Model independent determination of the CKM angle γ in $B^{\pm} \to (K^+K^-\pi^+\pi^-)_D h^{\pm}$ decays

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B2OC Meeting

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Outline

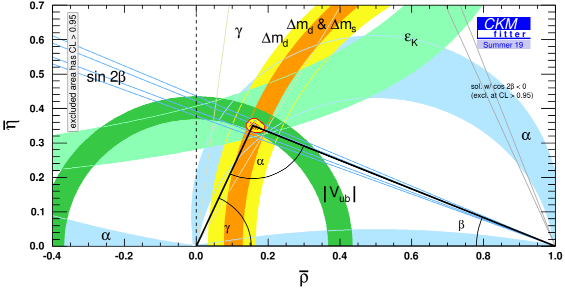
Introduction to the CKM angle γ

γ and the unitary triangle

- CP violation in SM is described by the Unitary Triangle

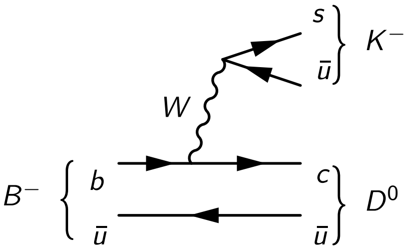
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- Only CKM angle accessible at tree level $\implies$
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark
  - Compare with indirect measurements

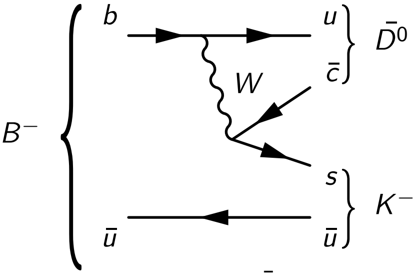


CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

Sensitivity through interference



(a) $B^- \to D^0 K^-$



(b) $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

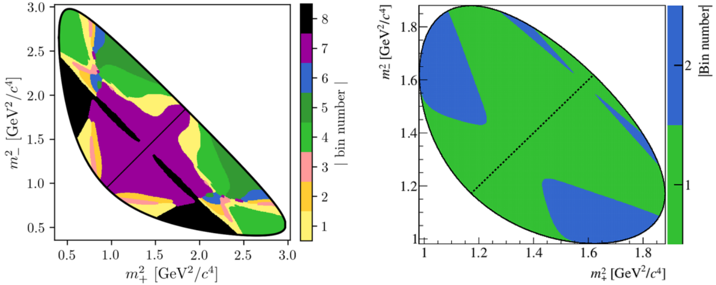
Measurement of γ from $B^{\pm} \to DK^{\pm}$, $D \to K^+K^-\pi^+\pi^-$

- First proposed by J. Rademacker and G. Wilkinson
  - arXiv:hep-ph/0611272
  - Amplitude model by FOCUS
  - Expected γ precision with 1000 candidates: 14°
- CLEO amplitude analysis
  - arXiv:1201.5716
  - Expected γ precision with 2000 candidates: 11°
- State of the art amplitude analysis by LHCb :
  - LHCb-PAPER-2018-041
  - Use to develop efficient binning scheme

Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

Binned measurement of γ

- Final measurement will be model-independent
  - Poor binning reduces statistical sensitivity $\rightarrow$ No bias!
- Need strong phases of D decay $\rightarrow$ Measure at BESIII
- LHCb-PAPER-2020-019: $B^{\pm} \rightarrow Dh^{\pm}$, $D \rightarrow K_S^0 h^+ h^-$
  - Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^{\circ}$



The BPGGSZ method

- $B^\pm \to Dh^\pm$ amplitude:

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

- $\mathcal{A}(D^0)$ and $\mathcal{A}(\bar{D}^0)$ depend on D phase space
- Strong-phase difference of D^0 and $\bar{D}^0$ decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

The BPGGSZ method

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right)$, $\quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi})$, $y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi})$ $\quad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

Binning scheme

Binning scheme requirements

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^\pm \to D^0 h^\pm$ and $B^\pm \to \bar{D}^0 h^\pm$

How to bin a 5-dimensional phase space?

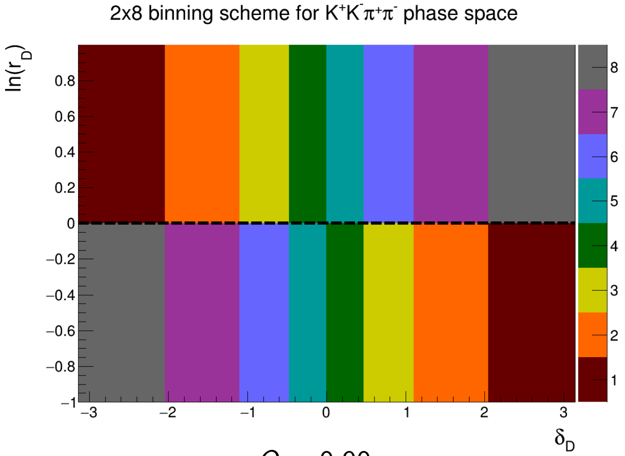
- Generate C++ code for LHCb amplitude model using AmpGen[1]
- For each $B^\pm$ candidate, calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along δ_D and r_D, maximize Q-value to optimize

[1] AmpGen by Tim Evans

Binning scheme

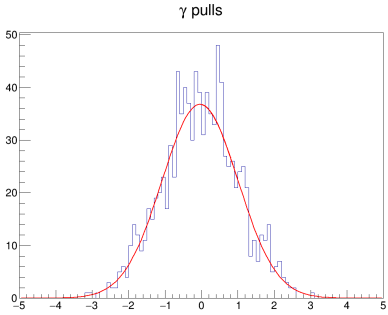


$Q = 0.90$

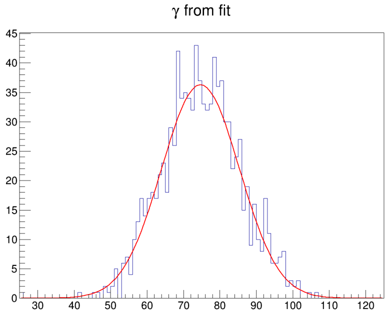
Bins $i < 0$ on top, $i > 0$ below

γ precision benchmark

- Generate 2000 $B^{\pm} \to DK^{\pm}$ candidates using LHCb model in AmpGen
- Fit back with same model using AmpGen



(a) Pull of γ

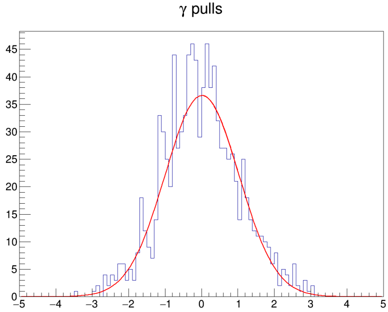


(b) Fitted γ values

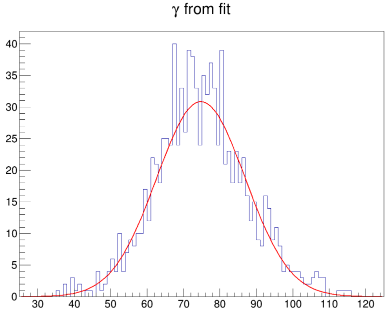
Precision of γ in unbinned fit: 11°

Study of γ precision

- Binned fit setup: Optimized 2×8 bins
- Fit same AmpGen samples, using c_i, s_i and F_i from LHCb model



(a) Pull of γ



(b) Fitted γ values

Precision of γ in binned fit: 12°
Consistent with unbinned fit and Q-value

$B^{\pm} \to (K^+K^-\pi^+\pi^-)_D h^{\pm}$ selection

Samples

- Data sample: Full Run 1 and 2
- MC samples: Full Run 1 and 2 excluding 2015
  - AmpGen model
  - Large filtered samples

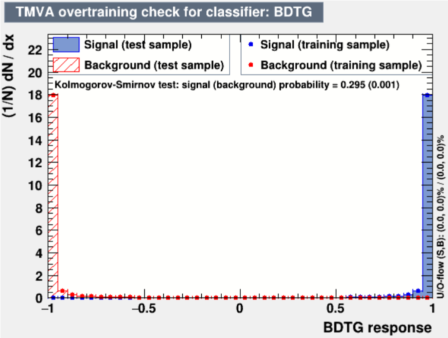
Stripping lines

StrippingB2D0PiD2HHHHBeauty2CharmLineDecision
StrippingB2D0KD2HHHHBeauty2CharmLineDecision

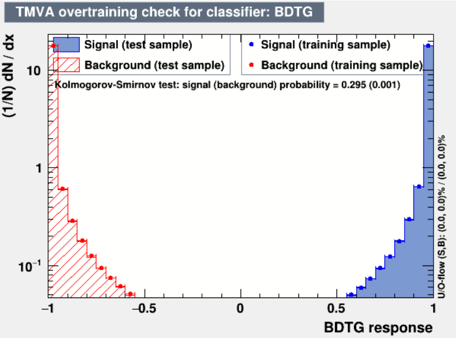
Boosted Decision Tree

- BDTG from TMVA Toolkit
- Signal sample: $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$ MC samples
- Background sample: Data sample with $m_{B^{\pm}}^{\mathrm{DTF}} \in [5800, 7000]\mathrm{MeV}$
- Random, equal sized test and training samples

BDT training results



(a) BDT output

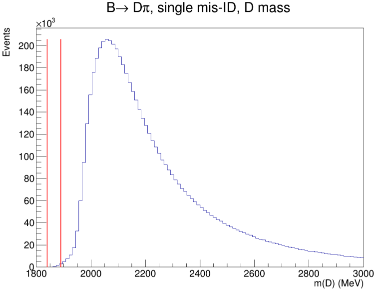


(b) BDT output on a logarithmic scale

Backgrounds

$D \to K\pi\pi\pi$ mis-ID background

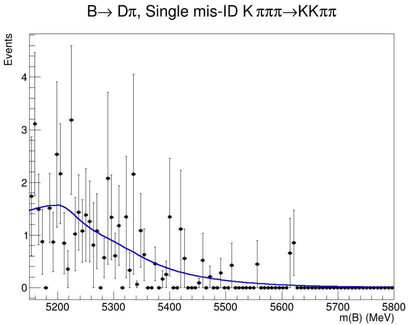
- $B^{\pm} \to Dh^{\pm}$, $D \to K\pi\pi\pi$
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID: $\pi\pi K\pi \to KK\pi\pi$
- Use LHCb MC generated with AmpGen, reweight with PIDCalib2



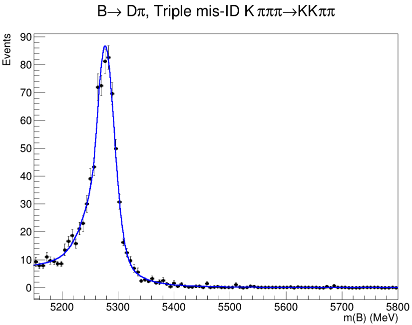
(a) Single mis-ID **(b)** Triple mis-ID

Figure 5: D invariant mass

$D \to K\pi\pi\pi$ mis-ID background



(a) Single mis-ID



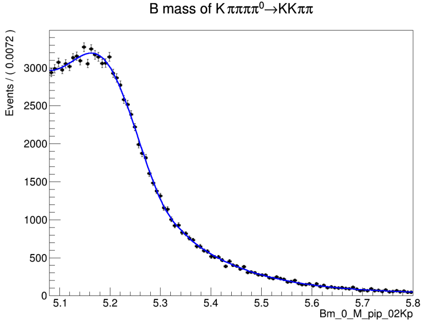
(b) Triple mis-ID

Figure 6: B invariant mass

Conclusion: Negligible impact, include in systematics

$D \to K\pi\pi\pi\pi^0$ mis-ID background

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi[\pi^0]$
- π^0 not reconstructed $\to$ Lower D mass
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi \to$ Higher D mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

Global fit

Signal parameterisation

- PDF shape parameterization identical to LHCb-ANA-2020-001
- Signal: Gaussian + Modified Cruijff
- Shape fixed from MC, yield and width floated
- Combinatorial background (exponential)
- Partially reconstructed background (dini shapes)

$$f_{\mathrm{MG}}(m|m_B,\sigma,\alpha_L,\alpha_R,\beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m = m - m_B < 0 \\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m = m - m_B > 0 \end{cases}$$

Global fit

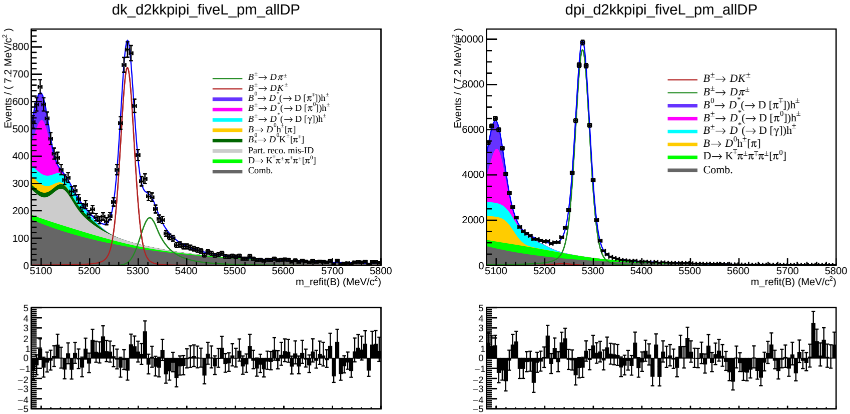


Figure 7: $B^\pm \to DK^\pm$ channel (left) and $B^\pm \to D\pi^\pm$ channel (right)

- $B^\pm \to DK^\pm$ yield: 3543 ± 75
- $B^\pm \to D\pi^\pm$ yield: $47\,503 \pm 260$

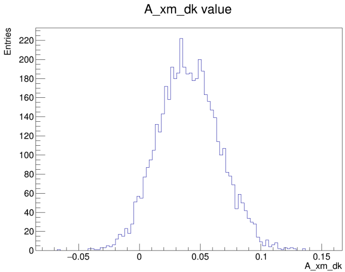
Binned CP fit

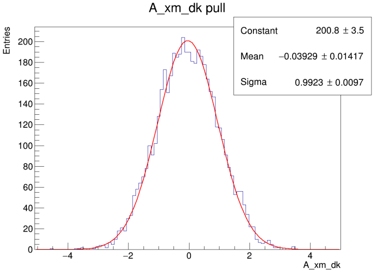
Binned CP fit

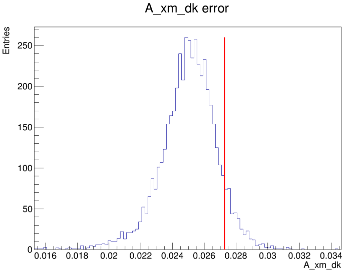
- Use 2×8 bins
- c_i and s_i calculated using MC integration of LHCb amplitude model
- Fit for CP observables
- PDF shape parameters fixed from global fit
- Yield of signal, low mass partially reconstructed background and combinatorial background floated
- Fractional yields F_i floated

$$\mathcal{R}_i = \begin{cases} F_i, & i = -8 \\ F_i / \sum_{j \geq i}, & -8 < i \leq +8 \end{cases}$$

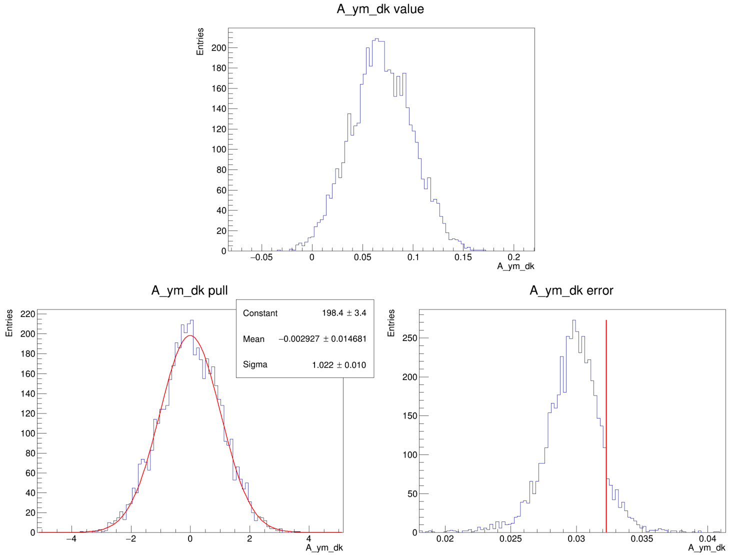
CP observables result: x_-^{DK}



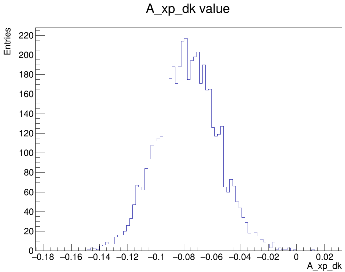


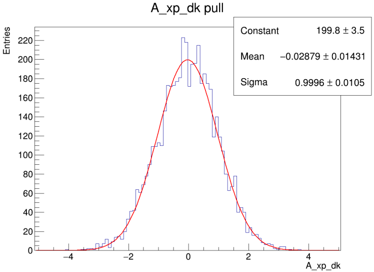


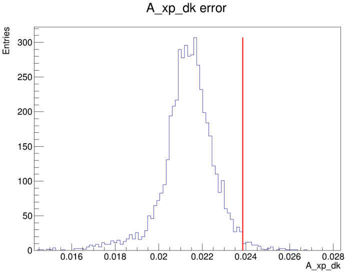
CP observables result: y_-^{DK}



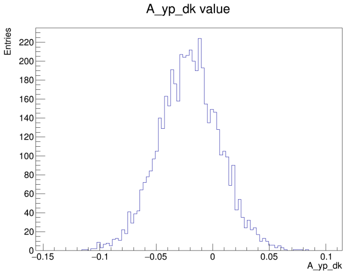
CP observables result: x_+^{DK}

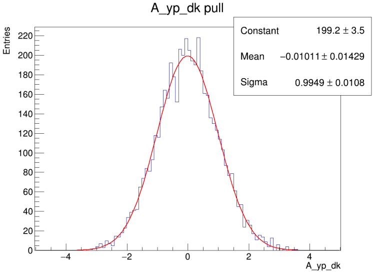


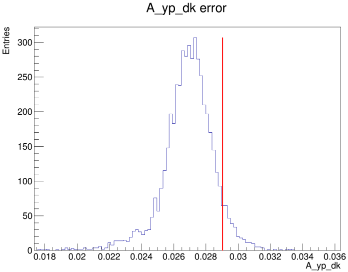




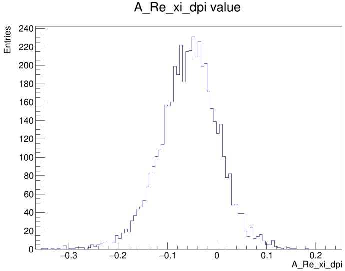
CP observables result: y_+^{DK}

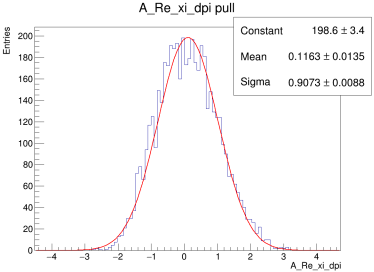


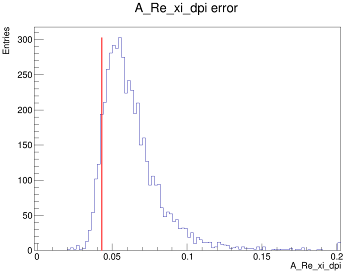




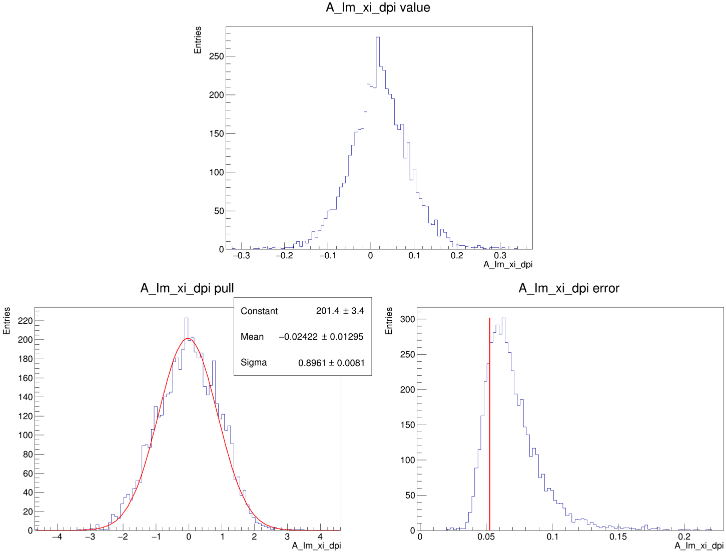
CP observables result: $x_{\xi}^{D\pi}$







CP observables result: $y_\xi^{D\pi}$



Systematic uncertainties

c_i and s_i systematic uncertainty

- Uncertainty of c_i and s_i in BESIII analysis (mostly statistical)
- Largest systematic uncertainty
- Take uncertainties from $D \to 4\pi$ strong phase analysis and extrapolate to $20\,\mathrm{fb}^{-1}$
- Smear c_i and s_i and do many fits to data

Summary of all systematic uncertainties

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.73	3.23	2.38	2.90	4.30	5.27
c_i, s_i	0.66	1.55	0.32	1.31	1.73	1.03
$B^\pm \to D\mu\nu$ background	0.04	0.03	0.02	0.15	0.30	0.10
$D \to K(X)l\nu_l$ background	0.15	0.05	0.11	0.03	0.35	0.25
$D \to K\pi\pi\pi$ background	0.17	0.03	0.04	0.01	0.46	0.18
Λ_b background	0.09	0.11	0.00	0.18	0.16	0.21
Bin dependent mass shape	0.21	0.05	0.17	0.01	0.37	0.11
Fit bias	0.19	0.03	0.16	0.04	0.30	0.16
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
Low mass physics effects	0.05	0.09	0.05	0.18	0.41	0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Total LHCb systematic	0.39	0.17	0.27	0.30	0.92	0.65
Total systematic	0.77	1.55	0.41	1.34	1.96	1.22

Summary and conclusion

Summary of CP observables

- Measured CP observables:

$$
\begin{aligned}
x_-^{DK} &= (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2}, \\
y_-^{DK} &= (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2}, \\
x_+^{DK} &= (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2}, \\
y_+^{DK} &= (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2}, \\
x_\xi^{D\pi} &= (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2}, \\
y_\xi^{D\pi} &= (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2},
\end{aligned}
$$

- Note: Currently using c_i and s_i from the LHCb model
- Publication strategy: Publish current results together with binned yields $\rightarrow$ Redo fit to obtain model-independent CP observables once c_i and s_i from BESIII are available

Interpretation in terms of γ

- Interpret in terms of physics parameters:

$$
\begin{aligned}
\gamma &= (x.x^{+14}_{-15})^\circ, \\
\delta_B^{DK} &= (x.x^{+15}_{-14})^\circ, \\
r_B^{DK} &= x.x^{+0.019}_{-0.018}, \\
\delta_B^{D\pi} &= (x.x^{+117}_{-63})^\circ, \\
r_B^{D\pi} &= x.x^{+0.0052}_{-0.0024}.
\end{aligned}
$$

- Next steps:
  - ANA note ready
  - Will update uncertainty of c_i and s_i with model-dependent values

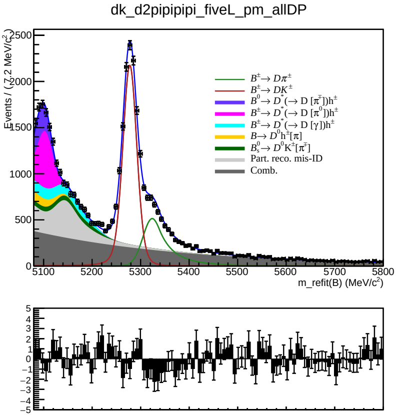
Bonus measurement

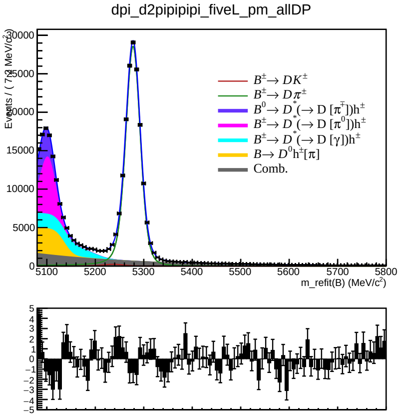
- The mode $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$ very similar
- Run this through same selection (including BDT)
- Can measure GLW CP observables as additional constraints on γ:

$$\begin{aligned} A_h =& \frac{\Gamma(B^- \to Dh^-) - \Gamma(B^+ \to Dh^+)}{\Gamma(B^- \to Dh^-) + \Gamma(B^+ \to Dh^+)}, \\ R_{\mathrm{CP}} =& \frac{R(4\pi)}{R(K3\pi)}, \\ R =& \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}. \end{aligned}$$

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi$ yields provided by Tim Evans

Global fit of $B^{\pm} \to Dh^{\pm}$, $D \to \pi^+\pi^-\pi^+\pi^-$





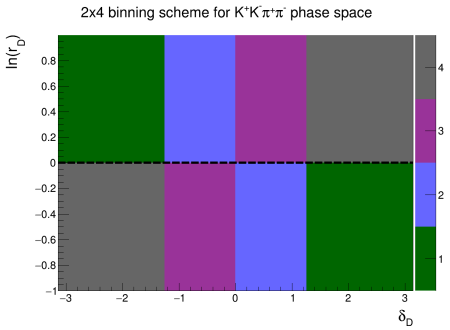
$$\frac{\mathrm{Yield}(\pi\pi\pi\pi)}{\mathrm{Yield}(KK\pi\pi)} = \frac{161\,900 \pm 455}{54\,039 \pm 256} = 2.996 \pm 0.017$$

Compare with PDG: 3.06 ± 0.16

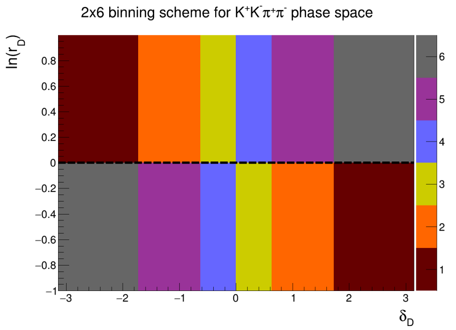
Thank you!

Backup

Binning scheme

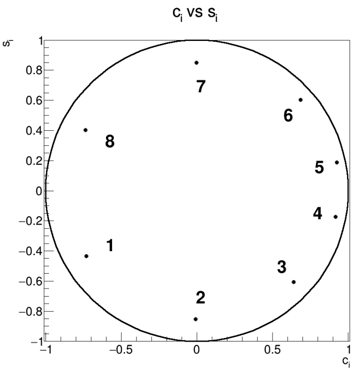


(a) $Q = 0.85$

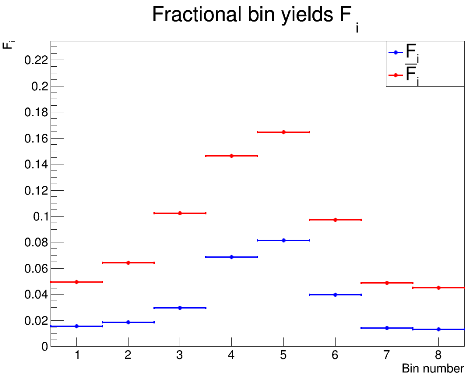


(b) $Q = 0.89$

c_i, s_i and F_i

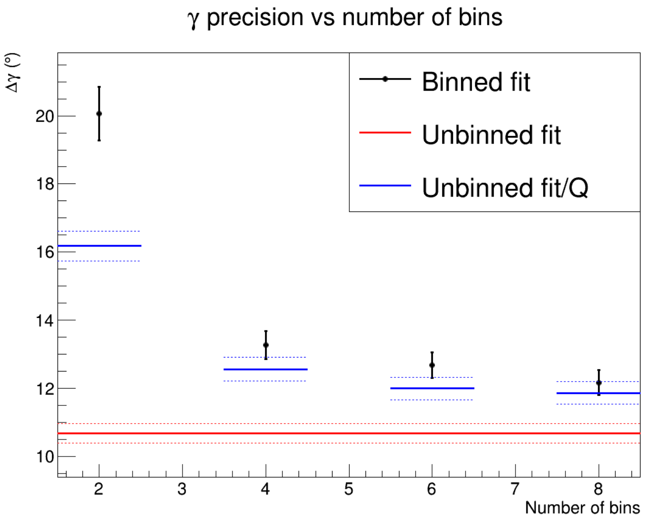


c_i vs s_i
s_i
c_i
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−0.5
0.5
7
6
8
5
4
1
3
2



Fractional bin yields F_i
F_i
F̄_i
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Bin number

Comparison of binned fit precision with unbinned fit



Trigger requirements

Run 1 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackAllL0Decision_TOS) and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or Bu_Hlt2Topo3BodyBBDTDecision_TOS or Bu_Hlt2Topo4BodyBBDTDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackMVADecision_TOS or Bu_Hlt1TwoTrackMVADecision_TOS) and (Bu_Hlt2Topo2BodyDecision_TOS or Bu_Hlt2Topo3BodyDecision_TOS or Bu_Hlt2Topo4BodyDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)

Initial cuts

Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100\text{GeV}$
3	Bachelor has RICH	True
4	D invariant mass	$[1839.84, 1889.84]\text{MeV}$
5	$B^{\pm}$ invariant mass	$[5080, 5800]\text{MeV}$
6	$K^{\pm}$ daughter PID	> -10
7	$\pi^{\pm}$ daughter PID	< 20

Final cuts

Rectangular cuts after BDT

Number	Variable description	Cut
8	$K^{\pm}$ bachelor PID	> 4
9	$\pi^{\pm}$ bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	$K^{\pm}$ PID	> 0
13	K_S^0 mass veto	$[477, 507]$MeV

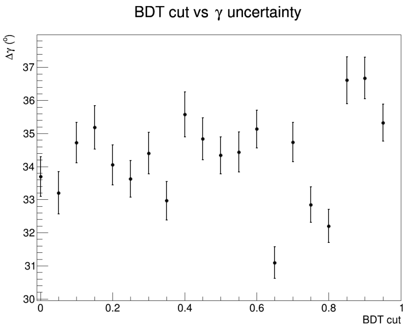
BDT training variables

Name	Rank (%)	Description
`log(D0_RHO_BPV)`	7.7	D radial distance to beamline
`log(Bu_FDCHI2_OWNPV)`	6.3	$B^{\pm}$ flight distance χ^2
`log(Bu_RHO_BPV)`	6.1	$B^{\pm}$ radial distance to beamline
`log(Bach_PT)`	6.1	Bachelor transverse momentum
`Bu_PTASY_1.5`	5.3	$B^{\pm}$ asymmetry parameter
`log(1-D0_DIRA_BPV)`	5.0	Angle between PV and D
`log(Bu_IPCHI2_OWNPV)`	4.8	$B^{\pm}$ impact parameter χ^2
`log(1-Bu_DIRA_BPV)`	4.7	Angle between PV and $B^{\pm}$
`log(h[1,2]_PT)`	4.4	$K^{\pm}$ transverse momentum
`Bu_MAXDOCA`	4.4	$B^{\pm}$ distance of closest approach
`log(Bach_IPCHI2_OWNPV)`	4.1	Bachelor impact parameter χ^2

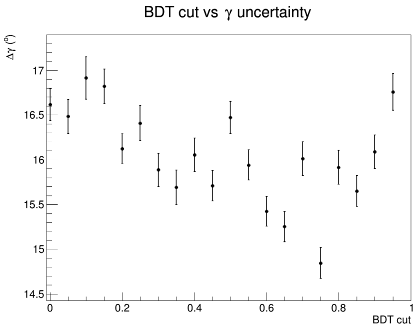
BDT training particles

Name	Rank (%)	Description
log(Bu_constD0PV_D0_P)	3.7	D momentum from DTF
log(D0_VTXCHI2DOF)	3.3	$D0$ vertex fit χ^2
log(h[3,4]_IPCHI2_OWNPV)	3.3	$\pi^\pm$ impact parameter χ^2
log(D0_IPCHI2_OWNPV)	3.2	D impact parameter χ^2
log(h[3,4]_PT)	3.2	$\pi^\pm$ transverse momentum
log(Bu_PT)	2.8	$B^\pm$ transverse momentum
log(h[1,2]_P)	2.8	$K^\pm$ momentum
log(Bach_P)	2.7	Bachelor momentum
log(Bu_constD0PV_P)	2.6	$B^\pm$ momentum from DTF
log(h[1,2]_IPCHI2_OWNPV)	2.5	$K^\pm$ impact parameter χ^2
D0_MAXDOCA	2.5	D distance of closest approach
log(Bu_VTXCHI2DOF)	2.0	$B^\pm$ vertex fit χ^2
log(h[3,4]_P)	1.9	$\pi^\pm$ momentum

BDT optimization study



(a) Run 1



(b) Run 2

- Run 1: Pick BDT working point at 0.65
- Run 2: Pick BDT working point at 0.75

Partially reconstructed background

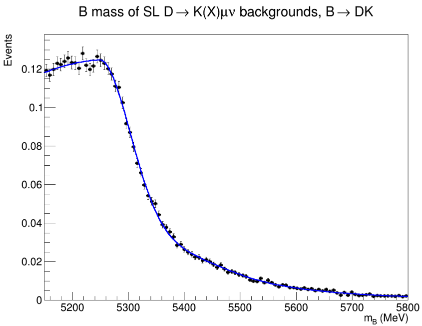
- $B^{\pm} \to D\pi^{\pm}$:
  1. $B^{\pm} \to (D^{*0} \to D^0[\pi^0])\pi^{\pm}$
  2. $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])\pi^{\pm}$
  3. $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]\pi^{\pm}$
  4. $B^{\pm} \to (D^{*0} \to D^0[\gamma])\pi^{\pm}$

- $B^{\pm} \to DK^{\pm}$:
  1. $B^{\pm} \to (D^{*0} \to D^0[\pi^0])K^{\pm}$
  2. $B^0 \to (D^{*\mp} \to D^0[\pi^{\mp}])K^{\pm}$
  3. $B^{\pm(0)} \to D^0[\pi^{0(\mp)}]K^{\pm}$
  4. $B^{\pm} \to (D^{*0} \to D^0[\gamma])K^{\pm}$
  5. $B_s^0 \to \bar{D}^0[\pi^+]K^-$
  6. Mis-ID from partially reconstructed $B^{\pm} \to D\pi^{\pm}$ channel

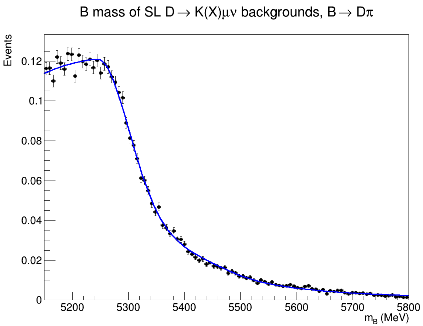
D semileptonic backgrounds

- $B^{\pm} \to Dh^{\pm}$, $D \to K(X)l\nu$, $K(X) \to K\pi\pi$
  - $K_1(1270)$
  - $K_1(1400)$
  - $K^*(1410)$
  - $K^*(1680)$
  - $K_2^*(1430)$
- Single mis-ID: $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID: $K\pi\pi\mu \to KK\pi\pi$
- Generate Rapidsim samples, reweight with PIDCalib2

D semileptonic backgrounds



(a) $B \to DK$



(b) $B \to D\pi$

Conclusion: Negligible impact, include in systematics

Remaining systematic uncertainties

Different strategies for evaluating systematic uncertainties:

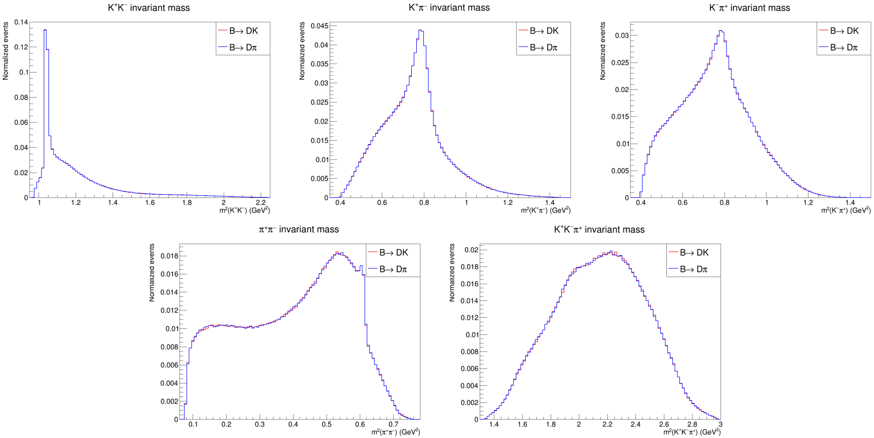
- Generate toy datasets with systematics, fit with default model and take the bias as a systematic:
  - Small backgrounds ($D \to K(X)l\nu_l$, $D \to K\pi\pi\pi$, $B \to Dl\nu_l$, Λ_b)
  - Bin dependent mass shape
  - Low mass physics effects
- Do multiple fits to data while smearing parameters:
  - c_i and s_i
  - Mass shape
  - Fixed yield fractions
  - PID efficiency
- Fit bias: Take bias toys as systematic uncertainty

Efficiency related systematics

Efficiency related systematics:

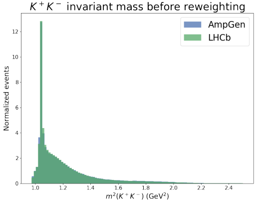
- Difference in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ phase space acceptance
- Efficiency correction of c_i and s_i

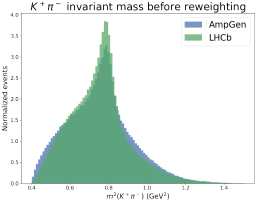
Efficiency differences between $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$

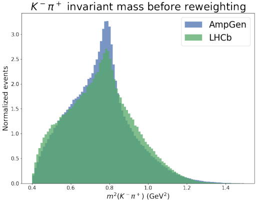


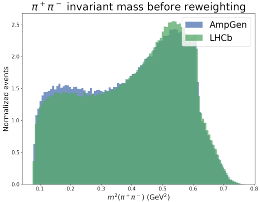
Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered

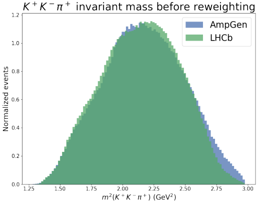
Efficiency correction of c_i and s_i





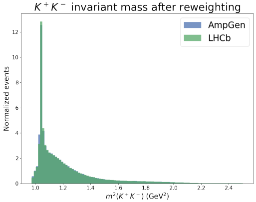


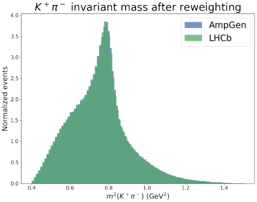


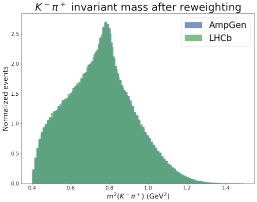


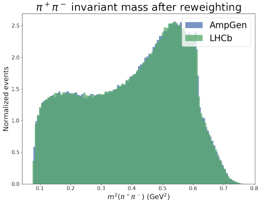
Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

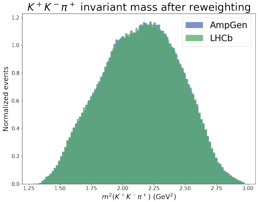
Efficiency correction of c_i and s_i











After reweighing, use weights to recalculate c_i and s_i

Conclusion: Efficiency correction of c_i and s_i is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered