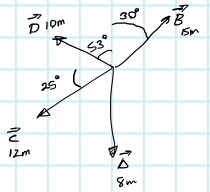
Semana 1

viernes, 23 de agosto de 2024 12:29

Ejercicio: $2\vec{A} + 3\vec{B} - 4\vec{C} =$

2 y 3r cuadrante + 180°



$\vec{A} = -8\hat{j}$ $\qquad -16\hat{j}$

$\vec{B} = 15\cos 60°\,\hat{i} + 15\sin 60°\,\hat{j}$ $\qquad 45\cos 60°\,\hat{i} + 45\sin 60°\,\hat{j}$

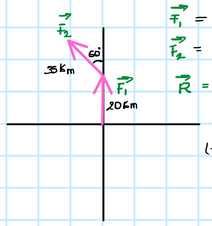
$\vec{C} = 12\cos 205°\,\hat{i} + 12\sin 205°\,\hat{j}$ $\qquad -48\cos 205°\,\hat{i} - 48\sin 205°\,\hat{j}$

$\vec{R} = 66\,m\,\hat{i} + 43{,}26\,m\,\hat{j}$

$\|\vec{R}\| = 78{,}914\,m$

$\tan\theta = \frac{43{,}26}{66} =$ $\quad \theta = 33{,}243°$

Ejercicio:



$\vec{F_1} = 20\hat{j}$

$\vec{F_2} = 35\cos 150°\,\hat{i} + 35\sin 150°$

$\vec{R} = -30{,}31\,Km\,\hat{i} + 37{,}5\,Km\,\hat{j}$

$\|\vec{R}\| = 48{,}2176\,Km$

$(\tan^{-1}\theta) = \frac{37{,}5}{-30{,}31}$ $\quad \theta = 180° + (-51{,}0525)$

$\theta = 128{,}947°$

Ejercicio: $r_1 = (15\hat{i} + 30\hat{j} + 12\hat{k})$ cm

$r_2 = (23\hat{i} - 14\hat{j} - 5\hat{k})$ cm

$r_3 = (-13\hat{i} + 15\hat{j})$ cm

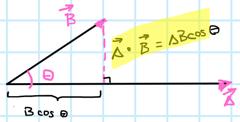
$R = (25\hat{i} + 31\hat{j} + 7\hat{j})$ cm

$\|R\| = 40{,}44$ cm

Producto Punto (Escalar)

$\vec{A}\cdot\vec{B} = (A_x, A_y, A_z)\cdot(B_x, B_y, B_z)$ $\qquad \therefore\ AB\cos\theta = (A_xB_x, A_yB_y, A_zB_z)$

$= (A_xB_x, A_yB_y, A_zB_z)$ $\qquad \cos\theta = \frac{(A_xB_x, A_yB_y, A_zB_z)}{AB}$

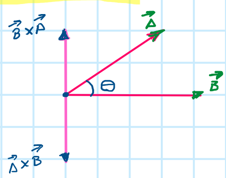


$\vec{A}\cdot\vec{B} = AB\cos\theta$

Producto Cruz (Vectorial)

$$\vec{A}\times\vec{B} = \begin{array}{ccc} (+) & (-) & (+) \\ \hat{i} & \hat{j} & \hat{k} \end{array} \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{array}{l} \hat{i}\,(A_yB_z - A_zB_y) \\ -\hat{j}\,(A_xB_z - A_zB_x) \\ \hat{k}\,(A_xB_y - A_yB_x) \end{array}$$

$|\vec{A}\times\vec{B}| = AB\sin\theta$



"Se va tachando cada columna y la primera fila."

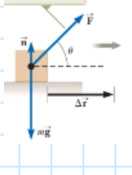
Ejercicio $\vec{A} = 2\hat{i} + 3\hat{j}$ $\qquad \vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

$\vec{B} = -\hat{i} + 2\hat{j}$

① Trabajo (escalar)

Cantidad de energía que se transfiere y cambia la posición.

1. Constante $\qquad$ Unidad: (J) Joules

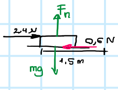


$W = F\cdot\Delta r\cdot\cos\theta$

$= \vec{F}\cdot\vec{\Delta r}$

$\cos\theta$:
- $+ \rightarrow 0° \le \theta < 90°$ A favor
- $- \rightarrow 90° < \theta \le 180°$ En contra
- $0 \rightarrow 90°$ Perpendicular

Ejercicio 1:



a) $W = 2{,}4\cdot 1{,}5\cdot\cos 0° = 3{,}6\,J$

b) $W = 0{,}6\cdot 1{,}5\cos 180° = -0{,}9\,J$

c) $W_{F_n} = 0\,J$

④ Teorema de Trabajo neto y Energía cinética

$W_{TOT} = K_f - K_i = \Delta K$ $\qquad K = \frac{1}{2}mv^2$ Energía cinética

Ej. 1,5 kg $\quad A \rightarrow 3{,}21$ m/s $\quad B \rightarrow 1{,}25$ m/s

a) $W_{TOTAL} = \Delta K = \frac{1}{2}1{,}5(1{,}25^2 - 3{,}21^2) = -6{,}5562\,J$

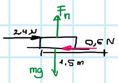
b) $-0{,}750 = \frac{1}{2}1{,}5(V_f^2 - 1{,}25^2) \rightarrow V_f = 0{,}75$ m/s

c) $+0{,}750 = \frac{1}{2}\cdot 1{,}5(V_c^2 - 1{,}25^2) \rightarrow V_f = 1{,}6$ m/s

Ej. 8 kg $\quad A \rightarrow 4$ m/s $\quad A + 2{,}5 \rightarrow 6$ m/s

$W_{neto} = \frac{1}{2}8\cdot(6^2 - 4^2) = 80\,J$

$80 = F\cdot 2{,}5 \quad F = 32\,N$



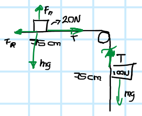
a) $W = 2{,}4 \cdot 1{,}5 \cdot \cos 0° = 3{,}6\ J$

b) $W = 0{,}6 \cdot 1{,}5 \cos 180° = -0{,}9\ J$

c) $W_{F_n} = 0\ J$

d) $W_{mg} = 0\ J$

Ejercicio 2:



100N:

A) $W_{mg} = 100 \cdot 0{,}75 \cdot \cos 0° = 75\ J$

B) $W_T = 100 \cdot 0{,}75 \cos 180° = -75\ J$

200N:

A) $W_{mg} = F_{mg} \cdot 0{,}75 \cdot \cos 90° = 0$

B) $W_T = 100 \cdot 0{,}75 \cos 0° = 75\ J$

C) $W_{F_r} = 100 \cdot 0{,}75 \cos 180° = -75\ J$

D) $W_{F_n} = F_{F_n} \cdot 0{,}75 \cdot \cos 90° = 0$

$W_{TOTAL\ (100N)} = 0\ J$

$W_{TOTAL\ (20N)} = 0\ J$

② Potencia

2.1. Potencia promedio $P_{med} = \frac{\Delta W}{\Delta t}$

t: segundos

2.2. Potencia instantánea $P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$

Ejercicio. $P_s = \frac{\Delta W}{\Delta t}$ $P = P_s(250000 \times 365 \times 24 \times 3600)$

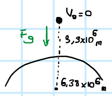
$P_{med} = \frac{P}{0{,}2} = 3{,}942 \times 10^{13} P_s$

③ Trabajo fuerza variable

$$W = \int_{x_1}^{x_2} F_x\,dx = \text{Área}$$

$$W_F = \int_{x_i}^{x_f} (F_x \hat{i} + F_y \hat{j}) \cdot (dx\,\hat{i})$$

Ej.



$g = \frac{-GM_T}{y^2}\hat{j}$ $G = 6{,}67$

$F_g = mg = \frac{2800 \cdot (-6{,}67)\, 5{,}97 \times 10^{24}}{y^2}$

$= \frac{-1{,}11 \times 10^{40}}{y^2}\hat{j}$

$W_{F_g} = F_g \cdot 3{,}3 \times 10^6$

$$W_{F_g} = \int_{y_i}^{y_f} \frac{-1{,}11 \times 10^{40}}{y^2}\hat{j}\,(dy\,\hat{j}) = -1{,}11 \times 10^{40} \int_{y_i}^{y_f} \frac{dy}{y^2}$$

$$-1{,}11 \times 10^{40} \left(-\frac{1}{y} \Big|_{9{,}68 \times 10^6}^{6{,}38 \times 10^6} \right) = 5{,}03 \times 10^{32}\ J$$

Ej. 8 kg A → 4 m/s A + 2,5 → 6 m/s

$W_{neto} = \frac{1}{2} 8 \cdot (6^2 - 4^2) = 80\ J$

$80 = F \cdot 2{,}5$ $F = 32\ N$

Ej. $F_s = -\alpha x^2$ $\alpha = 12\ N/m^2$

a) $\vec{r}_i = (0{,}1; 0)\,m$ $\Delta\vec{r} = (0, 0{,}4)\,m$

$\vec{r}_f = (0{,}1; 0{,}4)\,m$

$W_1 = \int_{0,1}^{0,1} -\alpha x^2\,dx = 0\ J$

b) $\vec{r}_i = (0{,}1; 0)\,m$ $\Delta\vec{r} = (0{,}2; 0)\,m$

$\vec{r}_f = (0{,}3; 0)\,m$

$$W_2 = \int_{0,1}^{0,3} -12x^2\,dx = -12\frac{x^3}{3}\Big|_{0,1}^{0,3} = -4(0{,}3^3 - 0{,}1^3) = -0{,}104\ J$$

c) $$W_3 = \int_{0,3}^{0,1} -12x^2\,dx = -4(0{,}1^3 - 0{,}3^3) = 0{,}104\ J$$

d) $W_T = W_2 + W_3$ $W_T = -0{,}104 + 0{,}104 = 0\ J$

①

$F_x = -Kx$

$a_x = \frac{d^2x}{dt^2} = -\frac{K}{m}x = -\omega^2 x$

Desplazamiento en el MAS

$x = A\cos(\omega t + \phi)$

Amplitud

$\sqrt{\frac{K}{m}}$ Frec. angular

Ángulo de fase

Frecuencias $f = \frac{1}{T}$ $T = \frac{1}{f}$

$\omega = 2\pi f = \frac{2\pi}{T}$

Ej ① $f = 6{,}7$ $f = \frac{1}{T}$ $T = \frac{1}{6{,}7 \times 10^6} = 0{,}15 \times 10^{-6}\ s$

$\omega = 2\pi f = 2\pi(6{,}7 \times 10^6\ Hz) = 42\,097\,341\ rad/s$

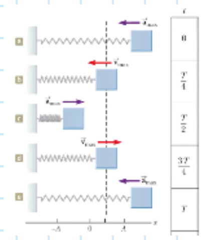
Velocidad MAS

$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$ $v_{max} = \pm\omega A$

Aceleración MAS

$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$ $a_{max} = \pm\omega^2 A$

Ej ②



Ej ③

Energía del oscilador armónico simple $E = \frac{1}{2}KA^2$

Ej ④ $K = 20$ $m = 0{,}5\,kg$

a) $v_{max} = \omega A = \sqrt{\frac{K}{m}}(0{,}03\,m) = \sqrt{\frac{20}{0{,}5}}(0{,}03) = 0{,}19\ m/s$

b) $v = \pm\sqrt{\frac{K}{m}(A^2 - x^2)}$