

Definition 1 Let $P \in \mathbb{N}^*$. We call the d -SBD coefficients on $(a, 1)$ of order P $\alpha_1^*, \alpha_2^*, \dots, \alpha_P^*$ the minimizers of the quantity

$$Q(\alpha_1, \dots, \alpha_P) = \int_a^1 r \left(\frac{-1}{r^2} - \sum_{p=1}^P \alpha_p \rho_p^2 J_0''(\rho_p r) \right)^2.$$

where ρ_1, \dots, ρ_P are the roots of J_0 .

Theorem 1 There exists L_1 and L_2 such that for all $a \in (0, 1)$, for all $P \in \mathbb{N}$.

$$\left| \frac{1}{r} - 1 - \left(\sum_{p=1}^P \alpha_p^* \rho_p J_0'(\rho_p r) - \sum_{p=1}^P \alpha_p^* \rho_p J_0'(\rho_p) \right) \right| \leq L_1 e^{-L_2 a P}$$

Lemma 1 Let $e \in H^2(B)$ a radial function. Then it holds

$$|e'(r) - e'(1)| \leq \sqrt{|\log r|} \int_r^1 r e''(r)^2$$