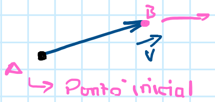
Vectores

lunes, 19 de agosto de 2024 11:20

① Qué es? Es un objeto matemático que tiene dirección y magnitud. Se representa por un segmento de recta orientado.

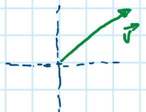


B → Punto final $B=(c,d)$

A → Punto inicial $A=(a,b)$

$\vec{v} = \langle c-a,\ d-b \rangle$ — Final, Inicial

② Módulo de un vector



$|\vec{v}| = \sqrt{a^2+b^2}$

③ Vector unitario → Su módulo es 1 $|\vec{v}| = 1$

$\hat{i} = \langle 1;0 \rangle \qquad \hat{j} = \langle 0;1 \rangle$

Para hallar el vector unitario de un vector:

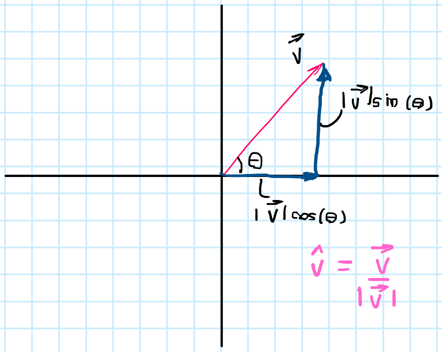
Vector unitario de un vector $= \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\langle a;b \rangle}{k} = \left\langle \dfrac{a}{k};\dfrac{b}{k} \right\rangle$

Ej:

$\vec{v} = \langle 3,4 \rangle$

$|\vec{v}| = 5$

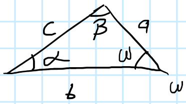
$\dfrac{\langle 3;4 \rangle}{5} = \left\langle \dfrac{3}{5};\dfrac{4}{5} \right\rangle$



$\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$

Ley de senos:

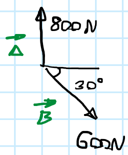
$\dfrac{a}{sen\,\alpha} = \dfrac{b}{sen\,\beta} = \dfrac{c}{sen}$



Método del paralelogramo

$|A+B|^2 = |A|^2 + |B|^2 + 2|A||B|\cos\theta$

①



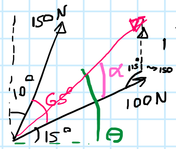
$800^2 + 600^2 + 2(600)(800)\cos(120°)$

$|A+B|^2 = 520000$

$|A+B| = 721{,}11$

$\vec{A} = \langle 0,800 \rangle \qquad \vec{B} = \langle 600\cos(330°),\ 600\sin(330°) \rangle$

②



$|\vec{R}|^2 = 150^2 + 100^2 + 2(150)(100)\cos(65°)$

$|R| = 212{,}55$

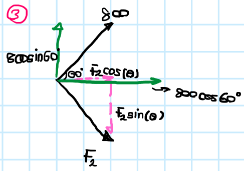
$\dfrac{150}{\sin\alpha} = \dfrac{212{,}55}{\sin 115°} \to \sin\alpha = \dfrac{150}{212{,}55}\cdot\sin(115°)$

$\sin\alpha = 0{,}6395$

$\alpha = 39{,}7617$

$\theta = \alpha + 15° = 54{,}76°$

③



$800\sin 60° - |F_2|\sin\theta = 0$

$800\sin 60° = |F_2|\sin\theta$

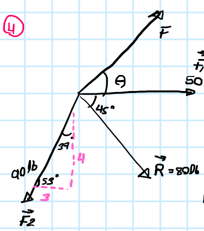
$\dfrac{800\sin 60°}{\sin\theta} = |F_2|$

$\sin\theta = 1$

$\theta = 90°$

$800\sin 60° = |F_2|$

④



$\vec{F} = \langle |\vec{F}|\cos\theta,\ |\vec{F}|\sin\theta \rangle$

$\vec{F_1} = \langle 50, 0 \rangle$

$\vec{F_2} = \langle -54, -72 \rangle$

$\vec{R} = \langle 80\cos(315°),\ 80\sin(315°) \rangle$

$|\vec{F}|\cos\theta + 50 - 54 = 80\cos(315°)$

$|\vec{F}|\cos\theta = 80\cos(315°) + 4$

$|\vec{F}|\sin\theta + 0 - 72 = 80\sin(315°)$

$|\vec{F}|\sin\theta = 80\sin(315°) + 72$

$|\vec{F}| = 62{,}5\ N$

$\dfrac{|\vec{F}|\sin\theta}{|\vec{F}|\cos\theta} = \dfrac{80\sin(315°)+72}{80\cos(315°)+4} \to \tan\theta = 0{,}2547$

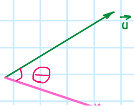
$\theta = 14{,}2935°$

① Producto Escalar

$\vec{u} = \langle u_1, u_2, u_3 \rangle \qquad \vec{u}\cdot\vec{v} = u_1 v_1 + u_2 v_2$

$\vec{v} = \langle v_1, v_2, v_3 \rangle \qquad \vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cos\theta$

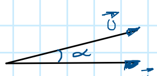
Ej.



Sirve para hallar el ángulo entre dos vectores.

Actividad Previa 1:

① Producto escalar — Siempre da un escalar



$\vec{u}\cdot\vec{v} = u_1\cdot v_1 + u_2 v_2$

$\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cos\alpha$

$\vec{u}\cdot\vec{v} > 0$ Ángulo agudo

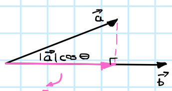
$\vec{u}\cdot\vec{v} < 0$ Ángulo obtuso — α

$\vec{a} = \langle a_1, a_2, a_3 \rangle$

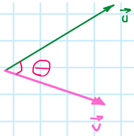
$\vec{b} = \langle b_1, b_2, b_3 \rangle$

$\vec{a}\cdot\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$= \|\vec{a}\|\|\vec{b}\|\cos\theta$



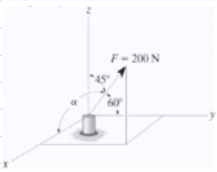
Ej.



Sirve para hallar el ángulo entre dos vectores.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}$$

Ej.



$\|F\| = 200$

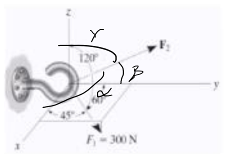
$\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

$\cos^2 60° + \cos^2 45° + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - 0{,}25 - 0{,}5 \qquad \cos\alpha = 0{,}25$

Ej.

Dos vectores actúan sobre el gancho que se muestra en la figura. Especifique la magnitud de $\mathbf{F_2}$ y sus ángulos directores coordenados, de modo que el vector resultante $\mathbf{F_R}$ actúe a lo largo del eje y positivo y tenga una magnitud de 800 N.



$F_1 = \langle 300\cos 45°,\ 150,\ -150\rangle$

$F_2 = \langle -300\cos 45°,\ 650,\ 150\rangle \qquad |F_2| = 700\ N$

$\cos\alpha = \frac{-300\cos 45°}{700} \quad \cos\beta = \frac{650}{700} \quad \cos\gamma = \frac{150}{700}$

$\alpha = 107{,}64° \qquad \beta = 21{,}79° \qquad \gamma = 77.63°$

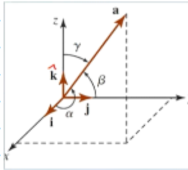
$\vec{u}\cdot\vec{v} > 0$ Ángulo agudo

$\vec{u}\cdot\vec{v} < 0$ Ángulo obtuso — α

$\vec{u}\cdot\vec{v} = 0$ Ángulo recto

② Cosenos directores

$a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$



$\vec{a}\cdot\hat{k} = |\vec{a}|\cdot|\hat{k}|\cdot\cos\gamma$

$\vec{a}\cdot\hat{k} = |\vec{a}|\cos\gamma$

$\vec{a} = |\vec{a}|\cos\gamma$

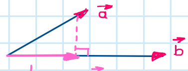
- $\cos\gamma = \frac{a_3}{|\vec{a}|}$
- $\cos\beta = \frac{a_2}{|\vec{a}|}$
- $\cos\alpha = \frac{a_1}{|\vec{a}|}$

$$\frac{\vec{a}}{\|a\|} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$

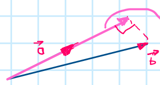
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

③ Proyección y componentes

PROYECCIÓN (Vector)



$\text{proy}_{\vec{b}}\vec{a}$ Proyección de $\vec{a}$ en $\vec{b}$



$\text{proy}_{\vec{a}}\vec{b}$ Proyección de $\vec{b}$ en $\vec{a}$

COMPONENTES (Magnitud)

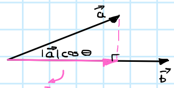
$\text{Comp}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|b\|} \ \wedge \ \text{Proy}_{\vec{b}}\vec{a} = \text{comp}_{\vec{b}}\vec{a}\cdot\frac{\vec{b}}{\|\vec{b}\|}$

$a = 4i + j \qquad b = 2i + 3j$

$\text{Comp}_{\vec{b}}\vec{a} \ \frac{11}{\sqrt{13}} \qquad \text{proy}_{\vec{b}}\vec{a} = \frac{11}{\sqrt{13}} \cdot \frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$

$\frac{11}{\sqrt{13}}\left(\frac{2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{j}\right)$

$\frac{22}{13}\hat{i} + \frac{33}{13}\hat{j}$

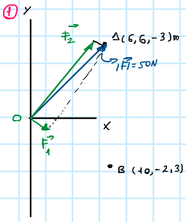


Componente vectorial $\text{proy}_{\vec{b}}\vec{a}$ (Vector) $\longrightarrow \text{proy}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}\cdot\frac{\vec{b}}{\|\vec{b}\|} = \text{Comp}_{\vec{b}}\vec{a}\cdot\frac{\vec{b}}{\|\vec{b}\|}$

Proyección escalar $\text{Comp}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|}$ (Magnitud)

$\hat{b}$: vector unitario de b.

Ej ①



$\vec{F_1} = \text{proy}_{\vec{OB}}\vec{F} = \frac{\vec{OB}\cdot\vec{F}}{\|\vec{OB}\|}\cdot\frac{\vec{OB}}{\|\vec{OB}\|}$

$\vec{F} = |\vec{F}|\cdot\hat{OA} = \qquad \vec{OB} = 10i - 2j + 3k$

$\vec{OA} = 6\hat{i} + 6\hat{j} - 3\hat{k} \qquad \|\vec{OB}\| = \sqrt{113}$

$\hat{OA} = \frac{6}{9}\hat{i} + \frac{6}{9}\hat{j} - \frac{3}{9}\hat{k}$

$\vec{F} = 50\cdot\frac{6}{9}i + 50\cdot\frac{6}{9}j - 50\cdot\frac{3}{9}k$

$= \frac{100}{3}i + \frac{100}{3}j - \frac{50}{3}k$

$\vec{F_1}\ \text{Proy}_{\vec{OB}}\vec{F} = \frac{\left(\frac{100}{3}i + \frac{100}{3}j - \frac{50}{3}k\right)(10i - 2j + 3k)}{\sqrt{113}^2}\cdot(10i - 2j + 3k)$

$\frac{\frac{1000}{3} - \frac{200}{3} - \frac{150}{3}}{113}\cdot(10i - 2j + 3k) = \frac{650}{339}(10i - 2j + 3k)$

$+\hat{i} \quad -\hat{j} \quad +\hat{k}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & b_2 \\ a_2 & b_2 \end{vmatrix}$$

$$\vec{a}\times\vec{b} = (a_2 b_3 - a_3 b_2)\hat{i} - (a_1 b_3 - a_3\cdot b_1)\hat{j} + (a_1 b_2 - b_2 a_2)\hat{k}$$

$$\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin\theta$$