Sistemas de ecuaciones lineales

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

a_{ij} : coeficientes

con $1 \le i \le m$; $1 \le j \le n$

b_i : términos constantes

x_j : incógnitas o variables

Matrices y sistemas de ecuaciones lineales

- Matriz: arreglo rectangular de números reales

$$\begin{cases} x+2y-z=4 \\ 2x+5y+2z=9 \\ x+4y+7z=6 \end{cases}$$

$$\boldsymbol{A}=\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{2} & \mathbf{5} & \mathbf{2} \\ \mathbf{1} & \mathbf{4} & \mathbf{7} \end{pmatrix}$$

$$\boldsymbol{A'}=\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-1} & \mathbf{4} \\ \mathbf{2} & \mathbf{5} & \mathbf{2} & \mathbf{9} \\ \mathbf{1} & \mathbf{4} & \mathbf{7} & \mathbf{6} \end{pmatrix}$$

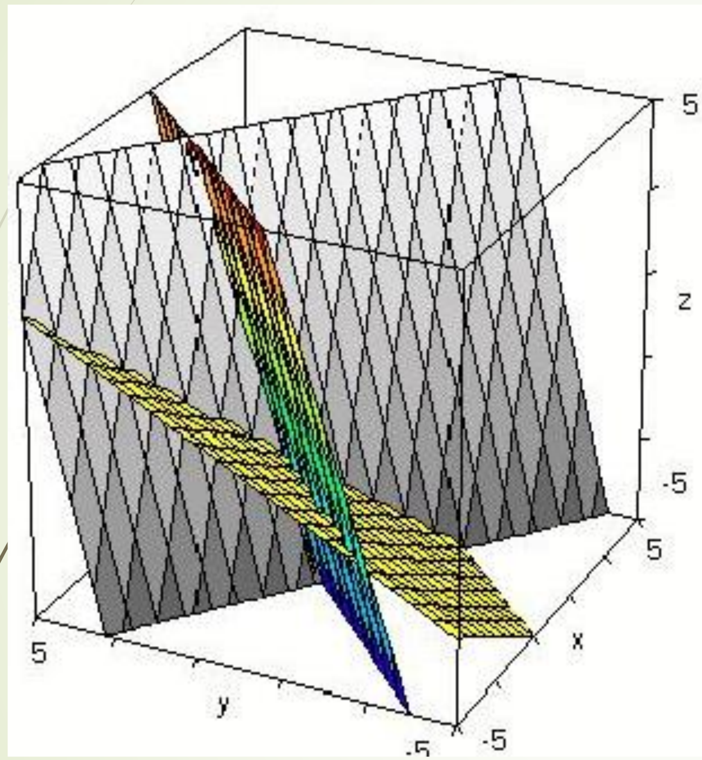
- Matriz de coeficientes (A): matriz que tiene por elementos los coeficientes de las incógnitas del sistema de ecuaciones lineales

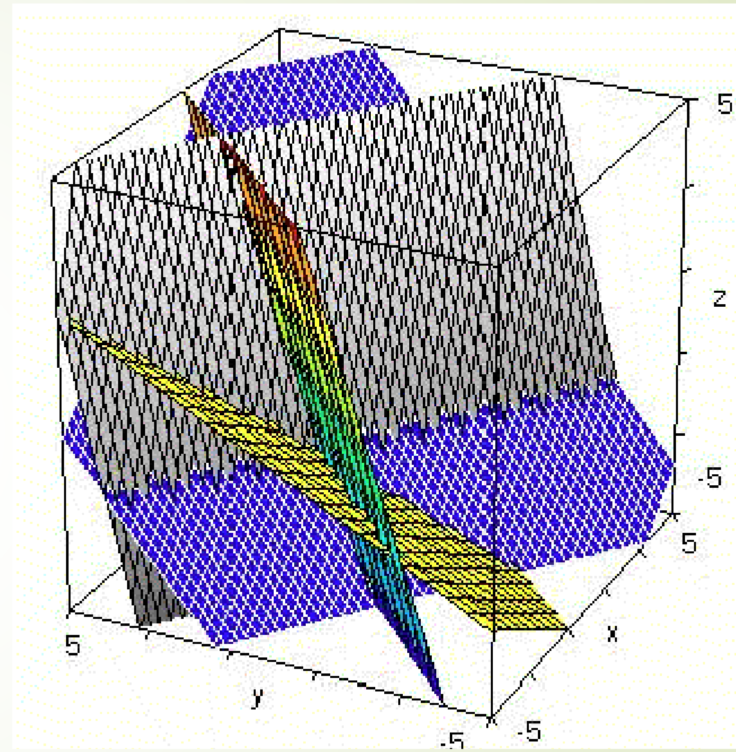
- Matriz aumentada o ampliada (A´): es la matriz de los coeficientes ampliada con la columna de términos independientes.

Operaciones elementales con renglones

- Multiplicar (o dividir) un renglón por un número distinto de cero.
- Sumar un múltiplo no nulo de un renglón a otro renglón.
- Intercambiar dos renglones.

Sistemas equivalentes





$$\begin{cases} 3x_1 + 4x_2 - x_3 = 3 \\ 3x_1 - 3x_2 + x_3 = -8 \\ x_1 - x_2 + 2x_3 = -6 \end{cases} \qquad \begin{cases} 3x_1 + 4x_2 - x_3 = 3 \\ 3x_1 - 3x_2 + x_3 = -8 \\ x_1 - x_2 + 2x_3 = -6 \\ 6x_1 + 8x_2 - 7x_3 = 16 \end{cases}$$

Se llaman sistemas equivalentes a aquellos que presentan el mismo conjunto solución.

$x_1 = 2; x_2 = 1; x_3 = -3$

Ejemplo 1

$$\begin{cases} x + 2y + 3z = 4 \\ 3x - y + z = 1 \\ x + 3y + 2z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & -1 & 1 & 1 \\ 1 & 3 & 2 & 0 \end{array}\right) \qquad \begin{array}{l} F_2 \to F_2 + F_1 \cdot (-3) \\ F_3 \to F_3 + F_1 \cdot (-1) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -7 & -8 & -11 \\ 0 & 1 & -1 & -4 \end{array}\right) \qquad F_2 \rightleftarrows F_3$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & -4 \\ 0 & -7 & -8 & -11 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & -4 \\ 0 & -7 & -8 & -11 \end{array}\right) \quad F_3 \to F_3 + F_2 \cdot 7$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & -15 & -39 \end{array}\right)$$

$$\begin{cases} x + 2y + 3z = 4 \\ y - z = -4 \\ -15z = -39 \end{cases} \Rightarrow z = \frac{-39}{-15} \Longrightarrow \boxed{z = \frac{13}{5}}$$

$$y - \frac{13}{5} = -4 \Rightarrow y = -4 + \frac{13}{5} \Longrightarrow \boxed{y = -\frac{7}{5}}$$

$$x + 2 \cdot \left(-\frac{7}{5}\right) + 3 \cdot \frac{13}{5} = 4 \Rightarrow x = 4 - \frac{39}{5} + \frac{14}{5}$$

$$\boxed{C_s = \left\{\left(-1; -\frac{7}{5}; \frac{13}{5}\right)\right\}} \qquad \boxed{x = -1}$$

Ejemplo 2

$$\begin{cases} x+y+z=4 \\ -x-y+z=-1 \\ x+y+z=-1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \end{array}\right)$$

$$F_2 \to F_2 + F_1$$
$$F_3 \to F_3 + F_1 \cdot (-1)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -5 \end{array}\right)$$

$$\begin{cases} x+y+z=4 \\ 2z=3 \\ 0z=-5 \end{cases}$$

Sistema Incompatible

Ejemplo 3

$$\begin{cases} 3x_1 + 4x_2 - x_3 = 3 \\ 3x_1 - 3x_2 + x_3 = -8 \\ x_1 - x_2 + 2x_3 = -6 \end{cases}$$

$$\begin{pmatrix} \mathbf{3} & \mathbf{4} & \mathbf{-1} & \mathbf{|\ 3} \\ \mathbf{3} & \mathbf{-3} & \mathbf{1} & \mathbf{|-8} \\ \mathbf{1} & \mathbf{-1} & \mathbf{2} & \mathbf{|-6} \end{pmatrix} \quad \boldsymbol{F_1 \rightleftarrows F_3}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} & \mathbf{|-6} \\ \mathbf{3} & \mathbf{-3} & \mathbf{1} & \mathbf{|-8} \\ \mathbf{3} & \mathbf{4} & \mathbf{-1} & \mathbf{|\ 3} \end{pmatrix} \quad \begin{matrix} \boldsymbol{F_2 \to F_2 + F_1 \cdot (-3)} \\ \boldsymbol{F_3 \to F_3 + F_1 \cdot (-3)} \end{matrix}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} & \mathbf{|-6} \\ \mathbf{0} & \mathbf{0} & \mathbf{-5} & \mathbf{|10} \\ \mathbf{0} & \mathbf{7} & \mathbf{-7} & \mathbf{|\ 21} \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 2 & |-6 \\ 0 & 0 & -5 & |10 \\ 0 & 7 & -7 & |\ 21 \end{pmatrix} F_2 \rightleftarrows F_3$$

$$\begin{pmatrix} 1 & -1 & 2 & |-6 \\ 0 & 7 & -7 & |21 \\ 0 & 0 & -5 & |10 \end{pmatrix}$$

$$\begin{cases} x_1 - x_2 + 2x_3 = -6 \\ x_2 - x_3 = 3 \\ \boxed{x_3 = -2} \end{cases}$$

$$\begin{cases} 3x_1 + 4x_2 - x_3 = 3 \\ 3x_1 - 3x_2 + x_3 = -8 \\ x_1 - x_2 + 2x_3 = -6 \end{cases}$$

$$x_2 - (-2) = 3 \quad \boxed{x_2 = 1}$$

$$\begin{cases} 3 \cdot (-1) + 4 \cdot 1 - (-2) = 3 \\ 3 \cdot (-1) - 3 \cdot 1 + (-2) = -8 \\ -1 - 1 + 2 \cdot (-2) = -6 \end{cases}$$

$$x_1 - 1 + 2 \cdot (-2) = -6 \quad \boxed{x_1 = -1}$$

$$C_s = \{(-1; 1; -2)\}$$

Ejemplo 4

$$\begin{cases} x - 3y + 7z = 10 \\ 5x - y + z = 8 \\ x + 4y - 10z = -11 \end{cases}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{7} & | & \mathbf{10} \\ \mathbf{5} & \mathbf{-1} & \mathbf{1} & | & \mathbf{8} \\ \mathbf{1} & \mathbf{4} & \mathbf{-10} & | & \mathbf{-11} \end{pmatrix} \quad \begin{array}{l} \boldsymbol{F_2 \to F_2 + F_1 \cdot (-5)} \\ \boldsymbol{F_3 \to F_3 + F_1 \cdot (-1)} \end{array}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{7} & | & \mathbf{10} \\ \mathbf{0} & \mathbf{14} & \mathbf{-34} & | & \mathbf{-42} \\ \mathbf{0} & \mathbf{7} & \mathbf{-17} & | & \mathbf{-21} \end{pmatrix} \quad \boldsymbol{F_2 \to F_2 \cdot \left(\frac{1}{14}\right)}$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{7} & | & \mathbf{10} \\ \mathbf{0} & \mathbf{1} & -\frac{\mathbf{17}}{\mathbf{7}} & | & \mathbf{-3} \\ \mathbf{0} & \mathbf{7} & \mathbf{-17} & | & \mathbf{-21} \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 7 & 10 \\ 0 & 1 & -\frac{17}{7} & -3 \\ 0 & 7 & -17 & -21 \end{array}\right) \quad F_3 \to F_3 + F_2 \cdot (-7)$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 7 & 10 \\ 0 & 1 & -\frac{17}{7} & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x - 3y + 7z = 10 \\ \quad y - \frac{17}{7}z = -3 \end{cases} \qquad y = -3 + \frac{17}{7}z$$

$$x - 3 \cdot \left(-3 + \frac{17}{7}z\right) + 7z = 10$$

$$x + 9 - \frac{51}{7}z + 7z = 10$$

$$x = 1 + \frac{2}{7}z$$

$$\boldsymbol{x = 1 + \frac{2}{7}z} \qquad \boldsymbol{y = -3 + \frac{17}{7}z}$$

$$\boldsymbol{z = k} \qquad \boldsymbol{x = 1 + \frac{2}{7}k} \qquad \boldsymbol{y = -3 + \frac{17}{7}k}$$

$$\boldsymbol{C_s = \left\{\left(1 + \frac{2}{7}k; -3 + \frac{17}{7}k; k\right) / \, k \in \mathbb{R}\right\}}$$

$$\boldsymbol{k = 7} \qquad \boldsymbol{x = 3} \qquad \boldsymbol{y = 14} \qquad \boldsymbol{z = 7}$$

$$\begin{cases} x - 3y + 7z = 10 \\ 5x - y + z = 8 \\ x + 4y - 10z = -11 \end{cases} \qquad \begin{cases} \mathbf{3} - 3 \cdot \mathbf{14} + 7 \cdot \mathbf{7} = 10 \\ 5 \cdot \mathbf{3} - \mathbf{14} + \mathbf{7} = 8 \\ \mathbf{3} + 4 \cdot \mathbf{14} - 10 \cdot \mathbf{7} = -11 \end{cases}$$