Determination of the CKM angle γ in $B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$ decays

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B2OC Meeting

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Outline

- 1 Introduction to the CKM angle γ
- 2 Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode
- Binning scheme
- 4 $B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$ selection

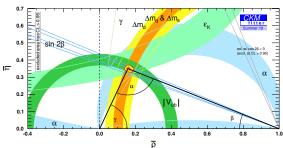
- Backgrounds
- 6 Fit to data
- Systematic uncertainty
- Summary and conclusion

γ and the unitary triangle

ullet Unitarity of CKM matrix: $V_{ud}V_{ub}^*+V_{cd}V_{cb}^*+V_{td}V_{tb}^*=0 \Longrightarrow$

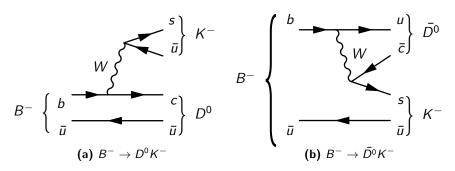
$$\gamma = \mathrm{arg} \Big(- \frac{V_{ud} \, V_{ub}^*}{V_{cd} \, V_{cb}^*} \Big)$$

- Only CKM angle accessible at tree level ⇒
 - Negligible theoretical uncertainties
 - Ideal Standard Model benchmark
 - Compare with indirect measurements



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

Sensitivity through interference



- ullet Superposition of D^0 and $ar{D^0}$
- ullet b o uar cs and b o car us interference o Sensitivity to γ

$$\mathcal{A}(B^{-}) = \mathcal{A}(D^{0}) + r_{B}e^{i(\delta_{B}-\gamma)}\mathcal{A}(\bar{D^{0}})$$

$$\mathcal{A}(B^{+}) = \mathcal{A}(\bar{D^{0}}) + r_{B}e^{i(\delta_{B}+\gamma)}\mathcal{A}(D^{0})$$

Measurement of γ from $B^{\pm} \to DK^{\pm}$, $D \to K^+K^-\pi^+\pi^-$

- First proposed by J. Rademacker and G. Wilkinson
 - arXiv:hep-ph/0611272
 - Amplitude model by FOCUS
 - Expected γ precision with 1000 candidates: 14 $^{\circ}$
- CLEO amplitude analysis
 - arXiv:1201.5716
 - ullet Expected γ precision with 2000 candidates: 11°
- State of the art amplitude analysis by LHCb :
 - LHCb-PAPER-2018-041

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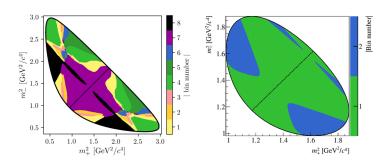
Use to develop efficient binning scheme

The $D \to K^+ K^- \pi^+ \pi^-$ decay

Binned
$$\gamma$$
 analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

Binned measurement of γ

- Final measurement will be model-independent
 - \bullet Poor binning reduces statistical sensitivity \to No bias!
- Need strong phases of D decay \rightarrow Measure at BESIII
- LHCb-PAPER-2020-019: $B^\pm o Dh^\pm$, $D o K^0_S h^+ h^-$
 - Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^{\circ}$



The BPGGSZ method

• $B^{\pm} \rightarrow Dh^{\pm}$ amplitude:

$$\begin{split} \mathcal{A}(B^-) &= \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D^0}) \\ \mathcal{A}(B^+) &= \mathcal{A}(\bar{D^0}) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0) \end{split}$$

- ullet $\mathcal{A}(D^0)$ and $\mathcal{A}(ar{D^0})$ depend on D phase space
- ullet Strong-phase difference of D^0 and $ar{D^0}$ decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

Event yield in bin i

$$\begin{split} N_i^- &= h_{B^-} \Big(F_i + \big(x_-^2 + y_-^2 \big) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} \big(x_- c_i + y_- s_i \big) \Big) \\ N_{-i}^+ &= h_{B^+} \Big(F_i + \big(x_+^2 + y_+^2 \big) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} \big(x_+ c_i + y_+ s_i \big) \Big) \end{split}$$

The BPGGSZ method

Event yield in bin i

$$\begin{split} N_i^- &= h_{B^-} \big(F_i + (x_-^2 + y_-^2) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i) \big) \\ N_{-i}^+ &= h_{B^+} \big(F_i + (x_+^2 + y_+^2) \bar{F}_i + 2 \sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i) \big) \end{split}$$

- CP observables:
 - $\begin{array}{l} \bullet \ \ x_{\pm}^{DK} = r_B^{DK} \cos \left(\delta_B^{DK} \pm \gamma \right), \quad \ y_{\pm}^{DK} = r_B^{DK} \sin \left(\delta_B^{DK} \pm \gamma \right) \\ \bullet \ \ x_{\xi}^{D\pi} = \operatorname{Re}(\xi^{D\pi}), \ \ y_{\xi}^{D\pi} = \operatorname{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{B\pi}}{r_D^{DK}} e^{i(\delta_B^{D\pi} \delta_B^{DK})} \right) \end{array}$
- Fractional bin yield:
 - $\bullet \ \ F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$

- Floated in the fit, mostly constrained by $B^\pm o D\pi^\pm$
- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D^0})| \cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi |\mathcal{A}(\bar{D^0})|^2}} \quad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D^0})| \sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi |\mathcal{A}(\bar{D^0})|^2}}$$

Binning Scheme

Binning scheme

Binning scheme requirements

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^\pm \to D^0 h^\pm$ and $B^\pm \to \bar{D^0} h^\pm$

How to bin a 5-dimensional phase space?

- Generate C++ code for LHCb amplitude model using AmpGen¹
- For each B^{\pm} candidate, calculate

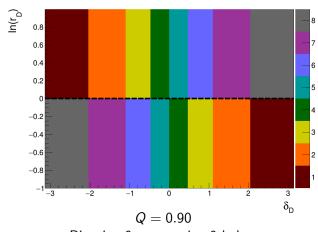
$$\frac{A(D^0)}{A(\bar{D^0})} = r_D e^{i\delta_D}$$

• Bin along δ_D and r_D , maximize Q-value to optimize

¹AmpGen by Tim Evans

Binning scheme

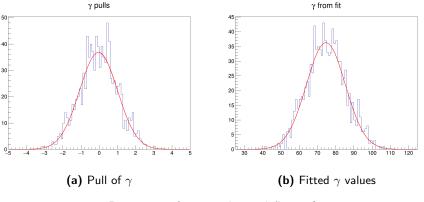
2x8 binning scheme for $K^+K^-\pi^+\pi^-$ phase space



Bins i < 0 on top, i > 0 below

γ precision benchmark

- ullet Generate 2000 $B^\pm o DK^\pm$ candidates using LHCb model in AmpGen
- Fit back with same model using AmpGen



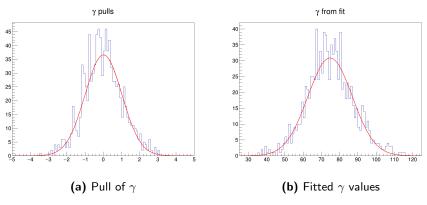
Precision of γ in unbinned fit: 11°

Study of γ precision

Binned fit setup: Optimized 2 × 8 bins

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• Fit same AmpGen samples, using c_i , s_i and F_i from LHCb model



Precision of γ in binned fit: 12° Consistent with unbinned fit and Q-value

$$B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$$
 selection

$$B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$$
 selection

Samples

- Data sample: Full Run 1 and 2
- MC samples: Full Run 1 and 2 excluding 2015
 - AmpGen model
 - Large filtered samples

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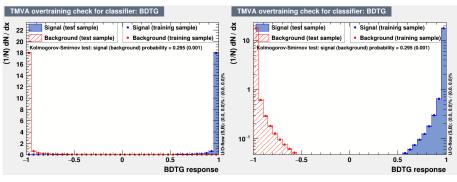
Stripping lines

StrippingB2D0PiD2HHHHBeauty2CharmLineDecision StrippingB2D0KD2HHHHBeauty2CharmLineDecision

Boosted Decision Tree

- BDTG from TMVA Toolkit
- Signal sample: $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ MC samples
- Background sample: Data sample with $m_{B^{\pm}}^{\text{DTF}} \in [5800, 7000] \text{MeV}$
- Random, equal sized test and training samples

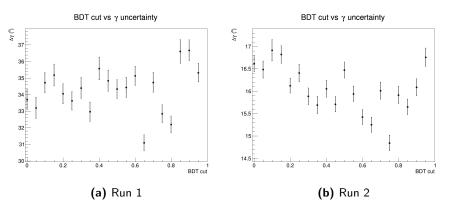
BDT training results



(a) BDT output

(b) BDT output on a logarithmic scale

BDT optimization study



- Run 1: Pick BDT working point at 0.65
- Run 2: Pick BDT working point at 0.75

Backgrounds

Backgrounds

$D o K\pi\pi\pi$ mis-ID background

- $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K\pi\pi\pi$
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID: $\pi\pi K\pi \to KK\pi\pi$

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Use LHCb MC generated with AmpGen, reweight with PIDCalib2

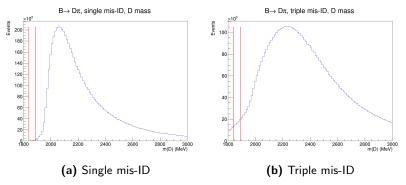


Figure 6: D invariant mass

$D \to K \pi \pi \pi$ mis-ID background

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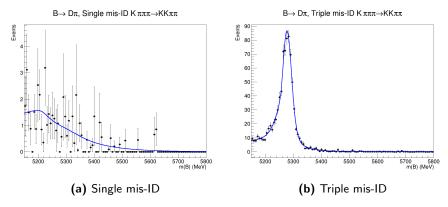


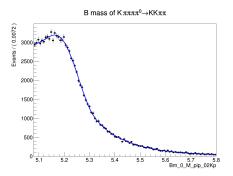
Figure 7: B invariant mass

Conclusion: Negligible impact, include in systematics

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$D o K\pi\pi\pi\pi^0$ mis-ID background

- $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K\pi\pi\pi[\pi^0]$
- π^0 not reconstructed \to Lower D mass
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi \to Higher\ D$ mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

Martin Tat

Global fit

Global fit



Signal parameterisation

- PDF shape parameterization identical to LHCb-ANA-2020-001
- Signal: Gaussian + Modified Cruijff
- Shape fixed from MC, yield and width floated
- Exponential background

$$f_{\text{MG}}(m|m_B,\sigma,\alpha_L,\alpha_R,\beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m=m-m_B<0\\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m=m-m_B>0 \end{cases}$$

Partially reconstructed background

- $B^{\pm} \rightarrow D\pi^{\pm}$:
 - **1** $B^{\pm} \to (D^{*0} \to D^0[\pi^0])\pi^{\pm}$
 - ② $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])\pi^{\pm}$
 - **3** $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]\pi^{\pm}$
 - **4** $B^{\pm} \to (D^{*0} \to D^0[\gamma])\pi^{\pm}$
- $B^{\pm} \rightarrow DK^{\pm}$:
 - **1** $B^{\pm} \to (D^{*0} \to D^0[\pi^0])K^{\pm}$
 - ② $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])K^{\pm}$
 - **3** $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]K^{\pm}$
 - **4** $B^{\pm} \to (D^{*0} \to D^{0}[\gamma])K^{\pm}$

- **5** $B_s^0 \to \bar{D^0}[\pi^+]K^-$
- **10** Mis-ID from partially reconstructed $B^\pm o D\pi^\pm$ channel

Global fit

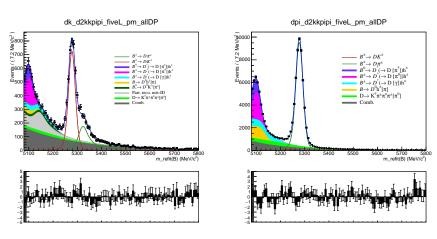


Figure 8: $B^{\pm} \rightarrow DK^{\pm}$ channel (left) and $B^{\pm} \rightarrow D\pi^{\pm}$ channel (right)

- $B^{\pm} \rightarrow DK^{\pm}$ yield: 3543 ± 75
- $B^{\pm} \to D\pi^{\pm}$ yield: 47 503 \pm 260 Sneha Malde

Binned CP fit

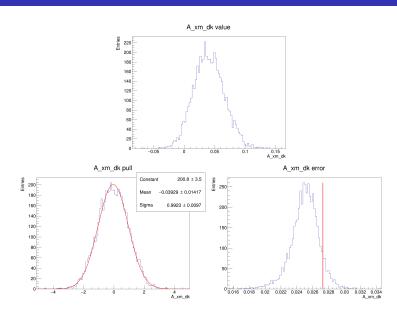
Binned CP fit

Binned CP fit

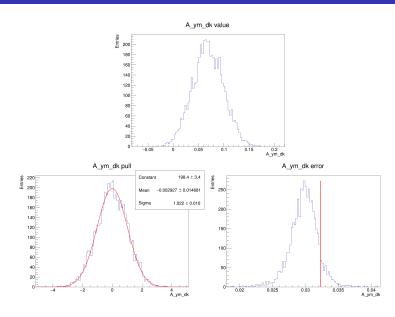
- Use 2×8 bins
- c_i and s_i calculated using MC integration of LHCb amplitude model
- Fit for CP observables
- PDF shape parameters fixed from global fit
- Yield of signal, low mass partially reconstructed background and combinatorial background floated
- Fractional yields F; floated

$$\mathcal{R}_{i} = \begin{cases} F_{i}, & i = -8 \\ F_{i} / \sum_{j \geq i}, -8 < i \leq +8 \end{cases}$$

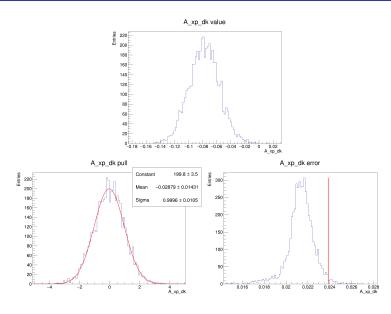
CP observables result: x_{-}^{DK}



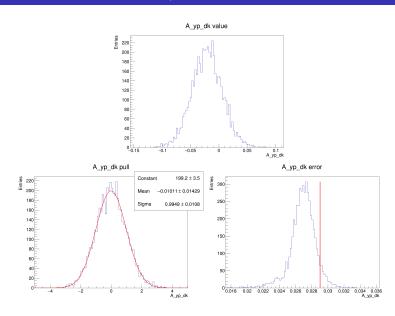
CP observables result: y_{-}^{DK}



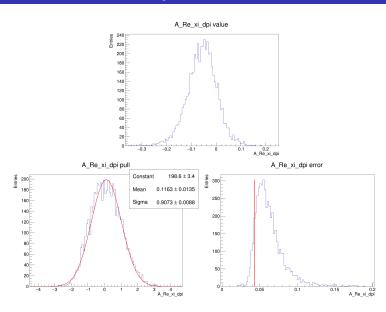
CP observables result: x_{\pm}^{DK}



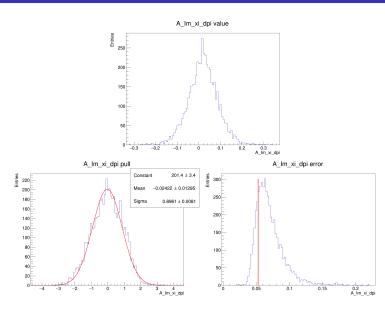
CP observables result: y_+^{DK}



CP observables result: $x_{\xi}^{D\pi}$



$\overline{\sf CP}$ observables result: $y_\xi^{D\pi}$



Systematic uncertainties

Systematic uncertainties

c; and s; systematic uncertainty

- Uncertainty of c_i and s_i in BESIII analysis (mostly statistical)
- Largest systematic uncertainty
- Take uncertainties from $D \to 4\pi$ strong phase analysis and extrapolate to $20 \, \mathrm{fb}^{-1}$
- Smear c_i and s_i and do many fits to data

Remaining systematic uncertainties

Different strategies for evaluating systematic uncertainties:

- Generate toy datasets with systematics, fit with default model and take the bias as a systematic:
 - Small backgrounds $(D \to K(X)l\nu_l, D \to K\pi\pi\pi, B \to Dl\nu_l, \Lambda_b)$
 - Bin dependent mass shape
 - Low mass physics effects
- Do multiple fits to data while smearing parameters:
 - c; and s;
 - Mass shape
 - Fixed yield fractions

- PID efficiency
- Fit bias: Take bias toys as systematic uncertainty

Summary of all systematic uncertainties

Source	χ_{-}^{DK}	y_{-}^{DK}	x_+^{DK}	y_+^{DK}	$x_{\xi}^{D\pi}$	$y_{\xi}^{D\pi}$
Statistical	2.73	3.23	2.38	2.90	4.30	5.27
C_i, S_i	0.66	1.55	0.32	1.31	1.73	1.03
$B^\pm o D\mu u$ background	0.04	0.03	0.02	0.15	0.30	0.10
$D o K(X) I u_I$ background	0.15	0.05	0.11	0.03	0.35	0.25
$D o K\pi\pi\pi$ background	0.17	0.03	0.04	0.01	0.46	0.18
Λ_b background	0.09	0.11	0.00	0.18	0.16	0.21
Bin dependent mass shape	0.21	0.05	0.17	0.01	0.37	0.11
Fit bias	0.19	0.03	0.16	0.04	0.30	0.16
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
Low mass physics effects	0.05	0.09	0.05	0.18	0.41	0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Total LHCb systematic	0.39	0.17	0.27	0.30	0.92	0.65
Total systematic	0.77	1.55	0.41	1.34	1.96	1.22

Summary and conclusion

Summary and conclusion

Summary of CP observables

Measured CP observables:

$$\begin{split} x_{-}^{DK} = & (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2}, \\ y_{-}^{DK} = & (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2}, \\ x_{+}^{DK} = & (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2}, \\ y_{+}^{DK} = & (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2}, \\ x_{\xi}^{D\pi} = & (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2}, \\ y_{\xi}^{D\pi} = & (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2}, \end{split}$$

- Note: Currently using c_i and s_i from the LHCb model
- Publication strategy: Publish current results together with binned yields \rightarrow Redo fit to obtain model-independent CP observables once c_i and s_i from BESIII are available

Interpretation in terms of γ

• Interpret in terms of physics parameters:

$$\begin{split} \gamma &= (x.x_{-15}^{+14})^{\circ}, \\ \delta_B^{DK} &= (x.x_{-14}^{+15})^{\circ}, \\ r_B^{DK} &= x.x_{-0.018}^{+0.019}, \\ \delta_B^{D\pi} &= (x.x_{-63}^{+117})^{\circ}, \\ r_B^{D\pi} &= x.x_{-0.0024}^{+0.0052}. \end{split}$$

Bonus measurement

- The mode $B^\pm o Dh^\pm$, $D o \pi^+\pi^-\pi^+\pi^-$ very similar
- Run this through same selection (including BDT)
- Can measure GLW CP observables as additional constraints on γ :

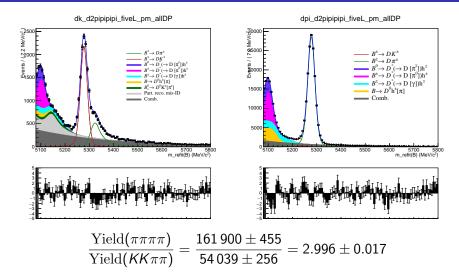
$$A_{h} = \frac{\Gamma(B^{-} \to Dh^{-}) - \Gamma(B^{+} \to Dh^{+})}{\Gamma(B^{-} \to Dh^{-}) + \Gamma(B^{+} \to Dh^{+})},$$

$$R_{\text{CP}} = \frac{R(4\pi)}{R(K3\pi)},$$

$$R = \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}.$$

• $B^\pm \to D h^\pm$, $D \to K \pi \pi \pi$ yields provided by Tim Evans

Global fit of $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow \pi^{+}\pi^{-}\pi^{+}\pi^{-}$



Compare with PDG: 3.06 ± 0.16

Thank you!

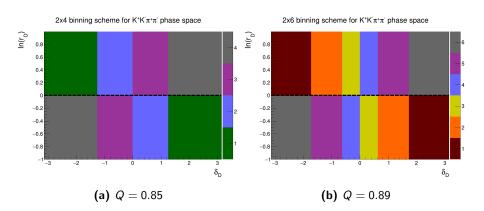
Thank you!

Backup

Backup

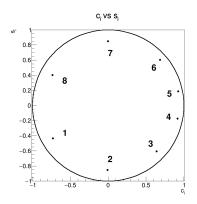


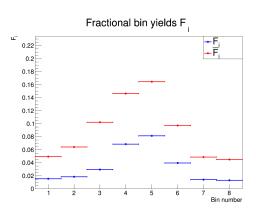
Binning scheme



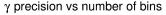


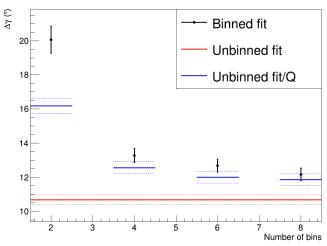
c_i , s_i and F_i





Comparison of binned fit precision with unbinned fit





Trigger requirements

Run 1 trigger	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS)
requirements	and (Bu_Hlt1TrackAllLODecision_TOS)
	and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or
	Bu_Hlt2Topo3BodyBBDTDecision_TOS or
	Bu_Hlt2Topo4BodyBBDTDecision_TOS or
	Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS)
requirements	and (Bu_Hlt1TrackMVADecision_TOS or
	Bu_Hlt1TwoTrackMVADecision_TOS)
	and (Bu_Hlt2Topo2BodyDecision_TOS or
	Bu_Hlt2Topo3BodyDecision_TOS or
	Bu_Hlt2Topo4BodyDecision_TOS or
	Bu_Hlt2IncPhiDecision_TOS)

Initial cuts

Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100 {\sf GeV}$
3	Bachelor has RICH	True
4	D invariant mass	[1839.84, 1889.84]MeV
5	B^\pm invariant mass	[5080, 5800]MeV
6	${\mathcal K}^\pm$ daughter PID	> -10
7	π^\pm daughter PID	< 20

Final cuts

Rectangular cuts after BDT

Number	Variable description	Cut
8	${\it K}^{\pm}$ bachelor PID	> 4
9	π^\pm bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	\mathcal{K}^{\pm} PID	> 0
13	$\mathcal{K}_{\mathcal{S}}^{0}$ mass veto	[477, 507]MeV

BDT training variables

Name	Rank (%)	Description
log(DO_RHO_BPV)	7.7	D radial distance to beamline
log(Bu_FDCHI2_OWNPV)	6.3	B^\pm flight distance χ^2
log(Bu_RHO_BPV)	6.1	B^\pm radial distance to beamline
log(Bach_PT)	6.1	Bachelor transverse momentum
Bu_PTASY_1.5	5.3	B^\pm asymmetry parameter
log(1-DO_DIRA_BPV)	5.0	Angle between PV and $\it D$
log(Bu_IPCHI2_OWNPV)	4.8	B^\pm impact parameter χ^2
log(1-Bu_DIRA_BPV)	4.7	Angle between PV and B^\pm
log(h[1,2]_PT)	4.4	${\it K}^{\pm}$ transverse momentum
Bu_MAXDOCA	4.4	B^\pm distance of closest approach
log(Bach_IPCHI2_OWNPV)	4.1	Bachelor impact parameter χ^2

BDT training particles

Name	Rank (%)	Description
log(Bu_constDOPV_DO_P)	3.7	D momentum from DTF
log(DO_VTXCHI2DOF)	3.3	D0 vertex fit χ^2
log(h[3,4]_IPCHI2_OWNPV)	3.3	π^{\pm} impact parameter χ^2
log(DO_IPCHI2_OWNPV)	3.2	D impact parameter χ^2
log(h[3,4]_PT)	3.2	π^{\pm} transverse momentum
log(Bu_PT)	2.8	B^\pm transverse momentum
log(h[1,2]_P)	2.8	\mathcal{K}^{\pm} momentum
log(Bach_P)	2.7	Bachelor momentum
log(Bu_constDOPV_P)	2.6	B^\pm momentum from DTF
log(h[1,2]_IPCHI2_OWNPV)	2.5	K^{\pm} impact parameter χ^2
DO_MAXDOCA	2.5	D distance of closest approach
log(Bu_VTXCHI2DOF)	2.0	B^{\pm} vertex fit χ^2
log(h[3,4]_P)	1.9	π^{\pm} momentum

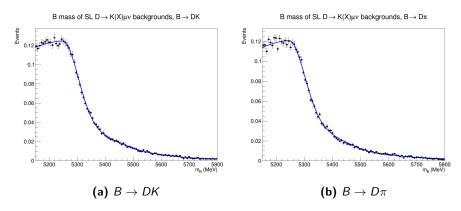
D semileptonic backgrounds

- $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K(X)I\nu$, $K(X) \rightarrow K\pi\pi$
 - $K_1(1270)$
 - $K_1(1400)$
 - K*(1410)
 - K*(1680)
 - $K_2^*(1430)$
- Single mis-ID: $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID: $K\pi\pi\mu \to KK\pi\pi$

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Generate Rapidsim samples, reweight with PIDCalib2

D semileptonic backgrounds



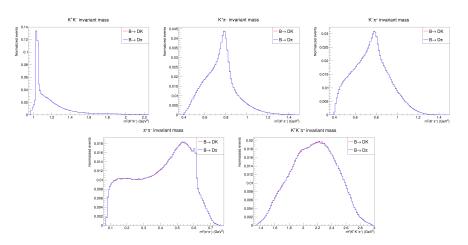
Conclusion: Negligible impact, include in systematics

Efficiency related systematics

Efficiency related systematics:

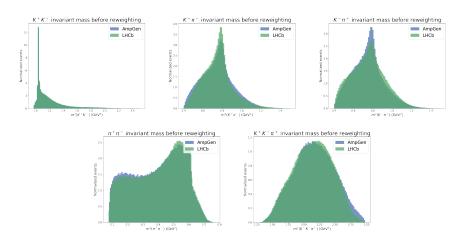
- ullet Difference in $B^\pm o DK^\pm$ and $B^\pm o D\pi^\pm$ phase space acceptance
- Efficiency correction of c_i and s_i

Efficiency differences between $B^\pm o DK^\pm$ and $B^\pm o D\pi^\pm$



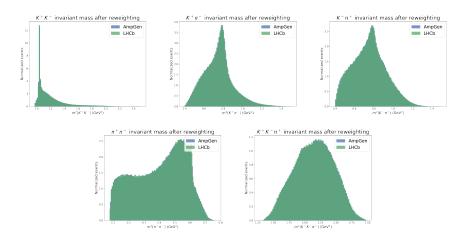
Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered

Efficiency correction of c_i and s_i



Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

Efficiency correction of c_i and s_i



After reweighing, use weights to recalculate c_i and s_i Conclusion: Efficiency correction of c_i and s_i is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered