Determination of the CKM angle γ in $B^{\pm} \to (K^{+}K^{-}\pi^{+}\pi^{-})_{D}h^{\pm}$ decays

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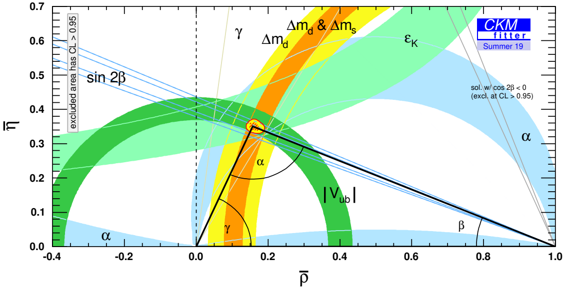
Outline

γ and the unitary triangle

- Unitarity of CKM matrix: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \implies$

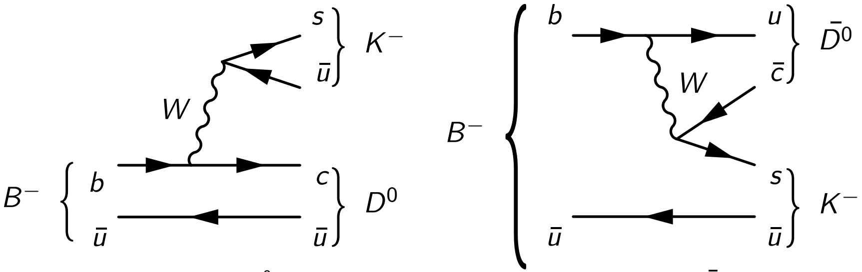
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- Only CKM angle accessible at tree level $\implies$
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark
  - Compare with indirect measurements



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

Sensitivity through interference



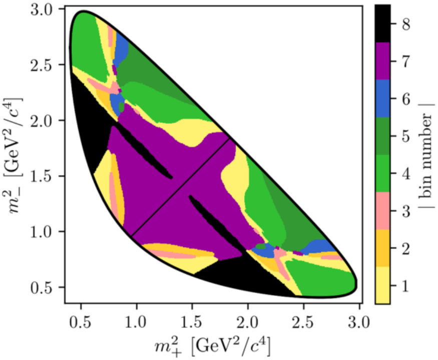
(a) $B^- \to D^0 K^-$ **(b)** $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

Binned measurement of γ

- Enhance sensitivity through binning of phase space
- Need strong phases of D decay $\rightarrow$ Measure at BESIII!
- LHCb-PAPER-2020-019: $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K_S^0 h^+ h^-$
  - Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^{\circ}$



Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

The $D \to K^+K^-\pi^+\pi^-$ decay

Measurement of γ from $B^\pm \to DK^\pm$, $D \to K^+K^-\pi^+\pi^-$

- First proposed by J. Rademacker and G. Wilkinson
  - arXiv:hep-ph/0611272
  - Amplitude model by FOCUS
  - Expected γ precision with 1000 candidates: 14°
- CLEO amplitude analysis
  - arXiv:1201.5716
  - Expected γ precision with 2000 candidates: 11°
- State of the art amplitude analysis by LHCb LHCb-PAPER-2018-041:
  - Used to develop efficient binning scheme in this analysis
  - Final measurement will be model-independent
  - Poor binning reduces statistical sensitivity $\to$ No bias!

The BPGGSZ method

- $B^{\pm} \to Dh^{\pm}$ amplitude:

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

- $\mathcal{A}(D^0)$ and $\mathcal{A}(\bar{D}^0)$ depend on D phase space
- Strong-phase difference of D^0 and $\bar{D}^0$ decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i \bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i \bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

The BPGGSZ method

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

Binning scheme

Binning scheme requirements

A binning scheme must satisfy the following:

- Avoid dilution of strong phases when integrating over bins
- Enhance interferences between $B^\pm \to D^0 h^\pm$ and $B^\pm \to \bar{D}^0 h^\pm$ amplitudes

How to bin a 5-dimensional phase space?

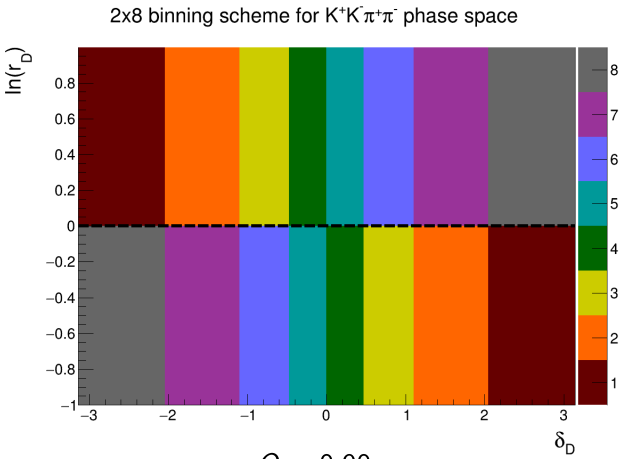
- Generate C++ code for LHCb amplitude model using AmpGen[1]
- For each $B^\pm$ candidate, calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along δ_D and r_D, maximize Q-value to optimize

[1] AmpGen by Tim Evans

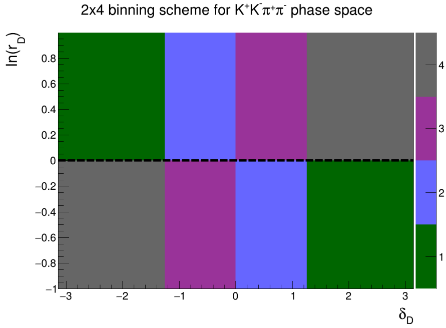
Binning scheme



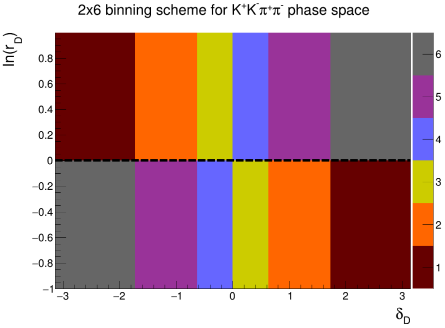
$Q = 0.90$

Bins $i > 0$ on top, $i < 0$ below

Binning scheme



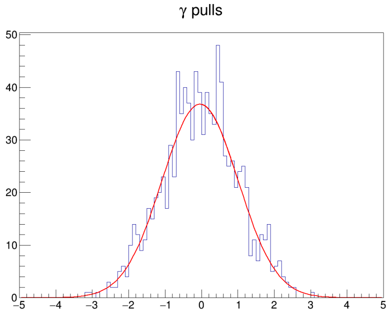
(a) $Q = 0.85$



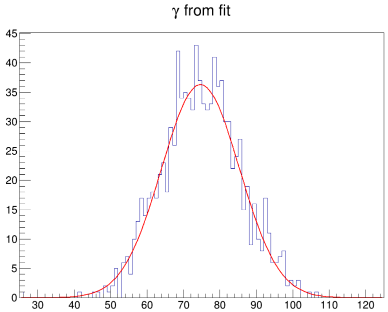
(b) $Q = 0.89$

Study of γ precision

- Generate 2000 $B^{\pm} \to DK^{\pm}$ candidates using LHCb model in AmpGen
- Fit back with same model using AmpGen



(a) Pull of γ

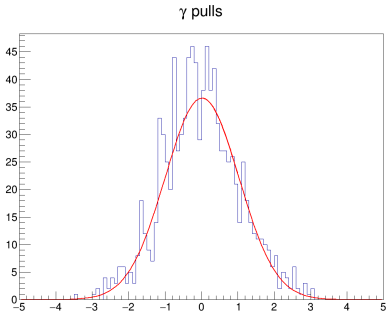


(b) Fitted γ values

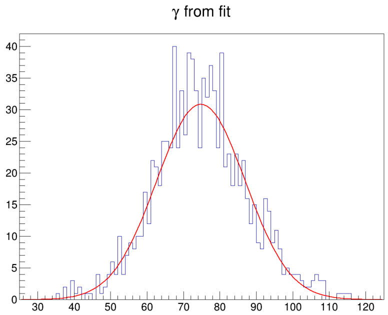
Precision of γ in unbinned fit: 11°

Study of γ precision

- Binned fit setup: Optimized 2×8 bins
- Fit same AmpGen samples, using c_i, s_i and F_i from LHCb model



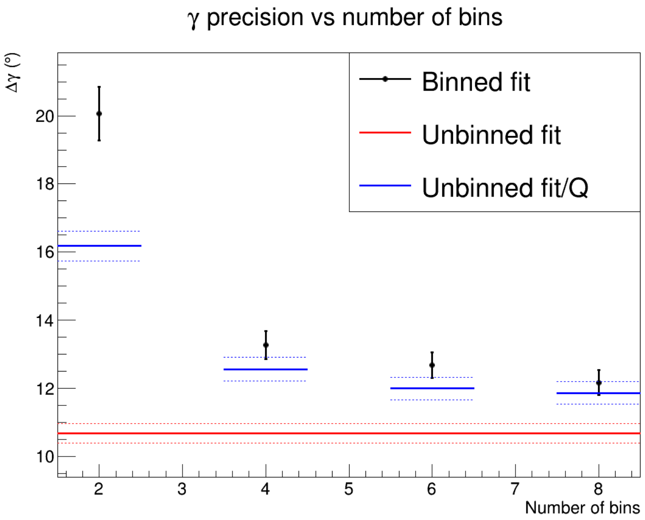
(a) Pull of γ



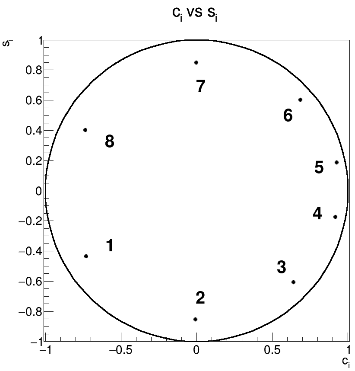
(b) Fitted γ values

Precision of γ in unbinned fit: 12°
Consistent with unbinned fit and Q-value

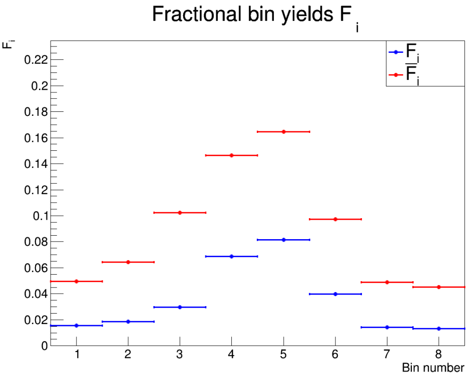
Comparison of binned fit precision with unbinned fit



c_i, s_i and F_i



c_i vs s_i
s_i
c_i
1
2
3
4
5
6
7
8



Fractional bin yields F_i
F_i
Bin number

$B^\pm \to (K^+K^-\pi^+\pi^-)_D h^\pm$ selection

Samples

- Stripping lines:
  - StrippingB2D0PiD2HHHHBeauty2CharmLineDecision
  - StrippingB2D0KD2HHHHBeauty2CharmLineDecision
- Data samples: 2011-2018 (2011-2012 not processed yet)
- MC samples: 2011-2018 (excluding 2015), filtered, AmpGen

Initial cuts

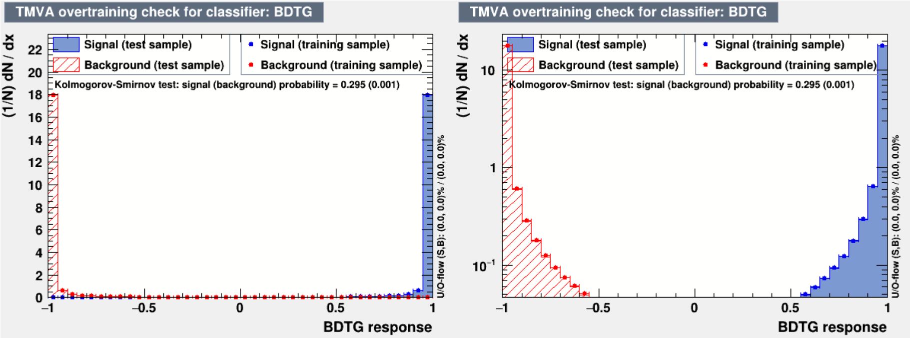
Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100\text{GeV}$
3	Bachelor has RICH	True
4	D invariant mass	$[1839.84, 1889.84]\text{MeV}$
5	$B^{\pm}$ invariant mass	$[5080, 5800]\text{MeV}$
6	$K^{\pm}$ daughter PID	> -10
7	$\pi^{\pm}$ daughter PID	< 20

Boosted Decision Tree

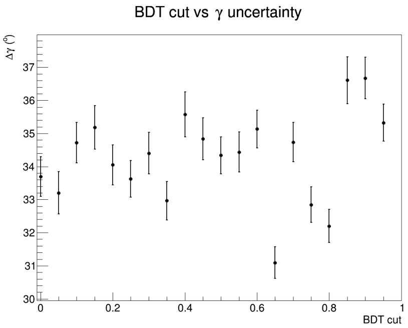
- BDTG from TMVA Toolkit
- Signal sample: $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ MC samples
- Background sample: $B^\pm \to D\pi^\pm$ using $m_{B^\pm}^{\mathrm{DTF}} \in [5800, 7000]\mathrm{MeV}$
- Random, equal sized test and training samples

BDT training results

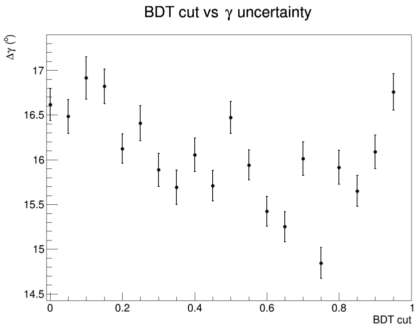


(a) BDT output **(b)** BDT output on a logarithmic scale

BDT optimization study



(a) Run 1



(b) Run 2

- Run 1: Pick BDT working point at 0.65
- Run 2: Pick BDT working point at 0.75

Final cuts

Rectangular cuts after BDT

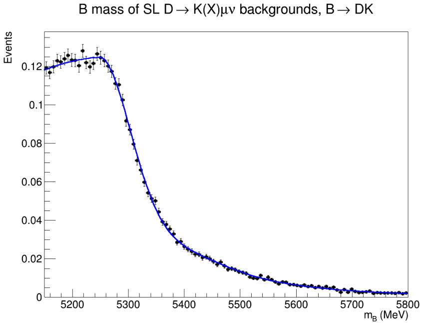
Number	Variable description	Cut
8	$K^{\pm}$ bachelor PID	> 4
9	$\pi^{\pm}$ bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	$K^{\pm}$ PID	> 0
13	K_S^0 mass veto	$[477, 507]$MeV

Backgrounds

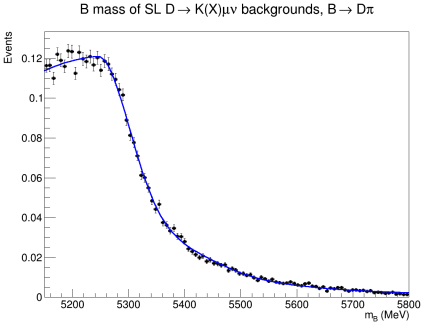
D semileptonic backgrounds

- $B^{\pm} \to Dh^{\pm}$, $D \to K(X)l\nu$, $K(X) \to K\pi\pi$
  - $K_1(1270)$
  - $K_1(1400)$
  - $K^*(1410)$
  - $K^*(1680)$
  - $K_2^*(1430)$
- Single mis-ID: $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID: $K\pi\pi\mu \to KK\pi\pi$
- Generate Rapidsim samples, reweight with PIDCalib2

D semileptonic backgrounds



(a) $B \to DK$

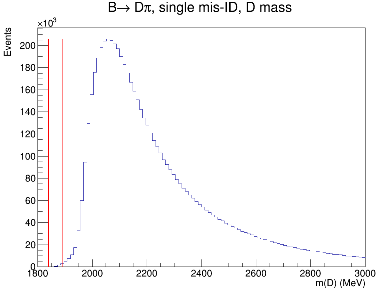


(b) $B \to D\pi$

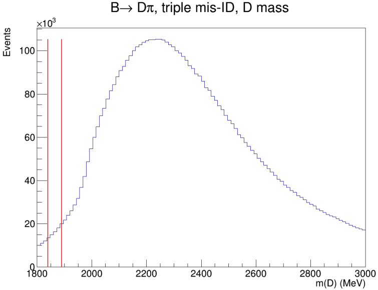
Conclusion: Negligible impact, include in systematics

$D \to K\pi\pi\pi$ mis-ID background

- $B^{\pm} \to Dh^{\pm}$, $D \to K\pi\pi\pi$
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID: $\pi\pi K\pi \to KK\pi\pi$
- Use LHCb MC generated with AmpGen, reweight with PIDCalib2



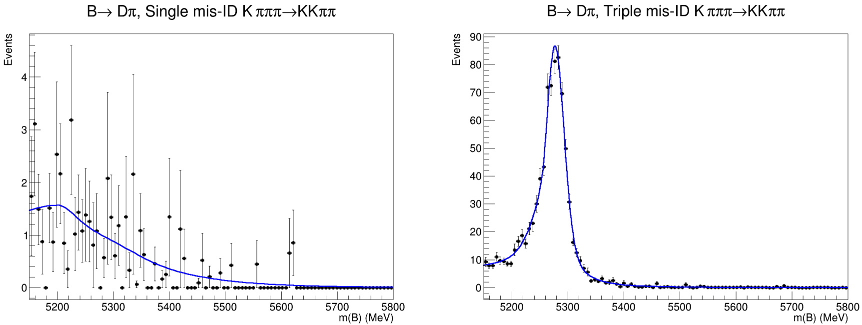
(a) Single mis-ID



(b) Triple misID

Figure 9: D invariant mass

$D \to K\pi\pi\pi$ mis-ID background



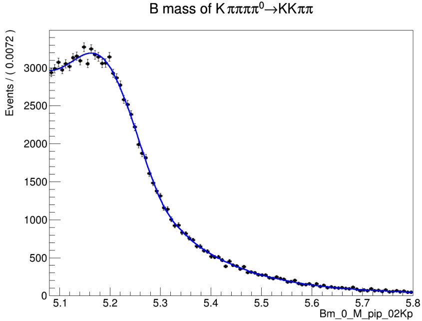
(a) Single mis-ID **(b)** Triple misID

Figure 10: B invariant mass

Conclusion: Negligible impact, include in systematics

$D \to K\pi\pi\pi^0$ mis-ID background

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi[\pi^0]$
- π^0 not reconstructed $\to$ Lower D mass
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi \to$ Higher D mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

Global fit

Signal parameterisation

- PDF shape parameterization identical to LHCb-ANA-2020-001
- Signal: Gaussian + Modified Cruijff
- Shape fixed from MC, yield and width floated
- Exponential background

$$f_{\mathrm{MG}}(m|m_B,\sigma,\alpha_L,\alpha_R,\beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m = m - m_B < 0 \\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m = m - m_B > 0 \end{cases}$$

Partially reconstructed background

- $B^{\pm} \rightarrow D\pi^{\pm}$:
  1. $B^{\pm} \rightarrow (D^{*0} \rightarrow D^0[\pi^0])\pi^{\pm}$
  2. $B^0 \rightarrow (D^{*\mp} \rightarrow D^0[\pi^{\mp}])\pi^{\pm}$
  3. $B^{\pm(0)} \rightarrow D^0[\pi^{0(\mp)}]\pi^{\pm}$
  4. $B^{\pm} \rightarrow (D^{*0} \rightarrow D^0[\gamma])\pi^{\pm}$
- $B^{\pm} \rightarrow DK^{\pm}$:
  1. $B^{\pm} \rightarrow (D^{*0} \rightarrow D^0[\pi^0])K^{\pm}$
  2. $B^0 \rightarrow (D^{*\mp} \rightarrow D^0[\pi^{\mp}])K^{\pm}$
  3. $B^{\pm(0)} \rightarrow D^0[\pi^{0(\mp)}]K^{\pm}$
  4. $B^{\pm} \rightarrow (D^{*0} \rightarrow D^0[\gamma])K^{\pm}$
  5. $B_s^0 \rightarrow \bar{D}^0[\pi^+]K^-$
  6. Mis-ID from partially reconstructed $B^{\pm} \rightarrow D\pi^{\pm}$ channel

Global fit

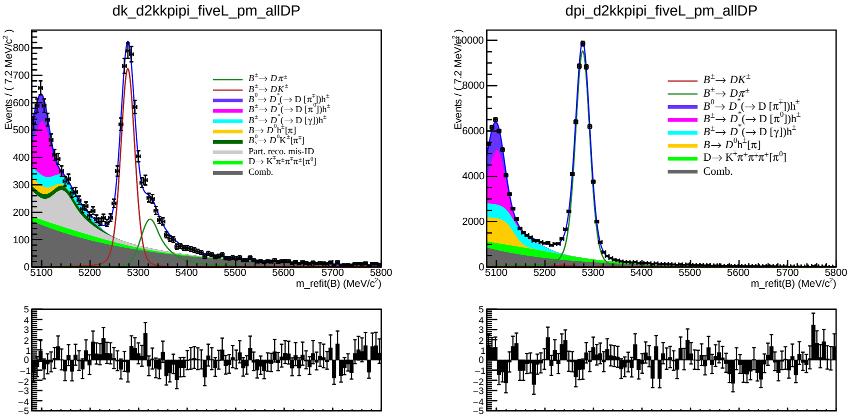


Figure 11: Global fit of $B^\pm$ mass distribution for the $DK^\pm$ channel (left) and $D\pi^\pm$ channel (right)

- $B^\pm \to DK^\pm$ yield: 3543 ± 75
- $B^\pm \to D\pi^\pm$ yield: $47\,503 \pm 260$

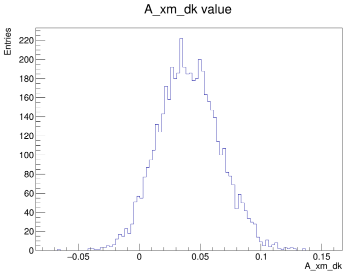
Binned CP fit

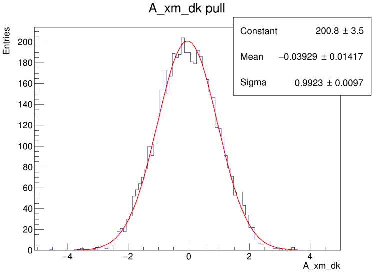
Binned CP fit

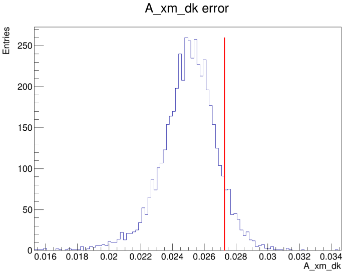
- Use 2×8 bins
- c_i and s_i calculated using MC integration of LHCb amplitude model
- Fit for CP observables
- PDF shape parameters fixed from global fit
- Yield of signal, low mass partially reconstructed background and combinatorial background floated
- Fractional yields F_i floated

$$\mathcal{R}_i = \begin{cases} F_i, & i = -8 \\ F_i / \sum_{j \geq i}, & -8 < i \leq +8 \end{cases}$$

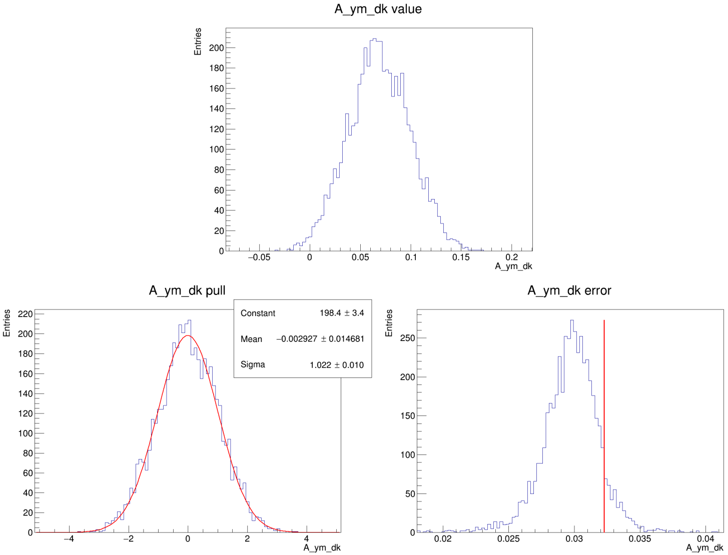
CP observables result: x_-^{DK}



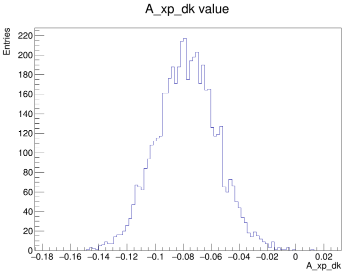


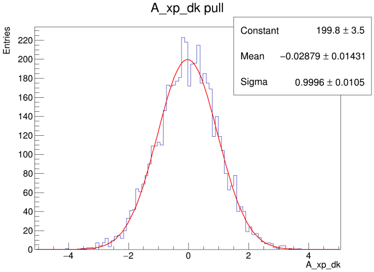


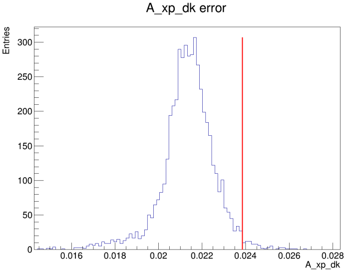
CP observables result: y_-^{DK}



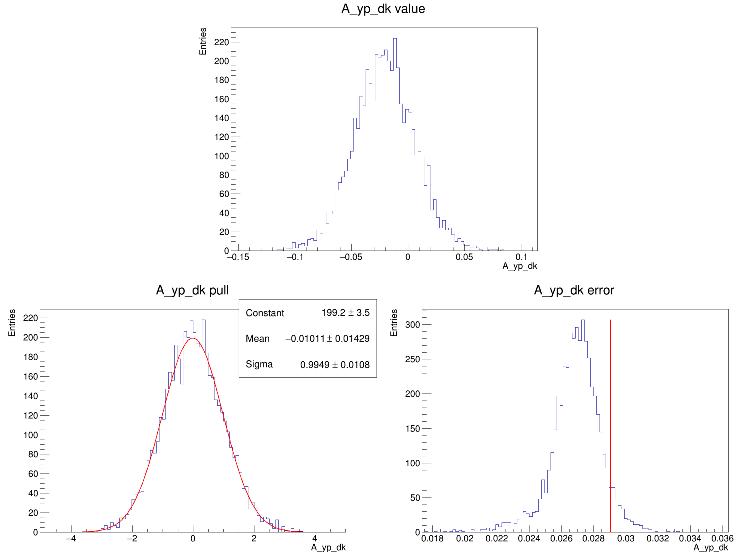
CP observables result: x_+^{DK}



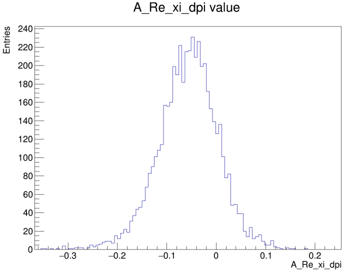


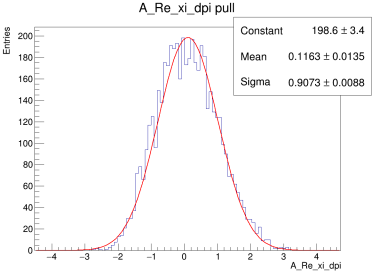


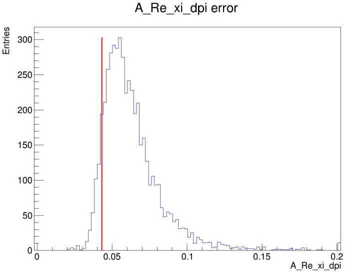
CP observables result: y_+^{DK}



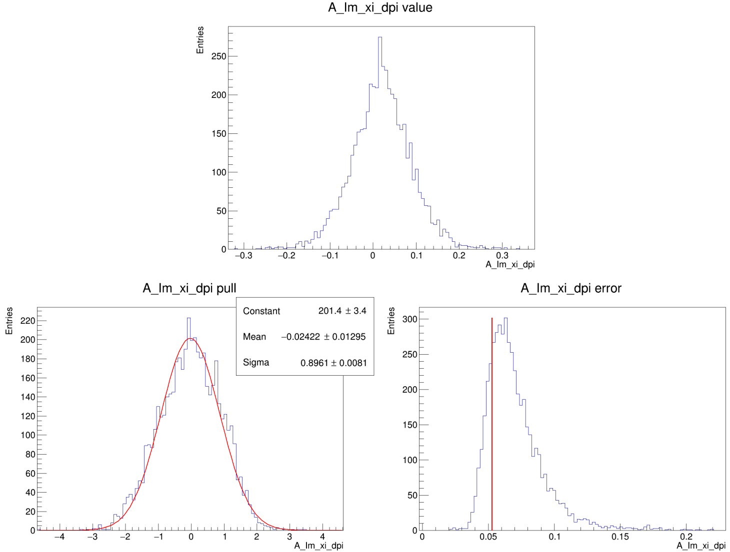
CP observables result: $x_\xi^{D\pi}$







CP observables result: $y_{\xi}^{D\pi}$



Systematics

Summary of all systematics

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.73	3.23	2.38	2.90	4.30	5.27
c_i, s_i	0.66	1.55	0.32	1.31	1.73	1.03
$D \to K(X)l\nu_l$ background	-0.15	0.05	-0.11	0.03	0.35	-0.25
$D \to K\pi\pi\pi$ background	-0.17	0.03	-0.04	0.01	0.46	-0.18
Λ_b background	0.09	0.11	-0.00	-0.18	0.16	0.21
Bin dependent mass shape	-0.21	-0.05	-0.17	0.01	0.37	-0.11
Low mass physics effects	0.05	-0.09	-0.05	-0.18	0.41	-0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Fit bias	0.19	0.03	0.16	-0.04	0.30	-0.16
Total LHCb systematic	0.39	0.17	0.27	0.26	0.87	0.64
Total systematic	0.76	1.55	0.41	1.33	1.93	1.21

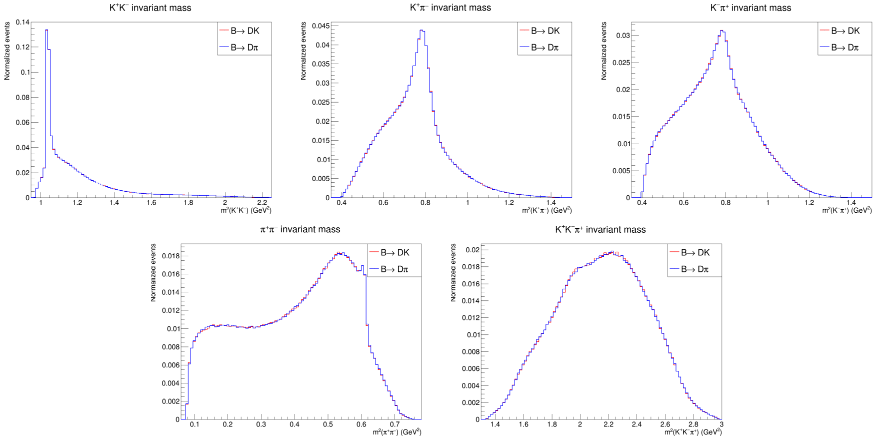
c_i and s_i systematic uncertainty

- Originate from (mostly statistical) uncertainty of c_i and s_i in BESIII analysis
- By far the largest systematic uncertainty
- Take uncertainties from $D \to 4\pi$ strong phase analysis and extrapolate to $20\,\mathrm{fb}^{-1}$
- Smear c_i and s_i and do many fits to data

Remaining systematic uncertainties

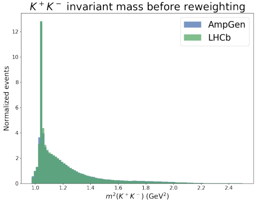
- For these systematics, generate toy datasets, fit with default model and take the bias as a systematic:
  - Small backgrounds ($D \to K(X)l\nu_l$, $D \to K\pi\pi\pi$, Λ_b)
  - Bin dependent mass shape
  - Low mass physics effects
- For these systematics, do multiple fits to data while smearing parameters:
  - Mass shape
  - Fixed yield fractions
  - PID efficiency
- Fit bias: From toys

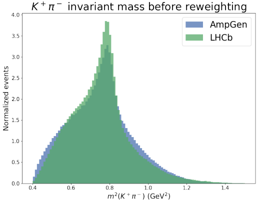
Efficiency difference between $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pi}$

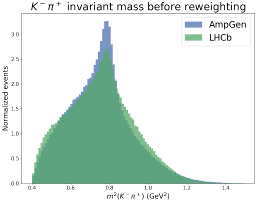


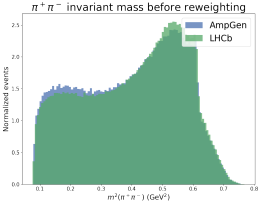
Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered

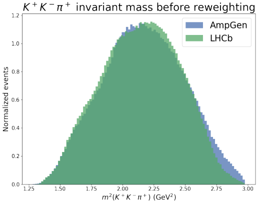
Efficiency correction of c_i and s_i





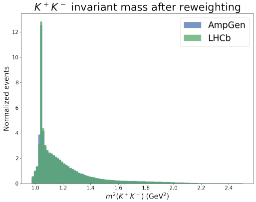


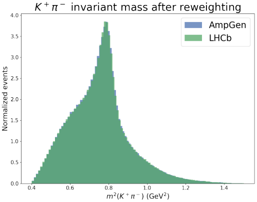


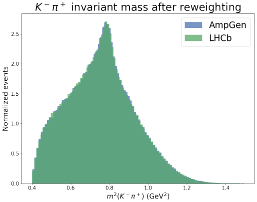


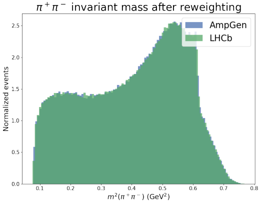
Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

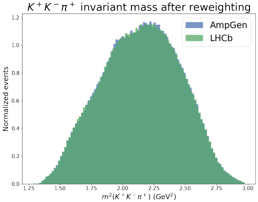
Efficiency correction of c_i and s_i











After reweighing, use weights to recalculate c_i and s_i
Conclusion: Efficiency correction of c_i and s_i is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered

Summary and conclusion

Interpretation in terms of γ

- Measured CP observables:

$$
\begin{aligned}
x_-^{DK} &= (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2},\\
y_-^{DK} &= (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2},\\
x_+^{DK} &= (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2},\\
y_+^{DK} &= (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2},\\
x_\xi^{D\pi} &= (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2},\\
y_\xi^{D\pi} &= (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2},
\end{aligned}
$$

- Note: Currently CP observables are determined using c_i and s_i from the LHCb model
- Publication strategy: Publish current results together with binned yields $\rightarrow$ Redo fit to obtain model-independent CP observables once c_i and s_i from BESIII are available

Interpretation in terms of γ

- Interpret in terms of physics parameters:

$$
\begin{aligned}
\gamma &= (x.x^{+14}_{-15})^\circ, \\
\delta_B^{DK} &= (x.x^{+15}_{-14})^\circ, \\
r_B^{DK} &= x.x^{+0.019}_{-0.018}, \\
\delta_B^{D\pi} &= (x.x^{+117}_{-63})^\circ, \\
r_B^{D\pi} &= x.x^{+0.0052}_{-0.0024}.
\end{aligned}
$$

Bonus measurement

- The mode $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$ very similar
- Run this through same selection (including BDT)
- Can measure GLW CP observables as additional constraints on γ:

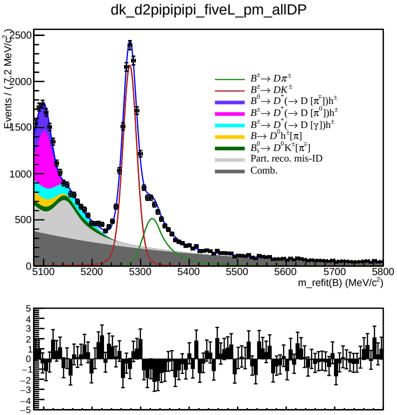
$$A_h = \frac{\Gamma(B^- \to Dh^-) - \Gamma(B^+ \to Dh^+)}{\Gamma(B^- \to Dh^-) + \Gamma(B^+ \to Dh^+)},$$

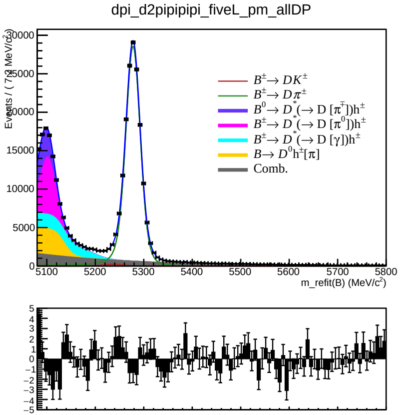
$$R_{\mathrm{CP}} = \frac{R(4\pi)}{R(K3\pi)},$$

$$R = \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}.$$

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi$ yields provided by Tim Evans

Global fit of $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$





$$\frac{\text{Yield}(\pi\pi\pi\pi)}{\text{Yield}(KK\pi\pi)} = \frac{161\,900 \pm 455}{54\,039 \pm 256} = 2.996 \pm 0.017$$

Compare with PDG: 3.06 ± 0.16

Backup

Thank you!

Thank you!

Trigger requirements

Run 1 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackAllL0Decision_TOS) and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or Bu_Hlt2Topo3BodyBBDTDecision_TOS or Bu_Hlt2Topo4BodyBBDTDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackMVADecision_TOS or Bu_Hlt1TwoTrackMVADecision_TOS) and (Bu_Hlt2Topo2BodyDecision_TOS or Bu_Hlt2Topo3BodyDecision_TOS or Bu_Hlt2Topo4BodyDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)

BDT training variables

Name	Rank (%)	Description
log(D0_RHO_BPV)	7.7	D radial distance to beamline
log(Bu_FDCHI2_OWNPV)	6.3	$B^{\pm}$ flight distance χ^2
log(Bu_RHO_BPV)	6.1	$B^{\pm}$ radial distance to beamline
log(Bach_PT)	6.1	Bachelor transverse momentum
Bu_PTASY_1.5	5.3	$B^{\pm}$ asymmetry parameter
log(1-D0_DIRA_BPV)	5.0	Angle between PV and D
log(Bu_IPCHI2_OWNPV)	4.8	$B^{\pm}$ impact parameter χ^2
log(1-Bu_DIRA_BPV)	4.7	Angle between PV and $B^{\pm}$
log(h[1,2]_PT)	4.4	$K^{\pm}$ transverse momentum
Bu_MAXDOCA	4.4	$B^{\pm}$ distance of closest approach
log(Bach_IPCHI2_OWNPV)	4.1	Bachelor impact parameter χ^2

BDT training particles

Name	Rank (%)	Description
log(Bu_constD0PV_D0_P)	3.7	D momentum from DTF
log(D0_VTXCHI2DOF)	3.3	$D0$ vertex fit χ^2
log(h[3,4]_IPCHI2_OWNPV)	3.3	$\pi^{\pm}$ impact parameter χ^2
log(D0_IPCHI2_OWNPV)	3.2	D impact parameter χ^2
log(h[3,4]_PT)	3.2	$\pi^{\pm}$ transverse momentum
log(Bu_PT)	2.8	$B^{\pm}$ transverse momentum
log(h[1,2]_P)	2.8	$K^{\pm}$ momentum
log(Bach_P)	2.7	Bachelor momentum
log(Bu_constD0PV_P)	2.6	$B^{\pm}$ momentum from DTF
log(h[1,2]_IPCHI2_OWNPV)	2.5	$K^{\pm}$ impact parameter χ^2
D0_MAXDOCA	2.5	D distance of closest approach
log(Bu_VTXCHI2DOF)	2.0	$B^{\pm}$ vertex fit χ^2
log(h[3,4]_P)	1.9	$\pi^{\pm}$ momentum