Model-independent determination of the CKM angle γ in $B^{\pm} \to (K^+K^-\pi^+\pi^-)_D h^{\pm}$ decays

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Outline

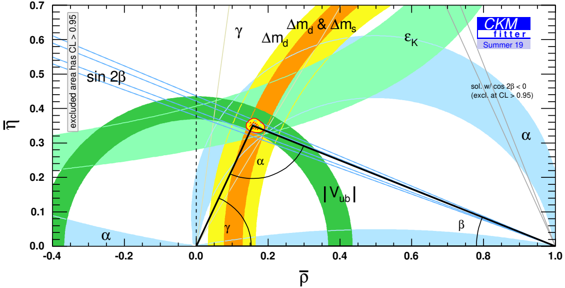
Introduction to the CKM angle γ

γ and the unitary triangle

- CP violation in SM is described by the Unitary Triangle

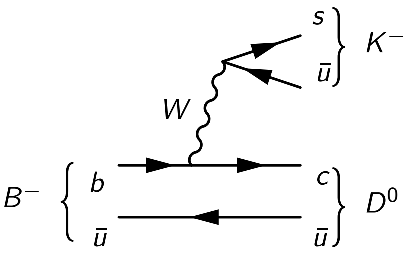
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- Only CKM angle accessible at tree level $\implies$
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark
  - Compare with indirect measurements

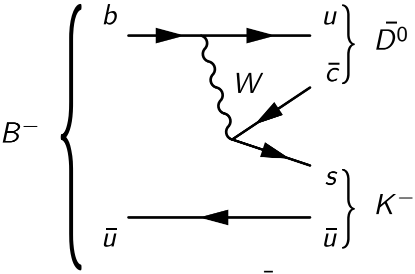


CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

Sensitivity through interference



(a) $B^- \to D^0 K^-$



(b) $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

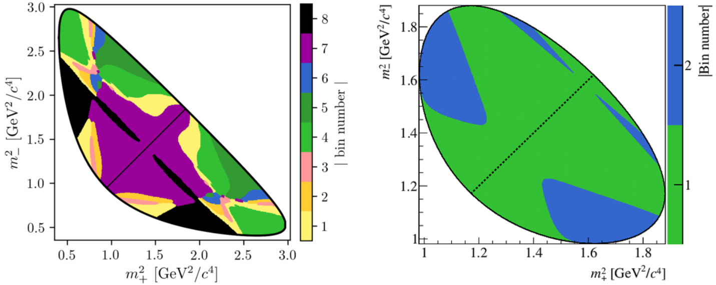
Measurement of γ from $B^\pm \to DK^\pm$, $D \to K^+K^-\pi^+\pi^-$

- First proposed by J. Rademacker and G. Wilkinson
  - arXiv:hep-ph/0611272
  - Amplitude model by FOCUS
  - Expected γ precision from amplitude fit with 1000 candidates: 14°
- CLEO amplitude analysis
  - arXiv:1201.5716
  - Expected γ precision from amplitude fit with 2000 candidates: 11°
- State of the art amplitude analysis by LHCb:
  - LHCb-PAPER-2018-041
  - Use to develop efficient binning scheme

Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

Binned measurement of γ

- Final measurement will be model-independent
  - Poor binning reduces statistical sensitivity $\rightarrow$ No bias!
- Need strong phases of D decay $\rightarrow$ Measure at BESIII
- LHCb-PAPER-2020-019: $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K_S^0 h^+ h^-$
  - Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^{\circ}$



The BPGGSZ method

- $B^\pm \to Dh^\pm$ amplitude:

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)}\mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)}\mathcal{A}(D^0)$$

- $\mathcal{A}(D^0)$ and $\mathcal{A}(\bar{D}^0)$ depend on D phase space
- Strong-phase difference of D^0 and $\bar{D}^0$ decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

The BPGGSZ method

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude averaged strong phases can be obtained from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

Binning scheme

Binning scheme requirements

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^{\pm} \to D^0 h^{\pm}$ and $B^{\pm} \to \bar{D}^0 h^{\pm}$

How to bin a 5-dimensional phase space?

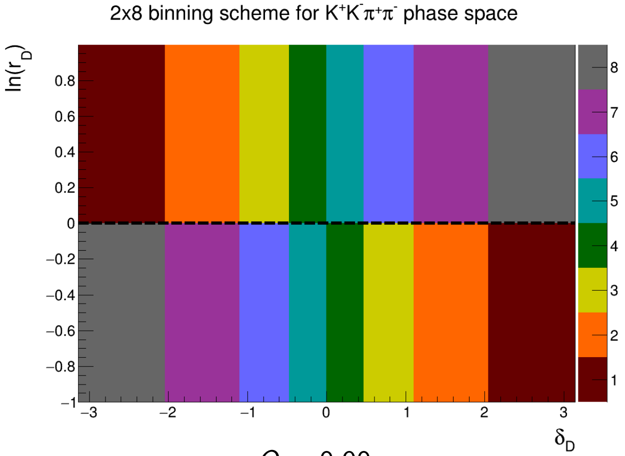
- Generate C++ code for LHCb amplitude model using AmpGen[1]
- For each $B^{\pm}$ candidate, calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along δ_D and r_D, maximize Q-value to optimize

[1] AmpGen by Tim Evans

Binning scheme

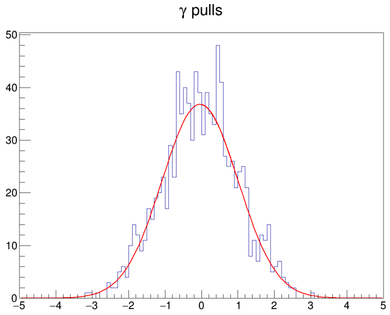


$Q = 0.90$

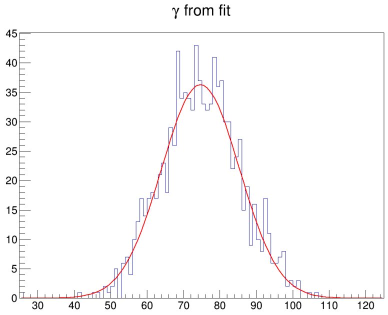
Bins $i < 0$ on top, $i > 0$ below

γ precision benchmark

- Generate 2000 $B^\pm \to DK^\pm$ candidates using LHCb model in AmpGen
- Fit back with same model using AmpGen



(a) Pull of γ

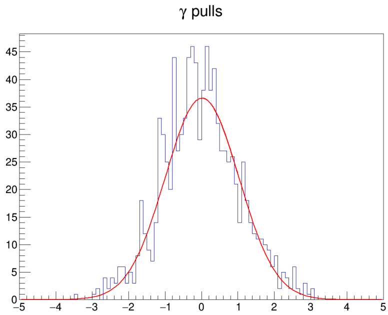


(b) Fitted γ values

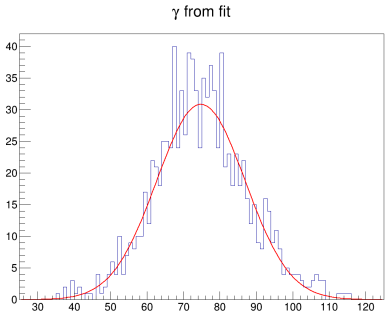
Precision of γ in unbinned fit: 11°

Study of γ precision

- Binned fit setup: Optimized 2×8 bins
- Fit same AmpGen samples, using c_i, s_i and F_i from LHCb model



(a) Pull of γ



(b) Fitted γ values

Precision of γ in binned fit: 12°
Consistent with unbinned fit and Q-value

Global fit

Signal parameterisation

- PDF shape parameterization identical to LHCb-ANA-2020-001
- Signal: Gaussian + Modified Cruijff
- Shape fixed from MC, yield and width floated
- Combinatorial background (exponential)
- Partially reconstructed background (dini shapes)

$$f_{\mathrm{MG}}(m|m_B, \sigma, \alpha_L, \alpha_R, \beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m = m - m_B < 0 \\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m = m - m_B > 0 \end{cases}$$

Global fit

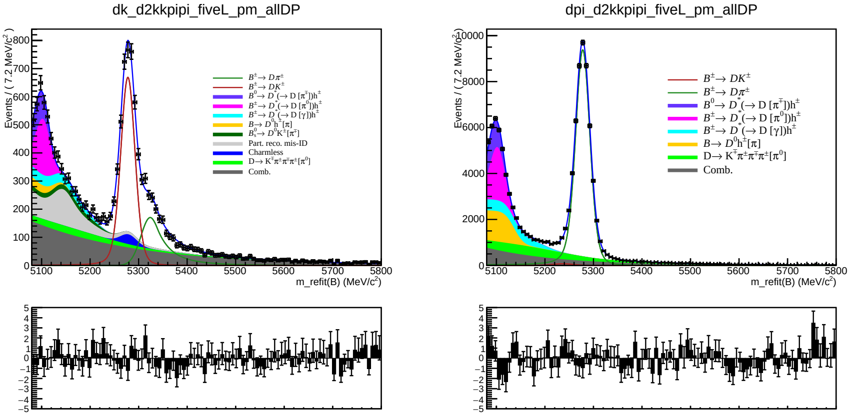


Figure 4: $B^\pm \to DK^\pm$ channel (left) and $B^\pm \to D\pi^\pm$ channel (right)

- $B^\pm \to DK^\pm$ yield: 3306 ± 75
- $B^\pm \to D\pi^\pm$ yield: $46\,695 \pm 256$

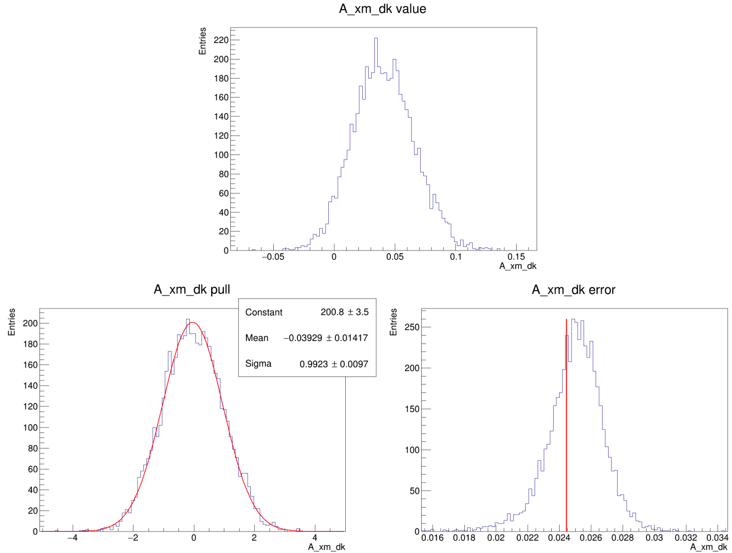
Binned CP fit

Binned CP fit

- Use 2×8 bins
- c_i and s_i not available yet $\implies$
  - Calculate using MC integration of LHCb amplitude model for now
- Fit for CP observables
- PDF shape parameters fixed from global fit
- Yield of signal, low mass partially reconstructed background and combinatorial background floated
- Fractional yields F_i floated

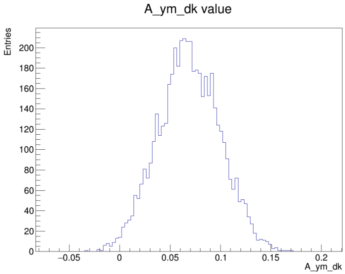
$$\mathcal{R}_i = \begin{cases} F_i, & i = -8 \\ F_i / \sum_{j \geq i} F_j, & -8 < i \leq +8 \end{cases}$$

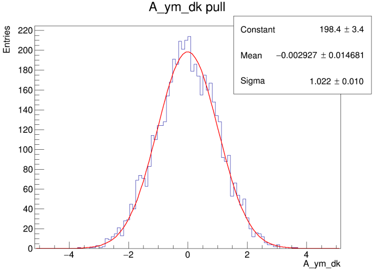
CP observables result: x_-^{DK}

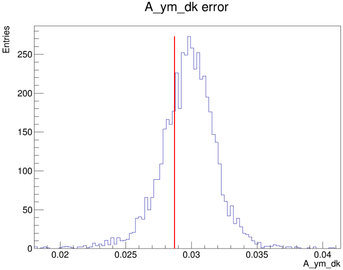


Red line: Uncertainty in fit to data

CP observables result: y_-^{DK}

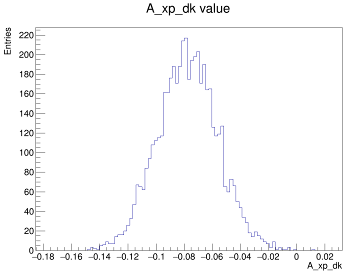


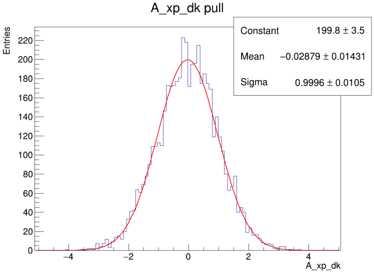


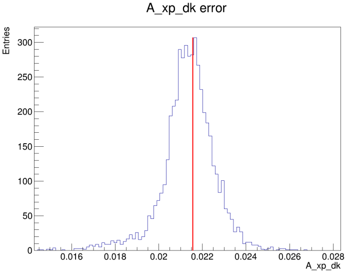


Red line: Uncertainty in fit to data

CP observables result: x_+^{DK}

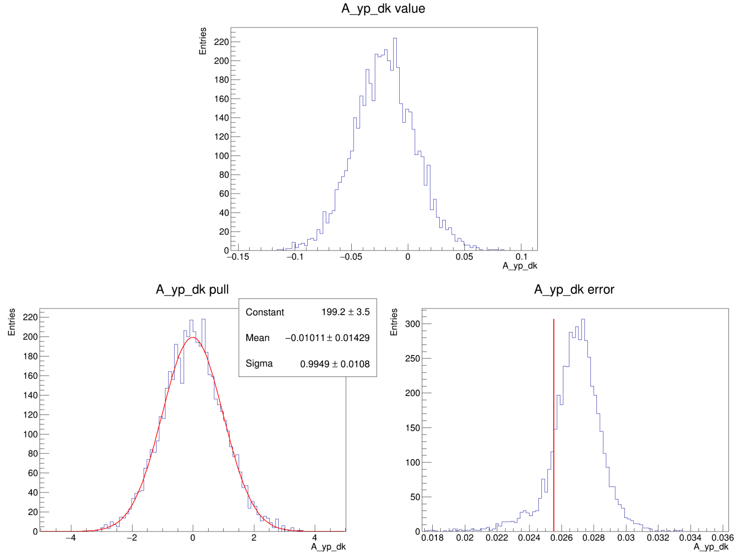






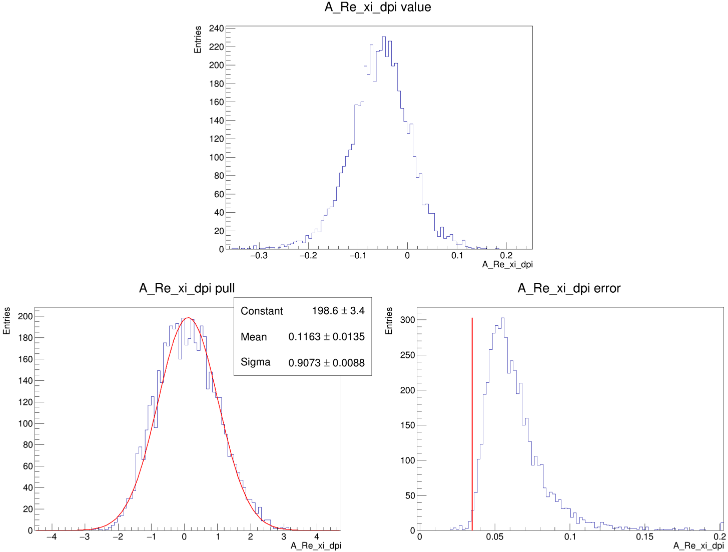
Red line: Uncertainty in fit to data

CP observables result: y_+^{DK}



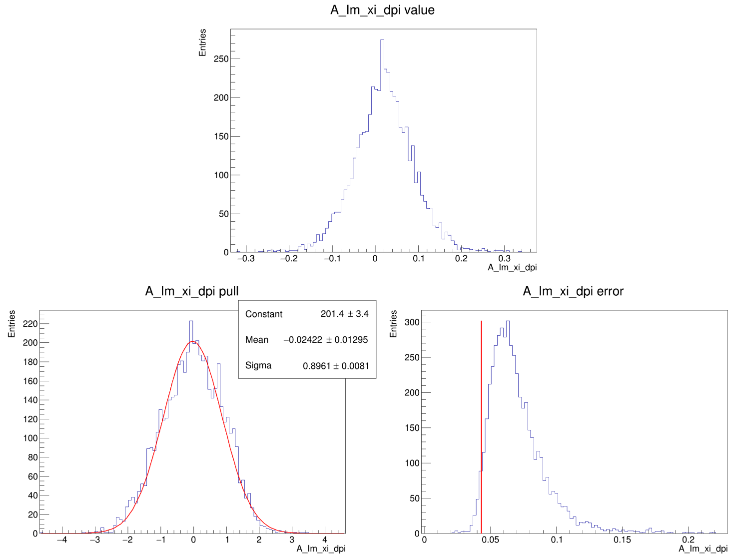
Red line: Uncertainty in fit to data

CP observables result: $x_\xi^{D\pi}$



Red line: Uncertainty in fit to data

CP observables result: $y_\xi^{D\pi}$



Red line: Uncertainty in fit to data

Summary of $KK\pi\pi$ analysis

Summary of CP observables

- Measured CP observables:

$$
\begin{aligned}
x_-^{DK} =& (x.x \pm 2.4 \pm 0.4 \pm 0.7) \times 10^{-2}, \\
y_-^{DK} =& (x.x \pm 2.9 \pm 0.2 \pm 1.6) \times 10^{-2}, \\
x_+^{DK} =& (x.x \pm 2.2 \pm 0.3 \pm 0.3) \times 10^{-2}, \\
y_+^{DK} =& (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2}, \\
x_\xi^{D\pi} =& (x.x \pm 3.5 \pm 0.9 \pm 1.7) \times 10^{-2}, \\
y_\xi^{D\pi} =& (x.x \pm 4.3 \pm 0.6 \pm 1.0) \times 10^{-2},
\end{aligned}
$$

- Note: Currently using c_i and s_i from the LHCb model
- Publication strategy: Publish current results together with binned yields $\rightarrow$ Redo fit to obtain model-independent CP observables once c_i and s_i from BESIII are available
- Will update c_i and s_i with model-dependent uncertainties

Bonus measurement:

Quasi-GLW observables with $B^{\pm} \to Dh^{\pm}$, $D \to \pi^+\pi^-\pi^+\pi^-$

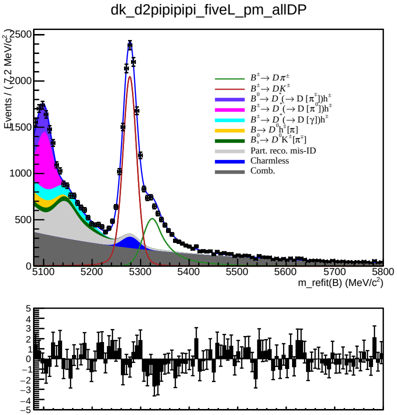
Bonus measurement

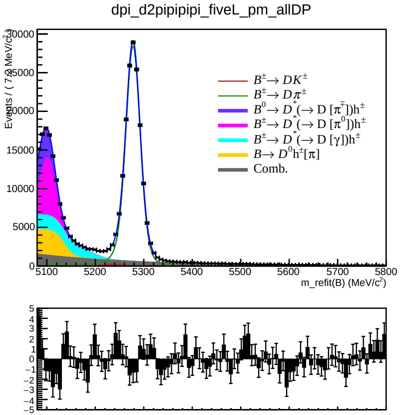
- The mode $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$ very similar
- Run this through <u>same</u> selection (including BDT)
- Can measure quasi-GLW observables of $\pi^+\pi^-\pi^+\pi^-$ and $K^+K^-\pi^+\pi^-$ as additional constraints on γ:

$$\begin{aligned} A_h &= \frac{\Gamma(B^- \to Dh^-) - \Gamma(B^+ \to Dh^+)}{\Gamma(B^- \to Dh^-) + \Gamma(B^+ \to Dh^+)}, \\ R_{\mathrm{CP}} &= \frac{R(4\pi)}{R(K3\pi)}, \\ R &= \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}. \end{aligned}$$

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi$ yields provided by Tim Evans

Global fit of $B^{\pm} \to Dh^{\pm}$, $D \to \pi^+\pi^-\pi^+\pi^-$





$$\frac{\mathrm{Yield}(\pi\pi\pi\pi)}{\mathrm{Yield}(KK\pi\pi)} = \frac{161\,900 \pm 455}{54\,039 \pm 256} = 2.996 \pm 0.017$$

Compare with PDG: 3.06 ± 0.16

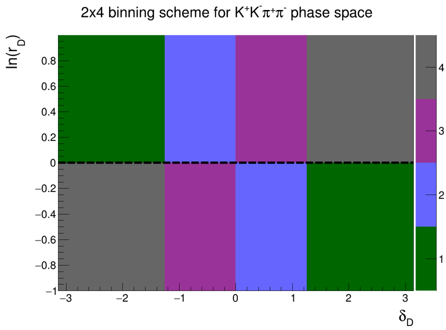
Conclusion

Conclusion

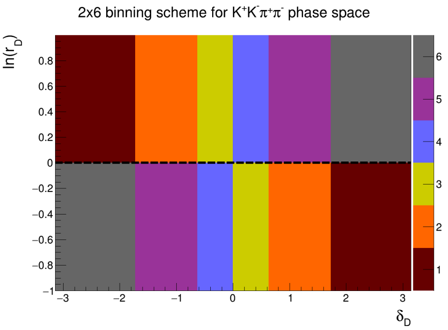
- Binned γ analysis of $B^{\pm} \to (K^+K^-\pi^+\pi^-)_D h^{\pm}$ is mostly complete, with promising results
- Add quasi-GLW observables of $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$ to further constrain γ
- Will publish model-dependent measurement for now, update with model-independent results when c_i and s_i from BESIII are available

Thank you!

Backup

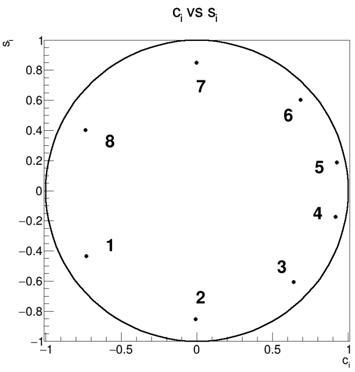

2x4 binning scheme for K⁺K⁻π⁺π⁻ phase space
ln(r_D)
δ_D
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−3
−2
−1
0
1
2
3
4
3
2
1

(a) $Q = 0.85$

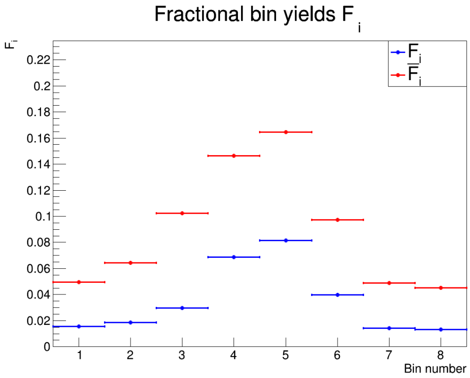

2x6 binning scheme for K⁺K⁻π⁺π⁻ phase space
ln(r_D)
δ_D
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−3
−2
−1
0
1
2
3
6
5
4
3
2
1

(b) $Q = 0.89$

c_i, s_i and F_i

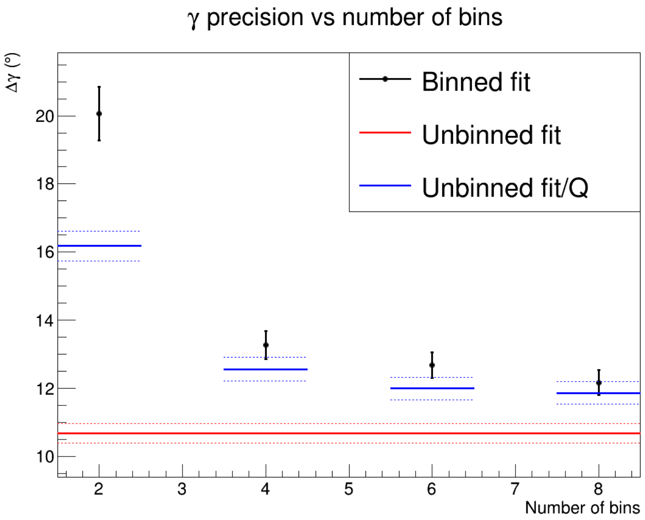


c_i vs s_i
s_i
c_i
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−0.5
0.5
7
6
8
5
4
1
3
2



Fractional bin yields F_i
F_i
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Bin number
F_i
F̄_i

Comparison of binned fit precision with unbinned fit



Trigger requirements

Run 1 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackAllL0Decision_TOS) and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or Bu_Hlt2Topo3BodyBBDTDecision_TOS or Bu_Hlt2Topo4BodyBBDTDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackMVADecision_TOS or Bu_Hlt1TwoTrackMVADecision_TOS) and (Bu_Hlt2Topo2BodyDecision_TOS or Bu_Hlt2Topo3BodyDecision_TOS or Bu_Hlt2Topo4BodyDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)

Initial cuts

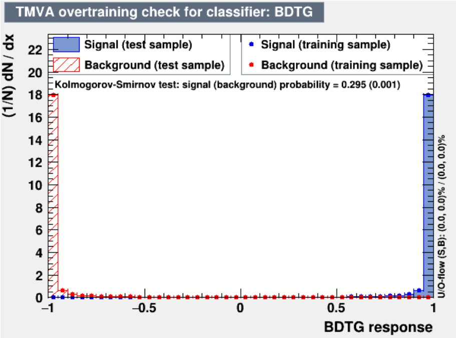
Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100\text{GeV}$
3	Bachelor has RICH	True
4	D invariant mass	$[1839.84, 1889.84]\text{MeV}$
5	$B^{\pm}$ invariant mass	$[5080, 5800]\text{MeV}$
6	$K^{\pm}$ daughter PID	> -10
7	$\pi^{\pm}$ daughter PID	< 20

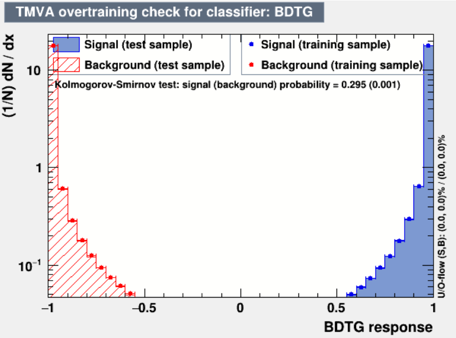
Boosted Decision Tree

- BDTG from TMVA Toolkit
- Signal sample: $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$ MC samples
- Background sample: Data sample with $m_{B^{\pm}}^{\mathrm{DTF}} \in [5800, 7000]\mathrm{MeV}$
- Random, equal sized test and training samples

BDT training results



(a) BDT output



(b) BDT output on a logarithmic scale

Final cuts

Rectangular cuts after BDT

Number	Variable description	Cut
8	$K^{\pm}$ bachelor PID	> 4
9	$\pi^{\pm}$ bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	$K^{\pm}$ PID	> 0
13	K_S^0 mass veto	$[477, 507]$MeV

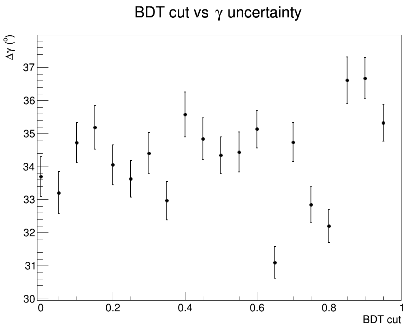
BDT training variables

Name	Rank (%)	Description
log(D0_RHO_BPV)	7.7	D radial distance to beamline
log(Bu_FDCHI2_OWNPV)	6.3	$B^{\pm}$ flight distance χ^2
log(Bu_RHO_BPV)	6.1	$B^{\pm}$ radial distance to beamline
log(Bach_PT)	6.1	Bachelor transverse momentum
Bu_PTASY_1.5	5.3	$B^{\pm}$ asymmetry parameter
log(1-D0_DIRA_BPV)	5.0	Angle between PV and D
log(Bu_IPCHI2_OWNPV)	4.8	$B^{\pm}$ impact parameter χ^2
log(1-Bu_DIRA_BPV)	4.7	Angle between PV and $B^{\pm}$
log(h[1,2]_PT)	4.4	$K^{\pm}$ transverse momentum
Bu_MAXDOCA	4.4	$B^{\pm}$ distance of closest approach
log(Bach_IPCHI2_OWNPV)	4.1	Bachelor impact parameter χ^2

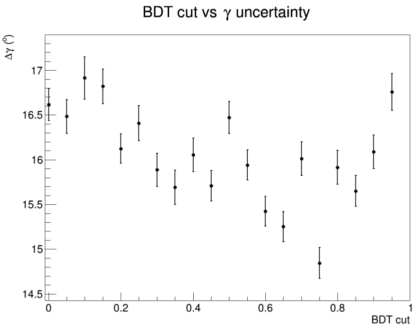
BDT training particles

Name	Rank (%)	Description
`log(Bu_constD0PV_D0_P)`	3.7	D momentum from DTF
`log(D0_VTXCHI2DOF)`	3.3	$D0$ vertex fit χ^2
`log(h[3,4]_IPCHI2_OWNPV)`	3.3	$\pi^\pm$ impact parameter χ^2
`log(D0_IPCHI2_OWNPV)`	3.2	D impact parameter χ^2
`log(h[3,4]_PT)`	3.2	$\pi^\pm$ transverse momentum
`log(Bu_PT)`	2.8	$B^\pm$ transverse momentum
`log(h[1,2]_P)`	2.8	$K^\pm$ momentum
`log(Bach_P)`	2.7	Bachelor momentum
`log(Bu_constD0PV_P)`	2.6	$B^\pm$ momentum from DTF
`log(h[1,2]_IPCHI2_OWNPV)`	2.5	$K^\pm$ impact parameter χ^2
`D0_MAXDOCA`	2.5	D distance of closest approach
`log(Bu_VTXCHI2DOF)`	2.0	$B^\pm$ vertex fit χ^2
`log(h[3,4]_P)`	1.9	$\pi^\pm$ momentum

BDT optimization study



(a) Run 1



(b) Run 2

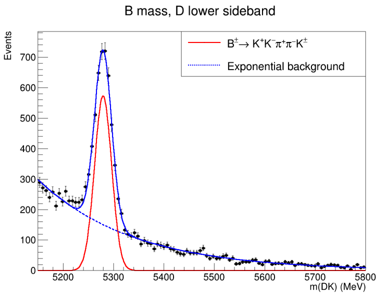
- Run 1: Pick BDT working point at 0.65
- Run 2: Pick BDT working point at 0.75

Partially reconstructed background

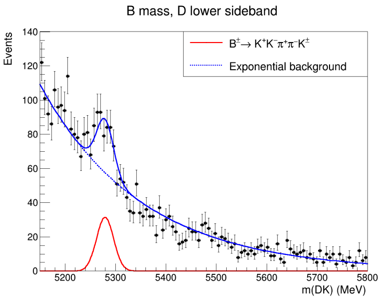
- $B^\pm \to D\pi^\pm$:
  1. $B^\pm \to (D^{*0} \to D^0[\pi^0])\pi^\pm$
  2. $B^0 \to (D^{*\mp} \to D^0[\pi^\mp])\pi^\pm$
  3. $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]\pi^\pm$
  4. $B^\pm \to (D^{*0} \to D^0[\gamma])\pi^\pm$
- $B^\pm \to DK^\pm$:
  1. $B^\pm \to (D^{*0} \to D^0[\pi^0])K^\pm$
  2. $B^0 \to (D^{*\mp} \to D^0[\pi^\mp])K^\pm$
  3. $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]K^\pm$
  4. $B^\pm \to (D^{*0} \to D^0[\gamma])K^\pm$
  5. $B_s^0 \to \bar{D}^0[\pi^+]K^-$
  6. Mis-ID from partially reconstructed $B^\pm \to D\pi^\pm$ channel

Charmless background

- $B \to KK\pi\pi K$ background in $B \to DK$ channel
- Flight significance cut at 2
- Fix remaining background with Gaussian shape of lower sideband



(a) No FS cut

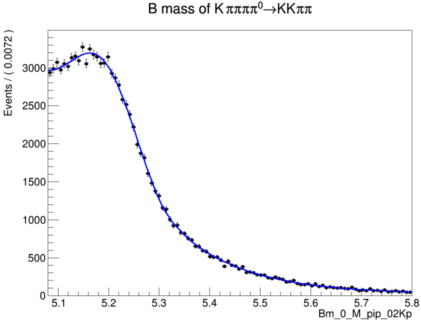


(b) FS cut at 2

Figure 15: B invariant mass in lower D sideband

$D \to K\pi\pi\pi\pi^0$ mis-ID background

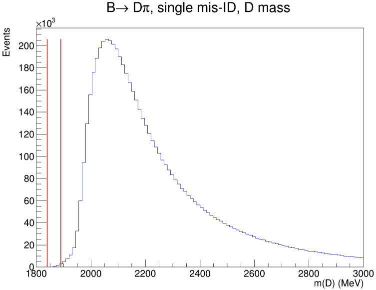
- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi[\pi^0]$
- π^0 not reconstructed $\to$ Lower D mass
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi \to$ Higher D mass
- Generate RapidSim samples, reweight with PIDCalib2



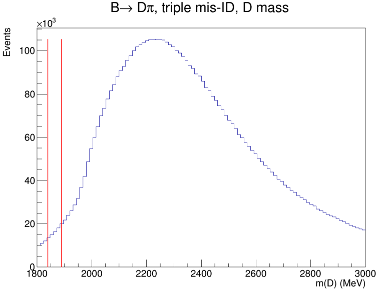
Conclusion: Fix shape from RapidSim, allow yield to float

$D \to K\pi\pi\pi$ mis-ID background

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi$
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID: $\pi\pi K\pi \to KK\pi\pi$
- Use LHCb MC generated with AmpGen, reweight with PIDCalib2



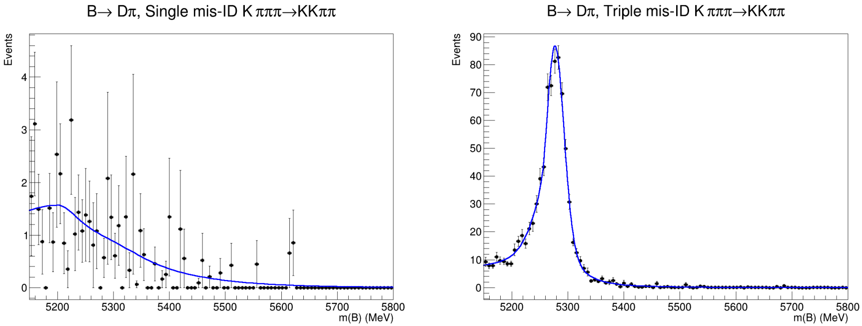
(a) Single mis-ID



(b) Triple mis-ID

Figure 16: D invariant mass

$D \to K\pi\pi\pi$ mis-ID background



(a) Single mis-ID **(b)** Triple mis-ID

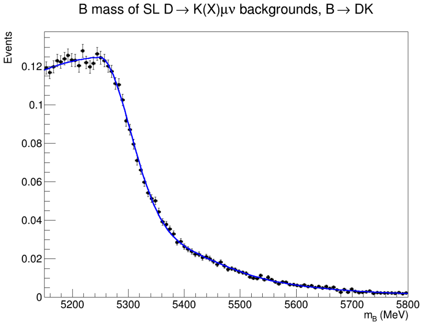
Figure 17: B invariant mass

Conclusion: Negligible impact, include in systematics

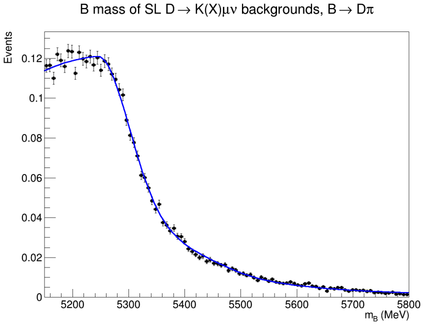
D semileptonic backgrounds

- $B^\pm \to Dh^\pm$, $D \to K(X)l\nu$, $K(X) \to K\pi\pi$
  - $K_1(1270)$
  - $K_1(1400)$
  - $K^*(1410)$
  - $K^*(1680)$
  - $K_2^*(1430)$
- Single mis-ID: $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID: $K\pi\pi\mu \to KK\pi\pi$
- Generate RapidSim samples, reweight with PIDCalib2

D semileptonic backgrounds



(a) $B \to DK$



(b) $B \to D\pi$

Conclusion: Negligible impact, include in systematics

c_i and s_i systematic uncertainty

- c_i and s_i uncertainty will come from BESIII analysis
  - Mostly statistical in origin
- Largest systematic uncertainty
- Use estimated uncertainties corresponding to $20\,\mathrm{fb}^{-1}$ at $\psi(3770)$
- Smear c_i and s_i and do many fits to data

Remaining systematic uncertainties

Different strategies for evaluating systematic uncertainties:

- Generate toy datasets with systematics, fit with default model and take the bias as a systematic:
  - Small backgrounds ($D \to K(X)l\nu_l$, $D \to K\pi\pi\pi$, $B \to Dl\nu_l$, Λ_b)
  - Bin dependent mass shape
  - Low mass physics effects

- Do multiple fits to data while smearing parameters:
  - c_i and s_i
  - Mass shape
  - Fixed yield fractions
  - PID efficiency

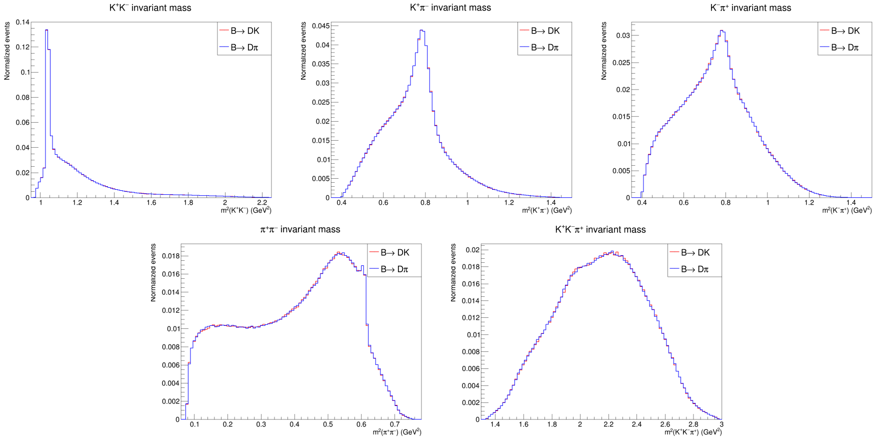
- Fit bias: Take bias toys as systematic uncertainty

Efficiency related systematics

Efficiency related systematics:

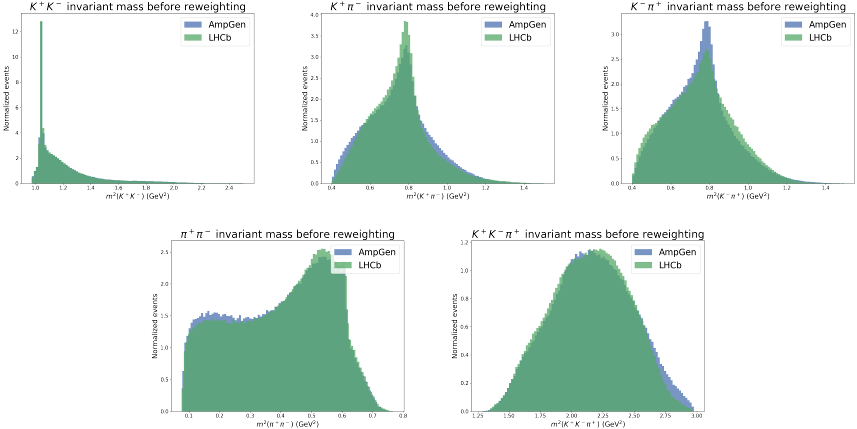
- Difference in $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$ phase space acceptance
- Efficiency correction of c_i and s_i

Efficiency differences between $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$



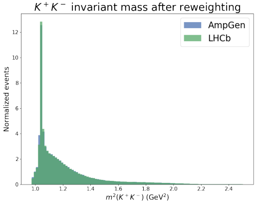
Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered

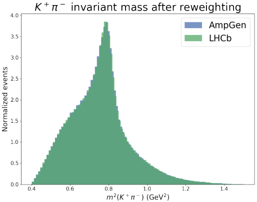
Efficiency correction of c_i and s_i

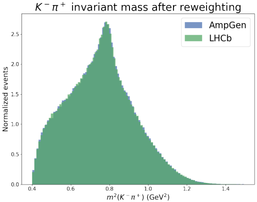


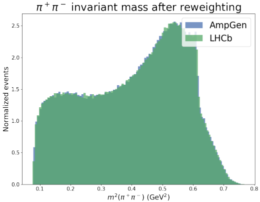
Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

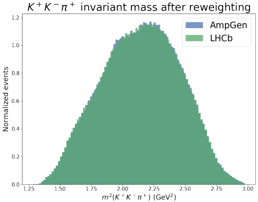
Efficiency correction of c_i and s_i











After reweighing, use weights to recalculate c_i and s_i
Conclusion: Efficiency correction of c_i and s_i is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered

Summary of all systematic uncertainties

Uncertainties of CP observables in units of 10^{-2}

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.44	2.87	2.16	2.87	3.47	4.30
c_i, s_i	0.66	1.55	0.32	1.31	1.73	1.03
$B^\pm \to D\mu\nu$ background	0.04	0.03	0.02	0.15	0.30	0.10
$D \to K(X)l\nu_l$ background	0.15	0.05	0.11	0.03	0.35	0.25
$D \to K\pi\pi\pi$ background	0.17	0.03	0.04	0.01	0.46	0.18
Λ_b background	0.09	0.11	0.00	0.18	0.16	0.21
Bin dependent mass shape	0.21	0.05	0.17	0.01	0.37	0.11
Fit bias	0.19	0.03	0.16	0.04	0.30	0.16
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
Low mass physics effects	0.05	0.09	0.05	0.18	0.41	0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Total LHCb systematic	0.39	0.17	0.27	0.30	0.92	0.65
Total systematic	0.77	1.55	0.41	1.34	1.96	1.22