

- (1) Digit-Number definition with base b :

$$m = d_n d_{n-1} \dots d_1 d_0 . d_{-1} \dots d_{-m}$$

- (2) Converting from base b to base 10:

$$N_{10} = b^{-m} \cdot \sum_{k=0}^{n+m} d_{k-m} \cdot b^k$$

- (3) Converting from base 10 to b :

$$x = \text{int}(x) + \text{dec}(x)$$

$$\text{int}(x) \uparrow, \quad \text{dec}(x) \downarrow$$

- (4) Representation of binary number:

$$x = (-1)^s \cdot m \cdot b^e, \quad \text{where: } e \in [L, U]$$

$$m = 0.c_1 c_2 \dots c_t, \quad b = \text{baza}$$

$$t = \text{dolžine mantise}, \quad c_i = i\text{-ta številka}$$

$$P(b, t, L, U) = \text{representation set}$$

$$\text{osnovna zaok. napaka: } u = \frac{b^{1-t}}{2}$$

- (5) IEEE Rounding for $b = 2$:

$c_{(t+1)}$	$c_{(t+2)}$	Round Up
0	0	Never
0	1	Never
1	0	if $c_t = 1$
1	1	Always

- (6) IEEE standards, $\circ \in \{+, -, \cdot, /\}$:

$fl(\cdot)$ - floating point evaluation

$$fl(x \circ y) = (x \circ y)(1 + \delta), \quad |\delta| < |u|$$

$$fl(\sqrt{x}) = \sqrt{x}(1 + \delta), \quad |\delta| < |u|$$

- (7) Absolute & Relative Error:

$$\Delta x = |x - \hat{x}|, \quad \delta = \frac{|x - \hat{x}|}{|x|}$$

- (8) Errors types:

$$D_n = |y - \bar{y}|, \quad D_m = |\bar{y} - \tilde{y}|, \quad D_z = |\tilde{y} - \hat{y}|$$

- (9) Stability problems:

- Odštevanja števil enake velikost
- Deljenje s števili blizu 0

- (10) Relative-Stability definitions:

$$\text{Direktna: } |\delta| = \left| \frac{fl(x) - x}{x} \right| \leq C \cdot u \subseteq \mathcal{O}(u)$$

$$\text{Obratna: } fl(f(x)) = f(\tilde{x}), \quad |\tilde{x} - x| \leq \mathcal{O}(u)|x|$$

Pri obratni stabilnosti, preverimo ali

lahko vsako napako interpretiramo

kot perturbacijo vhoda.

(1) **Fixed point iteration :**

$f(x) = 0$, use $g(x_r) = x_{r+1}$ or $g(x) = x + f(x)$

α_i je i -ta ničla funkcije f

Privlačne točke: $|g'(x)| < 1$

Odbojne točke: $|g'(x)| > 1$

Neznana situacija: $|g'(x)| = 1$

Error es. : $|x_{r+1} - \alpha| \leq \frac{m}{1-m} |g(x_r) - g(x_{r-1})|$

$m = \sup_{x \in I} g'(x) < 1$, g je skrčitev, $\alpha \in I$

Red konvergence je prvi odvod: $g^{(n)}(\alpha) \neq 0$

Konvergenca:

- $x_{r+1} - x_r = f(x_r) > 0, \quad \forall r : x_r < \alpha$

- $x_{r+1} - x_r = f(x_r) < 0, \quad \forall r : x_r > \alpha$

(2) **Newton-Rapson Method:**

$$g(x_r) = x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$g'(x_r) = \frac{f(x) \cdot f''(x_r)}{(f'(x_r))^2}$$

Red Konvergence:

Ničla = α , kratnost ničle = m_α

- $m_\alpha = 1 \implies$ vsaj kvadratnična

- $m_\alpha > 1 \implies \lim_{x \rightarrow \alpha} g'(x) = 1 - \frac{1}{m}$

Konvergenca:

- $x_{r+1} - x_r = \frac{f(x_r)}{f'(x_r)} > 0, \quad \forall r : x_r < \alpha$

- $x_{r+1} - x_r = \frac{f(x_r)}{f'(x_r)} < 0, \quad \forall r : x_r > \alpha$

(3) **Modified Newton-Rapson Method:**

$$G(x_r) = x_{r+1} = a \cdot x_r - b \frac{f(x_r)}{f'(x_r)}$$

$$G'(x_r) = a - b + b g'(x_r)$$

vsaj kvadratnična konv. $a = 1$ in $b = m_\alpha$

$$\lim_{x \rightarrow \alpha} G' = 0, (a = 1, b = m_\alpha)$$

Izrek o konvergenči : Naj bo f na intervalu

$I = [a, \infty)$ dvakrat zvezno odvedljiva,

naračajoča in konveksna funkcija, ki ima ničlo

$\alpha \in I$. Potem je α edina ničla f na I in za

vsak $x_o \in I$ tangetna metoda konvergira k α .

(1) **LU-brez-pivotiranja:**

$$A = LU, Ax = b, LUx = b, Ly = b, Ux = y$$

$$\text{Big-O: } \frac{2n^3}{3} + \mathcal{O}(n^2)$$

(2) **LU-delno-pivotiranje:**

$$PLUx = Pb, \det(A) = \det(U)$$

$$P = I \cdot \prod_{i=1}^{n-1} P_i, \prod_{i=1}^{n-1} L_i P_i \cdot A = U$$