Heuristic Opt. Techniques - Assignment 1 Report

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Summary

We chose a simple two-step algorithm that constructs an initial solution. First, the ordering is fixed by looking at the order of the vertices. We found out that the algorithm gives the best results when we place the vertices with the highest degree at the top and bottom of the spine. Then, the edges are one by one added to the page with the minimal new crossings. As for the randomization, we found ways to extends these steps to allow for a parameterized uncertainty factor (similar to the α factor in GRASP). Our results showed that randomization improved our results for almost all our test instances.

Implementation

In the following subchapters we will describe details of the implementation.

Solution representation

Internally, we represent a solution as the order of its vertices, an integer array, and the lists of edges assigned to pages.

Additionally, we developed a new data structure that stores the active edges at each vertex. For example, a edge (0,7) would be active at all vertices between the vertex 0 and vertex 7. Even though this increases space consumption, we are able to add a new edge by only checking the active edges at the two end points and can compute the number of newly introduced crossings in logarithmic time (since we ordered the active edges by their remaining “length”).

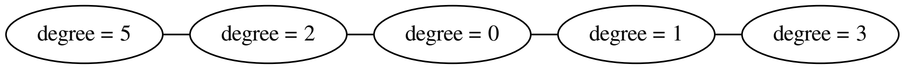
Two step algorithm

We decided that our construction heuristic should first fix the vertex order and afterwards determine a good page partition for this ordering. This allowed us to find clear rules for the construction at each step, as we will explain in the following sections. A downside of this method is that it does not allow to avoid certain crossings encountered during the heuristic by changing the vertex order. This could be alleviated by adding a local search procedure, ideally searching for both changes in vertex ordering and page partition.

Furthermore, fixing the vertices first allows us to create the active edge data structure (see above), and thus lets us very efficiently check whether adding an edge would introduce new crossings. We think that this operation is used very often (and also used in further extensions like local search) and therefore this approach seems to be the most promising.

Deterministic Construction

Our deterministic construction looks at the degree of each vertex to determine its order. Initially, our idea was to have vertices with higher degree nearer the middle of the “spine”.



The idea was that since most edges will be incident to the vertices in the middle, there would be less crossings. However, as we tested the algorithm, we found out that that the reverse order, namely that the vertices with higher degree are on the top and bottom of the spine, results in less crossings, as in the graph above. We are unable to explain this behavior, but in almost all test cases the reverse order leads to a better result.

For the edge partitioning, we implemented a first fit algorithm that chooses the first page with which the new edge introduces no new crossings. If no such page exists, the one with the least introduced crossings is chosen.

Randomized construction

For the randomization, we tried to find meaningful ways to introduced a controlled randomization factor into each step, that does not automatically regress to a random search. For this purpose, we used a parameter α. $\alpha = 0.0$ means the algorithm proceeds as the deterministic case, with $\alpha = 1.0$ essentially reducing each to step to be almost completely random. Every value in between gradually changes the behavior.

For the vertex ordering, a random offset for the degree count is added. The vertices are then sorted by degree and offset together.

As for the edges, we first collect all edges to be added and then shuffle the order in which the first fit procedure from above is considered. We experimented with also adding edges randomly (depending on α), but the results proved unsatisfactory.

Evaluation

Our tests were run only on our private machines. We acquired the information for the cluster fairly late and by that time had already started testing. The testing was done on a laptop with an Intel i7-4500U @ 1.8 GHz and 8 GB of RAM running Ubuntu 16.04 x64 and a desktop PC with an Intel Core 2 Quad Q6600 and 6GB of RAM on Windows 10 x64. We used the Java framework from TUWEL as a starting point and implemented it in Java 8.

Our best results can be seen on the tool provided to submit solutions. We found that the best results were almost always achieved by the randomized variant (except for instance 10, where our best result stems from the deterministic version). When sampling for 10 values, however, it can be seen that the randomized construction usually is worse on average. The minimum is almost always lower, however.

Instance name	Det. Construction	Rand. Mean	Rand. Std. Deviation	Rand. Minimal
Instance 1	18	18.5	3.75	9
Instance 2	59	68.6	10.67	6
Instance 3	114	132.9	26.49	57
Instance 4	215	160.02	26.31	49
Instance 5	91	89.2	23.4	30
Instance 6	6732459	4498075	211133	4064964
Instance 7	156906	157854	5740.4	131553
Instance 8	915429	633363	19867	582977
Instance 9	1529691	789895	71314	573669
Instance 10	107333	121714	3317.5	116638

Figure 1: Test results of crossing counts. 10 samples were used for mean and deviation

We assume the most obvious way to improve our results would be the implementation of a local search. Since we implemented an incremental way to detect crossings (the most computationally expensive part of our heuristic), we could quickly introduce changes to the page partition (though not the vertex order) and check for improvements.