Model-independent measurement of γ using $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ decays

Sneha Malde [1], Jonas Rademacker [2], **Martin Tat**[1,4], Ben Westhenry [2], Mark Whitehead [3], Guy Wilkinson [1]

[1]University of Oxford, [2]University of Bristol, [3]University of Glasgow, [4]Heidelberg University

15th April 2025

Outline

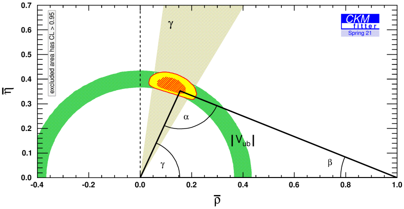
Acknowledgements

Many thanks to all WG and RC reviewers!

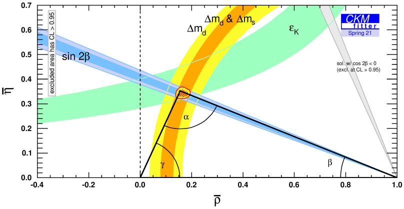
- B2OC WG review: Anton, Wenbin and Resmi
- RC: Francesco and Xiao-Rui

Introduction to γ and *CP* violation

- CPV in SM is described by the Unitary Triangle, with angles α, β, γ
- The angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is very important:
  1. Negligible theoretical uncertainties: Ideal SM benchmark
  2. Accessible at tree level: Indirectly probe New Physics that enter loops
  3. Compare with a global CKM fit: Is the Unitary Triangle a triangle?



(a) Tree level: $\gamma = \left(72.1^{+5.4}_{-5.7}\right)^\circ$

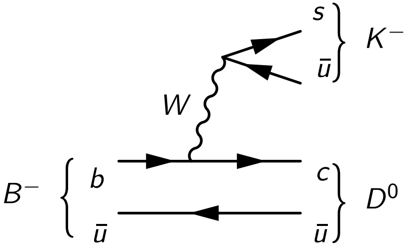


(b) Loop level: $\gamma = \left(65.5^{+1.1}_{-2.7}\right)^\circ$

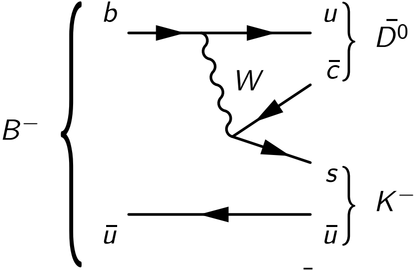
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), updated results and plots available at: http://ckmfitter.in2p3.fr

Sensitivity through interference

Measure γ through interference effects in $B^{\pm} \to DK^{\pm}$



Favoured $B^- \to D^0 K^-$



Suppressed $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
  - Consider $D^0/\bar{D}^0$ decays to the same final state, such as $D \to K^+K^-$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}_B\left(\mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)}\mathcal{A}_{\bar{D}^0}\right)$$
$$\mathcal{A}(B^+) = \mathcal{A}_B\left(\mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)}\mathcal{A}_{D^0}\right)$$

Multi-body charm decays

In this presentation, four-body charm decays are considered:

$$D^0 \to K^+K^-\pi^+\pi^-$$

$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Note: Such decays have a five-dimensional phase space!

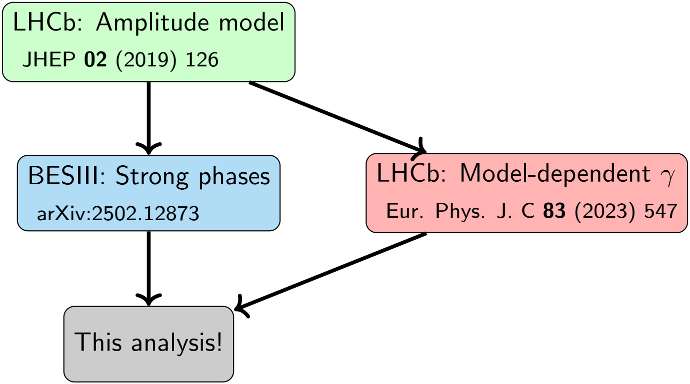
Degrees of freedom for an N-body decay

$$\begin{aligned}
&\quad 4N \text{ (momentum components)} \\
&- N\ (E_i^2 - p_i^2 = m_i^2) \\
&- 4 \text{ (energy-momentum conservation)} \\
&- 3 \text{ (choice of frame)} \\
\hline
&= 3N - 7 \text{ degrees of freedom}
\end{aligned}$$

Previous studies of γ with $B^{\pm} \to DK^{\pm}$, $D \to K^+K^-\pi^+\pi^-$

1. First proposed by J. Rademacker and G. Wilkinson:
   - Physics Letters B **647** (2007) 400
   - Amplitude model by FOCUS
   - Expected γ precision from amplitude fit with 1000 candidates: 14°
2. CLEO amplitude analysis:
   - Phys. Rev. D **85** (2012) 122002
   - Expected γ precision from amplitude fit with 2000 candidates: 11°
3. State of the art amplitude analysis by LHCb:
   - JHEP **02** (2019) 126
4. Model-dependent measurement by LHCb:
   - Eur. Phys. J. C **83** 547 (2023)
   - Optimised binning scheme using LHCb amplitude model

Inputs to $D^0 \to K^+K^-\pi^+\pi^-$ mode



Previous studies of γ with $B^{\pm} \to DK^{\pm}$, $D \to \pi^+\pi^-\pi^+\pi^-$

1. CLEO amplitude analysis:
   - JHEP **05** (2017) 143
2. CLEO-c strong-phase measurement:
   - JHEP **01** (2018) 144
   - Expected γ statistical (systematic) precision with 2×5 bins is 9.7° (7.4°)
3. For this LHCb publication:
   - New amplitude model from BESIII Chin. Phys. C **48** (2024) 083001
   - $\to$ Motivation to perform c_i/s_i measurements using a new binning scheme Phys. Rev. D **110** (2024) 112008
   - Results presented from both CLEO-c and BESIII binning schemes
   - Selection unified between Bristol and Oxford

Inputs to $D^0 \to \pi^+\pi^-\pi^+\pi^-$ mode

CLEO-c: Amplitude model
JHEP **05** (2017) 143

BESIII: Amplitude model
Chin. Phys. C **48** (2024) 083001

CLEO-c: Strong phases
JHEP **01** (2018) 144

BESIII: Strong phases
Phys. Rev. D **110** (2024) 112008

This analysis!

Selection of $B^\pm \to DK^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$

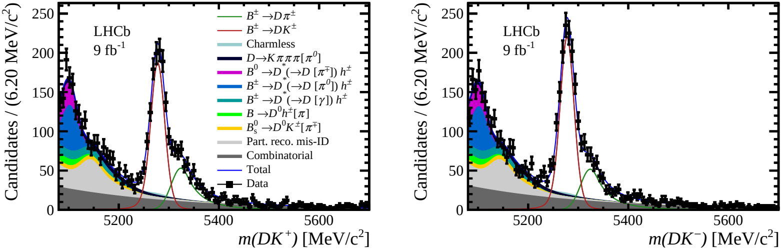
Selection of $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ is mostly similar to that in phase-space integrated measurement Eur. Phys. J. C **83** 547 (2023)

- Minor differences adapted for binned measurement:

- Flight significance cut loosened from 4 to 2
  - Phase-space binned analysis is less sensitive to charmless background

- Hadrons from D^0: `ProbNNpi*(1-ProbNNk) > 0.05`
  - Highly efficient at rejecting combinatorial background
  - Phase-space binned analysis benefits from higher purity in low-yield bins

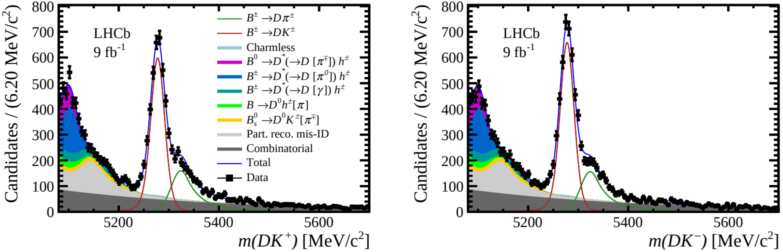
Phase-space integrated study of γ:
Charged asymmetries measured for $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ in Eur. Phys. J. C **83** 547 (2023)



$D \to K^+K^-\pi^+\pi^-$

- $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ asymmetries:
  - $D \to K^+K^-\pi^+\pi^-$: $\mathcal{A} = 0.095 \pm 0.023 \pm 0.002$
  - $D \to \pi^+\pi^-\pi^+\pi^-$: $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

Phase-space integrated study of γ:
Charged asymmetries measured for $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ in Eur. Phys. J. C **83** 547 (2023)



$D \to \pi^+\pi^-\pi^+\pi^-$

- $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ asymmetries:
  - $D \to K^+K^-\pi^+\pi^-$: $\mathcal{A} = 0.095 \pm 0.023 \pm 0.002$
  - $D \to \pi^+\pi^-\pi^+\pi^-$: $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

Multi-body D decays

Main focus of this talk: Discuss phase-space binned analysis of
$D \to h^+ h^- \pi^+ \pi^-$

- Strong-phase difference δ_D is a function of phase space
- Compare yields of B^+ and B^- and determine the asymmetry in local phase space regions, known as phase-space bins

$$
\begin{tikzcd}
 & D^0 K^- \arrow[dr, bend left, "\mathcal{A}_{D^0}(\Phi)"] & \\
B^- \arrow[ur, bend left, "\mathcal{A}_B"] \arrow[dr, bend right, "\mathcal{A}_B r_B e^{i(\delta_B - \gamma)}"'] & & DK^- \\
 & \bar{D}^0 K^- \arrow[ur, bend right, "\mathcal{A}_{\bar{D}^0}(\Phi)"'] &
\end{tikzcd}
$$

$$
\begin{aligned}
|\mathcal{A}(B^-)|^2 \propto\ & |\mathcal{A}_{D^0}(\Phi)|^2 + r_B^2 |\mathcal{A}_{\bar{D}^0}(\Phi)|^2 \\
& + 2 r_B |\mathcal{A}_{D^0}(\Phi)||\mathcal{A}_{\bar{D}^0}(\Phi)| \cos(\delta_B - \gamma + \delta_D)
\end{aligned}
$$

The BPGGSZ method

Event yield in bin i

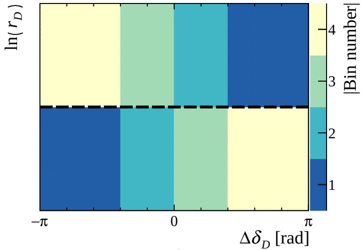
$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude-averaged strong phases:

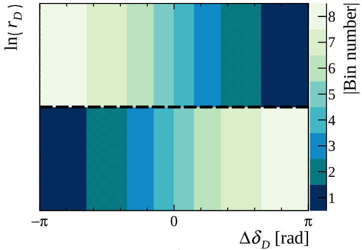
$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

- Interpretation of γ from the multi-body charm decays require external inputs of the charm strong-phase differences
- Measure model-independent strong-phases at a charm factory, such as BESIII, using an optimised binning scheme



4 bins



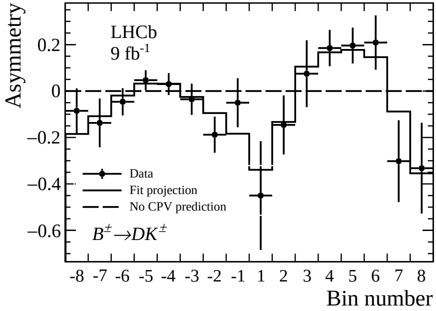
8 bins

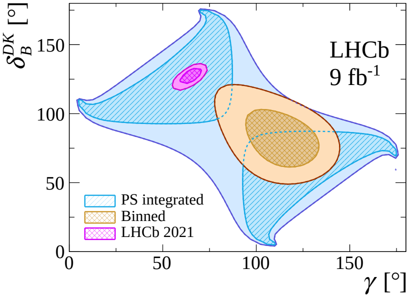
$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

Model-dependent measurement with $D \to K^+K^-\pi^+\pi^-$

From the phase-space binned asymmetries, we obtain:

$$\gamma = (116^{+12}_{-14})^\circ$$





Eur. Phys. J. C **83**, 547 (2023)

How will this evolve with model-independent BESIII inputs? Will the 3σ tension reduce?

Motivation for this analysis

What is new from previous analysis of $K^+K^-\pi^+\pi^-$?

- New binned strong-phase analyses of $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ have recently been made public by BESIII
  - $D^0 \to KK\pi\pi$: arXiv:2502.12873
  - $D^0 \to \pi\pi\pi\pi$: Phys. Rev. D **110** (2024) 112008
  - For $D \to \pi^+\pi^-\pi^+\pi^-$, these improve in precision on earlier binned study made with CLEO-c data JHEP **01** (2018) 144
- Make first binned model-independent measurement with $D \to K^+K^-\pi^+\pi^-$, updating earlier LHCb model-dependent analysis
- Use same strategy for $D \to \pi^+\pi^-\pi^+\pi^-$ with a joint Oxford-Bristol selection
- After checking for compatibility, perform joint analysis

BESIII preliminary $D^0 \to K^+K^-\pi^+\pi^-$ strong-phase results

First binned strong-phase analysis of $D^0 \to K^+K^-\pi^+\pi^-$, which uses the 2×4 binning scheme with 20 fb^{-1} $\psi(3770)$ data

$c_1 = -0.22 \pm 0.08 \pm 0.01$

$s_1 = -0.47 \pm 0.22 \pm 0.04$

$c_2 = +0.79 \pm 0.04 \pm 0.01$

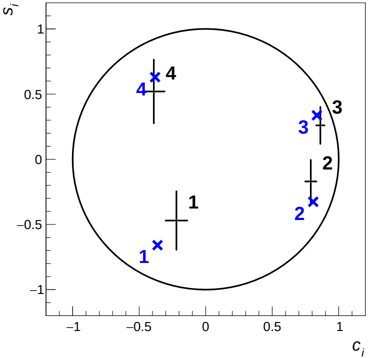
$s_2 = -0.17 \pm 0.16 \pm 0.04$

$c_3 = +0.862 \pm 0.029 \pm 0.008$

$s_3 = +0.26 \pm 0.14 \pm 0.02$

$c_4 = -0.39 \pm 0.08 \pm 0.01$

$s_4 = +0.52 \pm 0.24 \pm 0.04$

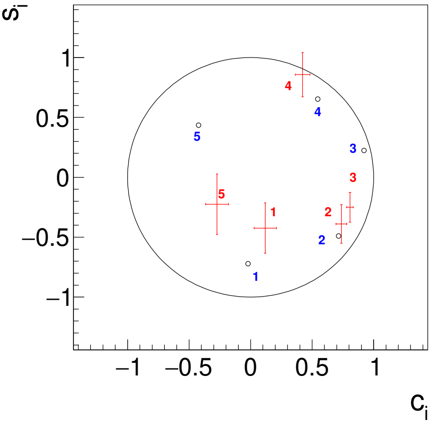


Measured values (black) are consistent and close to LHCb model predictions (blue), so central values are not expected to change much

BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

Small differences between model prediction and measurement, but data points are generally close to the unit circle

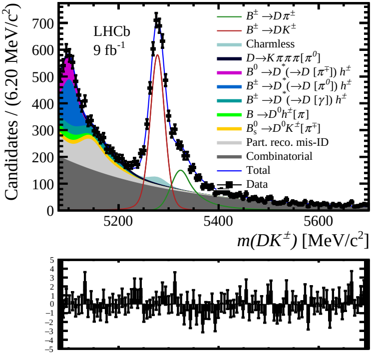
$$c_1 = +0.12 \pm 0.09 \pm 0.02$$
$$s_1 = -0.42 \pm 0.21 \pm 0.04$$
$$c_2 = +0.74 \pm 0.04 \pm 0.02$$
$$s_2 = -0.39 \pm 0.16 \pm 0.06$$
$$s_3 = -0.25 \pm 0.12 \pm 0.03$$
$$c_3 = +0.81 \pm 0.03 \pm 0.01$$
$$c_4 = +0.42 \pm 0.06 \pm 0.02$$
$$s_4 = +0.86 \pm 0.19 \pm 0.07$$
$$c_5 = -0.27 \pm 0.09 \pm 0.03$$
$$s_5 = -0.22 \pm 0.25 \pm 0.08$$

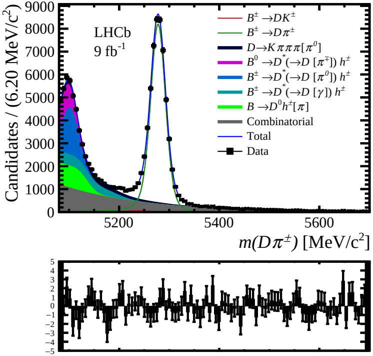


The HyperPlot software is used (binary lookup tree in 5D phase space)

Global fit

Global fit of $K^+K^-\pi^+\pi^-$ remains as in model-dependent publication:

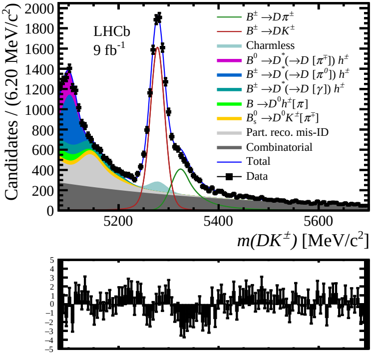


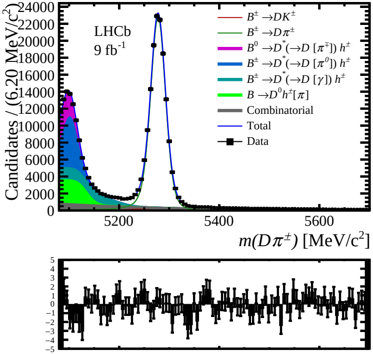


- $B^\pm \to [K^+K^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 3280 ± 41
  - $B^\pm \to D\pi^\pm$: 47610 ± 231

Global fit

Global fit of $\pi^+\pi^-\pi^+\pi^-$ has a good fit quality:





- $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 9172 ± 110
  - $B^\pm \to D\pi^\pm$: 132246 ± 394

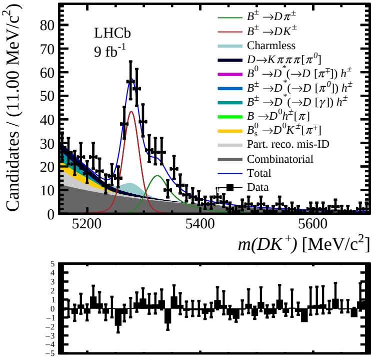
CP fit

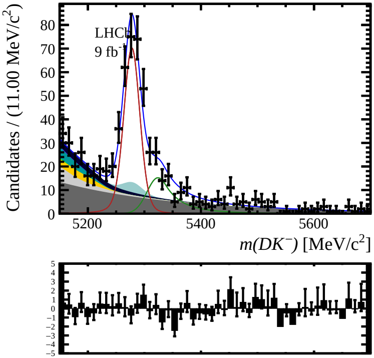
After global fit, perform a "CP fit" to study CP violation:

- Split candidates by:
  1. B^+ and B^- charges
  2. $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays
  3. D phase-space bins
- Combinatorial and low-mass backgrounds are floating in each category
- Parameterise signal yields in terms of $x_\pm^{DK}$, $y_\pm^{DK}$, $x_\xi^{D\pi}$, $y_\xi^{D\pi}$
- $2N - 1$ floating F_i parameters
- c_i and s_i are Gaussian constrained

CP fit bin asymmetry

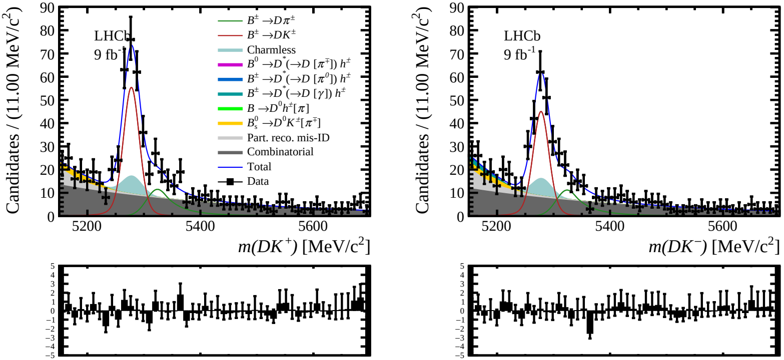
Example of bin asymmetry in $D \to K^+K^-\pi^+\pi^-$ bin -3:





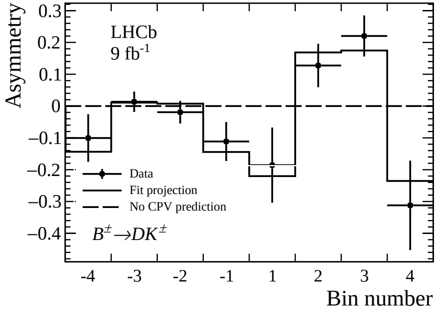
CP fit bin asymmetry

Example of bin asymmetry in $D \to \pi^+\pi^-\pi^+\pi^-$ bin $+5$:

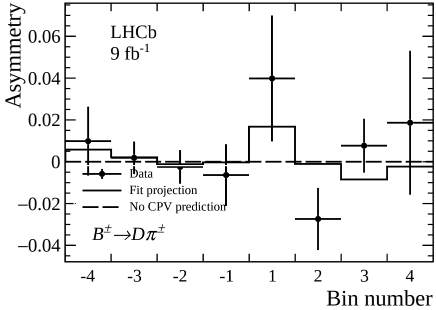


Bin asymmetries

$B^{\pm} \to [K^+ K^- \pi^+ \pi^-]_D h^{\pm}$ bin asymmetries



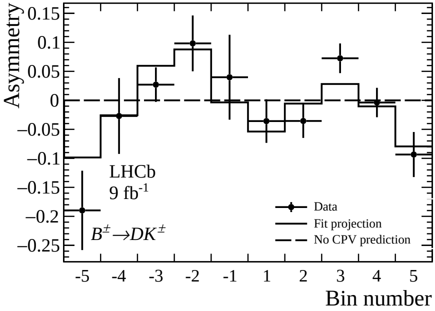
$B^{\pm} \to DK^{\pm}$



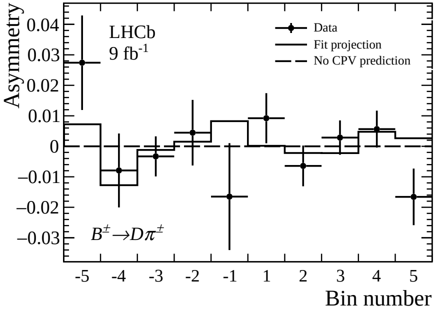
$B^{\pm} \to D\pi^{\pm}$

Bin asymmetries

$B^{\pm} \to [\pi^+\pi^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries



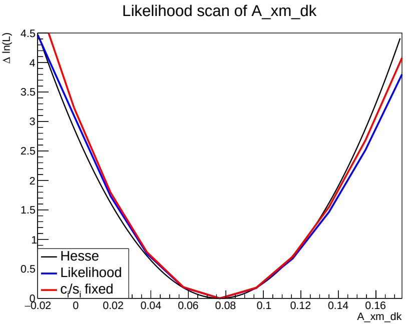
$B^{\pm} \to DK^{\pm}$



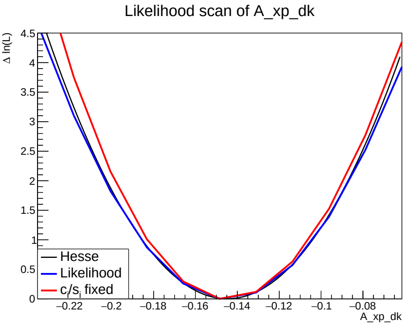
$B^{\pm} \to D\pi^{\pm}$

Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

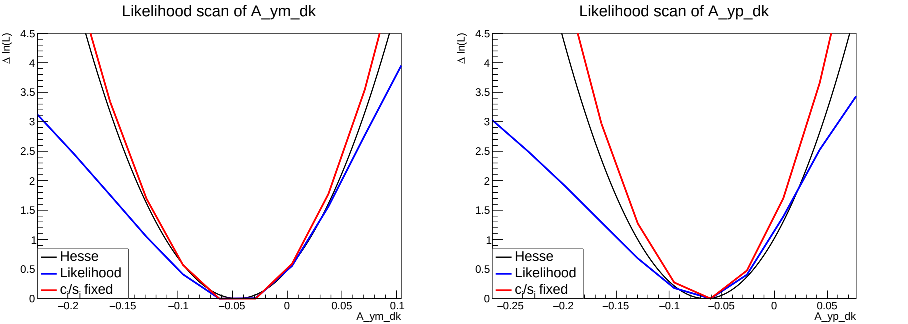


x_{+}^{DK}

$$D^0 \to K^+K^-\pi^+\pi^-$$

Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK} y_{+}^{DK}

$D^0 \to K^+ K^- \pi^+ \pi^-$

Likelihood scan of CP observables

What do the likelihood scans tell us?

- Uncertainties from c_i and s_i are significant, which justifies Gaussian constraining c_i and s_i
- New strategy:
  1. Produce a likelihood function from CP fit
  2. Interpret CP observables in terms of γ, etc
  3. Must profile all nuisance parameters (F_i, c_i, s_i, backgrounds yields, normalisation constants)
  4. Provide direct measurements of γ, δ_B and r_B

Summary of LHCb internal systematic uncertainties

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
CP violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic *b*-hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \to K^-\pi^+\pi^-\pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \to pD\pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \to K^-\pi^+\pi^-\pi^+\pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Fit bias	0.06	0.05	0.13	0.02	0.06	0.13
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57

Give systematic uncertainties in terms of CP observables (not γ) since these are more Gaussian and better behaved

Interpretation strategy

From CP fit, we have a (negative log) likelihood function with nuisance parameters n_k:

$$\mathcal{L}(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}, \{n_k\})$$

Express in terms of physics parameters:

$$\mathcal{L}(\gamma, \delta_B^{DK}, r_B^{DK}, \delta_B^{D\pi}, r_B^{D\pi}, \{n_k\})$$

In this step, also add a Gaussian smearing term on CP observables to account for internal LHCb systematics (see backup)

Interpretation results

Results from interpretation of $K^+K^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

Model independent

$$\begin{aligned}
\gamma =& (121 \pm 16)^\circ \\
\delta_B^{DK} =& (74 \pm 14)^\circ \\
r_B^{DK} =& (12.1 \pm 3.0) \times 10^{-2} \\
\delta_B^{D\pi} =& (243 \pm 116)^\circ \\
r_B^{D\pi} =& (1 \pm 6) \times 10^{-3}
\end{aligned}$$

Model dependent

$$\begin{aligned}
\gamma =& (116^{+12}_{-14})^\circ \\
\delta_B^{DK} =& (81^{+14}_{-13})^\circ \\
r_B^{DK} =& (11.0 \pm 2.0) \times 10^{-2} \\
\delta_B^{D\pi} =& (298^{+62}_{-118})^\circ \\
r_B^{D\pi} =& (4^{+5}_{-4}) \times 10^{-3}
\end{aligned}$$

Central value of γ remains high...

... it seems that the large tension with the LHCb global result $\gamma = (64.6 \pm 2.8)^\circ$ remains

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

$K^+K^-\pi^+\pi^-$	$\pi^+\pi^-\pi^+\pi^-$
$\gamma = (121 \pm 16)^\circ$	$\gamma = (45 \pm 10)^\circ$
$\delta_B^{DK} = (74 \pm 14)^\circ$	$\delta_B^{DK} = (115 \pm 9)^\circ$
$r_B^{DK} = (12.1 \pm 3.0) \times 10^{-2}$	$r_B^{DK} = (9.4 \pm 1.9) \times 10^{-2}$
$\delta_B^{D\pi} = (243 \pm 116)^\circ$	$\delta_B^{D\pi} = (194 \pm 74)^\circ$
$r_B^{D\pi} = (1 \pm 6) \times 10^{-3}$	$r_B^{D\pi} = (0 \pm 4) \times 10^{-3}$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

$K^+K^-\pi^+\pi^-$

$$\begin{aligned}
\gamma =& (121 \pm 16)^\circ \\
\delta_B^{DK} =& (74 \pm 14)^\circ \\
r_B^{DK} =& (12.1 \pm 3.0) \times 10^{-2} \\
\delta_B^{D\pi} =& (243 \pm 116)^\circ \\
r_B^{D\pi} =& (1 \pm 6) \times 10^{-3}
\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$

$$\begin{aligned}
\gamma =& (45 \pm 10)^\circ \\
\delta_B^{DK} =& (115 \pm 9)^\circ \\
r_B^{DK} =& (9.4 \pm 1.9) \times 10^{-2} \\
\delta_B^{D\pi} =& (194 \pm 74)^\circ \\
r_B^{D\pi} =& (0 \pm 4) \times 10^{-3}
\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

...but how Gaussian are these uncertainties?

Interpretation results

We can also compare the statistical sensitivity of $\pi^+\pi^-\pi^+\pi^-$ between the CLEO-c and BESIII binning schemes (keep c_i and s_i fixed)

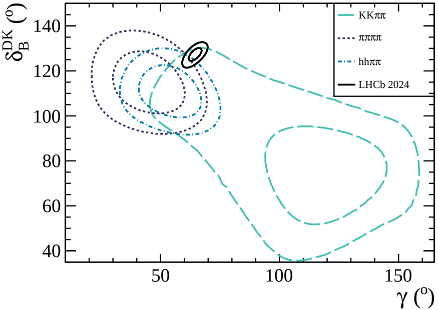
BESIII	CLEO-c
$\gamma = (47 \pm 10)^\circ$	$\gamma = (51 \pm 20)^\circ$
$\delta_B^{DK} = (113 \pm 9)^\circ$	$\delta_B^{DK} = (109 \pm 19)^\circ$
$r_B^{DK} = (9.2 \pm 1.6) \times 10^{-2}$	$r_B^{DK} = (6.5 \pm 1.8) \times 10^{-2}$
$\delta_B^{D\pi} = (208 \pm 58)^\circ$	$\delta_B^{D\pi} = (310 \pm 508)^\circ$
$r_B^{D\pi} = (3.9 \pm 2.7) \times 10^{-3}$	$r_B^{D\pi} = (0 \pm 5) \times 10^{-3}$

Very good agreement!

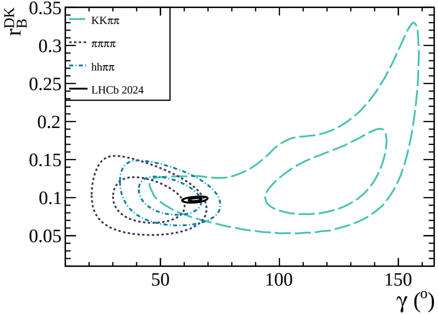
BESIII binning scheme, which has more bins and values of c_i and s_i further from the origin, performs better

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}

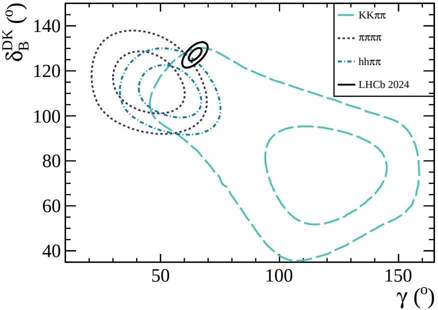


γ vs r_B^{DK}

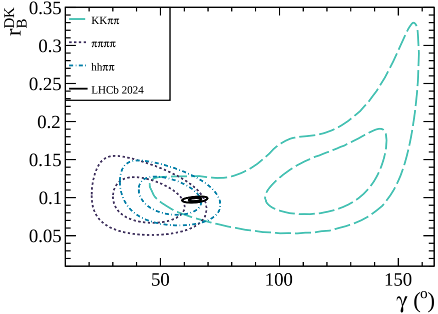
When all biases, correlations and non-Gaussian uncertainties are accounted for, the tension with the LHCb average has reduced significantly

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}



γ vs r_B^{DK}

However, with all the non-Gaussian behaviour, are we sure these contours cover 68% and 95% ?

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

At each scan point of γ, perform these fits to data:

1. Fit with all parameters floating, and save the log-likelihood χ^2
2. Fit with γ fixed to scan point, and save χ^2_{fix}
3. Calculate $\Delta\chi^2_{\mathrm{data}} = \chi^2_{\mathrm{fix}} - \chi^2$

We expect $\Delta\chi^2_{\mathrm{data}}$ to become large as we move away from best-fit value, but without direct knowledge of underlying PDF, we cannot determine any confidence intervals from this

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a “brute-force” approach to assigning a confidence interval

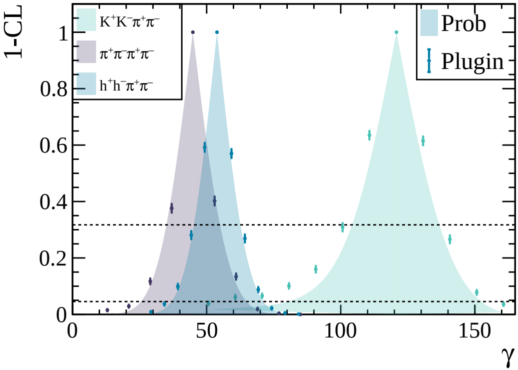
At each scan point of γ, perform these fits to toy:

1. Fix γ to scan point and generate 1000 toys
2. Perform fits to each toy, with γ both floating and fixed
3. Calculate $\Delta\chi^2_{\mathrm{toy}}$

At each scan point, the fraction of toys with $\Delta\chi^2_{\mathrm{toy}} > \Delta\chi^2_{\mathrm{data}}$ is equal to $1 - \mathrm{CL}$, and the exact 68% confidence interval can then be obtained using an interpolation between points

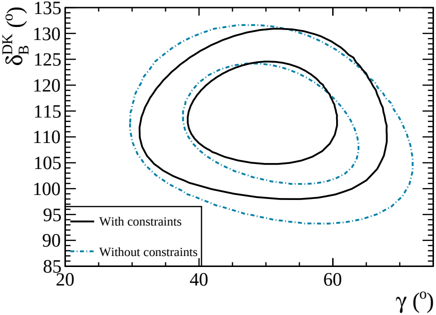
Plugin/Feldman-Cousins method

LHCb average within 2σ of $D \to K^+K^-\pi^+\pi^-$ Plugin result
Combined fit shows good agreement between Plugin and Prob scans

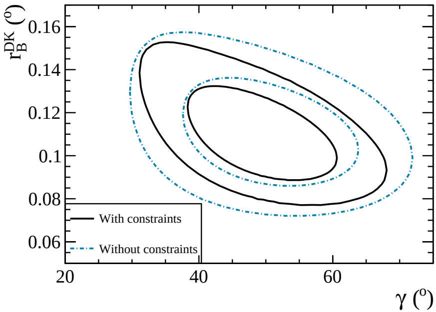


Combined fit result: $\gamma = (53.9^{+9.5}_{-8.9})^\circ$
One of the most precise single measurements of γ!

We can add phase-space integrated observables as a constraint:



γ vs δ_B^{DK}

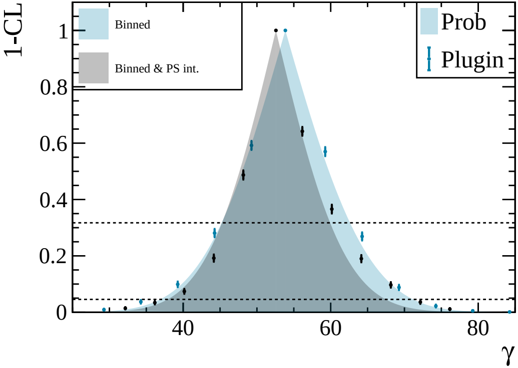


γ vs r_B^{DK}

The global asymmetries contain useful information!

Combining phase-space binned and integrated results

Run Plugin with phase-space integrated constraints:



Final measurement: $\gamma = (52.6^{+8.5}_{-6.4})^{\circ}$

Conclusion

1. Binned model-independent measurement of γ with $B^{\pm} \to [h^+h^-\pi^+\pi^-]_D h^{\pm}$ has been performed
    - Result: $\gamma = (53.9^{+9.5}_{-8.9})^{\circ}$
2. Can also combine with existing phase-space integrated measurements
    - Result: $\gamma = (52.6^{+8.5}_{-6.4})^{\circ}$
3. 3σ tension in $D \to K^+K^-\pi^+\pi^-$ has reduced

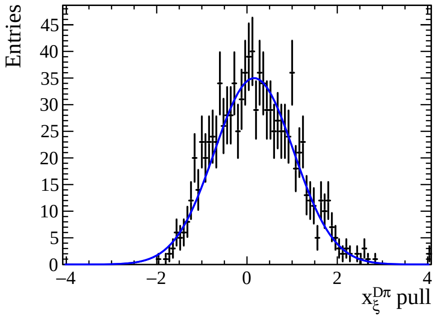
Future prospects

- Statistically limited measurement, but s_i uncertainties are large
- $\pi^+\pi^-\pi^+\pi^-$ inputs will become more precise:
  - Current analysis uses $3\,\text{fb}^{-1}$
  - Future updates will use the full $20\,\text{fb}^{-1}$ data set
- Minor improvements from BESIII are expected with $K^+K^-\pi^+\pi^-$:
  - Current analysis already uses $20\,\text{fb}^{-1}$
  - Charm mixing studies can improve s_i precision
- We believe the analysis is ready for approval to go to paper
  - You can find the TWiki here

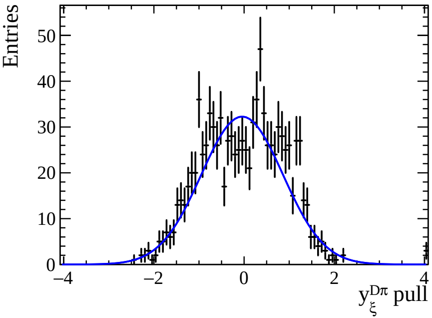
Thanks for your attention!

Backup: CP fit toy studies

In toy studies biases in $D\pi$ observables are consistent with model-dependent analysis



$x_\xi^{D\pi}$

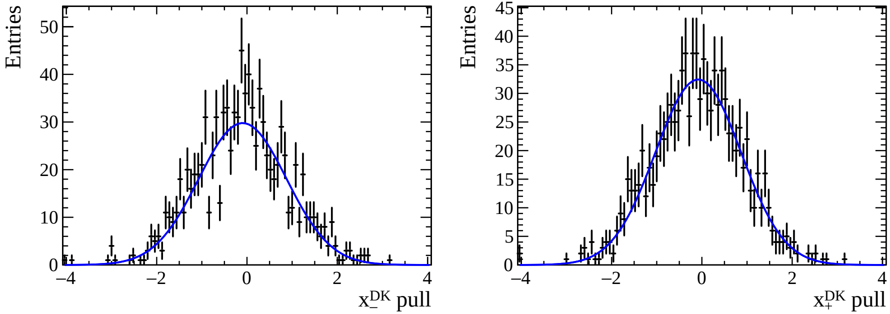


$y_\xi^{D\pi}$

$D^0 \to K^+K^-\pi^+\pi^-$

Backup: CP fit toy studies

Minor biases in $x_{\pm}^{DK}$ are seen but can be corrected for...

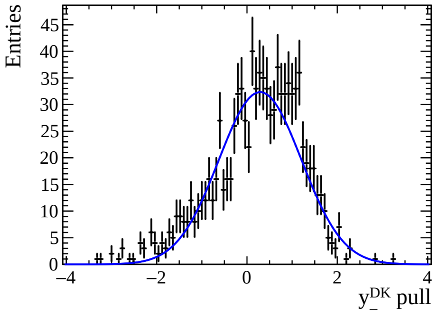


x_-^{DK} x_+^{DK}

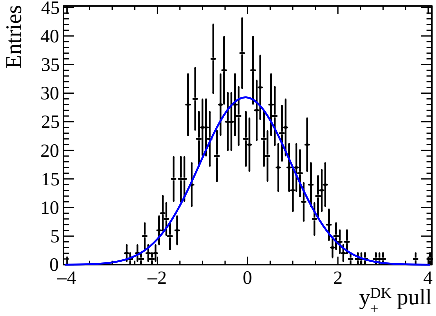
$$D^0 \to K^+K^-\pi^+\pi^-$$

Backup: CP fit toy studies

...but $y_{\pm}^{DK}$ pulls are now slightly asymmetric!



y_{-}^{DK}

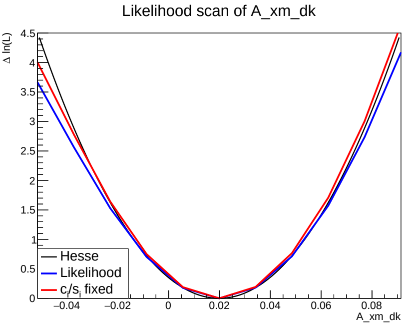


y_{+}^{DK}

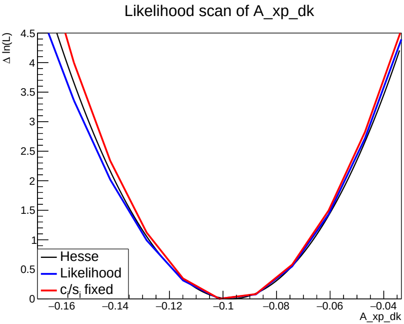
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: Likelihood scan of CP observables

$x_\pm^{DK}$ agree well between likelihood scan and Hesse approximation



x_-^{DK}

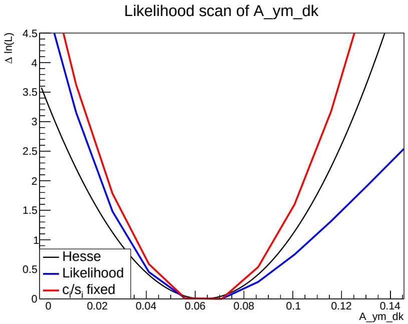


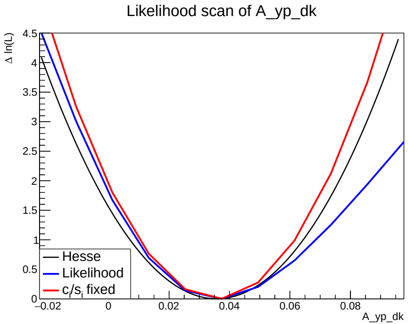
x_+^{DK}

$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



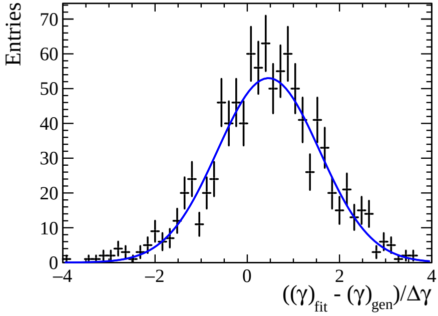


y_{-}^{DK} $\quad$ y_{+}^{DK}

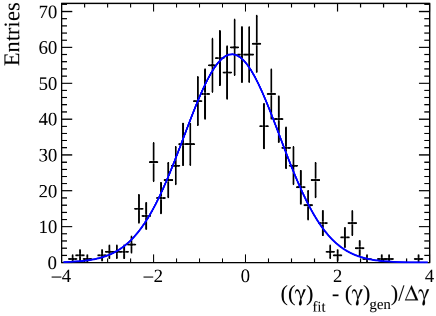
$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do <u>not</u> expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$



$\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

Indeed, small but significant biases are observed!