

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}, \dots\right)$$

```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$

# INTELLIGENCE ARTIFICIELLE

## Minimax, Alpha-beta, MCTS

Stéphane BONNEVAY

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Laboratoire ERIC  
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```

pretreatments();
lBestSolution = Stream
.iterate(lLowerBoundPattern, x -> x + 1)
.limit(1)
.map(x -> compute(x))
.min((sol1, sol2) -> (int) (sol1.getGlobalFitness() - sol2
.getGlobalFitness()))
.get();
return lBestSolution;

```

### Jeux à deux joueurs

Information complète : chaque joueur connaît ses possibilités d'action, les possibilités d'action de son adversaire, ainsi que les gains résultants des actions



Objectif : implémenter des IA qui jouent contre un joueur humain :

- Minimax
- Alpha-beta
- MCTS (Monte Carlo Tree Search)
- AlphaZero → cours « Deep Learning »

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min \left( \frac{1}{4} \right)$$

```
or (Item lCurrentItem :  
  lIdItem = lCurrentItem  
  lItemConstraint = new  
  for (int j = 0; j < p  
    lCurrentPattern =  
    lItemConstraint[j]  
  }  
  constraints.add(new L  
    Relationship.
```

$$: \prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$

### Tic-tac-toe

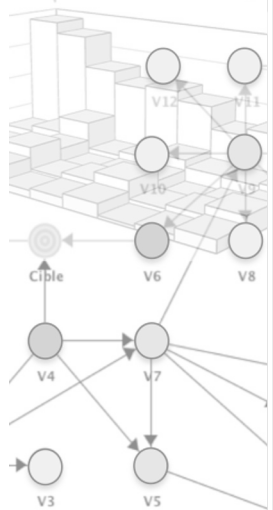
x joue

o joue

x joue

état de la partie

Se fixer une  
profondeur  
(hauteur de  
l'arbre)  
maximum

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$


```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.
  )

```

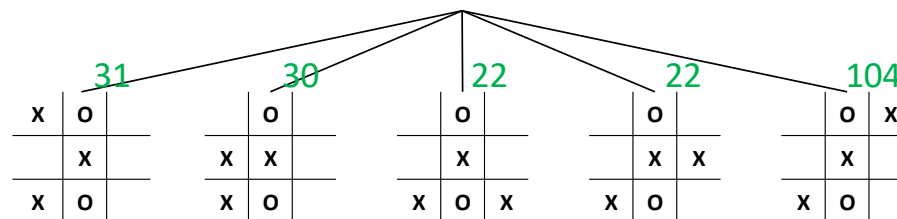
$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)} \right)$$

## Fonction d'évaluation pour le Tic-tac-toe

Calcul d'un score pour chacune des 3 lignes, 3 colonnes et 2 diagonales :

Score = +100	si 3 « x » alignés	(-100 pour « o »)
Score = +10	si 2 « x » alignés et 1 case vide	(-10 pour « o »)
Score = +1	si 1 « x » et 2 cases vides	(-1 pour « o »)

Evaluation = somme des 8 scores

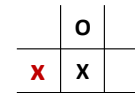


### Tic-tac-toe

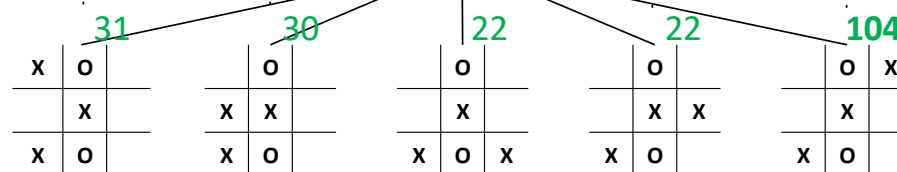
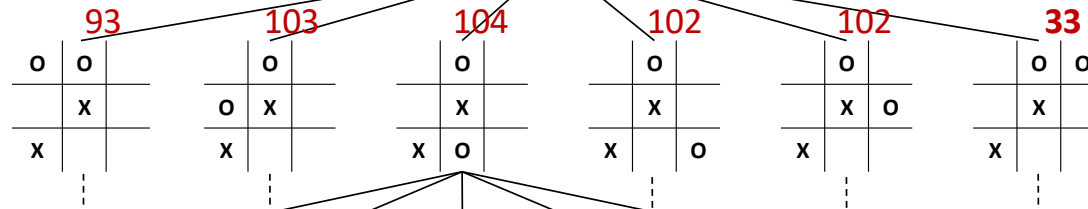
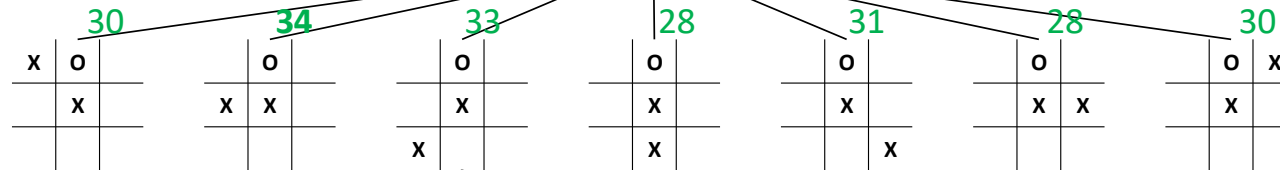
max (x)

min (o)

max (x)



état de la partie



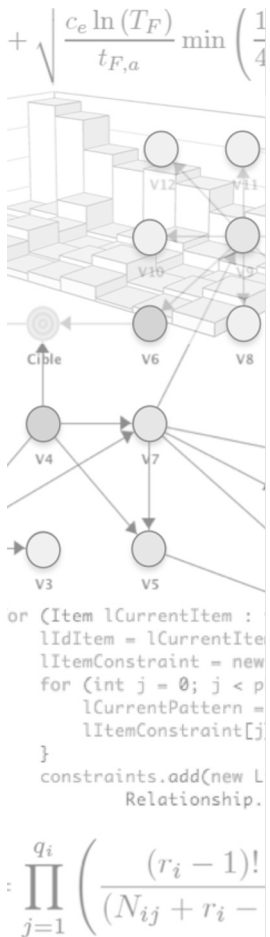
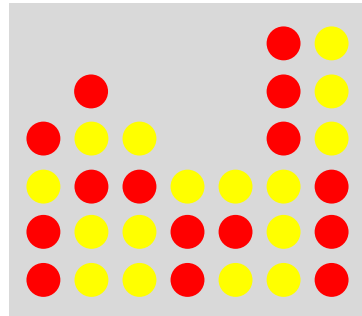
Évaluation des  
feuilles

On suppose  
que tout le  
monde joue  
de manière  
optimale

### Fonction d'évaluation

La qualité de l'IA dépend de la profondeur de l'arbre mais beaucoup de la fonction d'évaluation !

Pour le « Puissance 4 », quelle pourrait être la fonction d'évaluation ?

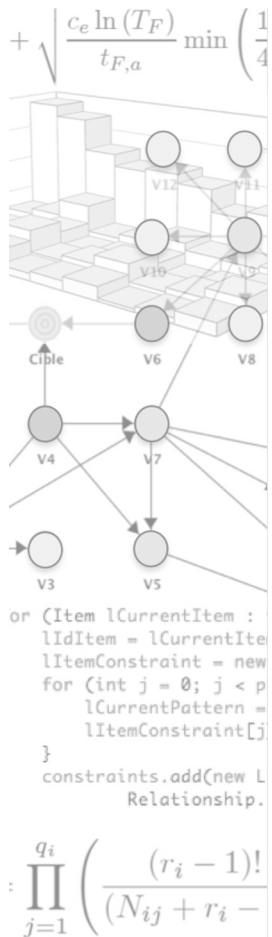
Par exemple :

Pour chaque joueur, faire la somme des points calculés comme suit :

- 4 pions alignés : 1000 points
- 3 pions et 1 case vide alignés : 50 points
- 2 pions et 2 cases vides alignés : 5 points
- 1 pion et 3 cases vides alignés : 1 point

Faire la différence des scores des deux joueurs.

## Code



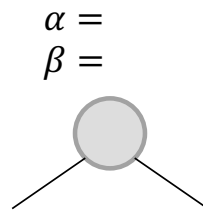
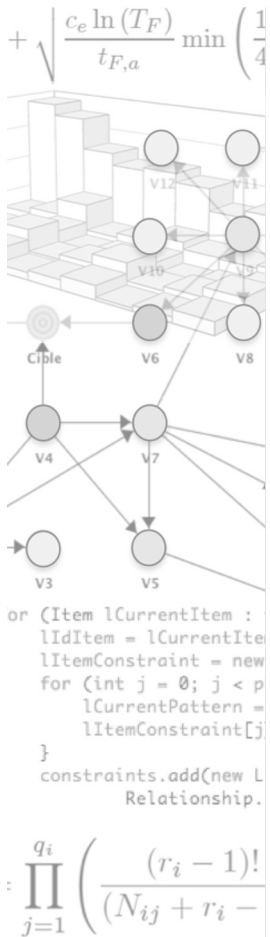
```
function MINIMAX(racine, maxProfondeur)
    eval, action = JOUEURMAX(racine, maxProfondeur)
    return action
end function
```

```
function JOUEURMAX(n, p)
    if n est une feuille ou p = 0 then
        return EVAL(n), _null_
    end if
    u = -∞ et a = _null_
    for all f fils de n (obtenu par une action a_f) do
        eval, . = JOUEURMIN(f, p - 1)
        if eval > u then
            u = eval et a = a_f
        end if
    end for
    return u, a
end function
```

```
function JOUEURMIN(n, p)
    if n est une feuille ou p = 0 then
        return EVAL(n), _null_
    end if
    u = +∞ et a = _null_
    for all f fils de n (obtenu par une action a_f) do
        eval, . = JOUEURMAX(f, p - 1)
        if eval < u then
            u = eval et a = a_f
        end if
    end for
    return u, a
end function
```

### Alpha-beta = Minimax + élagage (pruning)

Objectif : diminuer la taille de l'arbre de l'algorithme Minimax



$\alpha$  : meilleure valeur de la branche pour le joueur Max  
 $\alpha$  n'est mis à jour que sur les nœuds Max

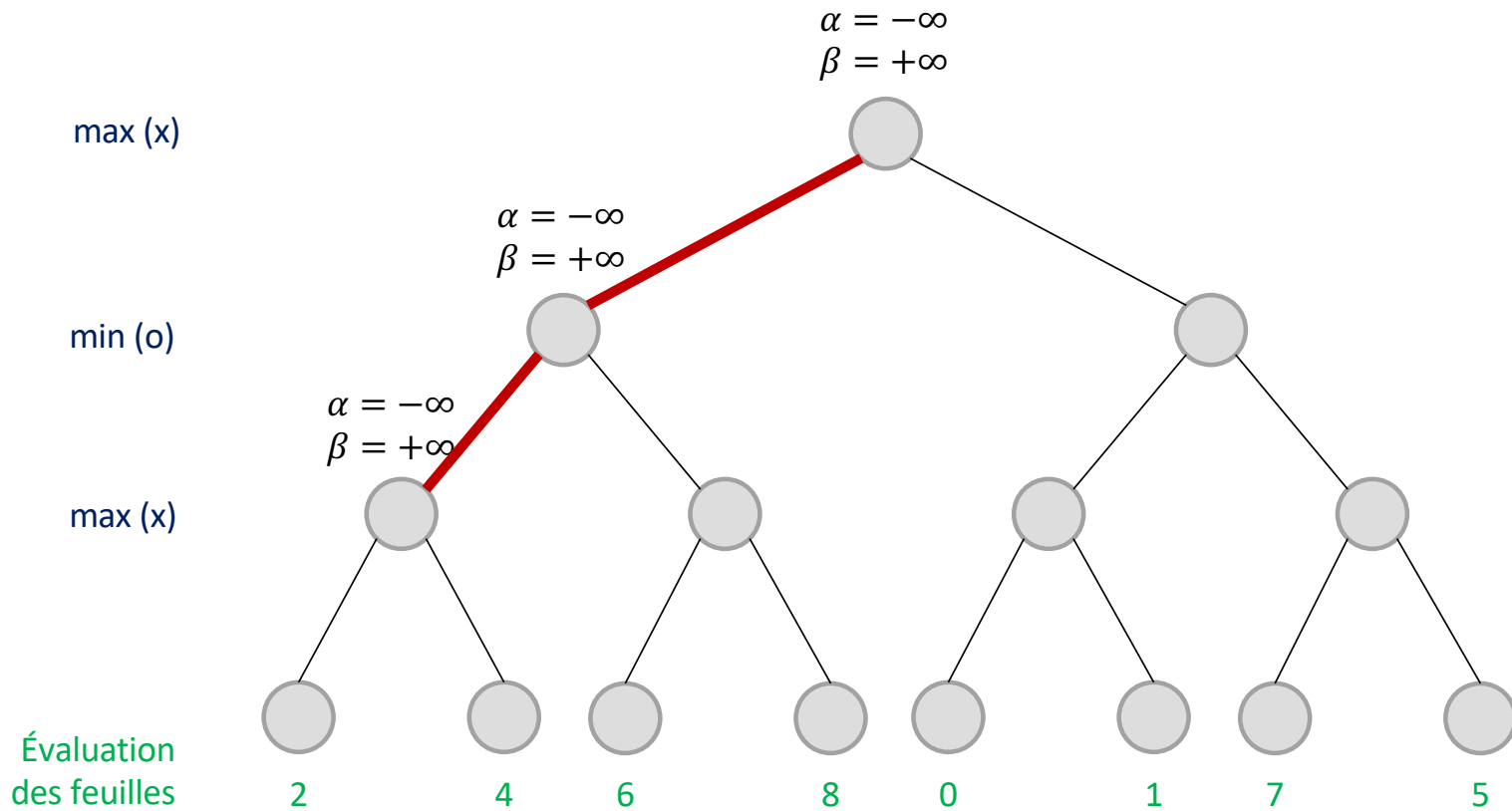
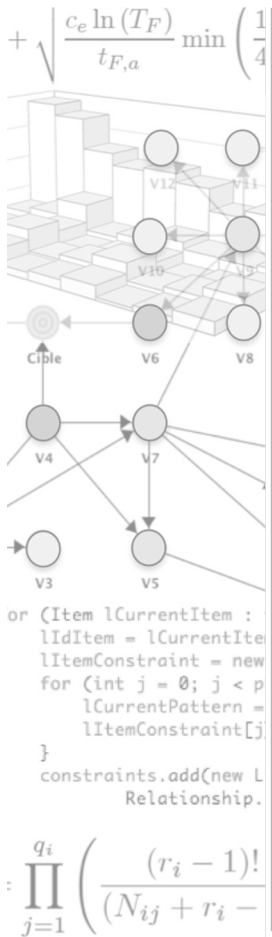
$\beta$  : meilleure valeur de la branche pour le joueur Min  
 $\beta$  n'est mis à jour que sur les nœuds Min

Initialisation :  $\alpha = -\infty$   
 $\beta = +\infty$

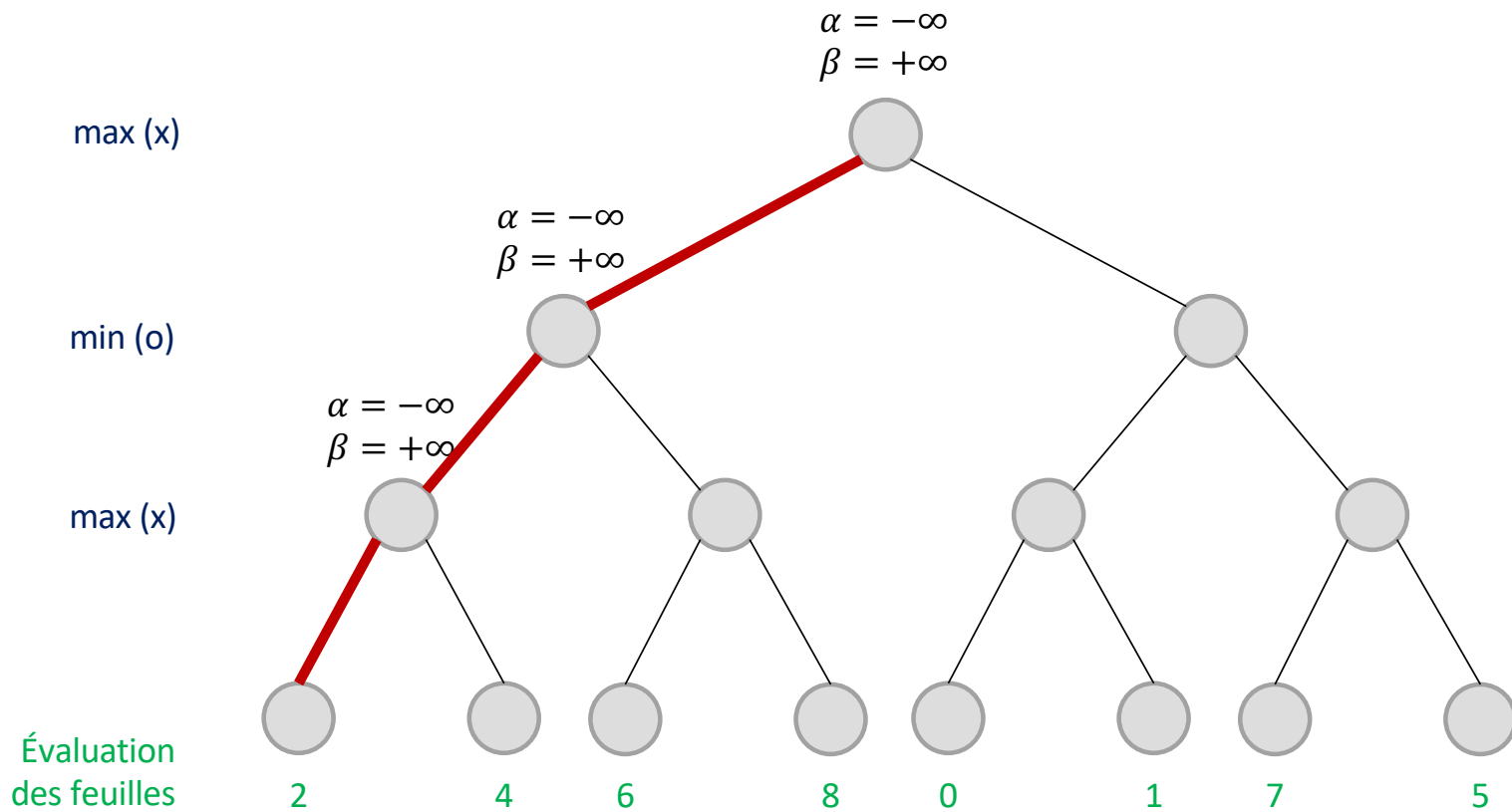
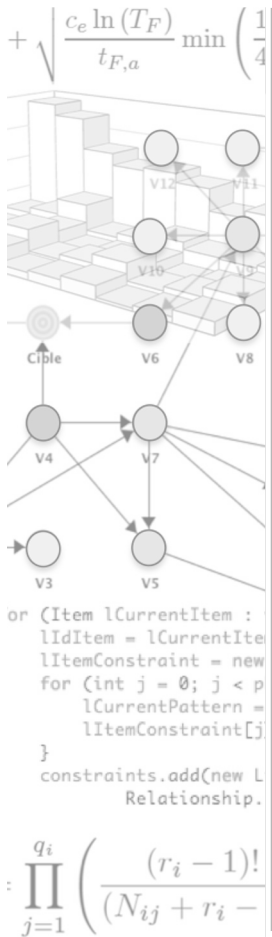
Règle : si  $\alpha \geq \beta$ , on élague la branche



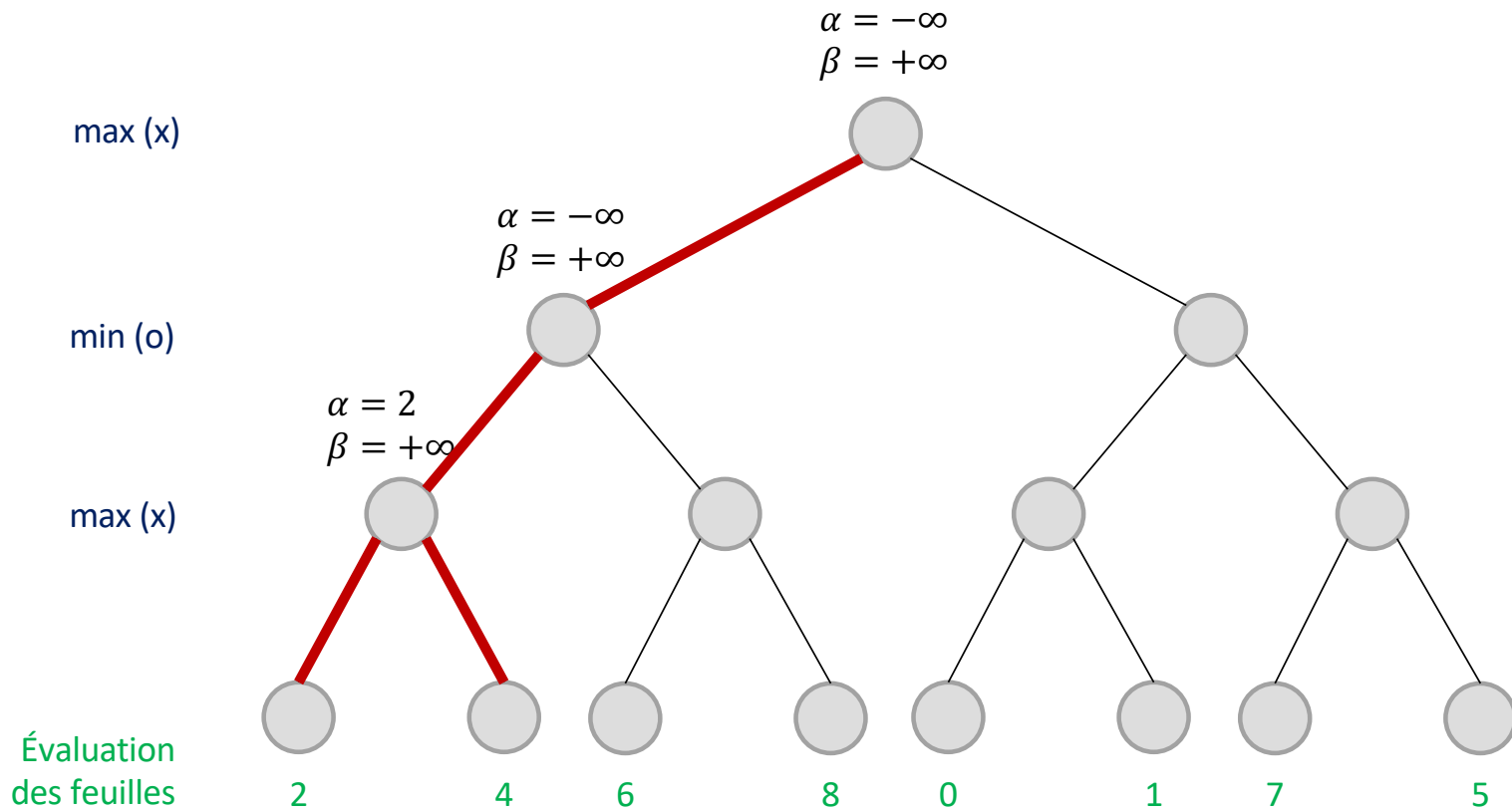
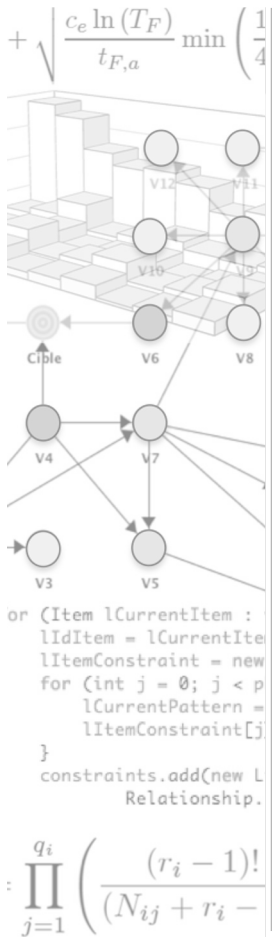
Alpha-beta = Minimax + élagage (pruning)



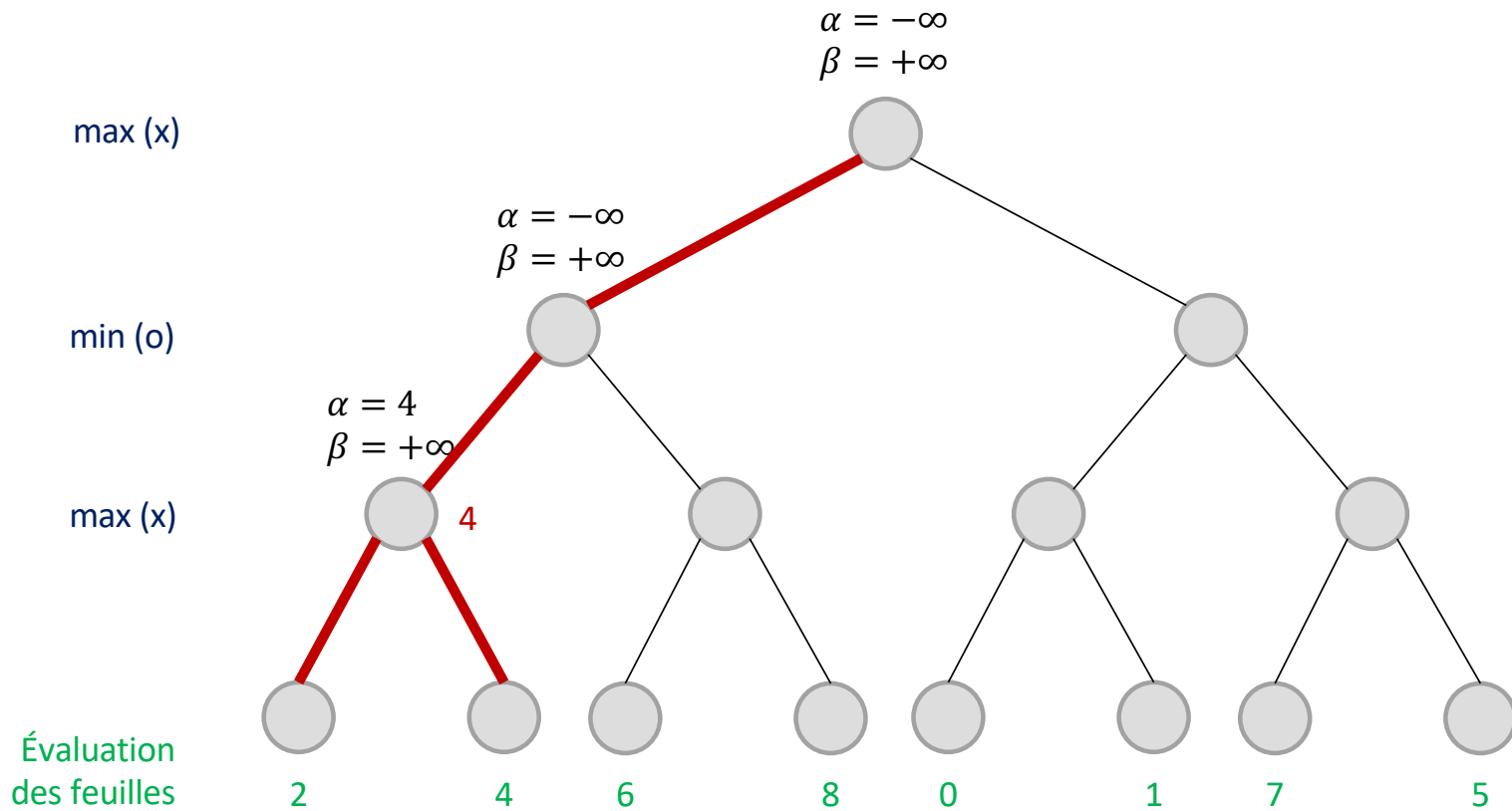
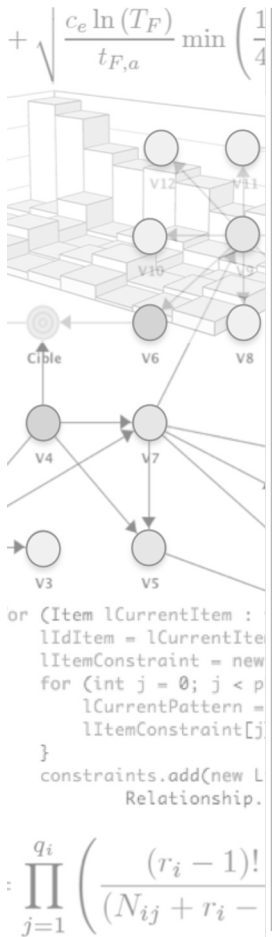
Alpha-beta = Minimax + élagage (pruning)



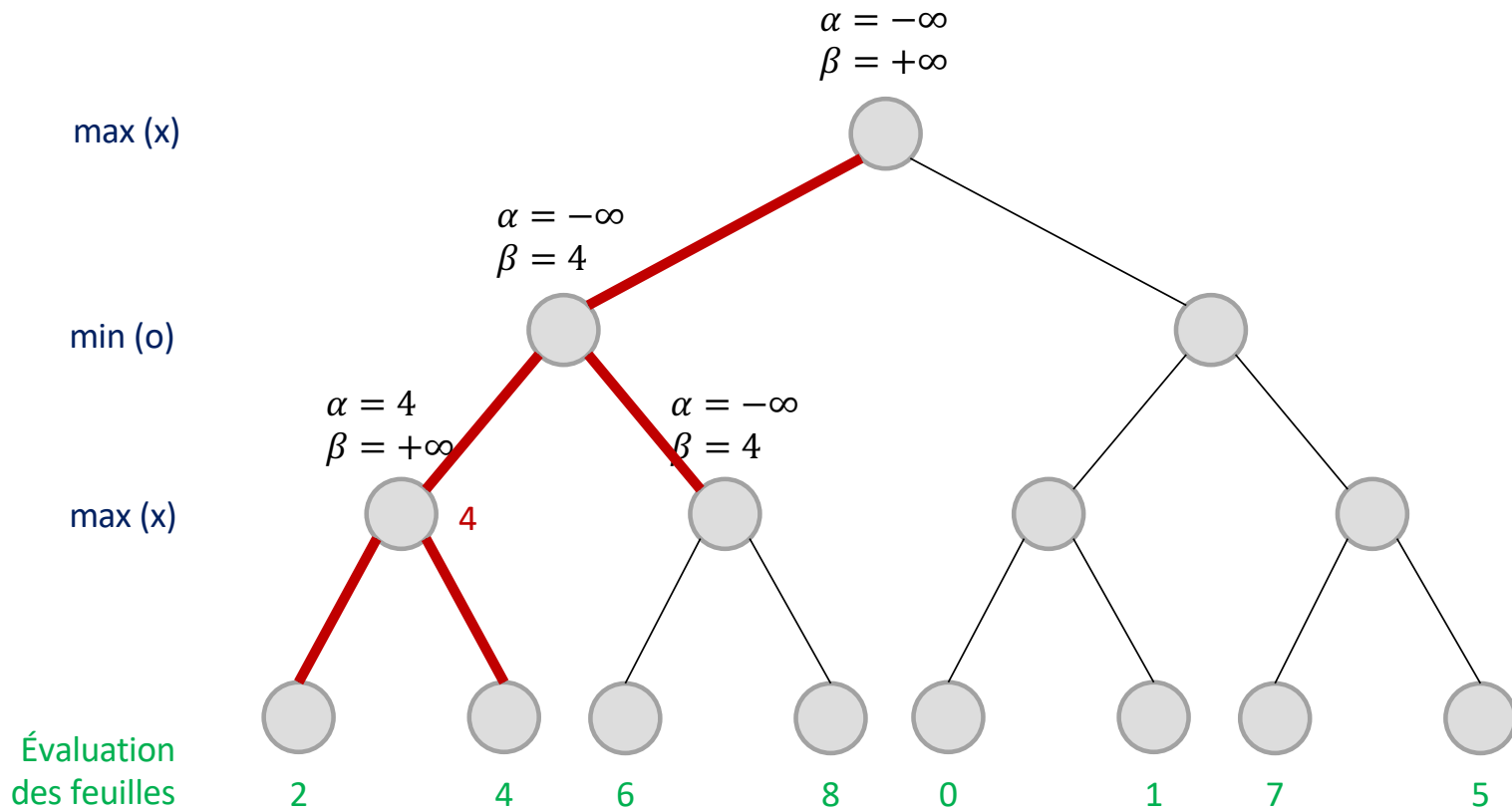
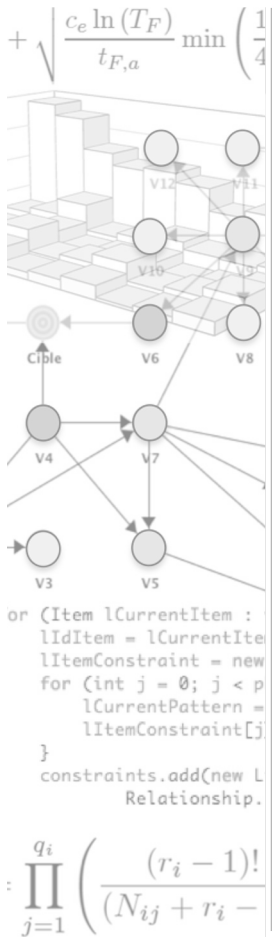
Alpha-beta = Minimax + élagage (pruning)



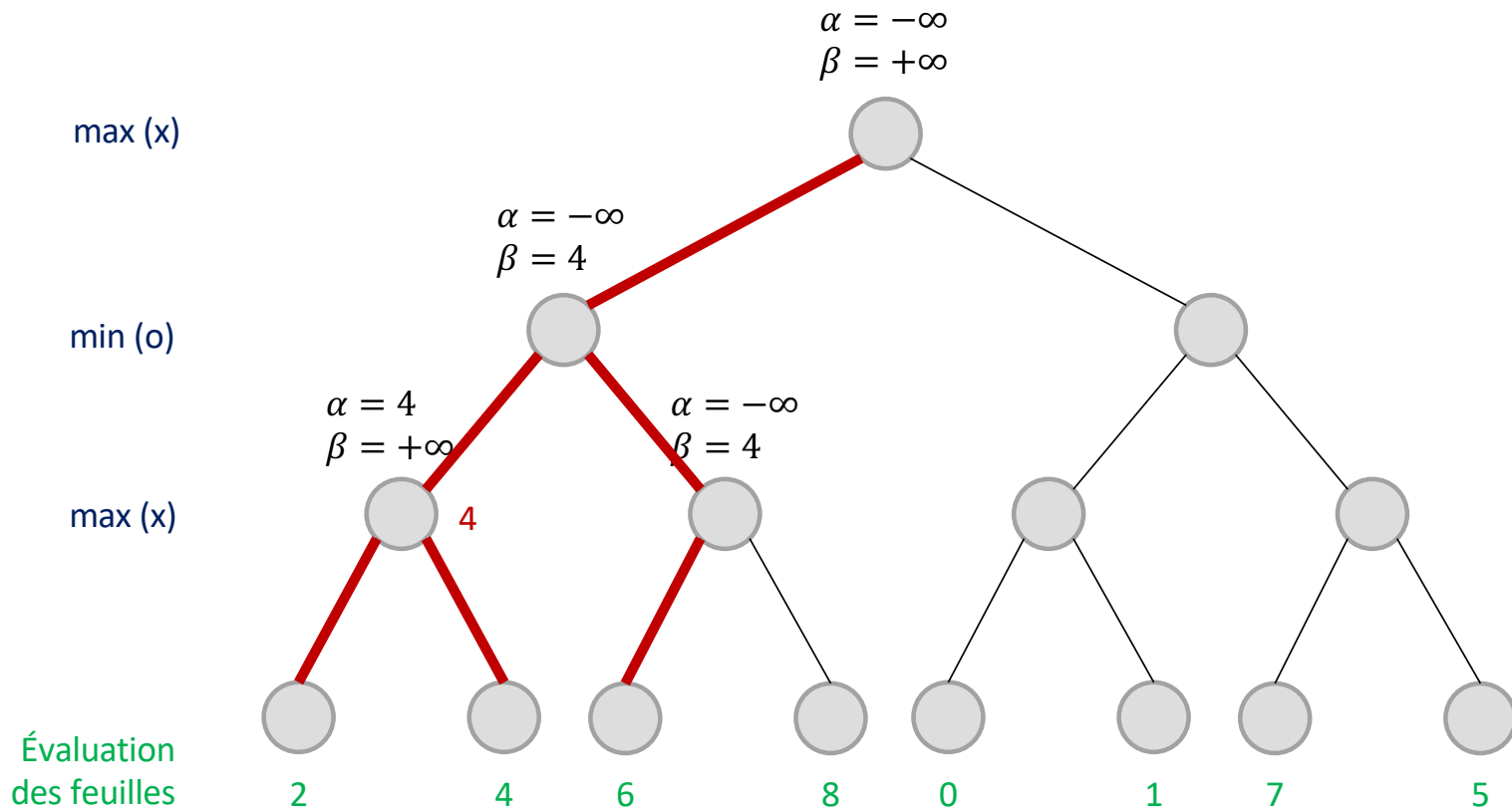
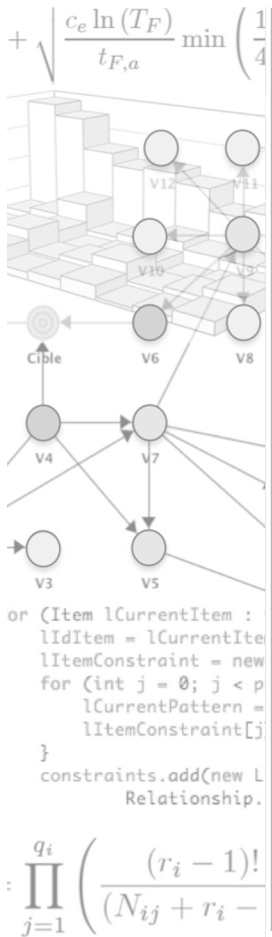
Alpha-beta = Minimax + élagage (pruning)



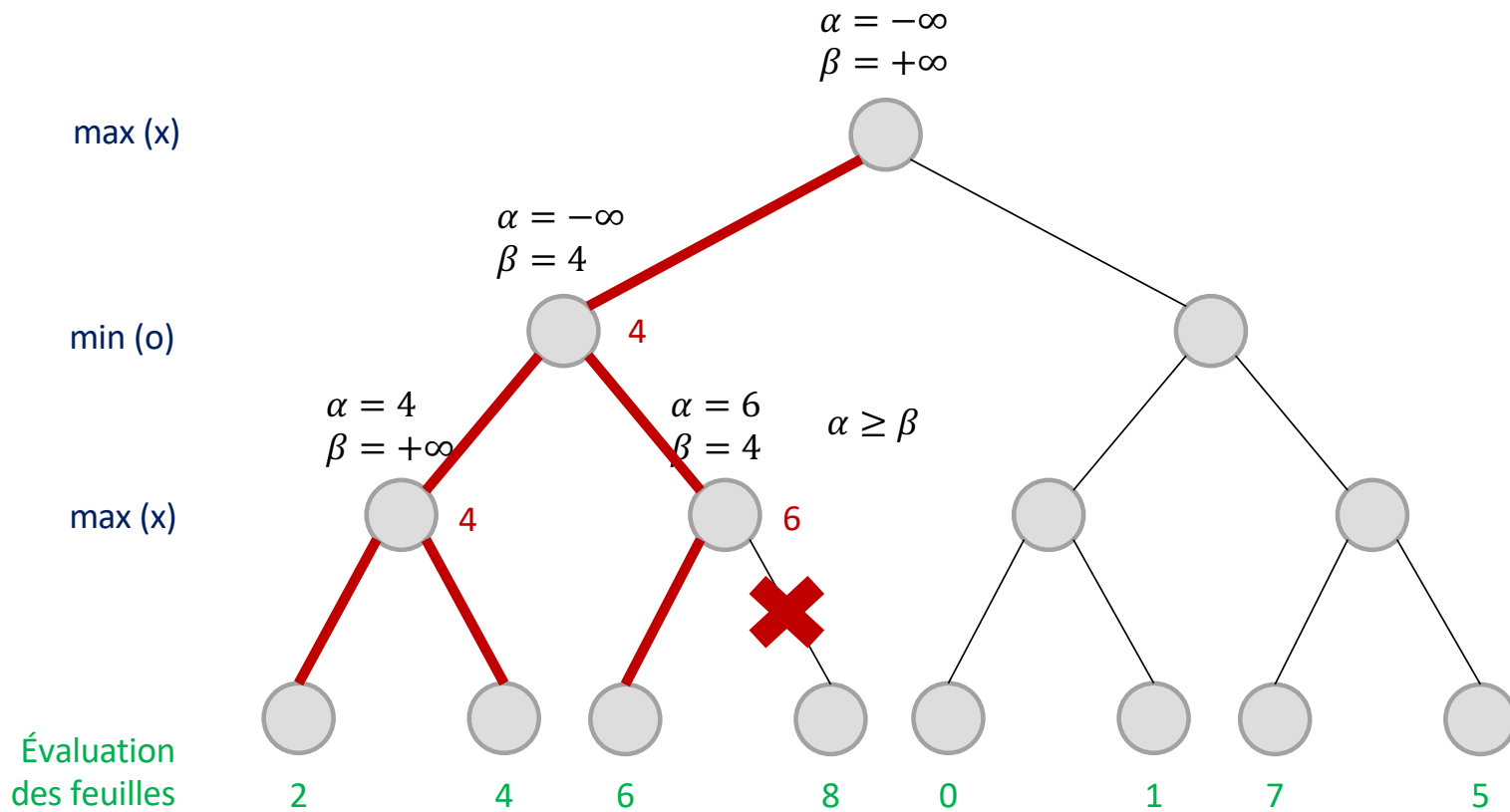
Alpha-beta = Minimax + élagage (pruning)



Alpha-beta = Minimax + élagage (pruning)

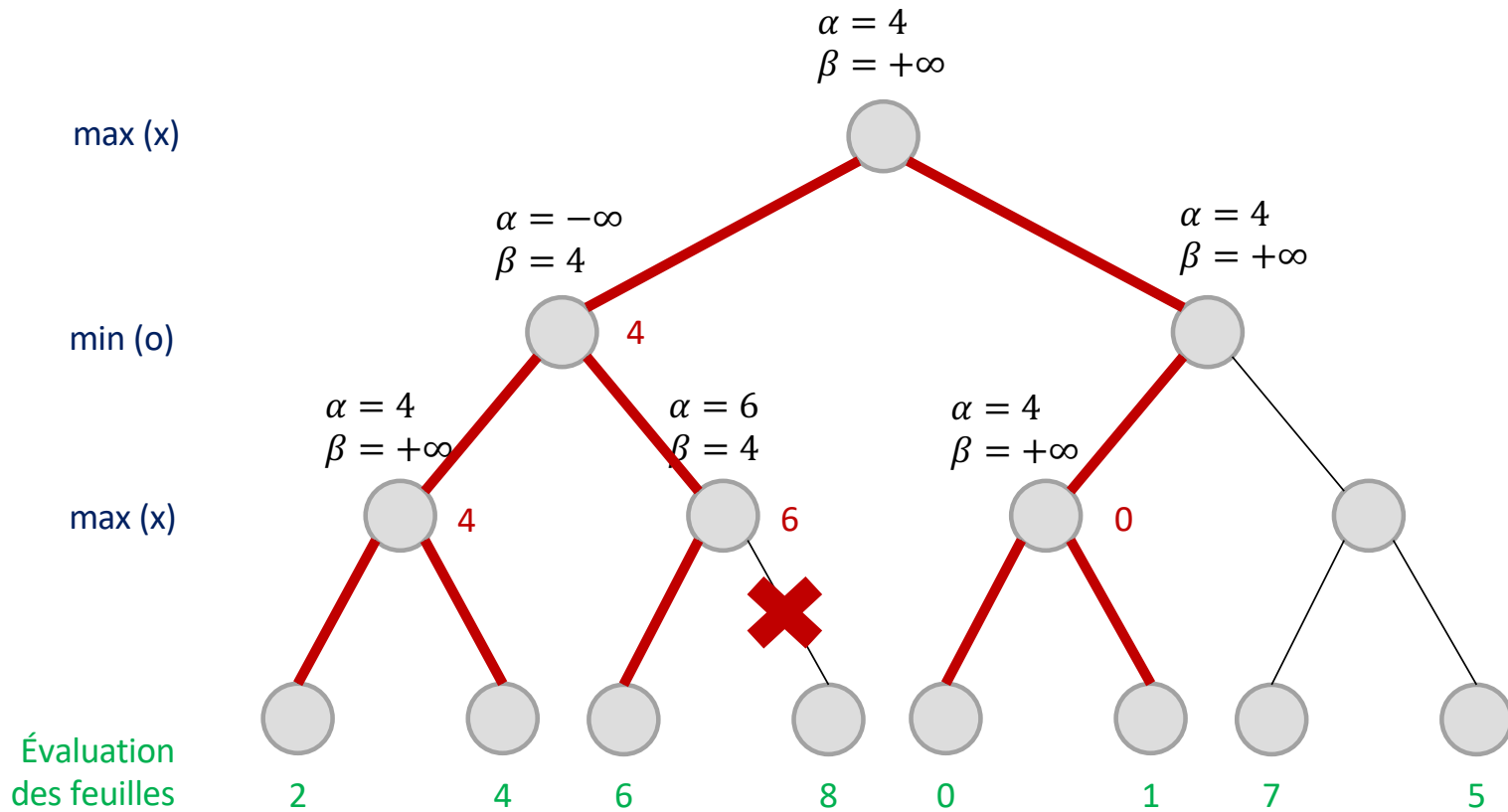
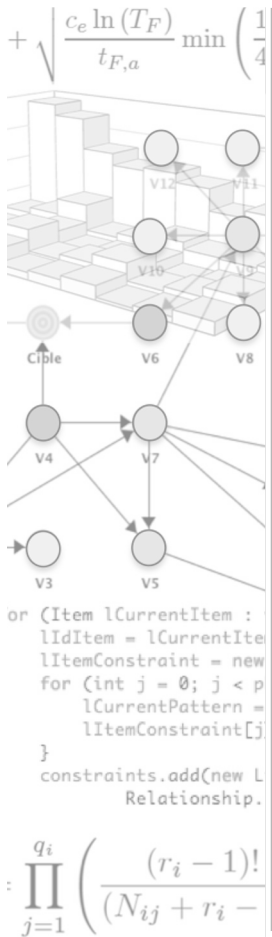


## Alpha-beta = Minimax + élagage (pruning)



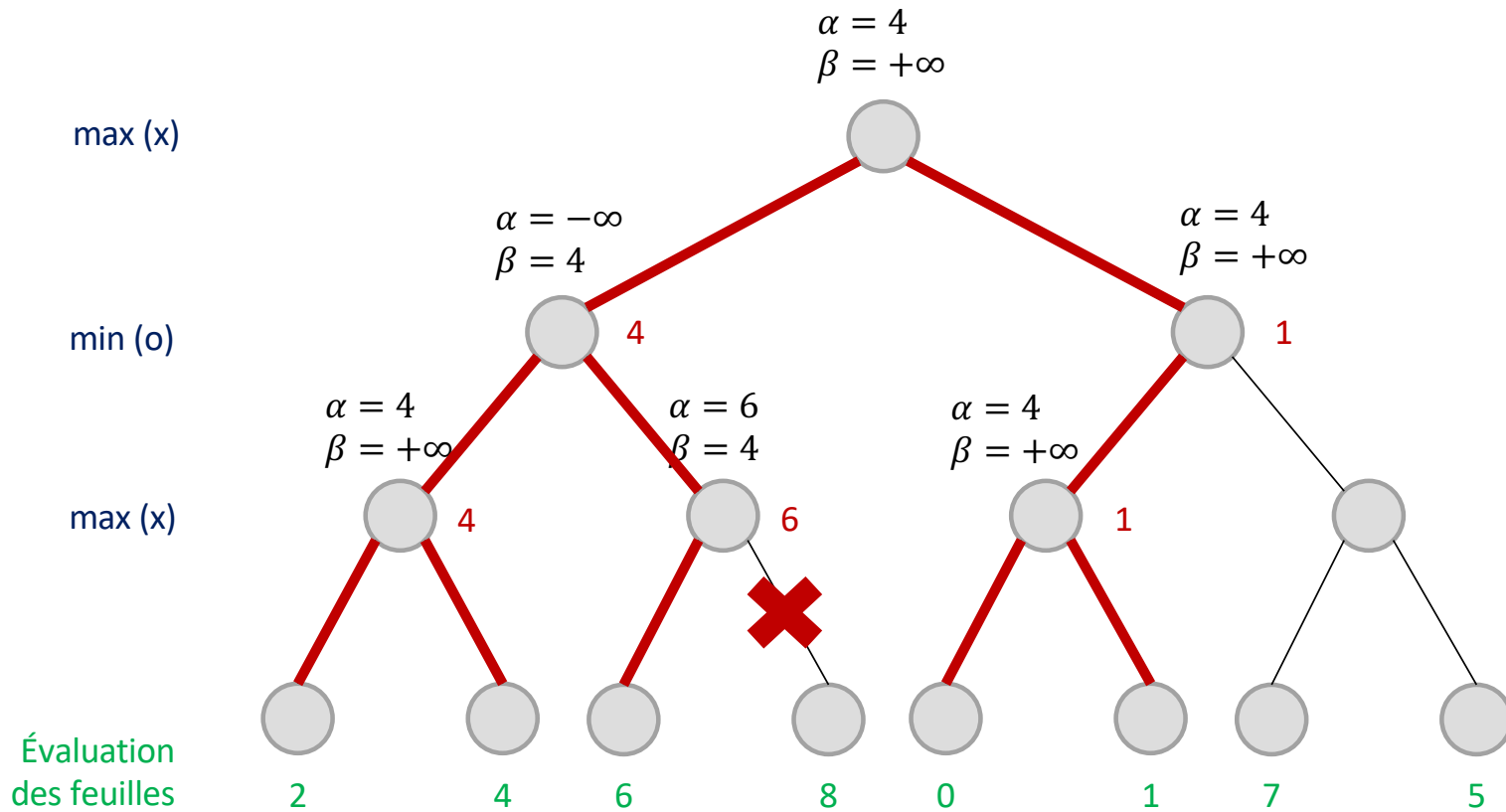
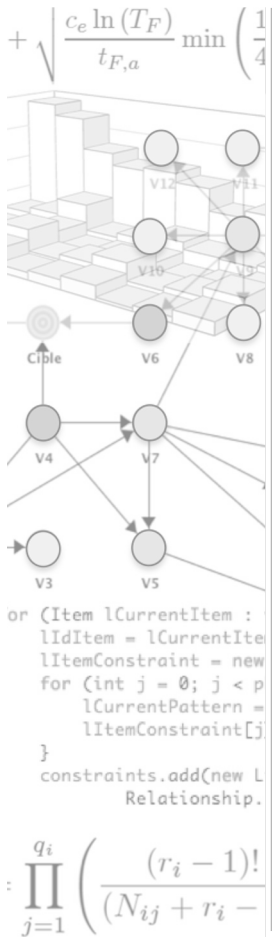
## Évaluation des feuilles

Alpha-beta = Minimax + élagage (pruning)

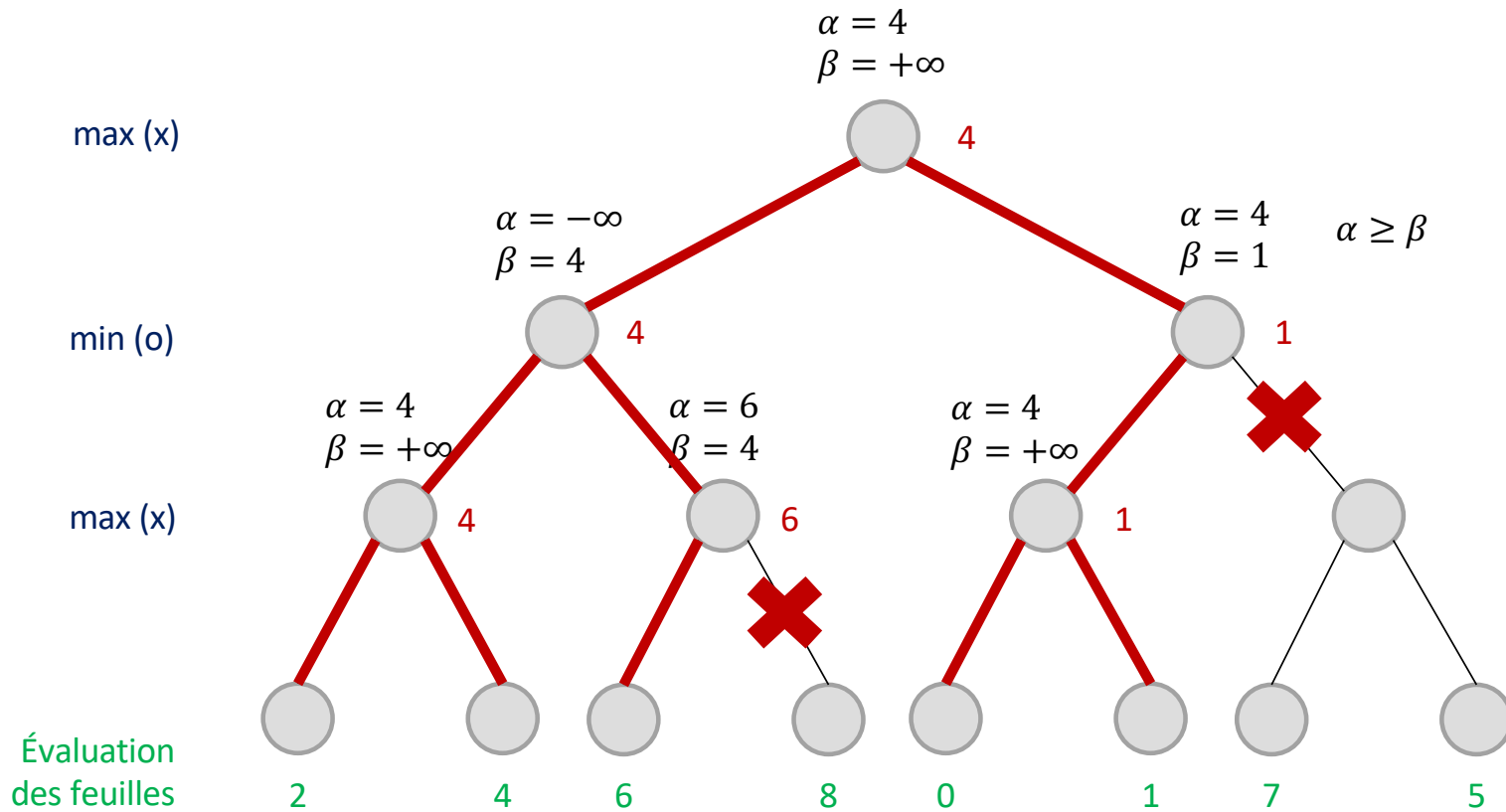




## Alpha-beta = Minimax + élagage (pruning)



## Alpha-beta = Minimax + élagage (pruning)



## Évaluation des feuilles

Stéphane BONNEVAY – Polytech Lyon

## Code

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$

```

function ALPHA-BETA(racine, maxProfondeur)
  eval, action = JOUEURMAX(racine, maxProfondeur,  $-\infty$ ,  $+\infty$ )
  return action
end function

```

```

function JOUEURMAX(n, p,  $\alpha$ ,  $\beta$ )
  if n est une feuille ou p = 0 then
    return EVAL(n), null
  end if
  u =  $-\infty$  et a = null
  for all f fils de n (obtenu par une action af) do
    eval, . = JOUEURMIN(f, p - 1,  $\alpha$ ,  $\beta$ )
    if eval > u then
      u = eval et a = af
    end if
    if u ≥  $\beta$  then
      return u, a
    end if
     $\alpha$  = max( $\alpha$ , u)
  end for
  return u, a
end function

```

```

function JOUEURMIN(n, p,  $\alpha$ ,  $\beta$ )
  if n est une feuille ou p = 0 then
    return EVAL(n), null
  end if
  u =  $+\infty$  et a = null
  for all f fils de n (obtenu par une action af) do
    eval, . = JOUEURMAX(f, p - 1,  $\alpha$ ,  $\beta$ )
    if eval < u then
      u = eval et a = af
    end if
    if u ≤  $\alpha$  then
      return u, a
    end if
     $\beta$  = min( $\beta$ , u)
  end for
  return u, a
end function

```

## Monte Carlo Tree Search

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

```

or (Item lCurrentItem :
    lIdItem = lCurrentItem
    lItemConstraint = new
    for (int j = 0; j < p
        lCurrentPattern =
        lItemConstraint[j]
    }
    constraints.add(new L
        Relationship.

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)} \right)$$

Stéphane BONNEVAY – Polytech Lyon

IEEE TRANSACTIONS ON COMPUTATIONAL INTELLIGENCE AND AI IN GAMES, VOL. 4, NO. 1, MARCH 2012

1

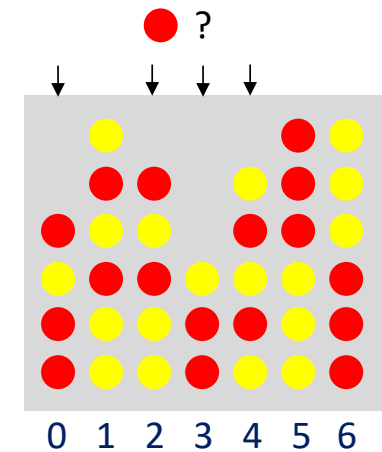
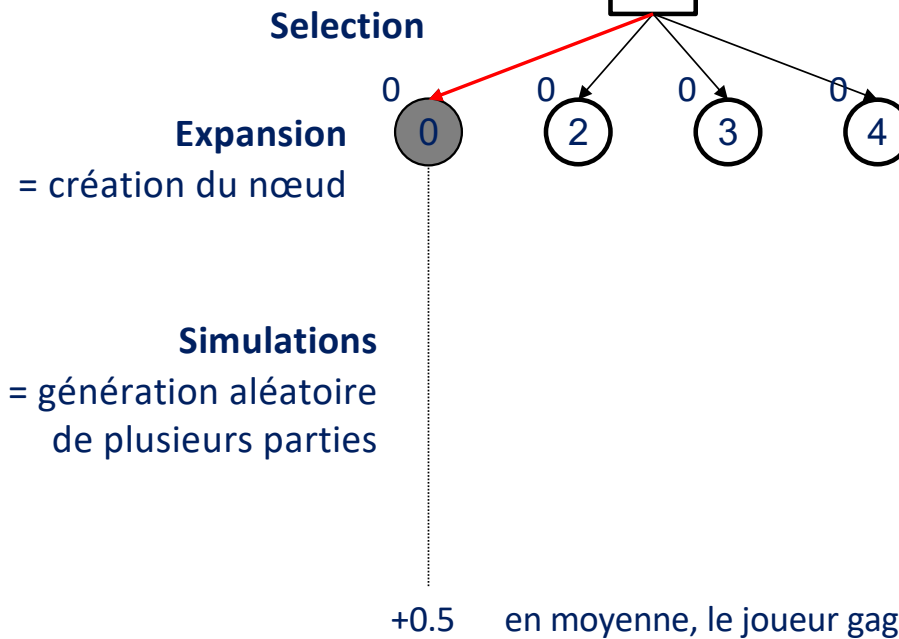
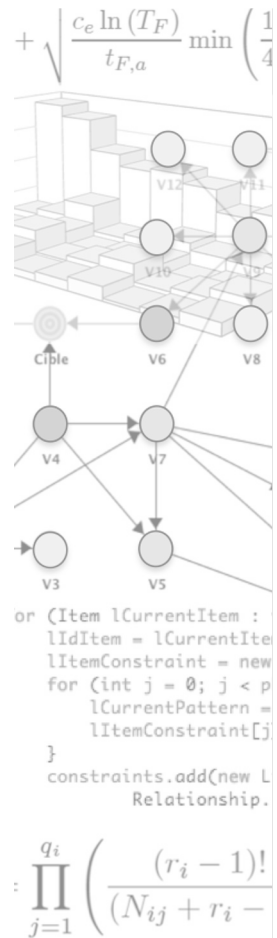
## A Survey of Monte Carlo Tree Search Methods

Cameron Browne, *Member, IEEE*, Edward Powley, *Member, IEEE*, Daniel Whitehouse, *Member, IEEE*, Simon Lucas, *Senior Member, IEEE*, Peter I. Cowling, *Member, IEEE*, Philipp Rohlfshagen, Stephen Tavener, Diego Perez, Spyridon Samothrakis and Simon Colton

**Abstract**—Monte Carlo Tree Search (MCTS) is a recently proposed search method that combines the precision of tree search with the generality of random sampling. It has received considerable interest due to its spectacular success in the difficult problem of computer Go, but has also proved beneficial in a range of other domains. This paper is a survey of the literature to date, intended to provide a snapshot of the state of the art after the first five years of MCTS research. We outline the core algorithm's derivation, impart some structure on the many variations and enhancements that have been proposed, and summarise the results from the key game and non-game domains to which MCTS methods have been applied. A number of open research questions indicate that the field is ripe for future work.

**Index Terms**—Monte Carlo Tree Search (MCTS), Upper Confidence Bounds (UCB), Upper Confidence Bounds for Trees (UCT), Bandit-based methods, Artificial Intelligence (AI), Game search, Computer Go.

### Monte Carlo Tree Search



### Monte Carlo Tree Search

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

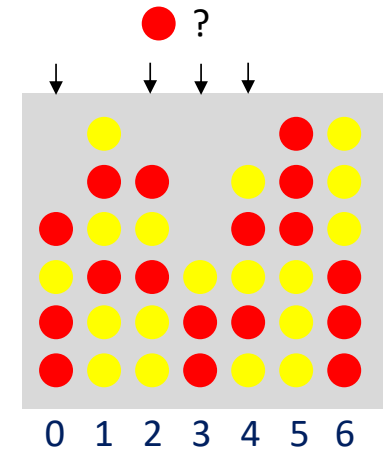
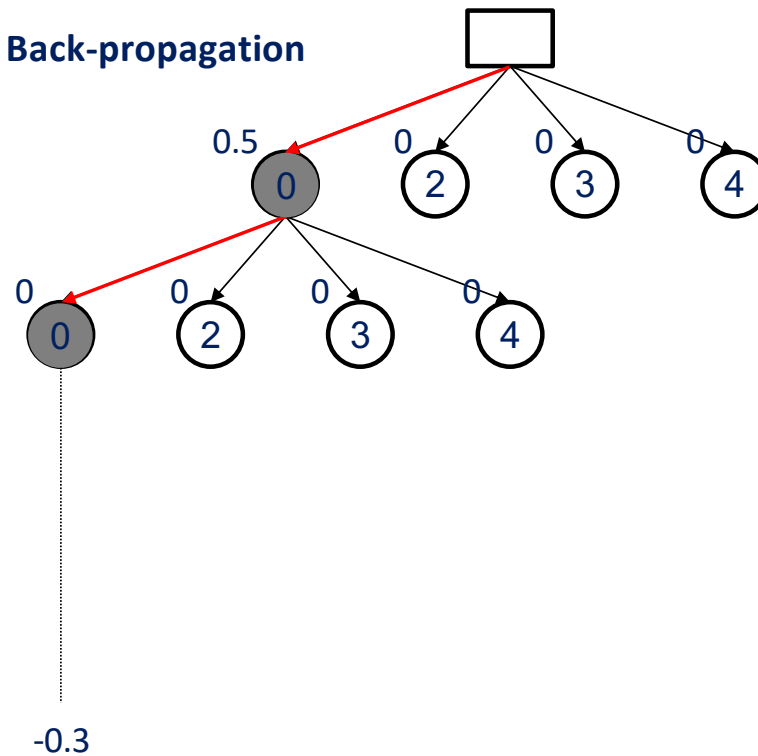
```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.
  )

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$

#### Back-propagation



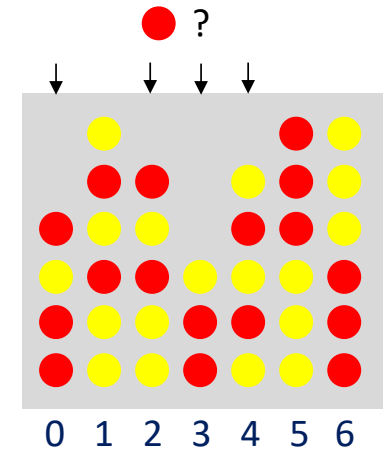
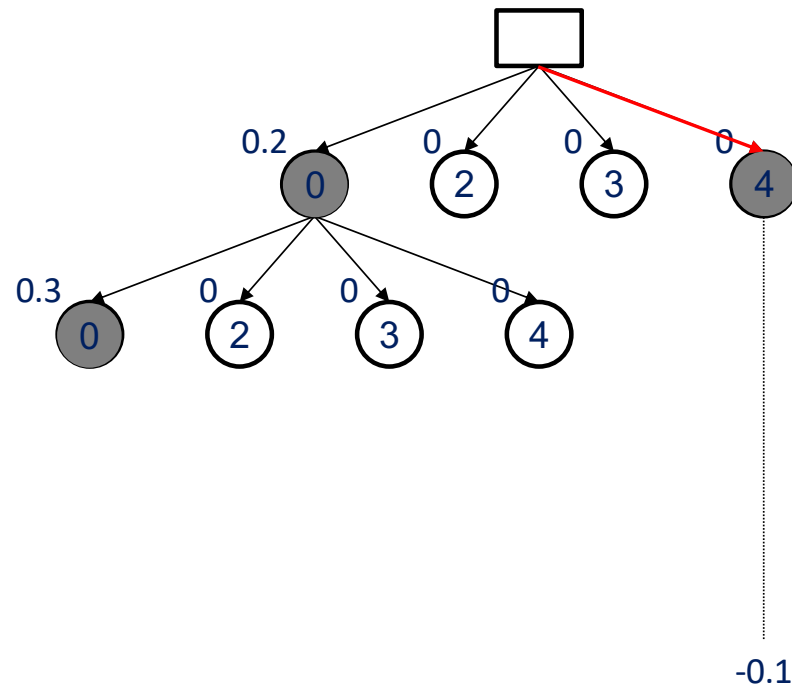
### Monte Carlo Tree Search

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.
  )

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)} \right)$$


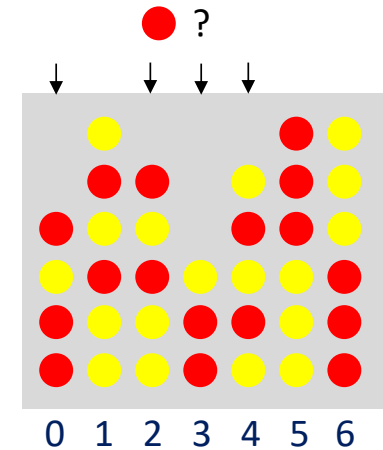
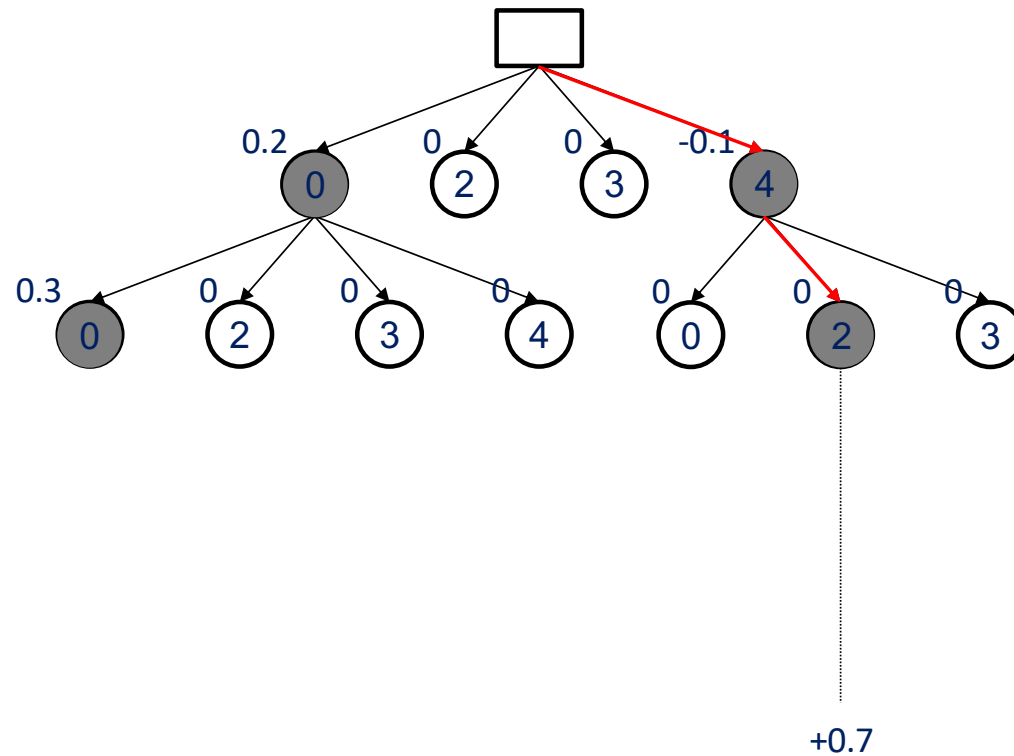
### Monte Carlo Tree Search

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
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  for (int j = 0; j < p
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  }
  constraints.add(new L
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  )

```

$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$


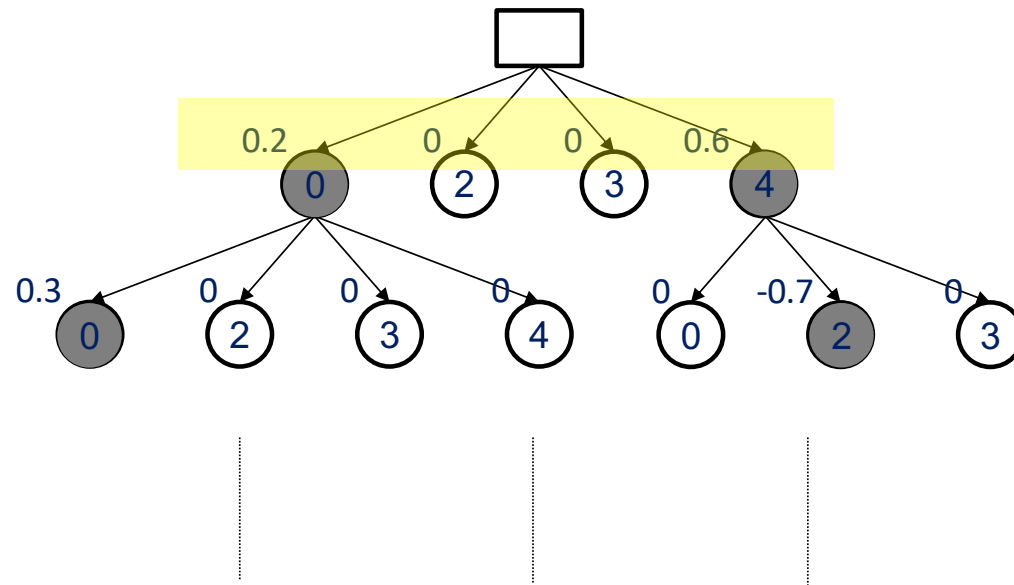


### Monte Carlo Tree Search

$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

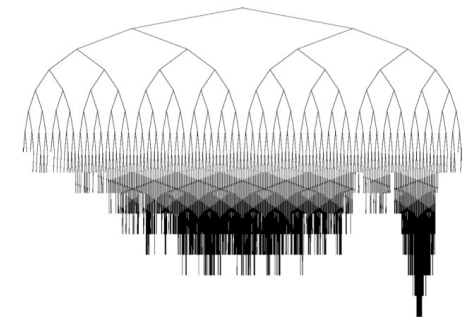
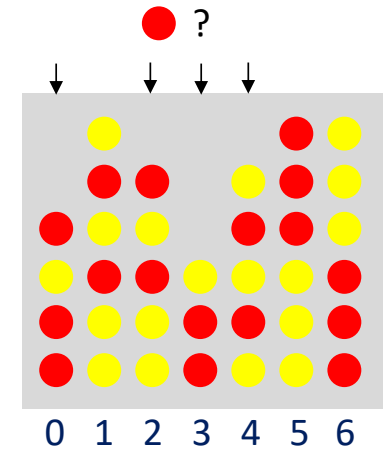
```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
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  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.
  
```

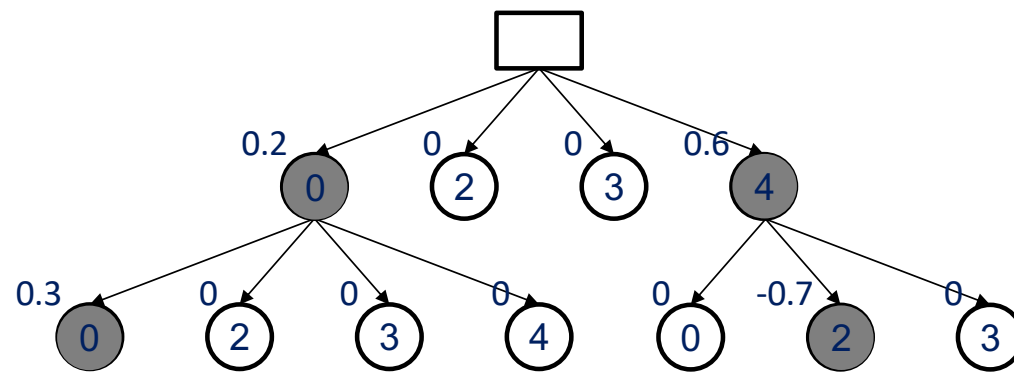
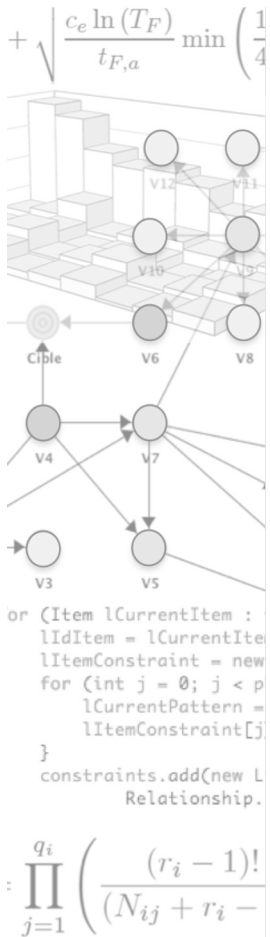
$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)!} \right)$$


poursuivre le processus ...

à la fin choisir l'action avec la plus grande valeur



### Monte Carlo Tree Search



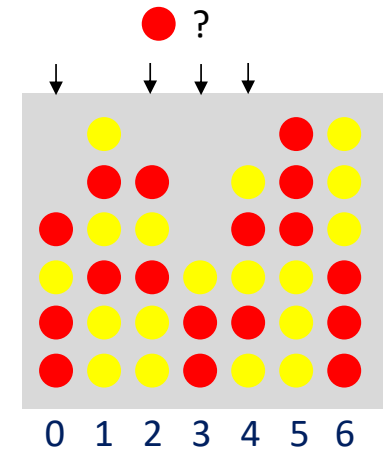
Sélection :

$$\frac{Q(v')}{N(v')} + c \sqrt{\frac{2 \ln N(v)}{N(v')}}$$

/ intensification

\ diversification

La valeur de C influence la forme de l'arbre et la qualité des résultats



## Code

### 3.3 Upper Confidence Bounds for Trees (UCT)

This section describes the most popular algorithm in the MCTS family, the *Upper Confidence Bound for Trees* (UCT)

```

function UCTSEARCH( $s_0$ )
  create root node  $v_0$  with state  $s_0$ 
  while within computational budget do
     $v_l \leftarrow$  TREEPOLICY( $v_0$ )
     $\Delta \leftarrow$  DEFAULTPOLICY( $s(v_l)$ )
    BACKUP( $v_l, \Delta$ )
  return  $a(\text{BESTCHILD}(v_0, 0))$ 

```

```

function TREEPOLICY( $v$ )
  while  $v$  is nonterminal do
    if  $v$  not fully expanded then
      return EXPAND( $v$ )
    else
       $v \leftarrow$  BESTCHILD( $v, C_p$ )
  return  $v$ 

```

```

function EXPAND( $v$ )
  choose  $a \in$  untried actions from  $A(s(v))$ 
  add a new child  $v'$  to  $v$ 
    with  $s(v') = f(s(v), a)$ 
    and  $a(v') = a$ 
return  $v'$ 

```

```

function BESTCHILD( $v, c$ )
  return  $\arg \max_{v' \in \text{children of } v} \frac{Q(v')}{N(v')} + c \sqrt{\frac{2 \ln N(v)}{N(v)}}$ 

```

```

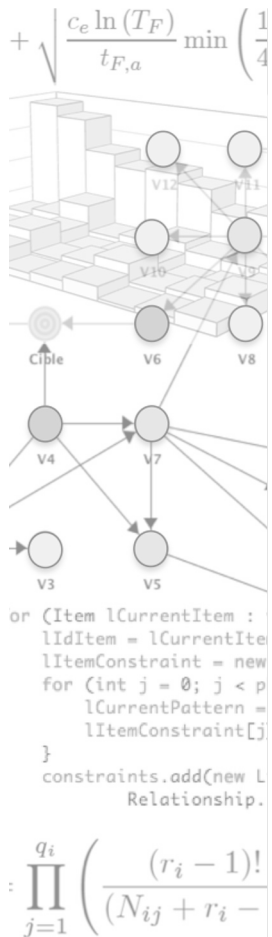
function DEFAULTPOLICY( $s$ )
  while  $s$  is non-terminal do
    choose  $a \in A(s)$  uniformly at random
     $s \leftarrow f(s, a)$ 
  return reward for state  $s$ 

```

```

function BACKUP( $v, \Delta$ )
  while  $v$  is not null do
     $N(v) \leftarrow N(v) + 1$ 
     $Q(v) \leftarrow Q(v) + \Delta$ 
     $\Delta \leftarrow -\Delta$ 
     $v \leftarrow \text{parent of } v$ 

```





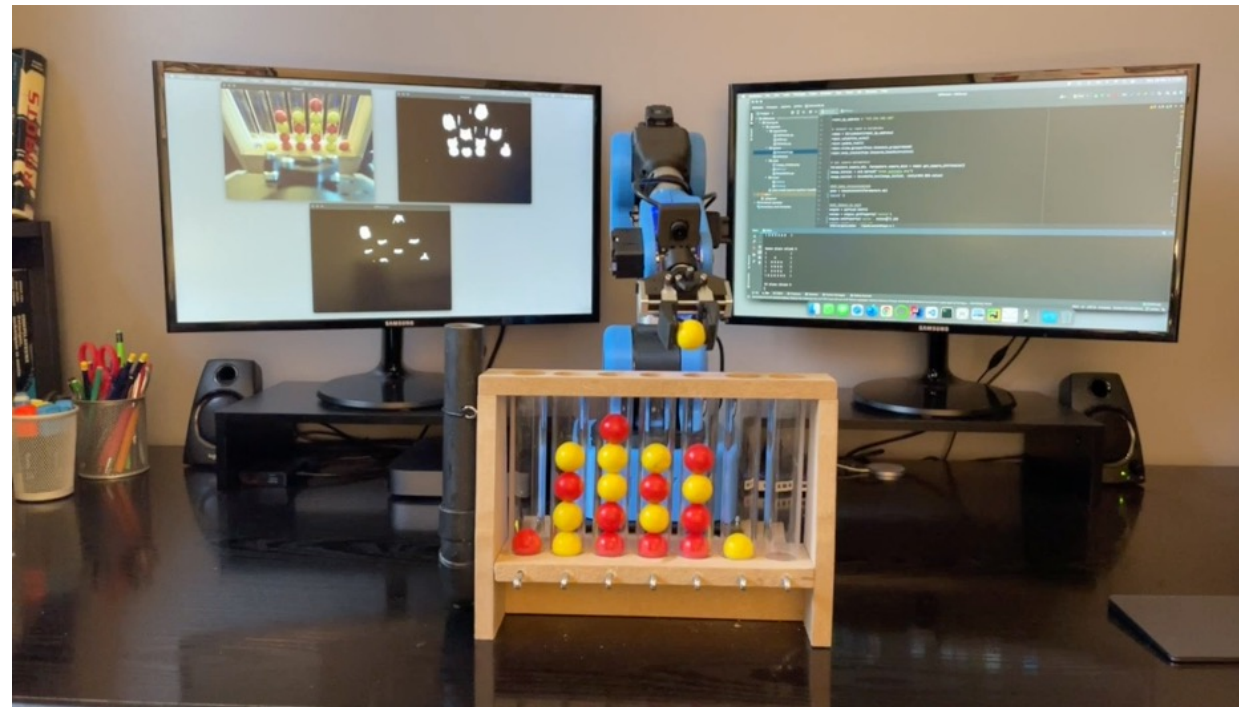
$$+ \sqrt{\frac{c_e \ln(T_F)}{t_{F,a}}} \min\left(\frac{1}{4}\right)$$

```

or (Item lCurrentItem :
  lIdItem = lCurrentItem
  lItemConstraint = new
  for (int j = 0; j < p
    lCurrentPattern =
    lItemConstraint[j]
  }
  constraints.add(new L
    Relationship.

```

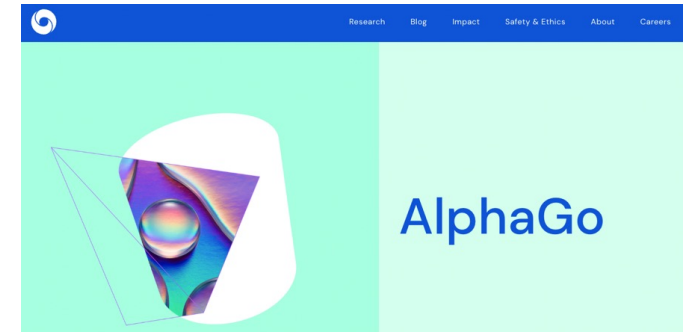
$$\prod_{j=1}^{q_i} \left( \frac{(r_i - 1)!}{(N_{ij} + r_i - 1)} \right)$$



## AlphaGo



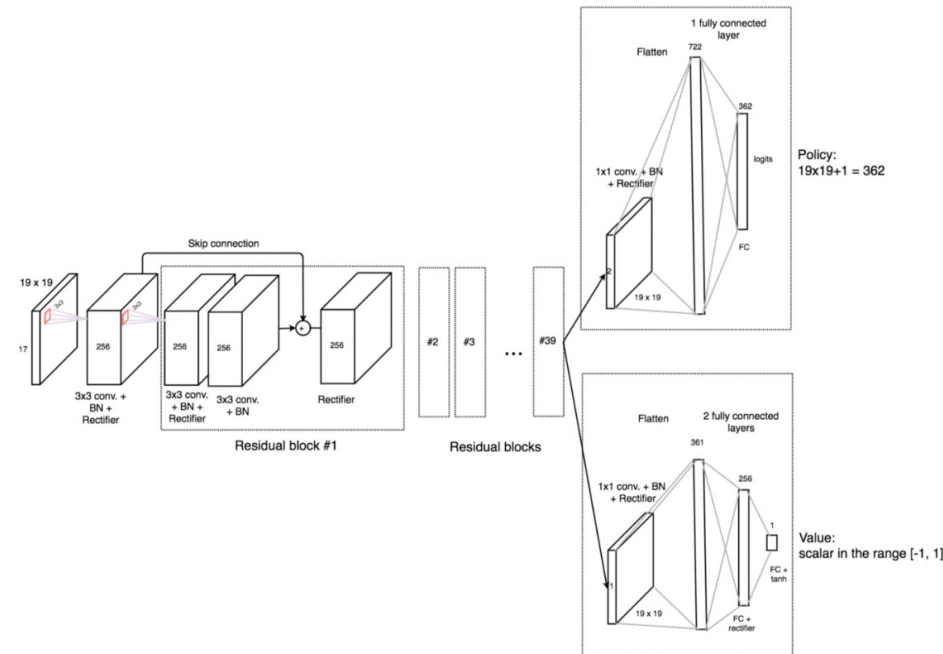
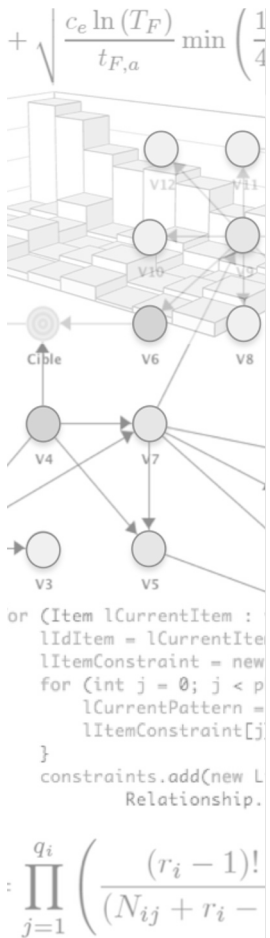
<https://www.youtube.com/watch?v=WXuK6gekU1Y>



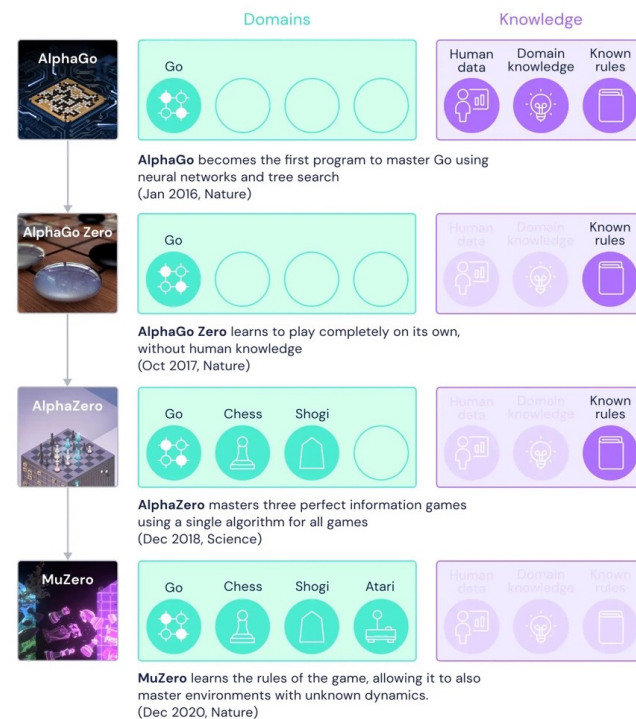
<https://www.deepmind.com/research/highlighted-research/alphago>

## AlphaGo Zero

Idée : faire apprendre la **policy** et la **value function** à un réseau de neurones



## AlphaGo, AlphaGo Zero, AlphaZero, MuZero



## AlphaZero

- Généralisation de AlphaGo Zero à d'autres jeux que le jeu de Go
- Globalement même architecture que AlphaGo Zero
- Par contre, il faut adapter l'entrée du NN en fonction du jeu
- Pas d'évaluateur : c'est le même NN qui est amélioré au cours du temps