

离散数学概述

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分班教学
授课内容可能有出入, 不会影响考试

平时作业 *vs.* 期中测试 *vs.* 期末测试

3 : 3 : 4

4 : 3 : 3

弹性制



`n5.disalg.cn`

每周四晚上发布作业 下周四 23:59 前提交作业

每次作业按 10 分计算

迟交: 周三前向助教登记, 可延长两天, 总分 8 分

非必要, 不点名

非必要, 不迟到

非必要, 不迟到

尽量吃早餐, 但不可以在教室吃早餐

~~非/必/要/~~，不抄袭；一经发现，后果严重

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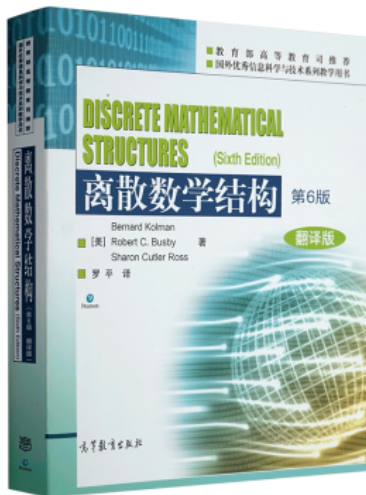
当次作业计零分；总评扣十分

QQ 群号: 612 868 020

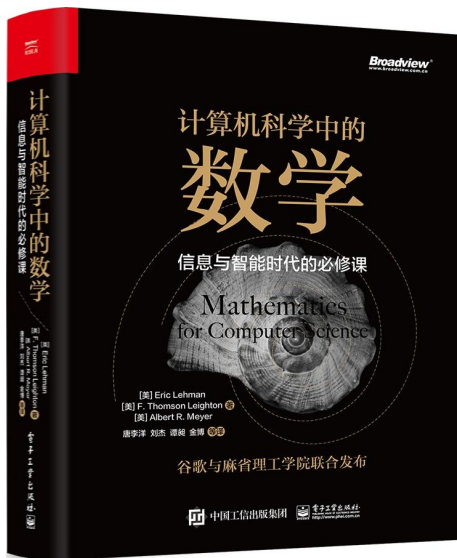


作业助教: 裴一凡、戴若石、肖江

技术支持: 唐瑞泽



教材不重要, 听讲更重要



其它参考书随课程进度安排



离散数学

离散数学

研究离散对象及其关系的数学分支 (大杂烩)

四大主题: 逻辑、集合论、图论、抽象代数 (群论)

支流遍布: 组合与计数、数论、(离散) 概率

关于离散数学, 学长纷纷表示:

我太难了

啥用没有

真得有那么难吗?

真得有那么难吗?

确实蛮难的: 知识点多而分散

真得没啥用吗?

真得没啥用吗?

太基础, 用了但不自觉 (逻辑)

浅尝辄止, 想用但用不上 (群论)

将离散数学看作一门语言，一套工具

培养形式化描述问题的能力

培养做严格证明的能力

Axiomatic Set Theory (ZFC)



Ernst Zermelo (1871–1953)



Abraham Fraenkel (1891–1965)



- (1) To draw a straight line from any point to any point.
- (2) To extend a finite straight line continuously in a straight line.
- (3) To describe a circle with any center and radius.
- (4) That all right angles are equal to one another.
- (5) The parallel postulate.

Theorem (Dov Jarden (1953))

$$\exists a, b \in \mathbb{R} \setminus \mathbb{Q} : a^b \in \mathbb{Q}.$$

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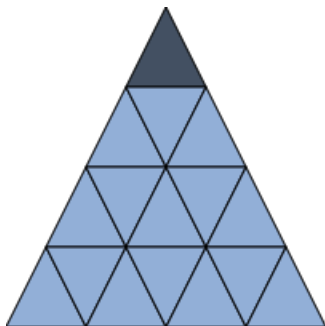
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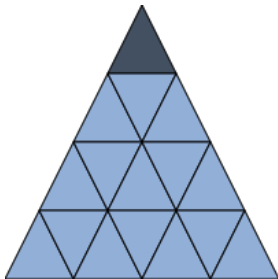
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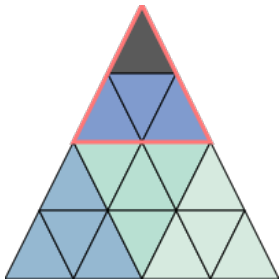
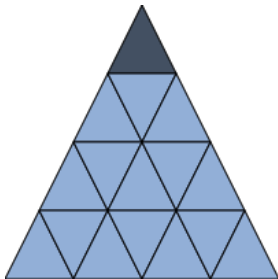
Q : 这是构造性证明吗? 这是反证法吗?

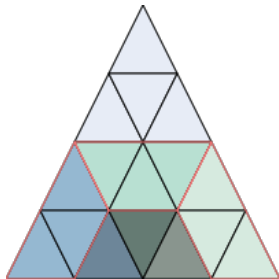
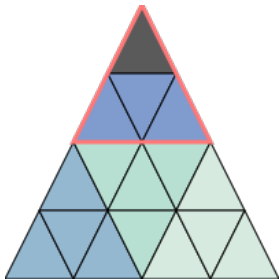
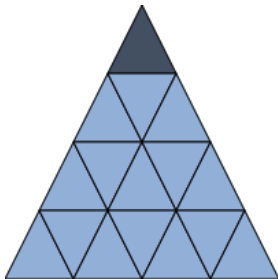
覆盖问题

假设 n 是正整数。一个等边三角形被切成 4^n 个全等小等边三角形, 其中一个角上的小等边三角形被移除。请证明, 剩余区域可被如下图形覆盖:









Base Case:

Induction Hypothesis:

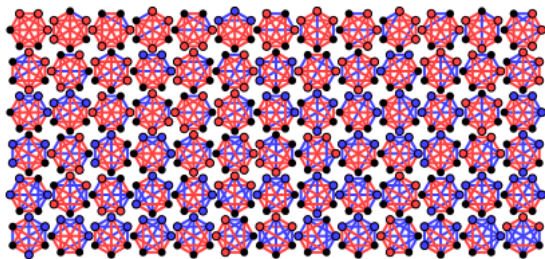
Induction Step: ... by induction hypothesis ...

Theorem on Friends and Strangers

At any party with at least 6 people, there are 3 people who are all either mutual acquaintances or mutual strangers.

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In Terms of **Graph Theory**.

(Is there a **monochromatic** triangle in any 2-coloring of K_6 ?)

Theorem on Friends and Strangers

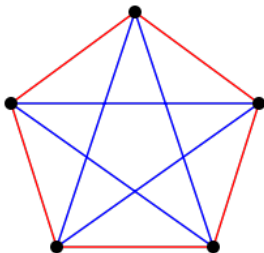
Is there a **monochromatic** triangle in any 2-coloring of K_6 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?

Theorem on Friends and Strangers

Is there a **monochromatic** triangle in any 2-coloring of K_5 ?



Ramsey theory

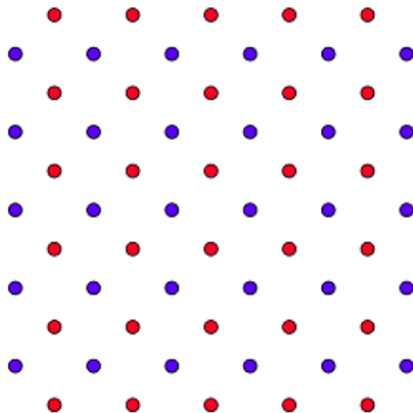
How **big** must the **structure** be
to ensure that it has a given interesting **property**?

Ramsey theory

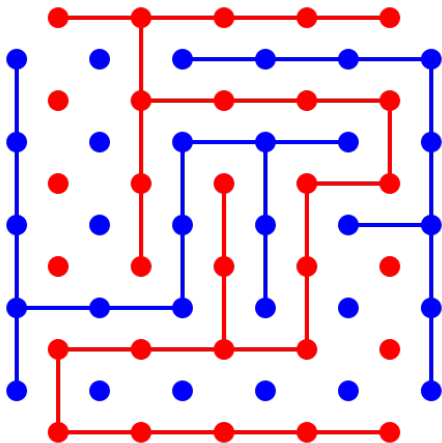
How **big** must the **structure** be
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Bridg-It Game (David Gale, 1958)



5×6 vs. 6×5



5×6 vs. 6×5

Let's Play with it!

Let's Analyze it!

Will Bridg-It **end in a tie**?

Will Bridg-It **end in a tie**?

No! By **contradiction**.

Does **Player 2** have a **winning strategy**?

Does **Player 2** have a **winning strategy**?

No! By the **strategy stealing argument**.

Does **Player 1** have a **winning strategy**?

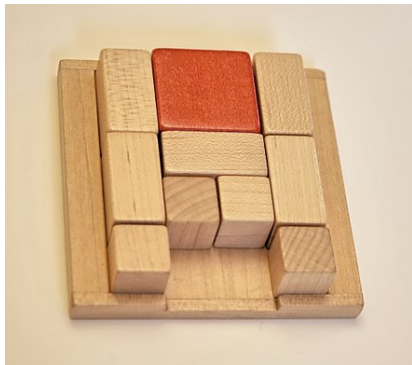
Does **Player 1** have a **winning strategy**?

Yes! It uses **spanning trees** in **graph theory**.



STAY TUNED

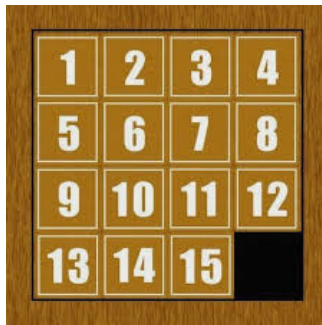
Klotski Puzzle (华容道)



Klotski Puzzle (华容道; 中国版本)



15 Puzzle (数字华容道)





Is it solvable?

How to solve it?

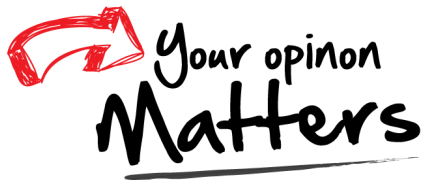
How to solve it?

It uses **permutation groups** in **group theory**.



STAY TUNED

Thank
You!



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