

**Definition 1.** Let the  $M$ -transform be defined by

$$\mathcal{M}\phi(\xi) := \sqrt{2} \int_0^{+\infty} \cos(\xi\sqrt{u}) \frac{\phi(u)}{\sqrt{u}} du$$

**Definition 2.** For any function  $\phi$  defined on  $\mathbb{R}^+$ , let  $C$  the operator defined by

$$C\phi(t) = \phi(t^2), \quad t \in \mathbb{R}$$

For any function even function  $\phi$  defined on  $\mathbb{R}$ ,  $C^{-1}\phi$  is a function defined on  $\mathbb{R}^+$  by

$$C^{-1}\phi(u) = \phi(\sqrt{u})$$

**Definition 3.** We note  $\mathcal{S}(\sqrt{\mathbb{R}^+})$  the space of  $\phi$  such that  $C\phi \in \mathcal{S}(\mathbb{R})$ . Let  $S_p(\mathbb{R})$  the subspace of even functions that belong to the Schwartz class.

**Proposition 1.**  $C\mathcal{S}(\sqrt{\mathbb{R}^+}) = S_p(\mathbb{R})$

**Proposition 2.** The operator  $\mathcal{M} : \mathcal{S}(\sqrt{\mathbb{R}^+}) \longrightarrow S_p(\mathbb{R})$  is a bijection, with inverse

$$\mathcal{M}^{-1}f = \sqrt{2} \int_0^{+\infty} \cos(\xi\sqrt{u}) f(\xi) d\xi$$