Model-independent measurement of the CKM angle γ in $B^{\pm} \to [h^+h^-\pi^+\pi^-]_D h^{\pm}$ decays

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Outline

1. Introduction to γ and CP violation
2. Analysis
3. Main changes during WG review
4. Conclusion and future prospects

Acknowledgements

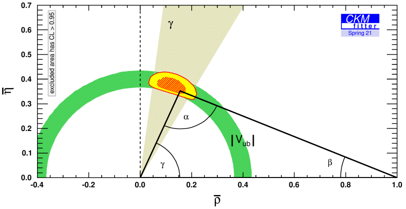
Thanks to:

- Anton Poluektov
- Wenbin Qian
- Resmi Puthumanaillam

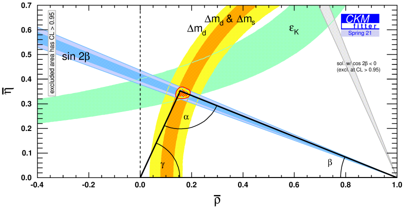
for their helpful comments, suggestions and feedback during WG review

Introduction to γ and CP violation

- CPV in SM is described by the Unitary Triangle, with angles α, β, γ
- The angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is very important:
  1. Negligible theoretical uncertainties: Ideal SM benchmark
  2. Accessible at tree level: Indirectly probe New Physics that enter loops
  3. Compare with a global CKM fit: Is the Unitary Triangle a triangle?



(a) Tree level: $\gamma = \left(72.1^{+5.4}_{-5.7}\right)^\circ$

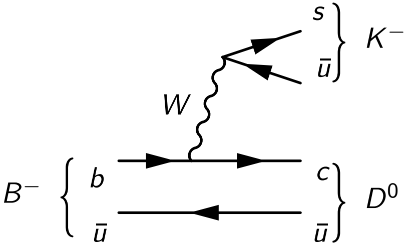


(b) Loop level: $\gamma = \left(65.5^{+1.1}_{-2.7}\right)^\circ$

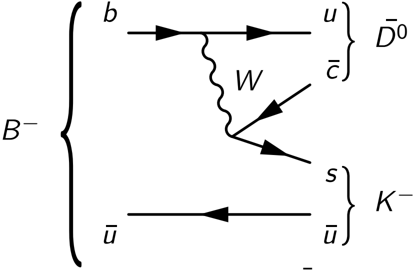
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), updated results and plots available at: http://ckmfitter.in2p3.fr

Sensitivity through interference

Measure γ through interference effects in $B^\pm \to DK^\pm$



Favoured $B^- \to D^0 K^-$



Suppressed $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
  - Consider $D^0/\bar{D}^0$ decays to the same final state, such as $D \to K^+K^-$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}_B\left(\mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)}\mathcal{A}_{\bar{D}^0}\right)$$
$$\mathcal{A}(B^+) = \mathcal{A}_B\left(\mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)}\mathcal{A}_{D^0}\right)$$

The BPGGSZ method

Event yield in bin i

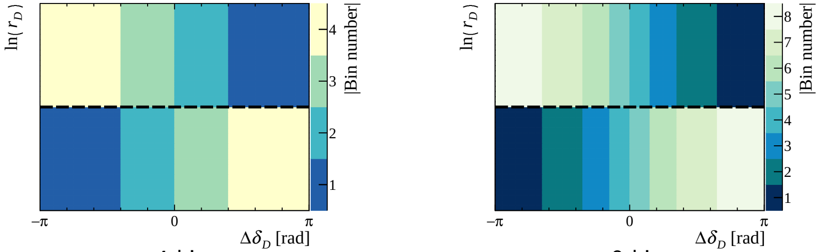
$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_-c_i + y_-s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+c_i + y_+s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude-averaged strong phases:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

- Interpretation of γ from the multi-body charm decays require external inputs of the charm strong-phase differences
- Measure model-independent strong-phases at a charm factory, such as BESIII, using an optimised binning scheme



4 bins 8 bins

$D^0 \to K^+K^-\pi^+\pi^-$ binning scheme

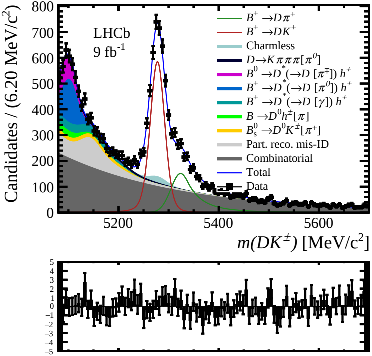
External strong-phase inputs

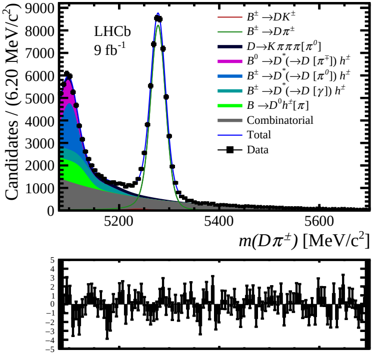
During B2OC WG review, preliminary results from BESIII have been used

- Final numbers for $D^0 \to K^+K^-\pi^+\pi^-$ are not public yet
- Breaking news: $D^0 \to \pi^+\pi^-\pi^+\pi^-$ is now available: arXiv:2408.16279

During B2OC WG review, the analysis made use of preliminary strong-phase results from the BESIII collaboration. We thank the BESIII management for the privilege of being allowed to show these measurements in internal LHCb meetings. We note that the results for $D^0 \to K^+K^-\pi^+\pi^-$ are not yet public, and the results presented on the next slide are not to be shown outside LHCb.

Global fit of $K^+K^-\pi^+\pi^-$ remains as in model-dependent publication:

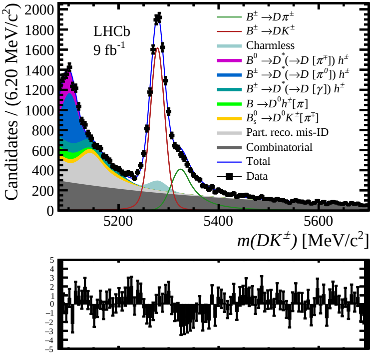


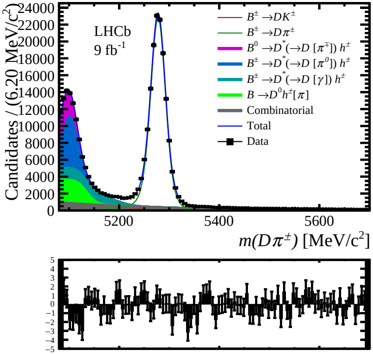


- $B^\pm \to [K^+K^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 3304 ± 42
  - $B^\pm \to D\pi^\pm$: 47894 ± 235

Global fit

Global fit of $\pi^+\pi^-\pi^+\pi^-$ has a good fit quality:





- $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 9211 ± 112
  - $B^\pm \to D\pi^\pm$: 132654 ± 398

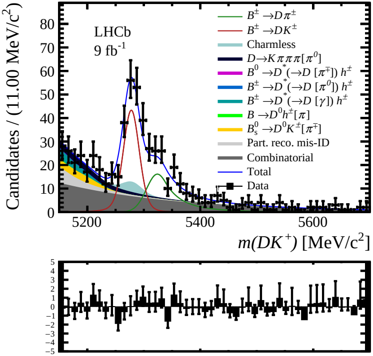
CP fit

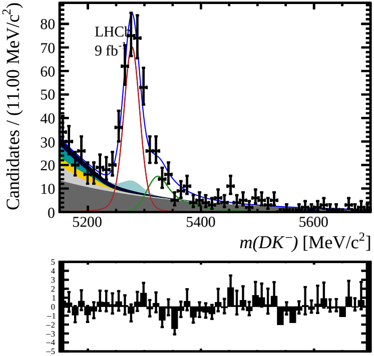
After global fit, perform a "CP fit" to study CP violation:

- Split candidates by:
  1. B^+ and B^- charges
  2. $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays
  3. D phase-space bins
- Combinatorial and low-mass backgrounds are floating in each category
- Parameterise signal yields in terms of $x_\pm^{DK}$, $y_\pm^{DK}$, $x_\xi^{D\pi}$, $y_\xi^{D\pi}$
- $2N - 1$ floating F_i parameters
- c_i and s_i are Gaussian constrained

CP fit bin asymmetry

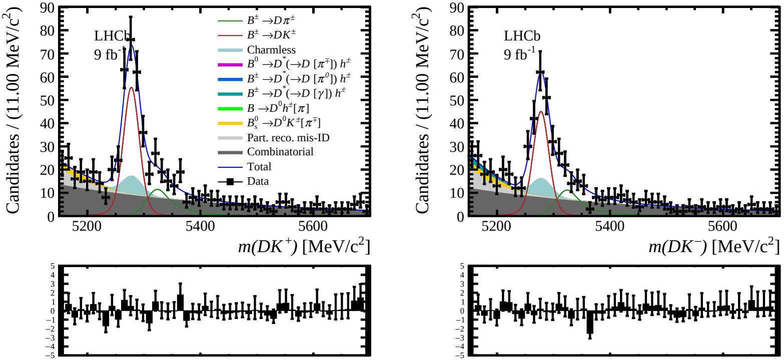
Example of bin asymmetry in $D \to K^+K^-\pi^+\pi^-$ bin -3:





CP fit bin asymmetry

Example of bin asymmetry in $D \to \pi^+\pi^-\pi^+\pi^-$ bin $+5$:



Treatment of non-Gaussian uncertainties

Study of the profile likelihoods show non-Gaussian behaviour induced by s_i uncertainties

- This justifies Gaussian constraining c_i and s_i
- Strategy:
  1. Produce a likelihood function from CP fit
  2. Interpret CP observables in terms of γ, etc
  3. Must profile all nuisance parameters (F_i, c_i, s_i, backgrounds yields, normalisation constants)
  4. Provide direct measurements of γ, δ_B and r_B

Interpretation strategy

From CP fit, we have a (negative log) likelihood function with nuisance parameters n_k:

$$\mathcal{L}(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}, \{n_k\})$$

Express in terms of physics parameters:

$$\mathcal{L}(\gamma, \delta_B^{DK}, r_B^{DK}, \delta_B^{D\pi}, r_B^{D\pi}, \{n_k\})$$

In this step, also add a Gaussian smearing term on CP observables to account for internal LHCb systematics (see backup)

Interpretation results

Results from interpretation of $K^+K^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

Model independent

$$\begin{aligned}\gamma =& (119 \pm 14)^\circ \\ \delta_B^{DK} =& (80 \pm 12)^\circ \\ r_B^{DK} =& (11.4 \pm 2.3) \times 10^{-2} \\ \delta_B^{D\pi} =& (253 \pm 62)^\circ \\ r_B^{D\pi} =& (3 \pm 7) \times 10^{-3}\end{aligned}$$

Model dependent

$$\begin{aligned}\gamma =& (116^{+12}_{-14})^\circ \\ \delta_B^{DK} =& (81^{+14}_{-13})^\circ \\ r_B^{DK} =& (11.0 \pm 2.0) \times 10^{-2} \\ \delta_B^{D\pi} =& (298^{+62}_{-118})^\circ \\ r_B^{D\pi} =& (4^{+5}_{-4}) \times 10^{-3}\end{aligned}$$

Central value of γ remains high...

... it seems that the large tension with the LHCb global result $\gamma = (63.8^{+3.5}_{-3.7})^\circ$ remains

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

$K^+K^-\pi^+\pi^-$

$$\begin{aligned}\gamma =&(119 \pm 14)^\circ\\ \delta_B^{DK} =&(80 \pm 12)^\circ\\ r_B^{DK} =&(11.4 \pm 2.3) \times 10^{-2}\\ \delta_B^{D\pi} =&(253 \pm 62)^\circ\\ r_B^{D\pi} =&(3 \pm 7) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$

$$\begin{aligned}\gamma =&(45 \pm 9)^\circ\\ \delta_B^{DK} =&(114 \pm 9)^\circ\\ r_B^{DK} =&(9.5 \pm 1.9) \times 10^{-2}\\ \delta_B^{D\pi} =&(176 \pm 111)^\circ\\ r_B^{D\pi} =&(0.8 \pm 1.9) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

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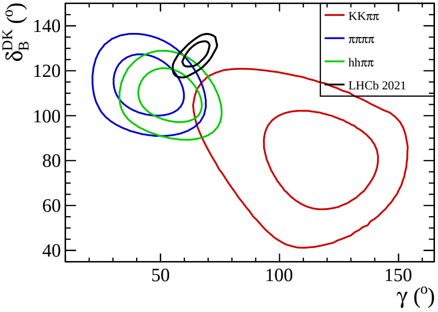
$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

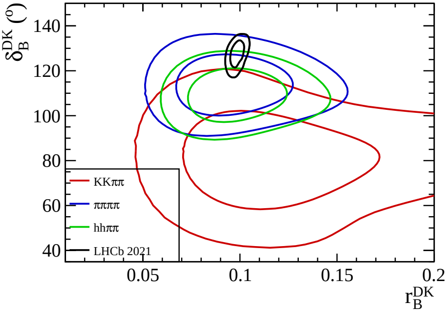
...but how Gaussian are these uncertainties?

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}

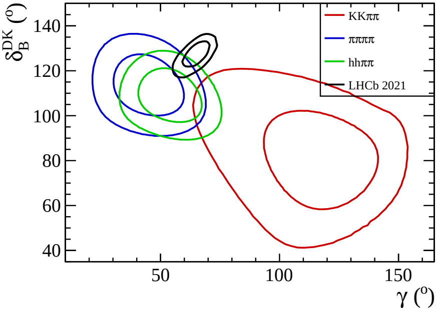


r_B^{DK} vs δ_B^{DK}

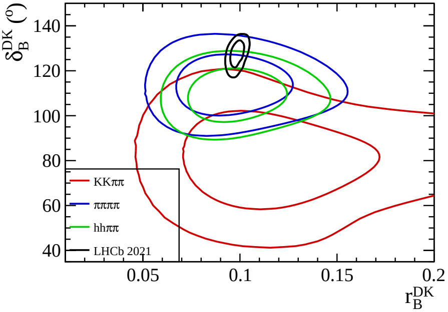
When all biases, correlations and non-Gaussian uncertainties are accounted for, the tension with the LHCb average has reduced significantly

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ 2σ contours overlap



γ vs δ_B^{DK}



r_B^{DK} vs δ_B^{DK}

However, with all the non-Gaussian behaviour, are we sure these contours cover 68% and 95% ?

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

At each scan point of γ, perform these fits to data:

1. Fit with all parameters floating, and save the log-likelihood χ^2
2. Fit with γ fixed to scan point, and save χ^2_{fix}
3. Calculate $\Delta\chi^2_{\text{data}} = \chi^2_{\text{fix}} - \chi^2$

We expect $\Delta\chi^2_{\text{data}}$ to become large as we move away from best-fit value, but without direct knowledge of underlying PDF, we cannot determine any confidence intervals from this

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

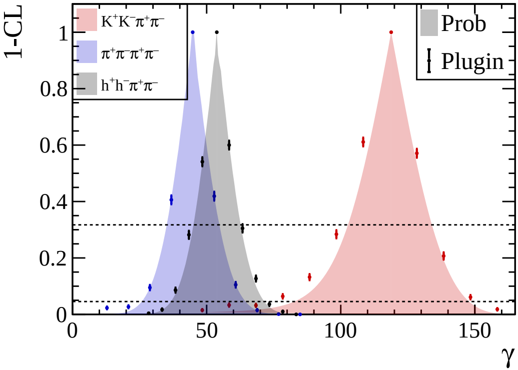
At each scan point of γ, perform these fits to toy:

1. Fix γ to scan point and generate 1000 toys
2. Perform fits to each toy, with γ both floating and fixed
3. Calculate $\Delta\chi^2_{\mathrm{toy}}$

At each scan point, the fraction of toys with $\Delta\chi^2_{\mathrm{toy}} > \Delta\chi^2_{\mathrm{data}}$ is equal to $1 - \mathrm{CL}$, and the exact 68% confidence interval can then be obtained using an interpolation between points

Plugin/Feldman-Cousins method

LHCb average within 2σ of $D \to K^+K^-\pi^+\pi^-$ Plugin result
Combined fit shows good agreement between Plugin and Prob scans



Combined fit result: $\gamma = (54.0^{+10.2}_{-9.5})^\circ$
Third most precise single measurement of γ in $B^\pm$ decays

Analysis progression since last presentation:

1. Selection between Oxford and Bristol groups have been merged
2. Many selection cuts are now applied before BDT, instead of after
3. All internal LHCb internal systematic uncertainties evaluated
4. Combination of phase-space binned and integrated results

Important points that were discussed during review:

- Correlation between binned and integrated results
- Efficiency corrections to c_i and s_i

Changes from previous presentation:

- Several loose cuts on p, p_T, χ^2_{IP}, $\chi^2_F D$, etc.
- Significant improvement in purity:
  - ProbNNpi $\times$ (1 $-$ ProbNNk) > 0.05
  - Applied to all $D^0 \to \pi^+\pi^-\pi^+\pi^-$ daughters
  - Suppresses combinatorial background (not mis-ID)

Changing order of cuts

Selection “workflow” before review:

1. Initial cuts
   - Triggers
   - RICH information
   - Mass range
   - Etc

2. BDT
   - Combinatorial background
   - Optimised for γ sensitivity

3. Final cuts
   - PID
   - Flight significance
   - Opening angle
   - K_S^0 veto

Changing order of cuts

Selection "workflow" after review:

1. Initial cuts
   - Triggers
   - RICH information
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   - Flight significance
   - Opening angle
   - K_S^0 veto
2. BDT
   - Combinatorial background
   - Optimised for γ sensitivity
3. Final cuts
   - PID

Minimal change in final results

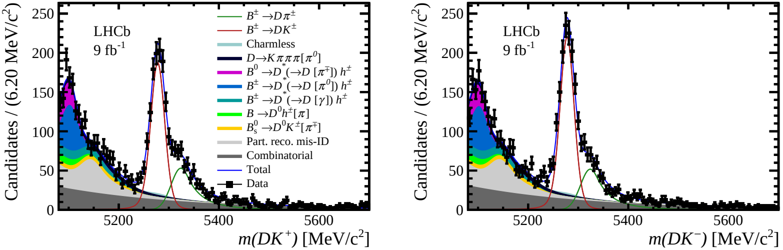
Summary of LHCb internal systematic uncertainties

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
CP violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic *b*-hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \to K^-\pi^+\pi^-\pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \to pD\pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \to K^-\pi^+\pi^-\pi^+\pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57

Numbers for $\pi^+\pi^-\pi^+\pi^-$ are very similar

Phase-space integrated CP observables

Phase-space integrated study of γ:
Charged asymmetries measured for $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ in Eur. Phys. J. C **83** 547 (2023)

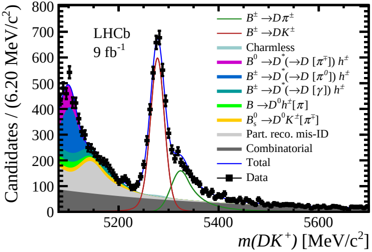


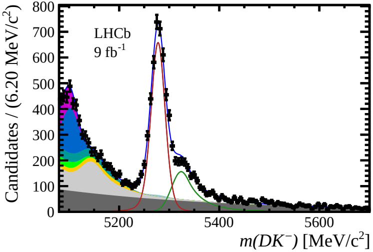
$D \to K^+K^-\pi^+\pi^-$

- $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ asymmetries:
  - $D \to K^+K^-\pi^+\pi^-$: $\mathcal{A} = 0.095 \pm 0.023 \pm 0.002$
  - $D \to \pi^+\pi^-\pi^+\pi^-$: $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

Phase-space integrated CP observables

Phase-space integrated study of γ:
Charged asymmetries measured for $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ in Eur. Phys. J. C **83** 547 (2023)





$D \to \pi^+\pi^-\pi^+\pi^-$

- $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ asymmetries:
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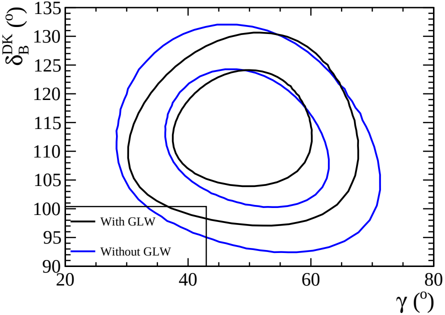
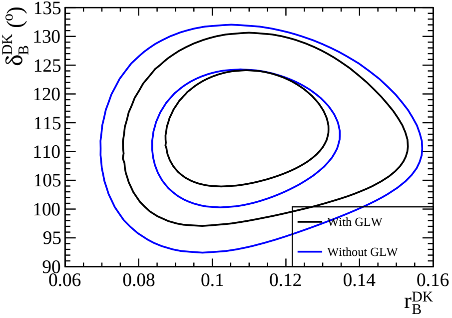
  - $D \to \pi^+\pi^-\pi^+\pi^-$: $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

Combining phase-space binned and integrated results

We can add phase-space integrated observables as a constraint:



γ vs δ_B^{DK}

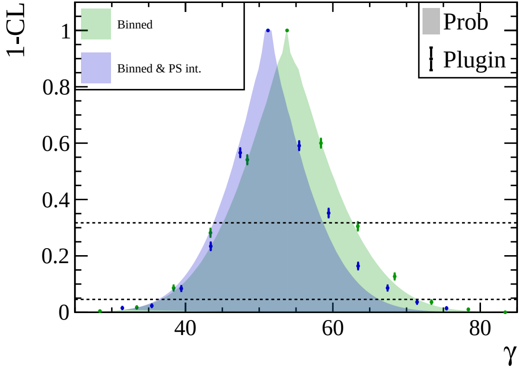


r_B^{DK} vs δ_B^{DK}

The global asymmetries contain useful information!

Combining phase-space binned and integrated results

Run Plugin with phase-space integrated constraints:



Final measurement: $\gamma = (51.2^{+8.9}_{-6.5})^{\circ}$

Additional work

Additional work foreseen during RC review:

1. Update with final values of $D^0 \to K^+K^-\pi^+\pi^-$ strong phases from BESIII (minimal changes expected)

2. Hope to evaluate central value and statistical uncertainty with CLEO-c binning

Neither of these things are expected to change the results presented today

Conclusion

1. Binned model-independent measurement of γ with $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ has been performed: $\gamma = (54.0^{+10.2}_{-9.5})^\circ$
   - External strong-phase inputs from BESIII
2. Combination with integrated results: $\gamma = (51.2^{+8.9}_{-6.5})^\circ$
3. 3σ tension in $D \to K^+K^-\pi^+\pi^-$ has reduced

Future prospects

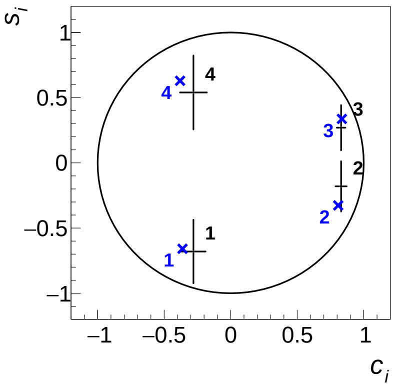
- We see no showstoppers for this analysis, which will provide important constraints to GammaCombo in the near future
- BESIII results for $K^+K^-\pi^+\pi^-$ will be available imminently
- We would like this analysis to move to RC
  - You can find the TWiki here
  - Link to gitlab and EOS path to tuples can also be found here

Thanks for your attention!

Backup: BESIII preliminary $D^0 \to K^+K^-\pi^+\pi^-$ strong-phase results

First binned strong-phase analysis of $D^0 \to K^+K^-\pi^+\pi^-$, which uses the 2×4 binning scheme with 16 fb^{-1} $\psi(3770)$ data

$$c_1 = -0.28 \pm 0.09 \pm 0.01$$
$$s_1 = -0.68 \pm 0.24 \pm 0.04$$
$$c_2 = +0.83 \pm 0.04 \pm 0.01$$
$$s_2 = -0.18 \pm 0.19 \pm 0.03$$
$$c_3 = +0.83 \pm 0.03 \pm 0.01$$
$$s_3 = +0.27 \pm 0.17 \pm 0.03$$
$$c_4 = -0.28 \pm 0.10 \pm 0.01$$
$$s_4 = +0.54 \pm 0.28 \pm 0.04$$

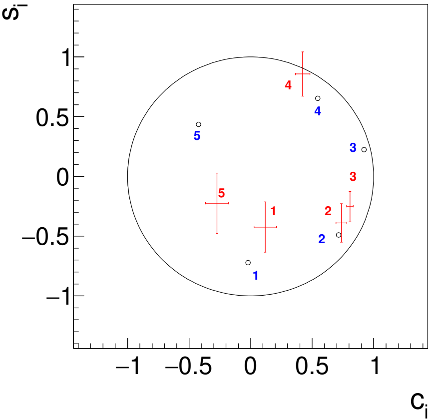


Measured values (black) are consistent and close to LHCb model predictions (blue), so central values are not expected to change much

Backup: BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

Small differences between model prediction and measurement, but data points are generally close to the unit circle

$$c_1 = +\,0.12 \pm 0.09 \pm 0.02$$
$$s_1 = -\,0.42 \pm 0.21 \pm 0.04$$
$$c_2 = +\,0.74 \pm 0.04 \pm 0.02$$
$$s_2 = -\,0.39 \pm 0.16 \pm 0.06$$
$$s_3 = -\,0.25 \pm 0.12 \pm 0.03$$
$$c_3 = +\,0.81 \pm 0.03 \pm 0.01$$
$$c_4 = +\,0.42 \pm 0.06 \pm 0.02$$
$$s_4 = +\,0.86 \pm 0.19 \pm 0.07$$
$$c_5 = -\,0.27 \pm 0.09 \pm 0.03$$
$$s_5 = -\,0.22 \pm 0.25 \pm 0.08$$



The HyperPlot software is used (binary lookup tree in 5D phase space)

Backup: BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

- Binned strong-phase analysis of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ uses the 2×5 "optimal" binning scheme with 3 fb^{-1} $\psi(3770)$
- Earlier CLEO-c analysis with 0.8 fb^{-1} JHEP **01** (2018) 144
- New BESIII analysis uses a new binning scheme optimised with a BESIII amplitude model arXiv:2312.02524
  - Amplitude model constructed from a larger data set
  - In principle more sensitive
- Two binning schemes are available:
  - We use the more sensitive "optimal" binning with $Q = 0.85$
  - The other "equal δ" binning has $Q = 0.80$

Backup: Global fit

How do we include the $\pi^+\pi^-\pi^+\pi^-$ mode?

- We have already studied $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ for phase-space integrated measurement
  1. Different D daughter PID cuts in stripping
  2. No $D \to K\pi\pi\pi\pi^0$ background
  3. Charmless background recalculated using the sideband
  4. Use same BDT
  5. No additional peaking backgrounds

- Sort candidates into phase-space bins using BESIII binning scheme

- Can fit separately or simultaneously with $K^+K^-\pi^+\pi^-$

Backup: Strong-phase parameters in CP fit

Why are c_i and s_i Gaussian constrained?

- Previous BPGGSZ analyses have kept c_i and s_i fixed
  1. c_i and s_i uncertainties are added as a systematic through smearing
  2. Convenient for calculating correlations between different analyses
  3. Appropriate when c_i and s_i uncertainties are small
- In four-body analyses, uncertainties on γ from c_i and s_i are almost the same size as the statistical uncertainty
- Large s_i uncertainties introduces non-Gaussian uncertainties on $y_\pm$
- γ/δ_B move significantly when fixing s_i instead of constraining them
- These effects are largest for $K^+K^-\pi^+\pi^-$, but are also seen in $\pi^+\pi^-\pi^+\pi^-$ and in the combined fit

Backup: CP fit categories

Summary of free parameters in the CP fit:

$K^+K^-\pi^+\pi^-$

$2 \times 2 \times 2 \times 4 = 32$ categories

- 6 CP observables
- 7 F_i parameters
- 8 c_i and s_i parameters
- 32 combinatorial yields
- 32 low mass yields
- 4 global normalisations
- Total: 89 parameters

$\pi^+\pi^-\pi^+\pi^-$

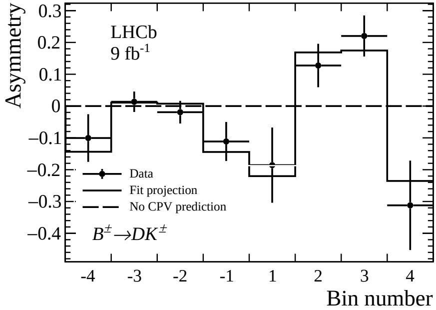
$2 \times 2 \times 2 \times 5 = 40$ categories

- 6 CP observables
- 9 F_i parameters
- 10 c_i and s_i parameters
- 40 combinatorial yields
- 40 low mass yields
- 4 global normalisations
- Total: 109 parameters

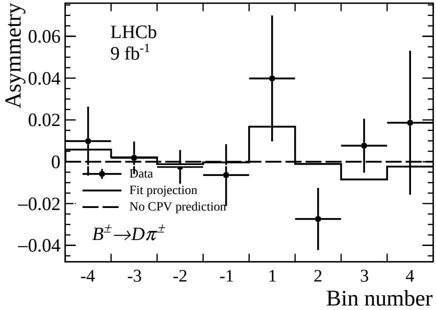
In a combined fit where CP observables are shared, there are $89 + 109 - 6 = 192$ parameters

Backup: Bin asymmetries

$B^\pm \to [K^+K^-\pi^+\pi^-]_D h^\pm$ bin asymmetries



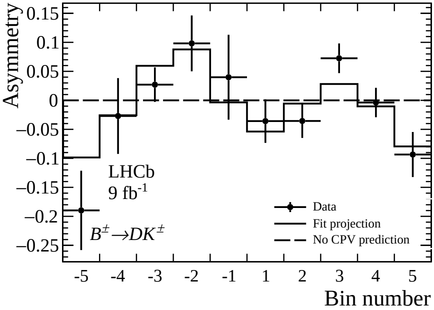
$B^\pm \to DK^\pm$



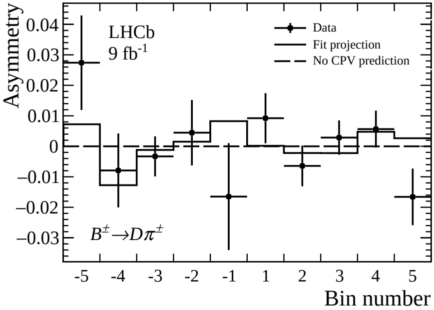
$B^\pm \to D\pi^\pm$

Backup: Bin asymmetries

$B^{\pm} \to [\pi^+\pi^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries

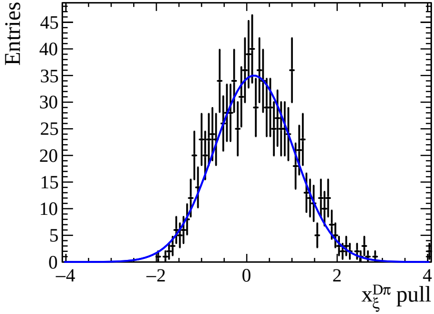


$B^{\pm} \to DK^{\pm}$

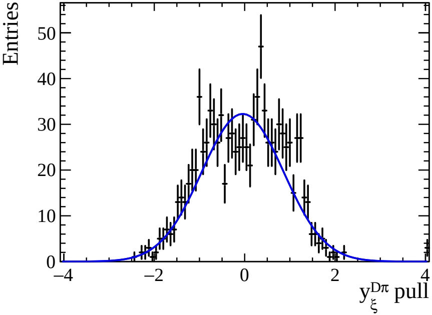


$B^{\pm} \to D\pi^{\pm}$

In toy studies biases in $D\pi$ observables are consistent with model-dependent analysis



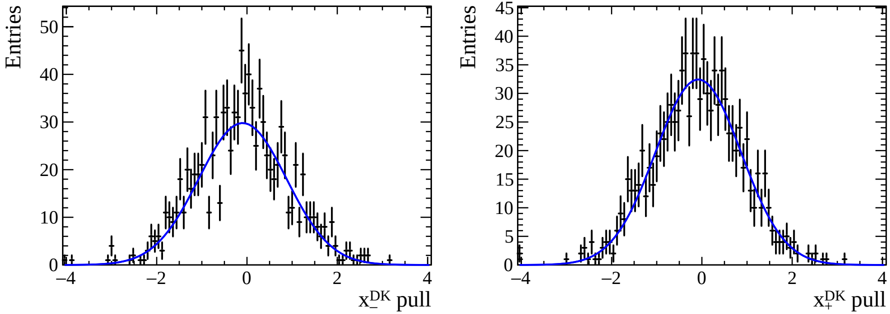
$x_\xi^{D\pi}$



$y_\xi^{D\pi}$

$D^0 \to K^+K^-\pi^+\pi^-$

Minor biases in $x_{\pm}^{DK}$ are seen but can be corrected for...

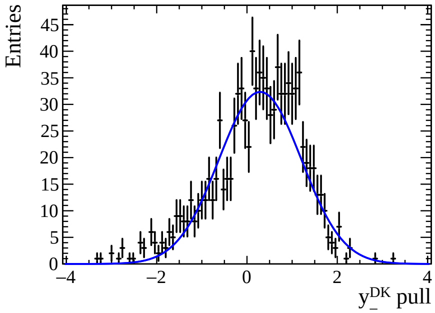


x_{-}^{DK} x_{+}^{DK}

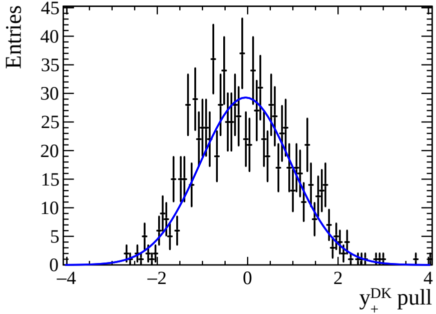
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: CP fit toy studies

...but $y_{\pm}^{DK}$ pulls are now slightly asymmetric!



y_{-}^{DK}

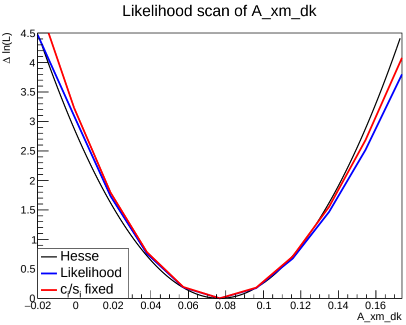


y_{+}^{DK}

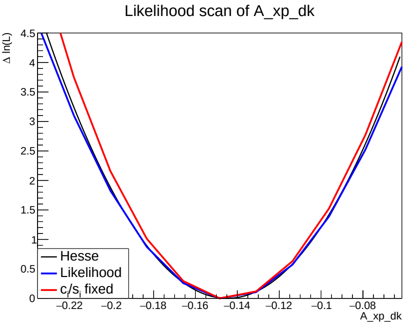
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

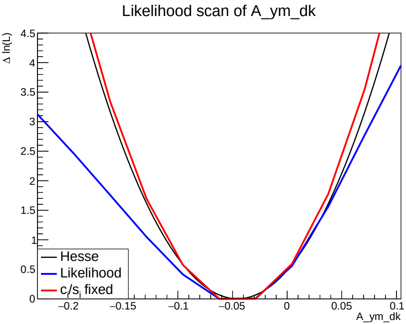


x_{+}^{DK}

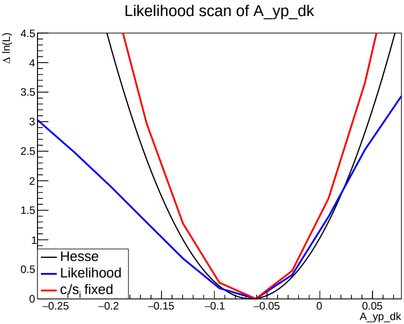
$$D^0 \to K^+K^-\pi^+\pi^-$$

Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK}

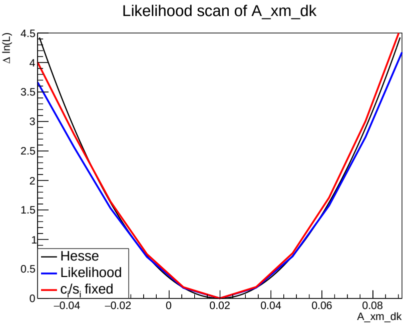


y_{+}^{DK}

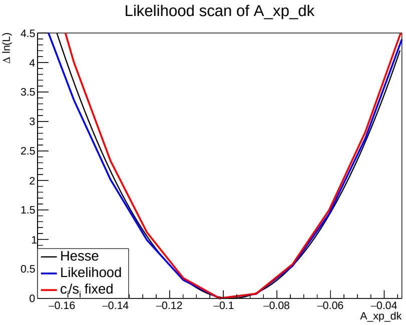
$D^0 \to K^+K^-\pi^+\pi^-$

Backup: Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

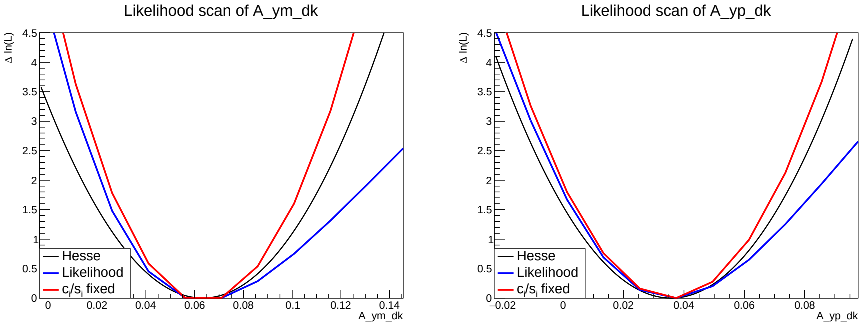


x_{+}^{DK}

$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ

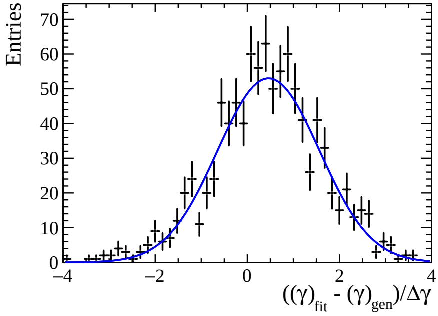


y_{-}^{DK} y_{+}^{DK}

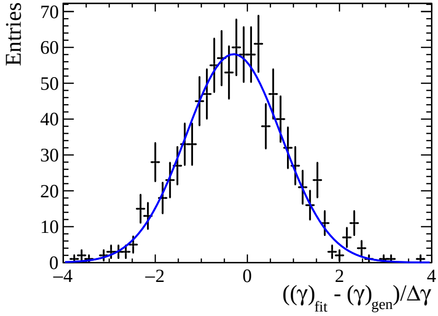
$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do not expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$



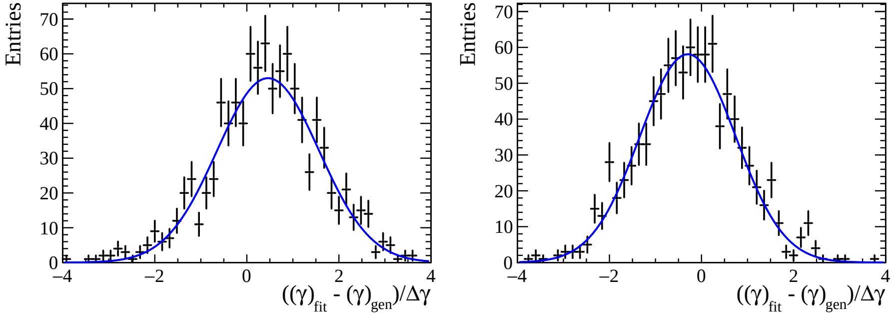
$\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

Indeed, small but significant biases are observed!

Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do <u>not</u> expect these to behave very Gaussian...



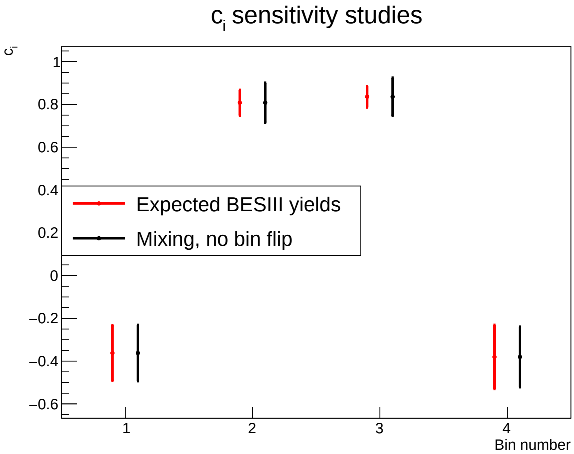
$K^+K^-\pi^+\pi^-$ $\qquad$ $\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

The absolute bias corrections are:

Backup: Charm mixing studies with multi-body decays

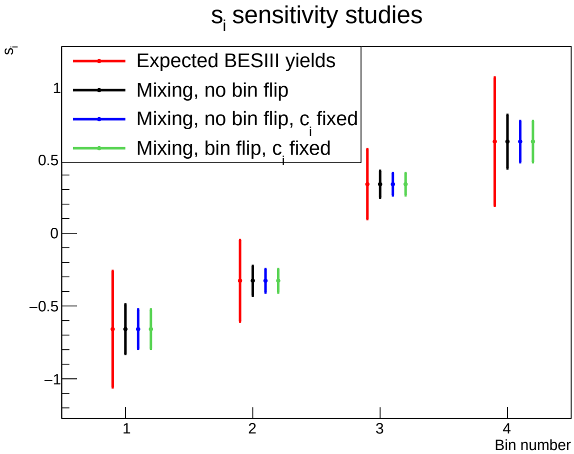
Sensitivity to c_i: Similar between BESIII and charm mixing at LHCb



- BESIII yields equivalent to $8\,\mathrm{fb}^{-1}$ of $\psi(3770)$
- 4 million $D \to K^+K^-\pi^+\pi^-$ candidates in mixing analysis

Backup: Charm mixing studies with multi-body decays

Sensitivity to s_i: Significant improvements expected!



- BESIII yields equivalent to $8\,\mathrm{fb}^{-1}$ of $\psi(3770)$
- 4 million $D \to K^+K^-\pi^+\pi^-$ candidates in mixing analysis