Model-independent measurement of γ using $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ decays

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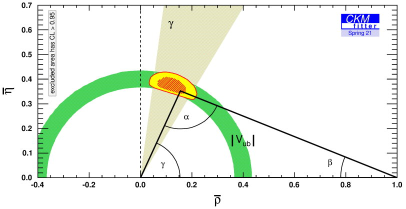
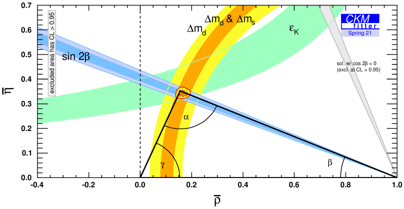
Outline

Introduction to γ and CP violation

- CPV in SM is described by the Unitary Triangle, with angles α, β, γ
- The angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is very important:
  1. Negligible theoretical uncertainties: Ideal SM benchmark
  2. Accessible at tree level: Indirectly probe New Physics that enter loops
  3. Compare with a global CKM fit: Is the Unitary Triangle a triangle?



(a) Tree level: $\gamma = \left(72.1^{+5.4}_{-5.7}\right)^\circ$

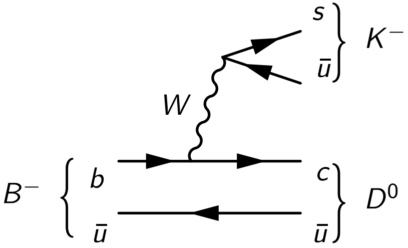


(b) Loop level: $\gamma = \left(65.5^{+1.1}_{-2.7}\right)^\circ$

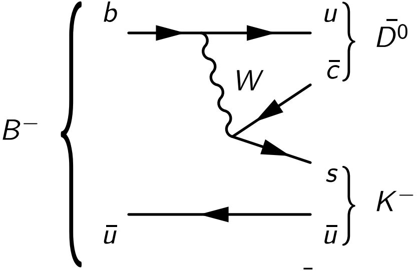
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), updated results and plots available at: http://ckmfitter.in2p3.fr

Sensitivity through interference

Measure γ through interference effects in $B^{\pm} \to DK^{\pm}$



Favoured $B^- \to D^0 K^-$



Suppressed $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
  - Consider $D^0/\bar{D}^0$ decays to the same final state, such as $D \to K^+K^-$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}_B\left(\mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)}\mathcal{A}_{\bar{D}^0}\right)$$
$$\mathcal{A}(B^+) = \mathcal{A}_B\left(\mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)}\mathcal{A}_{D^0}\right)$$

Multi-body charm decays

In this presentation, four-body charm decays are considered:

$$D^0 \to K^+ K^- \pi^+ \pi^-$$

$$D^0 \to \pi^+ \pi^- \pi^+ \pi^-$$

Note: Such decays have a five-dimensional phase space!

Degrees of freedom for an N-body decay

$$\begin{aligned}
&\ 4N \text{ (momentum components)} \\
&- N\ (E_i^2 - p_i^2 = m_i^2) \\
&- 4 \text{ (energy-momentum conservation)} \\
&- 3 \text{ (choice of frame)} \\
\hline
&= 3N - 7 \text{ degrees of freedom}
\end{aligned}$$

Previous studies of γ with $B^{\pm} \to DK^{\pm}$, $D \to K^+K^-\pi^+\pi^-$

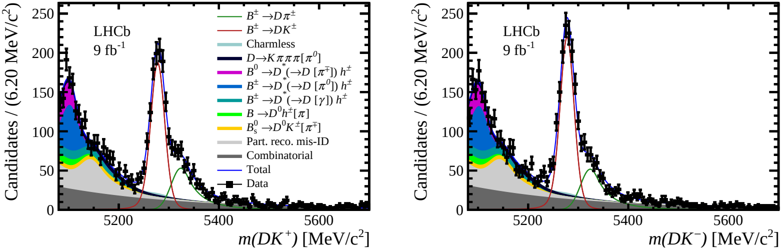
1. First proposed by J. Rademacker and G. Wilkinson:
   - Physics Letters B **647** (2007) 400
   - Amplitude model by FOCUS
   - Expected γ precision from amplitude fit with 1000 candidates: 14°
2. CLEO amplitude analysis:
   - Phys. Rev. D **85** (2012) 122002
   - Expected γ precision from amplitude fit with 2000 candidates: 11°
3. State of the art amplitude analysis by LHCb:
   - JHEP **02** (2019) 126
4. Model-dependent measurement by LHCb:
   - Eur. Phys. J. C **83** 547 (2023)
   - Optimised binning scheme using LHCb amplitude model

Previous studies of γ with $B^\pm \to DK^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$

1. CLEO amplitude analysis:
   - JHEP **05** (2017) 143
2. CLEO-c strong-phase measurement:
   - JHEP **01** (2018) 144
   - Expected γ statistical (systematic) precision with 2×5 bins is 9.7° (7.4°)
3. Recent work by Bristol group:
   - B2OC meeting, 18th April 2024
4. New BESIII amplitude model:
   - arXiv:2312.02524
   - Joint effort between Oxford and Bristol group using a new, optimal binning scheme

Phase-space integrated CP observables

Phase-space integrated study of γ:
Charged asymmetries measured for $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ in Eur. Phys. J. C **83** 547 (2023)



$D \to K^+K^-\pi^+\pi^-$

- $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ asymmetries:
  - $D \to K^+K^-\pi^+\pi^-$: $\mathcal{A} = 0.095 \pm 0.023 \pm 0.002$
  - $D \to \pi^+\pi^-\pi^+\pi^-$: $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

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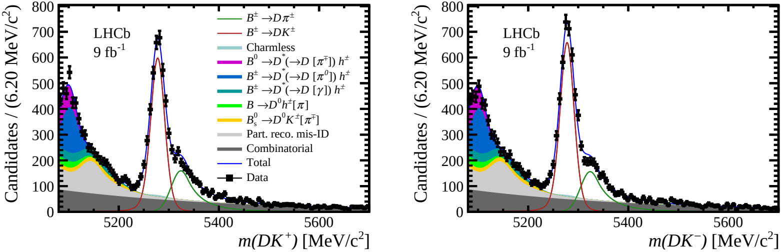

$D \to \pi^+\pi^-\pi^+\pi^-$

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Multi-body D decays

Main focus of this talk: Discuss phase-space binned analysis of
$D \to h^+ h^- \pi^+ \pi^-$

- Strong-phase difference δ_D is a function of phase space
- Compare yields of B^+ and B^- and determine the asymmetry <u>in local phase space regions</u>, known as phase-space bins

$$\begin{tikzcd}
 & D^0 K^- \arrow[dr, bend left, "\mathcal{A}_{D^0}(\Phi)"] & \\
B^- \arrow[ur, bend left, "\mathcal{A}_B"] \arrow[dr, bend right, "\mathcal{A}_B r_B e^{i(\delta_B - \gamma)}"'] & & DK^- \\
 & \bar{D}^0 K^- \arrow[ur, bend right, "\mathcal{A}_{\bar{D}^0}(\Phi)"'] &
\end{tikzcd}$$

$$\begin{aligned}
|\mathcal{A}(B^-)|^2 \propto\ & |\mathcal{A}_{D^0}(\Phi)|^2 + r_B^2 |\mathcal{A}_{\bar{D}^0}(\Phi)|^2 \\
& + 2 r_B |\mathcal{A}_{D^0}(\Phi)||\mathcal{A}_{\bar{D}^0}(\Phi)| \cos(\delta_B - \gamma + \delta_D)
\end{aligned}$$

The BPGGSZ method

Event yield in bin i

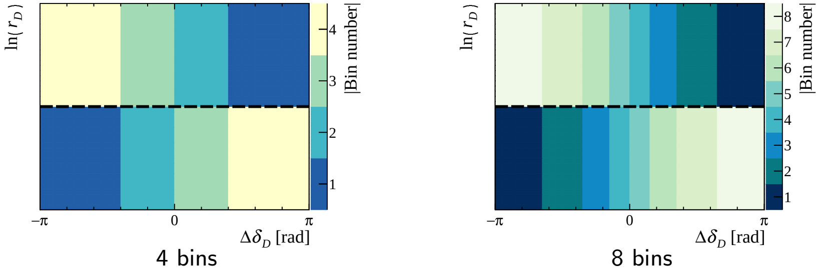
$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude-averaged strong phases:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

- Interpretation of γ from the multi-body charm decays require external inputs of the charm strong-phase differences
- Measure model-independent strong-phases at a charm factory, such as BESIII, using an optimised binning scheme

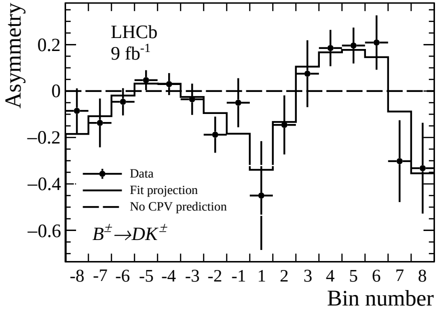


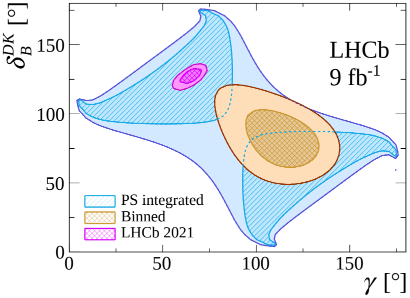
$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

Model-dependent measurement with $D \to K^+K^-\pi^+\pi^-$

From the phase-space binned asymmetries, we obtain:

$$\gamma = (116^{+12}_{-14})^\circ$$





Eur. Phys. J. C **83**, 547 (2023)

How will this evolve with model-independent BESIII inputs? Will the 3σ tension reduce?

Motivation for this analysis

What is new from previous analysis of $K^+K^-\pi^+\pi^-$?

- Binned strong-phase analyses of $D \to K^+K^-\pi^+\pi^-$ (Oxford) and $D \to \pi^+\pi^-\pi^+\pi^-$ (Oxford-USTC) are emerging from BESIII
  - For $D \to \pi^+\pi^-\pi^+\pi^-$, these improve in precision on earlier binned study made with CLEO-c data JHEP **01** (2018) 144
- Make first binned model-independent measurement with $D \to K^+K^-\pi^+\pi^-$, updating earlier LHCb model-dependent analysis
- Clone analysis for $D \to \pi^+\pi^-\pi^+\pi^-$ with same 4-body selection
- After checking for compatibility, perform joint analysis

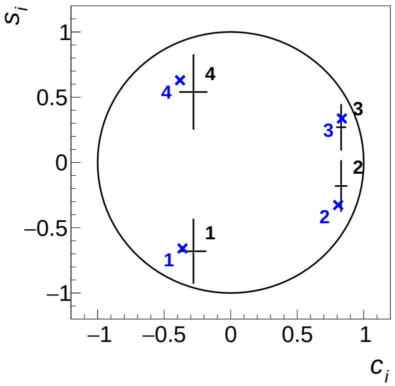
Motivation for this analysis

The results shown in this presentation make use of strong-phase results from the BESIII collaboration. They derive from mature analyses led by Oxford and USTC, but some of these are not yet public, and remain preliminary in nature. They are not to be shown outside LHCb. We thank the BESIII management for the privilege of being allowed to use them during the review of this measurement.

BESIII preliminary $D^0 \to K^+K^-\pi^+\pi^-$ strong-phase results

First binned strong-phase analysis of $D^0 \to K^+K^-\pi^+\pi^-$, which uses the 2×4 binning scheme with 16 fb^{-1} $\psi(3770)$ data

$$c_1 = -0.28 \pm 0.09 \pm 0.01$$
$$s_1 = -0.68 \pm 0.24 \pm 0.04$$
$$c_2 = +0.83 \pm 0.04 \pm 0.01$$
$$s_2 = -0.18 \pm 0.19 \pm 0.03$$
$$c_3 = +0.83 \pm 0.03 \pm 0.01$$
$$s_3 = +0.27 \pm 0.17 \pm 0.03$$
$$c_4 = -0.28 \pm 0.10 \pm 0.01$$
$$s_4 = +0.54 \pm 0.28 \pm 0.04$$

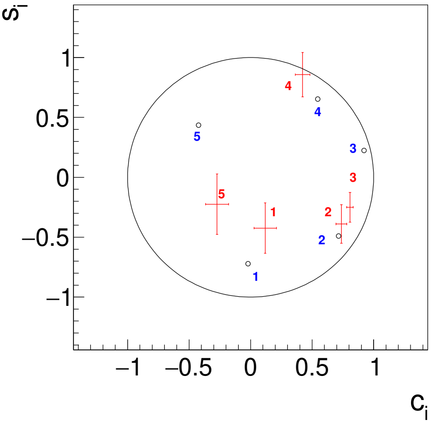


Measured values (black) are consistent and close to LHCb model predictions (blue), so central values are not expected to change much

BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

Small differences between model prediction and measurement, but data points are generally close to the unit circle

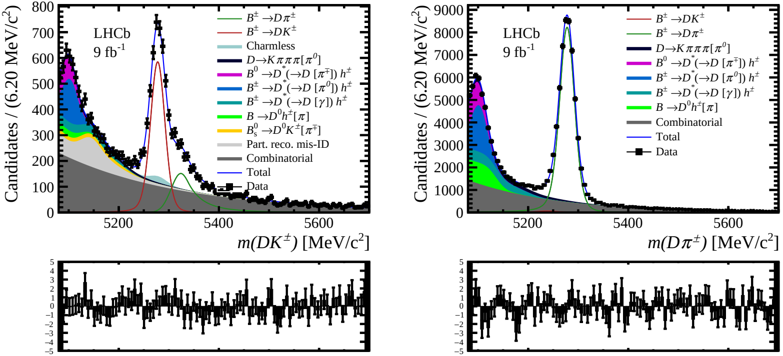
$$c_1 = +0.12 \pm 0.09 \pm 0.02$$
$$s_1 = -0.42 \pm 0.21 \pm 0.04$$
$$c_2 = +0.74 \pm 0.04 \pm 0.02$$
$$s_2 = -0.39 \pm 0.16 \pm 0.06$$
$$s_3 = -0.25 \pm 0.12 \pm 0.03$$
$$c_3 = +0.81 \pm 0.03 \pm 0.01$$
$$c_4 = +0.42 \pm 0.06 \pm 0.02$$
$$s_4 = +0.86 \pm 0.19 \pm 0.07$$
$$c_5 = -0.27 \pm 0.09 \pm 0.03$$
$$s_5 = -0.22 \pm 0.25 \pm 0.08$$



The HyperPlot software is used (binary lookup tree in 5D phase space)

Global fit

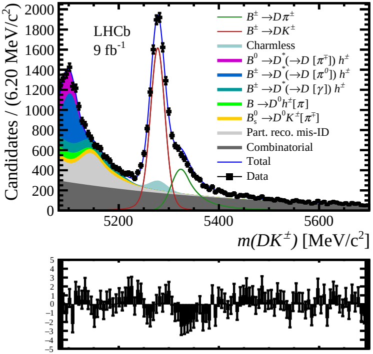
Global fit of $K^+K^-\pi^+\pi^-$ remains as in model-dependent publication:

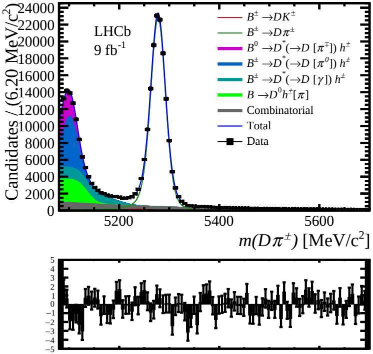


- $B^\pm \to [K^+K^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 3304 ± 42
  - $B^\pm \to D\pi^\pm$: 47894 ± 235

Global fit

Global fit of $\pi^+\pi^-\pi^+\pi^-$ has a good fit quality:





- $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 9211 ± 112
  - $B^\pm \to D\pi^\pm$: 132654 ± 398

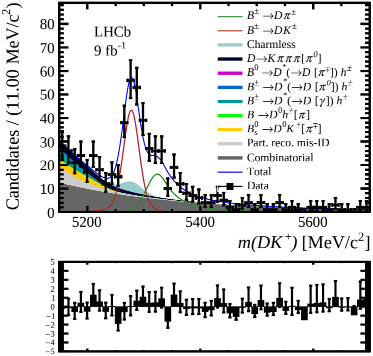
CP fit

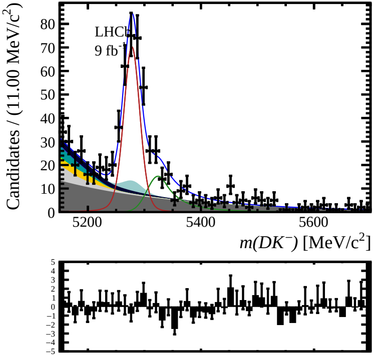
After global fit, perform a "CP fit" to study CP violation:

- Split candidates by:
  1. B^+ and B^- charges
  2. $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays
  3. D phase-space bins
- Combinatorial and low-mass backgrounds are floating in each category
- Parameterise signal yields in terms of $x_\pm^{DK}$, $y_\pm^{DK}$, $x_\xi^{D\pi}$, $y_\xi^{D\pi}$
- $2N - 1$ floating F_i parameters
- c_i and s_i are Gaussian constrained

CP fit bin asymmetry

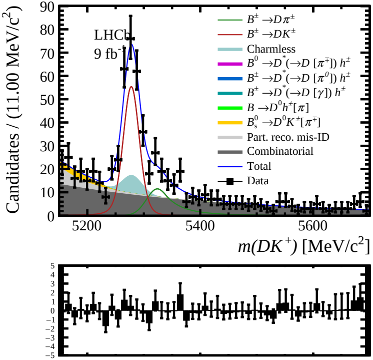
Example of bin asymmetry in $D \to K^+K^-\pi^+\pi^-$ bin -3:

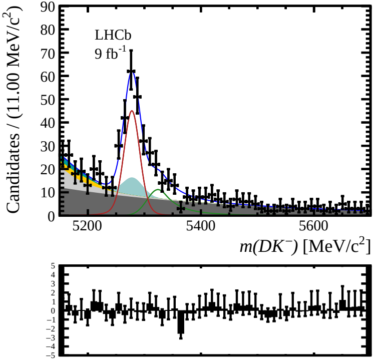




CP fit bin asymmetry

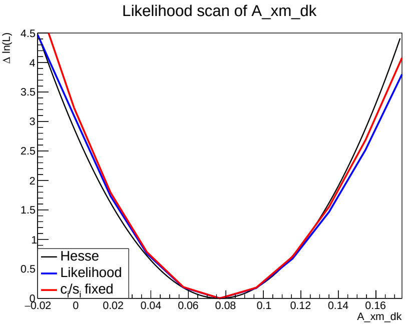
Example of bin asymmetry in $D \to \pi^+\pi^-\pi^+\pi^-$ bin $+5$:



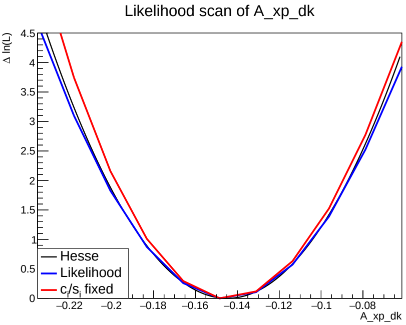


Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

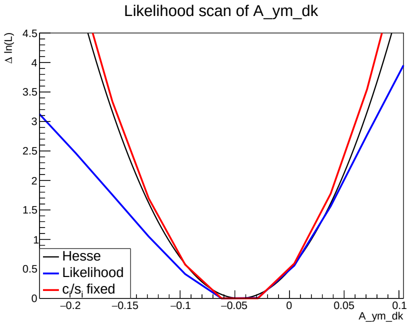


x_{+}^{DK}

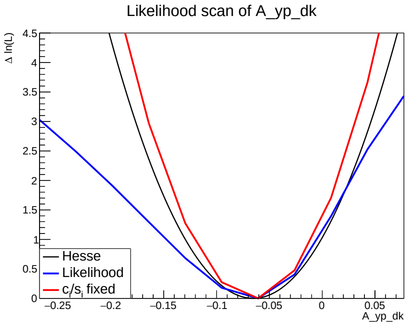
$D^0 \rightarrow K^+K^-\pi^+\pi^-$

Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK}



y_{+}^{DK}

$D^0 \rightarrow K^+K^-\pi^+\pi^-$

Likelihood scan of CP observables

What do the likelihood scans tell us?

- Uncertainties from c_i and s_i are significant, which justifies Gaussian constraining c_i and s_i
- New strategy:
  1. Produce a likelihood function from CP fit
  2. Interpret CP observables in terms of γ, etc
  3. Must profile all nuisance parameters (F_i, c_i, s_i, backgrounds yields, normalisation constants)
  4. Provide direct measurements of γ, δ_B and r_B

Interpretation strategy

From CP fit, we have a (negative log) likelihood function with nuisance parameters n_k:

$$\mathcal{L}(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}, \{n_k\})$$

Express in terms of physics parameters:

$$\mathcal{L}(\gamma, \delta_B^{DK}, r_B^{DK}, \delta_B^{D\pi}, r_B^{D\pi}, \{n_k\})$$

In this step, also add a Gaussian smearing term on CP observables to account for internal LHCb systematics (see backup)

Interpretation results

Results from interpretation of $K^+K^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

Model independent

$$
\begin{aligned}
\gamma =&(119 \pm 14)^\circ \\
\delta_B^{DK} =&(80 \pm 12)^\circ \\
r_B^{DK} =&(11.4 \pm 2.3) \times 10^{-2} \\
\delta_B^{D\pi} =&(253 \pm 62)^\circ \\
r_B^{D\pi} =&(3 \pm 7) \times 10^{-3}
\end{aligned}
$$

Model dependent

$$
\begin{aligned}
\gamma =&(116^{+12}_{-14})^\circ \\
\delta_B^{DK} =&(81^{+14}_{-13})^\circ \\
r_B^{DK} =&(11.0 \pm 2.0) \times 10^{-2} \\
\delta_B^{D\pi} =&(298^{+62}_{-118})^\circ \\
r_B^{D\pi} =&(4^{+5}_{-4}) \times 10^{-3}
\end{aligned}
$$

Central value of γ remains high...

... it seems that the large tension with the LHCb global result $\gamma = (63.8^{+3.5}_{-3.7})^\circ$ remains

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

$K^+K^-\pi^+\pi^-$

$$\begin{aligned}\gamma =&(119 \pm 14)^\circ\\ \delta_B^{DK} =&(80 \pm 12)^\circ\\ r_B^{DK} =&(11.4 \pm 2.3) \times 10^{-2}\\ \delta_B^{D\pi} =&(253 \pm 62)^\circ\\ r_B^{D\pi} =&(3 \pm 7) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$

$$\begin{aligned}\gamma =&(45 \pm 9)^\circ\\ \delta_B^{DK} =&(114 \pm 9)^\circ\\ r_B^{DK} =&(9.5 \pm 1.9) \times 10^{-2}\\ \delta_B^{D\pi} =&(176 \pm 111)^\circ\\ r_B^{D\pi} =&(0.8 \pm 1.9) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

Interpretation results

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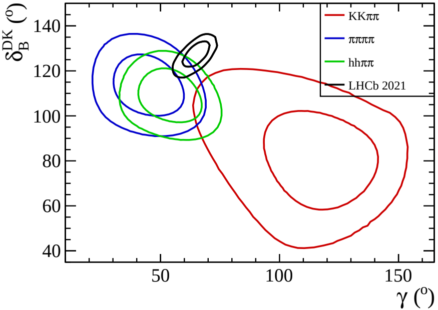
$$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

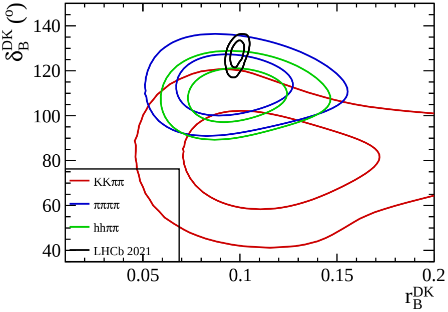
...but how Gaussian are these uncertainties?

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}

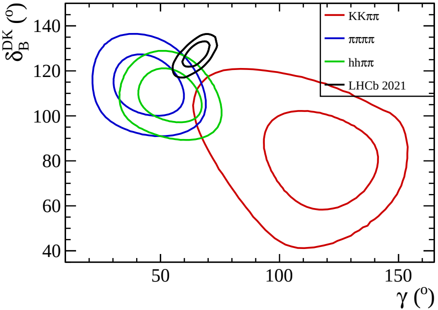


r_B^{DK} vs δ_B^{DK}

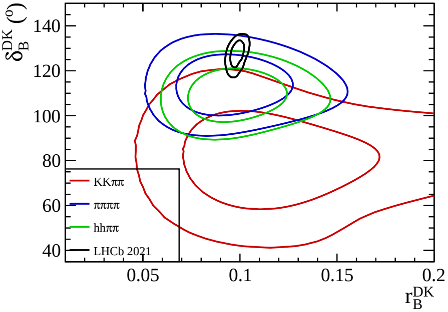
When all biases, correlations and non-Gaussian uncertainties are accounted for, the tension with the LHCb average has reduced significantly

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}



r_B^{DK} vs δ_B^{DK}

However, with all the non-Gaussian behaviour, are we sure these contours cover 68% and 95% ?

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

At each scan point of γ, perform these fits to data:

1. Fit with all parameters floating, and save the log-likelihood χ^2
2. Fit with γ fixed to scan point, and save χ^2_{fix}
3. Calculate $\Delta\chi^2_{\mathrm{data}} = \chi^2_{\mathrm{fix}} - \chi^2$

We expect $\Delta\chi^2_{\mathrm{data}}$ to become large as we move away from best-fit value, but without direct knowledge of underlying PDF, we cannot determine any confidence intervals from this

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

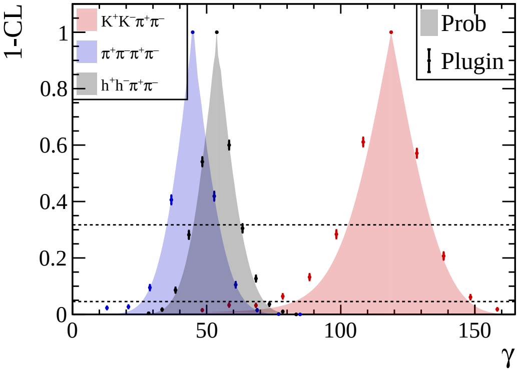
At each scan point of γ, perform these fits to toy:

1. Fix γ to scan point and generate 1000 toys
2. Perform fits to each toy, with γ both floating and fixed
3. Calculate $\Delta\chi^2_{\mathrm{toy}}$

At each scan point, the fraction of toys with $\Delta\chi^2_{\mathrm{toy}} > \Delta\chi^2_{\mathrm{data}}$ is equal to $1 - \mathrm{CL}$, and the exact 68% confidence interval can then be obtained using an interpolation between points

Plugin/Feldman-Cousins method

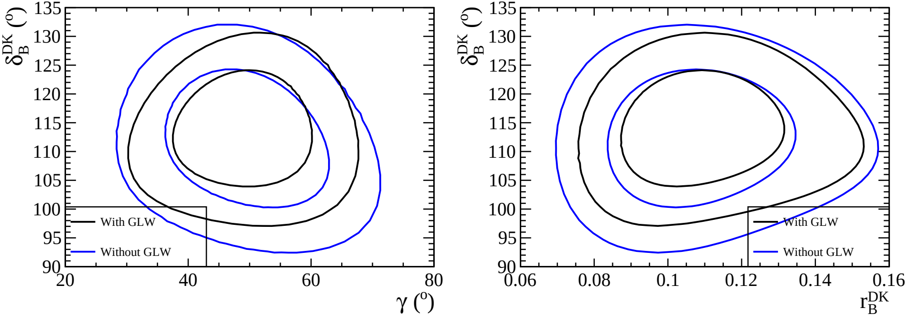
LHCb average within 2σ of $D \to K^+K^-\pi^+\pi^-$ Plugin result
Combined fit shows good agreement between Plugin and Prob scans



Combined fit result: $\gamma = (54.0^{+10.2}_{-9.5})^\circ$
Third most precise single measurement of γ in $B^\pm$ decays

Combining phase-space binned and integrated results

We can add phase-space integrated observables as a constraint:

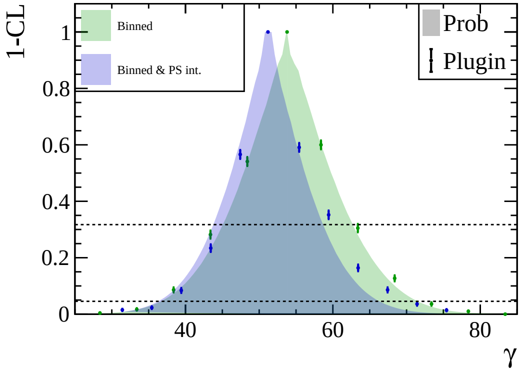


γ vs δ_B^{DK} $\quad$ r_B^{DK} vs δ_B^{DK}

The global asymmetries contain useful information!

Combining phase-space binned and integrated results

Run Plugin with phase-space integrated constraints:



Final measurement: $\gamma = (51.2^{+8.9}_{-6.5})^\circ$

Conclusion

1. Binned model-independent measurement of γ with $B^{\pm} \to [h^+h^-\pi^+\pi^-]_D h^{\pm}$ has been performed: $\gamma = (54.0^{+10.2}_{-9.5})^{\circ}$
   - Uses BESIII strong-phase inputs, some of which are not yet public, but are expected to become so on timescale of a few months
2. Combination with integrated results: $\gamma = (51.2^{+8.9}_{-6.5})^{\circ}$
3. 3σ tension in $D \to K^+K^-\pi^+\pi^-$ has reduced

Future prospects

- Statistically limited measurement, but s_i uncertainties are large
  - $\pi^+\pi^-\pi^+\pi^-$: s_i uncertainties will improve with full BESIII data set
  - $K^+K^-\pi^+\pi^-$: Charm mixing studies can improve s_i precision
- The analysis is approaching WG approval
- You can find the TWiki here

Thanks for your attention!

Backup: BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

- Binned strong-phase analysis of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ uses the 2×5 "optimal" binning scheme with 3 fb^{-1} $\psi(3770)$
- Earlier CLEO-c analysis with 0.8 fb^{-1} JHEP **01** (2018) 144
- New BESIII analysis uses a new binning scheme optimised with a BESIII amplitude model arXiv:2312.02524
  - Amplitude model constructed from a larger data set
  - In principle more sensitive
- Two binning schemes are available:
  - We use the more sensitive "optimal" binning with $Q = 0.85$
  - The other "equal δ" binning has $Q = 0.80$

Backup: Global fit

How do we include the $\pi^+\pi^-\pi^+\pi^-$ mode?

- We have already studied $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ for phase-space integrated measurement
  1. Different D daughter PID cuts in stripping
  2. No $D \to K\pi\pi\pi\pi^0$ background
  3. Charmless background recalculated using the sideband
  4. Use same BDT
  5. No additional peaking backgrounds
- Sort candidates into phase-space bins using BESIII binning scheme
- Can fit separately or simultaneously with $K^+K^-\pi^+\pi^-$

Backup: Strong-phase parameters in CP fit

Why are c_i and s_i Gaussian constrained?

- Previous BPGGSZ analyses have kept c_i and s_i fixed
  1. c_i and s_i uncertainties are added as a systematic through smearing
  2. Convenient for calculating correlations between different analyses
  3. Appropriate when c_i and s_i uncertainties are <u>small</u>
- In four-body analyses, uncertainties on γ from c_i and s_i are almost the same size as the statistical uncertainty
- Large s_i uncertainties introduces non-Gaussian uncertainties on $y_\pm$
- γ/δ_B move significantly when fixing s_i instead of constraining them
- These effects are largest for $K^+K^-\pi^+\pi^-$, but are also seen in $\pi^+\pi^-\pi^+\pi^-$ and in the combined fit

Backup: CP fit categories

Summary of free parameters in the CP fit:

$K^+K^-\pi^+\pi^-$

$2 \times 2 \times 2 \times 4 = 32$ categories

- 6 CP observables
- 7 F_i parameters
- 8 c_i and s_i parameters
- 32 combinatorial yields
- 32 low mass yields
- 4 global normalisations
- Total: 89 parameters

$\pi^+\pi^-\pi^+\pi^-$

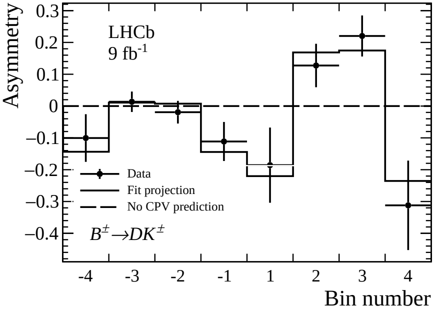
$2 \times 2 \times 2 \times 5 = 40$ categories

- 6 CP observables
- 9 F_i parameters
- 10 c_i and s_i parameters
- 40 combinatorial yields
- 40 low mass yields
- 4 global normalisations
- Total: 109 parameters

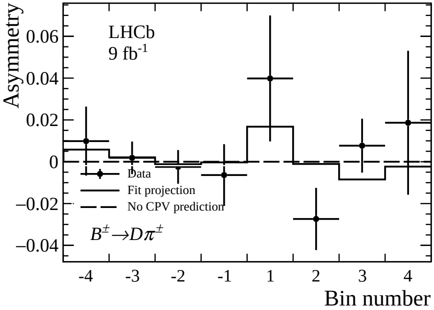
In a combined fit where CP observables are shared, there are $89 + 109 - 6 = 192$ parameters

Backup: Bin asymmetries

$B^{\pm} \to [K^+K^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries



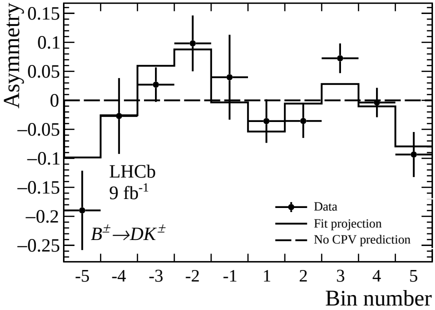
$B^{\pm} \to DK^{\pm}$



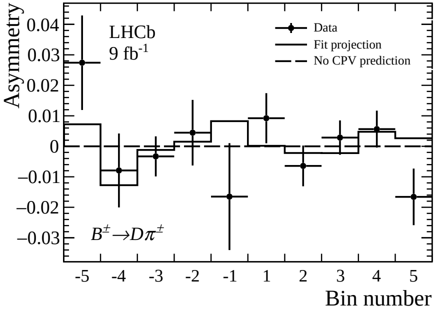
$B^{\pm} \to D\pi^{\pm}$

Backup: Bin asymmetries

$B^{\pm} \to [\pi^+\pi^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries

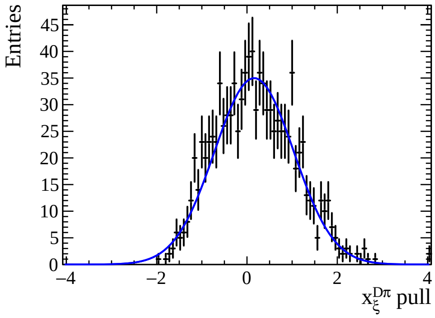


$B^{\pm} \to DK^{\pm}$

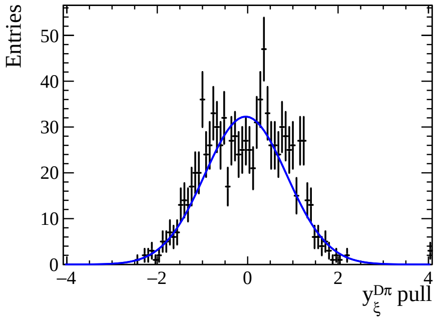


$B^{\pm} \to D\pi^{\pm}$

In toy studies biases in $D\pi$ observables are consistent with model-dependent analysis



$x_\xi^{D\pi}$

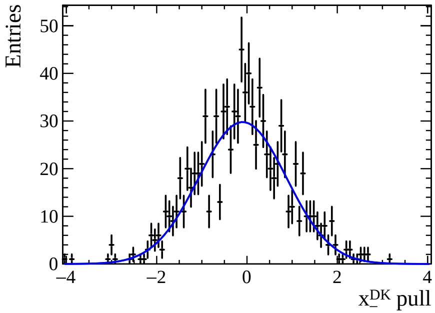


$y_\xi^{D\pi}$

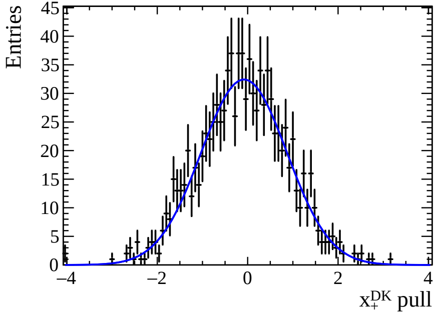
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: CP fit toy studies

Minor biases in $x_\pm^{DK}$ are seen but can be corrected for...



x_-^{DK}

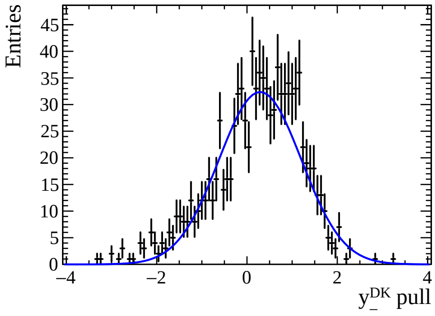


x_+^{DK}

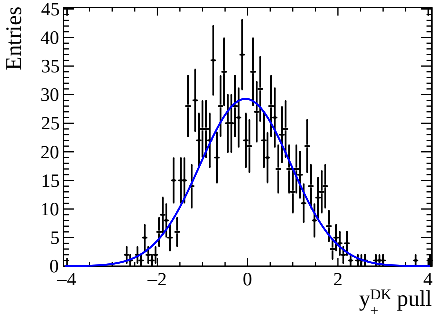
$D^0 \to K^+ K^- \pi^+ \pi^-$

Backup: CP fit toy studies

...but $y_{\pm}^{DK}$ pulls are now slightly asymmetric!



y_{-}^{DK}

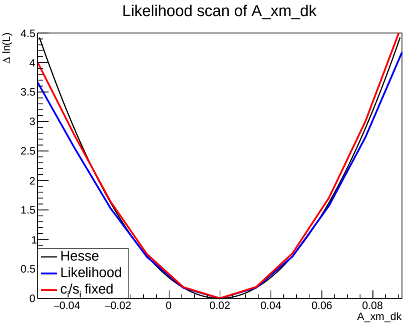


y_{+}^{DK}

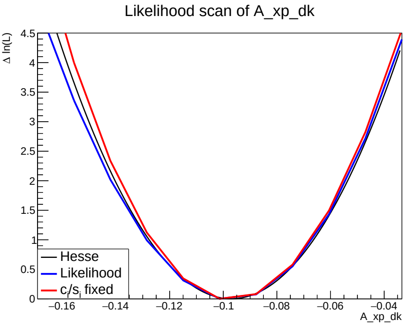
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: Likelihood scan of CP observables

$x_\pm^{DK}$ agree well between likelihood scan and Hesse approximation



x_-^{DK}

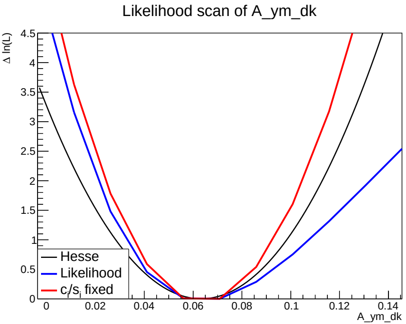


x_+^{DK}

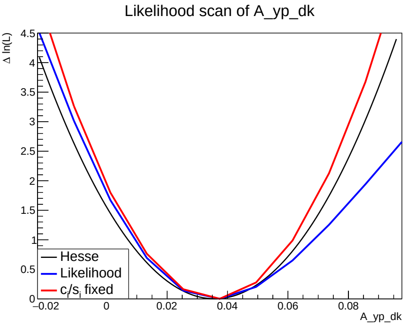
$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK}



y_{+}^{DK}

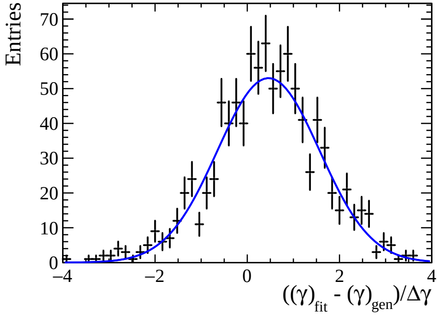
$D^0 \to \pi^+\pi^-\pi^+\pi^-$

Backup: Summary of LHCb internal systematic uncertainties

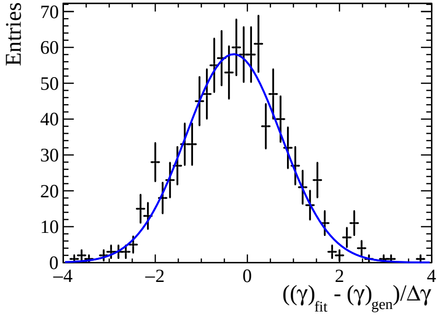
Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
CP violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic *b*-hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \to K^-\pi^+\pi^-\pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \to pD\pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \to K^-\pi^+\pi^-\pi^+\pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Fit bias	0.06	0.05	0.13	0.02	0.06	0.13
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57

Give systematic uncertainties in terms of CP observables (not γ) since these are more Gaussian and better behaved

We can perform toy studies on the interpretation fit, but we do <u>not</u> expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$



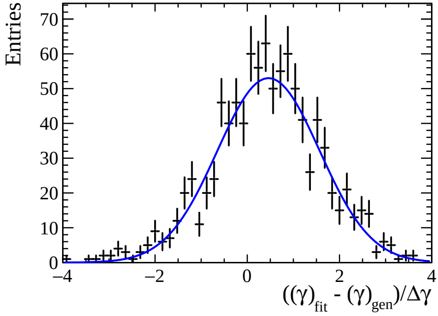
$\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

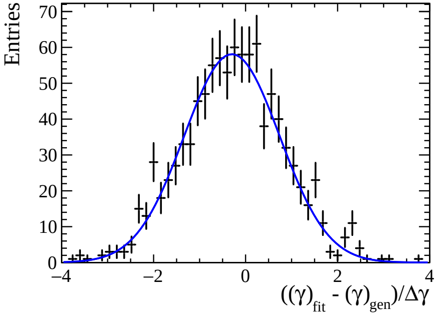
Indeed, small but significant biases are observed!

Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do not expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$



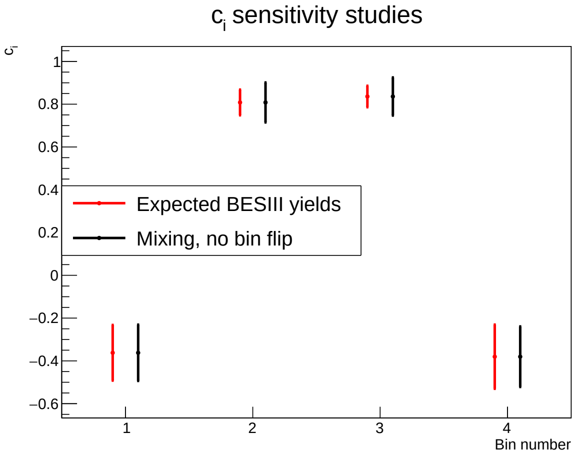
$\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

The absolute bias corrections are:

Backup: Charm mixing studies with multi-body decays

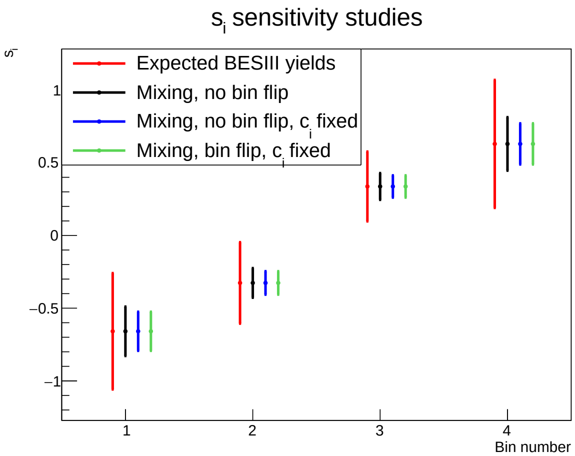
Sensitivity to c_i: Similar between BESIII and charm mixing at LHCb



- BESIII yields equivalent to $8\,\text{fb}^{-1}$ of $\psi(3770)$
- 4 million $D \to K^+K^-\pi^+\pi^-$ candidates in mixing analysis

Backup: Charm mixing studies with multi-body decays

Sensitivity to s_i: Significant improvements expected!



- BESIII yields equivalent to $8\,\mathrm{fb}^{-1}$ of $\psi(3770)$
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