

# A study of CP violation in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D h^\pm$ and $B^\pm \rightarrow (\pi^+ \pi^- \pi^+ \pi^-)_D h^\pm$ decays

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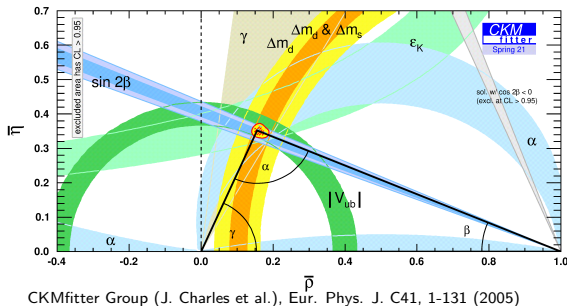


# Thank you

- Big thank you to all reviewers!
- B2OC WG reviewers/convenors:
  - Anton Poluektov
  - Nathan Philip Jurik
  - Paras Naik
- RC reviewers:
  - Lucia Grillo
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# Introduction and motivation

- Aim of this analysis: Measure CP observables in  $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D h^\pm$  decays using full Run 1 and 2 ( $9 \text{ fb}^{-1}$ )
  - First study of CP violation in this channel
  - Enhance sensitivity through sophisticated binning of 5D phase space
  - Provide input for future model-independent measurement of  $\gamma$
- Also measure phase space inclusive CP observables for  $D \rightarrow K^+K^-\pi^+\pi^-$ , as well as  $D \rightarrow \pi^+\pi^-\pi^+\pi^-$



# Introduction and motivation

- $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$  was first proposed by J. Rademacker and G. Wilkinson
  - [Phys. Lett. B647 \(2007\) 400](#)
  - Expected  $\gamma$  precision from FOCUS amplitude model with 1000  $B^\pm \rightarrow DK^\pm$  candidates:  $14^\circ$
- Recent state of the art amplitude analysis by LHCb:
  - [JHEP 02 \(2019\) 126](#)
  - Develop a suitable binning scheme
- No measurement of strong phases  $c_i$  and  $s_i$  exists today
  - Current BESIII dataset is  $8 \text{ fb}^{-1}$
  - Allows for a direct measurement of  $D \rightarrow K^+ K^- \pi^+ \pi^-$  strong phases
  - Final  $\gamma$  measurement will be model independent

# Theory of BPGGSZ method

- $B^\pm \rightarrow Dh^\pm$  amplitude:

$$\begin{aligned}\mathcal{A}(B^-) &= \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0) \\ \mathcal{A}(B^+) &= \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)\end{aligned}$$

- $\mathcal{A}(D^0)$  and  $\mathcal{A}(\bar{D}^0)$  depend on  $D$  phase space
- Strong-phase difference of  $D^0$  and  $\bar{D}^0$  decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

## Event yield in bin $i$

$$\begin{aligned}N_{-i}^- &= h_{B^-} \left( F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i) \right) \\ N_{-i}^+ &= h_{B^+} \left( F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i) \right)\end{aligned}$$

## Event yield in bin $i$

$$N_i^- = h_{B^-} (F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i))$$

$$N_i^+ = h_{B^+} (F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i))$$

- CP observables:

- $x_{\pm}^{DK} = r_B^{DK} \cos(\delta_B^{DK} \pm \gamma), \quad y_{\pm}^{DK} = r_B^{DK} \sin(\delta_B^{DK} \pm \gamma)$
- $x_{\xi}^{D\pi} = \text{Re}(\xi^{D\pi}), \quad y_{\xi}^{D\pi} = \text{Im}(\xi^{D\pi}) \quad \left( \xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})} \right)$

- Fractional bin yield:

- $F_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$
- Floated in the fit, mostly constrained by  $B^{\pm} \rightarrow D\pi^{\pm}$

- Amplitude averaged strong phases:

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}, \quad s_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \sin(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}$$

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between  $B^\pm \rightarrow D^0 h^\pm$  and  $B^\pm \rightarrow \bar{D}^0 h^\pm$

How to bin a 5-dimensional phase space?

- Generate C++ code for LHCb amplitude model using AmpGen<sup>1</sup>
- For each  $B^\pm$  candidate, calculate

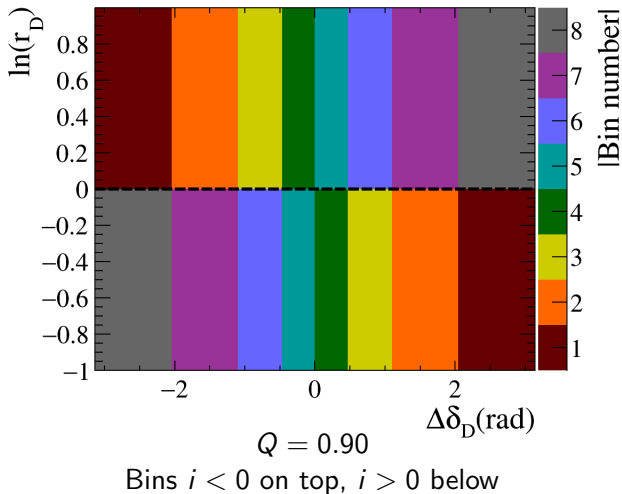
$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along  $\delta_D$  and  $r_D$ , maximize  $Q$ -value to optimize
  - $Q = 1$  corresponds to unbinned limit

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<sup>1</sup>AmpGen by Tim Evans

# Binning scheme





# Phase-space inclusive CP observables

- Statistically independent analysis without phase space binning
  - BPGGSZ looks at relative bin yields
  - Quasi-GLW observables depend on absolute yields

- Charge asymmetry:

$$A_h = \frac{\Gamma(B^- \rightarrow Dh^-) - \Gamma(B^+ \rightarrow Dh^+)}{\Gamma(B^- \rightarrow Dh^-) + \Gamma(B^+ \rightarrow Dh^+)}$$

- $B \rightarrow DK$  vs  $B \rightarrow D\pi$  double ratio:

$$R_{CP} = \frac{R_{hh\pi\pi}}{R_{K\pi\pi\pi}}, \quad R = \frac{\Gamma(B^- \rightarrow DK^-) + \Gamma(B^+ \rightarrow DK^+)}{\Gamma(B^- \rightarrow D\pi^-) + \Gamma(B^+ \rightarrow D\pi^+)}$$

- Measure  $A_h$  and  $R_{CP}$  for  $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D h^\pm$  and  $B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$

## CP observables and physics parameters

$$A_h = \frac{2r_B^{Dh}(2F_+ - 1) \sin(\delta_B^{Dh}) \sin(\gamma)}{1 + (r_B^{Dh})^2 + 2r_B^{Dh}(2F_+ - 1) \cos(\delta_B^{Dh}) \cos(\gamma)},$$

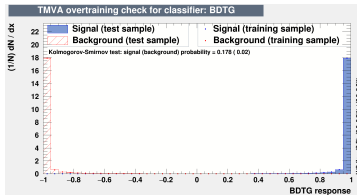
$$R_{\text{CP}} = 1 + (r_B^{Dh})^2 + 2r_B^{Dh}(2F_+ - 1) \cos(\delta_B^{Dh}) \cos(\gamma)$$

- Need CP-even fraction  $F_+$  to interpret in terms of physics parameters
- For  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ :
  - $F_+ = 0.736$
  - Calculated from the model
- For  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ :
  - $F_+ = 0.735 \pm 0.016$
  - Recent measurement by BESIII [arXiv:2208.10098](https://arxiv.org/abs/2208.10098)

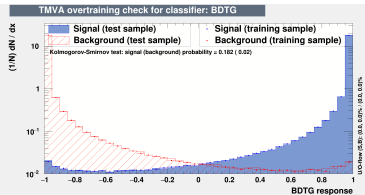
- Strong phases  $c_i$  and  $s_i$  have not been published by BESIII yet
- We would like to publish a model dependent result initially
  - Focus of the paper should be on CP violation, not  $\gamma$
  - Bin yields and correlation matrices will be provided in the appendix
- When  $c_i$  and  $s_i$  become available, the result can be updated to yield a model independent measurement of  $\gamma$ , which can be included in the next combination

# Selection

- 1 Initial cuts: Trigger requirements, mass cuts, bachelor  $p$ , etc
  - $D$  mass window: 25 MeV
  - $B^\pm$  mass fit range: [5080, 5700] MeV
- 2 BDT: Efficient combinatorial background rejection
  - Signal sample: MC with AmpGen model
  - Background sample: Data from  $m_B \in [5800, 7000]$  MeV
  - Pick BDT cut that minimises statistical sensitivity of  $\gamma$
- 3 Final cuts: PID cuts, flight significance cuts,  $K_S$  veto, etc
  - PIDK cut at 4 to separate  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D\pi^\pm$
  - Flight significance at 2 to reduce charmless backgrounds



(a) BDT output



(b) BDT output on a log scale

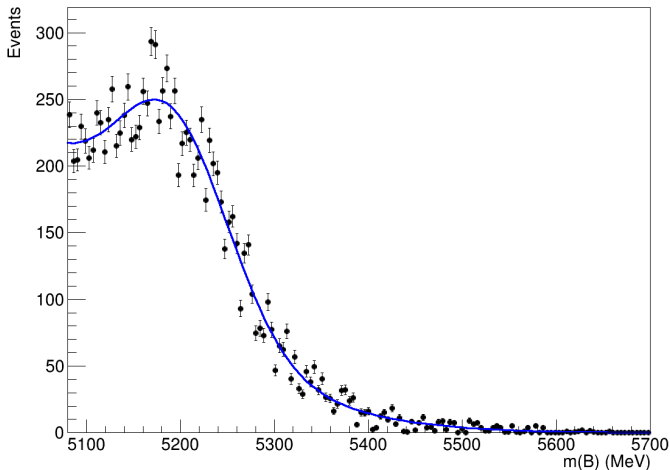
# Summary of backgrounds

- Partially reconstructed  $B$  decays
  - Modelled in the fit with HILL/HORNSdini shapes
- Combinatorial
  - Floating exponential
- CF  $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$  and semi-leptonic  $D^0 \rightarrow K^- (X) l^+ \nu_l$  background
  - Small, assign systematic
- Charmless
  - Fix from sideband
- CF  $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$  background
  - Not negligible!

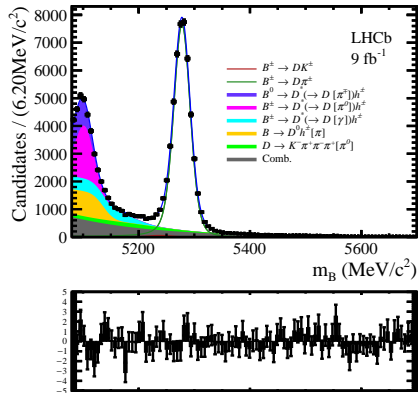
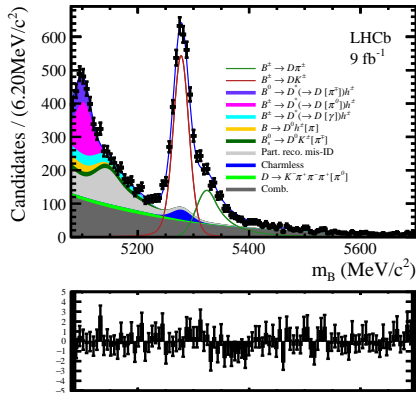
# $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$ partially reconstructed mis-ID

- Missing  $\pi^0$  and  $\pi \rightarrow K$  mis-ID
- Float yield relative to  $B \rightarrow D^* h$  background

B mass of  $K\pi\pi\pi^0 \rightarrow KK\pi\pi$



# Global fit



**Figure 2:**  $B^\pm \rightarrow DK^\pm$  channel (left) and  $B^\pm \rightarrow D\pi^\pm$  channel (right)

- $B^\pm \rightarrow DK^\pm$  yield:  $3026 \pm 38$
- $B^\pm \rightarrow D\pi^\pm$  yield:  $44\,349 \pm 218$

# Fit split by charge

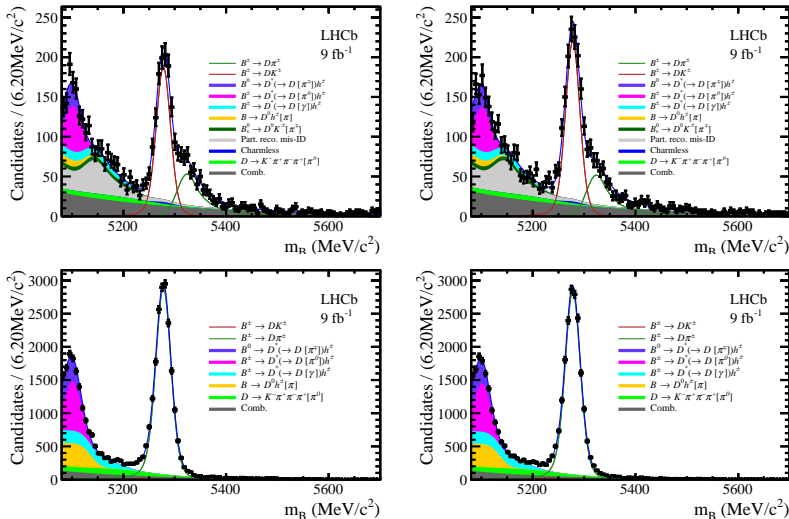


Figure 3:  $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-) D h^\pm$



# Fit split by charge

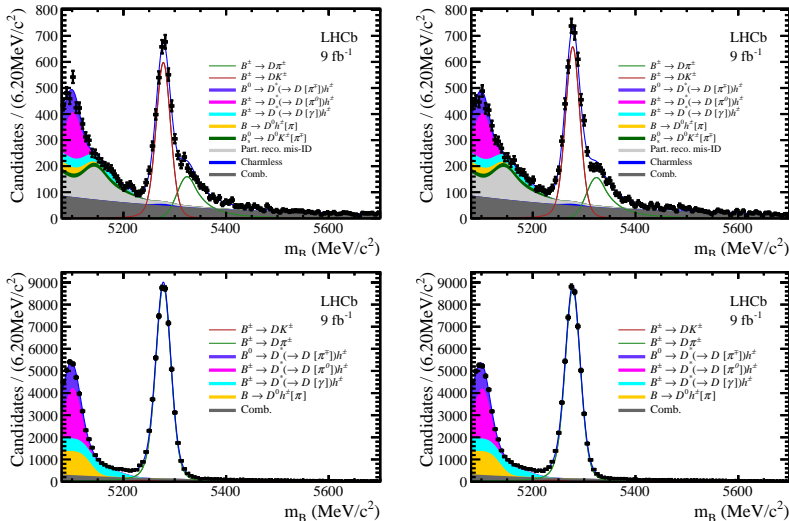
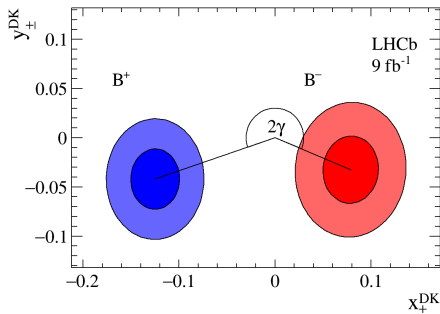


Figure 4:  $B^\pm \rightarrow (\pi^+\pi^-\pi^+\pi^-)Dh^\pm$

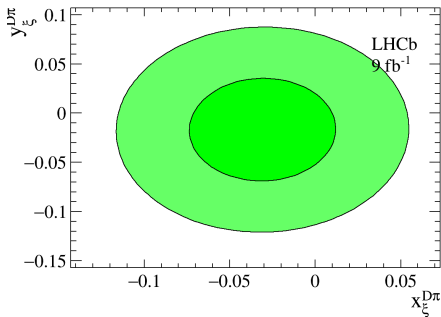
# CP fit setup

- Fix mass shape from global fit
- Split by  $B^\pm$  charge and  $D$  phase space bins
  - Simultaneous fit with 64 categories
- Signal yields parameterised in terms of CP observables  $x_\pm^{DK}, y_\pm^{DK}$   
 $x_\xi^{D\pi}, y_\xi^{D\pi}$  (6 parameters)
- Fractional bin yields  $F_i$  are floating (15 parameters)
- Combinatorial background yield floated in each bin (64 parameters)
- Total partially reconstructed background yield is floated in each bin (64 parameters)
- Normalisation of each charge and  $B^\pm$  decay is floated (4 parameters)
- In total: 153 free parameters

# CP fit results



(a)  $x_\pm^{DK}$  vs  $y_\pm^{DK}$



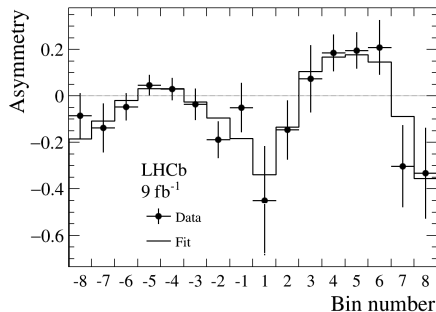
(b)  $x_\pm^{D\pi}$  vs  $y_\pm^{D\pi}$

$$x_\pm^{DK} = r_B \cos(\delta_B \pm \gamma)$$

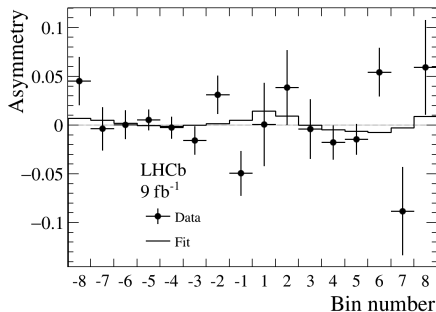
$$y_\pm^{DK} = r_B \sin(\delta_B \pm \gamma)$$

- Shown are the  $2\sigma$  contours of fitted CP observables
- The distinct  $B^\pm \rightarrow DK^\pm$  contours indicate CP violation, while the  $B^\pm \rightarrow D\pi^\pm$  mode has very low sensitivity to CP violation

# Fractional bin asymmetries



(a)  $B^\pm \rightarrow DK^\pm$



(b)  $B^\pm \rightarrow D\pi^\pm$

- Useful cross check to compare measured bin asymmetries against bin asymmetries predicted by the fitted CP observables
- The  $B^\pm \rightarrow DK^\pm$  mode show non-zero bin asymmetries, and the non-trivial distribution is driven by the change in strong phases across phase space

- Dominant  $c_i/s_i$  systematic uncertainty due to model dependence
  - Strategy: Generate toys with  $c_i/s_i$  from older CLEO model, fit with  $c_i/s_i$  from LHCb model
  - Will be replaced when BESIII results become available
- All internal systematic uncertainties are much smaller than the statistical uncertainties

# Summary of all BPGGSZ systematic uncertainties

Uncertainties of BPGGSZ CP observables in units of  $10^{-2}$

Source	$x_-^{DK}$	$y_-^{DK}$	$x_+^{DK}$	$y_+^{DK}$	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
CP violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic $b$ -hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \rightarrow K^- \pi^+ \pi^- \pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \rightarrow p D \pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Fit bias	0.06	0.05	0.13	0.02	0.06	0.13
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57
$c_i, s_i$	0.35	3.64	1.74	1.29	0.14	1.10
Total systematic	0.51	3.67	1.78	1.33	0.72	1.24

# Summary of all quasi-GLW systematic uncertainties

Uncertainties of quasi-GLW CP observables in units of  $10^{-2}$

Source	$A_K^{KK\pi\pi}$	$A_\pi^{KK\pi\pi}$	$A_K^{\pi\pi\pi\pi}$	$A_\pi^{\pi\pi\pi\pi}$	$R_{CP}^{KK\pi\pi}$	$R_{CP}^{\pi\pi\pi\pi}$
Statistical	23.5	5.5	13.3	3.1	24.2	14.3
Charmless background	1.2	$< 0.1$	0.4	$< 0.1$	13.9	8.5
External parameters	1.0	0.7	1.0	0.7	4.0	4.0
Fixed yield fractions	0.1	$< 0.1$	0.1	$< 0.1$	1.3	1.4
Mass shape	0.3	$< 0.1$	0.2	$< 0.1$	3.1	3.1
PID efficiency	0.1	$< 0.1$	0.1	$< 0.1$	2.5	1.6
Total systematic	1.6	0.7	1.1	0.7	15.1	10.1

# Summary of measured CP observables

- Measured binned CP observables:

$$x_{-}^{DK} = (7.9 \pm 2.9 \pm 0.4 \pm 0.4) \times 10^{-2}$$

$$y_{-}^{DK} = (-3.3 \pm 3.4 \pm 0.4 \pm 3.6) \times 10^{-2}$$

$$x_{+}^{DK} = (-12.5 \pm 2.5 \pm 0.3 \pm 1.3) \times 10^{-2}$$

$$y_{+}^{DK} = (-4.2 \pm 3.1 \pm 0.3 \pm 1.3) \times 10^{-2}$$

$$x_{\xi}^{D\pi} = (-3.1 \pm 4.3 \pm 0.7 \pm 0.1) \times 10^{-2}$$

$$y_{\xi}^{D\pi} = (-1.7 \pm 5.2 \pm 0.6 \pm 1.1) \times 10^{-2}$$

- Measured inclusive CP observables:

$$A_K^{KK\pi\pi} = 0.093 \pm 0.023 \pm 0.002$$

$$A_{\pi}^{KK\pi\pi} = -0.009 \pm 0.006 \pm 0.001$$

$$A_K^{\pi\pi\pi\pi} = 0.060 \pm 0.013 \pm 0.001$$

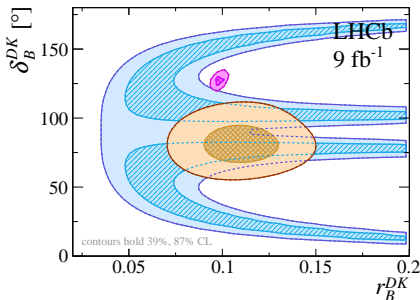
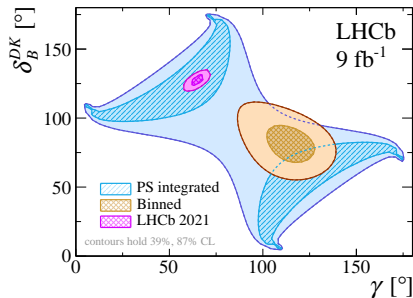
$$A_{\pi}^{\pi\pi\pi\pi} = -0.0082 \pm 0.0031 \pm 0.0007$$

$$R_{\text{CP}}^{KK\pi\pi} = 0.974 \pm 0.024 \pm 0.015$$

$$R_{\text{CP}}^{\pi\pi\pi\pi} = 0.978 \pm 0.014 \pm 0.010$$



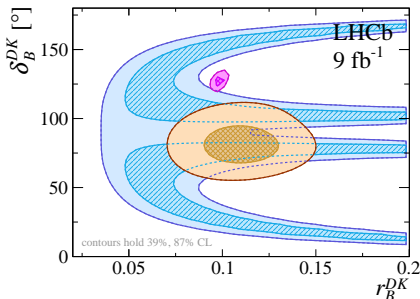
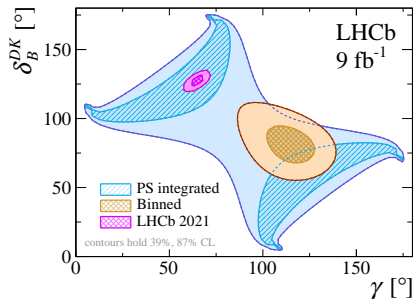
# Interpretation



Results from binned fit:

$$\begin{aligned}\gamma &= (116_{-14}^{+12})^\circ \\ \delta_B^{DK} &= (81_{-13}^{+14})^\circ \\ r_B^{DK} &= 0.110_{-0.020}^{+0.020} \\ \delta_B^{D\pi} &= (298_{-118}^{+62})^\circ \\ r_B^{D\pi} &= 0.0041_{-0.0041}^{+0.0055}\end{aligned}$$

# Interpretation



- Value of  $\gamma$  from binned analysis is somewhat high, but falls within the  $3\sigma$  contours
- Phase space inclusive measurement are compatible with our expectations
- Interpretation may evolve when full model independent inputs are available

# Conclusion

- First study of CP violation has been performed in  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  in bins of phase space
- Phase space inclusive measurement for  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  and  $B^\pm \rightarrow [\pi^+ \pi^- \pi^+ \pi^-]_D h^\pm$
- This publication is model dependent, but strong phases will become available from BESIII soon
  - Sufficient information will be provided in the paper to allow a model independent update when  $c_i$  and  $s_i$  become available
- Complete paper draft is available
- Link to:
  - [TWiki](#)
  - [ANA note](#)
  - [Paper draft](#) (also uploaded to Indico)

## Thanks for listening!