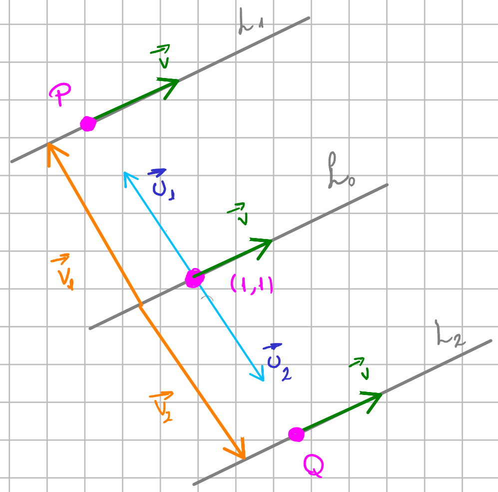
Tarea Semanal ①

Pregunta ①

Dato: $\bullet\ \mathcal{L} = \{(1,1) + t(1,5),\ t \in \mathbb{R}\}$

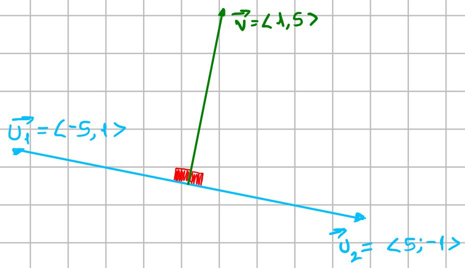


Sol:

$\vec{v} = \langle 1,5 \rangle$

$\vec{U_1} = \langle -5,1 \rangle$

$\vec{U_2} = \langle 5,-1 \rangle$



Unitarios:

$\vec{\mu_{u_1}} = \langle \frac{-5}{\sqrt{26}}, \frac{1}{\sqrt{26}} \rangle$ $\longrightarrow$ Los multiplicamos $\times 5$ $\quad \vec{V_1} = \langle \frac{-25}{\sqrt{26}}; \frac{5}{\sqrt{26}} \rangle \quad \|\vec{V_1}\| = 5$

$\vec{\mu_{u_2}} = \langle \frac{5}{\sqrt{26}}, \frac{-1}{\sqrt{26}} \rangle$ $\longrightarrow$ Los multiplicamos $\times 5$ $\quad \vec{V_2} = \langle \frac{25}{\sqrt{26}}; \frac{-5}{\sqrt{26}} \rangle \quad \|\vec{V_2}\| = 5$

Ahora sumamos al punto de partida los vectores $\vec{V_1}$ y $\vec{V_2}$.

$$P = \left((1,1) + \left(\frac{-25}{\sqrt{26}}; \frac{5}{\sqrt{26}}\right)\right) = (-3{,}903;\ 1{,}981)$$

$$Q = \left((1,1) + \left(\frac{25}{\sqrt{26}}; \frac{-5}{\sqrt{26}}\right)\right) = (5{,}903;\ 0{,}0194)$$

$\therefore$ $\mathcal{L}_1 = (-3{,}903;\ 1{,}981) + t\langle 1,5 \rangle$

$\mathcal{L}_2 = (5{,}903;\ 0{,}0194) + t\langle 1,5 \rangle$

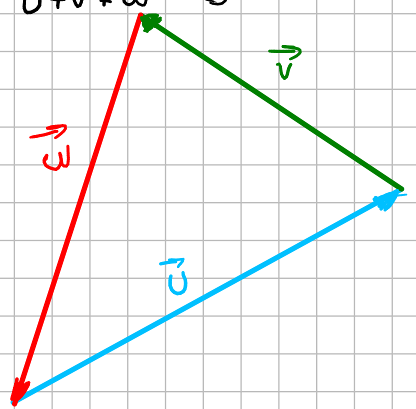
Pregunta ②

Datos: $\|\vec U\| = 2,\ \|\vec V\| = 5,\ \|\vec w\| = 6$

Método del paralelogramo:

$$(\vec A + \vec B)^2 = |\vec A|^2 + |\vec B|^2 + 2|\vec A||\vec B| \cos\theta$$

$\vec U + \vec v + \vec w = 0$



Sol: $\vec v + \vec w = -\vec U$

$\|\vec v + \vec w\| = \|-\vec U\|$

$\|\vec v + \vec w\|^2 = \|\vec U\|^2$

$\therefore\ 4 = \|\vec v\|^2 + \|\vec w\|^2 + 2\underbrace{\|\vec v\|\|\vec w\| \cos\theta}$

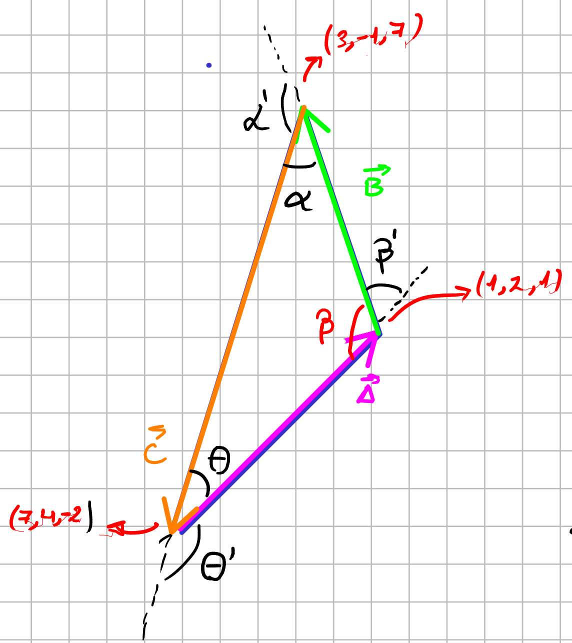
$4 = 25 + 36 + 2(\vec v \cdot \vec w)$

$4 = 61 + 2(\vec v \cdot \vec w)$

$-\frac{57}{2} = \vec v \cdot \vec w$

PREGUNTA 3

DATOS: $A=(1,2,1)$ $B=(3,-1,7)$ $C=(7,4,-2)$



Sol:

$\vec{A} = \langle (1,2,1)-(7,4,-2)\rangle = \langle -6,-2,3\rangle$ $\quad |\vec{A}| = 7$

$\vec{B} = \langle (3,-1,7)-(1,2,1)\rangle = \langle 2,-3,6\rangle$ $\quad |\vec{B}| = 7$ $\Big\}$ Isósceles

$\vec{C} = \langle (7,4,-2)-(3,-1,7)\rangle = \langle 4,5,-9\rangle$ $\quad |\vec{C}| = \sqrt{122}$

- $\vec{A}\cdot\vec{B} = -12+6+18 = 12 = |\vec{A}||\vec{B}|\cos\beta'$ $\quad \cos\beta' = \frac{12}{7\cdot 7}$ $\quad \beta' = 75{,}8241^\circ$ $\quad \beta = 180-\beta'$

  $\beta = 104{,}176^\circ$

- $\vec{A}\cdot\vec{C} = -24-10-27 = -61 = |\vec{A}||\vec{C}|\cos\theta'$ $\quad \cos\theta' = \frac{-61}{7\sqrt{122}}$ $\quad \theta' = 142{,}09$ $\quad \theta = 180-\theta'$

  $\theta = 37{,}912^\circ$

- $\vec{B}\cdot\vec{C} = 8-15-54 = -61 = |\vec{B}||\vec{C}|\cos\alpha'$ $\quad \cos\alpha' = \frac{-61}{7\sqrt{122}}$ $\quad \alpha' = 142{,}09$ $\quad \alpha = 180-\alpha'$

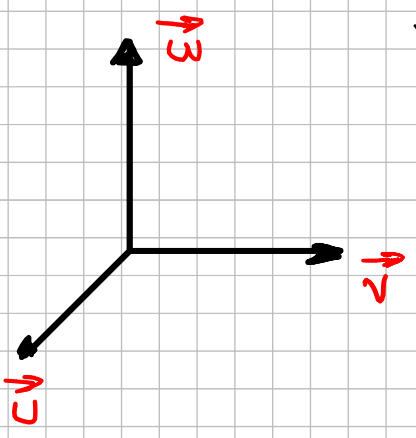
  $\alpha = 37{,}912^\circ$

PREGUNTA 4

$\vec{v} = \langle x,y,z\rangle$

DATOS:

- $\vec{u} = \langle 1,2,3\rangle$
- $|\vec{v}|^2 = \frac{3}{2}$
- $\vec{w} = \vec{u}\times\vec{v} = \langle -4,-1,2\rangle$



Sol:

$$\vec{b} = \frac{\vec{c}\times\vec{a}}{\|\vec{a}\|^2}$$

$$\vec{w}\times\vec{u} = \vec{v}\cdot S$$

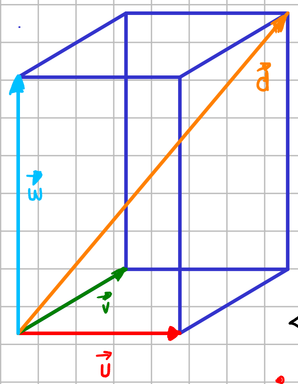
$$\vec{w}\times\vec{u} = \begin{array}{ccc} i & -j & k \end{array}$$
$$\begin{vmatrix} -4 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= i(-3-4) - j(-12-2) + k(-8+1)$$

$$-7i + 14j - 7k$$

$$\vec{v} = \frac{\langle -7, 14, -7\rangle}{\|\sqrt{14}\|^2} = \left\langle -\frac{1}{2}, 1, -\frac{1}{2}\right\rangle$$

PREGUNTA 5



Sol:

$r\langle 1,0,0\rangle = \vec{v} = \langle r,0,0\rangle \rightarrow \langle 4,0,0\rangle$

$s\langle 2,3,0\rangle = \vec{u} = \langle 2s,3s,0\rangle \rightarrow \langle 4,6,0\rangle$

$t\langle -4,-5,-6\rangle = \vec{w} = \langle -4t,-5t,-6t\rangle \rightarrow \langle -8,-10,-12\rangle$

- $\vec{v}+\vec{u}+\vec{w} = \vec{d} = \langle 0,-4,-12\rangle$

$\langle r+2s-4t,\ 3s-5t,\ -6t\rangle = \langle 0,-4,-12\rangle$

- $r+2s-4t=0 \quad r+2(2)-4(2)=0 \quad r=4$
- $3s-5t=-4 \quad 3s-5(2)=-4 \quad 3s=6 \quad s=2$
- $-6t=-12 \quad t=2$

VOLUMEN DE PARALELEPIPEDO

$|(A\times B)\cdot C| = \text{Volumen} = |(\vec{u}\times\vec{v})\cdot\vec{w}|$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & -j & k \\ 4 & 6 & 0 \\ 4 & 0 & 0 \end{vmatrix} = i(0)-j(0)+k(-24) = -24k = \langle 0,0,-24\rangle$$

$\vec{w}\cdot(\vec{u}\times\vec{v}) = -8(0)+(-10)0-24(-12)$

$= 0+0+288 \quad \therefore \quad \text{Volumen} = 288\ u^3$

PREGUNTA 6

DATOS: $x+y+2z=10 \quad \ldots(1)$

$x-2y+4z=5 \quad \ldots(2)$

$-2x+y+z=2 \quad \ldots(3)$

Sol:

- $(1)+(2)+(3) = 0x+0y+7z=17 \quad z=17/7$
- $(1)-(2) = 0x+3y-2z=5$

$3y-2\left(\frac{17}{7}\right)=5 \quad 3y=5+\frac{34}{7}$

$3y=\frac{35+34}{7}=\frac{69}{7} \quad y=23/7$

- $x+23/7+2\left(\frac{17}{7}\right)=10 \quad x+\frac{57}{7}=10 \quad x=\frac{13}{7}$

PUNTO DE INTERSECCIÓN:

$\left(\frac{13}{7}, \frac{23}{7}, \frac{17}{7}\right)$

1) Plano 1 · Plano 2 = $1\cdot1+1(-2)+2(4)=7$

Plano 1 · Plano 3 = $1(-2)+1(1)+2(1)=1$

Plano 2 · Plano 3 = $1(-2)+(-2)1+4(1)=0$

Esto significa que el plano 2 y el plano 3 son perpendiculares.