

- (1) Digit-Number definition with base  $b$ :  
 $m = d_n d_{n-1} \dots d_1 d_0 . d_{-1} \dots d_{-m}$

- (2) Converting from base  $b$  to base 10:  
 $N_{10} = b^{-m} \cdot \sum_{k=0}^{n+m} d_{k-m} \cdot b^k$

- (3) Converting from base 10 to  $b$ :  
 $x = \text{int}(x) + \text{dec}(x)$   
 $\text{int}(x) \uparrow, \text{dec}(x) \downarrow$

- (4) Representation of binary number:  
 $x = (-1)^s \cdot m \cdot b^e$ , where:  $e \in [L, U]$   
 $m = 0.c_1 c_2 \dots c_t$ ,  $b$  = baza  
 $t$  = dolžine mantise,  $c_i = i$ -ta številka  
 $P(b, t, L, U)$  = representation set  
osnovna zaok. napaka:  $u = \frac{b^{L-t}}{2}$

- (5) IEEE Rounding for  $b = 2$ :

$c_{(t+1)}$	$c_{(t+2)}$	Round Up
0	0	Never
0	1	Never
1	0	if $c_t = 1$
1	1	Always

- (6) IEEE standards,  $\circ \in \{+, -, \cdot, /\}$ :  
 $fl(\cdot)$  - floating point evaluation  
 $fl(x \circ y) = (x \circ y)(1 + \delta)$ ,  $|\delta| < |u|$   
 $fl(\sqrt{x}) = \sqrt{x}(1 + \delta)$ ,  $|\delta| < |u|$

- (7) Absolute & Relative Error:

$$\Delta x = |x - \hat{x}|, \quad \delta = \frac{|x - \hat{x}|}{|x|}$$

- (8) Errors types:

$$D_n = |y - \bar{y}|, D_m = |\bar{y} - \tilde{y}|, D_z = |\tilde{y} - \hat{y}|$$

- (9) Stability problems:

- Odštevanja števil enake velikost
- Deljenje s števili blizu 0

- (10) Relative-Stability definitions:

Direktna:  $|\delta| = \left| \frac{fl(x) - x}{x} \right| \leq C \cdot u \subseteq \mathcal{O}(u)$   
Obratna:  $fl(f(x)) = f(\tilde{x})$ ,  $|\tilde{x} - x| \leq \mathcal{O}(u)|x|$   
Pri obratni stabilnosti, preverimo ali lahko vsako napako interpretiramo kot perturbacijo vhoda.

- (1) **Fixed point iteration** :

$f(x) = 0$ , use  $g(x_r) = x_{r+1}$  or  $g(x) = x + f(x)$   
 $\alpha_i$  je  $i$ -ta ničla funkcije  $f$   
Privlačne točke:  $|g'(x)| < 1$   
Odbojne točke:  $|g'(x)| > 1$   
Neznana situacija:  $|g'(x)| = 1$   
Error es. :  $|x_{r+1} - \alpha| \leq \frac{m}{1-m} |g(x_r) - g(x_{r-1})|$   
 $m = \sup_{x \in I} |g'(x)| < 1$ ,  $g$  je skrčitev,  $\alpha \in I$   
Red konvergence je prvi odvod:  $g^{(n)}(\alpha) \neq 0$   
Konvergenca:

- $x_{r+1} - x_r = f(x_r) > 0$ ,  $\forall r : x_r < \alpha$
- $x_{r+1} - x_r = f(x_r) < 0$ ,  $\forall r : x_r > \alpha$

- (2) **Newton-Rapson Method**:

$$g(x_r) = x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$g'(x_r) = \frac{f(x_r) f''(x_r)}{(f'(x_r))^2}$$

Red Konvergence:

Ničla =  $\alpha$ , kratnost ničle =  $m_\alpha$

- $m_\alpha = 1 \implies$  vsaj kvadratnična
- $m_\alpha > 1 \implies \lim_{x \rightarrow \alpha} g'(x) = 1 - \frac{1}{m}$

Konvergenca:

- $x_{r+1} - x_r = \frac{f(x_r)}{f'(x_r)} > 0$ ,  $\forall r : x_r < \alpha$
- $x_{r+1} - x_r = \frac{f(x_r)}{f'(x_r)} < 0$ ,  $\forall r : x_r > \alpha$

- (3) **Modified Newton-Rapson Method**:

$$G(x_r) = x_{r+1} = a \cdot x_r - b \frac{f(x_r)}{f'(x_r)}$$

$$G'(x_r) = a - b \frac{f''(x_r)}{f'(x_r)}$$

vsaj kvadratnična konv.  $a = 1$  in  $b = m_\alpha$

$\lim_{x \rightarrow \alpha} G' = 0$ , ( $a = 1$ ,  $b = m_\alpha$ )

**Izrek o konvergenca** : Naj bo  $f$  na intervalu  $I = [a, \infty)$  dvakrat zvezno odvedljiva, naračajoča in konveksna funkcija, ki ima ničlo  $\alpha \in I$ . Potem je  $\alpha$  edina ničla  $f$  na  $I$  in za vsak  $x_0 \in I$  tangetna metoda konvergira k  $\alpha$ .

- (1) **LU-brez-pivotiranja**:

$$A = LU, Ax = b, LUx = b, Ly = b, Ux = y$$

$$\text{Big-O: } \frac{2n^3}{3} + \mathcal{O}(n^2)$$

- (2) **LU-delno-pivotiranje**:

$$PLUx = Pb, \det(A) = \det(U)$$

$$P = I \cdot \prod_{i=1}^{n-1} P_i, \prod_{i=1}^{n-1} L_i P_i \cdot A = U$$