# Determination of the CKM angle $\gamma$ in $B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$ decays

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**B2OC** Meeting

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#### Outline

- **1** Introduction to the CKM angle  $\gamma$
- 2 Binned  $\gamma$  analysis of the  $D \to K^+K^-\pi^+\pi^-$  mode
- Binning scheme
- A  $B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$  selection

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- Backgrounds
- 6 Fit to data
- Systematics
- Summary and conclusion



#### $\gamma$ and the unitary triangle

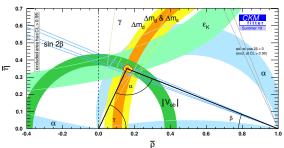
• Unitarity of CKM matrix:  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \implies$ 

$$\gamma = \mathrm{arg} \Big( - \frac{\mathit{V_{ud}} \mathit{V_{ub}^*}}{\mathit{V_{cd}} \mathit{V_{cb}^*}} \Big)$$

- Only CKM angle accessible at tree level ⇒
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark

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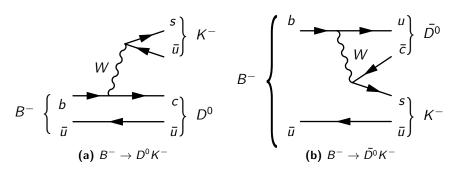
Compare with indirect measurements



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

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## Sensitivity through interference

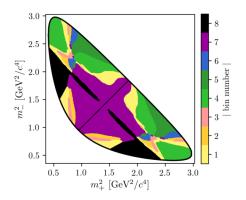


- ullet Superposition of  $D^0$  and  $ar{D^0}$
- ullet b o uar cs and b o car us interference o Sensitivity to  $\gamma$

$$\mathcal{A}(B^{-}) = \mathcal{A}(D^{0}) + r_{B}e^{i(\delta_{B}-\gamma)}\mathcal{A}(\bar{D^{0}})$$
  
$$\mathcal{A}(B^{+}) = \mathcal{A}(\bar{D^{0}}) + r_{B}e^{i(\delta_{B}+\gamma)}\mathcal{A}(D^{0})$$

## Binned measurement of $\gamma$

- Enhance sensitivity through binning of phase space
- Need strong phases of D decay  $\rightarrow$  Measure at BESIII!
- LHCb-PAPER-2020-019:  $B^\pm \to D h^\pm$ ,  $D \to K_S^0 h^+ h^-$ 
  - Single most precise measurement:  $\gamma = (68.7^{+5.2}_{-5.1})^{\circ}$



## The $D \to K^+ K^- \pi^+ \pi^-$ decay

Binned 
$$\gamma$$
 analysis of the  $D \to K^+K^-\pi^+\pi^-$  mode

## The $D \to K^+K^-\pi^+\pi^-$ decay

## Measurement of $\gamma$ from $B^{\pm} \to DK^{\pm}$ , $D \to K^+K^-\pi^+\pi^-$

- First proposed by J. Rademacker and G. Wilkinson
  - arXiv:hep-ph/0611272
  - Amplitude model by FOCUS
  - Expected  $\gamma$  precision with 1000 candidates: 14 $^{\circ}$
- CLEO amplitude analysis
  - arXiv:1201.5716
  - ullet Expected  $\gamma$  precision with 2000 candidates:  $11^\circ$
- State of the art amplitude analysis by LHCb LHCb-PAPER-2018-041:
  - Used to develop efficient binning scheme in this analysis
  - Final measurement will be model-independent
  - $\bullet$  Poor binning reduces statistical sensitivity  $\to$  No bias!

#### The BPGGSZ method

•  $B^{\pm} \rightarrow Dh^{\pm}$  amplitude:

$$\begin{split} \mathcal{A}(B^-) &= \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D^0}) \\ \mathcal{A}(B^+) &= \mathcal{A}(\bar{D^0}) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0) \end{split}$$

- ullet  $\mathcal{A}(D^0)$  and  $\mathcal{A}(ar{D^0})$  depend on D phase space
- ullet Strong-phase difference of  $D^0$  and  $ar{D^0}$  decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

#### Event yield in bin i

$$N_{i}^{-} = h_{B^{-}} \Big( F_{i} + (x_{-}^{2} + y_{-}^{2}) \bar{F}_{i} + 2 \sqrt{F_{i} \bar{F}_{i}} (x_{-} c_{i} + y_{-} s_{i}) \Big)$$

$$N_{-i}^{+} = h_{B^{+}} \Big( F_{i} + (x_{+}^{2} + y_{+}^{2}) \bar{F}_{i} + 2 \sqrt{F_{i} \bar{F}_{i}} (x_{+} c_{i} + y_{+} s_{i}) \Big)$$

#### The BPGGSZ method

#### Event yield in bin i

$$\begin{split} N_i^- &= h_{B^-} \big( F_i + (x_-^2 + y_-^2) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i) \big) \\ N_{-i}^+ &= h_{B^+} \big( F_i + (x_+^2 + y_+^2) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i) \big) \end{split}$$

- CP observables:
  - $\mathbf{x}_{\pm}^{DK} = r_{B}^{DK} \cos(\delta_{B}^{DK} \pm \gamma), \quad \mathbf{y}_{\pm}^{DK} = r_{B}^{DK} \sin(\delta_{B}^{DK} \pm \gamma)$ •  $\mathbf{x}_{\varepsilon}^{D\pi} = \text{Re}(\xi^{D\pi}), \ \mathbf{y}_{\varepsilon}^{D\pi} = \text{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_{B}^{D\pi}}{r_{\rho DK}} e^{i(\delta_{B}^{D\pi} - \delta_{B}^{DK})}\right)$
- Fractional bin yield:
  - $\bullet \ F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_i \int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by  $B^\pm o D\pi^\pm$
- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D^0})| \cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi |\mathcal{A}(\bar{D^0})|^2}} \quad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D^0})| \sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi |\mathcal{A}(\bar{D^0})|^2}}$$

## Binning Scheme

## Binning scheme

## Binning scheme requirements

## A binning scheme must satisfy the following:

- Avoid dilution of strong phases when integrating over bins
- Enhance interferences between  $B^\pm o D^0 h^\pm$  and  $B^\pm o \bar{D^0} h^\pm$  amplitudes

## How to bin a 5-dimensional phase space?

- Generate C++ code for LHCb amplitude model using AmpGen<sup>1</sup>
- For each  $B^{\pm}$  candidate, calculate

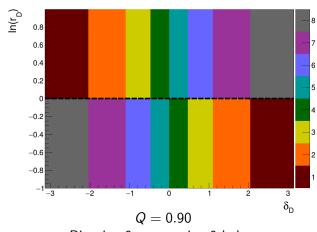
$$\frac{A(D^0)}{A(\bar{D^0})} = r_D e^{i\delta_D}$$

• Bin along  $\delta_D$  and  $r_D$ , maximize Q-value to optimize

<sup>&</sup>lt;sup>1</sup>AmpGen by Tim Evans

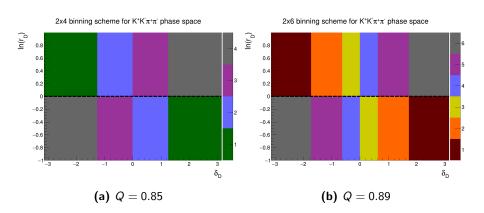
## Binning scheme

2x8 binning scheme for  $K^+K^-\pi^+\pi^-$  phase space



Bins i > 0 on top, i < 0 below

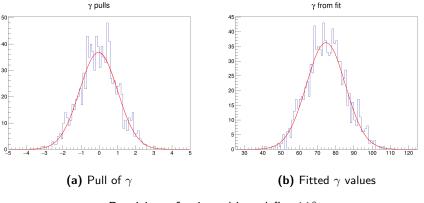
## Binning scheme





## Study of $\gamma$ precision

- ullet Generate 2000  $B^\pm o DK^\pm$  candidates using LHCb model in AmpGen
- Fit back with same model using AmpGen



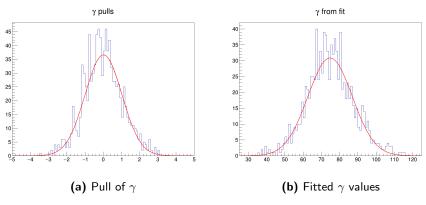
Precision of  $\gamma$  in unbinned fit:  $11^{\circ}$ 

## Study of $\gamma$ precision

• Binned fit setup: Optimized  $2 \times 8$  bins

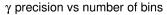
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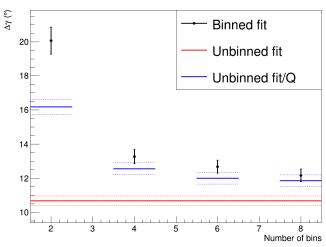
• Fit same AmpGen samples, using  $c_i$ ,  $s_i$  and  $F_i$  from LHCb model



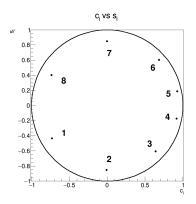
Precision of  $\gamma$  in unbinned fit: 12° Consistent with unbinned fit and Q-value

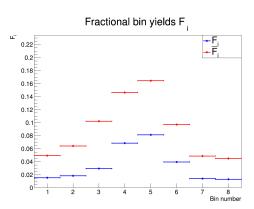
## Comparison of binned fit precision with unbinned fit





## $c_i$ , $s_i$ and $F_i$





$$B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$$
 selection

$$B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$$
 selection

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## Samples

- Stripping lines:
  - $\bullet \ Stripping B2D0PiD2HHHHBeauty 2 Charm Line Decision$
  - StrippingB2D0KD2HHHHBeauty2CharmLineDecision
- Data samples: 2011-2018 (2011-2012 not processed yet)
- MC samples: 2011-2018 (excluding 2015), filtered, AmpGen

#### Initial cuts

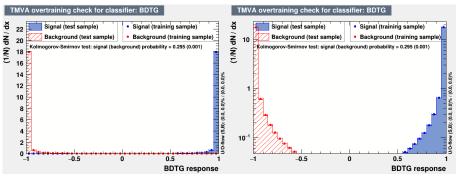
#### Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100 {\sf GeV}$
3	Bachelor has RICH	True
4	D invariant mass	[1839.84, 1889.84]MeV
5	$B^\pm$ invariant mass	[5080, 5800]MeV
6	${\mathcal K}^\pm$ daughter PID	> -10
7	$\pi^\pm$ daughter PID	< 20

#### **Boosted Decision Tree**

- BDTG from TMVA Toolkit
- Signal sample:  $B^\pm \to DK^\pm$  and  $B^\pm \to D\pi^\pm$  MC samples
- Background sample:  $B^{\pm} \to D\pi^{\pm}$  using  $m_{B^{\pm}}^{\mathsf{DTF}} \in [5800, 7000] \mathsf{MeV}$
- Random, equal sized test and training samples

### BDT training results



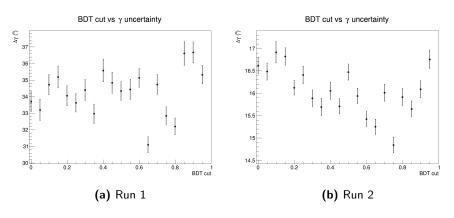
(a) BDT output

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(b) BDT output on a logarithmic scale

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## BDT optimization study



- Run 1: Pick BDT working point at 0.65
- Run 2: Pick BDT working point at 0.75

#### Final cuts

#### Rectangular cuts after BDT

Number	Variable description	Cut
8	${\it K}^{\pm}$ bachelor PID	> 4
9	$\pi^\pm$ bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	$\mathcal{K}^{\pm}$ PID	> 0
13	$K_S^0$ mass veto	[477, 507]MeV

## Backgrounds

## Backgrounds

## D semileptonic backgrounds

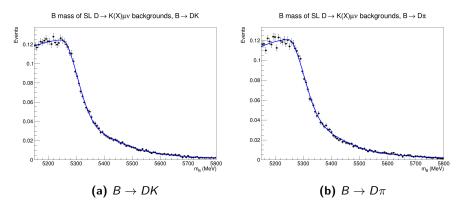
- $B^{\pm} \rightarrow Dh^{\pm}$ ,  $D \rightarrow K(X)I\nu$ ,  $K(X) \rightarrow K\pi\pi$ 
  - $K_1(1270)$
  - $K_1(1400)$
  - K\*(1410)
  - K\*(1680)
  - $K_2^*(1430)$
- Single mis-ID:  $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID:  $K\pi\pi\mu \to KK\pi\pi$

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Generate Rapidsim samples, reweight with PIDCalib2

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## D semileptonic backgrounds



Conclusion: Negligible impact, include in systematics

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## $D o K\pi\pi\pi$ mis-ID background

- $B^{\pm} \rightarrow Dh^{\pm}$ ,  $D \rightarrow K\pi\pi\pi$
- Single mis-ID:  $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID:  $\pi\pi K\pi \to KK\pi\pi$

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Use LHCb MC generated with AmpGen, reweight with PIDCalib2

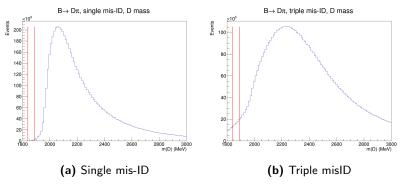


Figure 9: D invariant mass

## $D o K\pi\pi\pi$ mis-ID background

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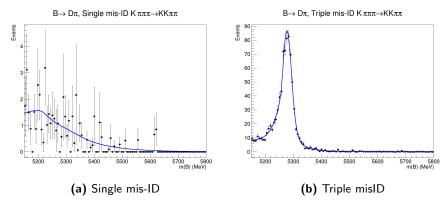
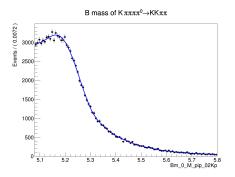


Figure 10: B invariant mass

Conclusion: Negligible impact, include in systematics

## $D \to K\pi\pi\pi\pi^0$ mis-ID background

- $B^{\pm} \rightarrow Dh^{\pm}$ ,  $D \rightarrow K\pi\pi\pi[\pi^0]$
- $\pi^0$  not reconstructed  $\rightarrow$  Lower D mass
- Single mis-ID:  $K\pi\pi\pi \to KK\pi\pi \to Higher D$  mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

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Guy Wilkinson

#### Global fit

## Global fit



## Signal parameterisation

- PDF shape parameterization identical to LHCb-ANA-2020-001
- Signal: Gaussian + Modified Cruijff
- Shape fixed from MC, yield and width floated
- Exponential background

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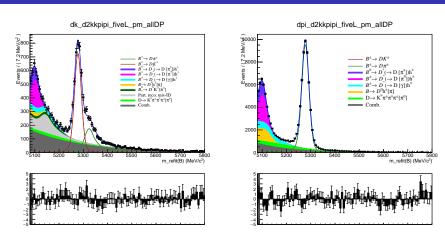
$$f_{\text{MG}}(m|m_B,\sigma,\alpha_L,\alpha_R,\beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m=m-m_B<0\\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m=m-m_B>0 \end{cases}$$

## Partially reconstructed background

- $B^{\pm} \rightarrow D\pi^{\pm}$ :

  - 2  $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])\pi^{\pm}$
  - **3**  $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]\pi^{\pm}$
  - **4**  $B^{\pm} \to (D^{*0} \to D^{0}[\gamma])\pi^{\pm}$
- $B^{\pm} \rightarrow DK^{\pm}$ :
  - **1**  $B^{\pm} \to (D^{*0} \to D^0[\pi^0])K^{\pm}$
  - ②  $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])K^{\pm}$
  - **3**  $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]K^{\pm}$
  - **4**  $B^{\pm} \to (D^{*0} \to D^{0}[\gamma])K^{\pm}$
  - **5**  $B_s^0 \to \bar{D^0}[\pi^+]K^-$
  - **10** Mis-ID from partially reconstructed  $B^\pm o D\pi^\pm$  channel

#### Global fit



**Figure 11:** Global fit of  $B^{\pm}$  mass distribution for the  $DK^{\pm}$  channel (left) and  $D\pi^{\pm}$  channel (right)

- $B^{\pm} \rightarrow DK^{\pm}$  yield: 3543  $\pm$  75
- $B^{\pm} \rightarrow D\pi^{\pm}$  yield:  $47503 \pm 260$

#### Binned CP fit

## Binned CP fit

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#### Binned CP fit

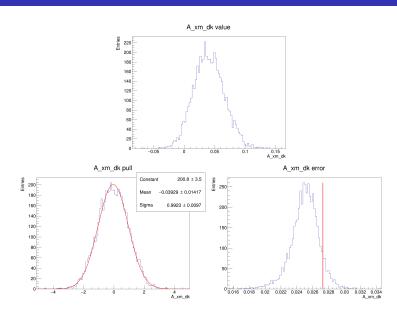
- Use  $2 \times 8$  bins
- $\bullet$   $c_i$  and  $s_i$  calculated using MC integration of LHCb amplitude model
- Fit for CP observables
- PDF shape parameters fixed from global fit
- Yield of signal, low mass partially reconstructed background and combinatorial background floated
- Fractional yields F<sub>i</sub> floated

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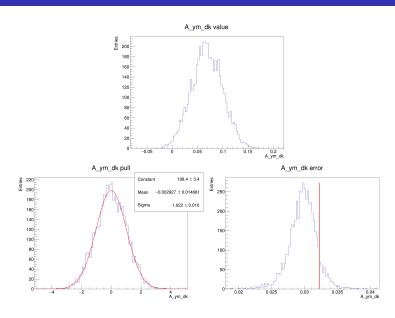
$$\mathcal{R}_i = \begin{cases} F_i, & i = -8 \\ F_i / \sum_{j \ge i}, -8 < i \le +8 \end{cases}$$

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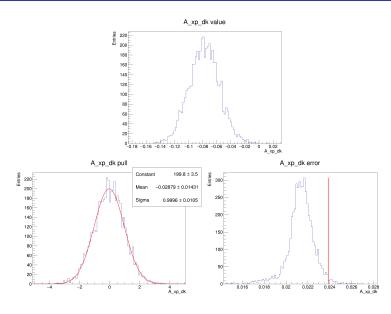
### CP observables result: $x_{-}^{DK}$



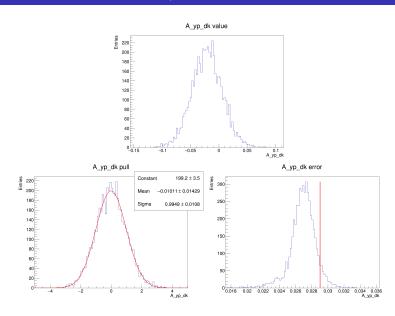
### CP observables result: $y_{-}^{DK}$



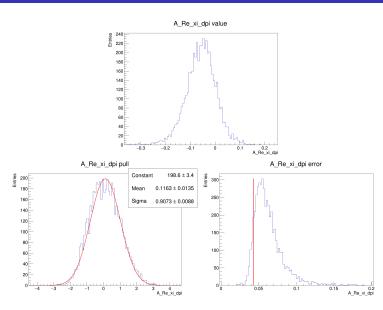
### CP observables result: $x_{\pm}^{DK}$



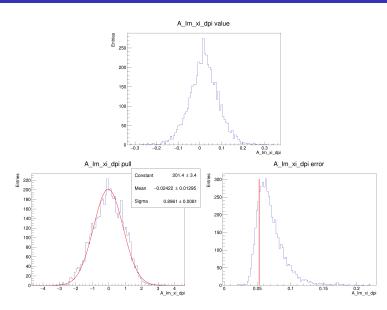
## CP observables result: $y_+^{DK}$



### CP observables result: $x_{\xi}^{D\pi}$



### $\overline{\sf CP}$ observables result: $y_\xi^{D\pi}$



#### Systematics

## Systematics



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#### Summary of all systematics

$x_{-}^{DK}$	$y_{-}^{DK}$	$x_{+}^{DK}$	$y_+^{DK}$	$x_{\xi}^{D\pi}$	$y_{\xi}^{D\pi}$
2.73	3.23	2.38	2.90	4.30	5.27
0.66	1.55	0.32	1.31	1.73	1.03
-0.15	0.05	-0.11	0.03	0.35	-0.25
-0.17	0.03	-0.04	0.01	0.46	-0.18
0.09	0.11	-0.00	-0.18	0.16	0.21
-0.21	-0.05	-0.17	0.01	0.37	-0.11
0.05	-0.09	-0.05	-0.18	0.41	-0.48
0.03	0.03	0.02	0.02	0.04	0.01
0.02	0.03	0.02	0.02	0.01	0.01
0.03	0.03	0.02	0.02	0.04	0.01
0.19	0.03	0.16	-0.04	0.30	-0.16
0.39	0.17	0.27	0.26	0.87	0.64
0.76	1.55	0.41	1.33	1.93	1.21
	2.73 0.66 -0.15 -0.17 0.09 -0.21 0.05 0.03 0.02 0.03 0.19 0.39	2.73 3.23 0.66 1.55 -0.15 0.05 -0.17 0.03 0.09 0.11 -0.21 -0.05 0.05 -0.09 0.03 0.03 0.02 0.03 0.03 0.03 0.19 0.03 0.39 0.17	2.73     3.23     2.38       0.66     1.55     0.32       -0.15     0.05     -0.11       -0.17     0.03     -0.04       0.09     0.11     -0.00       -0.21     -0.05     -0.17       0.05     -0.09     -0.05       0.03     0.03     0.02       0.02     0.03     0.02       0.03     0.03     0.02       0.19     0.03     0.16       0.39     0.17     0.27	2.73     3.23     2.38     2.90       0.66     1.55     0.32     1.31       -0.15     0.05     -0.11     0.03       -0.17     0.03     -0.04     0.01       0.09     0.11     -0.00     -0.18       -0.21     -0.05     -0.17     0.01       0.05     -0.09     -0.05     -0.18       0.03     0.03     0.02     0.02       0.02     0.03     0.02     0.02       0.03     0.03     0.02     0.02       0.19     0.03     0.16     -0.04       0.39     0.17     0.27     0.26	2.73     3.23     2.38     2.90     4.30       0.66     1.55     0.32     1.31     1.73       -0.15     0.05     -0.11     0.03     0.35       -0.17     0.03     -0.04     0.01     0.46       0.09     0.11     -0.00     -0.18     0.16       -0.21     -0.05     -0.17     0.01     0.37       0.05     -0.09     -0.05     -0.18     0.41       0.03     0.03     0.02     0.02     0.04       0.02     0.03     0.02     0.02     0.01       0.03     0.03     0.02     0.02     0.04       0.19     0.03     0.16     -0.04     0.30       0.39     0.17     0.27     0.26     0.87

#### c<sub>i</sub> and s<sub>i</sub> systematic uncertainty

- Originate from (mostly statistical) uncertainty of  $c_i$  and  $s_i$  in BESIII analysis
- By far the largest systematic uncertainty
- Take uncertainties from  $D \to 4\pi$  strong phase analysis and extrapolate to 20 fb $^{-1}$
- Smear  $c_i$  and  $s_i$  and do many fits to data

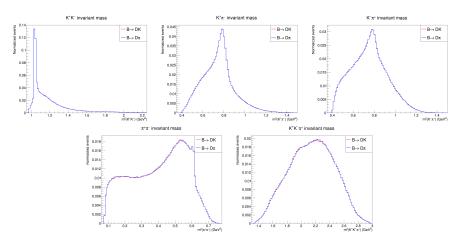
#### Remaining systematic uncertainties

- For these systematics, generate toy datasets, fit with default model and take the bias as a systematic:
  - Small backgrounds  $(D \to K(X) l \nu_l, D \to K \pi \pi \pi, \Lambda_b)$
  - Bin dependent mass shape
  - Low mass physics effects
- For these systematics, do multiple fits to data while smearing parameters:
  - Mass shape
  - Fixed yield fractions

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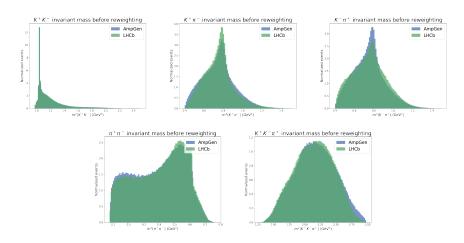
- PID efficiency
- Fit bias: From toys

#### Efficiency difference between $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pi}$



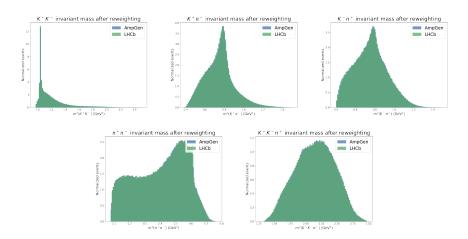
Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered

#### Efficiency correction of $c_i$ and $s_i$



Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

#### Efficiency correction of $c_i$ and $s_i$



After reweighing, use weights to recalculate  $c_i$  and  $s_i$ Conclusion: Efficiency correction of  $c_i$  and  $s_i$  is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered

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#### Summary and conclusion

### Summary and conclusion

#### Interpretation in terms of $\gamma$

Measured CP observables:

$$\begin{split} x_{-}^{DK} &= (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2}, \\ y_{-}^{DK} &= (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2}, \\ x_{+}^{DK} &= (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2}, \\ y_{+}^{DK} &= (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2}, \\ x_{\xi}^{D\pi} &= (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2}, \\ y_{\xi}^{D\pi} &= (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2}, \end{split}$$

- Note: Currently CP observables are determined using c; and s; from the LHCb model
- Publication strategy: Publish current results together with binned vields  $\rightarrow$  Redo fit to obtain model-independent CP observables once c; and s; from BESIII are available

### Interpretation in terms of $\gamma$

• Interpret in terms of physics parameters:

$$\begin{split} \gamma &= (x.x_{-15}^{+14})^{\circ}, \\ \delta_B^{DK} &= (x.x_{-14}^{+15})^{\circ}, \\ r_B^{DK} &= x.x_{-0.018}^{+0.019}, \\ \delta_B^{D\pi} &= (x.x_{-63}^{+117})^{\circ}, \\ r_B^{D\pi} &= x.x_{-0.0024}^{+0.0052}. \end{split}$$

#### Bonus measurement

- The mode  $B^\pm o Dh^\pm$ ,  $D o \pi^+\pi^-\pi^+\pi^-$  very similar
- Run this through <u>same</u> selection (including BDT)
- Can measure GLW CP observables as additional constraints on  $\gamma$ :

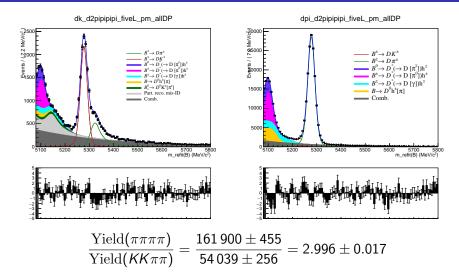
$$A_{h} = \frac{\Gamma(B^{-} \to Dh^{-}) - \Gamma(B^{+} \to Dh^{+})}{\Gamma(B^{-} \to Dh^{-}) + \Gamma(B^{+} \to Dh^{+})},$$

$$R_{\text{CP}} = \frac{R(4\pi)}{R(K3\pi)},$$

$$R = \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}.$$

ullet  $B^\pm o Dh^\pm$ ,  $D o K\pi\pi\pi$  yields provided by Tim Evans

#### Global fit of $B^{\pm} \rightarrow Dh^{\pm}$ , $D \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$



Compare with PDG:  $3.06 \pm 0.16$ 

#### Thank you!

Thank you!

Sneha Malde

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#### Backup

# Backup



### Trigger requirements

Run 1 trigger	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS)
requirements	and (Bu_Hlt1TrackAllLODecision_TOS)
	and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or
	Bu_Hlt2Topo3BodyBBDTDecision_TOS or
	Bu_Hlt2Topo4BodyBBDTDecision_TOS or
	Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS)
requirements	and (Bu_Hlt1TrackMVADecision_TOS or
	Bu_Hlt1TwoTrackMVADecision_TOS)
	and (Bu_Hlt2Topo2BodyDecision_TOS or
	Bu_Hlt2Topo3BodyDecision_TOS or
	Bu_Hlt2Topo4BodyDecision_TOS or
	Bu_Hlt2IncPhiDecision_TOS)

### BDT training variables

Name	Rank (%)	Description
log(DO_RHO_BPV)	7.7	D radial distance to beamline
log(Bu_FDCHI2_OWNPV)	6.3	$B^\pm$ flight distance $\chi^2$
log(Bu_RHO_BPV)	6.1	$B^\pm$ radial distance to beamline
log(Bach_PT)	6.1	Bachelor transverse momentum
Bu_PTASY_1.5	5.3	$B^\pm$ asymmetry parameter
log(1-D0_DIRA_BPV)	5.0	Angle between PV and $\it D$
log(Bu_IPCHI2_OWNPV)	4.8	$B^\pm$ impact parameter $\chi^2$
log(1-Bu_DIRA_BPV)	4.7	Angle between PV and $B^\pm$
log(h[1,2]_PT)	4.4	${\it K}^{\pm}$ transverse momentum
Bu_MAXDOCA	4.4	$B^\pm$ distance of closest approach
log(Bach_IPCHI2_OWNPV)	4.1	Bachelor impact parameter $\chi^2$

#### BDT training particles

Name	Rank (%)	Description
log(Bu_constDOPV_DO_P)	3.7	D momentum from DTF
log(D0_VTXCHI2D0F)	3.3	$D0$ vertex fit $\chi^2$
log(h[3,4]_IPCHI2_OWNPV)	3.3	$\pi^{\pm}$ impact parameter $\chi^2$
log(DO_IPCHI2_OWNPV)	3.2	$D$ impact parameter $\chi^2$
log(h[3,4]_PT)	3.2	$\pi^{\pm}$ transverse momentum
log(Bu_PT)	2.8	$B^\pm$ transverse momentum
log(h[1,2]_P)	2.8	$\mathit{K}^{\pm}$ momentum
log(Bach_P)	2.7	Bachelor momentum
log(Bu_constDOPV_P)	2.6	$B^\pm$ momentum from DTF
log(h[1,2]_IPCHI2_OWNPV)	2.5	$K^\pm$ impact parameter $\chi^2$
DO_MAXDOCA	2.5	D distance of closest approach
log(Bu_VTXCHI2DOF)	2.0	$B^{\pm}$ vertex fit $\chi^2$
log(h[3,4]_P)	1.9	$\pi^{\pm}$ momentum