

Triangulations And- Blowups For Toric- Varieties

A package to compute triangulations and
blowups of toric varieties

2019.02.09

9 February 2019

Martin Bies

Martin Bies

Email: martin.bies@alumni.uni-heidelberg.de

Homepage: <https://www.ulb.ac.be/sciences/ptm/pmif/people.html>

Address: Physique Théorique et Mathématique

Université Libre de Bruxelles

Campus Plaine - CP 231

Building NO - Level 6 - Office O.6.111

1050 Brussels

Belgium

Contents

1	Introduction	3
1.1	What is the goal of the TriangulationsAndBlowupsForToricVarieties package?	3
2	Toric varieties from grading	4
2.1	Constructors	4
2.2	Toric varieties from grading: Examples	4
3	Blowups of toric varieties	6
3.1	Constructors	6
3.2	Blowups of toric varieties: Examples	6
	Index	10

Chapter 1

Introduction

1.1 What is the goal of the `TriangulationsAndBlowupsForToricVarieties` package?

TriangulationsAndBlowupsForToricVarieties provides data structures to compute triangulations and blowups of toric varieties.

Chapter 2

Toric varieties from grading

2.1 Constructors

2.1.1 ToricVarietyFromGrading (for IsList)

▷ `ToricVarietyFromGrading(a, list, of, lists, of, integers)` (operation)
Returns: a variety
Creates a toric variety from a grading of Cox ring

2.2 Toric varieties from grading: Examples

2.2.1 Example for constructing the projective 2-space from grading

Example

```
gap> g := [[1,1,1]];
[ [ 1,1,1 ] ]
gap> v1 := ToricVarietyFromGrading( g );
[ <A toric variety of dimension 2> ]
gap> CoxRing( v1[ 1 ] );
Q[x_1,x_2,x_3]
(weights: [ 1, 1, 1 ])
```

2.2.2 Example for constructing the conifold from grading

Example

```
gap> g2 := [[1,1,-1,-1]];
[ [ 1,1,-1,-1 ] ]
gap> v2 := ToricVarietyFromGrading( g2 );
[ <A toric variety of dimension 3>, <A toric variety of dimension 3> ]
gap> CoxRing( v2[ 1 ] );
Q[x_1,x_2,x_3,x_4]
(weights: [ 1, -1, -1, 1 ])
gap> Display( SRIdeal( v2[ 1 ] ) );
x_2*x_3

A (left) ideal generated by the entry of the above matrix
(graded, degree of generator: -2)
```

```
gap> Display( SRIdeal( v2[ 2 ] ) );  
x_1*x_4
```

A (left) ideal generated by the entry of the above matrix

(graded, degree of generator: 2)

Chapter 3

Blowups of toric varieties

3.1 Constructors

3.1.1 BlowupOfToricVariety (for IsToricVariety, IsList, IsString)

▷ `BlowupOfToricVariety(a, toric, variety, a, list, and, a, string)` (operation)

Returns: a variety

The arguments are a toric variety `variety`, a string `s` which specifies the locus to be blown up and a string which specifies how to name the new blowup coordinate. Based on this, this method creates the blowup of a toric variety. This process rests on 3.3.17 in CLS and performs a star sub-division of the fan.

3.1.2 SequenceOfBlowupsOfToricVariety (for IsToricVariety, IsList)

▷ `SequenceOfBlowupsOfToricVariety(a, toric, variety, and, a, list)` (operation)

Returns: a variety

The arguments are a toric variety `variety` and a list of lists. Each entry of this list must contain the information for one blowup, i.e. be made up of the two lists used as input for the method `BlowupOfToricVariety`. This method then performs this sequence of blowups and returns the corresponding toric variety.

3.2 Blowups of toric varieties: Examples

3.2.1 Blowups of C3

Example

```
gap> rays := [ [1,0,0], [0,1,0], [0,0,1] ];
[ [1,0,0], [0,1,0], [0,0,1] ]
gap> max_cones := [ [1,2,3] ];
[ [1,2,3] ]
gap> fan := Fan( rays, max_cones );
<A fan in |R^3>
gap> C3 := ToricVariety( rays, max_cones, [[0],[0],[0]], "x1,x2,x3" );
<A toric variety of dimension 3>
gap> B1C3 := BlowupOfToricVariety( C3, "x1,x2,x3", "u0" );
<A toric variety of dimension 3>
gap> [ IsComplete( B1C3 ), IsSimplicial( B1C3 ), IsSmooth( B1C3 ) ];
```

```

[ false, true, true ]
gap> B2C3 := BlowupOfToricVariety( B1C3, "x1,u0", "u1" );
<A toric variety of dimension 3>
gap> Rank( ClassGroup( B2C3 ) );
3
gap> B3C3 := BlowupOfToricVariety( B2C3, "x1,u1", "u2" );
<A toric variety of dimension 3>
gap> CoxRing( B3C3 );
Q[x3,x2,x1,u0,u1,u2]
(weights: [ ( 0, 1, 0, 0 ), ( 0, 1, 0, 0 ), ( 0, 1, 1, 1 ),
( 0, -1, 1, 0 ), ( 0, 0, -1, 1 ), ( 0, 0, 0, -1 ) ])

```

3.2.2 Blowups of P3

Example

```

gap> rays := [ [1,0,0], [0,1,0], [0,0,1], [-1,-1,-1] ];
[ [1,0,0], [0,1,0], [0,0,1], [-1,-1,-1] ]
gap> max_cones := [ [1,2,3], [1,2,4], [1,3,4], [2,3,4] ];
[ [1,2,3], [1,2,4], [1,3,4], [2,3,4] ]
gap> fan := Fan( rays, max_cones );
<A fan in |R^3>
gap> P3 := ToricVariety( rays, max_cones, [[1],[1],[1],[1]], "x1,x2,x3,x4" );
<A toric variety of dimension 3>
gap> B1P3 := BlowupOfToricVariety( P3, "x1,x2,x3", "u0" );
<A toric variety of dimension 3>
gap> [ IsComplete( B1P3 ), IsSimplicial( B1P3 ), IsSmooth( B1P3 ) ];
[ true, true, true ]
gap> B2P3 := BlowupOfToricVariety( B1P3, "x1,u0", "u1" );
<A toric variety of dimension 3>
gap> Rank( ClassGroup( B2C3 ) );
3
gap> B3P3 := BlowupOfToricVariety( B2P3, "x1,u1", "u2" );
<A toric variety of dimension 3>
gap> CoxRing( B3P3 );
Q[x4,x3,x2,x1,u0,u1,u2]
(weights: [ ( 1, 0, 0, 0 ), ( 1, 1, 0, 0 ), ( 1, 1, 0, 0 ),
( 1, 1, 1, 1 ), ( 0, -1, 1, 0 ), ( 0, 0, -1, 1 ), ( 0, 0, 0, -1 ) ])

```

3.2.3 Blowups of a generalized Hirzebruch 3-fold

Example

```

gap> vars := "u,s,v,t,r";
"u,s,v,t,r"
gap> rays := [ [0,0,-1], [1,0,0], [0,1,0], [-1,-1,-17], [0,0,1] ];
[ [0,0,-1], [1,0,0], [0,1,0], [-1,-1,-17], [0,0,1] ]
gap> cones := [ [1,2,3], [1,2,4], [1,3,4], [2,3,5], [2,4,5], [3,4,5] ];
[ [1,2,3], [1,2,4], [1,3,4], [2,3,5], [2,4,5], [3,4,5] ]
gap> weights := [ [1,-17], [0,1], [0,1], [0,1], [1,0] ];
[ [1,-17], [0,1], [0,1], [0,1], [1,0] ]
gap> H3fold := ToricVariety( rays, cones, weights, vars );
<A toric variety of dimension 3>
gap> B1H3fold := BlowupOfToricVariety( H3fold, "u,s", "u1" );
<A toric variety of dimension 3>

```



```

gap> b15 := BlowupOfToricVariety( b14, "u4,v", "d" );
<A toric variety of dimension 5>
gap> final_space := BlowupOfToricVariety( b15, "u3,u5", "u15" );
<A toric variety of dimension 5>

```

3.2.5 Sequences of blowups

Example

```

gap> final_space2 := SequenceOfBlowupsOfToricVariety( base,
> [ ["x,y,u","u1"],
> ["x,y,u1","u2"],
> ["y,u1","u3"],
> ["y,u2","u4"],
> ["u2,u3","u5"],
> ["u1,u3","u6"],
> ["u2,u4","u7"],
> ["u3,u4","u8"],
> ["u4,u5","u9"],
> ["u5,u8","u10"],
> ["u4,u8","u11"],
> ["u4,u9","u12"],
> ["u8,u9","u13"],
> ["u9,u11","u14"],
> ["u4,v","d"],
> ["u3,u5","u15"] ] );
<A toric variety of dimension 5>
gap> [ IsComplete( final_space2 ), IsSimplicial( final_space2 ), IsSmooth( final_space2 ) ];
[ true, true, false ]

```

Index

BlowupOfToricVariety
for IsToricVariety, IsList, IsString, [6](#)

SequenceOfBlowupsOfToricVariety
for IsToricVariety, IsList, [6](#)

ToricVarietyFromGrading
for IsList, [4](#)