 2020. After that, each age group that counts Kristin decreases. The count in 2010 was approximately 20 million people for 30 – 34 years old. The count of the age group 70 – 74 years old in 2050 is projected to be approximately 16 million people. The decrease in the count of people in each age group after 2020 is due to the number of deaths.

Assessment Ideas:

Assessment Task:

Consider the following assessment and assessment ideas for this lesson. Students should have access to a computer lab and Excel or a compatible spreadsheet program to complete the rest of this module. Provide time for students to work with the first introduction of the recursive model. An Excel file entitled "The 1 Country.xls" is designed to illustrate the implications of changes in age groups resulting from the population factors over time. This file is included in the Projection Files section of the Teacher Edition.

After providing the file to all students with access to a computer, draw attention to the initial set-up of the imaginary country (the "1 Country"). All population factors and counts for 2010 and 2015 and subsequent years are 1. Ask students to explain what a population factor of 1 means. The only cells that are not 1 are the cells that will be impacted by the Foundation Factor introduced in the next lesson, or age groups that do not have a count in the previous 5-year period. Discuss the following 3 options (all provided on the Student Edition for this lesson) for students to explore with the spreadsheet. Consider assigning a different option to each student or small group of students.

Option 1:

Before you change the values in a specific cell, describe the total population of this imaginary country from 2010 to 2050.

What if it was determined that a person in this country who was 36 years old at the start of 2015 was not counted? Enter the value of 2 in the appropriate cell of the spreadsheet for "1 Country?"

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the 2015 count for the age group 35 – 39 changes the population factor for that age group to 2. This indicates that every previous count for this age group is multiplied by 2 (or population doubles in those cells). The overall population increases to 22 for 2015. Although cells increase as a result of this change over time, the top cells also change to 0 (add 1 to a cell and subtract 1 to another cell). The total population remains at 22 for each 5-year period. The domino effect is visible by observing the changes of the cells based on the change in the population factor. Ask students what this indicates about a country if the 2 represented 2 million people.

Option 2:

What if it was determined that a person in this country who was 36 years old at the start of 2010 was not counted? Enter a 2 in the appropriate cell of the spreadsheet for "1 Country"?

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the 2010 count for the age group 35 – 39 years changes the population factor for that age group to 0.5. The overall population goes down to 1.5 people during each 5 years (not possible unless the values are represented by a different unit – for example, what if the values represented 1 million people?) The domino effect is again visible. The changes would be a result that as the counts moved forward, the population factor indicated that less people were counted due to death or moving out of the country.

Option 3:

What if a person who was 36 years old moved into the country and was counted in the country at the start of 2020? Enter a 2 in the appropriate cell of the spreadsheet for "1 Country?"

Answer the same questions for this scenario.

1. Describe the total population of this imaginary country from 2010 to 2015.
2. What cells changed in the spreadsheet over time?
3. Explain why the cells you identified changed.

Redo your entry and set the count in the cell you changed back to 1.

The entry of 2 in the cell counting the 35 – 39 years old people in 2020 does not change the population factor – only the count in that one cell. The additional person carries through each 5 years, or the count of this person is noted in the next age group. The overall population increased by 1 from the population values before any changes were made. The domino effect is visible with this change.

Wrap up this assessment task with the following question:

- What is the connection of the calculations in this lesson to falling dominos?

Students suggest that the connection to dominos is that as one calculation is completed to estimate the count for an age group, several connected calculations need to be redone (like *falling dominos). The process continues like the impact of one falling domino causing another domino to fall and then another.*

If a spreadsheet application is not available, direct students to complete an **Exit Summary (Handout 13)** for this lesson.

Teaching Notes

Lesson 12

Completing the Recursive Model with the Foundation Layer

Overview:

This lesson completes the development of the Recursive Model. Counts for each of the other age groups in 2020 and beyond were derived in Lesson 11 by multiplying the appropriate population factors and the counts of the connected age groups. The foundation count, or the count of the 0 – 4 years old age group in 2020, has a major impact on a country's future. The previous lesson provided an estimate of the total of the 2020 population except for the people who were born or moved into the country within the ages of 0 – 4 years old after the start of 2015. The "hole" (or blank cell) left by the unknown estimate of the 0 – 4 years old is filled in this lesson by making a second important assumption, namely, that the proportion of the total population who were 0 – 4 years old at the start of 2015 will be the same proportion at the start of 2020. This assumption is continued to the conclusion of this model, or 2050.

The domino effect is again visible. Filling the hole of the foundation age group in 2020 is then passed to the next iteration of estimating counts. In the same way, the next iteration for 2025, and then the other years highlighted in the handout, are derived. The handout that students complete has color-coded the cells representing the characters in Kristin's story. Students are encouraged to think about how the projected estimates (or the numbers) will impact these characters in the future. Throughout this module, reflecting on the impact of "crunching the numbers" on the characters was intentional.

This lesson further pushes students' thinking to levels 3 and 4 of the **Modeling Continuum**. An alignment of the problems in this lesson to the **Modeling Continuum** are suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 1, 2,3	Problems: 4, 5, 6	Problems: 7, 8, 9, 10, 11, 12, 13, 14, 15, 16	Problems: 17, 18, 19, 20, 21

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 12 for each student. This lesson also requires 1 additional handout for students to complete the problems, **Handout 6: *United States 2010 – 2050 (Student Edition)*.** Provide an electronic or printed copy of the handout.

Launch:

Start this lesson by discussing the data story, **Kristin's Story – Chapter 7**. This chapter highlights new characters who are important in the problems. Estimating the count of children who are born or under the age of 5 is derived differently in the recursive model. As mentioned in previous lessons, the young people in that category are not connected to a previous age group. They are the people who will be listed in the next population summary as 0 – 4 years old, but at the present, they are not counted. Incorporating this group of people into the model poses an interesting challenge and is the goal of this lesson. Who are these children, and how do they impact the future shape of the country?

Implementation Ideas:

After students read the data story, direct them to complete the problems individually or in small groups. Summaries or highlights of several problems are provided to help determine if students are understanding the material. Provide opportunities for students to verbally summarize their responses or answers of the problems. This lesson addresses an important feature of the recursive model that will impact the age group estimates for several decades.

Consider organizing a bulletin board highlighting the images of people who are considered the decision-making leaders, the entertainment or sports leaders, or the general cultural leaders of the country. Also consider including in the display advertisements designed to sell clothes, furniture, cars, food, etc. (Collect either printed ads or images from online advertisements.) Ask students to identify the target age groups of these ads. Would people 65 years old or older be interested in the ads? Would people in their 20's be interested in the items or services these ads are trying to sell? The dominate age groups (or the age groups with the greatest count of people) are a driving force of a country's economy and culture. Often the entertainment industry primarily targets the dominant age groups. If an age group is targeted that is not the dominant age group, then the challenge is even more critical that the targeted age groups are motivated by the ads. Continue these discussions throughout this lesson and the lessons that follow.

Lesson 12 – Problems

1. Adeline was born in 2012. What is her age at the start of 2015?
   She would be turning 3 years old in 2015.

2. In what age group would Adeline be counted in the 2015 histogram?

   She would be counted in the 0 – 4 years old age group of 2015.

3. Estimate the number of young people counted in that age group.
   Based on the histogram, there were approximately 20 million people in that age group.

4. Based on Kristin's story, what do you know about Mathew's young sister and Dominic's younger brother at the start of 2015?

   They were born after 2015 and before 2020. As a result, they will not be counted on a population graph until 2020.

5. Most population graphs prepared by the United States Census Bureau use the 5-year age groups represented in the above histogram. There are people not counted in the 2015 histograms who are part of the population from 2015 to 2019. Describe these people.

   The people who are not counted in the 2015 population would be people born during 2015 to 2019 or people under 5 years old during that time who moved into the country. They would be in the age group 0 – 4 of the 2020 graph as that graph counts the people who are 0 – 4 years old at the start of 2020.

6. Assume a histogram of the actual count of people was prepared in 2015 by the United States Census Bureau. If a histogram is prepared every 5 years by the Census Bureau of the actual count, when will the people described in problem 5 be represented on a United States population graph?

   The people described in question 5 would be counted in the 0 – 4 years old age group of 2020.

There were several blanks cells in **Handout 5** that could not be completed as there was no estimate of the projected number of people born in the 0 – 4 age group after 2015. **Handout 6** begins to fill in these blank cells. This ***new*** group of people, or the ***Foundation Layer***, is the first domino that impacts a country's shape looking forward. A country's shape can drastically change from events that impact the count of people born in a 5-year period.

7. If the actual count of people in the 0 – 4 age group in 2020 turns out to be larger than what people expected, what might explain the larger number of people?

   One explanation is a higher birth rate. If more births occurred during 2015 to 2019, then the count would be higher. Another explanation is possibly more people moved into the country who were 0 – 4 years old at the start of 2020. Possibly more immigrants, or possibly more adoptions from outside of the United States occurred during that period of time.

8. In a similar way, if the actual count of people in the 0 – 4 age group in 2020 is less than what people expected, what might explain the smaller number of people?

   Just the opposite would explain a lower count. A lower birth rate resulting in fewer births. It is also possible that fewer people of that age were moving into the country.

9. **Handout 6** indicates that the total number of people in the United States in 2010 was 309.35 million people. It also indicates there were 20.19 million people who were estimated to be 0 – 4 years old. What is the proportion of the population in 2010 who were counted in the 0 – 4 age group? Express this proportion to the nearest thousandth.

   $\frac{20.19}{309.35}$ *approximately equals 0.065*

10. Based on the above proportion, what is the percent of the people in the United States who were counted in the 0 – 4 age group at the start of 2010?

    The above proportion indicates that approximately 6.5% of the population in 2010 were 0 – 4 years old.

11. In the same way, use **Handout 6** to derive the proportion of the United States population who were counted in the 0 – 4 age group at the start of 2015.

    ***Handout 6** indicates there were 19.91 million people in 2015 who were 0 – 4 years old. The total population of the United States in 2015 was 320.91 million. Therefore, the proportion of people who were counted in the 0 – 4 age group at the start of 2015 is:*
    $\frac{19.91}{320.91}$, *this proportion is approximately equal to 0.062.*

12. What is the percent of the United States population who were counted in the 0 – 4 age group at the start of 2015?

    The percent of the population who were counted in the 0 – 4 age group is approximately 6.2%.

13. What is the difference in the count of people who were 0 – 4 years old in 2010 to the count of people who were 0 – 4 years old in 2015? What might be an explanation for this difference in the count of people born in the 5 years before 2010 and the count of people born in the 5 years before 2015?

    The approximate difference in the count is 20.19 million people – 19.91 million people or approximately 0.28 million people. This difference represents approximately 280,000 people. The difference is a result of fewer births (lower birth rate) from 2010 to 2014, or fewer children moving into the country who were 0 – 4 years old prior to 2015.

14. An estimate of the count of people in 2020 who were 5 years old or older to 100+ years was 311.21 million people. Explain how this estimate was determined using **Handout 6**. Is 311.21 million people the total population of the country in 2020? Explain your answer.

    This estimate was determined by adding the counts of people in all of the age groups from 5 – 9 years old to 100+ years old. The estimates for each age group were determined first by multiplying the 2015 counts by the population factors for the connected age groups. This sum is not the total population in 2020 as it is missing the count of people 0 – 4 years old.

15. Consider the following equation:

    $$\frac{x}{x+311.21} = 0.062$$

    If the above equation is used to determine an estimate of the count of people 0 – 4 years old in 2020, answer the following:

    a. What does 0.062 represent in this equation? What does 311.21 represent?
    The proportion of the 2015 population who were approximately 0 – 4 years old is 0.062. Assume the same proportion of the population in 2020 is approximately 0 – 4 years old. 311.21 million people represents the sum of the population who are estimated to be 5 – 9 years old to 100+ years old in 2020.

    b. What does x represent in this equation?

    x represents the count of people 0 – 4 years old at the start of 2020 in millions of people.

    c. Solve for x to the nearest hundredth.

    x = 0.062(x + 311.21) => x = 0.062x + 19.30 => x - 0.062x = 19.30

    - ⇨ *0.938x = 19.30*
    - ⇨ *x = 19.30/0.938 which is approximately 20.57 million people.*

d. If the count of people in the 5 – 9 years old age group to the 100+ age group is greater than 311.21 millions of people, what happens to the value of x if the population factor stays the same? What happens to the count of the total population?
The value of x also increases. Either use the equation to point this out to students or consider changing the value of 311.21 to a larger value but keeping the foundation factor at 0.062. Direct students to solve for x based on the increased value. The total population also increases as a result of an increase in x.

16. Your solution to the above equation is an estimate of the 0 - 4 years old age group in the 2020 population. Place this value in the appropriate cell of **Handout 6**. What is your estimate of the total population of the United States in 2020? Indicate in the space below how you derived the value of the total population. Also enter this value in the appropriate cell of **Handout 6**.

    *Students place the value 20.57 million to the 0 – 4 years old age group for 2020. They also add this value to the 311.21 million people estimating the total population at 331.78 million people. Verify correct entry using the Teacher Edition of **Handout 6.***

17. Given your estimate of the 0 - 4 age group for 2020, estimate the number of people in the 5 – 9 age group for the 2025 population group by multiplying the count derived for the 0 – 4 age group by the Population Factor for the connecting age groups of 0 – 4 to 5 – 9 years old. Indicate in the space below how you derived this value. Also enter this value in the appropriate cell of **Handout 6**.

    *The estimate of the number of people in the 5 – 9 years old age group for 2025 is 20.57 x 1.014. This product is approximately 20.87 million people. Place this estimate in the proper cell of **Handout 6**. (Estimates can differ from this result by +/- 0.01 due to round off procedures.)*

18. Revise the equation in problem 15 to estimate the count of people in the 0 – 4 age group for 2025 population using the same foundation factor of 0.062 for 2015 and 2020. Place your estimate in the proper cell of **Handout 6**. Why was it necessary to derive the estimate for the 5 – 9 age group before you derived the above estimate for the 0 – 4 age group?

    Students use the same equation that was set-up in problem 15. After estimating the count of the 5 – 9 years old age group in the previous problem, they calculate the sum of the population for age groups 5 – 9 years old to 100+ years old. That sum is approximately 320.67 millions of people. Using the equation, students solve for x or the population of the 0 – 4 years old in 2025:

    $$\frac{x}{x+320.67} = 0.062 \Rightarrow x = 0.062(x + 320.67) \Rightarrow x = 0.062x + 19.88$$

    $\Rightarrow$ *0.938x = 19.88 => x = 19.88/0.938 which is approximately 21.19 or 21.20*

    Enter 21.20 in the blank cell for the count of 0 – 4 years old in 2025.

19. Derive estimates for the remaining blank cells in **Handout 6**. (Remember your estimates can differ from some of the listed estimates by + 0.01 or - 0.01.)

    *Verify correct entries for the blank cells using the Teacher Edition of **Handout 6.***

20. After all blank cells have been filled, Identify the age group with the greatest number of people for each of the following:

Year	Age group with the greatest number of people	Percent of the country within age group (nearest tenth of a percent)
2010	*45 – 49 age group*	$\frac{22.64}{309.35}$ *or 0.073 = 7.3%*
2015	*20 – 24 age group*	$\frac{22.69}{320.91}$ *or 0.071 = 7.1%*
2020	*25 – 29 age group*	$\frac{23.42}{331.78}$ *or 0.071 = 7.1%*
2025	*30 – 34 age group*	$\frac{23.94}{341.87}$ *or 0.070 = 7.0%*
2030	*35 – 39 age group*	$\frac{24.23}{350.87}$ *or 0.069 = 6.9%*
2035	*40 – 44 age group*	$\frac{24.33}{358.55}$ or 0.068 or 6.8%
2040	*45 – 49 age group*	$\frac{24.20}{364.94}$ *or 0.066 or 6.6%*
2045	*50 – 54 age group*	$\frac{23.82}{370.45}$ *or 0.064 or 6.4%*
2050	*25 – 29 age group*	$\frac{23.69}{375.67}$ *= 0.063 or 6.3%*

Note: After students complete the above table, ask them what they observed about the count and percent of the age group that has the greatest number of people. In general, the count slightly declines over each 5 years (there are a few exceptions), and the percent of the country within the dominant age group is also declining. This general pattern is also addressed in the next lesson.

21. Why might a person who was 22 years old at the start of 2015 be highlighted in an online commercial over a person who was 42 years old at the start of 2015?
A person in 2015 who was 22 years old belonged to the age group that had the largest count of people (with approximately 7.1% of the population). A person 42 years old belonged to an age group that had less people. In addition, there were also higher counts of people older than a person 42 years old. (Notice the slight dip of the age groups around 42 years old in the histogram.) The higher counts of people influence the target audience of a commercial, as well as what movies or other form of entertainment are produced, what form of transportation is established, and what type of housing options are created including the size of a house or apartment, or preferences for renting or owning a house. A country's economy and cultural are linked to those age groups who make up the greater percent of the country's population.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of this lesson.

The country of Awesome recorded that there were 1,000 people 0 – 4 years old at the start of 2015. Awesome also recorded a total population of 25,000 people at the start of 2015.

a. What is the value of the foundation factor for Awesome at the beginning of 2015?

b. Adeline lives in Awesome and turned 3 years old in 2015. Was she one of the 1,000 people counted in the 0 – 4 years old age group? Explain.

c. If the total population in Awesome at the start of 2020 is 30,000 people, and if the foundation factor did not change, estimate the count of 0 – 4 years old in 2020?

d. Adeline's friend Dominic was born in 2015. Is Dominic one of the 1,000 people counted in the 0 – 4 years old age group? Explain.

Comments on the Assessment Task:

With 1000 people in the 0 – 4 years old age group, there would be a 0.04 or 4% foundation factor for 2015. Students derive the foundation factor by setting up the proportion of 1000/25000.

Adeline would be counted in the 0 – 4 years old age group as she was 2 years old at the start of 2015.

An estimate of the 0 – 4 years old age group in 2020 would be 1,200 people, or 4% of 30,000 people.

Finally, Dominic would not be counted in the 2015 summary of the population as he was not born at the start of 2015.

Additional Assessment Summary:

This lesson highlights the second type of factor used to estimate future counts of the population, namely the ***foundation factor***. Discuss with students why the term "foundation factor" is a good description for this important value. Also, ask students why this factor is important in projecting future counts. The previous description of falling dominos could be used again to explain the impact of the 0 – 4 years old age group. As the projection for the 0 – 4 years old age group was determined in one 5-year period, all of the other age groups dependent on that estimate can be estimated in the next 5 -year periods.

Summarize with students the completed **Handout 6**. Highlight with them any patterns they observe in the handout, such as the summary mentioned in question 20. For each year except 2050, the age group with the most people kept getting older by 5 years from the previous 5-year age group. Yet, the percent in the age group with the most people slightly declined. Many other patterns can be identified that show how the population ages. Consider asking students to describe how the country might change for a person 22 years old in 2015.

Consider directing students to complete an **Exit Summary** (or Handout 13) for this lesson.

Teaching Notes

Lesson 13

"The More Things Change, the More things Stay the Same"

Overview:

This lesson focuses on the changes in the shape of the United States population based on the recursive model. Students derive summaries of age groups highlighted in the graphs. Students also reflect on the possible changes for people living in the country if the future counts derived from the model are accurate.

The problems and questions extract data from either the graphs or **Handout 6**. This lesson continues to direct students to apply their modeling tools to summarize changes in the age groups overtime. Students also summarize the impact of these changes on the country's population distribution.

An alignment of the problems in this lesson to the **Modeling Continuum** is suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems: 7	Problems: 1, 4, 5, 6, 8, 9, 10, 11, 12, 13	Problems: 2, 3, 7	

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 13 for each student (either printed or online). This lesson requires one additional handout for students to use to complete the problems, **Handout 6: *United States 2010 – 2050.*** Students use the completed copy of Handout 6 from Lesson 12.

Launch:

Ask students what they think the United States population might look like in 2050. Ask students how old they will be in 2050. What if people in the age groups who are younger are projected to be a larger segment of the population in 2050? How might a growing count of younger age groups impact the important decisions facing the country? What level of education might the younger age groups obtain? What careers or jobs will exist in 2050? Will people prefer owning a home or prefer renting? What if older age groups will be a larger segment of the 2050 population? Will they require more health care, or more housing arrangements for seniors? After a brief discussion and a list of the reflections involving these questions, direct students to complete the problems.

Implementation Ideas:

The problems are focused on understanding the projections derived from the recursive model. The overall changes begin to smooth out over time as the graphs begin to lose some of the distinctive highs and lows of the earlier graphs. Allow students to record their responses to the problems and also to verbally summarize what they observe in the graphs.

The problems are more subtle than several of the previous lessons, so writing out their responses and talking about them are important. Periodic discussion about the problems will help determine if the end result of the recursive model, or the projection for 2050, makes sense. Use the graphs as a way to summarize the story of how the characters in this module (and highlighted by the colors in the graphs) become part of older age groups. Note how Abbey and Adeline's age groups are not that different in the projections for 2050. Also note how Kristin's age group, and more noticeably her parent's age group, begin to reflect the changes in the counts due to dying. (If the colors are difficult to distinguish, please point out the age groups of each character in 2015, and have students identify the age groups of the characters for the intervening years.)

Lesson 1 – Problems

1. In what way does Raphine's decision to leave the country in 2018 change the count of people in the United States?
   Raphine moved to the United States in 2015. He would have been counted as an immigrant and would have changed the count of the 2020 population for his age group by a +1. His decision to return to Kenya in 2018, however, would change the count of the 2020 population by a -1. As a result of his moving into the country and then out of the country, Raphine did not change the count of the 2020 population.

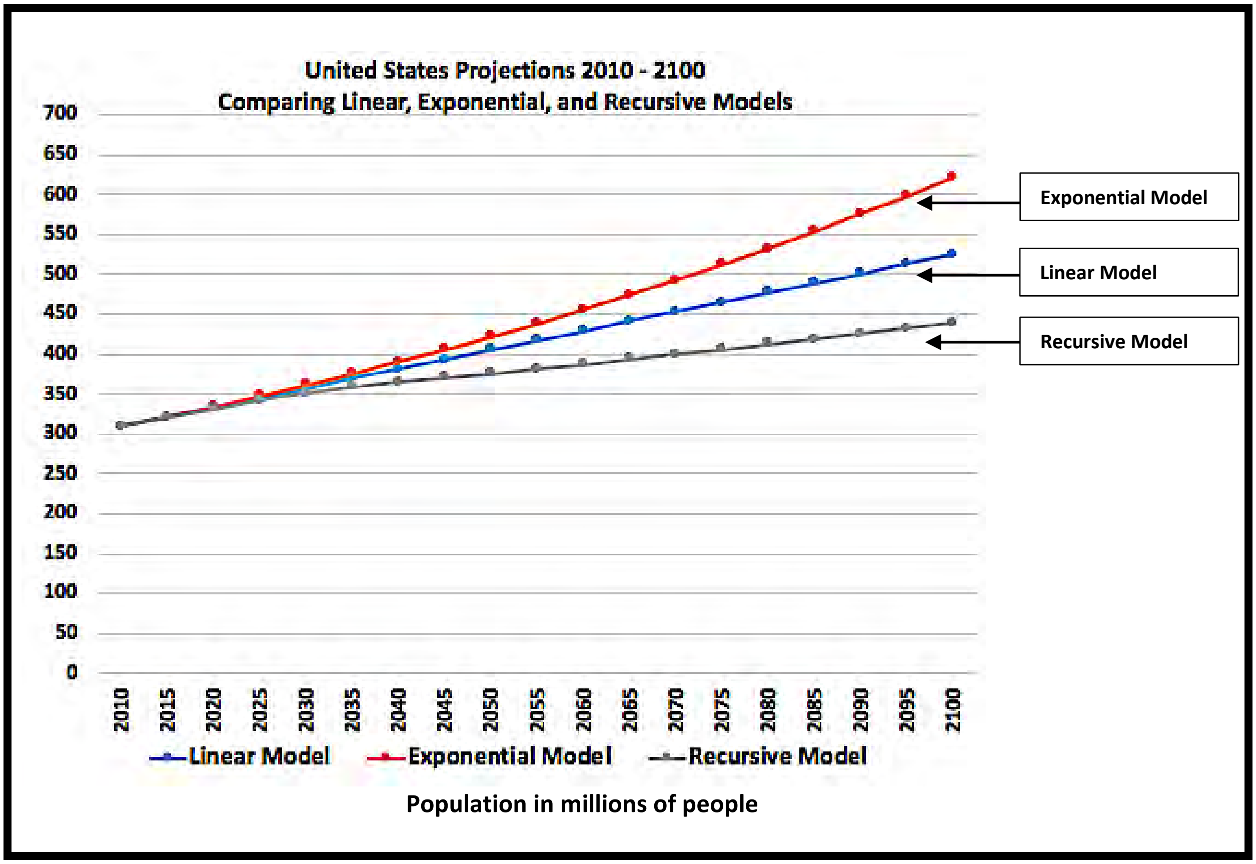
   A general discussion regarding home ownership is included in the lesson. Have a brief discussion regarding home ownership based on students' answer to problem 2, and its possible impact on an economic. Outside research might be considered as this topic is frequently mentioned as a change in the culture of the United States as more and more Millennials are deciding to rent rather than own a home. The impact may be significant.

2. If the age groups in 2050 who are under 40 years old generally do not buy a house, do you think the effect on the country's economy would be significant? Why or why not?

   A major component of the United States economy in 2015 was connected to home ownership, particularly by people 35 years old or younger. Decisions that might change the percent of people who own a house will impact the nation's economy. The recursive model is projecting a growing number of people under 35 years old by 2050, especially noteworthy when compared to similar age groups in 2035, 2040 and 2045. Other goods and services in 2050 may be considered as important factors in discussing the strengths or weaknesses of an economy.

3. What products or services might be considered important in 2050 based on the population distribution?

   Lesson 12 indicates that in 2050 the most dominant age group will be 25 – 29 years old. Several age groups close to these ages are projected to have nearly the same counts. This age group is a change from the dominant age groups projected in 2035, 2040, and 2045. What people in these age groups consider important may be the driving force of the economy in 2050. If they are similar to people in these age groups in 2015 and 2020, smaller houses, renting instead of owning a home, more energy efficient cars, fewer children and the goods and services that go along with smaller families, may be the important factors of the country's economy. Students may also note that the oldest age groups (90-year-old or older) in 2050 are projected to increase when compared to 2040 and 2045. This projected change will also affect the economy of the country.



4. The population projections for 5-year intervals are plotted for each of the models from 2010 to 2100. Answer the following.

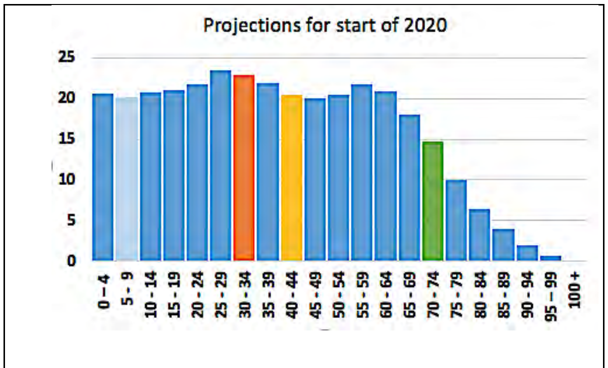
   a. Which model would result in the greatest change in the total population from 2010 to 2100? Explain your answer.

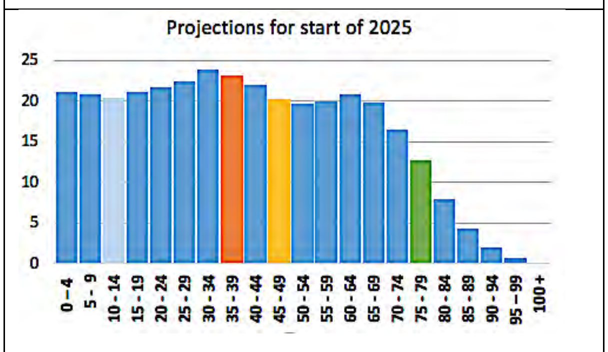
      The graph of the exponential model indicates the greatest increases in the counts. The differences in counts between the exponential model and the recursive or linear models becomes even greater over time.

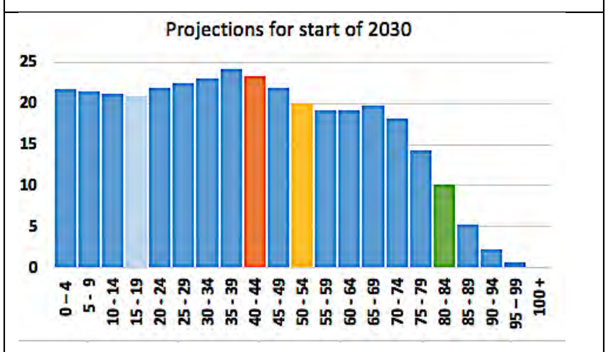
   b. Which model would result in the least change in the total population from 2010 to 2100? Explain you answer.

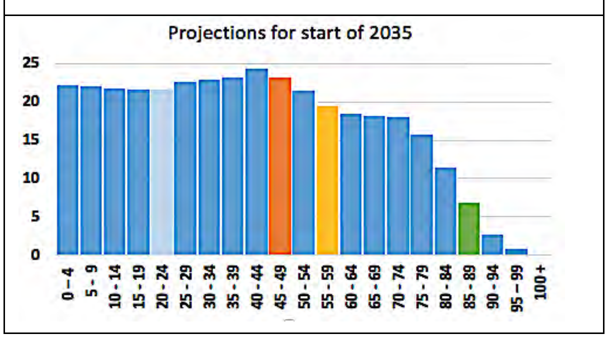
      The graph of the recursive model indicates the smallest increases over time. The recursive model's projections suggest that at some point in the future the population counts increase slightly from year to year. The graph indicates the recursive model eventually looks similar to a linear model with a slope less than the linear model derived in Lesson 8.

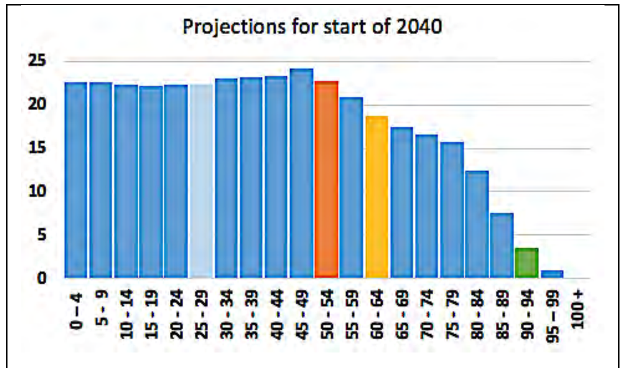
Several answers to the following questions require students to study the changes in the histograms over time:

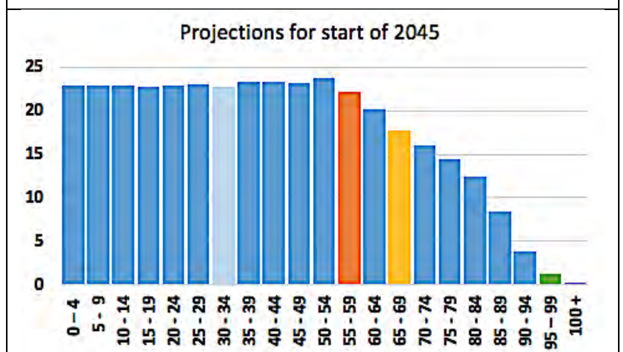


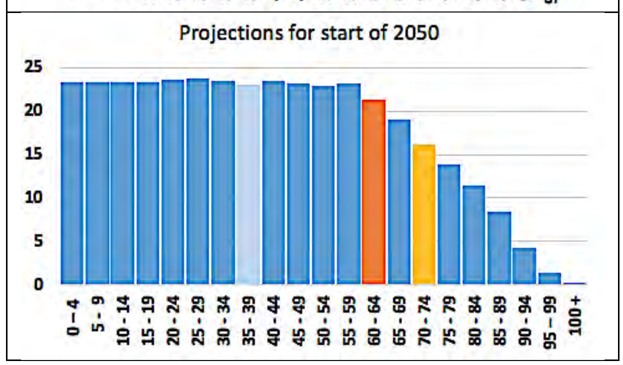












Population in millions of people for all graphs

Key:

Adeline (niece)	
Abbey (sister)	
Kristin	
Parent	

5. Answer the following:

   a. Identify at least 2 summaries of the 2025 graph that indicate it followed the 2020 graph.

      • The most dominant age group in 2020 is projected to be the 25 – 29 years old age group. The most dominant age group in 2025 is projected to be the 30 – 35 years old age group. This dominant age group aged by 5 years.

      • The 45 – 49 age group in 2020 is an interesting age group to watch in the graphs. It represents a type of "valley" with older age groups close to it having greater counts, and younger age groups close to it also having greater counts. The same visual of a "valley" is displayed in 2025 for the age group 50 – 54 years old. The pattern continued as people aged by 5 years.

   b. Identify at least 2 summaries of the 2030 graph that indicate it followed the 2025 graph.

      • The most dominant age group in 2025 is projected to be the 30 – 34 years old age group. The most dominant age group in 2030 is projected to be the 35 – 39 years old age group. The dominant age group aged by 5 years.

      • There is a similar build-up to the dominant age group. For 2025, several age groups younger that 30 – 34 years old are projected to each have increased counts until the 30 – 34 age group. This pattern is also observed in the 2030 histogram, except the age groups are projected to increase to the dominant age group of 35 – 39 years old.

   c. In what way is the population distribution as outlined in the histogram for 2035 similar to the population distribution as outlined in the histogram for 2030?

      A similar pattern of age groups increasing in count and then decreasing is observed in both histograms. The overall visual pattern for the 2035 graph is essentially similar to the pattern of the 2030 graph except shifted over by 5 years.

6. Kristin commented that the graphs for 2040, 2045, 2050 begin to look like a rectangle followed by a downward slope at the end.

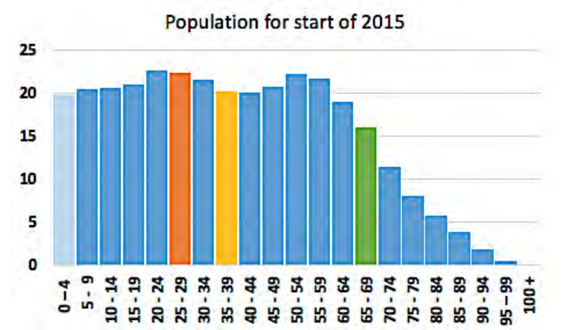
   a. What summaries of the age groups of the 2050 graph make it look like a rectangle from age groups 0 – 4 years old to 55 – 59 years old?

      The counts for the age groups 0 – 4 years old to the 55 – 59 years old have similar counts. The variations in the counts of these age groups are very small.

   b. What do you think is the reason for the downward slope that begins at 60 – 64 years old?

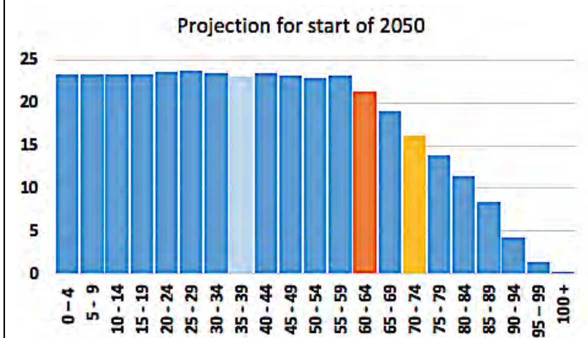
      The downward slope that begins with the 60 – 64 years old age group is due to deaths over time.

7. The following graphs provide an overall comparison of the changes of the country and the age groups from the start of 2015 to the start of 2050. The counts of the age groups are based on the counts you completed for **Handout 6**. Calculate the percent of the country's population in each of the age intervals requested for 2015 and 2050:



Estimated count of people 0 – 24 years old:
104.78 million people
Estimated count of people 25 – 49 years old:
105.29 million people
Estimated count of people 50 – 74 years old:
90.63 million people
Estimated count of people 75 – 100+ years old:
20.21 million people
Estimated total population:
320.91 million people

Country's Layers
Estimated percent of the country who are 0 – 24 years old:
104.78 / 320.91 or approximately 32.7%
Estimated percent of the country who are 25 – 49 years old:
105.29 / 320.91 or approximately 32.8%
Estimated percent of the country who are 50 – 74 years old:
90.63 / 320.91 or approximately 28.2%
Estimated percent of the country who are 75 – 100+ years old:
20.21 / 320.91 or approximately 6.3%



Projected count of people 0 – 24 years old:
116.72 million people
Projected count of people 25 – 49 years old:
116.77 million people
Projected count of people 50 – 74 years old:
102.64 million people
Projected count of people 75 – 100+ years old:
39.53 million people
Projected total population:
375.67 million people

Country's Layers
Projected percent of the country who are 0 – 24 years old:
116.72 / 375.67 or approximately 31.1%
Projected percent of the country who are 25 – 49 years old:
116.77 / 375.67 or approximately 31.1%
Projected percent of the country who are 50 – 74 years old:
102.64 / 375.67 or approximately 27.3%
Projected percent of the country who are 75 – 100+ years old:
39.53 / 375.67 or approximately 10.5%

8. Based on the above percent of the layers, describe the country's shape as bottom-layered, lower middle-layered, upper middle-layered, or top-layered for:

   a. 2015

      Lower middle-layered

b. 2050

The percent of the lower and middle layers (both lower and upper) are nearly the same in 2015 and 2050. Examining the actual estimated counts for the United States indicates the country is a lower middle-layered country in 2015 and 2020. The top layer is projected to be higher for 2050.

9. Use **Handout** 6 to estimate the 5-year age group in which the median age would be located for 2015.

   The proportion of the cumulative count to the total population is slightly more than half or 0.50 within the age group 35 – 39 years old. The median age would be an age within that interval. The United States Census Bureau reported the median age at 37.8 years for 2015.

10. Use **Handout** 6 to estimate the 5-year age group in which the median age would be located for 2050.

    The proportion of the cumulative count to the total population is slightly more than half or 0.50 within the age group 35 – 39 years old. The median age would be an age within that interval. Note that the median age has stayed the same. The median age of the country for the intervening years, however, was slightly higher. It is in 2050 that we begin to see the younger age groups gaining in count.

    There are several ways to estimate the interval that contains the median age. One way is to start adding the counts from the 25 – 29 years old age group and older, each time calculating the percent based on the total population. There was 31.1% of the population below the age of 24 years based on the summary students previously calculated, therefore, the median age would be found in age groups older than 24 years old.

11. Use **Handout** 6 to identify the age group that has the greatest projected increase in the count of people when comparing the 2015 to the 2050 counts. What is the percent increase of the 2050 count in this age group to the 2015 count of this age group?

    The greatest increase in the count of people is projected to be the 75 – 79 years old age group from 8.12 million to 13.93 million people, or an increase of 5.81 million people. After looking over the handout, students are expected to list age groups with estimated increases of several million people, and from that list, calculate the age group with the greatest increase.

12. Use **Handout** 6 to identify the age group that has the greatest projected decrease in the count of people when comparing the 2015 to the 2050 counts if any decreases exist. What is the percent decrease of the 2050 count to the 2015 count of this age group?

    There are no age groups that are projected to decrease in count. The smallest estimated increase in count is projected in the 100+ age group with a 0.16 million increase (or approximately 160,000 people).

13. Identify an age group that is projected to have the greatest percent increase of people when comparing the 2015 to the 2050 counts.

    The age group with the greatest percent increase is the age group 100+ years old. It is estimated that there will be an increase of 0.16 millions of people from 2015 to 2050 in this age group. If accurate, the percent increase from 2015 would be 200%. Remind students that although the count of this age group is projected to be small, the percent increase is projected to be the greatest. The difference of these two indicators of projected change in the population should be discussed.

Assessment Ideas:

Assessment Task:

Consider the following assessment task to determine a student's understanding of this lesson.

An online streaming company plans to produce a show that will be targeted for 55 – 59 years old. The company must make a profit with this show otherwise they will go out of business. Based on previous research, at most 20% of a targeted age group will watch a show. The company will need at least 4 million viewers to make a profit. Production of the show would only be possible at the start of 2025. If their research is accurate, in what years should the company release the show based on the data from the recursive model? Identify the years by using the column headings from Handout 6.

Do you think the company should trust the data? Explain your answer.

Comments on the Assessment Task:

Students derive that 4 million is 20% of 20 million. Therefore, students examine the counts projected for the age group 55 – 59 and determine that for the years 2040, 2045, or 2050, the estimated counts are greater than 20 million people (or 20.95 million for 2040, 22.19 million for 2045, and 23.20 million for 2050). The estimated counts are less than 20 million for the other years starting with 2025. Therefore, any of the yeas 2040, or 2045, or 2050 will have a potential number of 55 - 59 years old for the company to make a profit. In addition, point out to students (or ask them to point this out) that Abbey might be one of the viewers if the show is released in 2045.

Students are expected to comment that the years identified are the latter years of the model, and therefore, the counts may change due to events that alter population factors or the foundation factor. The further estimates are from the starting point of 2015, the more likely the estimates are not accurate. Therefore, there is some uncertainty with the years identified.

Additional comments for discussion:

This lesson highlights projected counts using the recursive model. The graphs for each of the intervening years from 2015 to 2050 show how various age groups that stood out in 2015 continue to stand out, but each group ages by 5 years in the intervening graphs. By 2050, most of the age groups (until age 60 and older) have similar projections. The "sameness" of the projections is what makes this model stand out as different, especially when compared to the overall count projected by the exponential model or the linear model. The recursive model is essentially the exponential in "chunks" with several age groups having smaller incremental changes over time. The counts of the total population of the recursive model display a leveling off from 2015 to 2050, although each projection of the total population shows an increase. Will this increase continue? Will the leveling off ultimately result in a stable impact or no changes in the total population of the country?

Changes in the population based on events that would change the intervening counts is the focus of Unit 4. "What if…?" events (for example, a decreasing number of immigrants, or war, or increasing employment opportunities that results in greater immigration) are examined in Unit 4 by changing the population factors or the foundation factor of the recursive model.

Teaching Notes

Lesson 14

Kenya, Japan, and the United States – Summing It Up

Overview:

This lesson summarizes the main points derived about the recursive model from the previous lessons. It also extends the interpretation of these summaries by asking questions that compare the 3 countries, the United States, Kenya, and Japan.

The problems and questions extract data from either the graphs or **Handouts 6, 7, and 8**. Students summarize and reflect on the impact of the data.

An alignment of the problems in this lesson to the **Modeling Continuum** is suggested in the following table:

Modeling Continuum Classification

Level 1	Level 2	Level 3	Level 4
Problems:	Problems: 1, 2,3, 8, 9, 10, 14, 21	Problems: 4, 5, 6, 7, 11, 12, 15, 16, 17, 18, 19, 20.	Problems: 13

Primary tools students use in this lesson to answer the above problems are:

Arithmetic operations, proportions, ratios, percent, interpreting proportions and percent, recursion. See the connection of these tools to high school standards in the ***Overview of the Module***.

Resources needed for this lesson:

Provide a copy of a complete Lesson 14 for each student (either printed or online). This lesson requires three additional handouts for students to use to complete the problems, **Handout 6: *United States 2010 – 2050.*** (Students should use the completed copy of Handout 6 from Lesson 12.) In addition students need **Handout 7*: Kenya 2010 – 2050*** and **Handout 8*: Japan 2010 – 2050***.

Launch:

Begin this lesson by reminding students that the future counts of a country are impacted by:

- people moving into the country (increasing a country's population),
- people moving out of the country (decreasing a country's population),
- people dying (decreasing a population), and of course
- birth (increasing a population).

The interaction of these factors results in new projections for not only the total population of a country, but also for the age groups discussed throughout this module. Direct students to examine the 2015 and 2050 histograms, and the handouts identified in this lesson, that summarize the counts for 2015 and the years leading to and including 2050 for the United States, Kenya, and Japan. Review with students what the handouts indicate, and then start the problems.

Implication Ideas:

This lesson is a review lesson and a type of overall assessment of the module for Unit 3. It highlights the components of the recursive model by looking at the 2015 population summaries and then the 2050 summaries projected by the model for the United States, Kenya, and Japan. Several questions are similar to questions asked in previous lessons; therefore, this lesson is not breaking new ground, but rather summarizing the complete implications if implementing the recursive model.

Allow students time to work individually or in small groups on the problems. Provide them opportunities to explain their answers and explanations as they work through the problems. If possible, consider assigning students a poster project involving one of the countries. On their posters, students should consider including a graph or a summary of a few of the major changes in their country's population as projected by the model or a summary of key descriptions that the model indicates will not change. Encourage them to use the 2015 and 2050 histograms to explain these projected changes along with copies of the population pyramids graphs found at the International Data Base (IDB) of the Census Bureau. Also encourage them to include some additional research to help others understand its past and present populations. Display posters and conduct a gallery walk.

Student responses or descriptions
Lesson 14 - Problems
The United States

1. What is the count of people who are projected to be 10 – 14 years old in 2025? What is the count of people who are projected to be 15 - 19 in 2030? What is projected to happen during those 5 years to change the count of people in the connected age groups?

   The count of people who are projected to be 10 – 14 years old in 2025 is 20.47 million people. The count of people who are project to be 15 – 19 in 2030 is 20.88 million people. The two age groups are connected age groups. Therefore, the increase in the number of people during these 5 years would be explained by immigration.

2. What is the projected count of people who will be 70 – 74 years old in 2040? What is the projected count of people who will be 75 – 79 years old in 2045? Explain what happened from 2040 to 2045 that changed the count of people in these connected layers?

   The count of people who are projected to be 70 – 74 years old in 2040 is 16.61 million people. The count of people who are projected to be 75 – 79 years old in 2045 is 14.44 million people. The decrease in the connected age group is likely a result of deaths.

3. What is the projected count of people 0 – 4 years old in 2030? What is the projected count of people 0 – 4 years old in 2035? Explain why the count of people in this age group is not predicted to stay the same.

   The count of children 0 – 4 years old in 2030 is projected to be 21.75 million people. The count of children 0 – 4 years old in 2035 is projected to be 22.33 million people. The recursive model projects more births during this time. Either the birthrate increases (people decided to have more children, therefore, the number of children increases), or the count of people who have children went up. (Typically people in the age groups of 20 – 40 years old have children and if the number of people in those age groups increased during the 5 years from 2025 to 2035, then there would likely be an increase in the number of births even if the birthrate did not change.)

Kenya

4. The Population Factor for the projected change in the 15 – 19 age group to the 20 – 24 age group is 0.988. The same population factor for the United States is 1.032. What does that indicate is different about the projected count of people in these connected age groups?

   A population factor that is greater than 1 indicates the connected age group increases in count. A population factor that is less than 1 indicates the connected age group decreases in count. Therefore, the count of people 20 – 24 years old in the next 5 years is projected to be greater than the count of people 15 – 19 years old in the United States. The count of people 20 – 24 years old is projected to be less than the count of people 15 – 19 years old in Kenya.

5. What is the projected count of people in Kenya in the 80 - 84 age group in 2040? What is the project count of people in the 85 – 89 age group in 2045? Explain what happened during the 5 years to change that count of people.
   The projected count of people in Kenya in the 80 – 84 years old age group in 2040 is 0.30 million people (or approximately 300,000 people). The projected count of people in Kenya in the 85 – 89 years old age group in 2045 is 0.15 million people (or approximately 150,000 people). The count of people declined significantly during those 5 years. The likely factor is death. There could also be some decline due to people moving out of the country.

6. None of Kenya's population factors are projected to be greater than 1.000. What does this indicate is different about the projections for Kenya and the United States?
   Population factors less than 1 indicate the population is declining, therefore, the count of people who die and the count of people who leave the country is greater than the count of people moving into the country. The United States' increase in population is due to immigration which is not the explanation for the increase in Kenya.

7. All of the Population Factors in Kenya are less than the Population Factors in the United States (except for the population factor of the age group 90-94 which is due to a round off of the small projected populations). What does the smaller Population Factors indicate about Kenya that is different than the United States?
   (Note: A special adjustment to the counts were needed to avoid a division by 0 in the population factors for the oldest 2 age groups. The estimated counts from the Census Bureau indicates a loss of the population during the five-years. Population Factors rounded to the next decimal place would be needed to reflect the small numbers and more accurate proportions.)
   Kenya proportionally decreases in count due to people dying or leaving the country. As indicated in previous answers, the United States has population factors greater than 1,

therefore, these age groups increase in count due to immigration. For the age groups connected by Population Factors less than 1, the decrease is connected to death and people leaving the country, which is proportionally less in the United States than in Kenya.

8. What is the Foundation Factor used to estimate the count of people in the 0 - 4 age group? Compare this to the Foundation Factor used in the United States. In what way is the projected population of Kenya changed by this different foundation factor?
   The Foundation Factor for Kenya used in the recursive model is 0.139 or approximately 13.9% of the population is projected to be 0 – 4 years old. The Foundation Factor for the United States is 0.062 or approximately 6.2% of the population is 0 – 4 years old. The difference in the Foundation Factors is significant and results in a projected increase in the population of Kenya. (Use this question to point out that all of the population factors in Kenya were less than or equal to 1 indicating that all connected age groups are projected to decrease in counts. The fact that the total population of Kenya is actually projected to increase is an indication of the large Foundation Factor. It will be highlighted in the last 2 lessons, however, that the recursive projections and the Census Bureau's projections for Kenya are different, with the Census Bureau's projections indicating a smaller rate of growth in Kenya's total population. Analyzing this difference is investigated in Lesson 16.)

9. What is the projected count of the 0 – 4 age group in 2030? What is the projected count of people for the 0 – 4 age group in 2035? Explain why the estimates for this age group are different.
   The projected count of 0 – 4 years old in 2030 is 9.01 million people. The projected count of 0 – 4 years old in 2035 is 10.08 million people. The differences are a result of an increased number of births from 2025 to 2035. This increased number of children within this age group is either due to a higher birthrate or more births from an increased count of people who have reached the age of having children.

10. Determine the projected percent increase in the population of Kenya from 2015 to 2050.
    The total population in 2015 is 45.93 million people. The total population in 2050 is 100.62 million people. The recursive model estimates that there will be an increase of 54.69 million people from 2015 to 2050. The proportion of change is $\frac{54.69}{45.93}$ *or 1.19, or an increase of 119%.*

Japan

11. The Population Factor for the projected change of the 10 – 14 age group to the 15 – 19 age group is 0.998. The population factor for the same connected age groups in the United States is 1.020. What does that indicate is different about the projected count of people who will be in these connected age groups?
    The population factor of 1.020 for the United States indicates the projected count of the connected age groups will increase due to immigration. The 0.998 population factor for Japan indicates the projected count of the connected age groups will decrease due to death or people moving out of the country. The population factor for Japan is close to 1.000, however, resulting in a very slight decrease.

12. None of Japan's population factors are projected to be greater than 1.000. What does this indicate is different about the projections for Japan and the United States?
    None of the connected age groups in Japan are projected to increase during a 5-year period. There are several population factors greater than 1 in the United States indicating several connected age groups are projected to increase during a 5-year period. The gains in the United States are anticipated due to immigration and the loss of population in Japan is anticipated due to death and fewer immigrants.

13. You are able to travel to a city in Japan. Do you expect to meet people who immigrated to Japan? Explain your answer.
    No. As explained above, the population factors are all less than or equal to 1 and indicate that most of the connected age groups decrease in counts due to death. There is no gain in connected age groups, therefore, little indication of immigration.

14. Identify the age groups in which the population factors for Japan are greater than the population factors for the United States.
    The population factors for all the connected age groups from 45 – 49 years old to 95 – 99 years old are greater in Japan than in the United States.

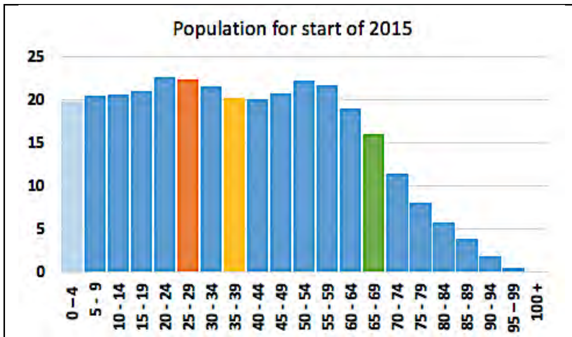
15. What does it indicate about the projected counts of people for connected age groups that have a greater population factor than the United States?
    Although the population factors are less than 1 in Japan, they are greater than than the United States population factors for several connected age groups in the United States that are less than 1. For those connected age groups, the proportion of the decrease in population in Japan is less than the proportion of decrease in population in the United States.

16. What is the Foundation Factor used to project the count of people in the 0 - 4 age group? Compare this to the Foundation Factor used in the United States. What does the different Foundation Factor for Japan indicate when comparing Japan to the United States? Kenya?
The recursive model indicates that the Foundation Factor in Japan is 0.042, or 4.2% of the population is projected to be 0 – 4 years old. The Foundation Factor in the United States is 0.062 indicating 6.2% of the population is projected to be 0 – 4 years old. The Foundation Factor in Kenya is 0.139 indicating 13.9% of the population in Kenya is projected to be 0 – 4 years old.

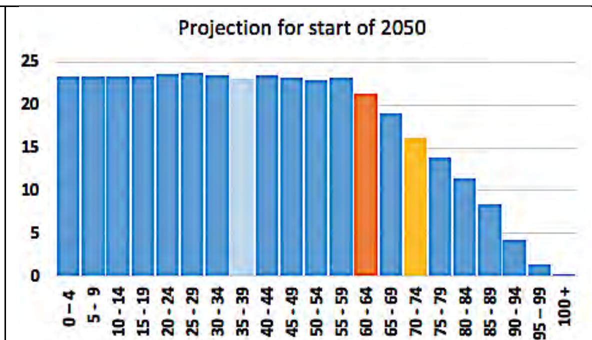
17. What is the projected count of the 0-4 age group for 2045? What is the projected count of the 0-4 age group for 2050? What does the difference in the projected counts tell us about Japan's future?
The projected count of children in Japan 0 – 4 years old in 2045 is 4.47 million people. The projected count of children in Japan 0 – 4 years old in 2050 is 4.29 million people. The projected decrease of 0.18 million people (or approximately 180,000 people) is due to fewer people who were of the age to have children or a declining birthrate.

Summaries

18. The United States



Population in millions of people

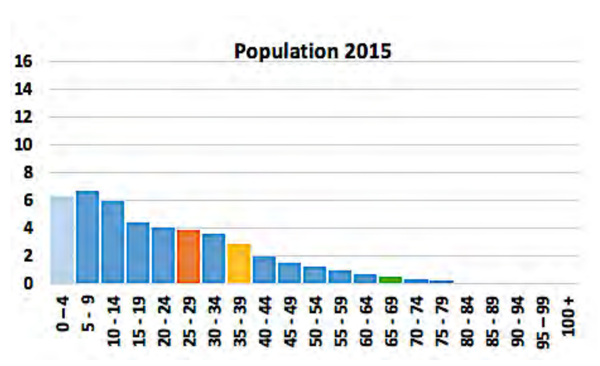


Population in millions of people

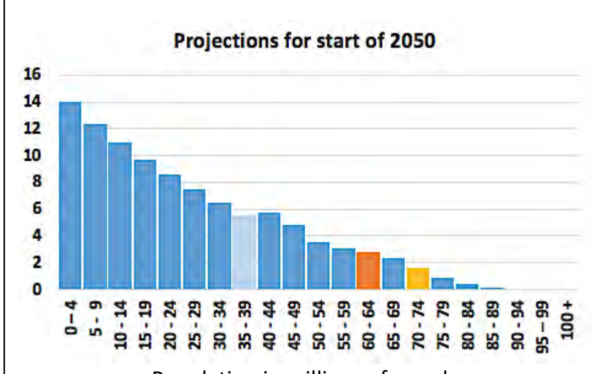
Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

The overall population of the United States is projected to increase from 2015 to 2050, with all age groups also projected to increase in counts. The increases are a result of immigration and the projected number of births. The projected increase in the number of births is likely more a result of an increase of the number of people who have children rather than an increase in the birthrate.

19. Kenya



Population in millions of people

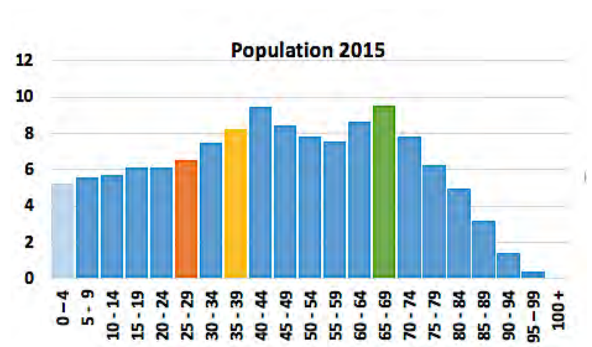


Population in millions of people

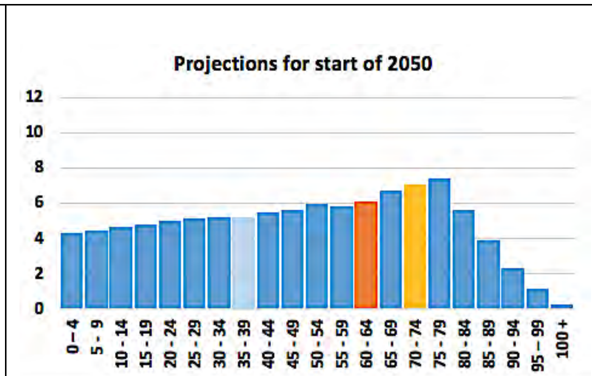
Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

The large Foundation Factor (0.130) of the 3 countries results in a major increase in the total population of Kenya with most population factors less than 1. The increase in the births is projected to result in more people than the loss due to deaths or people leaving the country.

20. Japan



Population in millions of people

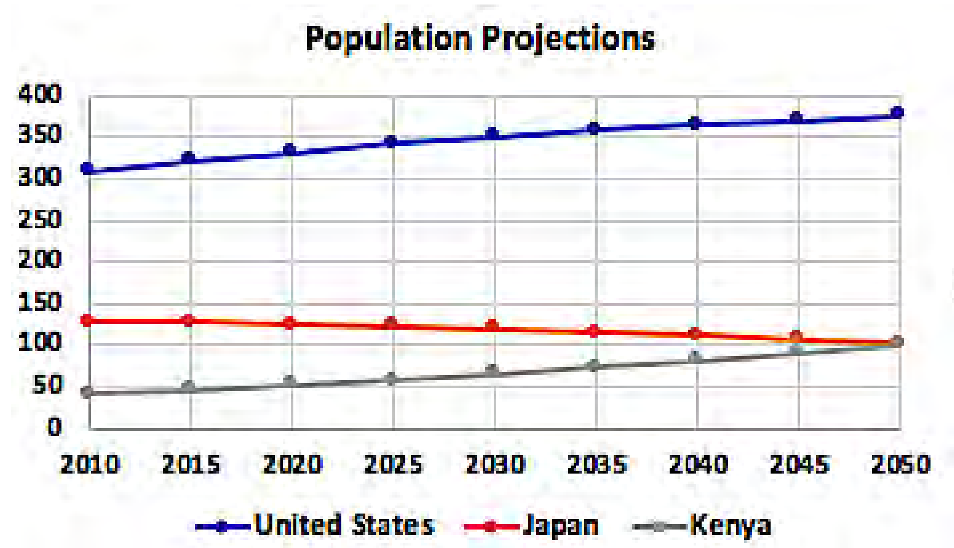


Population in millions of people

Summarize what happens from 2015 to 2050 if the assumptions about the changes in the population are accurate:

The total population of Japan decreases from 2015 to 2050. The decrease in the population is due to a small Foundation Factor and all population factors less than or equal to 1. As a result, the number of births is less than the count of people who leave the country or die. As all of the population factors are less than or equal to 1, there is little indication of immigration.

21. Use **Handouts 6, 7, and 8** to complete the following graph:
The following graph completes the graph started for students:



Population in millions of people

Give at least two reason why the recursive model used in making the above projections cannot continue without revisions.

The recursive model indicates an increase in the total population for Kenya and the United States. Although the projections seem to be leveling off for the United States, the total population is still projected to increase. An increase in the population cannot continue without some adjustments over time as the population would grow too large to sustain. In a similar way, the decrease indicated for Japan cannot continue or eventually the entire country would have 0 people.

Consider assigning as an independent research for students a summary report of "carrying capacity". The topic explores whether or not a country can adequately support its population. Discussions concerning this topic are especially important in sociology and research conducted in that discipline. This topic could be controversial for some students, so address this only if it adds to the general interest of the discussions with students.

Extension

The goal is for students to use their population values for My Country in the recursive model. A projection model entitled ***MyCountry Recursive Model.xisx*** *has been created to analyze the data for the fictitious country MyCountry. Recall that My Country is an example of a top-layered country. If students enter the data for the country data they created in Lesson 5, direct them to open the Excel file "The 1 Country".xisx and replace the population counts for 2010 and 2015* *with the counts they generated. Make sure they save the revised file by a different name as the 1-Country file will be used in the next 2 lessons.*

This extension is to be completed using the Excel file **"The 1 Country".xisx** or ***MyCountry Recursive Model.xisx***. Recall that the file **"The 1 Country".xisx** implemented the recursive model for a fictitious country, or "The 1 Country", in which the count for each of the age groups and the foundation and population factors were 1. Replace the counts of the age groups in 2010 and 2015 with the population counts of your country. Make sure you carefully save the revised spreadsheet using a different file name as "The 1 Country" file will also be used in Lessons 15 and 16. If directed by your teacher, use the Excel file ***MyCountry Recursive Model*** to answer the following questions or topics based on the fictitious country introduced in earlier lessons as My Country. Describe the results of applying the recursive model to your data or to the My Country data by completing the following summaries:

a. Shape of your country or My Country in 2010 (bottom-layered, lower middle-layered, upper middle-layered, top-layered):

b. Shape of your country or My Country in 2050:

c. Description of the histogram for 2015.

d. General description of the Population Factors (for example, age groups in which the factors were greater than 1, less than 1, or even equal to 1):

e. Value of your foundation factor for 2015. Change the foundation factors for all of the other years (2020, 2025, etc.) to this value.

f. Description of the histogram for 2050.

g. General summary of the population of your country in 2050:

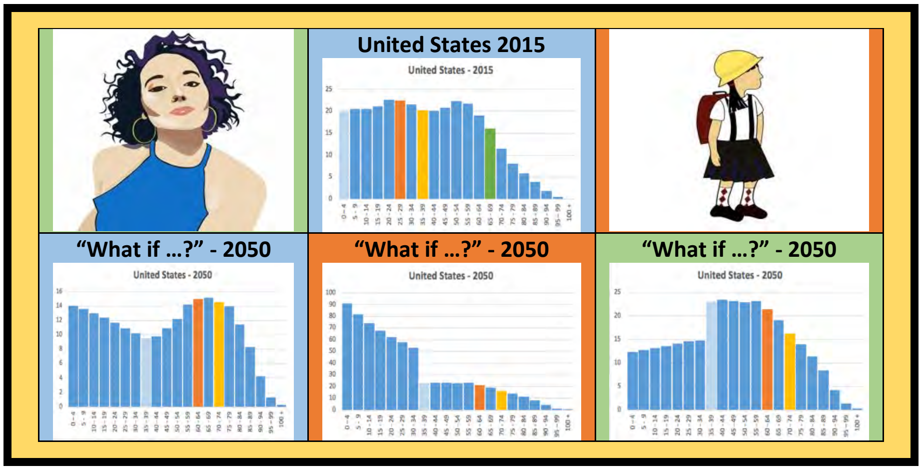
Assessment Ideas:

Lesson 14 is a type of unit assessment as this lesson concludes Unit 3 and the design of the recursive model. It was a model incorporating two actual data sets of a country's population as the anchors for summarizing change in 5-year intervals of time. Assuming (and this of course is the big assumption) that the proportion of the summarized changes in the anchor populations remain the same for the next several decades, what are the estimated future counts of this country by age groups? Will different age groups emerge as the dominant counts? Will the country change its shape as a result of different age groups increasing or decreasing in counts?

Problems 18, 19, 20 and 21 provide a basis for assessing students' ability to answer these questions. As a final set of wrap-up questions on these ideas, ask students to explain the role of the Population Factor in the recursive model. Also, ask them to explain the role of a Foundation Factor. If an adequate description is given to these two terms, ask them to explain or answer the following problems:

- Explain how Kenya is projected to experience more than double its population by 2050 with the recursive model, and yet each of its connecting age groups are projected to decline in counts.
- Will Japan eventually fade away?

Teaching Notes
Unit 4
“What if …?”



Introduction

“What if …?” are investigations that think about what might happen to our countries if events change the assumptions used to design our projection models. Each of the countries we studied had a past that resulted in their current shapes. Each of the countries had recent changes in their counts that may or may not continue when future counts are conducted. This unit is about remodeling our projection tools to think about changes in a population based on events that may or may not happen. This unit also evaluates the recursive model by looking at how well it did in estimating the more recent counts of people based on past counts. Should we trust this model?

What if there is a major drought? What if there are changes in laws and policies that limit the count of people coming into a country? What if there is a change in the birth rate? What if a country needs many more workers that require efforts to bring people from other countries into a country? What if …? What may happen to the generations that include Kristin or Raphine or Hana? “What if …?” are stories that are yet to be told.

Teaching Notes

Lesson 15

"What if …?" Scenarios

Implementation Ideas:

This lesson targets a student's work at Level 4 of the Modeling Continuum. The task presented in this lesson is open-ended and does not include a set of guided problems. It is presented and explained in the student's lesson as a task. Students are expected to rework the recursive model to match their interpretation of a scenario described in the lesson. The "What if…?" scenarios are designed to simulate events similar to the events discussed in ***Unit 2: Looking Back*** that altered the counts of age groups for several years. Students also analyze how the collective changes of several age groups alter the shape of a country as defined in this module.

Begin this lesson by providing time for students to read the opening section and the steps needed to complete the task. This section also provides an overview of why these scenarios are important. Discuss with students that the recursive model is ***not static*** and that selected factors can be altered to change the shape of a country's population. The task expects students to select a scenario, determine what factors of the recursive model need to be changed, justify why these factors should be changed, and finally interpret the altered counts of the age groups and the altered shape of the country.

This lesson expects students have access to the Excel files that implement the recursive model. The files are part of the resources available for this module. Download the 3 versions of the recursive file as identified in the section **Resources needed**. Provide access for students to the files as each student will need at least one of the files. The steps outlined in the student lesson indicate that students should load the file for the country they selected. As they enter their revised values for the factors, they should then save the revised file by a different name. In this way, students will maintain access to the original file in case they decide to restart the process with the original factors in place.

<u>It is important to provide feedback as students complete this task</u>. A rubric is included with these teaching notes to assist you in evaluating students' work and providing feedback. Given the more challenging nature of this lesson, revisions and additions to the rubric or a similar assessment tool are encouraged. It is also your decision on whether or not to share a rubric of this type with students before they select and plan out their scenario.

(Note that a few scenarios are highlighted with an *. Be aware of the sensitive nature of the scenarios identified with an *. Monitor the students who may select these scenarios and whether or not you might want to encourage them to select another option.)

Resources needed for this lesson:

Provide a copy of a complete Lesson 15 for each student. This lesson also directs students to have access to the following handouts they used in previous lessons:

Handout 6: The United States 2010 – 2050
Handout 7: Kenya 2010 – 2050
Handout 8: Japan 2010 – 2050

Students need to have access to the following Excel files:

USA Recursive Model.xlsx

Kenya Recursive Model.xlsx

Japan Recursive Model.xlsx

If Excel is not available, it is possible that a different spreadsheet program will also work with these files. It may be necessary, however, for you or the students to alter some of the entries in order for the files to work correctly.

Directions for the Lesson 15 task:

Provide students access to the Excel files ***USA Recursive Model***, ***Kenya Recursive Model***, or ***Japan Recursive Model***. Direct students to save any revised files on their computer or school network by a different file name as they will need access to the original spreadsheets in case they rethink their changes to the recursive model. Before they begin the task, allow them to play around with the spreadsheet files. What happens if they decrease the values of several population factors? What happens if they increase or decrease the foundation factor?

The spreadsheet files start off with the same values as **Handouts 6, 7, or 8** that were used in Unit 3. Direct students to use the handout of the country they selected as a way to plan what factors they will alter. If necessary, assist students in connecting the summaries provided in the handouts to the spreadsheets. After an appropriate time of simply experimenting with the spreadsheets, direct students to the **Scenario Planning Template** and the **Final Summary Report** for the country they selected. Discuss with students the directions provided in the student lesson and listed below.

Step 1: Read through the scenarios. Select one scenario to complete for this lesson. If time promotes, you may be asked to select another scenario and repeat the process.

Step 2: Review your country's current population distribution using **Handout 6** for the United States, **Handout 7** for Kenya, or **Handout 8** for Japan. Carefully examine the **Scenario Planning Template** for the country you selected that is included at the end of this lesson. Indicate on this template the population factors (if any) you would change, the new values you would assign to these factors, and a brief statement why you would change these factors based on the scenario you selected..

Step 3: In addition to possibly changing the population factors, decide if you would also change the foundation factors for your country and why. Identify your changes and your explanations on the **Scenario Planning Template.** Unlike the proposed changes to the population factors, you may recommend changing the foundation factor for certain periods. For example, you may decide to increase the foundation factor from its current value for the years 2020 to 2040, and then may decide to decrease it for the years 2045 to 2050.

Step 4: Load the Excel file that matches your country (***USA Recursive Model***, ***Kenya Recursive Model***, and ***Japan Recursive Model***). Make sure you save the revised file to your account or computer by a new name. Directions to save files will be provided by your teachers as computer networks have different procedures regarding saving files. This process will allow you to return to the original values of the recursive model in case of errors or revisions while completing this lesson.

Step 5: Enter the proposed revisions from the **Scenario Planning Template** on the Excel file. Note the outcomes for the age group and for the population totals as a result of your changes to the model. Also note the revised shape of the final histogram.

Step 6: Continue to either revise or alter population and foundation factors on the Excel file if you think your plan needs revisions. Make sure you also record the changes on the **Scenario Planning Template.**

As students complete the task and write their final report, provide feedback of the revised model. The following rubric indicates some of the important points to evaluate and include in your feedback to students.

Possible Rubric for assessing the task:

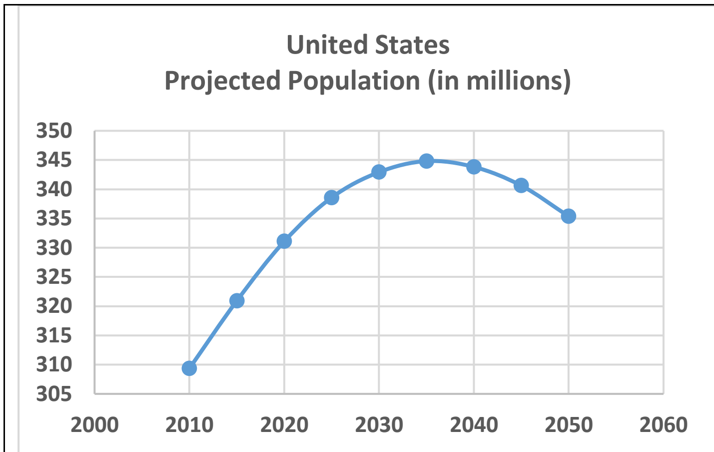
Name: ______________________________ Date: __________________

Criteria	0 Student did not address the criteria in the Scenario Planning Template or the Summary Final Report.	1 Student addressed the criteria in the Scenario Planning Template or Summary Final Report; however, no explanation was provided, or the explanation provided was unclear and not accurate in addressing the scenario.	2 Criteria were summarized in the Scenario Planning Template or the Summary Final Report. Student's explanations of revising the model were connected to the scenario.
1. Identifies if the scenario requires an increase or decrease or no change in the total population.			
2. Modifies selected population factors that result in increasing or decreasing the population and specific age groups affected by the change.			
3. Explains if changes are needed or not needed in the Foundation Level based on scenario.			
4. Describes if the population will increase or decrease or stay the same based on the foundation factor used.			
5. Provides an explanation and a sketch of the 2050 histogram based on the simulation in the Summary Final Report.			

Extension Problems

Each of the following problems contain graphs of the USA population from 2010 to 2050 as a result of changing the Population Factors for certain age groups, or changing the Foundation Factor for various years, or changing both. What if the following graphs summarize stories of the United States population from 2010 to 2050? What would you change in the recursive model and why you would make those changes to the recursive model to obtain each graph? If you have access to the recursive model for the United States (**USA Recursive Model**), make the changes you identified. Did your changes result in a graph similar to the graph in the problem? If your graph is not the same, what other changes might be considered?

Extension Problem A:

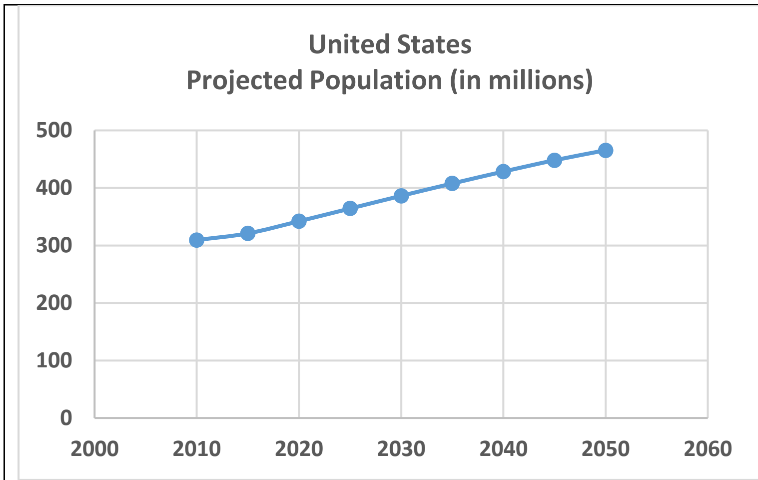


(graph)	What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem? *The graph is a result of changing the Foundation in 2020 to 0.060, 2025 to 0.055, 2030 to 0.050, 2035 to 0.048, 2040 to 0.045, 2045 to 0.043, and 2050 to 0.040. No Population Factors were changed.*

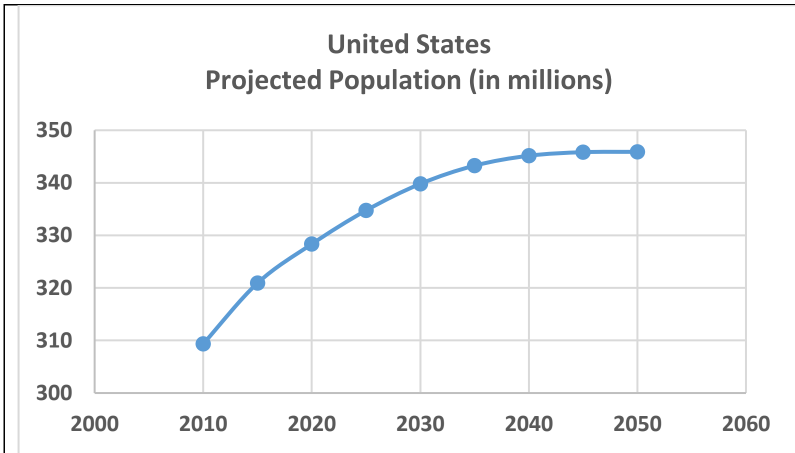
Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050.

The story would indicate a gradual decline in the percent of children in the group for each 5-year count. Essentially, the story is a gradual decline in the birthrate.

Extension Problem B:

	What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem? *This is a difficult problem for students. It is a result of changing all of the Population Factors to 1.00. The growth is major. Remind students that this change would indicate that essentially no one dies, therefore, the percent of children over time increases, and the domino effect (even with relatively low birthrates) is a major increase in the population for each 5-year count.*
Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050. *The story is about a major increase in the population of the United States. Students may investigation this change through increased Population Factors indicating immigration, or through Population Factors that are close to or equal to 1.00.*	

Extension Problem C:

	What changes would you make to the United States recursive model that would result in a graph similar to the one presented in this problem? *This graph is a result of changing all of the Population Factors greater than 1.00 to 1.00. These changes indicate reduction in immigration, but not elimination of immigration.*
Summarize the story of the United States if this graph is an accurate summary of the changes in the population from 2010 to 2050. *The story tells what happened if immigration were reduced over time.*	

Extension Problem D:

For this extension problem, you are in control. Make changes to the recursive model that tells a story of the population changes of the United States from 2010 to 2050. What if your changes were an accurate summary of what might happen in the United States? (If you do not have access the recursive mode spreadsheet file, sketch your graph and tell your story.)

	What changes would you make to the United States recursive model that would result in a graph similar to the one you produced?
Summarize the story of the United States if this graph you produced is an accurate summary of the changes in the population from 2010 to 2050.	

Consider the following variation for **Extension D**:

Direct students to write a short data-story in which the characters in their story (either fictious or real) are described in the present. Students are expected, however, to describe in their story various scenarios for the characters regarding their future plans (possibly having a family, possibly moving to another country, possibly adopting a child from another country, possibly retiring, etc.). Based on the present age of the characters and the scenarios, students create revisions to the recursive model that they think their characters represent in the larger view of the country's population in 2050.

Teaching Notes

Lesson 16

(Optional Lesson)

The United States Census Models and the Recursive Model

Implementation Ideas:

Provide time for students to read the introduction of this lesson. Conduct group discussions throughout the lesson as several questions ask students to identify subtle differences of the histograms of the United States, Japan, and Kenya. Students are directed to consider another type of "What if …?" scenario in this lesson, or ***what if*** revisions are needed to the recursive model to more closely match the US Census Bureau's projections? A country's possible future will unfold as we work with the models. The problems in this lesson require a little more precision than simply playing around with the recursive model. The model and the spreadsheet files designed to implement the model are used as tools to build specific outcomes that attempt to match the outcomes of the International Data Base (IDB).

This lesson begins by comparing the recursive models' outcomes to the outcomes from the Census Bureau. If the results reported by the Census Bureau are considered the most accurate projections of the United States, Kenya, and Japan's future, then what changes to the recursive models of each country are needed to match these projections? What changes should be considered to the population factors and the foundation factors that address assumptions about the future? Are more people likely to leave the country in the next several decades or are more people likely to move into the country? Are people more likely to live longer over the next several decades or are more people likely to die at an earlier age? Are more people indicating they plan to have children or are they indicating they are not planning to have children? And if yes, how many children are then planning to have? What might be the conditions of these countries that result in changes to the projections of the recursive model?

This lesson also addresses several serious questions for students to consider as they either revise the Foundation Factor or the Population Factors or both factors of the recursive model. They should make their suggestions for the revisions based on the comparison of the recursive model's predictions to the Census Bureau's predictions. Closely monitor the discussion.

The teaching notes for this lesson are designed to provide opportunities for discussion. As a wrap-up to this module, let the distinct differences, and the common concerns faced by the people in each country (United States, Kenya, or Japan), provide an opportunity for students to express their understanding of the key features of this module.

This lesson is designed for students to apply their own ideas and use the tools of the recursive model to build a specific outcome. The summaries provided in the teaching notes are intended to guide periodic discussions of the questions or problems. (In addition, this lesson evaluates a student's perseverance!)

Resources needed for this lesson:

Provide an online or printed copy of Lesson 16 for each student. This lesson also directs students to have access to the following handouts:

Handout 9: The United States Project Worksheet
Handout 10: Kenya Project Worksheet
Handout 11: Japan Project Worksheet

Handouts 9, 10, and 11 are essentially blank versions of the recursive model. The anchor years are included for each country. Population factors and foundation factors are blank. Students will complete their work on these handouts as directed in the lesson.

Students also need to have access to the following Excel files:

USA Recursive Model.xlsx

Kenya Recursive Model.xlsx

Japan Recursive Model.xlsx

Student responses or descriptions

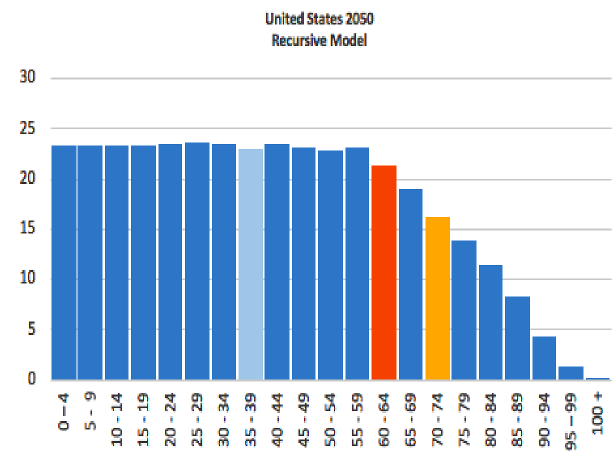
Lesson 16 - Problems

The first lesson of this module included the 2015 population pyramid graph of the United States compiled by the International Data Base (IDB) of the United States Census Bureau. This lesson was followed with problems that also analyzed the 2015 pyramid graphs of Kenya and Japan. The pyramid graphs for these countries are also included in this lesson.

The models students studied forecast future counts of these countries. The population pyramid graphs for 2015 and 2050 were obtained from the Census Bureau. The first pyramid graph is the familiar one used in several previous lessons that indicated the best estimates of the actual counts of people in 2015. The second pyramid graph is the estimate of the 2050 counts based on a population projection model used by the International Data Base (IDB) of the United States Census Bureau.

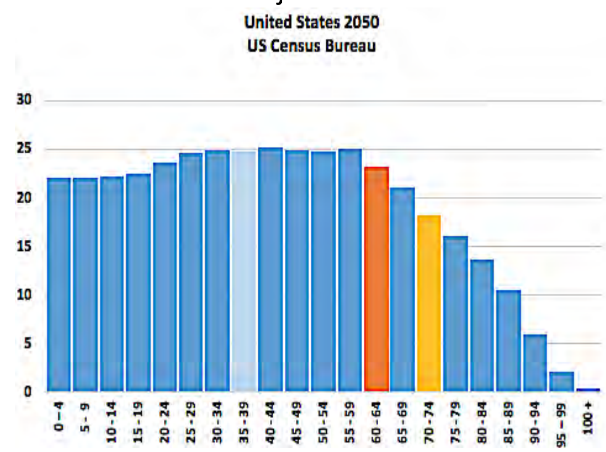
Discuss with students the following histograms. Are the estimates of the recursive and the Census Bureau models similar? If not, what are the differences in the models' predictions and the implications for the countries? These questions are investigated in this lesson.

Recursive Model Projections for 2050



Total population projected to be 375.67 million people

Census Projections for 2050



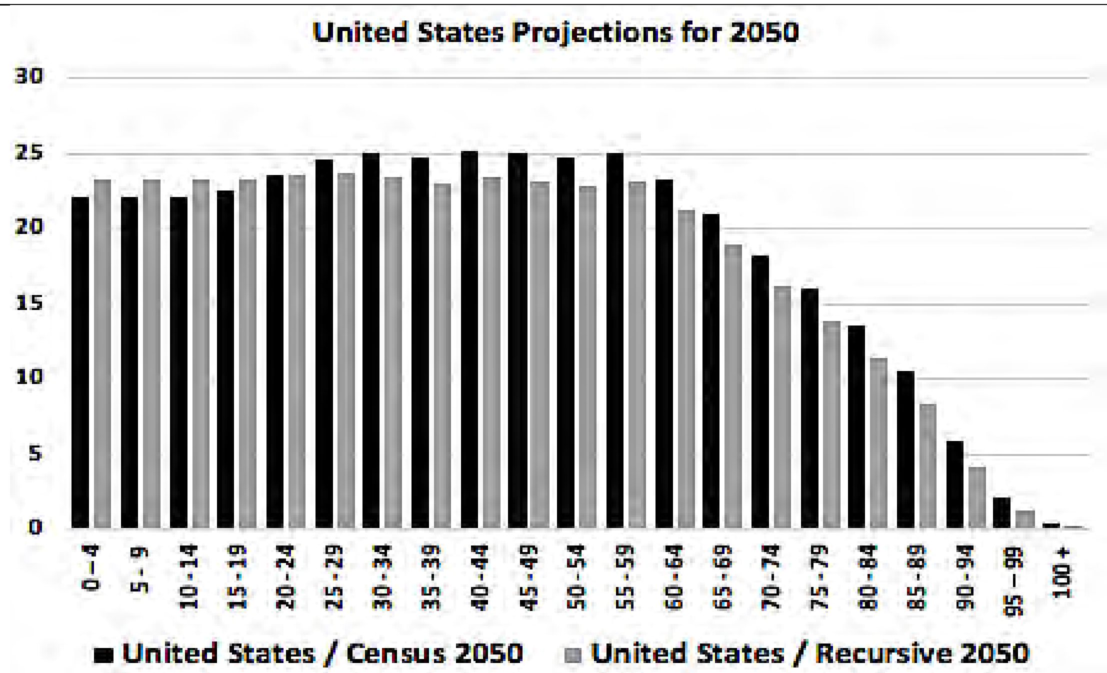
Total population projected to be 398.33 million people

1. Do you think the recursive projections and the Census projections are similar? Explain your answer.
   Overall, the two histograms have a similar pattern. The Census Bureau histogram is slightly more ragged and has a more obvious increase in counts of age groups from 15 – 19 years old to the 30 – 34 years old age group. Both histograms have a similar pattern for the older age groups when the counts decrease, although the Census Bureau counts of those age groups are greater. Based on the students' work in Lesson 15, there are events that the Census Bureau factored into their predictions, resulting in greater counts in the middle age groups. Consider using this lesson to discuss with students what might be some of these events. (For example, a decrease in the birth rate in the future, or people living longer, or an increase in the number of people immigrating to the United States.)

2. Of the two models, which model projects the largest estimated population for the United States in 2050? What is the population projected by that model?
   The US Census Bureau predicts the larger 2050 population at 398.44 million people compared to 375.67 million people predicted by the recursive model.

3. What is the percent increase of the Census model's prediction compared to the recursive model's prediction?
   The difference of the total populations of the two models is approximately 22.77 million people. The Census Bureau's prediction is approximately 6% greater than the recursive model. The proportion 22.77/375.67 is approximately equal to 6%.

The following graph is designed to compare the estimates of each model by age groups:



Population in millions of people

4. Study the age group estimates of the two models. Identify at least 3 age groups in which the recursive model's projections (color coded in light gray) are greater than the Census Bureau's projections (color coded in black).
   Answers will vary. Several of the younger age groups are greater in count (by about a million people or more) than the counts of the Census Bureau. Included in the age groups greater than the Census estimates are:
   0 – 4 years old, 5 – 9 years old, and
   10 – 14 years old are greater than the Census model.

5. Identify at least 3 age groups in which the recursive model's projections (color coded in light gray) are less than the Census predictions (color coded in black).
   Bars that shorter for particular age groups of the recursive model identify age groups that are projected to be less in count from the Census model. Several age groups in the middle of the age groups of the recursive model are less than the counts of the Census model, or:

   30 – 34 years, 35 – 39 years old, 40 – 44 years old, 45 – 49 years old,
   50 – 54 years old, 55 – 59 years old.

Several of the older age groups in the recursive model estimate a smaller count than the Census Bureau estimates:

75 – 79 years old, 80 – 84 years old, 85 – 89 years old, 90 – 94 years old, 95 – 99 years old, and 100+ years old are less than the Census model predictions.

6. What if we increase the population factor for the 35 – 39 years old age group to the 40 – 44 years old age group? Place a "+" in the location of this population factor on **Handout 9** as illustrated below. How does this change affect the counts in 2020? Answer the following questions related to this change of one population factor:
   a. Why will the estimated count for the 40 – 44 years old increase in 2020? Place a "+" in the 40 – 44 years old age group indicating that the count in this age group will be increased as a result of increasing the population factor.

35 - 39	+	20.08	20.31	
40 - 44		20.91	20.16	+

   The estimated count for the 40 – 44 years old age group increases as the population factor multiplied by the count of 35 – 39 years old to estimate the count of the 40 – 44 years old is greater.

   b. Why will the total count of the population also increase if we increase the population factor for people 35 – 39 years old? Place a "+" in the 2020 column representing the count of the total population.
   The estimate of the total population for 2020 will increase. A "+" should be added to the location recording the sum of the counts in the age groups. The total population increases as there is an increase in one age group.

7. Why will the estimated count for the 0 – 4 years old also increase in 2020? On **Handout 9**, place a "+" in the age groups representing the 0 – 4 years old. The "+" indicates that the count in this age group will also increase as a result of increasing the population factor of people who are 35 – 39 years old to 40 – 44 years old.

Age Groups	Population Factors	2010	2015	2020
0 – 4		20.19	19.91	+

The estimated count in the 0 – 4 years old age group increases as the total population increases. The estimated count of the 0 – 4 years old is 0.062 or 6.2% of the total population. If the total increases, then the estimated count in this age group increases.

It is very likely students will not recall the equation used to derive the count of the 0 – 4 years old age group (Lesson 12, problem 15). In that lesson, the count of the 0 – 4 years old age group was set-up as a proportion of the 0 – 4 years old age group to the total count of the population. This proportion was equal to the foundation factor. Therefore, if an increase in an age group also increases the total population, then the count in the 0 – 4 years age group would also have to increase so that the overall proportion (or the foundation factor) remains the same. It is appropriate to simply indicate to students that the model was designed with the assumption that if the counts in the 5 – 9 to 100+ age groups are increased, there will likely be more births resulting in an increased count in the 0 – 4 years old age group. A "+" in that cell indicates the count for that age group will increase. The handout students are completing should look like the following:

Handout 9: The United States Project Worksheet (Planning Worksheet)

		Foundation Factors:		
		0.065	0.062	
		Actual Counts:		Projections:
Age Groups	Population Factors	2010	2015	2020
0 – 4		20.19	19.91	+
5 - 9		20.33	20.48	
10 - 14		20.68	20.61	
15 - 19		21.98	21.09	
20 - 24		21.70	22.69	
25 - 29		21.15	22.40	
30 - 34		20.07	21.62	
35 - 39	+	20.08	20.31	
40 - 44		20.91	20.16	+
45 - 49		22.64	20.80	
50 - 54		22.35	22.29	
55 - 59		19.80	21.77	
60 - 64		16.99	19.04	
65 - 69		12.52	16.05	
70 - 74		9.34	11.48	
75 - 79		7.32	8.12	
80 - 84		5.76	5.80	
85 - 89		3.64	3.86	
90 - 94		1.47	1.85	
95 – 99		0.38	0.50	
100 +		0.05	0.08	
	Totals	309.35	320.91	+

Discussion with students:

Consider providing students a chance to verify the 3 anticipated changes to the 2020 population using the spreadsheet file ***USA Recursive Model.xlsx***. Direct students to load the spreadsheet file and change the population factor of the 35 – 39 years old age group to the 40 – 44 years old age group to 2.00. This altered factor indicates a 100% growth for the connecting age groups or a doubling of the count of people in the 40 – 44 years old age group in the next 5 years (very unrealistic). However, using a large value helps students identify the changes in the population based on changing one population factor. If students enter this value for the population factor, they should observe the following outcomes for the 2020 population:

- 40 – 44 years old has a count of 40.62 millions of people
- 0 – 4 years old has a count of 21.91 millions of people, and
- total population has a count of 353.35 millions of people. This total population count is approximately a 6.5% increase from the original estimate with no revisions.

The remaining age groups **for 2020** did not change.

You may discuss with students the effect of lowering a population factor. Direct students to change the population factor of the 35 – 39 years old to the 40 – 44 years old to 0.50. This factor indicates a 50% decrease, or a reduction by 1/2 for the connecting age groups (also unrealistic for a 5-year period). If students enter this value for the population factor, they should observe the following outcomes for the 2020 population:

- 40 – 44 years old has a count of 10.16 millions of people
- 0 – 4 years old has a count of 19.89 millions of people, and
- total population has a count of 320.87 millions of people. This total population count is approximately a 3.3% decrease from the original estimate with no revisions.

The remaining age groups for 2020 remained the same.

The observed changes should be used to help students verify their work with the handout. Changes in age groups from 2020 to 2050 using this one change in a population factor can be observed from the spreadsheet.

8. More age groups are affected than just the three identified in 2020. The increases represented by the "+" in 2020 also result in increases in the 2025 counts. Place a "+" in the age groups for 2025 that will increase as a result of the increases predicted in 2020.

   See the student handout that summarizes the increases if this one population factor is increased. Students could also observe the changes on the spreadsheet.

9. Continue this process. Place a "+" in the age groups of the table for 2030 to 2050 that will increase in the recursive model as a result of increasing the population factor for the 35 – 39 years old age group.
   A summary of Handout 9 is summarized below.

Handout 9: The United States Project Worksheet
(Planning Worksheet)

Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
		Foundation Factors:								
		0.065	0.062							
		Actual Counts:		Projections:						
0 – 4		20.19	19.91	+	+	+	+	+	+	+
5 - 9		20.33	20.48		+	+	+	+	+	+
10 - 14		20.68	20.61			+	+	+	+	+
15 - 19		21.98	21.09				+	+	+	+
20 - 24		21.70	22.69					+	+	+
25 - 29		21.15	22.40						+	+
30 - 34		20.07	21.62							+
35 - 39	+	20.08	20.31							
40 - 44		20.91	20.16	+	+	+	+	+	+	+
45 - 49		22.64	20.80		+	+	+	+	+	+
50 - 54		22.35	22.29			+	+	+	+	+
55 - 59		19.80	21.77				+	+	+	+
60 - 64		16.99	19.04					+	+	+
65 - 69		12.52	16.05						+	+
70 - 74		9.34	11.48							+
75 - 79		7.32	8.12							
80 - 84		5.76	5.80							
85 - 89		3.64	3.86							
90 - 94		1.47	1.85							
95 – 99		0.38	0.50							
100 +		0.05	0.08							
	Totals	309.35	320.91	+	+	+	+	+	+	+
	Key:	Adeline		Abbey		Kristin		Parent		

10. Summarize what age groups are projected to increase in 2050 as a result of the increase in the population factor of the 35 – 39 years old age group to the 40 – 44 years old age group.
The age groups from 0 – 4 years old to 30 – 34 years old will increase. An increase in those age groups would not provide a better match to the Census estimates as the recursive model estimated greater counts in those age groups before these changes. The actual increases will depend on the new value of the 35 – 39 years old population factor. Several of the middle age groups (40 – 44 years old to 70 – 74 years old will also increase. An increase in these age groups will more closely match the Census counts. The Census counts were greater than the recursive counts for those age groups, therefore an increase in the recursive model estimates would more closely match the Census estimates.

11. Will an increase in the age groups identified by the "+" suggest a closer match to the projections reported by the Census Bureau? Identify the age groups in 2050 that will increase in count and as a result more closely match the Census Bureau's projections.
Most of the age groups that were less than the Census model will increase in counts. As a result, several age groups are expected to more closely match the Census model. The challenge is to increase the counts in the older age groups and decrease the counts in the youngest age groups that the recursive model had estimated greater than the Census estimates.

What if an increase in the population factor of the 70 - 74 years old age group is considered?

12 In the same way that a "+" was used to indicate an increase for the 35 – 39 years old age group population factor, place a "+" in the population factor cell for the 70 – 74 years old age group population factor.
Monitor progress of student's handout.

13. Why was an increase in the population factor for 70 – 74 considered?
Increasing the counts in several of the older age groups is needed to match the Census model. By increasing the population factor, counts in the age groups that follow will also increase, and by 2050, several of the older age groups will be projected to increase in counts.

14. What additional age groups are increased as a result of increasing the population factor for the 70 – 74 years old population factor? Place a "+" in the age groups of the handout if the count of the age group will increase. If an age group already has a "+", place a second "+" next to it, or a "++" indicating two changes will increase the count in this cell. Will the increase in the 70 – 74 years old population factor result in a closer match to the Census model? Explain your answer.
The following table summarizes the changes if the two population factors identified in the above problems are increased. Several of the older age groups will increase in count which will more closely match the estimates of the Census Bureau.

Handout 9: The United States 2010 – 2050 (Planning Worksheet)

		Foundation Factors:								
		0.065	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
		Actual Counts:		Projections:						
Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
0 – 4		20.19	19.91	++	++	++	++	++	++	++
5 - 9		20.33	20.48		++	++	++	++	++	++
10 - 14		20.68	20.61			++	++	++	++	++
15 - 19		21.98	21.09				++	++	++	++
20 - 24		21.70	22.69					++	++	++
25 - 29		21.15	22.40						++	++
30 - 34		20.07	21.62							++
35 - 39	+	20.08	20.31							
40 - 44		20.91	20.16	+	+	+	+	+	+	+
45 - 49		22.64	20.80		+	+	+	+	+	+
50 - 54		22.35	22.29			+	+	+	+	+
55 - 59		19.80	21.77				+	+	+	+
60 - 64		16.99	19.04					+	+	+
65 - 69		12.52	16.05						+	+
70 - 74	+	9.34	11.48							+
75 - 79		7.32	8.12	+	+	+	+	+	+	+
80 - 84		5.76	5.80		+	+	+	+	+	+
85 - 89		3.64	3.86			+	+	+	+	+
90 - 94		1.47	1.85				+	+	+	+
95 – 99		0.38	0.50					+	+	+
100 +		0.05	0.08						+	+
	Totals	309.35	320.91	++	++	++	++	++	++	++
	Key:	Adeline		Abbey		Kristin		Parent		

15. Several of the younger age groups had a greater count in the recursive model than in the Census model. Let's consider decreasing the Foundation Factor in 2040 from the estimate used in the recursive model.
    a. Place a "-" in the location of the Foundation Factor for 2040. This indicates that a new foundation factor will be considered and that this factor will be less than the 6.2% used in the recursive model to derive the count of 0 - 4 years old.

Foundation Factors:						
0.065	**0.062**					**-**
Actual Counts:		**Projections:**				
2010	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**

    b. By decreasing the Foundation Factor, what age group is affected in 2040?
    The first age group to be affected will be the 0 – 4 years age group. The count in that age group will be decreased depending on the value of the lower percent of the population in the 0 – 4 years old age group. Place a "-" in the cell of the 0 – 4 years old age group.

    c. Although not an age group, what other count decreases as result of decreasing the 2040 foundation factor? Place a "-" in that location.
    The other location that will be decreased will be the cell representing the total population of the country in 2040. A "-" should also be placed in that cell.

    d. What additional age groups in 2045 will be impacted by a reduced count of 0 – 4 years old in 2040? Place a "-" in each age group in **Handout 9** that will decrease if the foundation factor for 2040 is decreased. For age groups that were previously identified as "+" or "++", add a "-" next to the list, or "+ -" or "+ + - ".
    Counts that will decrease in 2045 are the 0 – 4 years old as the total population value will be less. The 5 – 9 years old age group is derived by multiplying the population factor for the 0 – 4 years old by the 0 – 4 counts in 2040. As the 0 – 4 counts are estimated to decrease for 2040, so will the estimate for the connected age group of 5 – 9 years old in 2045. Also, the total population will decrease, therefore a "-" should be placed in that location.

16. Consider also reducing the Foundation Factors for 2045 and 2050.
    a. Place a "-" in the two locations of the Foundation Factors for 2045 and 2050. Also place a "-" in the age groups of 2045 and 2050 that will decrease as a result of decreasing the foundation factors in 2045 and 2050.
    See the completed table.
    b. Although not an age group, what else will decrease in count when the foundation factor for 2045 and 2050 is decreased? Place a "-" in each of these locations.
    See the completed table.

Handout 9: The United States 2010 – 2050
[Planning Worksheet

Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
	Foundation Factors:	0.065	0.062	0.062	0.062	0.062	0.062	-	-	-
		Actual Counts:		Projections:						
0 – 4		20.19	19.91	++	++	++	++	++ -	++ -	++ -
5 - 9		20.33	20.48		++	++	++	++	++ -	++ -
10 - 14		20.68	20.61			++	++	++	++	++ -
15 - 19		21.98	21.09				++	++	++	++
20 - 24		21.70	22.69					++	++	++
25 - 29		21.15	22.40						++	++
30 - 34		20.07	21.62							++
35 - 39	+	20.08	20.31							
40 - 44		20.91	20.16	+	+	+	+	+	+	+
45 - 49		22.64	20.80		+	+	+	+	+	+
50 - 54		22.35	22.29			+	+	+	+	+
55 - 59		19.80	21.77				+	+	+	+
60 - 64		16.99	19.04					+	+	+
65 - 69		12.52	16.05						+	+
70 - 74	+	9.34	11.48							+
75 - 79		7.32	8.12	+	+	+	+	+	+	+
80 - 84		5.76	5.80		+	+	+	+	+	+
85 - 89		3.64	3.86			+	+	+	+	+
90 - 94		1.47	1.85				+	+	+	+
95 – 99		0.38	0.50					+	+	+
100 +		0.05	0.08						+	+
	Totals	309.35	320.91	++	++	++	++	++ -	++ -	++ -
	Key:	Adeline		Abbey		Kristin		Parent		

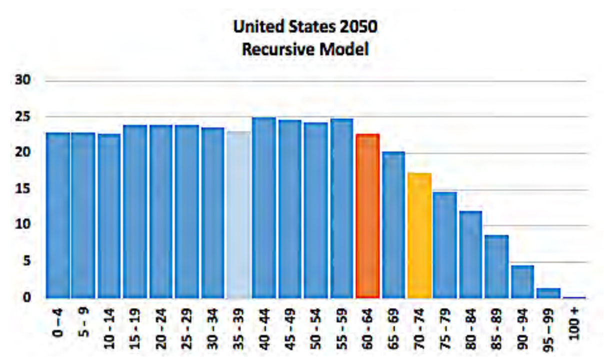
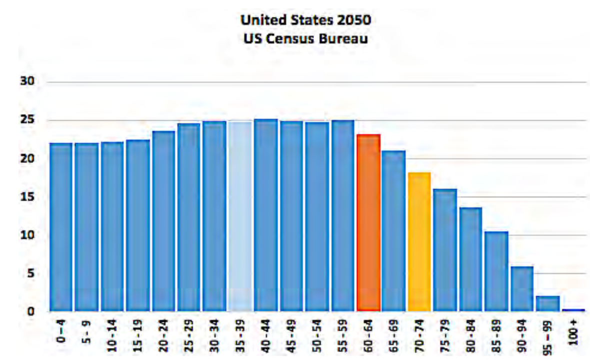
Cells that are identified in **Handout 9** for 2050 with a "+" or "++" are estimated to have an increase in counts. Cells that are identified as "-" are estimated to have a decrease in the counts. Cells that have a combination, or "+ -" or "+ + -" could increase or decrease depending on the value of the factors entered.

17. Use **Handout 9** to answer the following questions regarding the revised estimates for the 2050 age groups.
    a. What age groups will increase?
    Age groups 15 – 19 to 30 – 34 have a "++" designation. This designation indicates Increased counts from the original recursive model estimates. Also "+" for age groups 44 – 44 years old to 100+ increases are estimated. Age groups 0 – 4 years old to 10 – 14 years old are not clear. Depending on the actual revisions to the population factors or the foundation factors, we cannot determine whether or not the age groups increase or decrease.
    b. What age groups will decrease?
    No cells have strictly "-". Decreases could result for the age groups with a "+ - " or a "++-" in the cells.
    c. What age groups are unclear whether or not they will increase or decrease?
    Age groups that are "+ -" are unclear as to whether or not they will increase or decrease. This includes age groups 0 – 4 years old to 10 – 14 years old.
    d. If an age group remains blank, what does that indicate about the estimated count in 2050?
    Blank cells remain at the counts of the original recursive estimates.

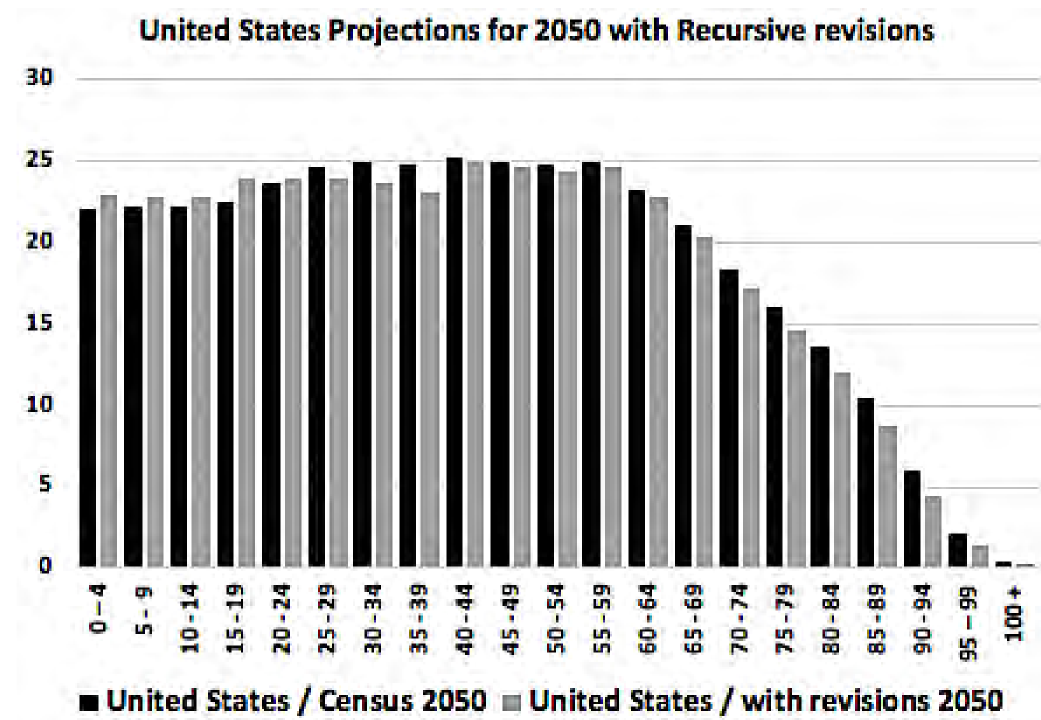
If you access to the file ***USA Recursive Model.xlsx,*** enter the following revisions to the recursive model:

Change the Population Factor for the 35 -39 years old to 1.055
Change the Population Factor for the 70 – 74 years old to 0.900
Change the Foundation Factors for 2040, 2045, and 2050 to 0.059.

The 2050 projections resulting from these revisions and the 2050 Census projections are summarized below:

Recursive model projections for 2050 with revisions:	Census model projections for 2050:
	
Total population: 387.5 million people	Total population: 398.33 million people

The following graph combines the above histograms to help us compare the two models. This graph compares the two models **after** the revisions were made to the recursive model.



18. Answer the following:
    a. Identify the age groups that have nearly equal projections.
    Age group 20 -24 is nearly equal for the two models.

    b. Identify the age groups where the recursive projections (color coded light gray) are noticeable greater than the Census projections (color coded black).
    Age groups 0 – 4 to 15 – 19 have greater counts in the recursive model. (Also, age group 20 – 24 has a slightly greater count for the recursive model.)

    c. Identify the age groups where the recursive projections are noticeable less than the Census projections.

    Age groups 25 – 29 to 100+ have noticeably less counts in the recursive model than the Census estimates.

    d. Did the changes in the population and foundation factors result in a better match of the recursive model to the Census model? Explain your answer.
    The recursive model and the Census model are a closer match, but still some adjustments should be considered.

19. Provide an explanation of events in the United States that would result in the changes entered to the recursive model.

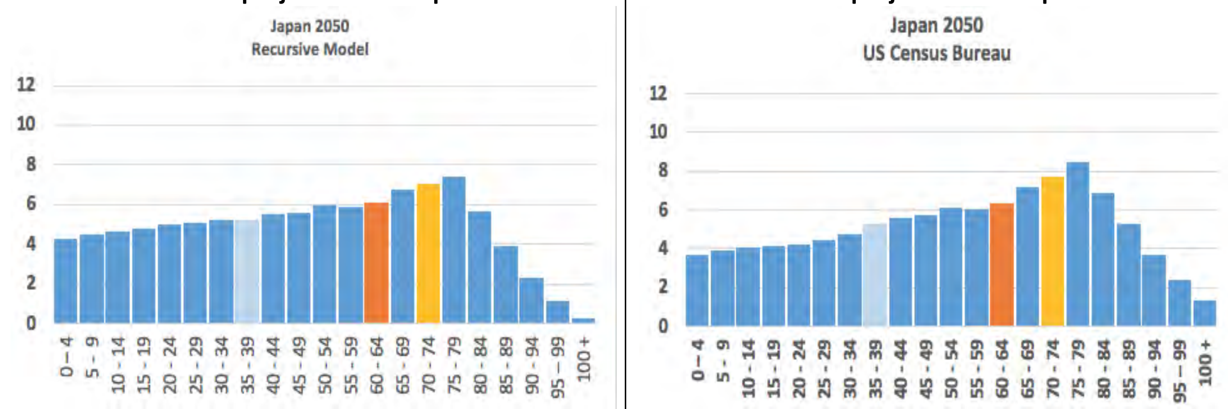
    The greater counts for the older age groups observed in the Census model indicate that more immigration and people living longer are built into their model. Fewer young people in the Census model indicates that birth rates or decreased foundation factors were also built into their model.

Consider other changes to the recursive model that you think will result in a closer match to the Census projections. If you have access to the spreadsheet file, revise either the population factors or the foundation factors that you think will result in a closer match of the two models. Keep track of your changes.

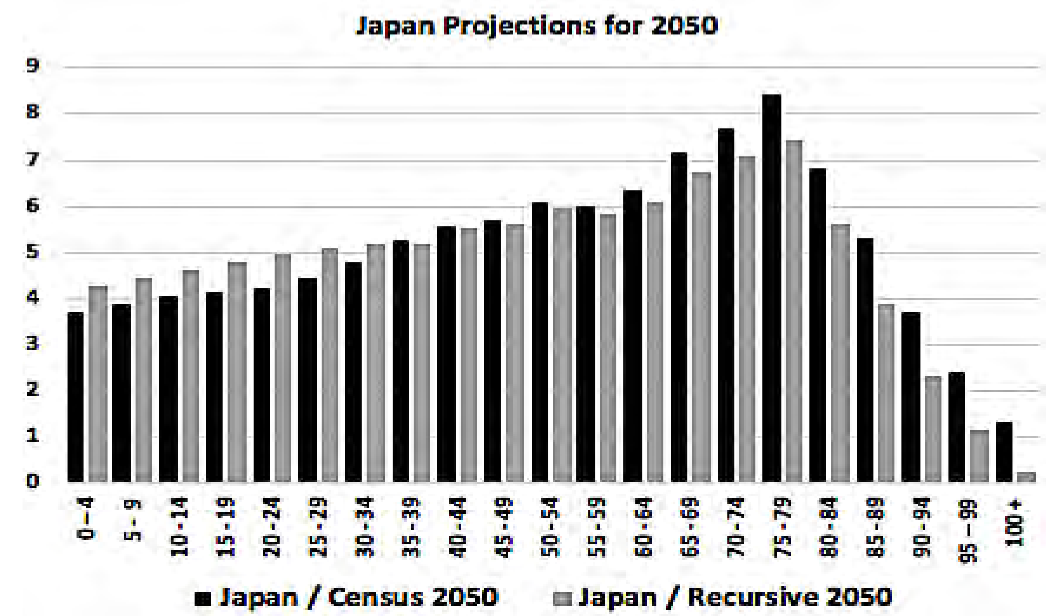
Japan

In a similar way, compare the projections from the recursive model to the projections of the United States Census Bureau's model for Japan's population:

Recursive model projections for Japan in 2050:	Census model projections for Japan in 2050:



Total population: 102.14 million people	Total population: 107.21 million people



20. Compare the two projection models of Japan's population in 2050 represented in the above histograms. Do you think the recursive projections and the Census projects are similar? Explain your answer.
    The two histograms are very similar. The overall shape (building up to the older age groups and then moving down) is similar. The total population for Japan is greater for the Census model. The counts in the older age groups are greater for the Census model

(75 – 59 years old to 100+ years old). The counts in the younger age groups are less (0 – 4 years old to 30 – 34 years old) in the Census model.

21. Derive the percent increase of the total population of Japan projected by the Census model to the total population projected by the recursive model.
    The difference in the total population is 5.07 million people. The Census model predicts the greater count. The percent difference is 5.07/ 102.14 or approximately 5%. The Census model's projection is 5% greater than the recursive model's projection.

22. Identify at least 3 age groups in which the recursive model projections (color coded in light gray) are greater than the Census Bureau's projections (color coded in black).
    Answers vary. Most of the age intervals in which the recursive model's projections were greater than the Census model's projections were the younger age groups (0 – 4 years old, 5 – 9 years old, 10 – 14 years old, 15 – 19 years old, and a few others).

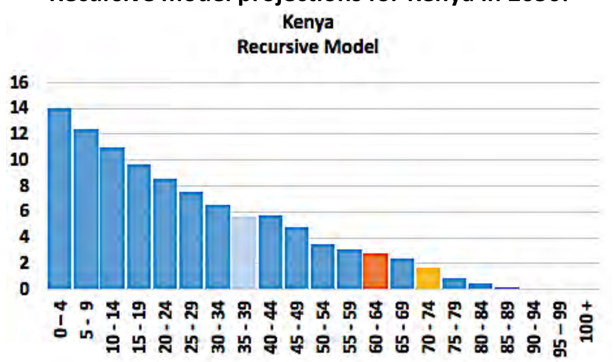
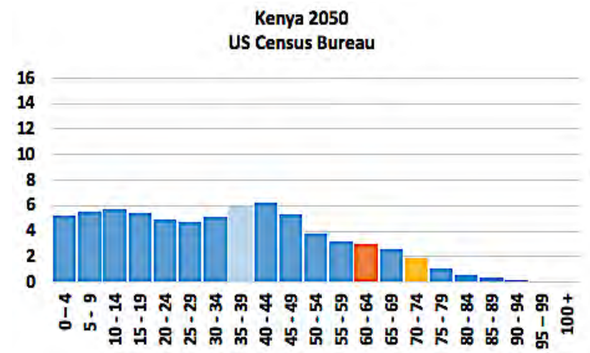
23. Identify at least 3 age groups in which the recursive model projections (color coded in light gray) are less than the Census projections (color coded in black).
    Answers will also vary in this problem. Most of the age intervals in which the recursive model's projections were less than the Census model's projections were the older age groups. Any of the age groups from 65 – 69 years old to 100+ years old are age groups in which the differences between the recursive model and the Census model are the most noticeable.

24. Given the option to change the population factors or the foundation factors in the spreadsheet **Japan Recursive Model**, identify and enter revisions to the recursive model that you think will result in similar projections to the Census projections.
    Discuss possibilities with students, especially if they are still unable to see the ripple effect of changing the factors. Consider discussing the following : Change the Foundation Factors to a slightly lower value at 2035 to 2050. This change will decrease the younger age groups. Increase the population factors for the older age groups to increase the counts in these age groups.

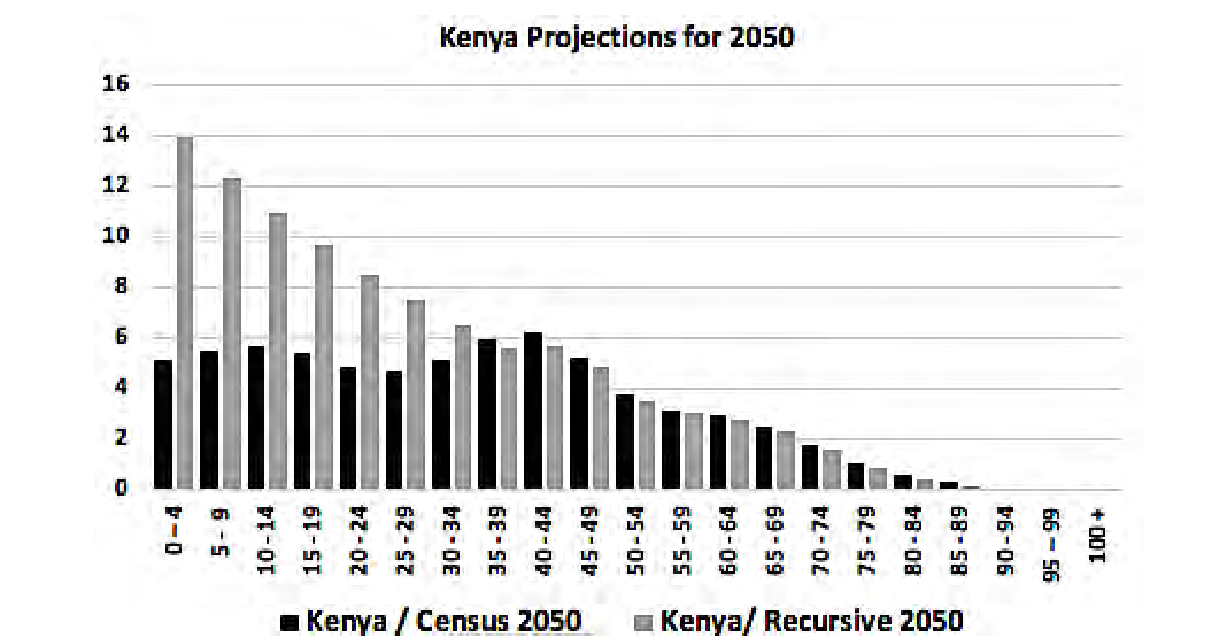
25. Based on the changes you proposed to the recursive model, what assumptions are you altering that you think will result in a better match to the Census Bureau's projections from 2020 to 2050 for Japan?
    Changes should be considered that decrease the younger age groups (or decrease selected Foundation factors) and increase the older age groups (or increase selected Population factors) in the recursive model.

Kenya

Finally, compare the projections from the recursive model to the projections of the United States Census Bureau's model for Kenya's population:

Recursive model projections for Kenya in 2050:	Census model projections for Kenya in 2050:
	
Total population: 100.62 million people	Total population: 70.76million people

26.



26. Compare the two histograms of Kenya. The first histogram was prepared from the estimates of the recursive model. The second histogram was prepared from the projections of the Census Bureau. Are they similar? Explain your answer.
    The Kenya predictions from the recursive model and the Census model are not similar. The graphs have little in common. The recursive model predicts a much larger population in 2050, and a more pronounced bottom-layered country. This is an excellent example in which major events are predicted for a country by the Census model.

27. Given the option to change the population factors and the foundation factors in the spreadsheet **Kenya Recursive Model**, what changes do you think will result in a better match of the recursive model and the Census model in 2050?

    The goal is to decrease the Foundation Factor and to increase the Population Factors. There are many options that could be explored. If students have access to the recursive model, changing several of these factors provide changes in the histogram that result in a lower total population and a greater count of the older population. If students do not have access to this model, then simply summarizing a decrease in the Foundation Factor and increases in the Population Factors is sufficient.

28. Based on the changes you proposed to the recursive model, what assumptions are you making that are different from the original assumptions in the recursive model from 2020 to 2050?

    The predicted changes in Kenya are linked to a decrease in the Foundation Factors, or the birth rate over time. The increase in the count of older people assumes that the people in the older age groups are anticipated to live longer.

29. Clearly the recursive model and the Census Bureau model differ the most for the Kenya predictions. Why do you think that the predictions by the Census Bureau for Kenya are different from the recursive model for Kenya?

    The Census model incorporates other factors that are assumed will happen in Kenya in the future. Namely, it is assumed Kenya will decrease its birth rate. In addition, people are anticipated to live longer.

Population projection models consider several other factors than just the population and foundation factors designed in the recursive model. Data obtained from surveys, death and birth records, health records, and several other data resources are also considered when building a population model. Each year in which the United States Census is conducted, a short survey is also distributed to a sample of households. This additional set of questions is referred to as the long-form. The questions people are asked varied from census to census. In addition, the questions are often met with political controversy.

30. Think of at least two questions you would include on a survey that might impact the assumptions included in a projection model. Indicate why you think the questions are related to projecting the future population counts.

    Answers will vary. Important questions in past census' included:

    > *Do you plan to have children or more children?*
    >
    > *Do you plan to continue your education?*
    >
    > *Where would you most like to live? (In the country? In the city?)*

Final problem for this project:

Identify and describe 3 reasons why an accurate population projection is important.

Students' answers to this question should link to the insights a projection model provides about the future of a country. Will a country grow? Is the country ready for this growth? What is the dominate age group in a country? Are the goods and services of a country reaching all age groups? Is there evidence of concerns in health care? As stated in this module from the beginning, people count.

Teaching Notes

Wrap-up of the People Count Stories

Implementation

The following projects or investigations are intended to be explored by students independently or in small groups. The directions for students in this wrap-up are less specific than many of the problems set-up in previous lessons. Consider these problems as independent or small group projects to conclude students' work with this module. You might consider, however, additional investigations involving other countries or other types of projection models. Projection models are extensively used in financial applications, as well as in estimating the counts of various endangered animals (for example, bees), estimating and analyzing the level of carbon emissions in the atmosphere, estimating the consumption of fresh water, etc. Designing models that analyze future outcomes represent one of the most important applications of mathematics in disciplines that rely on an understanding of the structures of mathematics (such as ratios, proportions, sequences, units).

Projects or Investigations

Consider completing one or more of the following projects to wrap-up your work with this module. Access to the spreadsheet files identified in the description of each project is needed to complete the project. You are expected to change the factors of the designated spreadsheet file to answer the questions or problems. A written summary of what you changed and why you made those changes is also expected.

Project 1: Estimating the Least Count of Immigration

Spreadsheet file needed to complete this project: USA Recursive Model

How many people during each of the 5-year periods from 2020 to 2050 are counted as immigrants if the recursive model is used to estimate future counts? Use the recursive model as designed in the spreadsheet ***USA Recursive Model*** to derive estimates of the least count of immigrants for each of the 5-years from 2020 to 2050. For example, what is an estimate of the least number of immigrants at the start of 2020 over the past 5 years if the recursive model is used to estimate future counts? In the same way, what is an estimate of the least number of immigrants at the start of 2025? 2030? … 2050?

Write a summary of how you revised the model and what were the estimates you obtained. Consider developing graphs to display the values over the 5-year intervals.

Any population factor greater than 1 indicates more people were added to the connecting age groups The only way more people could be added during a 5-year period is by immigration.

Although immigration also happened in connecting age groups with a population factor less than 1 (and also in the 0 – 4 years old age group), the counts in those age groups cannot be analyzed using the recursive model to estimate the counts of immigrants.

The best way to estimate the least count would be to add the increases in each of the connecting age groups that grew in count. This could be done by setting each population factor greater than 1 to the value of 1 (or slightly less). The resulting total population for that 5-year period would not include the increases in the age groups that had more people added by immigration. Compare the population totals obtained by revising the population factors to the totals obtained with no changes to the population factors. Encourage students to explain the changes they propose and their interpretation of the changes due to immigration.

The following table is provided to help evaluate students' estimates:
(Estimates are represented in millions of people)

	2020	2025	2030	2035	2040	2045	2050
No changes to population factors	331.78	341.87	350.87	358.55	364.94	370.45	375.67
Changes to population factors	328.32	334.74	339.50	343.37	345.15	345.83	345.89
Difference (an estimate of the least count of immigrants)	3.46	7.17	11.37	15.18	19.79	24.62	29.78

Project 2: Another Evaluation of the Recursive Model

Handout needed to complete this project: Handout 12: *Looking Back to Evaluate the Recursive Model (United States)*
Spreadsheet file needed to complete this project: Wrap-up Model.xlsx

Lesson 16 evaluated the recursive model by comparing (and revising) the projected estimates to the projections of the Census Bureau. The evaluation of which model is more accurate, however, requires waiting until 2050 at which point a census will be conducted.

Is there another way to evaluate the recursive model that would not require waiting until 2050? What if we used past counts provided by the Census Bureau (1980 and 1985), enter these counts in the recursive model, and then compare the projected results from the recursive model to the actual census counts reported in the census of 2010 and 2015?

The above plan provides an evaluation of the recursive model by looking back. Handout 12 provides the US Census counts (with estimates rounded off as indicated) for 1980 and 1985. The Excel spreadsheet **Wrap-up Model.xlsx** provides you the recursive model with columns set-up for the past. Each of these files are tools to assist you with the goals of this project. Enter the data from Handout 12 into the spreadsheet file to derive estimates for the US in 2010 and 2015. Are the estimates a good match to what was reported by the Census Bureau that summarized the census in 2010?

Write a summary that compares the estimates from the recursive model estimates to the reported Census counts. Indicate in your summary what you think happened in the country during those years that required changing the recursive model to match its outcomes to the reported census of 2010 and 2015. (For example, did more people die or leave the country during this time than the model projected? Were there more immigrants than anticipated by the model?)

*Provide students the spreadsheet file **Wrap-up Model**. In this spreadsheet, students will enter the 1980 and 1985 population counts provided with Handout 12. The spreadsheet will derive the population factors, the foundation factor, the age group counts, and the total population counts. Students are expected to compare the totals for 2010 and 2015 of the recursive model (starting with the year 1980) and the counts reported by the US Census Bureau.*

The following summary is provided to guide students' work:

	2010	**2015**
Using the **Wrap-up Model**	**301.68**	**312.94**
The Census values	**309.35**	**320.91**

The summary students develop should indicate that the Census values (obtained from an actual census) were greater than the recursive model's projections. The population factors derived from 2010 to 2015 are greater than the population factors from 1980 to 1985. The greater factors indicate there was more immigration, along with possibly people living longer prior to the 2010 and 2015 counts. Students should also note, however, that there is a difference in the foundation factors. The foundation factor for 1985 indicates that 0.075 or 7.5% of the population was 0 – 4 years old. The foundation factor for 2015 was 6.2%. Therefore, the increased counts in the intervening years are primarily a result of more immigration.

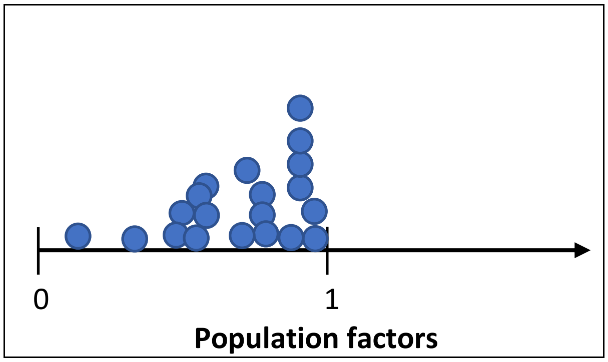
Project 3: Population Factors are not the Whole Story of a Country

Spreadsheet file needed to complete this project:

USA Recursive Model.xlsx

Two problems were presented in Lesson 10 that can be investigated further with the recursive model and the spreadsheet files. The first problem was the following:

1. Consider the following dot plot of the population factors of a fictitious country:



   a. What is the dominant explanation of change in the connecting age groups for a country represented by the above dot plot?

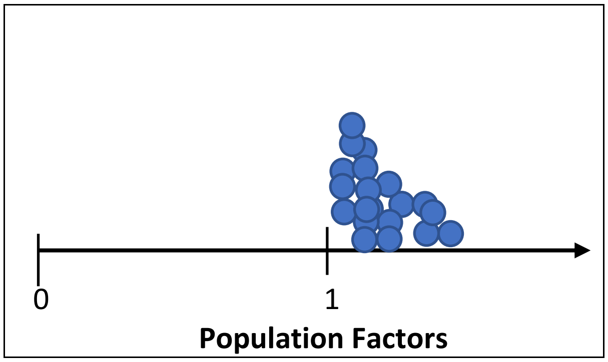
   b. Do you think it is possible for a country with the above population factors to have an increase in its total population during a 5-year period? Explain.

The dot plot indicates that all of the population factors derived in the recursive model were less than 1. Could the total population of country with the above population factors still grow over time? Design a fictitious country or obtain data for a real country to answer that question using the spreadsheet.

A country of this type was actually observed with the Kenya population. The large foundation factor (indicating a high birth rate) resulted in a large count of 0 – 4 years old for each of the 5-year intervals. The count was greater than the loss due to death or moving out of the country. If students create a fictitious country, counts that are more concentrated in the younger age groups and have all population factors less than 1 will emerge as examples. Remind students, however, that the more recent population summaries for Kenya indicated decreased counts in the younger age groups. As a result, Kenya's changes in the future are projected by the Census model to be different than the estimated counts using the recursive model.

The second problem in that lesson was the following:

2. Consider the following dot plot of the population factors for another fictitious country:



   a. What is the dominant explanation of change in the connecting age groups for the above dot plot?

   b. Do you think it is possible for a country with the above population factors to have a decrease in its total population during a 5-year period? Explain.

The dot plot for the second country indicates that all of the population factors derived were greater than 1. Could the total population of a country with the above population factors decline over time? Design a fictitious country or obtain data for a real country to answer that question using the spreadsheet file.

Write a report that indicates if you think countries could exist for each problem, and if yes, what were the counts and factors you used in setting up the population in these countries.

This scenario is more difficult to design, however, it is possible. A fictitious country in which the largest count of people are in the older age groups (with population factors slightly greater than 1) and a small foundation factor will result over time with a declining population. The loss of the older age groups to death at 100+ (as the design of the recursive model does not go beyond that age group) results in more people lost than gained by the other age groups and a small foundation factor. There is currently no country of this type, however, Japan's population is close.

Teaching Notes

People Count! (and their data stories)

Handouts and Spreadsheet Files

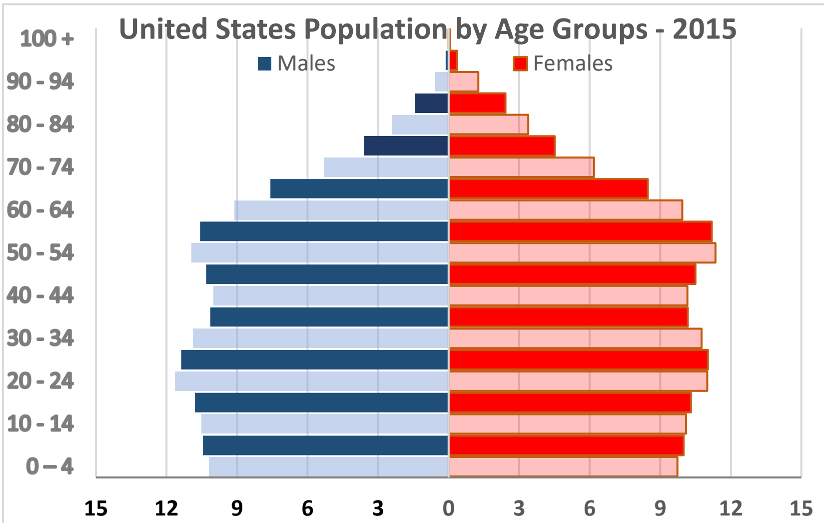
The following handouts are required for students to complete the problems. For each lesson, provide students a copy of the handouts that are identified in the opening sections of the Student Edition and the Teaching Notes.

Handout 1*: United States - 2015*
Handout 2*: Kenya - 2015*
Handout 3*: Japan - 2015*
Handout 4*: United States Connected Age Groups (Student Edition)*
Handout 4*: United States Connected Age Groups (Teacher Edition)*
Handout 5*:* Looking Forward for the United States *(Student Edition*
Handout 5*:* Looking Forward for the United States *(Teacher Edition*
Handout 6*: The United States 2010 – 2050 (Student Edition)*
Handout 6*: The United States 2010 – 2050 (Teacher Edition)*
Handout 7*: Kenya 2010 – 2050*
Handout 8*: Japan 2010 – 2050*
Handout 9*: The United States Project Worksheet*
Handout 10*:*: *Kenya Project Worksheet*
Handout 11: *Japan Project Worksheet*
Handout 12: *Looking Back to Evaluate the Recursive Model (United States)*
Handout 13: *Exit Summary*

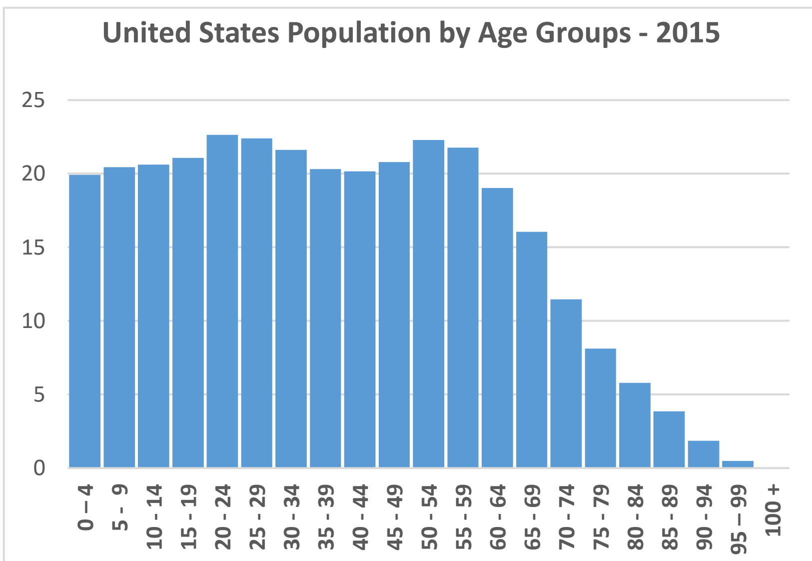
The following files have been created with the spreadsheet program EXCEL and are also identified in the opening sections of the Student Edition and the Teaching Notes for each lesson connected to a file. Students are provided an opportunity to interact with the recursive models and the resulting population projections by revising the data within the spreadsheet. Graphs are also embedded to help students understand the results of their revisions. **The files add opportunities for students as they complete the lessons but are not required to complete most of the lessons.**

"The 1 Country".xlsx
USA Recursive Model.xlxs
Kenya Recursive Model.xlxs
Japan Recursive Model.xlxs
MyCountry Recursive Model.xlxs
Wrap-up Model.xlxs

Handout 1: United States - 2015



Population in millions of people



Age Groups

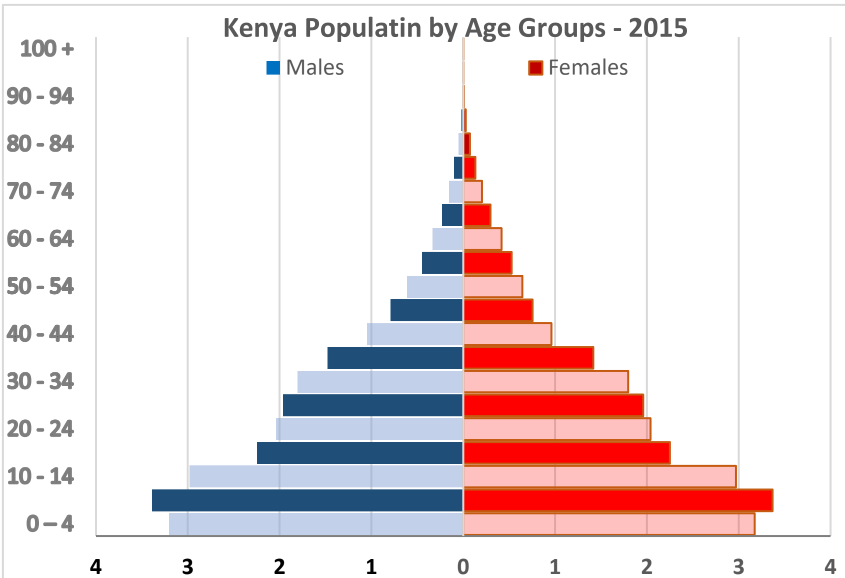
Population in millions of people

United States 2015

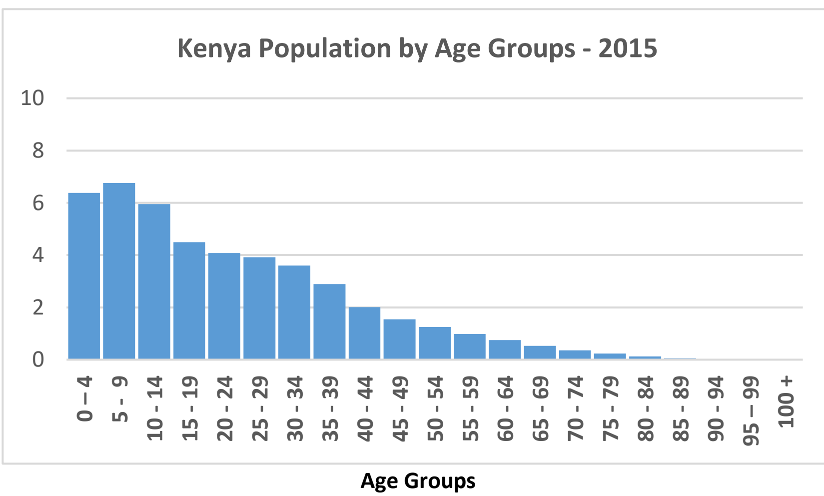
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	10,180,651	9,731,848	19,912,499
5-9	10,455,213	10,025,917	20,481,130
10-14	10,511,459	10,094,120	20,605,579
15-19	10,785,425	10,299,285	21,084,710
20-24	11,644,934	11,048,092	22,693,026
25-29	11,378,910	11,022,258	22,401,168
30-34	10,860,962	10,756,571	21,617,533
35-39	10,142,472	10,170,174	20,312,646
40-44	10,001,650	10,155,086	20,156,736
45-49	10,308,560	10,492,596	20,801,156
50-54	10,941,453	11,348,281	22,289,734
55-59	10,577,216	11,190,639	21,767,855
60-64	9,100,418	9,938,136	19,038,554
65-69	7,585,362	8,463,884	16,049,246
70-74	5,292,456	6,185,320	11,477,776
75-79	3,608,308	4,511,539	8,119,847
80-84	2,411,880	3,387,030	5,798,910
85-89	1,441,268	2,423,021	3,864,289
90-94	588,497	1,263,123	1,851,620
95-99	127,836	367,526	495,362
100+	15,105	62,137	77,242
Total	157,960,035	162,936,583	320,896,618

Handout 2: Kenya - 2015



Population in millions of people

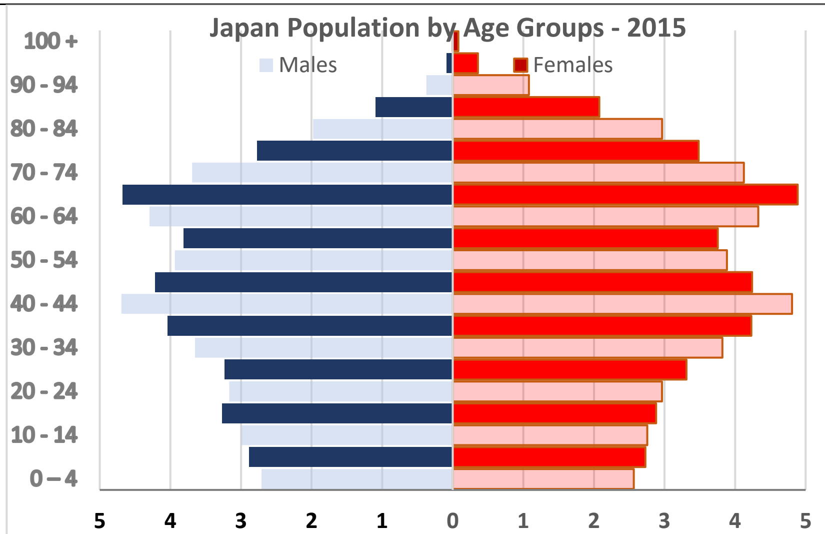


Population in millions of people

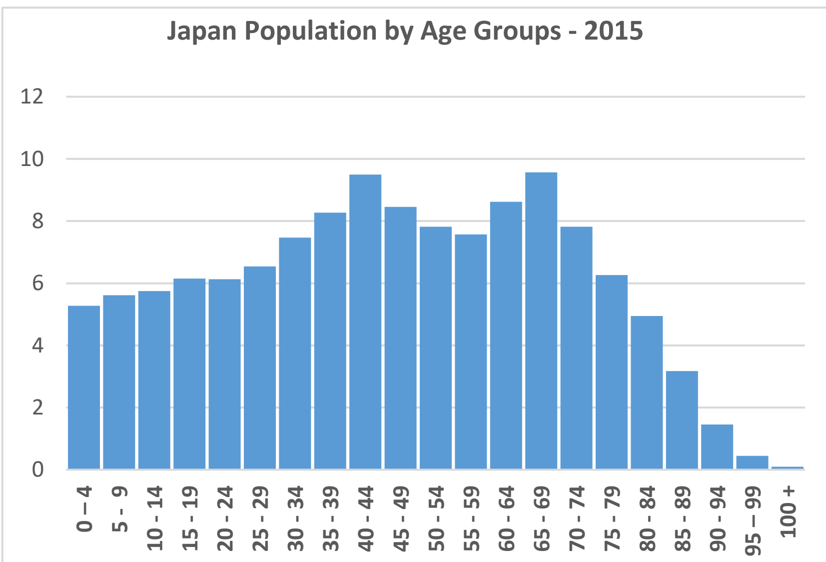
Kenya 2015
A Bottom-Layered Country

Age Group	Males	Females	Total
0-4	3,202,285	3,173,935	6,376,220
5-9	3,389,526	3,368,650	6,758,176
10-14	2,980,830	2,970,022	5,950,852
15-19	2,243,804	2,250,364	4,494,168
20-24	2,036,695	2,039,596	4,076,291
25-29	1,964,241	1,959,837	3,924,078
30-34	1,806,546	1,795,937	3,602,483
35-39	1,477,051	1,416,238	2,893,289
40-44	1,047,934	960,367	2,008,301
45-49	794,816	754,819	1,549,635
50-54	610,213	643,326	1,253,541
55-59	451,263	526,856	978,119
60-64	333,512	417,185	750,697
65-69	231,712	296,490	528,202
70-74	157,667	203,728	361,395
75-79	100,937	131,786	232,723
80-84	52,960	71,438	124,398
85-89	20,008	28,605	48,613
90-94	4,819	7,488	12,307
95-99	636	1,054	1,690
100+	45	78	123
Total	22,907,500	23,017,801	45,925,301

Handout 3: Japan - 2015



Population in millions of people



Population in millions of people

Japan 2015

An Upper Middle-Layered Country

Age Group	Males	Females	Total
0-4	2,707,598	2,565,400	5,272,998
5-9	2,882,367	2,729,721	5,612,088
10-14	2,992,683	2,756,585	5,749,268
15-19	3,269,467	2,881,921	6,151,388
20-24	3,167,481	2,964,887	6,132,368
25-29	3,231,011	3,311,576	6,542,587
30-34	3,650,718	3,821,036	7,471,754
35-39	4,037,243	4,230,960	8,268,203
40-44	4,692,392	4,807,537	9,499,929
45-49	4,216,126	4,241,648	8,457,774
50-54	3,936,931	3,885,016	7,821,947
55-59	3,810,051	3,755,442	7,565,493
60-64	4,294,784	4,328,875	8,623,659
65-69	4,679,676	4,887,715	9,567,391
70-74	3,693,158	4,124,955	7,818,113
75-79	2,775,712	3,484,536	6,260,248
80-84	1,980,330	2,966,549	4,946,879
85-89	1,094,666	2,077,183	3,171,849
90-94	373,121	1,079,680	1,452,801
95-99	84,247	357,337	441,584
100+	12,901	78,437	91,338
Total	61,582,663	65,336,996	126,919,659

Handout 4: United States Connected Age Groups (Student Edition)

Age group 2010 (Counted at the start of 2010)	**Connected Age Group in 2015** (Counted at the start of 2015)	**Population Factor Based on the Ratio of connected age groups**	**Decimal equivalent or Population Factor** (to the nearest thousandth)
0 – 4 20,189,589	5 – 9 20,481,130	$\frac{20,481,130}{20,189,589}$	1.014
5 – 9 20,331,807	10 – 14 20605579	$\frac{20,605,579}{20,331,807}$	1.013
10 – 14 20,681,215	15 – 19 21,084,710	$\frac{}{20,681,215}$	1.020
15 – 19 21,983,206	20 – 24 22,693,026	$\frac{22,693,026}{}$	1.032
20 – 24 21,704,549	25 – 29 22,401,168	$\frac{22,401,168}{21,704,549}$	1.032
25 – 29 21,145,232	30 – 34 21,617,533	$\frac{21,617,533}{21,145,232}$	
30 – 34 20,070,096	35 – 39 20,312,646	$\frac{20,312,646}{20,070,096}$	1.012
35 – 39 20,079,840	40 – 44 20,156,736	$\frac{}{20,079,840}$	
40 – 44 20,905,848	45 – 49 20,801,156	$\frac{20,801,156}{20,905,848}$	0.995
45 – 49 22,637,291	50 – 54 22,289,734	—	
50 – 54 22,353,471	55 – 59 21,767,855	$\frac{}{22,353,471}$	0.974
55 – 59 19,795,182	60 – 64 19,038,554	$\frac{19,038,554}{19,795,182}$	0.962
60 – 64 16,990,224	65– 69 16,049,246	$\frac{16,049,246}{16,990,224}$	0.945
65 – 69 12,521,439	70 – 74 11,477,776	$\frac{11,477,776}{12,521,439}$	0.917

70 – 74 9,336,583	75 – 79 8,119,847	$\frac{8,119,847}{}$	0.870
75 – 79 7,320,106	80– 84 5,798,910	$\frac{5,798,910}{7,320,106}$	
80 – 84 5,759,428	85 – 89 3,864,289	$\frac{3,864,289}{5,759,428}$	0.671
85 – 89 3,640,827	90– 94 1,851,620	$\frac{1,851,620}{}$	0.509
90 – 94 1,471,494	95 – 99 495,362	$\frac{495,362}{1,471,494}$	0.337
95 – 99 376,356	100 + 77,242	$\frac{}{376,356}$	0.205

Handout 4 – United States Connected Age Groups (Teacher Edition)

Age group 2010 (Counted at the start of 2010)	Connected Age Group in 2015 (Counted at the start of 2015)	Population Factor Based on the Ratio of connected age groups	Decimal equivalent or Population Factor (to the nearest thousandth)
0 – 4 20,189,589	5 – 9 20,481,130	$\frac{20{,}481{,}130}{20{,}189{,}589}$	1.014
5 – 9 20,331,807	10 – 14 20605579	$\frac{20{,}605{,}579}{20{,}331{,}807}$	1.013
10 – 14 20,681,215	15 – 19 21,084,710	$\frac{21{,}084{,}710}{20{,}681{,}215}$	1.020
15 – 19 21,983,206	20 – 24 22,693,026	$\frac{22{,}693{,}026}{21{,}983{,}206}$	1.032
20 – 24 21,704,549	25 – 29 22,401,168	$\frac{22{,}401{,}168}{21{,}704{,}549}$	1.032
25 – 29 21,145,232	30 – 34 21,617,533	$\frac{21{,}617{,}533}{21{,}145{,}232}$	*1.022*
30 – 34 20,070,096	35 – 39 20,312,646	$\frac{20{,}312{,}646}{20{,}070{,}096}$	1.012
35 – 39 20,079,840	40 – 44 20,156,736	$\frac{20{,}156{,}736}{20{,}079{,}840}$	*1.004*
40 – 44 20,905,848	45 – 49 20,801,156	$\frac{20{,}801{,}156}{20{,}905{,}848}$	0.995
45 – 49 22,637,291	50 – 54 22,289,734	$\frac{22{,}289{,}734}{22{,}637{,}291}$	*0.985*
50 – 54 22,353,471	55 – 59 21,767,855	$\frac{21{,}767{,}855}{22{,}353{,}471}$	0.974
55 – 59 19,795,182	60 – 64 19,038,554	$\frac{19{,}038{,}554}{19{,}795{,}182}$	0.962
60 – 64 16,990,224	65– 69 16,049,246	$\frac{16{,}049{,}246}{16{,}990{,}224}$	0.945
65 – 69 12,521,439	70 – 74 11,477,776	$\frac{11{,}477{,}776}{12{,}521{,}439}$	0.917

70 – 74 9,336,583	75 – 79 8,119,847	$\frac{8,119,847}{9,336,583}$	0.870
75 – 79 7,320,106	80– 84 5,798,910	$\frac{5,798,910}{7,320,106}$	*0.792*
80 – 84 5,759,428	85 – 89 3,864,289	$\frac{3,864,289}{5,759,428}$	0.671
85 – 89 3,640,827	90– 94 1,851,620	$\frac{1,851,620}{3,640,827}$	0.509
90 – 94 1,471,494	95 – 99 495,362	$\frac{495,362}{1,471,494}$	0.337
95 – 99 376,356	100 + 77,242	$\frac{77,242}{376,356}$	0.205

Handout 5: Looking Forward for the United States
Student Edition

		Actual Counts:		Projections:						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91							
5 - 9	1.014	20.33	20.48	20.20						
10 - 14	1.020	20.68	20.61	20.76	20.47					
15 - 19	1.032	21.98	21.09		21.17	20.88				
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55			
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25		
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	
35 - 39	1.004	20.08	20.31	21.88	23.17		23.25		23.34	
40 - 44	0.995	20.91	20.16	20.39		23.26		23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05			23.14	24.20	23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78		22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95		23.20
60 - 64	0.945	16.99	19.04	20.93		19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72		17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	
75 - 79	0.792	7.32	8.12	9.98	12.79		15.76		14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86		4.31	5.30	6.79	7.61	8.37	
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69		3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24

Key

Adeline		Kristin	
Abbey		Kristin’s mother	

Handout 5: Looking Forward for the United States
Teacher Edition

		Actual Counts:		Projections:						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91							
5 - 9	1.014	20.33	20.48	20.20						
10 - 14	1.020	20.68	20.61	20.76	20.47					
15 - 19	1.032	21.98	21.09	**21.02**	21.17	20.88				
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55			
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25		
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	
35 - 39	1.004	20.08	20.31	21.88	23.17	**24.23**	23.25	**23.17**	23.34	23.02
40 - 44	0.995	20.91	20.16	20.39	**21.97**	23.26	**24.33**	23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05	**20.28**	21.85	23.14	24.20	23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78	**23.82**	22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95	**22.19**	23.20
60 - 64	0.945	16.99	19.04	20.93	**20.88**	19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72	**18.12**	17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	**16.20**
75 - 79	0.792	7.32	8.12	9.98	12.79	**14.34**	15.76	**15.72**	14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86	**3.89**	4.31	5.30	6.79	7.61	8.37	**8.35**
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69	**3.45**	3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24

Key:

Adeline		Kristin		Abbey		Kristin's Mother	

Handout 6: The United States 2010 – 2050
Student Edition

		Foundation Factors:								
		0.065	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91			21.75	22.23	22.63	22.97	23.29
5 - 9	1.014	20.33	20.48	20.20			22.07	22.55	22.95	23.30
10 - 14	1.020	20.68	20.61	20.76	20.47	21.15	21.80	22.37	22.86	23.27
15 - 19	1.032	21.98	21.09	21.02	21.17	20.88	21.57	22.23	22.81	23.31
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55		22.95	23.55
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25	22.99	
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	23.50
35 - 39	1.004	20.08	20.31	21.88	23.17	24.23	23.25	23.17	23.34	23.02
40 - 44	0.995	20.91	20.16	20.39	21.97	23.26		23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05	20.28	21.85	23.14		23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78		22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95	22.19	23.20
60 - 64	0.945	16.99	19.04	20.93	20.88	19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72	18.12	17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	16.20
75 - 79	0.792	7.32	8.12	9.98	12.79	14.34	15.76	15.72	14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86	3.89	4.31	5.30	6.79	7.61	8.37	8.35
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69	3.45	3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24
	Totals	309.35	320.91		341.87	350.87	358.55	364.94	370.45	375.67
	Key:	Adeline		Abbey		Kristin		Parent		

Handout 6: The United States 2010 – 2050
Teacher Edition:

		Foundation Factors:								
		0.065	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**	**0.062**
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	1.014	20.19	19.91	**20.57**	**21.20**	21.75	22.23	22.63	22.97	23.29
5 - 9	1.014	20.33	20.48	20.20	**20.87**	**21.50**	22.07	22.55	22.95	23.30
10 - 14	1.020	20.68	20.61	20.76	20.47	21.15	21.80	22.37	22.86	23.27
15 - 19	1.032	21.98	21.09	21.02	21.17	20.88	21.57	22.23	22.81	23.31
20 - 24	1.032	21.70	22.69	21.77	21.70	21.86	21.55	**22.27**	22.95	23.55
25 - 29	1.022	21.15	22.40	23.42	22.47	22.40	22.56	22.25	22.99	**23.69**
30 - 34	1.012	20.07	21.62	22.90	23.94	22.97	22.90	23.06	22.74	23.50
35 - 39	1.004	20.08	20.31	21.88	23.17	24.23	23.25	23.17	23.34	23.02
40 - 44	0.995	20.91	20.16	20.39	21.97	23.26	**24.33**	23.34	23.26	23.43
45 - 49	0.985	22.64	20.80	20.05	20.28	21.85	23.14	**24.20**	23.22	23.14
50 - 54	0.974	22.35	22.29	20.48	19.74	19.97	21.51	22.78	**23.82**	22.86
55 - 59	0.962	19.80	21.77	21.71	19.95	19.23	19.45	20.95	22.19	23.20
60 - 64	0.945	16.99	19.04	20.93	20.88	19.18	18.49	18.71	20.15	21.34
65 - 69	0.917	12.52	16.05	17.99	19.78	19.72	18.12	17.47	17.67	19.03
70 - 74	0.869	9.34	11.48	14.72	16.49	18.13	18.08	16.61	16.02	16.20
75 - 79	0.792	7.32	8.12	9.98	12.79	14.34	15.76	15.72	14.44	13.93
80 - 84	0.670	5.76	5.80	6.43	7.91	10.14	11.36	12.49	12.46	11.45
85 - 89	0.508	3.64	3.86	3.89	4.31	5.30	6.79	7.61	8.37	8.35
90 - 94	0.340	1.47	1.85	1.96	1.98	2.19	2.69	3.45	3.87	4.25
95 – 99	0.205	0.38	0.50	0.63	0.67	0.67	0.75	0.92	1.17	1.32
100 +		0.05	0.08	0.10	0.13	0.14	0.14	0.15	0.19	0.24
	Totals	309.35	320.91	**331.78**	**341.87**	350.87	358.55	364.94	370.45	375.67

Handout 7: Kenya 2010 - 2050

		Foundation Factors:								
		0.168	0.139	0.139	0.139	0.139	0.139	0.139	0.139	0.139
		Actual Counts:		Projections:						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4	0.984	6.87	6.38	7.17	8.04	9.01	10.08	11.25	12.55	13.99
5 - 9	0.990	6.01	6.76	6.28	7.05	7.91	8.86	9.91	11.07	12.35
10 - 14	0.987	4.55	5.95	6.69	6.22	6.98	7.83	8.77	9.81	10.96
15 - 19	0.988	4.13	4.49	5.87	6.60	6.13	6.89	7.73	8.66	9.69
20 - 24	0.982	3.99	4.08	4.44	5.80	6.52	6.06	6.81	7.64	8.55
25 - 29	0.973	3.70	3.92	4.01	4.36	5.70	6.41	5.95	6.69	7.50
30 - 34	0.963	3.00	3.6	3.81	3.90	4.24	5.54	6.24	5.79	6.51
35 - 39	0.957	2.10	2.89	3.47	3.67	3.76	4.08	5.34	6.01	5.58
40 - 44	0.951	1.63	2.01	2.77	3.32	3.52	3.60	3.91	5.11	5.75
45 - 49	0.947	1.32	1.55	1.91	2.63	3.16	3.34	3.42	3.72	4.86
50 - 54	0.942	1.04	1.25	1.47	1.81	2.49	2.99	3.17	3.24	3.52
55 - 59	0.926	0.81	0.98	1.18	1.38	1.71	2.35	2.82	2.98	3.05
60 - 64	0.898	0.59	0.75	0.91	1.09	1.28	1.58	2.17	2.61	2.76
65 - 69	0.837	0.43	0.53	0.67	0.82	0.98	1.15	1.42	1.95	2.34
70 - 74	0.742	0.31	0.36	0.44	0.56	0.68	0.82	0.96	1.19	1.63
75 - 79	0.600	0.20	0.23	0.27	0.33	0.42	0.51	0.61	0.71	0.88
80 - 84	0.500	0.10	0.12	0.14	0.16	0.20	0.25	0.30	0.37	0.43
85 - 89	0.250	0.04	0.05	0.06	0.07	0.08	0.10	0.13	0.15	0.18
90 - 94	1.000	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.04
95 – 99	0.205	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.03
100 +		0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	Totals	40.84	45.93	51.57	57.85	64.80	72.48	80.96	90.32	100.62

Handout 8: Japan 2010 - 2050

Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
	Foundation Factors:									
		0.044	0.042	0.042	0.042	0.042	0.042	0.042	0.042	0.042
		Actual Counts:		Projections:						
0 – 4	0.996	5.63	5.27	5.26	5.15	5.01	4.84	4.66	4.47	4.29
5 - 9	0.998	5.76	5.61	5.25	5.24	5.13	4.99	4.82	4.64	4.46
10 - 14	0.998	6.16	5.75	5.60	5.24	5.23	5.12	4.98	4.81	4.63
15 - 19	1.000	6.13	6.15	5.74	5.59	5.23	5.22	5.11	4.97	4.81
20 - 24	0.998	6.55	6.13	6.15	5.74	5.59	5.23	5.22	5.11	4.97
25 - 29	0.996	7.50	6.54	6.12	6.14	5.73	5.58	5.23	5.22	5.11
30 - 34	0.996	8.30	7.47	6.51	6.10	6.12	5.71	5.56	5.20	5.19
35 - 39	0.995	9.55	8.27	7.44	6.49	6.07	6.09	5.69	5.54	5.19
40 - 44	0.993	8.52	9.50	8.23	7.40	6.46	6.04	6.06	5.66	5.51
45 - 49	0.989	7.91	8.46	9.43	8.17	7.35	6.41	6.00	6.02	5.62
50 - 54	0.984	7.69	7.82	8.36	9.33	8.08	7.27	6.34	5.93	5.95
55 - 59	0.975	8.84	7.57	7.70	8.23	9.18	7.95	7.15	6.24	5.84
60 - 64	0.965	9.92	8.62	7.38	7.51	8.03	8.95	7.75	6.98	6.08
65 - 69	0.946	8.27	9.57	8.32	7.12	7.24	7.75	8.64	7.48	6.73
70 - 74	0.909	6.89	7.82	9.05	7.86	6.73	6.85	7.32	8.17	7.07
75 - 79	0.845	5.86	6.26	7.10	8.22	7.14	6.12	6.22	6.65	7.42
80 - 84	0.742	4.27	4.95	5.29	6.00	6.95	6.03	5.17	5.26	5.62
85 - 89	0.604	2.40	3.17	3.67	3.93	4.46	5.16	4.48	3.84	3.90
90 - 94	0.427	1.03	1.45	1.92	2.22	2.37	2.69	3.12	2.71	2.32
95 – 99	0.205	0.33	0.44	0.62	0.82	0.95	1.01	1.15	1.33	1.16
100 +		0.05	0.09	0.09	0.13	0.17	0.19	0.21	0.24	0.27
	Totals	127.56	126.91	125.24	122.63	119.22	115.22	110.88	106.46	102.14

Handout 9: The United States Project Worksheet
(Planning worksheet)

		Foundation Factors:								
		0.065	**0.062**							
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4		20.19	19.91							
5 - 9		20.33	20.48							
10 - 14		20.68	20.61							
15 - 19		21.98	21.09							
20 - 24		21.70	22.69							
25 - 29		21.15	22.40							
30 - 34		20.07	21.62							
35 - 39		20.08	20.31							
40 - 44		20.91	20.16							
45 - 49		22.64	20.80							
50 - 54		22.35	22.29							
55 - 59		19.80	21.77							
60 - 64		16.99	19.04							
65 - 69		12.52	16.05							
70 - 74		9.34	11.48							
75 - 79		7.32	8.12							
80 - 84		5.76	5.80							
85 - 89		3.64	3.86							
90 - 94		1.47	1.85							
95 – 99		0.38	0.50							
100 +		0.05	0.08							
	Totals	309.35	320.91							
	Key:	Adeline		Abbey		Kristin		Parent		

Handout 10: Kenya Project Worksheet
(Planning worksheet)

Age Groups	Population Factors	2010	2015	2020	2025	2030	2035	2040	2045	2050
Foundation Factors:		0.168	0.139	0	0	0	0	0	0	0
		Actual Counts:		Projections:						
0 – 4		6.87	6.38							
5 - 9		6.01	6.76							
10 - 14		4.55	5.95							
15 - 19		4.13	4.49							
20 - 24		3.99	4.08							
25 - 29		3.70	3.92							
30 - 34		3.00	3.6							
35 - 39		2.10	2.89							
40 - 44		1.63	2.01							
45 - 49		1.32	1.55							
50 - 54		1.04	1.25							
55 - 59		0.81	0.98							
60 - 64		0.59	0.75							
65 - 69		0.43	0.53							
70 - 74		0.31	0.36							
75 - 79		0.20	0.23							
80 - 84		0.10	0.12							
85 - 89		0.04	0.05							
90 - 94		0.01	0.01							
95 – 99		0.01	0.01							
100 +		0.00	0.01							
	Totals	40.84	45.93							

Handout 11: Japan Project Worksheet
(Planning worksheet)

		Foundation Factors:								
		0.044	0.042							
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**2050**
0 – 4		5.63	5.27							
4 - 9		5.76	5.61							
10 - 14		6.16	5.75							
15 - 19		6.13	6.15							
20 - 24		6.55	6.13							
25 - 29		7.50	6.54							
30 - 34		8.30	7.47							
35 - 39		9.55	8.27							
40 - 44		8.52	9.50							
45 - 49		7.91	8.46							
50 - 54		7.69	7.82							
55 - 59		8.84	7.57							
60 - 64		9.92	8.62							
65 - 69		8.27	9.57							
70 - 74		6.89	7.82							
75 - 79		5.86	6.26							
80 - 84		4.27	4.95							
85 - 89		2.40	3.17							
90 - 94		1.03	1.45							
95 – 99		0.33	0.44							
100 +		0.05	0.09							
	Totals	127.56	126.91							

Handout 12: Looking Back to Evaluate the Recursive Model (United States)
Wrap-Up – Looking Back

		Foundation Factors:								
		Actual Counts:		**Projections:**						
Age Groups	**Population Factors**	**1980**	**1985**	**1990**	**1995**	**2000**	**2005**	**2010**	**2015**	**2020**
0 – 4		16.45	17.84							
5 - 9		16.60	16.66							
10 - 14		18.24	17.03							
15 - 19		21.11	18.73							
20 - 24		21.39	21.26							
25 - 29		19.69	21.67							
30 - 34		17.74	20.03							
35 - 39		14.08	17.60							
40 - 44		11.73	14.09							
45 - 49		11.05	11.61							
50 - 54		11.69	10.85							
55 - 59		11.61	11.23							
60 - 64		10.14	10.91							
65 - 69		8.81	9.34							
70 - 74		6.84	7.52							
75 - 79		4.83	5.51							
80 - 84		2.96	3.38							
85 - 89		1.58	1.77							
90 - 94		0.56	0.70							
95 – 99		0.12	0.18							
100 +		0.02	0.03							
	Totals	227.24	237.94							

Handout 13: People Count! (and their data stories)

Exit Summary

Name:

Lesson Number:

At your teacher's discretion, identify problems or questions in this lesson that you answered by using one or more of the levels of the Modeling Continuum. Within the column of the level or levels you identified, explain the steps you used to answer the questions or problems.

The Modeling Continuum

Level 1	Level 2	Level 3	Level 4
Identifying or extracting data from data sets or projections.	Summarizing data and projections from tables or graphs.	Interpreting the tools (for example, population factors, foundation factors, proportions) that are used to derive projections addressed in the lessons.	Reworking and modifying the tools used to make projections by addressing "What if …?" questions.
Answering questions directly from the presented data. Answering "What is …?" questions.	Summarizing data or outcomes in your own words. Answering "What is …?" questions using proportions, percent, or relative frequencies.	Answering questions or problems that require using the tools discussed in the lessons. Calculating new outcomes of a country's population based on changes in a country's immigration, births, and deaths.	Modifying the tools presented in the lessons that result in new population projections for real or fictitious countries. Answering questions that are a result of the modifications of a country's future population projections.

Level 1	Level 2	Level 3	Level 4
Problem:	**Problem:**	**Problem:**	**Problem:**

Henry's Quilt
Case Study Projects

Highlighting the population data of the United States, Kenya, and Japan in the opening lessons of this module were intentional. Each country provided a summary of a different shape as defined in the first 3 lessons of the module. Follow-up lessons were designed to provide a timeline of each country by tracing past shapes as well as possible future projections.

Students may express an interest in learning the data stories of other countries based on a similar study of their shapes and data. The cover of this module displays the 2015 population pyramid graphs of 16 other countries. In addition, a handout has been prepared for each of these countries that provides data to begin understanding the stories of these countries. The countries selected for this quilt have varying shapes, population factors, and foundation factors that provide connections to the past, present, and future population distributions.

After completing Lessons 1 to 3, students understand the connection of a population pyramid graph to the description of a country's shape. Lesson 4 uses this data to calculate numerical summaries (specifically the mean and median centers and a general description of spread) of the population distribution of a country. These opening lessons can be similarly used to organize the data for the other countries displayed on the quilt. Directions that connect these opening lessons to the countries displayed on the quilt are explained in the case study projects.

Students may also be interested in using the 2010 and 2015 population data for each country to apply in one of the projection models (the linear model, the exponential model, and the recursive model). The 2010 and 2015 data for each country are also included in the handouts that can be used to develop projection models using Lessons 8, 9, and 10 as a guide.

Consider the following Case Study Projects in expanding the connection of what was learned about the United States, Kenya, and Japan to the other countries displayed on the quilt.

Case Study Project: *Developing a Country Poster*
Direct students to individually or in small groups select one of the countries from Henry's Quilt. Provide the accompanying handouts for the countries they selected and included in this section of the module. The handouts include the classification of the countries based on the descriptions outlined in Lessons 1 to 3. If students select a bottom layered country, direct them to reuse Lesson 2. If students select a lower-middle layered country, direct them to reuse Lesson 1, and if they select an upper-middle layered country, direct them to reuse Lesson 3. Students develop answers to the same questions for the lesson using the data on the handout of their selected country. Consider highlighting selected questions from the lessons for students to summarize on a country poster. In addition, provide time for students to research

outside of class other summaries about the country they selected beyond the population data and to include some of these additional summaries on their poster.

Case Study Project: *Deriving a Country's Centers and Spread*
Provide students a blank copy of the templates in Lesson 4 that were used to estimate the mean and median ages, along with an estimate of the spread of the ages. Direct students to calculate the mean age, the median age, and the spread as outlined in Lesson 4 of their selected country, and to mark off these summaries using the histogram included on the handout for their country. Consider assigning students to write a summary of the description of a typical person in their country and other summaries that indicate the special features of their country's population distribution. (For example, the age group that has the most, or least, count of people, and what percent of the population is counted in the identified age groups.) Again, provide an opportunity for students to share with other students the visual they prepared from the histogram and the written summaries.

Case Study Project: *Interpreting Population and Foundation Layer Factors*
Provide students a handout of their selected country. The second page of the handout indicates the Population Factors derived in the same way the population factors were derived in Lesson 10 using the 2010 and 2015 age group counts. Direct students to summarize the factors based on the criteria of immigration, emigration, births, and deaths. Challenge students to write a summary of their country that indicates if they think their selected country has a noticeable immigrant population, if their country has a major count of births, and if there is an indication of an unusual loss of the population due to death. Also ask students to speculate that if their country is growing, do they think this country will continue to grow if these factors remained the same, or if their country is losing population, do they think their country will continue to lose population if these factors remained the same. Selected questions from Lessons 10, 11, and 12 would provide direction for students to summarize these questions.

Case Study Project: *Developing Projection Models*
Direct students to create a projection model for their selected country based on Lessons 8 (the linear model), Lesson 9 (the exponential model), or Lessons 10 to 12 (the recursive model). If students select the recursive model, provide them one of the spreadsheet files that they redesign to reflect the data of their selected country. Selected questions from the identified lessons could be used to provide direction for students in completing this case study.

Case Study Project: *Writing Kristin's Story*
Based on the population pyramid graphs or the histograms of their selected country, direct students to rewrite one of the Kristin stories (for example, ***Kristin's Story – Chapter 1*** in Lesson 1) assuming Kristin was the same age and lived in their selected country. Would Kristin's age group be a major age group in that country? Is there a significant count of people older than Kristin in this selected country, or younger than Kristin, or both? What do you think is the

decision with people approximately her age regarding whether or not to have children? Will people approximately her age be able to support the people in their parent's age group?

Case Study Project: *A Country Quilt Scavenger Hunt*

Place a copy of each of the 16 country handouts at various stations in a classroom. Arrange the stations so that small groups of students will be able to quickly scan the data on the handouts. Label each station as a Bottom Layered Country station, a Lower Middle-Layered Country station, or an Upper Middle-Layered Country station. Set a time limit (consider approximately 15 minutes) for students to answer the following questions individually or in small groups. (This is only a sampling of questions. Add other questions based on what students learned from the lessons completed in your class.)

Question 1: How many countries have more females than males?
Countries with more females than males: Chile, France, Germany, Greece, Italy, Jamaica, Mexico, South Africa. Countries with more males than females: China, Egypt, Greenland, India, Iran, Saudi Arabia

Question 2: Identify the countries in which the most count of people is in the 0 – 4 years old age group?
India, Egypt, and Mexico

Question 3: Identify the countries in which the most count of people is in the 50 – 54 years old age group?
Germany, Greenland

Question 4: Identify the countries that have no indication of immigration?
Chile (although there are a few population factors that are 1.00), China, Greenland, India, Iran, Jamaica, Mexico, Morocco

Question 5: Identify the countries that have at least 10% of their population younger than 5 years old?
Egypt

Question 6: Identify the countries in which the most count of people is in the 25 – 29 years old age group?
Morocco, China, Iran, Russia, South Africa

Question 7: Identify the countries in which more than 50% of the population older than 50 years old?
None of the countries have more than 50% of their population older than 50. (Germany comes closest.)

Question 8: Identify the country which has the lowest percent of the population in the 0 – 4 years old age group.
Germany with 4.2% of the population 0 – 4 years old

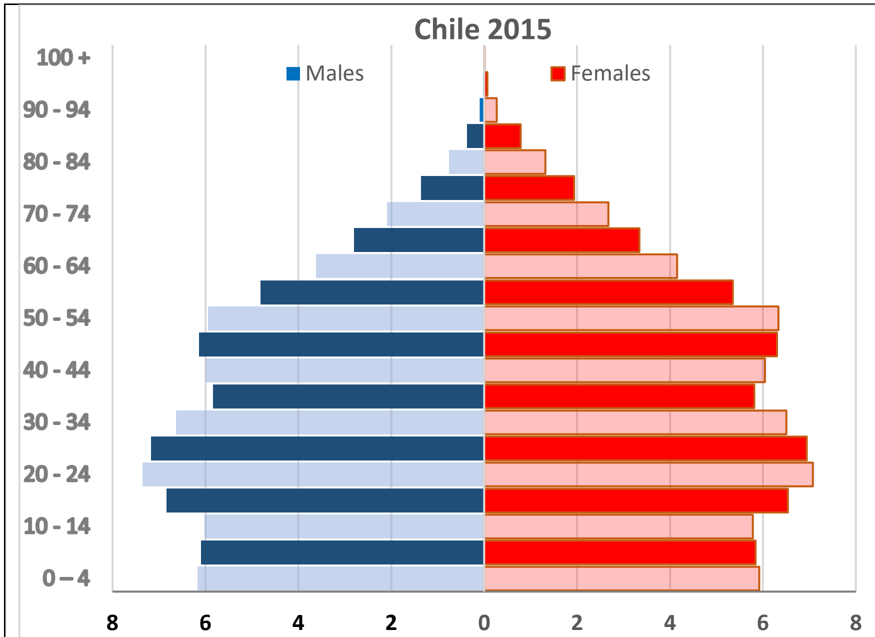
Question 9: Identify a country that less than 100 people who are 100 years old or older?
Greenland, Jamaica

Question 10: How many countries have more males than females in the 0 – 4 years old age group?
All countries displayed on the quilt have more males than females in the 0 – 4 years old age group.

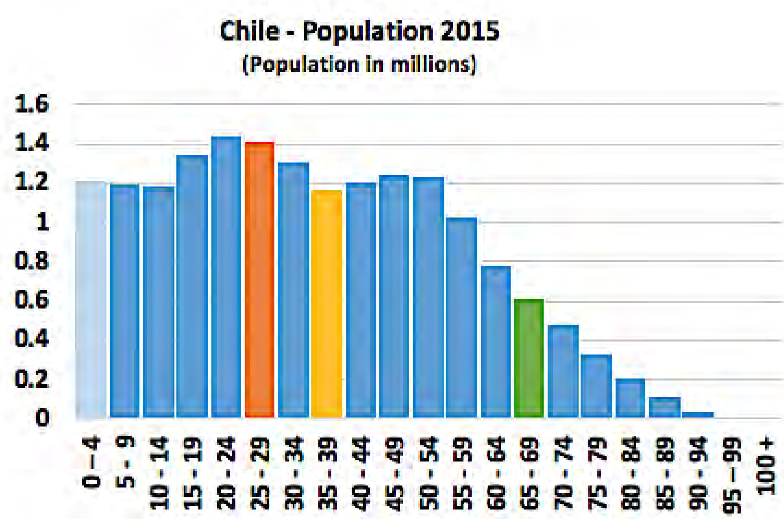
Handouts prepared for the following countries:

Greenland	**Saudi Arabia**	**Italy**	**Germany**
Mexico	**China**	**Iran**	**India**
Jamaica	**Egypt**	**Morocco**	**Chile**
Russia	**Greece**	**France**	**South Africa**

Handout: Chile



Population in hundred thousands of people



Age Groups

Chile 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	616,509	592,047	1,208,556
5-9	608,835	583,968	1,192,803
10-14	602,030	578,268	1,180,298
15-19	683,305	653,742	1,337,047
20-24	735,633	707,565	1,443,198
25-29	717,108	694,161	1,411,269
30-34	663,263	650,428	1,313,691
35-39	583,734	581,332	1,165,066
40-44	599,349	604,367	1,2037,16
45-49	613,192	629,963	1,243,155
50-54	594,357	633,404	1,227,761
55-59	481,026	535,295	1,016,321
60-64	361,320	415,279	776,599
65-69	279,987	334,420	614,407
70-74	209,438	267,385	476,823
75-79	135,705	193,811	329,516
80-84	75,135	131,703	206,838
85-89	36,288	78,302	114,590
90-94	9,377	27,058	36,435
95-99	1,807	7,141	8,948
100+	193	1,030	1,223
Total	8,607,591	8,900,669	17,508,260

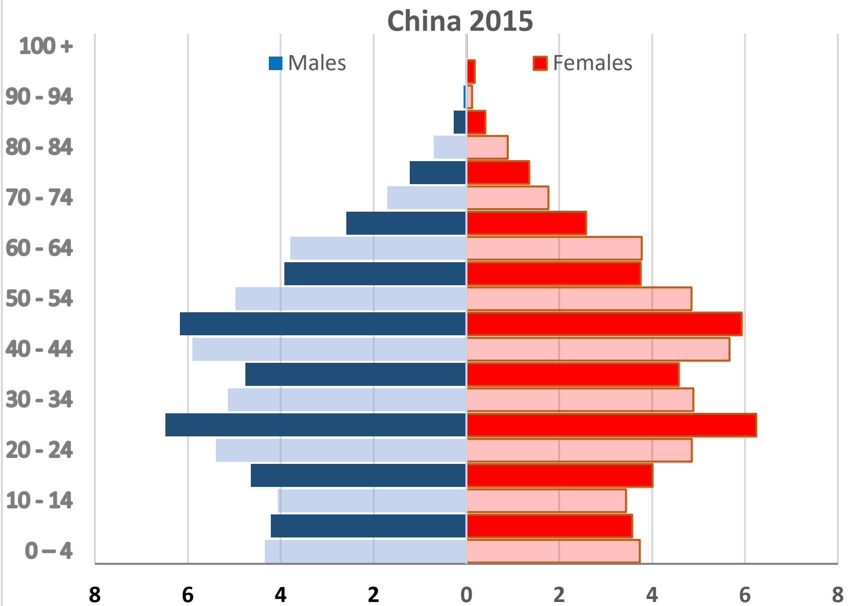
Population Estimates for Chile

United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

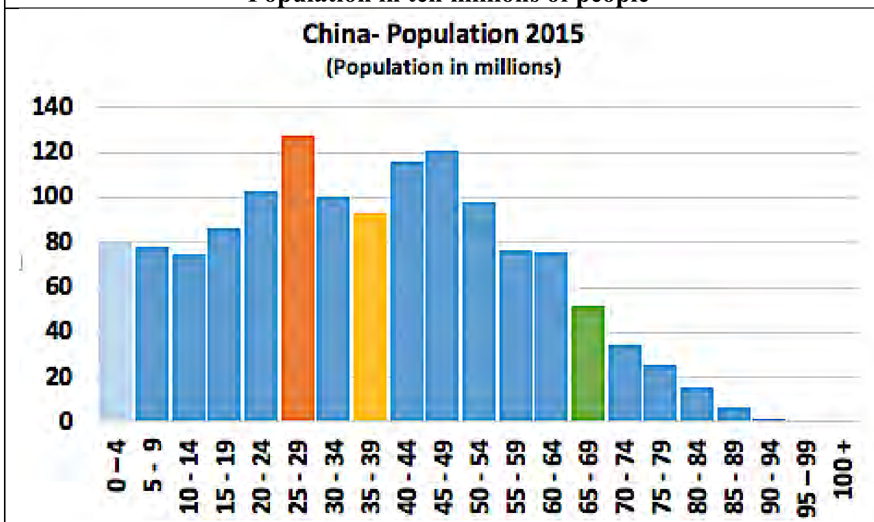
Age groups	Population 2010	Population 2015	Population Factors
0-4	1.19	1.21	1.000
5-9	1.18	1.19	1.000
10-14	1.34	1.18	1.000
15-19	1.44	1.34	1.000
20-24	1.41	1.44	1.000
25-29	1.32	1.41	0.992
30-34	1.17	1.31	1.000
35-39	1.21	1.17	0.992
40-44	1.25	1.2	0.992
45-49	1.25	1.24	0.984
50-54	1.04	1.23	0.981
55-59	0.81	1.02	0.963
60-64	0.65	0.78	0.938
65-69	0.53	0.61	0.906
70-74	0.39	0.48	0.846
75-79	28	0.33	0.008
80-84	0.19	0.21	0.579
85-89	0.79	0.11	0.051
90-94	0.03	0.04	0.333
95-99	0.01	0.01	0.100
100+	0.001	0.001	
Total	16.76	17.51	

Foundation Factor for 2015: 0.069
(approximately 6.9% of the population in 2015 is 0 – 4 years old)

Handout: China



Population in ten millions of people



Age Groups

China 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	43,425,977	37,343,325	80,769,302
5-9	42,127,236	35,698,617	77,825,853
10-14	40,592,924	34,368,323	74,961,247
15-19	46,455,317	40,069,270	86,524,587
20-24	53,925,386	48,546,029	102,471,415
25-29	64,785,579	62,373,427	127,159,006
30-34	51,373,587	48,879,469	100,253,056
35-39	47,630,765	45,708,582	93,339,347
40-44	58,992,298	56,664,766	115,657,064
45-49	61,721,245	59,317,959	121,039,204
50-54	49,737,321	48,473,098	98,210,419
55-59	39,162,479	37,527,188	76,689,667
60-64	37,936,123	37,759,365	75,695,488
65-69	25,883,268	25,784,470	51,667,738
70-74	17,077,724	176,70,012	34,747,736
75-79	12,133,609	13,474,513	25,608,122
80-84	7,067,943	8,898,009	15,965,952
85-89	2,718,754	4,082,755	6,801,509
90-94	619,779	1,215,859	1,835,638
95-99	68,521	1,784,53	246,974
100+	3,658	12,406	16,064
Total	703,439,493	664,045,895	1,367,485,388

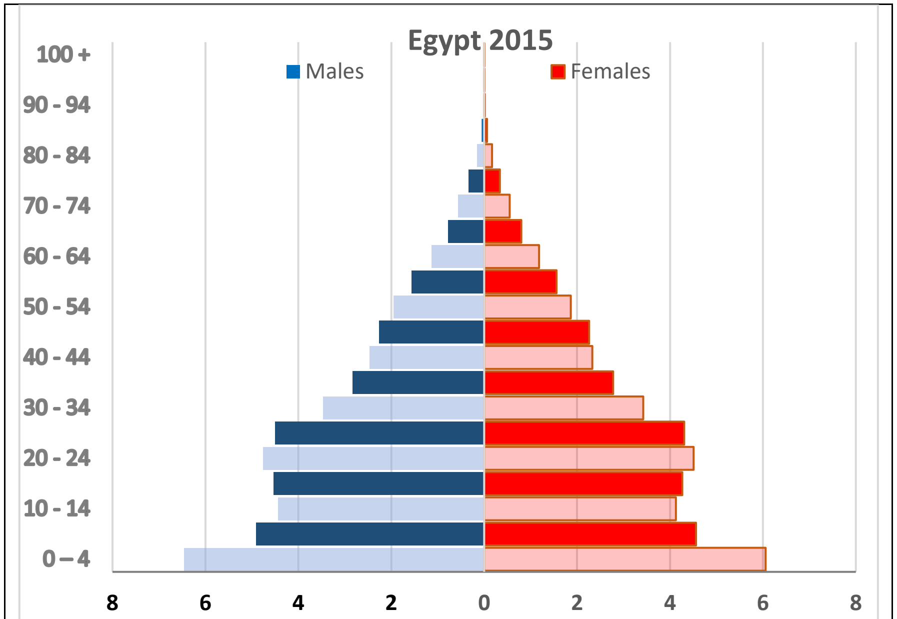
Population Estimates for China

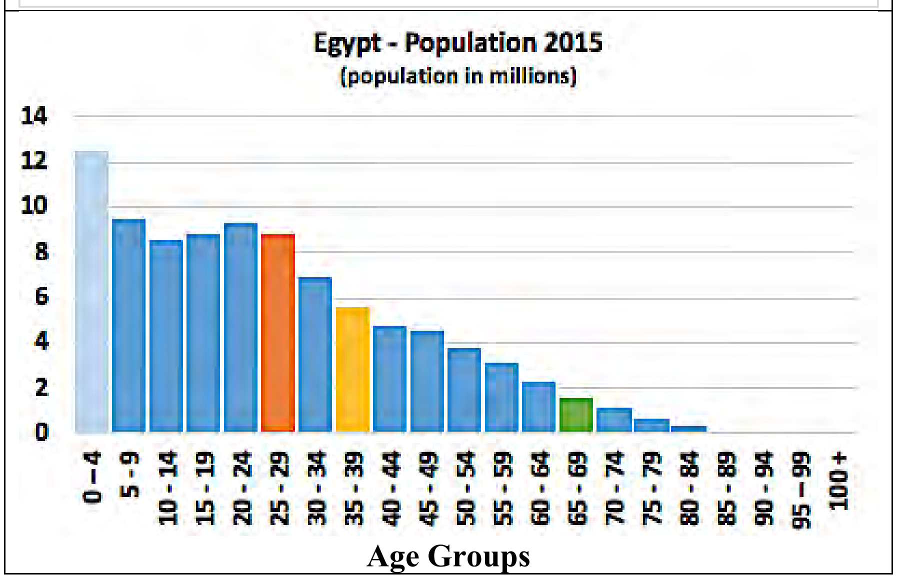
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	78.08	80.77	0.997
5-9	75.09	77.83	0.998
10-14	87.01	74.96	0.994
15-19	103.51	86.52	0.990
20-24	128.35	102.47	0.991
25-29	101.14	127.16	0.991
30-34	94.15	100.25	0.991
35-39	116.8	93.34	0.990
40-44	122.67	115.66	0.987
45-49	100.17	121.04	0.980
50-54	79.13	98.21	0.969
55-59	79.54	76.69	0.952
60-64	56.05	75.7	0.922
65-69	39.79	51.67	0.873
70-74	32.17	34.75	0.796
75-79	23.38	25.61	0.683
80-84	12.96	15.97	0.525
85-89	5.29	6.8	0.348
90-94	1.24	1.84	0.202
95-99	0.16	0.25	0.125
100+	0.01	0.02	
Total	1336.69	1367.49	

Foundation Factor for 2015: 0.059
(approximately 5.9% of the population in 2015 is
0 – 4 years old)

Handout: Egypt





Egypt 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	6,458,037	6,057,906	12,515,943
5-9	4,904,782	4,557,119	9,461,901
10-14	4,436,964	4,126,843	8,563,807
15-19	4,531,339	42,61,435	8,792,774
20-24	4,761,641	4,507,959	9,269,600
25-29	4,506,702	4,303,817	8,810,519
30-34	3,471,328	3,424,019	6,895,347
35-39	2,832,386	2,772,966	5,605,352
40-44	2,463,964	2,327,924	4,791,888
45-49	2,259,698	2,259,003	4,518,701
50-54	1,952,810	1,864,653	3,817,463
55-59	1,557,938	1,555,440	3,113,378
60-64	1,128,612	1,182,102	2,3107,14
65-69	773,282	801,544	1,574,826
70-74	560,239	549,226	1,109,465
75-79	332,897	337,961	670,858
80-84	155,809	169,848	325,657
85-89	54,604	64,611	119,215
90-94	11,448	14,807	26,255
95-99	1,280	1,846	3,126
100+	71	110	181
Total	47,155,831	45,141,139	92,296,970

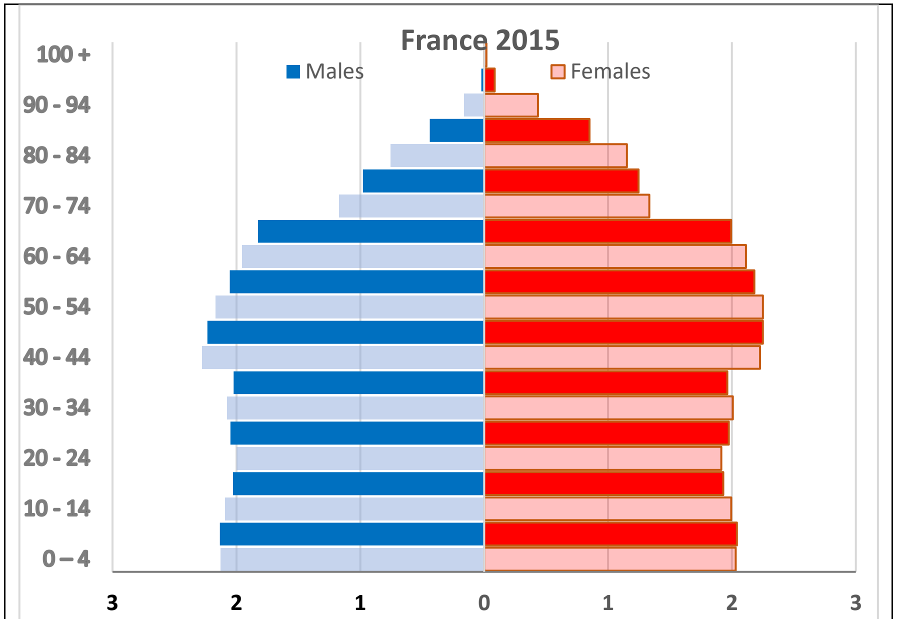
Population Estimates for Egypt

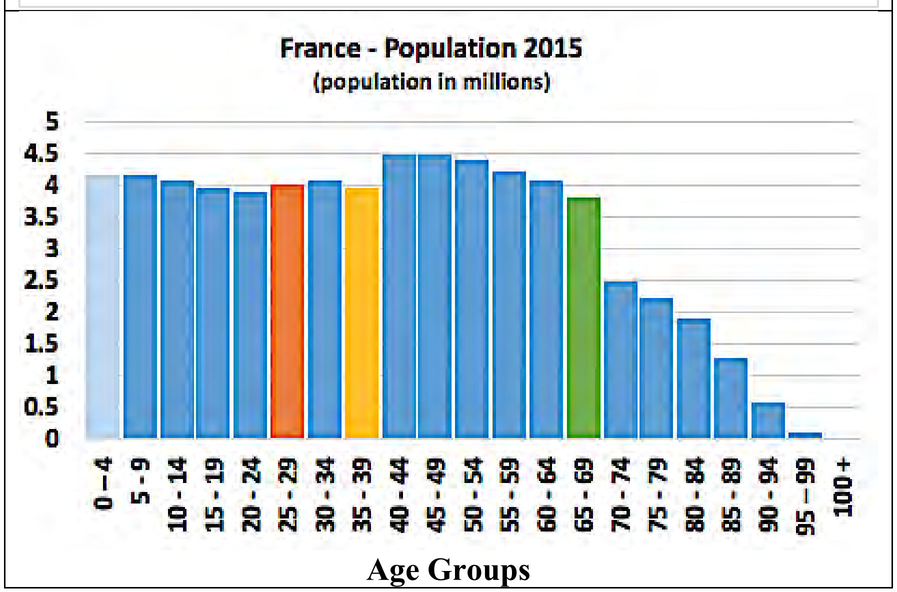
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	10.03	12.52	0.943
5-9	8.56	9.46	1.000
10-14	7.95	8.56	1.106
15-19	8.28	8.79	1.120
20-24	8.68	9.27	1.015
25-29	7.56	8.81	0.913
30-34	5.9	6.9	0.951
35-39	4.57	5.61	1.048
40-44	4.34	4.79	1.041
45-49	3.93	4.52	0.972
50-54	3.4	3.82	0.915
55-59	2.69	3.11	0.859
60-64	1.95	2.31	0.805
65-69	1.42	1.57	0.782
70-74	0.98	1.11	0.684
75-79	0.58	0.67	0.569
80-84	0.29	0.33	0.414
85-89	0.1	0.12	0.300
90-94	0.02	0.03	0.155
95-99	0.0024	0.0031	0.042
100+	0.0001	0.0001	
Total	81.24	92.3	

Foundation Factor for 2015: 0.136
(approximately 13.6% of the population in 2015 is 0 – 4 years old)

Handout: France





France 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	2,126,710	2,031,584	4,158,294
5-9	2,133,979	2,039,855	4,173,834
10-14	2,089,319	1,994,968	4,084,287
15-19	2,027,449	1,929,969	3,957,418
20-24	1,997,834	1,913,020	3,910,854
25-29	2,047,081	1,974,525	4,021,606
30-34	2,075,100	2,007,870	4,082,970
35-39	2,021,723	1,962,526	3,984,249
40-44	2,277,074	2,227,051	4,504,125
45-49	2,234,655	2,249,458	4,484,113
50-54	2,168,042	2,249,583	4,417,625
55-59	2,052,502	2,180,826	4,233,328
60-64	1,956,170	2,113,392	4,069,562
65-69	1,826,230	1,993,650	3,819,880
70-74	1,173,135	1,332,542	2,505,677
75-79	977,719	1,246,160	2,223,879
80-84	757,372	1,151,281	1,908,653
85-89	437,730	850,204	1,287,934
90-94	161,610	434,837	596,447
95-99	2,3043	85,010	108,053
100+	3,239	17,739	2,0978
Total	32,567,716	33,986,050	66,553,766

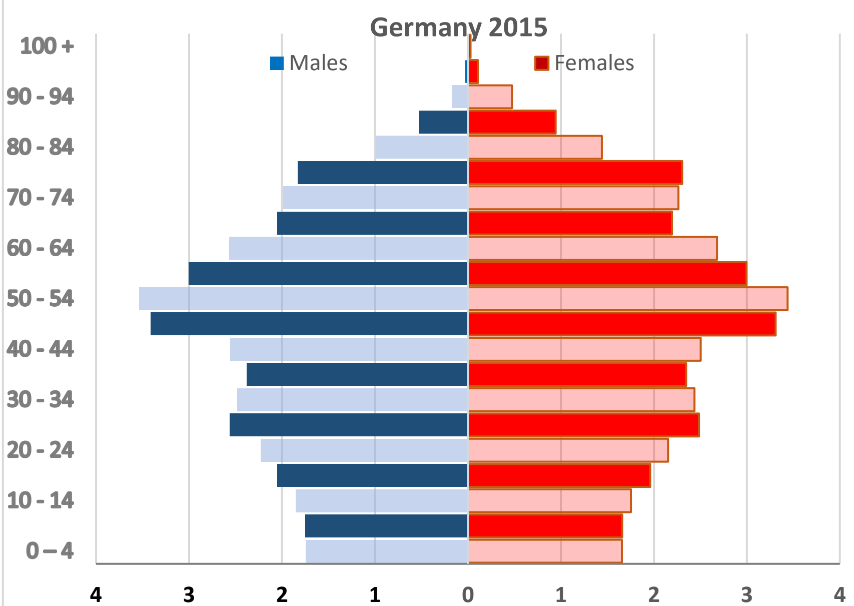
Population Estimates for France

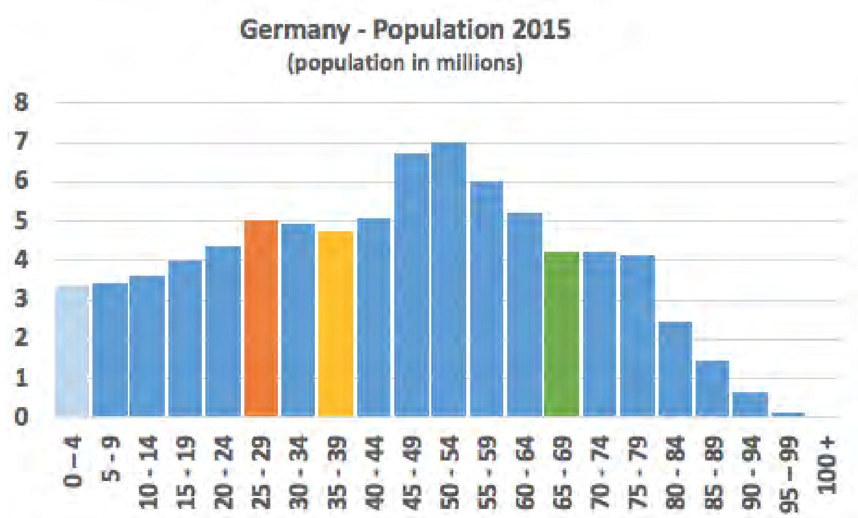
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	4.15	4.16	1.005
5-9	4.06	4.17	1.005
10-14	3.94	4.08	1.005
15-19	3.91	3.96	1.000
20-24	4.01	3.91	1.002
25-29	4.07	4.02	1.002
30-34	3.97	4.08	1.003
35-39	4.50	3.98	1.000
40-44	4.50	4.50	0.996
45-49	4.46	4.48	0.991
50-54	4.31	4.42	0.981
55-59	4.19	4.23	0.971
60-64	3.99	4.07	0.957
65-69	2.66	3.82	0.944
70-74	2.47	2.51	0.899
75-79	2.29	2.22	0.834
80-84	1.82	1.91	0.709
85-89	1.18	1.29	0.508
90-94	0.34	0.60	0.324
95-99	0.11	0.11	0.182
100+	0.02	0.02	
Total	64.94	66.55	

Foundation Factor for 2015: 0.063
(approximately 6.3% of the population in 2015 is
0 – 4 years old)

Handout: Germany





Germany 2015
An Upper Middle-Layered Country

Age Group	Males	Females	Total
0-4	1,745,339	1,655,569	3,400,908
5-9	1,750,952	1,659,400	3,410,352
10-14	1,849,795	1,753,102	3,602,897
15-19	2,053,482	1,961,171	4,014,653
20-24	2,226,480	2,152,575	4,379,055
25-29	2,562,953	2,485,113	5,048,066
30-34	2,479,396	2,436,660	4,916,056
35-39	2,381,425	2,346,824	4,728,249
40-44	2,556,793	2,503,539	5,060,332
45-49	3,414,717	3,309,674	6,724,391
50-54	3,538,896	3,438,122	6,977,018
55-59	3,006,143	2,996,276	6,002,419
60-64	2,565,551	2,678,828	5,244,379
65-69	2,052,017	2,194,170	4,246,187
70-74	1,984,877	2,264,255	4,249,132
75-79	1,832,823	2,304,763	4,137,586
80-84	991,381	1,439,965	2,431,346
85-89	520,987	943,475	1,464,462
90-94	170,528	474,048	644,576
95-99	32,092	106,970	139,062
100+	6,593	26,689	33,282
Total	39,723,220	41,131,188	80,854,408

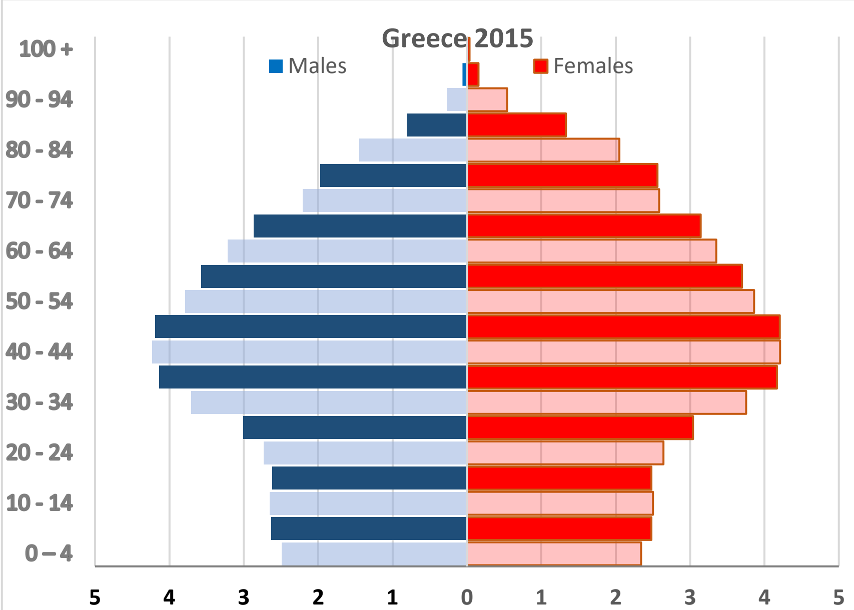
Population Estimates for Germany

United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

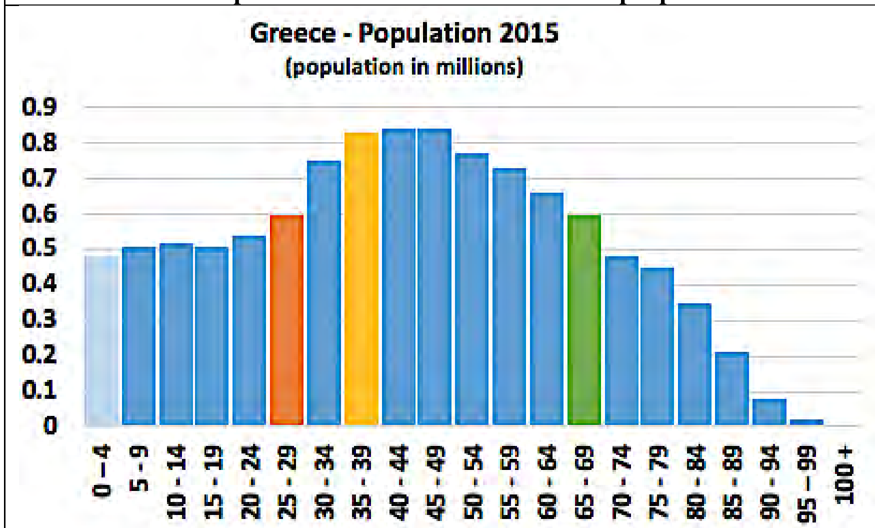
Age groups	Population 2010	Population 2015	Population Factors
0-4	3.4	3.4	1.003
5-9	3.6	3.41	1.000
10-14	3.97	3.6	1.010
15-19	4.23	4.01	1.035
20-24	4.91	4.38	1.029
25-29	4.89	5.05	1.006
30-34	4.75	4.92	0.996
35-39	5.1	4.73	0.992
40-44	6.79	5.06	0.990
45-49	7.09	6.72	0.984
50-54	6.15	6.98	0.976
55-59	5.46	6	0.960
60-64	4.5	5.24	0.944
65-69	4.65	4.25	0.914
70-74	4.79	4.25	0.864
75-79	3.15	4.14	0.771
80-84	2.3	2.43	0.635
85-89	1.33	1.46	0.481
90-94	0.41	0.64	0.341
95-99	0.15	0.14	0.200
100+	0.02	0.03	
Total	81.64	80.85	

Foundation Factor for 2015: 0.042
(approximately 4.2% of the population in 2015 is 0 – 4 years old)

Handout: Greece



Population in hundred thousands of people



Age Groups

Greece 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	248,864	234,148	483,012
5-9	263,293	247,885	511,178
10-14	265,490	250,104	515,594
15-19	261,926	247,973	509,899
20-24	272,929	264210	537139
25-29	300,953	303,833	604,786
30-34	370,740	375,381	746,121
35-39	413,598	416,866	830,464
40-44	422,978	421,002	843,980
45-49	419,393	420,629	840,022
50-54	379,170	386,076	765,246
55-59	357,636	369,627	727,263
60-64	321,397	335,206	656,603
65-69	286,769	314,367	601,136
70-74	220,728	258453	479,181
75-79	197,180	256,031	453,211
80-84	144,992	204,948	349,940
85-89	80,725	132,900	213,625
90-94	27,063	54,113	81,176
95-99	6,193	15,403	21,596
100+	1,086	3,385	4,471
Total	5,263,103	5,512,540	10,775,643

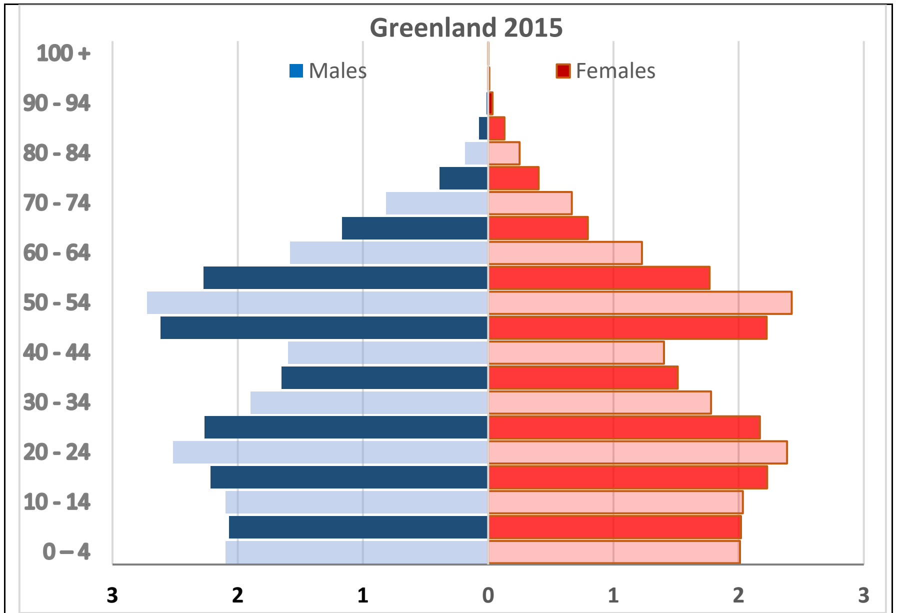
Population Estimates for Greece

United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

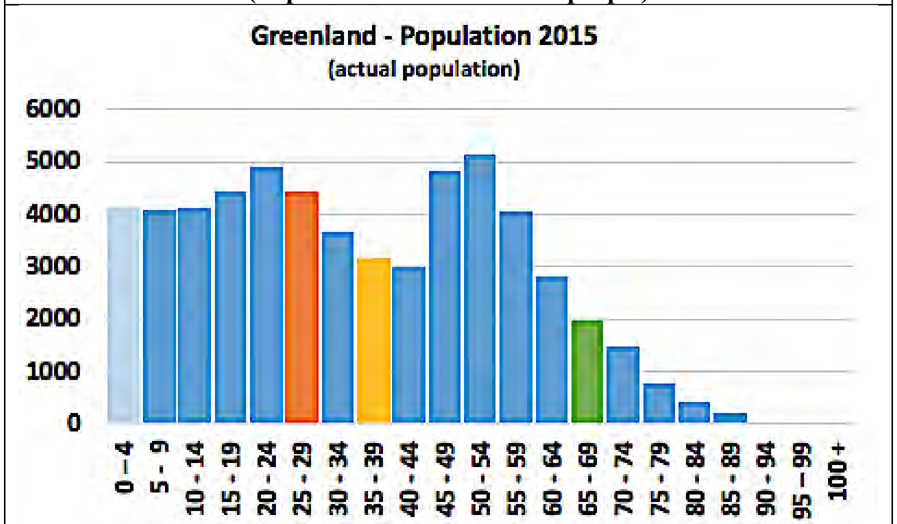
Age groups	Population 2010	Population 2015	Population Factors
0-4	0.51	0.48	1.000
5-9	0.51	0.51	1.020
10-14	0.51	0.52	1.000
15-19	0.53	0.51	1.019
20-24	0.58	0.54	1.034
25-29	0.73	0.6	1.027
30-34	0.82	0.75	1.012
35-39	0.84	0.83	1.000
40-44	0.84	0.84	1.000
45-49	0.77	0.84	1.000
50-54	0.74	0.77	0.986
55-59	0.68	0.73	0.971
60-64	0.63	0.66	0.952
65-69	0.53	0.6	0.906
70-74	0.53	0.48	0.849
75-79	0.46	0.45	0.761
80-84	0.33	0.35	0.636
85-89	0.16	0.21	0.506
90-94	0.059	0.081	0.339
95-99	0.02	0.02	0.200
100+	0.003	0.004	
Total	10.75	10.78	

Foundation Factor for 2015: 0.045
(approximately 4.5% of the population in 2015 is 0 – 4 years old)

Handout: Greenland



(Population in thousands of people)



Age Groups

Greenland 2015
A Bottom Layered Country

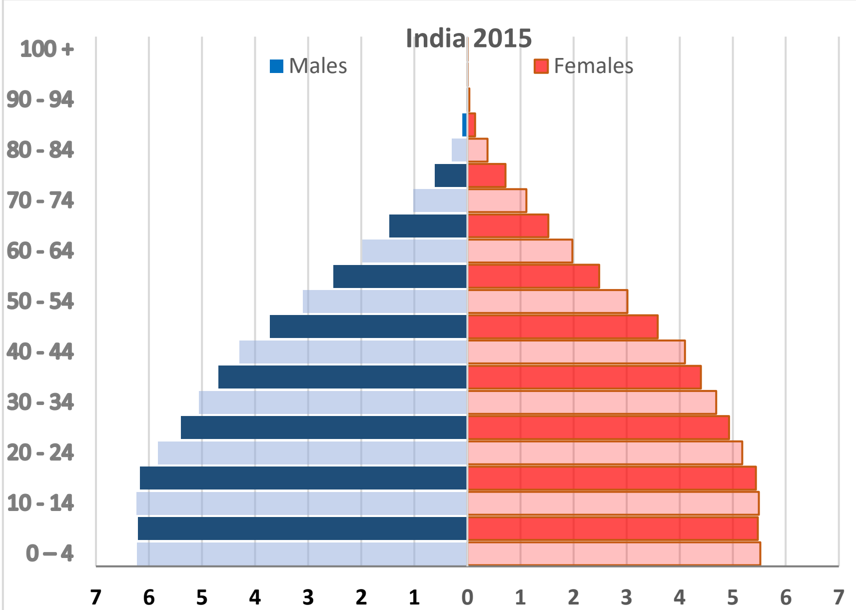
Age Group	Males	Females	Total
0-4	2,096	2,011	4,107
5-9	2,070	2,019	4,089
10-14	2,097	2,034	4,131
15-19	2,218	2,228	4,446
20-24	2,518	2,387	4,905
25-29	2,265	2,170	4,435
30-34	1,898	1,780	3,678
35-39	1,651	1,514	3,165
40-44	1,596	1,404	3,000
45-49	2,618	2,224	4,842
50-54	2,723	2,424	5,147
55-59	2,274	1,768	4,042
60-64	1,584	1,228	2,812
65-69	1,165	796	1,961
70-74	814	668	1,482
75-79	388	403	791
80-84	184	251	435
85-89	72	130	202
90-94	13	34	47
95-99	3	10	13
100+	1	2	3
Total	30,248	27,485	57,733

Population Estimates for Greenland
United States Census Bureau
International Data Base (IDB)

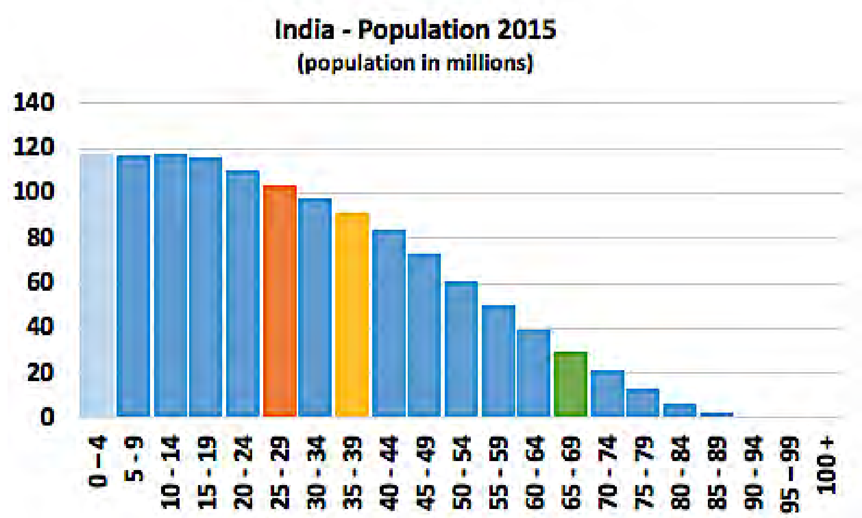
Age groups	Population 2010	Population 2015	Population Factors
0-4	4201	4107	0.973
5-9	4262	4089	0.969
10-14	4572	4131	0.972
15-19	5058	4446	0.970
20-24	4593	4905	0.966
25-29	3795	4435	0.969
30-34	3339	3678	0.948
35-39	3263	3165	0.919
40-44	5094	3000	0.951
45-49	5405	4842	0.952
50-54	4359	5147	0.927
55-59	3186	4042	0.883
60-64	2345	2812	0.836
65-69	1862	1961	0.796
70-74	1095	1482	0.722
75-79	675	791	0.644
80-84	369	435	0.547
85-89	115	202	0.409
90-94	39	47	0.333
95-99	8	13	0.375
100+	2	3	
Total	57637	57733	

Foundation Factor for 2015: 0.071
(approximately 7.1% of the population in 2015 is 0 – 4 years old)

Handout: India



Population in ten millions of people



Age Groups

India 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	62,247,958	55,201,830	117,449,788
5-9	62,083,654	54,709,227	116,792,881
10-14	62,403,725	54,924,811	117,328,536
15-19	61,658,005	54,358,774	116,016,779
20-24	58,275,712	51,794,339	110,070,051
25-29	54,027,716	49,325,095	103,352,811
30-34	50,626,742	46,886,799	97,513,541
35-39	46,883,656	44,005,831	90,889,487
40-44	42,957,020	40,998,776	83,955,796
45-49	37,195,010	35,862,419	73,057,429
50-54	31,010,226	30,158,528	61,168,754
55-59	25,237,863	24,836,947	50,074,810
60-64	19,755,519	19,783,390	39,538,909
65-69	14,732,277	15,245,211	29,977,488
70-74	10,225,569	11,116,174	21,341,743
75-79	6,182,820	7,186,619	13,369,439
80-84	2,961,309	3,791,789	6,753,098
85-89	987,533	1,441,249	2,428,782
90-94	201,144	344,180	545,324
95-99	21,808	44,431	66,239
100+	1,149	2,750	3,899
Total	649,676,415	602,019,169	1,251,695,584

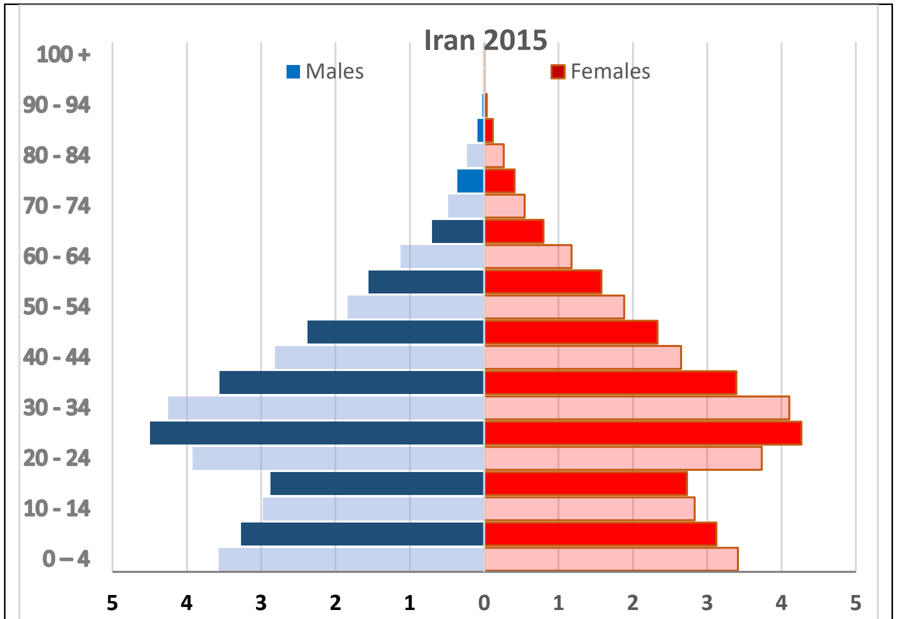
Population Estimates for India

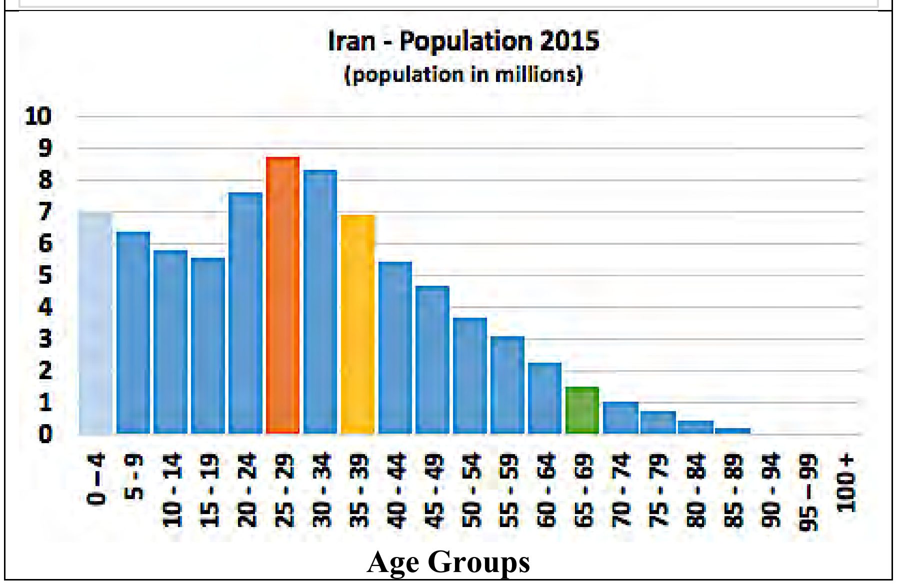
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	118.45	117.45	0.986
5-9	117.80	116.79	0.996
10-14	116.55	117.33	0.995
15-19	110.82	116.02	0.993
20-24	104.25	110.07	0.991
25-29	98.50	103.35	0.990
30-34	91.98	97.51	0.988
35-39	85.22	90.89	0.985
40-44	74.57	83.96	0.980
45-49	63.11	73.06	0.969
50-54	52.66	61.17	0.951
55-59	42.79	50.07	0.924
60-64	33.78	39.54	0.888
65-69	25.75	29.98	0.829
70-74	18.10	21.34	0.739
75-79	11.09	13.37	0.609
80-84	5.40	6.75	0.450
85-89	1.84	2.43	0.299
90-94	.39	.55	0.179
95-99	.04	.07	1.000
100+	.02	.04	
Total	1,173.11	1,251.70	

Foundation Factor for 2015: 0.093
(approximately 9.3% of the population in 2015 is 0 – 4 years old)

Handout: Iran





Iran 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	3,575,760	3,413,858	6,989,618
5-9	3,272,526	3,123,881	6,396,407
10-14	2,976,562	2,831,863	5,8084,25
15-19	2,870,362	2,728,739	5,599,101
20-24	3,923,035	3,734,442	7,657,477
25-29	4,495,945	4,267,497	8,763,442
30-34	4,251,893	4,105,454	8,357,347
35-39	3,558,766	3,392,840	6,951,606
40-44	2,815,881	2,648,866	5,464,747
45-49	2,375,599	2,332,263	4,707,862
50-54	1,839,801	1,883,606	3,723,407
55-59	1,554,005	1,577,685	3,131,690
60-64	1,127,572	1,175,467	2,303,039
65-69	704,433	797,913	1,502,346
70-74	483,918	54,4871	1,028,789
75-79	362,967	409,680	772,647
80-84	230,200	263,382	493,582
85-89	90,131	117,383	207,514
90-94	25,794	36,365	62,159
95-99	2,250	3,938	6,188
100+	182	402	584
Total	40,537,582	39,390,395	79,927,977

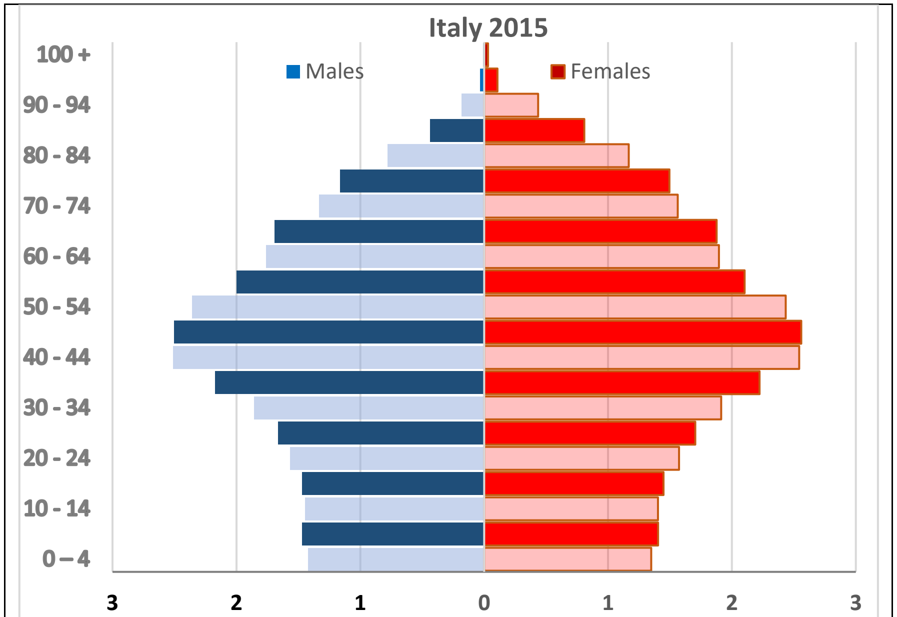
Population Estimates for Iran

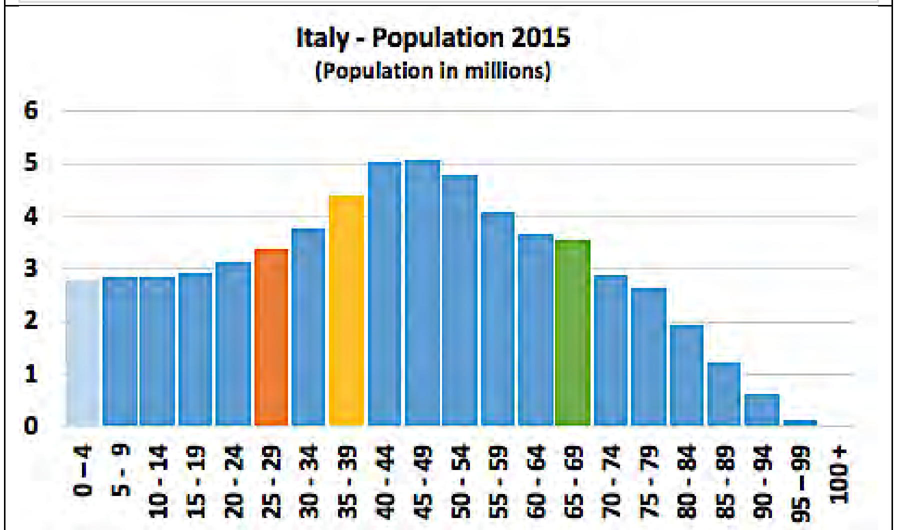
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	6.44	6.99	0.994
5-9	5.84	6.40	0.995
10-14	5.63	5.81	0.995
15-19	7.71	5.60	0.994
20-24	8.82	7.66	0.993
25-29	8.40	8.76	0.995
30-34	7.00	8.36	0.993
35-39	5.51	6.95	0.991
40-44	4.78	5.46	0.985
45-49	3.81	4.71	0.976
50-54	3.25	3.72	0.963
55-59	2.45	3.13	0.939
60-64	1.67	2.30	0.898
65-69	1.22	1.50	0.844
70-74	1.02	1.03	0.755
75-79	0.80	0.77	0.613
80-84	0.45	0.49	0.467
85-89	0.21	0.21	0.286
90-94	0.04	0.06	0.250
95-99	0.01	0.01	0.060
100+	0.0006	0.0006	
Total	75.03	79.93	

Foundation Factor for 2015: 0.087
(approximately 8.7% of the population in 2015 is 0 – 4 years old)

Handout: Italy





Italy 2015
A Lower Middle-Layered Country
(Population in millions to the nearest hundredth)

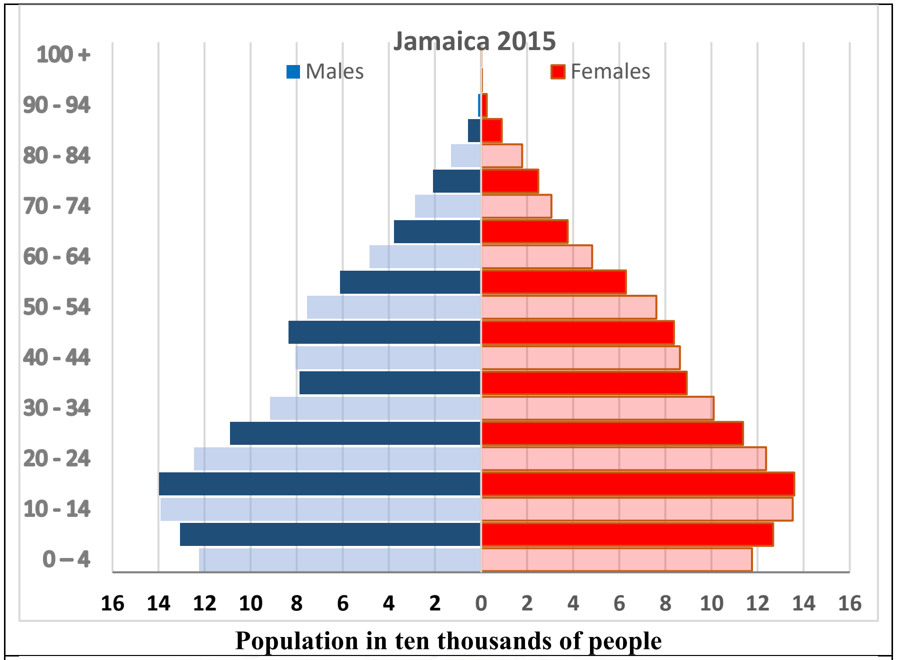
Age Group	Males	Females	Total
0-4	1,422,105	1,348320	2,770,425
5-9	1,470,920	1,403597	2,874,517
10-14	1,447,355	1,402820	2,850,175
15-19	1,468,171	1,447641	2,915,812
20-24	1,567,415	1,572943	3,140,358
25-29	1,662,332	1,703924	3,366,256
30-34	1,859,102	1,913078	3,772,180
35-39	2,171,637	2,222880	4,394,517
40-44	2,509,496	2,542653	5,052,149
45-49	2,505,344	2,559467	5,064,811
50-54	2,355,822	2,433973	4,789,795
55-59	1,997,639	2,102239	4,099,878
60-64	1,758,907	1,894951	3,653,858
65-69	1,693,814	1,876388	3,570,202
70-74	1,331,357	1,562453	2,893,810
75-79	1,163,117	1,495345	2,658,462
80-84	777,650	1,167028	1,944,678
85-89	437,544	808,236	1,245,780
90-94	181,979	435,619	617,598
95-99	33,990	107,338	141,328
100+	7,301	31,230	38,531
Total	29,822,997	32,032,123	61,855,120

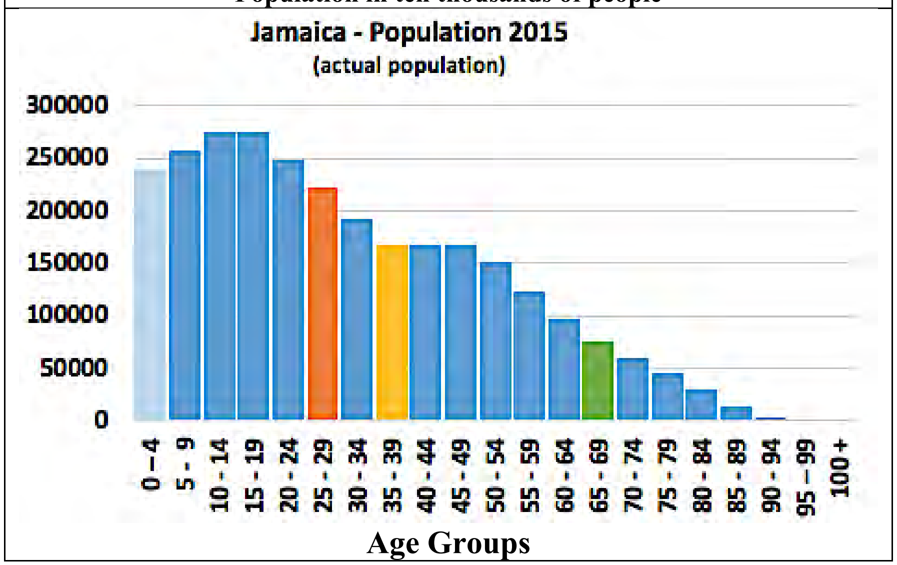
Population Estimates for Italy
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	2.84	2.77	1.011
5-9	2.78	2.87	1.025
10-14	2.78	2.85	1.050
15-19	2.99	2.92	1.050
20-24	3.15	3.14	1.070
25-29	3.55	3.37	1.062
30-34	4.18	3.77	1.050
35-39	4.9	4.39	1.031
40-44	4.99	5.05	1.014
45-49	4.77	5.06	1.004
50-54	4.13	4.79	0.993
55-59	3.75	4.1	0.973
60-64	3.74	3.65	0.955
65-69	3.12	3.57	0.926
70-74	3.04	2.89	0.875
75-79	2.48	2.66	0.782
80-84	1.85	1.94	0.676
85-89	1.16	1.25	0.534
90-94	0.37	0.62	0.378
95-99	0.15	0.14	0.267
100+	0.02	0.04	
Total	60.75	61.86	

Foundation Factor for 2015: 0.045
(approximately 4.5% of the population in 2015 is 0 – 4 years old)

Handout: Jamaica





Jamaica 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	122,483	117,554	240,037
5-9	130,711	126,776	257,487
10-14	139,109	135.244	274,353
15-19	139,672	135,939	275,611
20-24	124,663	123,678	248,341
25-29	109,020	113,626	222,646
30-34	91,550	100,939	192,489
35-39	78,808	89,256	168,064
40-44	80,476	86,278	166,754
45-49	83,676	83,637	167,313
50-54	75,609	76,062	151,671
55-59	61,268	62,878	124,146
60-64	48,464	48,189	96,653
65-69	37,871	37,587	75,458
70-74	28,750	30,515	59,265
75-79	20,937	24,792	45,729
80-84	12,943	17,801	30,744
85-89	5,705	8,929	14,634
90-94	1,358	2,486	3,844
95-99	191	416	607
100+	13	32	45
Total	1,393,277	1,422,614	2,815,891

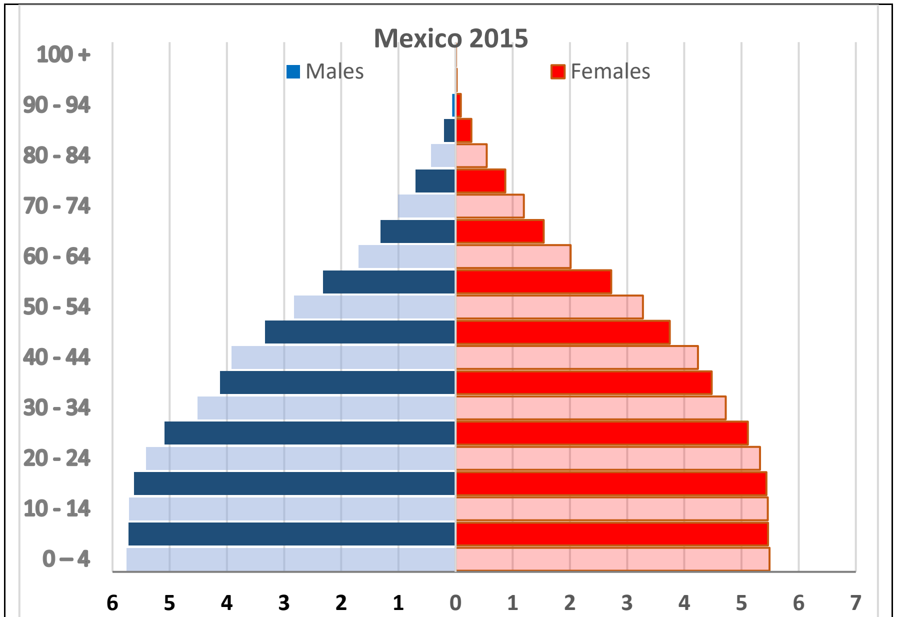
Population Estimates for Jamaica

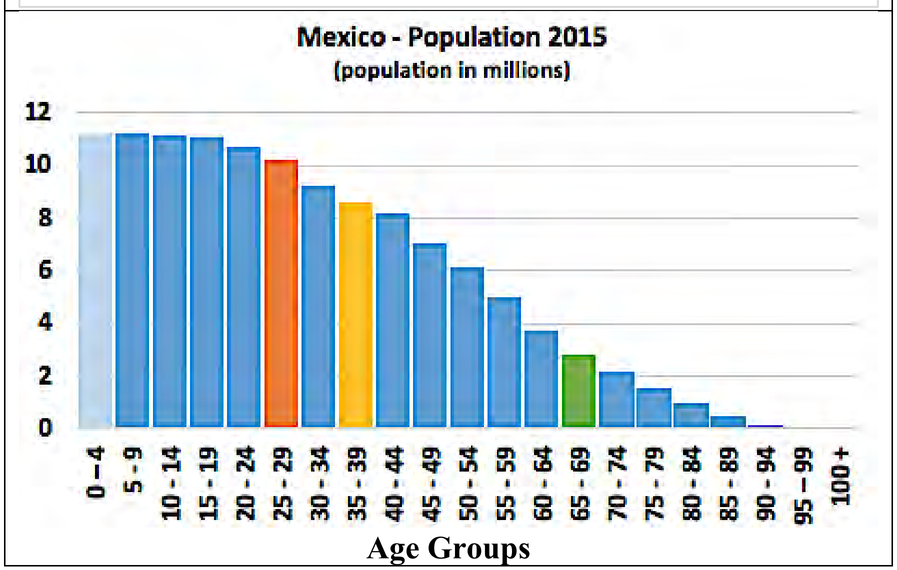
United States Census Bureau
International Data Base (IDB)
(actual population)

Age groups	Population 2010	Population 2015	Population Factors
0-4	264,646	240,037	0.973
5-9	286,146	257,487	0.959
10-14	300,641	274,353	0.917
15-19	280,072	275,611	0.887
20-24	259,555	248,341	0.858
25-29	229,372	222,646	0.839
30-34	196,234	192,489	0.856
35-39	181,994	168,064	0.916
40-44	171,072	166,754	0.978
45-49	151,675	167,313	1.000
50-54	124,517	151,671	0.997
55-59	98,831	124,146	0.978
60-64	83,071	96,653	0.908
65-69	67,236	75,458	0.881
70-74	54,334	59,265	0.842
75-79	44,487	45,729	0.691
80-84	28,775	30,744	0.509
85-89	12,018	14,634	0.320
90-94	3,406	3,844	0.178
95-99	517	607	0.087
100+	39	45	
Total	2,838,638	2,815,891	

Foundation Factor for 2015: 0.085
(approximately 8.5% of the population in 2015 is 0 – 4 years old)

Handout: Mexico





Mexico 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	5,753,392	5,490,615	11,244,007
5-9	5,718,574	5,461,539	11,180,113
10-14	5,706,361	5,460,183	11,166,544
15-19	5,618,908	5,435,388	11,054,296
20-24	5,408,656	5,324,058	10,732,714
25-29	5,091,053	5,112,092	10,203,145
30-34	4,513,592	4,724,855	9,238,447
35-39	4,115,290	4,480,589	8,595,879
40-44	3,912,839	4,238,800	8,151,639
45-49	3,328,785	3,743,839	7,072,624
50-54	2,823,786	3,276,470	6,100,256
55-59	2,318,798	2,722,236	5,041,034
60-64	1,698,923	2,012,155	3,711,078
65-69	1,309,000	1,543,547	2,852,547
70-74	1,005,109	1,193,904	2,199,013
75-79	702,356	868,207	1,570,563
80-84	423,504	544,596	968,100
85-89	198,581	275,882	474,463
90-94	59,211	89,581	148,792
95-99	10,953	17,642	28,595
100+	1,159	1,801	2,960
Total	59,718,830	62,017,979	121,736,809

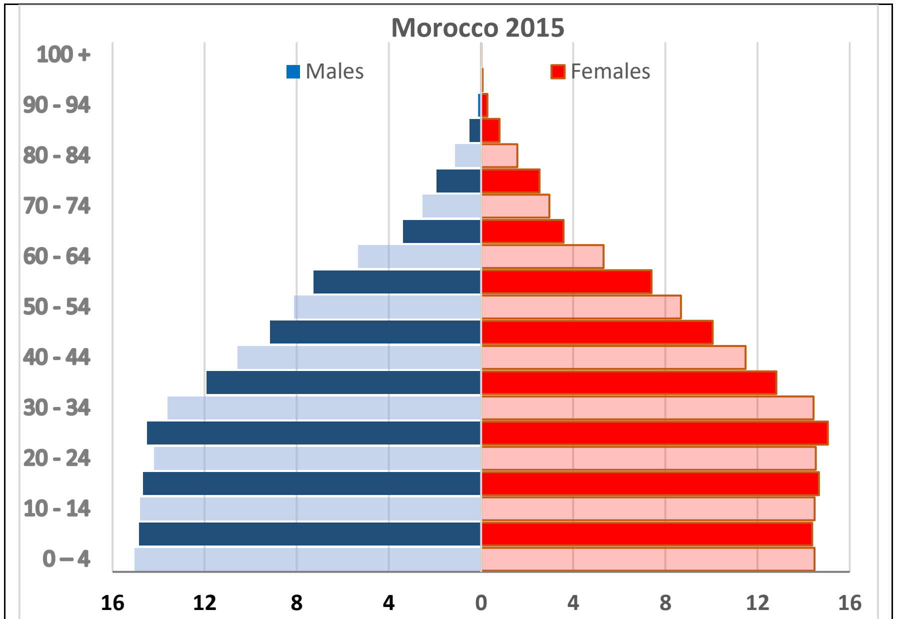
Population Estimates for Mexico

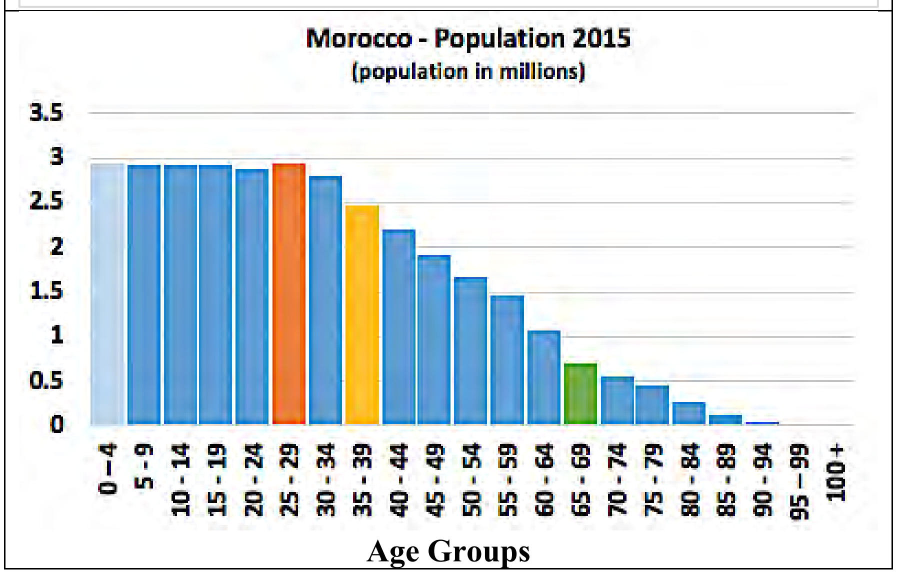
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	11.24	11.24	0.995
5-9	11.24	11.18	0.994
10-14	11.17	11.17	0.989
15-19	10.90	11.05	0.984
20-24	10.38	10.73	0.983
25-29	9.41	10.20	0.982
30-34	8.75	9.24	0.983
35-39	8.29	8.60	0.983
40-44	7.21	8.15	0.981
45-49	6.26	7.07	0.974
50-54	5.23	6.10	0.964
55-59	3.93	5.04	0.944
60-64	3.10	3.71	0.919
65-69	2.48	2.85	0.887
70-74	1.89	2.20	0.831
75-79	1.30	1.57	0.746
80-84	.79	.97	0.595
85-89	.36	.47	0.417
90-94	.12	.15	0.250
95-99	.02	.03	0.100
100+	.002	.002	
Total	114.06	121.74	

Foundation Factor for 2015: 0.092
(approximately 9.2% of the population in 2015 is 0 – 4 years old)

Handout: Morocco





Morocco 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	1,504,258	1,446,987	2,951,245
5-9	1,485,416	1,436,444	2,921,860
10-14	1,479,787	1,447,473	2,927,260
15-19	1,466,544	1,466,720	2,933,264
20-24	1,420,093	1,452,604	2,872,697
25-29	1,449,871	1,504,978	2,954,849
30-34	1,361,828	1,442,928	2,804,756
35-39	1,191,474	1,281,706	2,473,180
40-44	1,056,817	1,147,885	2,204,702
45-49	916,853	1,005,119	1,921,972
50-54	811,758	867,271	1,679,029
55-59	727,928	740,041	1,467,969
60-64	534,706	531,451	1,066,157
65-69	338,698	357,655	696,353
70-74	253,821	296,222	550,043
75-79	193,431	252,899	446,330
80-84	113,359	157,223	270,582
85-89	49,464	80,442	129,906
90-94	13,727	27,815	41,542
95-99	2,236	6,033	8,269
100+	164	570	734
Total	16,372,233	16,950,466	33,322,699

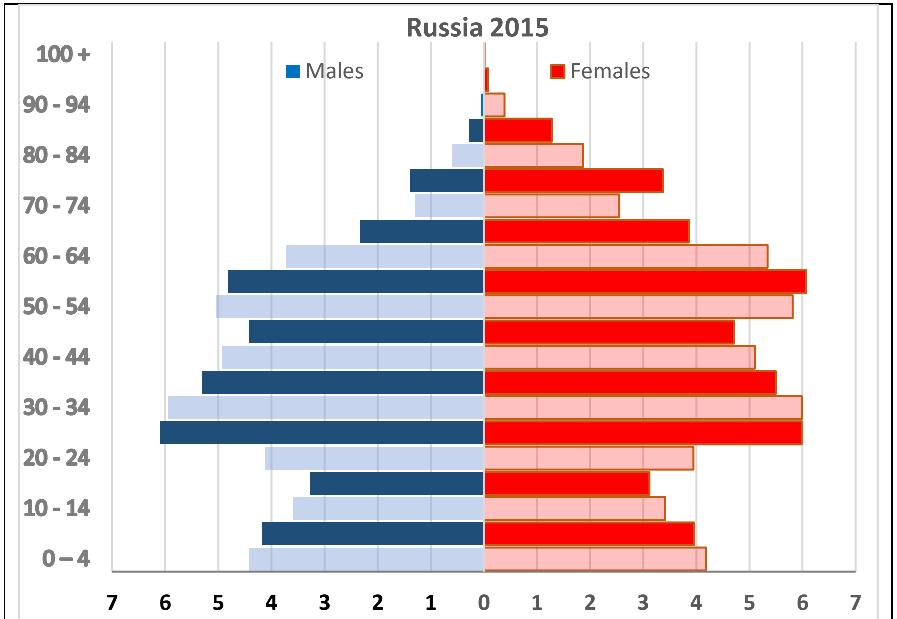
Population Estimates for Morocco

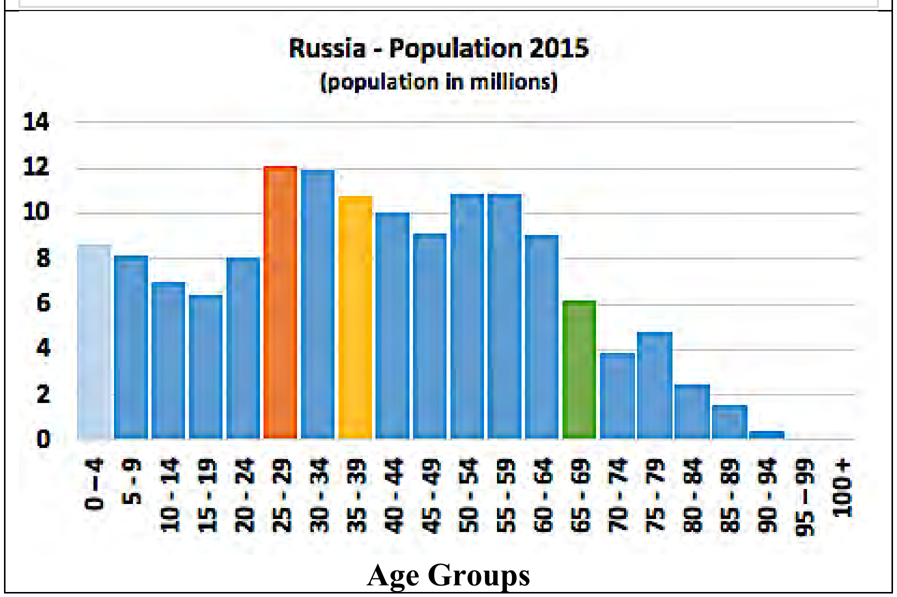
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	2.97	2.95	0.983
5-9	2.97	2.92	0.987
10-14	2.99	2.93	0.980
15-19	2.96	2.93	0.970
20-24	3.05	2.87	0.967
25-29	2.89	2.95	0.969
30-34	2.53	2.8	0.976
35-39	2.25	2.47	0.978
40-44	1.96	2.2	0.980
45-49	1.72	1.92	0.971
50-54	1.53	1.67	0.961
55-59	1.13	1.47	0.947
60-64	0.76	1.07	0.921
65-69	0.62	0.7	0.887
70-74	0.54	0.55	0.833
75-79	0.38	0.45	0.711
80-84	0.23	0.27	0.565
85-89	0.1	0.13	0.400
90-94	0.03	0.04	0.267
95-99	0.005	0.008	0.140
100+	0.0003	0.0007	
Total	31.63	33.32	

Foundation Factor for 2015: 0.089
(approximately 8.9% of the population in 2015 is 0 – 4 years old)

Handout: Russia





Russia 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	4,426,251	4,184,940	8,611,191
5-9	4,182,492	3,959,721	8,142,213
10-14	3,596,249	3,412,103	7,008,352
15-19	3,275,148	3,118,304	6,393,452
20-24	4,118,040	3,945,756	8,063,796
25-29	6,109,423	5,983,023	12,092,446
30-34	5,953,842	5,987,143	11,940,985
35-39	5,315,372	5,494,378	10,809,750
40-44	4,926,627	5,100,472	10,027,099
45-49	4,421,047	4,706,383	9,127,430
50-54	5,053,377	5,814,947	10,868,324
55-59	4,8115,41	6,06,6414	10,877,955
60-64	3,733,830	5,342,662	9,076,492
65-69	2,337,857	3,859,478	6,197,335
70-74	1,293,869	2,549,735	3,84,3604
75-79	1,383,292	3,369,360	4,752,652
80-84	607,580	1,864,982	2,472,562
85-89	289,117	1,280,368	1,569,485
90-94	54,341	388,030	442,371
95-99	11,088	79,856	90,944
100+	1,434	13,901	15,335
Total	65,901,817	76,521,956	142,423,773

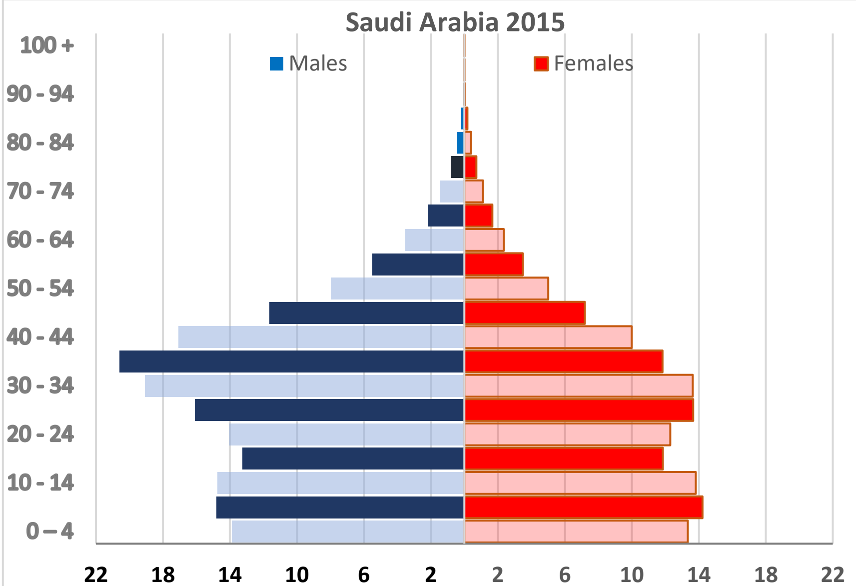
Population Estimates for Russia

United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

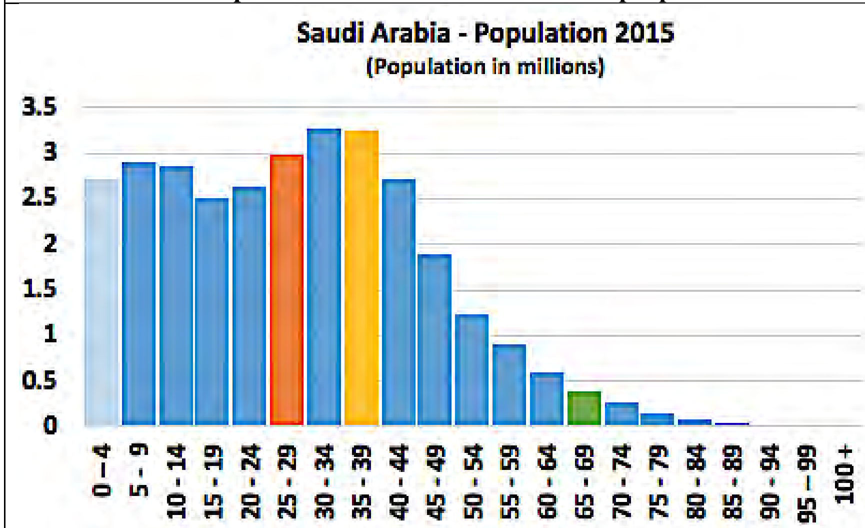
Age groups	Population 2010	Population 2015	Population Factors
0-4	8.13	8.61	1.001
5-9	6.98	8.14	1.004
10-14	6.36	7.01	1.005
15-19	8.00	6.39	1.008
20-24	12.04	8.06	1.004
25-29	11.96	12.09	0.998
30-34	10.89	11.94	0.993
35-39	10.16	10.81	0.987
40-44	9.35	10.03	0.976
45-49	11.32	9.13	0.960
50-54	11.56	10.87	0.941
55-59	9.98	10.88	0.910
60-64	7.00	9.08	0.886
65-69	4.60	6.20	0.835
70-74	6.29	3.84	0.755
75-79	3.73	4.75	0.662
80-84	2.83	2.47	0.555
85-89	1.00	1.57	0.440
90-94	.33	.44	0.273
95-99	.09	.09	0.222
100+	.01	.02	
Total	142.53	142.42	

Foundation Factor for 2015: 0.060
(approximately 6.0% of the population in 2015 is 0 – 4 years old)

Handout: Saudi Arabia



Population in hundred thousands of people



Age Groups

Saudi Arabia 2015
A Lower Middle-Layered Country

Age Group	Males	Females	Total
0-4	1,387,063	1,334,215	2,721,278
5-9	1,480,465	1,422,715	2,903,180
10-14	1,473,422	1,381,861	2,855,283
15-19	1,324,851	1,185,912	2,510,763
20-24	1,404,504	1,229,687	2,634,191
25-29	1,609,826	1,367,009	2,976,835
30-34	1,905,616	1,363,669	3,269,285
35-39	2,058,059	1,182,561	3,240,620
40-44	1,706,486	998,992	2,705,478
45-49	1,165,256	720,035	1,885,291
50-54	798,283	500,989	1,299,272
55-59	548,564	348,649	897,213
60-64	353,605	235,239	588,844
65-69	215,882	166,812	382,694
70-74	143,955	111,276	255,231
75-79	81,334	70,903	152,237
80-84	42,751	39,954	82,705
85-89	20,074	18,320	38,394
90-94	4,947	4,917	9,864
95-99	681	831	1,512
100+	48	67	115
Total	17,725,672	13,684,613	31,410,285

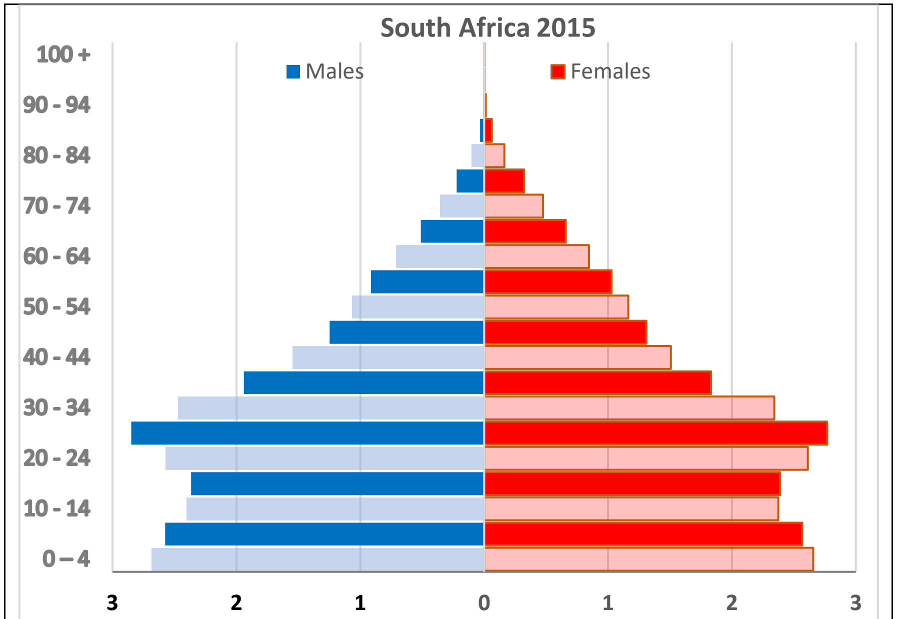
Population Estimates for Saudi Arabia

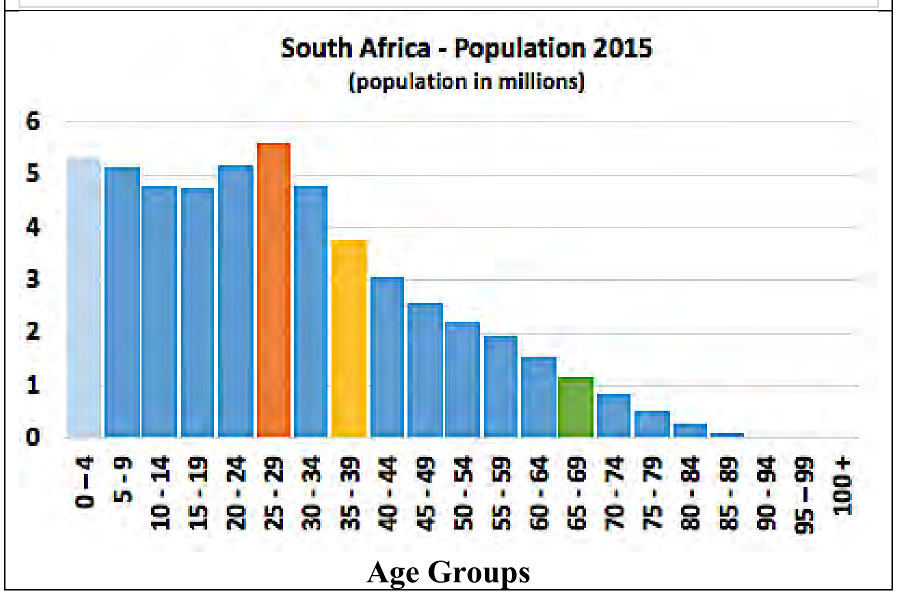
United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	2.67	2.72	1.086
5-9	2.78	2.9	1.029
10-14	2.66	2.86	0.944
15-19	2.61	2.51	1.008
20-24	2.76	2.63	1.080
25-29	3.03	2.98	1.079
30-34	2.96	3.27	1.095
35-39	2.53	3.24	1.071
40-44	1.9	2.71	0.995
45-49	1.39	1.89	0.885
50-54	0.94	1.23	0.957
55-59	0.6	0.9	0.983
60-64	0.42	0.59	0.905
65-69	0.31	0.38	0.839
70-74	0.18	0.26	0.833
75-79	0.12	0.15	0.667
80-84	0.07	0.08	0.571
85-89	0.03	0.04	0.333
90-94	0.01	0.01	0.200
95-99	0.001	0.002	0.115
100+	0.00007	0.00012	
Total	27.96	31.41	

Foundation Factor for 2015: 0.087
(approximately 8.7% of the population in 2015 is 0 – 4 years old)

Handout: South Africa





South Africa 2015
A Bottom Layered Country

Age Group	Males	Females	Total
0-4	2,683,200	2,656,056	5,339,256
5-9	2,575,157	2,567,782	5,142,939
10-14	2,401,816	2,374,175	4,775,991
15-19	2,366,777	2,389,659	4,756,436
20-24	2,570,392	2,612,542	5,182,934
25-29	2,849,468	2,770,118	5,619,586
30-34	2,472,440	2,341,736	4,814,176
35-39	1,938,825	1,832,001	3,770,826
40-44	1,545,027	1,507,072	3,052,099
45-49	1,247,048	1,309,197	2,556,245
50-54	1,067,615	1,163,298	2,230,913
55-59	915,352	1,028,904	1,944,256
60-64	712,831	846,042	1,558,873
65-69	510,105	659,226	1,169,331
70-74	355,726	475,474	831,200
75-79	221,472	323,428	544,900
80-84	103,108	163,098	266,206
85-89	33,830	62,405	96,235
90-94	6,640	13,870	20,510
95-99	707	1,784	2,491
100+	39	121	160
Total	26,577,575	27,097,988	53,675,563

Population Estimates for South Africa

United States Census Bureau
International Data Base (IDB)
(population in millions to the nearest hundredths)

Age groups	Population 2010	Population 2015	Population Factors
0-4	5.19	5.34	0.990
5-9	4.83	5.14	0.990
10-14	4.73	4.78	1.006
15-19	5.21	4.76	0.994
20-24	5.75	5.18	0.977
25-29	5.05	5.62	0.952
30-34	4.04	4.81	0.933
35-39	3.32	3.77	0.919
40-44	2.78	3.05	0.921
45-49	2.43	2.56	0.918
50-54	2.13	2.23	0.911
55-59	1.74	1.94	0.897
60-64	1.36	1.56	0.860
65-69	1.03	1.17	0.806
70-74	0.76	0.83	0.711
75-79	0.46	0.54	0.587
80-84	0.23	0.27	0.435
85-89	0.08	0.1	0.250
90-94	0.02	0.02	0.100
95-99	0.002	0.002	0.100
100+	0.0001	0.0002	
Total	51.12	53.68	

Foundation Factor for 2015: 0.099
(approximately 9.9% of the population in 2015 is 0 – 4 years old)