

Model-independent (sort of) determination of the CKM angle γ in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D h^\pm$ decays

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B2OC Meeting, WG sign off

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- 1 Summary of γ analysis in $B^\pm \rightarrow Dh^\pm$, $D \rightarrow h^+h^-\pi^+\pi^-$
- 2 Updates and improvements to analysis
 - $D \rightarrow K\pi\pi\pi\pi^0$ mis-ID background
 - Some minor changes and improvements
 - Systematic uncertainties
- 3 CP fit results, including central values (analysis not blind)
- 4 Summary of $KK\pi\pi$ analysis
- 5 Conclusion and next steps

Summary of γ analysis in

$$B^\pm \rightarrow Dh^\pm, D \rightarrow h^+h^-\pi^+\pi^-$$

Summary of γ analysis in $B^\pm \rightarrow Dh^\pm$, $D \rightarrow h^+h^-\pi^+\pi^-$

- First presented to B2OC WG on 4th November 2022
 - Slides [here](#)
 - TWiki page with ANA note [here](#)
- GGSZ: Analysis in phase space bins of $D^0 \rightarrow K^+K^-\pi^+\pi^-$
 - This publication: c_i/s_i from LHCb model [JHEP 02 \(2019\) 126](#)
 - Long term plan: Strong-phase analysis at BESIII currently ongoing where c_i/s_i are measured directly \implies Model independent analysis
- Quasi-GLW: Phase space integrated analysis of $D^0 \rightarrow h^+h^-\pi^+\pi^-$
 - $F_+^{4\pi} = 0.769 \pm 0.023$ from CLEO-c data [JHEP 01 \(2018\) 144](#)
 - $F_+^{KK\pi\pi} = 0.736$ from LHCb model
 - Preliminary BESIII analysis of F_+ shows good agreement with model

The BPGGSZ method

Event yield in bin i

$$N_i^- = h_{B^-} (F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i))$$

$$N_i^+ = h_{B^+} (F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i))$$

- CP observables:

- $x_{\pm}^{DK} = r_B^{DK} \cos(\delta_B^{DK} \pm \gamma), \quad y_{\pm}^{DK} = r_B^{DK} \sin(\delta_B^{DK} \pm \gamma)$
- $x_{\xi}^{D\pi} = \text{Re}(\xi^{D\pi}), \quad y_{\xi}^{D\pi} = \text{Im}(\xi^{D\pi}) \quad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})} \right)$

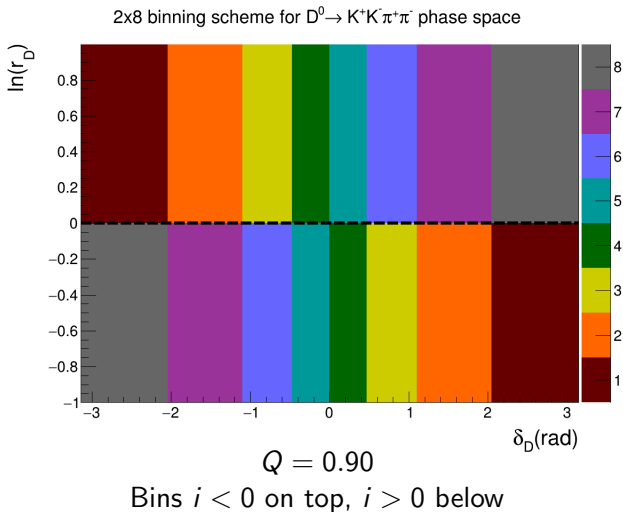
- Fractional bin yield:

- $F_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$
- Floated in the fit, mostly constrained by $B^{\pm} \rightarrow D\pi^{\pm}$

- Amplitude averaged strong phases:

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}, \quad s_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \sin(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}$$

Binning scheme



The quasi-GLW method

- Statistically independent analysis without phase space binning
 - BPGGSZ looks at relative bin yields
 - Quasi-GLW observables depend on absolute yields
- Charge asymmetry:

$$A_h = \frac{\Gamma(B^- \rightarrow Dh^-) - \Gamma(B^+ \rightarrow Dh^+)}{\Gamma(B^- \rightarrow Dh^-) + \Gamma(B^+ \rightarrow Dh^+)}$$

- $B \rightarrow DK$ vs $B \rightarrow D\pi$ double ratio:

$$R_{CP} = \frac{R_{hh\pi\pi}}{R_{K\pi\pi\pi}}, \quad R = \frac{\Gamma(B^- \rightarrow DK^-) + \Gamma(B^+ \rightarrow DK^+)}{\Gamma(B^- \rightarrow D\pi^-) + \Gamma(B^+ \rightarrow D\pi^+)}$$

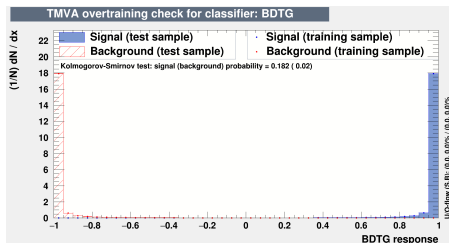
CP observables and physics parameters

$$A_h = \frac{2r_B^{Dh}(2F_+ - 1) \sin(\delta_B^{Dh}) \sin(\gamma)}{1 + (r_B^{Dh})^2 + 2r_B^{Dh}(2F_+ - 1) \cos(\delta_B^{Dh}) \cos(\gamma)},$$

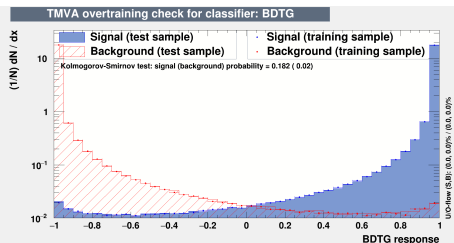
$$R_{CP} = 1 + (r_B^{Dh})^2 + 2r_B^{Dh}(2F_+ - 1) \cos(\delta_B^{Dh}) \cos(\gamma)$$

Selection

- 1 Initial cuts: Trigger requirements, mass cuts, bachelor p , etc
- 2 BDT: Efficient combinatorial background rejection
- 3 Final cuts: PID cuts, flight significance cuts, K_S veto, etc



(a) BDT output



(b) BDT output on a logarithmic scale

Global fit

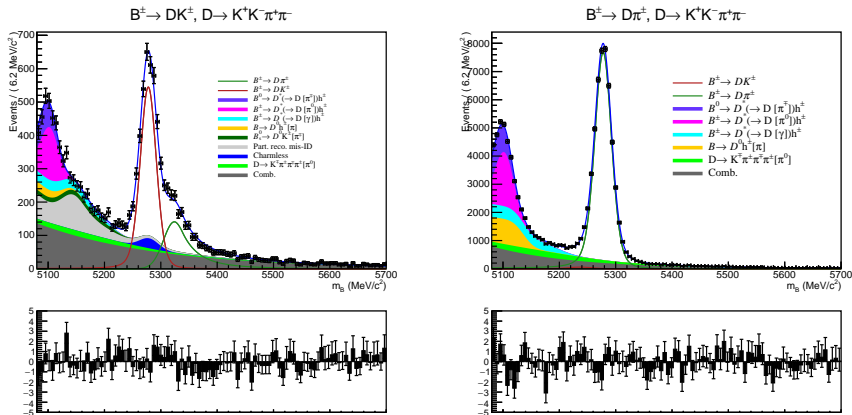


Figure 2: $B^\pm \rightarrow DK^\pm$ channel (left) and $B^\pm \rightarrow D\pi^\pm$ channel (right)

- $B^\pm \rightarrow DK^\pm$ yield: 3306 ± 75
- $B^\pm \rightarrow D\pi^\pm$ yield: $46\,695 \pm 256$

Fit split by charge

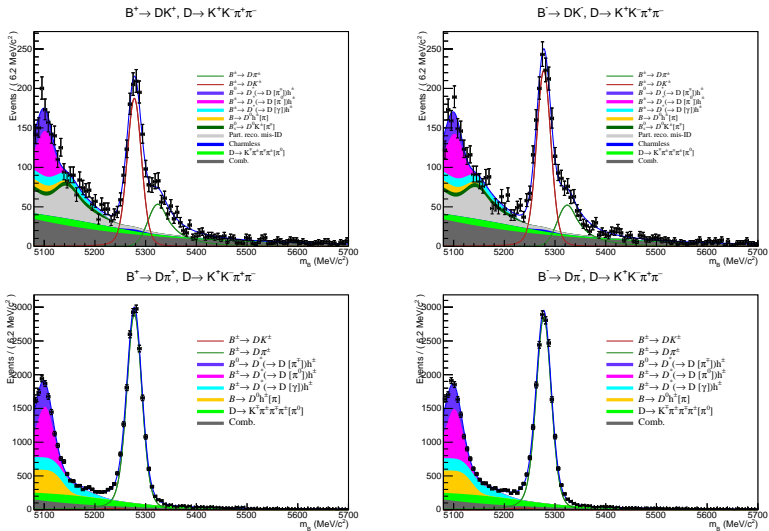


Figure 3: $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-) D h^\pm$

Fit split by charge

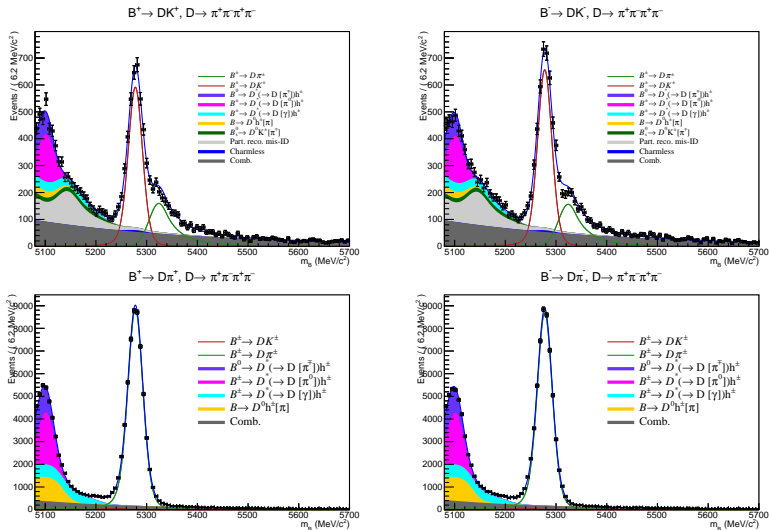


Figure 4: $B^\pm \rightarrow (\pi^+ \pi^- \pi^+ \pi^-) Dh^\pm$

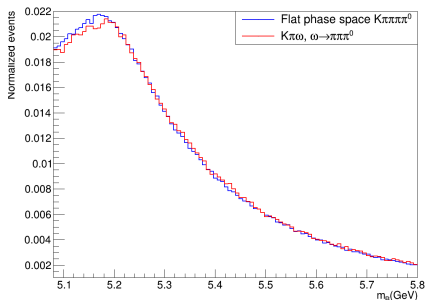
Updates and improvements to analysis

Thanks to Anton, Nathan, Paras
for reading the ANA note!

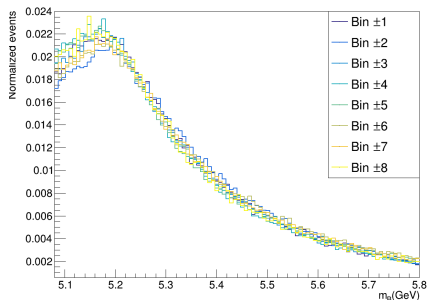
$D \rightarrow K\pi\pi\pi\pi^0$ mis-ID background

- Missing π^0 and $\pi \rightarrow K$ mis-ID
- Two changes:
 - ① Unclear how DTF affects mass shape \implies Requested LHCb MC
 - ② Float yield relative to $B \rightarrow D^*h$ background instead of all partially reconstructed background
- Cross checks: Model dependence and bin dependence

$K\pi\pi\pi^0$ mass shape model dependence



$K\pi\pi\pi^0$ mass shape by bins

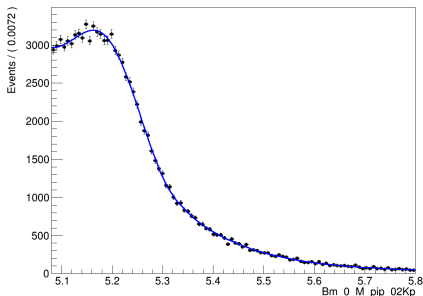


$D \rightarrow K\pi\pi\pi\pi^0$ mis-ID background

- Reweighting procedure:

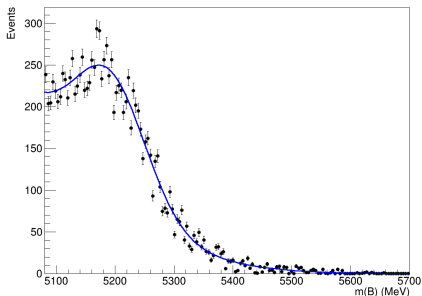
- ① Run through same reconstruction as $KK\pi\pi$
- ② Use $KK\pi\pi$ MC to calculate PID efficiency of $\text{PIDK} > -10$ as a function of p and η
- ③ Reweight each event to “undo” the PID cut from stripping
- ④ Reweight with PIDCalib2 to account for $\text{PIDK} > 0$ cut in selection

B mass of $K\pi\pi\pi^0 \rightarrow KK\pi\pi$



(a) RapidSim

B mass of $K\pi\pi\pi^0 \rightarrow KK\pi\pi$



(b) Full LHCb simulation

Some minor changes and improvements

- ① K_S veto for $D \rightarrow \pi\pi\pi\pi$ added to match the CLEO-c F_+ analysis
- ② Fit bias is corrected for
- ③ Fit range reduced from 5800 MeV to 5700 MeV
- ④ BPGGSZ and quasi-GLW correlations accounted for

Systematic uncertainties

- ① c_i/s_i systematic uncertainty from modelling
 - Strategy: Generate toys with c_i/s_i from older CLEO model, fit with c_i/s_i from LHCb model
 - Largest systematic uncertainty
 - Will be replaced when BESIII results become available
- ② Charmless background: Vary the total yield and fractional bin yields
- ③ $K\pi\pi\pi\pi^0$: Vary bin distribution from D^0 -like to $D^0 + \bar{D}^0$ -like
- ④ D mixing: Bias from toy fit
 - More or less negligible when F_i are floated
 - Relatively large if F_i are fixed
- ⑤ All quasi-GLW systematic studies finished

Summary of all BPGGSZ systematic uncertainties

Uncertainties of BPGGSZ CP observables in units of 10^{-2}

| Source | x_-^{DK} | y_-^{DK} | x_+^{DK} | y_+^{DK} | $x_\xi^{D\pi}$ | $y_\xi^{D\pi}$ |
|--|------------|------------|------------|------------|----------------|----------------|
| Statistical | 2.99 | 3.50 | 2.58 | 3.10 | 4.07 | 4.89 |
| c_i, s_i | 0.14 | 3.82 | 1.78 | 1.03 | 0.01 | 0.71 |
| $B^\pm \rightarrow D\mu\nu$ background | 0.07 | 0.06 | 0.08 | 0.30 | 0.17 | 0.00 |
| $D \rightarrow K(X)l\nu_l$ background | 0.08 | 0.00 | 0.73 | 0.14 | 0.27 | 0.44 |
| $D \rightarrow K\pi\pi\pi$ background | 0.25 | 0.00 | 0.73 | 0.06 | 0.07 | 0.27 |
| $D \rightarrow K\pi\pi\pi\pi^0$ background | 0.37 | 0.07 | 0.20 | 0.04 | 0.45 | 0.19 |
| Λ_b background | 0.10 | 0.06 | 0.06 | 0.26 | 0.15 | 0.07 |
| Bin dependent mass shape | 0.06 | 0.12 | 0.13 | 0.12 | 0.24 | 0.12 |
| Charmless background | 0.15 | 0.18 | 0.14 | 0.16 | 0.01 | 0.01 |
| Fit bias | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Fixed yield fractions | 0.02 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 |
| Low mass physics effects | 0.15 | 0.21 | 0.05 | 0.20 | 0.03 | 0.44 |
| Mass shape | 0.03 | 0.03 | 0.02 | 0.02 | 0.04 | 0.01 |
| PID Efficiency | 0.03 | 0.03 | 0.02 | 0.02 | 0.04 | 0.01 |
| D mixing | 0.00 | 0.02 | 0.01 | 0.02 | 0.00 | 0.00 |
| Total LHCb systematic | 0.52 | 0.32 | 1.08 | 0.52 | 0.63 | 0.72 |
| Total systematic | 0.54 | 3.83 | 2.08 | 1.15 | 0.63 | 1.01 |

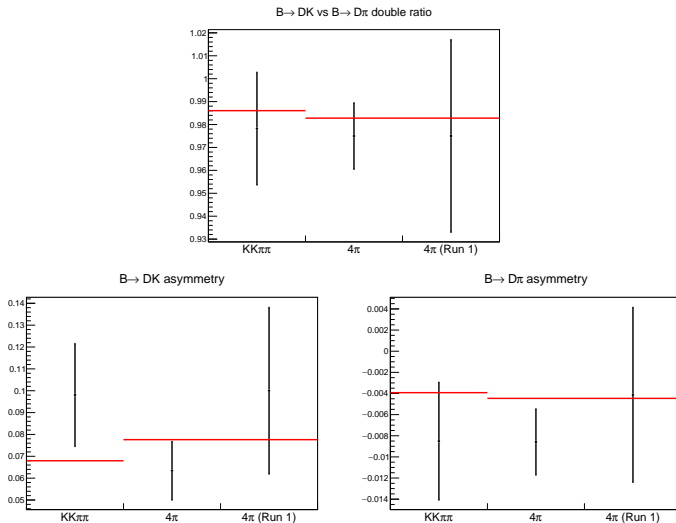
Summary of all quasi-GLW systematic uncertainties

Uncertainties of quasi-GLW CP observables in units of 10^{-2}

| Source | $A_K^{KK\pi\pi}$ | $A_\pi^{KK\pi\pi}$ | $A_K^{\pi\pi\pi\pi}$ | $A_\pi^{\pi\pi\pi\pi}$ | $R_{CP}^{KK\pi\pi}$ | $R_{CP}^{\pi\pi\pi\pi}$ |
|-----------------------|------------------|--------------------|----------------------|------------------------|---------------------|-------------------------|
| Statistical | 23.49 | 13.36 | 5.56 | 3.12 | 24.54 | 14.46 |
| Charmless background | 1.20 | 0.44 | 0.01 | 0.00 | 13.72 | 8.43 |
| External parameters | 0.98 | 0.99 | 0.74 | 0.74 | 3.98 | 3.96 |
| Fixed yield fractions | 0.11 | 0.08 | 0.02 | 0.00 | 1.32 | 1.44 |
| Mass shape | 0.27 | 0.20 | 0.03 | 0.02 | 3.11 | 3.05 |
| PID efficiency | 0.18 | 0.12 | 0.01 | 0.00 | 2.55 | 1.64 |
| Total systematic | 1.59 | 1.11 | 0.74 | 0.74 | 14.90 | 10.04 |

CP fit results

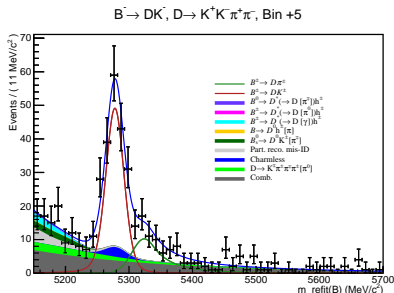
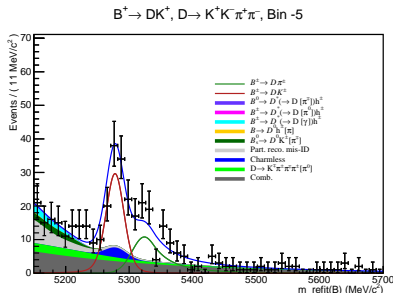
GLW CP observables



Red line: Prediction for CP observables using γ +charm combination

Binned CP fit

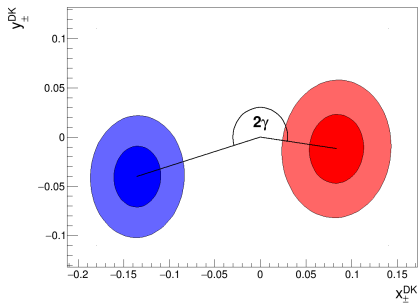
- PDF shape parameters fixed from global fit
- 2 charges, 2 B decays, 2×8 bins \implies 64 categories
- Nuisance parameters:
 - Combinatorial and low mass background (2×64 parameters)
 - F_i (15 parameters)
 - Yield normalization for each charge and B decay (4 parameters)
- 6 CP observables \implies 153 free parameters in total



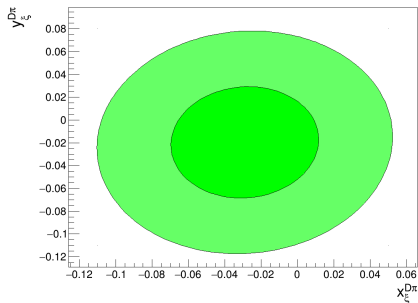
CP observables result

CP observables result

$B^\pm \rightarrow DK$ CP observables

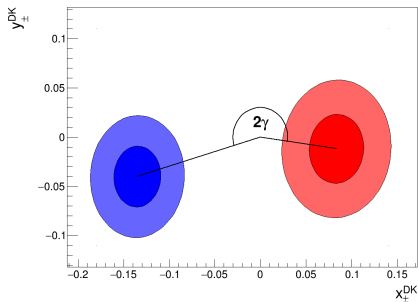


$B^\pm \rightarrow D\pi$ CP observables

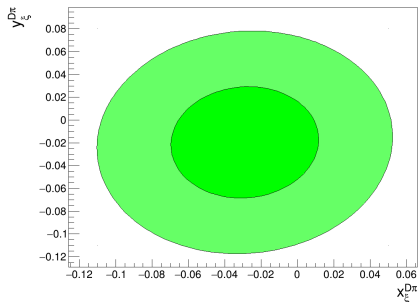


CP observables result

$B^\pm \rightarrow DK$ CP observables

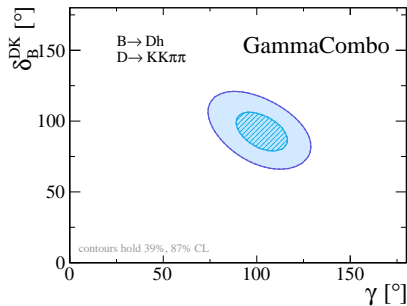
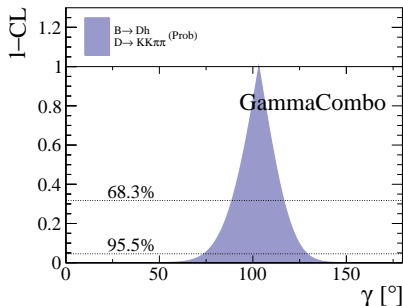


$B^\pm \rightarrow D\pi$ CP observables



Could there be a sign error in s_i ?

Interpretation



$$\gamma = (103 \pm 14)^\circ$$

$$\delta_B^{DK} = (92 \pm 14)^\circ$$

$$r_B^{DK} = 0.117 \pm 0.020$$

$$\delta_B^{D\pi} = (296 \pm 84)^\circ$$

$$r_B^{D\pi} = 0.004 \pm 0.005$$

Summary of $KK\pi\pi$ analysis

Summary of $KK\pi\pi$ analysis

- Measured CP observables:

$$x_-^{DK} = (8.3 \pm 3.0 \pm 0.5 \pm 0.1) \times 10^{-2},$$

$$y_-^{DK} = (-1.2 \pm 3.5 \pm 0.3 \pm 3.8) \times 10^{-2},$$

$$x_+^{DK} = (-13.5 \pm 2.6 \pm 1.1 \pm 1.8) \times 10^{-2},$$

$$y_+^{DK} = (-4.0 \pm 3.1 \pm 0.5 \pm 1.0) \times 10^{-2},$$

$$x_\xi^{D\pi} = (-2.9 \pm 4.1 \pm 0.6 \pm 0.0) \times 10^{-2},$$

$$y_\xi^{D\pi} = (-2.0 \pm 4.9 \pm 0.7 \pm 0.7) \times 10^{-2},$$

- Measured physics parameters:

$$\gamma = (103 \pm 14)^\circ$$

$$\delta_B^{DK} = (92 \pm 14)^\circ$$

$$r_B^{DK} = 0.117 \pm 0.020$$

$$\delta_B^{D\pi} = (296 \pm 84)^\circ$$

$$r_B^{D\pi} = 0.004 \pm 0.005$$

Conclusion and next steps

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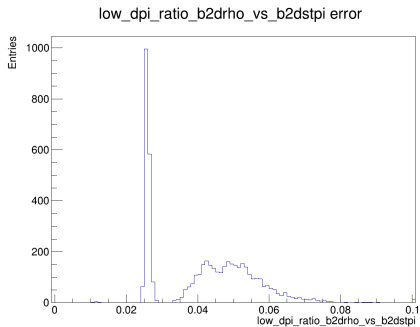
- Binned γ analysis of $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D h^\pm$ is ready for RC
- Quasi-GLW observables of $h^+ h^- \pi^+ \pi^-$ further constrains γ
- All systematics have been evaluated
- Next steps:
 - 1 Understand double peak error distribution in global fit toys
 - 2 Make sure s_i has the correct sign

Thank you!

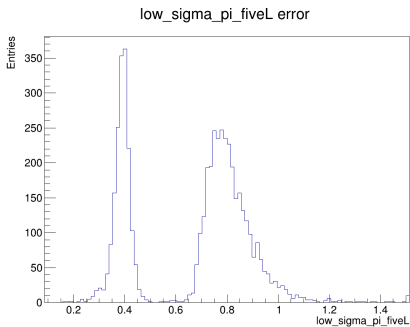
Thank you!

Backup

Double-peaked error distributions



(a) $f_{D\pi\pi}^{D\pi}$



(b) $\sigma_{\pi}^{\text{low}}$

s_i sign problem

- Amplitude model gives us: $A(\Phi) = \sum_k a_k S_k(\Phi)$
- Flavour-tagged LHCb data measures: $|A(\Phi)|^2$
- a_k phase is fixed from phase convention of $S_k(\Phi)$ lineshapes
- Physical observable: Interference fractions
 - Toy fit shows that LHCb and CLEO models are consistent
 - Thanks to Tim Evans for checking!

| Resonance | LHCb model phase (rad) | CLEO model (rad) |
|--|------------------------|------------------|
| $D^0 \rightarrow [\phi(1020)\rho^0]_{L=0}$ | 0 (fixed) | 0 (fixed) |
| $D^0 \rightarrow K_1(1400)^+ K^-$ | 1.05 | -1.79 |
| $D^0 \rightarrow K_1(1270)^+ K^-$ | 2.02 | -2.56 |

- BESIII data can determine sign uniquely!
- Reconstruct $KK\pi\pi$ vs $K_{S,L}\pi\pi$ double tags:

$$M_{i,j} \propto (K_i K'_{-j} + K_{-i} K'_j \mp 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j))$$

Cross checks:

- 1 Can use BESIII data with $KK\pi\pi$ vs $K_{S,L}\pi\pi$ tags to check s_i sign
- 2 Same fit code was used in $B \rightarrow Dh$, $D \rightarrow K_S\pi\pi$ analysis
- 3 Same code used to bin events in data and in calculation of c_i/s_i
- 4 Independent binned fit toys are consistent with AmpGen unbinned fit