Teoría de Conjuntos

Propiedades de las operaciones con conjuntos

Álgebra de Conjuntos
Propiedades más utilizadas

Idempotencia	
$A \cup A = A$	$A \cap A = A$
Conmutatividad	
$A \cup B = B \cup A$	$A \cap B = B \cap A$
Asociatividad	
$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributividad	
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Elemento Neutro	
$A \cup \varnothing = A$	$A \cap U = A$
Elemento Absorbente	
$A \cup U = U$	$A \cap \varnothing = \varnothing$
Absorción	
$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Involución	
$\overline{\overline{A}} = A$	
Leyes de De Morgan	
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
$A - (B \cup C) = (A - B) \cap (A - C)$	$A - (B \cap C) = (A - B) \cup (A - C)$

Propiedades de la Unión

- **Teorema 1**

  Dado los conjuntos A, B, C, D, y $\emptyset$, en un conjunto universal U, se cumplen las siguientes propiedades:

a) $A \subset (A \cup B) \wedge B \subset (A \cup B)$

b) $A \subset D \wedge B \subset D \Rightarrow (A \cup B) \subset D$

c) $A \cup A = A$ (Idempotencia)

d) $(A \cup B) \cup C = A \cup (B \cup C)$ (Asociativa)

e) $A \cup B = B \cup A$ (Conmutativa)

f) $A \cup \emptyset = A$ (Elemento Neutro)

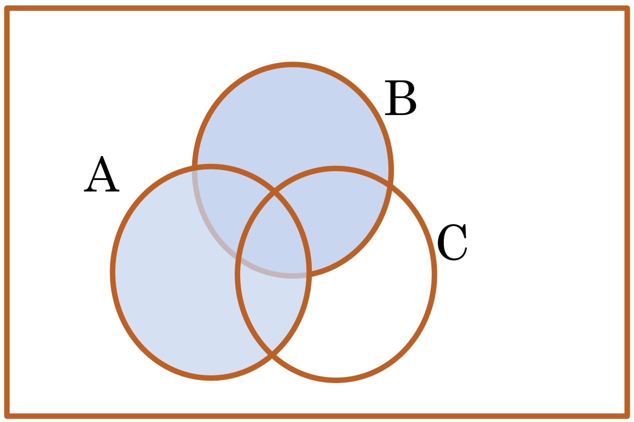
g) $A \subset B \Rightarrow A \cup B = B$

h) $A \cup U = U$ (Elemento Absorbente)

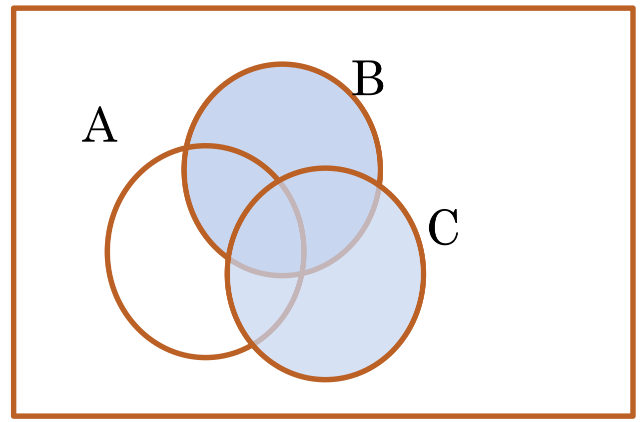
Gráficamente

- Asociativa

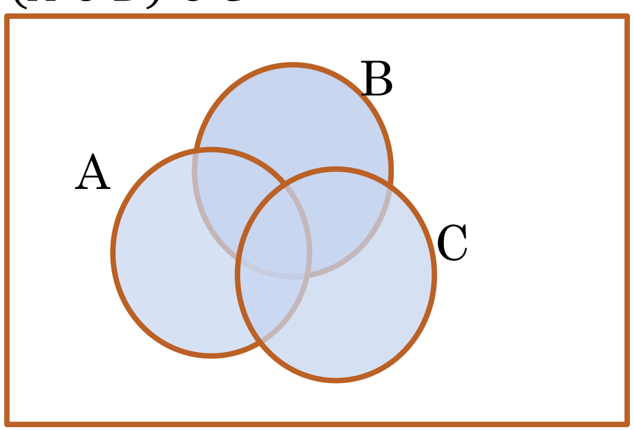
$A \cup B$



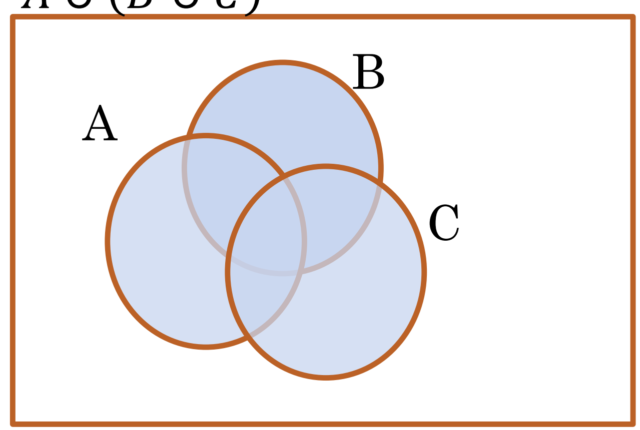
$B \cup C$



$(A \cup B) \cup C$

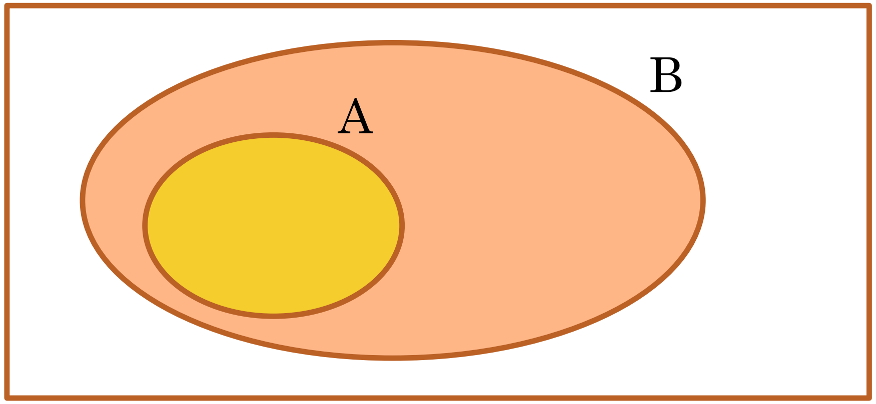


$A \cup (B \cup C)$

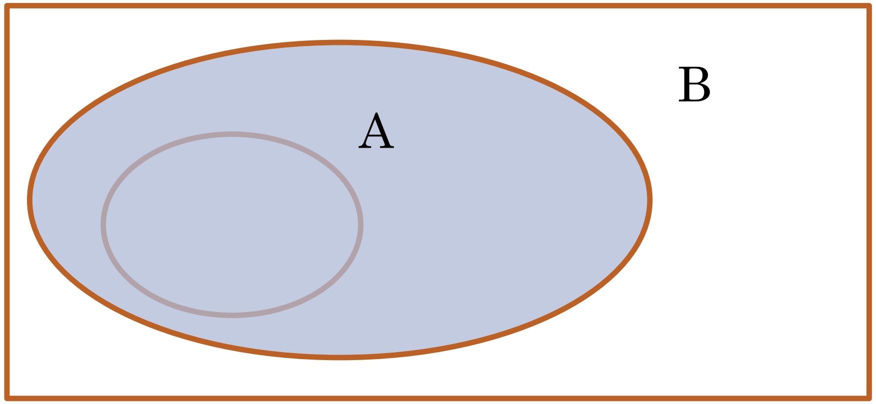


Gráficamente

- $A \subset B \Rightarrow A \cup B = B$



$A \subset B$



$A \cup B = \mathrm{B}$

PROPIEDADES DE LA INTERSECCIÓN

- **Teorema 2**

Dado los conjuntos A, B, C y $\emptyset$, en el conjunto universal U, se cumplen las siguientes propiedades:

a) $(A \cap B) \subset A \wedge (A \cap B) \subset B$

b) $A \cap A = A$ (Idempotencia)

c) $(A \cap B) \cap C = A \cap (B \cap C)$ (Asociativa)

d) $A \cap B = B \cap A$ (Conmutativa)

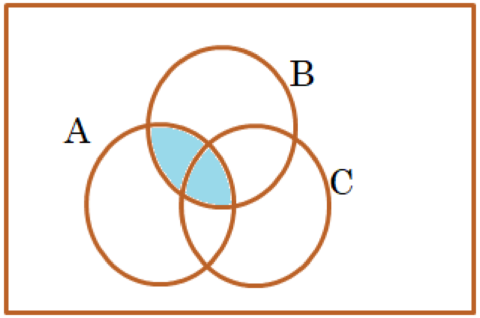
e) $A \subset B \Rightarrow A \cap B = A$

f) $A \cap \emptyset = \emptyset$ (Elemento Absorbente)

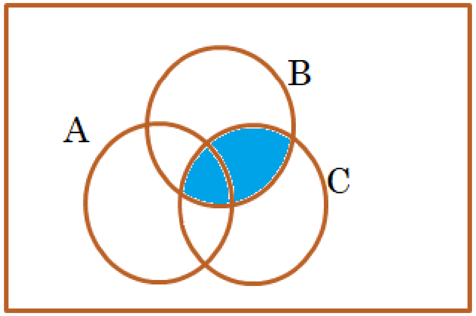
g) $A \cap U = A$ (Elemento Neutro)

Gráficamente

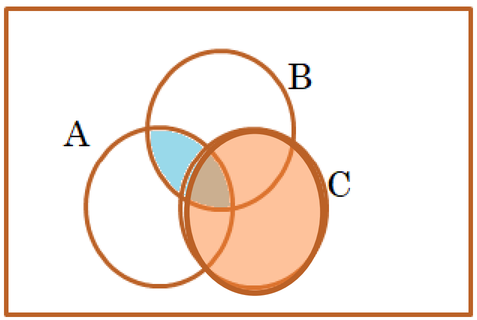
- Asociativa



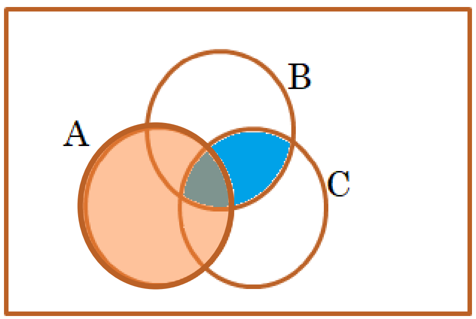
$A \cap B$



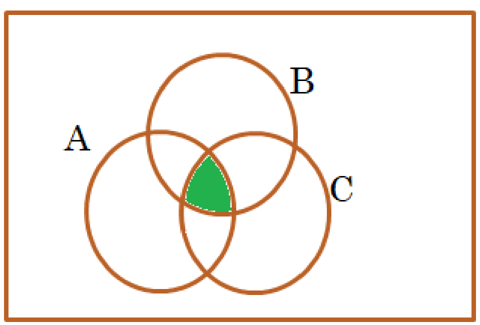
$B \cap C$



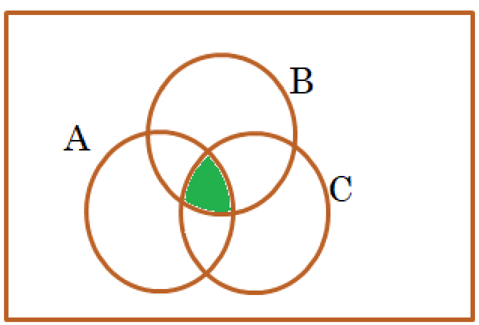
Pintamos C



Pintamos A



$(A \cap B) \cap C$



$A \cap (B \cap C)$

Propiedades distributivas

- **Teorema 3**

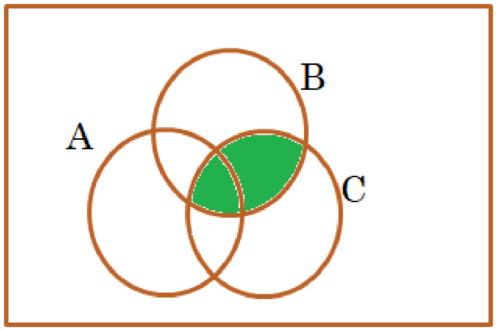
  Dado los conjuntos A, B y C se cumplen las siguientes propiedades distributivas:

  a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

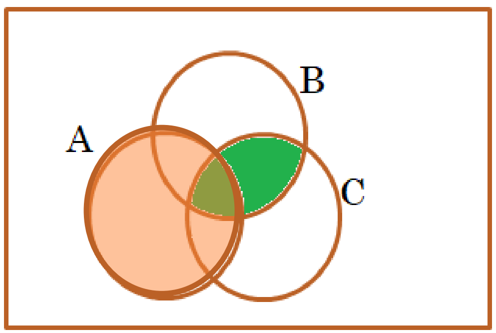
  b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

GRÁFICAMENTE

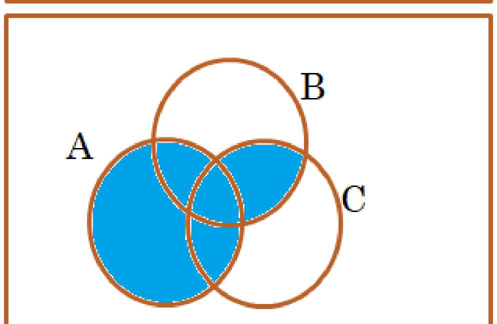
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



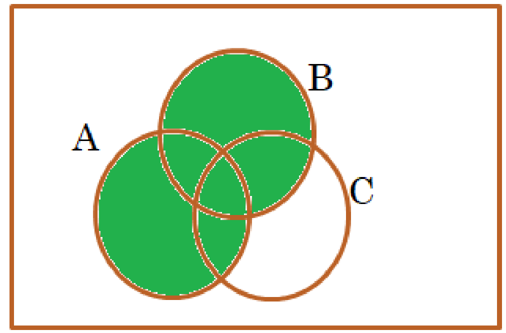
$B \cap C$



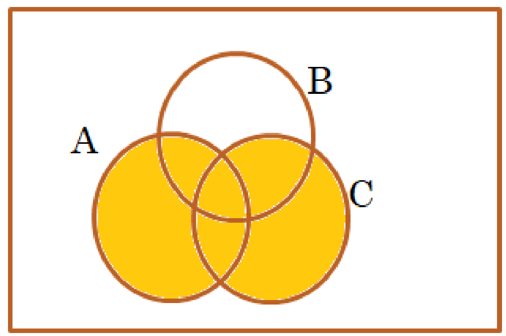
Pintamos A



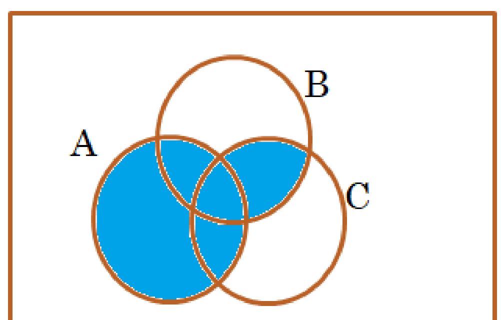
$A \cup (B \cap C)$



$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$

Propiedades de la diferencia de conjuntos

- **Teorema 4**

  Dado los conjuntos A, B, C, y $\emptyset$, se cumplen las siguientes propiedades:

a) $A - \emptyset = A$

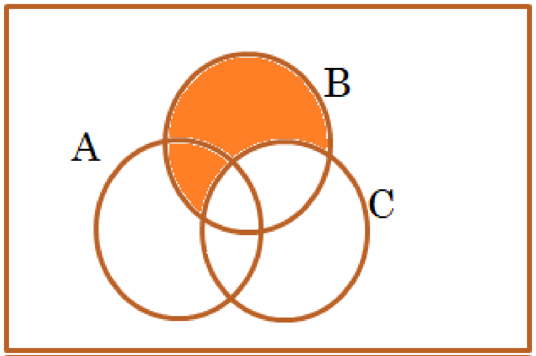
b) $A - A = \emptyset$

c) $A \cap (B - C) = (A \cap B) - (A \cap C)$

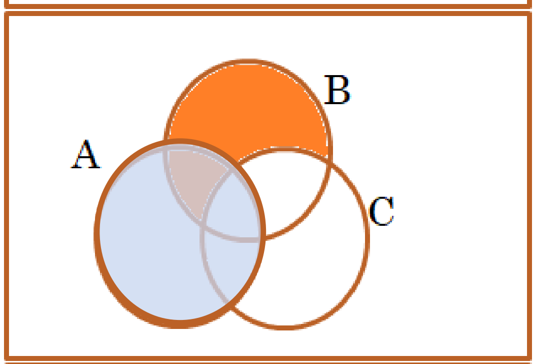
d) $A - B = (A \cup B) - B = A - (A \cap B) = A \cap \overline{\mathrm{B}}$

Gráficamente

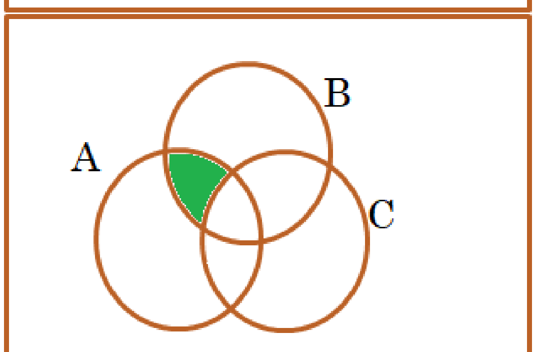
- $A \cap (B - C) = (A \cap B) - (A \cap C)$



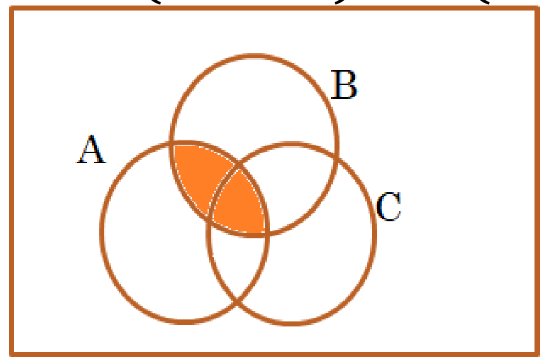
$B - C$



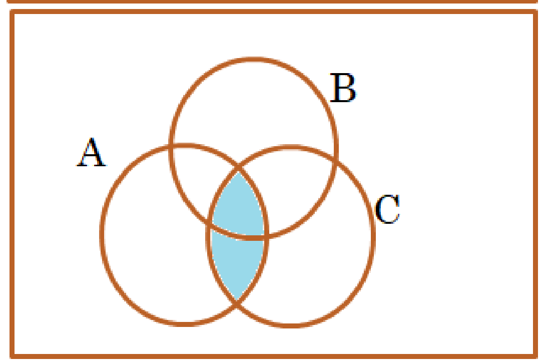
Pintamos A



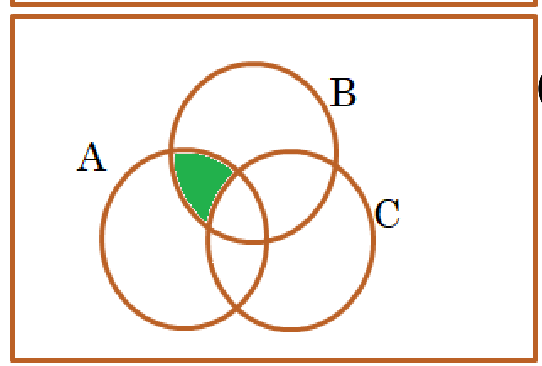
$A \cap (B - C)$



$A \cap B$



$A \cap C$



$(A \cap B) - (A \cap C)$

Propiedades del complemento

- **Teorema 5.**

  Dado los conjuntos $\emptyset$, $A \subset U$ y $B \subset U$, se cumplen las siguientes propiedades:

a) $\bar{\bar{A}} = A$

b) $A \subset B \Rightarrow \bar{B} \subset \bar{A}$;
$\bar{B} \subset \bar{A} \Rightarrow A \subset B$

c) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

d) $\overline{A \cup B} = \bar{A} \cap \bar{B}$

e) $A \cap \bar{A} = \emptyset$

f) $\mathrm{A} \cup \bar{A} = U$

g) $\bar{\emptyset} = U$ y $\bar{U} = \emptyset$.

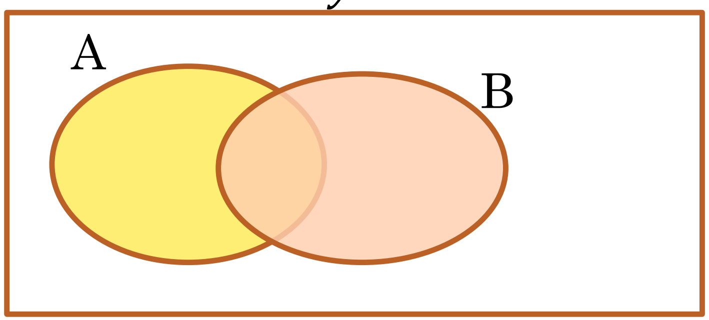
Gráficamente

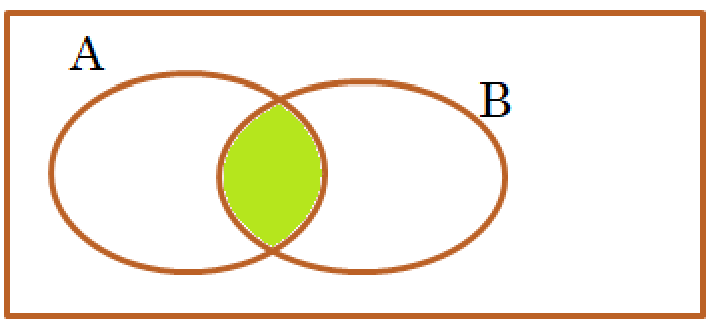
- Ley de De Morgan: El complemento de la intersección entre dos conjuntos es igual a la unión de los complementos de dichos conjuntos.

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

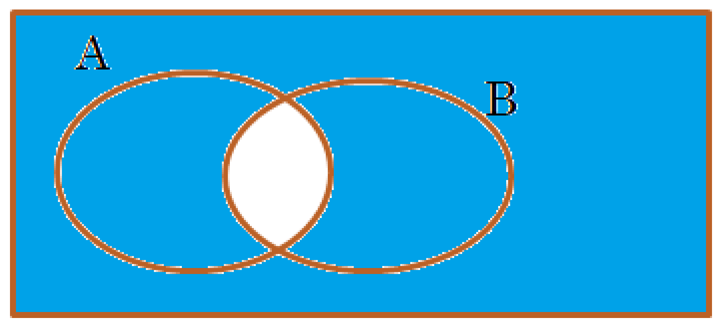
Comenzamos por trabajar gráficamente el primer miembro de la igualdad

$A \;\; y \;\; B$





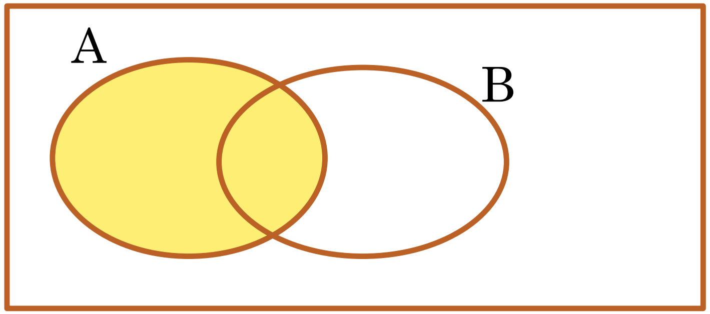
$A \cap B$



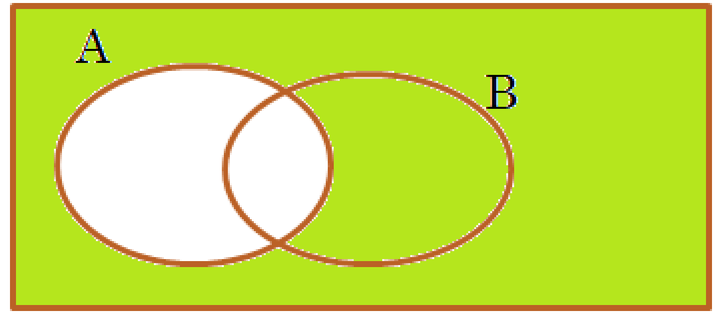
$\overline{A \cap B}$

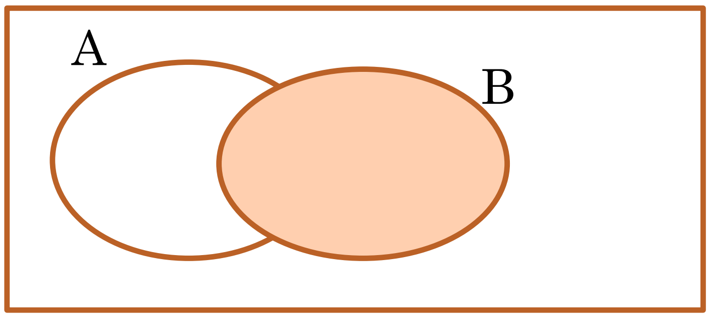
Trabajamos el segundo miembro de la igualdad

A

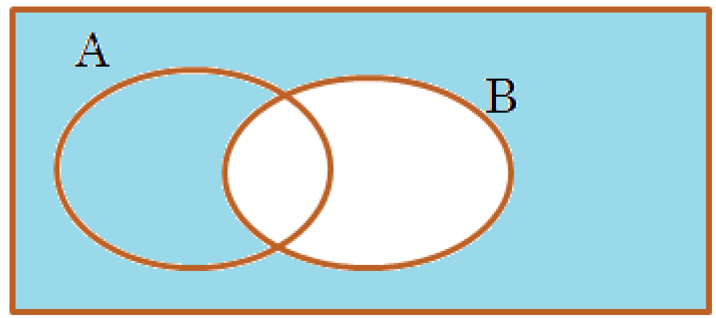


$\bar{A}$

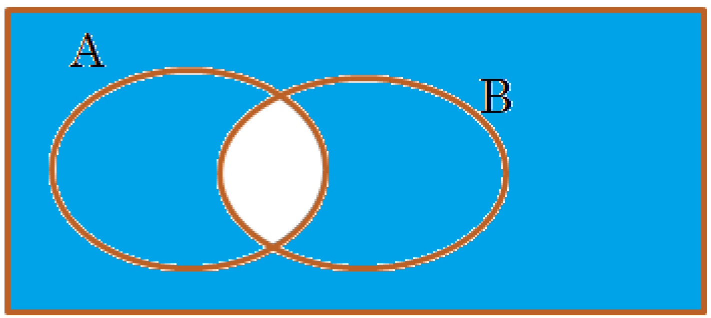




B



$\bar{B}$



$\bar{A} \cup \bar{B}$