

# Model-independent measurement of the CKM angle $\gamma$ in $B^\pm \rightarrow [h^+ h^- \pi^+ \pi^-]_D h^\pm$ decays

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# Outline

- 1 Introduction to  $\gamma$  and  $CP$  violation
- 2 Analysis
- 3 Main changes during WG review
- 4 Conclusion and future prospects
- 5 Strong-phase inputs from BESIII

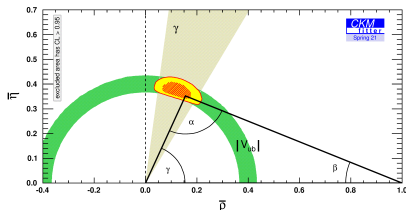
Thanks to:

- Anton Poluektov
- Wenbin Qian
- Resmi Puthumanaillam

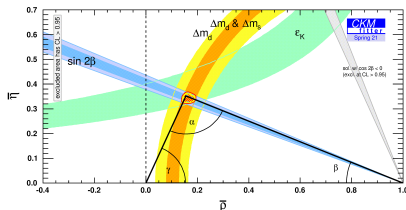
for their helpful comments, suggestions and feedback during WG review

# Introduction to $\gamma$ and $CP$ violation

- CPV in SM is described by the Unitary Triangle, with angles  $\alpha$ ,  $\beta$ ,  $\gamma$
- The angle  $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$  is very important:
  - 1 Negligible theoretical uncertainties: Ideal SM benchmark
  - 2 Accessible at tree level: Indirectly probe New Physics that enter loops
  - 3 Compare with a global CKM fit: Is the Unitary Triangle a triangle?



**(a) Tree level:**  $\gamma = (72.1^{+5.4}_{-5.7})^\circ$

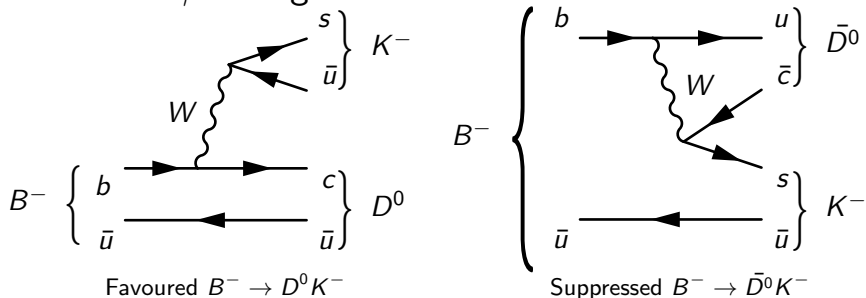


**(b) Loop level:**  $\gamma = (65.5^{+1.1}_{-2.7})^\circ$

CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), updated results and plots available at:  
<http://ckmfitter.in2p3.fr>

# Sensitivity through interference

Measure  $\gamma$  through interference effects in  $B^\pm \rightarrow DK^\pm$



- Superposition of  $D^0$  and  $\bar{D}^0$ 
  - Consider  $D^0/\bar{D}^0$  decays to the same final state, such as  $D \rightarrow K^+ K^-$
- $b \rightarrow u\bar{c}s$  and  $b \rightarrow c\bar{u}s$  interference  $\rightarrow$  Sensitivity to  $\gamma$

$$\mathcal{A}(B^-) = \mathcal{A}_B \left( \mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)} \mathcal{A}_{\bar{D}^0} \right)$$

$$\mathcal{A}(B^+) = \mathcal{A}_B \left( \mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0} \right)$$

# The BPGGSZ method

## Event yield in bin $i$

$$N_i^- = h_{B^-} (F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i))$$
$$N_i^+ = h_{B^+} (F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i))$$

- CP observables:

- $x_{\pm}^{DK} = r_B^{DK} \cos(\delta_B^{DK} \pm \gamma), \quad y_{\pm}^{DK} = r_B^{DK} \sin(\delta_B^{DK} \pm \gamma)$
- $x_{\xi}^{D\pi} = \text{Re}(\xi^{D\pi}), \quad y_{\xi}^{D\pi} = \text{Im}(\xi^{D\pi}) \quad \left( \xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})} \right)$

- Fractional bin yield:

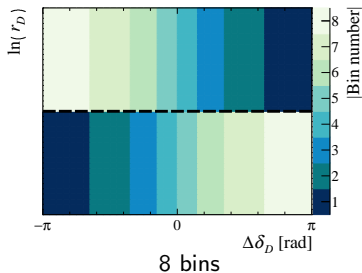
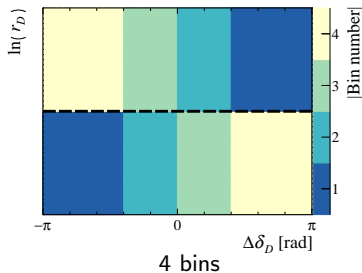
- $F_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$
- Floated in the fit, mostly constrained by  $B^{\pm} \rightarrow D\pi^{\pm}$

- Amplitude-averaged strong phases:

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}} \quad s_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \sin(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}$$

# $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ binning scheme

- Interpretation of  $\gamma$  from the multi-body charm decays require external inputs of the charm strong-phase differences
- Measure model-independent strong-phases at a charm factory, such as BESIII, using an optimised binning scheme



$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  binning scheme

# External strong-phase inputs

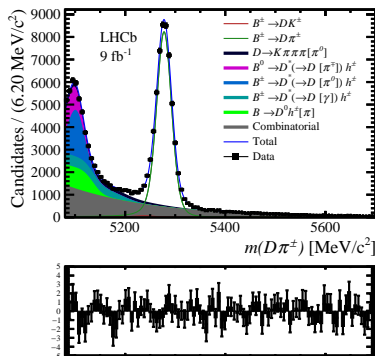
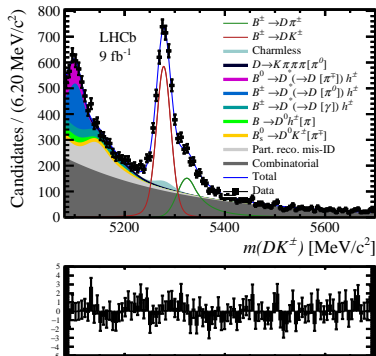
During B2OC WG review, preliminary results from BESIII have been used

- Final numbers for  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  are not public yet
- Breaking news:  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  is now available: [arXiv:2408.16279](https://arxiv.org/abs/2408.16279)

*During B2OC WG review, the analysis made use of preliminary strong-phase results from the BESIII collaboration. We thank the BESIII management for the privilege of being allowed to show these measurements in internal LHCb meetings. We note that the results for  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$  are not yet public, and the results presented on the next slide are not to be shown outside LHCb.*

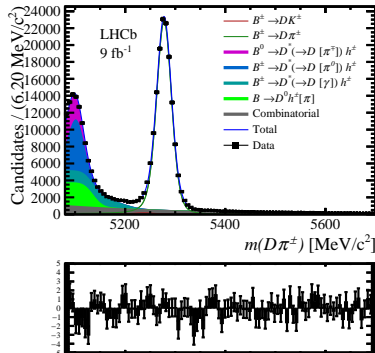
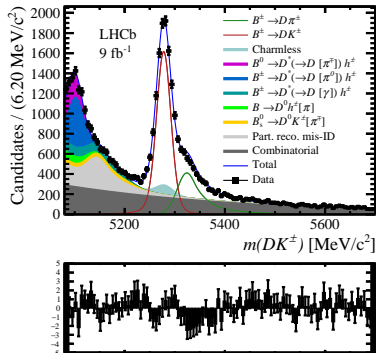


Global fit of  $K^+K^-\pi^+\pi^-$  remains as in model-dependent publication:



- $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D h^\pm$  signal yield:
  - $B^\pm \rightarrow DK^\pm$ :  $3304 \pm 42$
  - $B^\pm \rightarrow D\pi^\pm$ :  $47894 \pm 235$

Global fit of  $\pi^+\pi^-\pi^+\pi^-$  has a good fit quality:

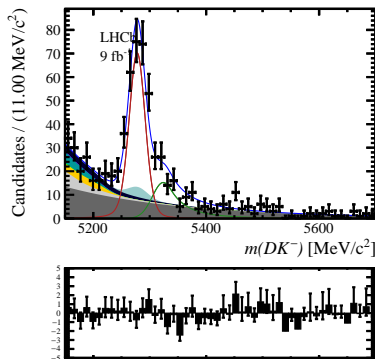
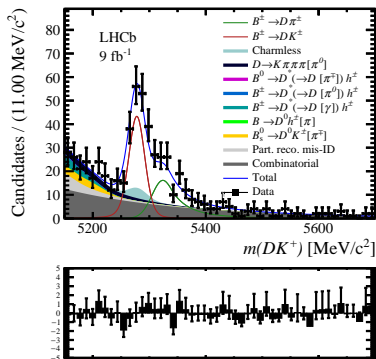


- $B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$  signal yield:
  - $B^\pm \rightarrow DK^\pm$ :  $9211 \pm 112$
  - $B^\pm \rightarrow D\pi^\pm$ :  $132654 \pm 398$

After global fit, perform a “CP fit” to study CP violation:

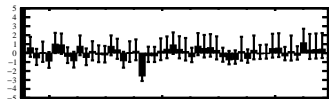
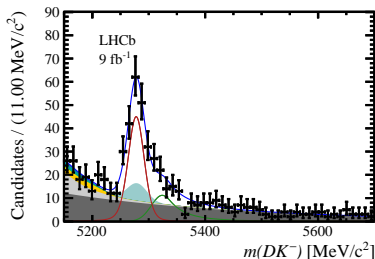
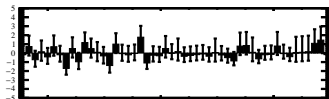
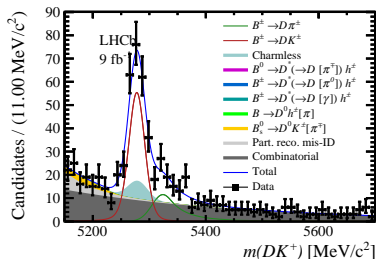
- Split candidates by:
  - ①  $B^+$  and  $B^-$  charges
  - ②  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D\pi^\pm$  decays
  - ③  $D$  phase-space bins
- Combinatorial and low-mass backgrounds are floating in each category
- Parameterise signal yields in terms of  $x_\pm^{DK}$ ,  $y_\pm^{DK}$ ,  $x_\xi^{D\pi}$ ,  $y_\xi^{D\pi}$
- $2N - 1$  floating  $F_i$  parameters
- $c_i$  and  $s_i$  are Gaussian constrained

Example of bin asymmetry in  $D \rightarrow K^+ K^- \pi^+ \pi^-$  bin -3:



# CP fit bin asymmetry

Example of bin asymmetry in  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  bin +5:



Study of the profile likelihoods show non-Gaussian behaviour induced by  $s_i$  uncertainties

- This justifies Gaussian constraining  $c_i$  and  $s_i$
- Strategy:
  - 1 Produce a likelihood function from CP fit
  - 2 Interpret CP observables in terms of  $\gamma$ , etc
  - 3 Must profile all nuisance parameters ( $F_i$ ,  $c_i$ ,  $s_i$ , backgrounds yields, normalisation constants)
  - 4 Provide direct measurements of  $\gamma$ ,  $\delta_B$  and  $r_B$

From CP fit, we have a (negative log) likelihood function with nuisance parameters  $n_k$ :

$$\mathcal{L}(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}, \{n_k\})$$

Express in terms of physics parameters:

$$\mathcal{L}(\gamma, \delta_B^{DK}, r_B^{DK}, \delta_B^{D\pi}, r_B^{D\pi}, \{n_k\})$$

In this step, also add a Gaussian smearing term on CP observables to account for internal LHCb systematics (see backup)

# Interpretation results

Results from interpretation of  $K^+K^-\pi^+\pi^-$ , after correcting for biases in central values (not uncertainties):

Model independent

$$\gamma = (119 \pm 14)^\circ$$

$$\delta_B^{DK} = (80 \pm 12)^\circ$$

$$r_B^{DK} = (11.4 \pm 2.3) \times 10^{-2}$$

$$\delta_B^{D\pi} = (253 \pm 62)^\circ$$

$$r_B^{D\pi} = (3 \pm 7) \times 10^{-3}$$

Model dependent

$$\gamma = (116^{+12}_{-14})^\circ$$

$$\delta_B^{DK} = (81^{+14}_{-13})^\circ$$

$$r_B^{DK} = (11.0 \pm 2.0) \times 10^{-2}$$

$$\delta_B^{D\pi} = (298^{+62}_{-118})^\circ$$

$$r_B^{D\pi} = (4^{+5}_{-4}) \times 10^{-3}$$

Central value of  $\gamma$  remains high...

... it seems that the large tension with the LHCb global result

$$\gamma = (63.8^{+3.5}_{-3.7})^\circ \text{ remains}$$



# Interpretation results

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$$K^+K^-\pi^+\pi^-$$

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$$\pi^+\pi^-\pi^+\pi^-$$

$$\gamma = (45 \pm 9)^\circ$$

$$\delta_B^{DK} = (114 \pm 9)^\circ$$

$$r_B^{DK} = (9.5 \pm 1.9) \times 10^{-2}$$

$$\delta_B^{D\pi} = (176 \pm 111)^\circ$$

$$r_B^{D\pi} = (0.8 \pm 1.9) \times 10^{-3}$$

$\pi^+\pi^-\pi^+\pi^-$  is in much better agreement with LHCb global result, but there is a tension with  $K^+K^-\pi^+\pi^-$ ...

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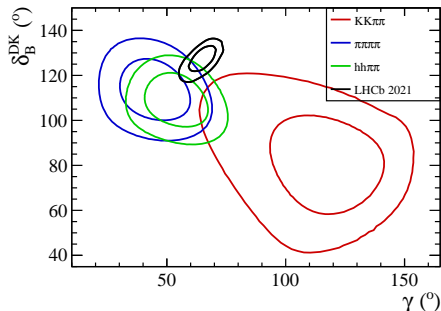
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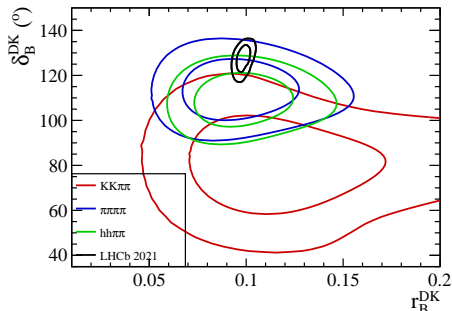
$\pi^+\pi^-\pi^+\pi^-$  is in much better agreement with LHCb global result, but there is a tension with  $K^+K^-\pi^+\pi^-$  ...  
...but how Gaussian are these uncertainties?

# Likelihood scan of interpretation fit

In fact, a likelihood scan shows that  $D \rightarrow K^+ K^- \pi^+ \pi^-$  and  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$   $2\sigma$  contours overlap



$\gamma$  vs  $\delta_B^{DK}$

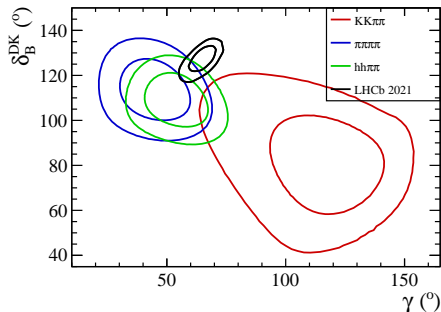


$r_B^{DK}$  vs  $\delta_B^{DK}$

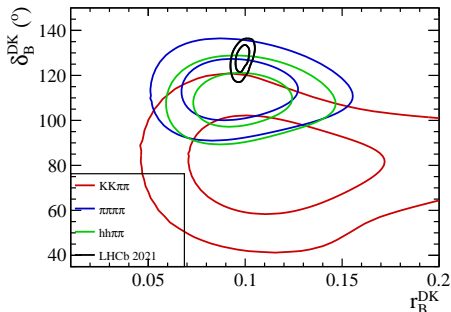
When all biases, correlations and non-Gaussian uncertainties are accounted for, the tension with the LHCb average has reduced significantly

# Likelihood scan of interpretation fit

In fact, a likelihood scan shows that  $D \rightarrow K^+ K^- \pi^+ \pi^-$  and  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$   $2\sigma$  contours overlap



$\gamma$  vs  $\delta_B^{DK}$



$r_B^{DK}$  vs  $\delta_B^{DK}$

However, with all the non-Gaussian behaviour, are we sure these contours cover 68% and 95% ?

Feldman-Cousins method, or Plugin, is a “brute-force” approach to assigning a confidence interval

At each scan point of  $\gamma$ , perform these fits to data:

- 1 Fit with all parameters floating, and save the log-likelihood  $\chi^2$
- 2 Fit with  $\gamma$  fixed to scan point, and save  $\chi_{\text{fix}}^2$
- 3 Calculate  $\Delta\chi_{\text{data}}^2 = \chi_{\text{fix}}^2 - \chi^2$

We expect  $\Delta\chi_{\text{data}}^2$  to become large as we move away from best-fit value, but without direct knowledge of underlying PDF, we cannot determine any confidence intervals from this

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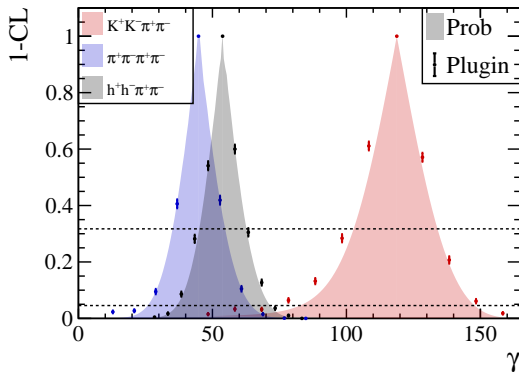
At each scan point of  $\gamma$ , perform these fits to toy:

- 1 Fix  $\gamma$  to scan point and generate 1000 toys
- 2 Perform fits to each toy, with  $\gamma$  both floating and fixed
- 3 Calculate  $\Delta\chi_{\text{toy}}^2$

At each scan point, the fraction of toys with  $\Delta\chi_{\text{toy}}^2 > \Delta\chi_{\text{data}}^2$  is equal to  $1 - \text{CL}$ , and the exact 68% confidence interval can then be obtained using an interpolation between points

# Plugin/Feldman-Cousins method

LHCb average within  $2\sigma$  of  $D \rightarrow K^+ K^- \pi^+ \pi^-$  Plugin result  
Combined fit shows good agreement between Plugin and Prob scans



Combined fit result:  $\gamma = (54.0^{+10.2}_{-9.5})^\circ$

Third most precise single measurement of  $\gamma$  in  $B^\pm$  decays

## Analysis progression since last presentation:

- ① Selection between Oxford and Bristol groups have been merged
- ② Many selection cuts are now applied before BDT, instead of after
- ③ All internal LHCb internal systematic uncertainties evaluated
- ④ Combination of phase-space binned and integrated results

## Important points that were discussed during review:

- Correlation between binned and integrated results
- Efficiency corrections to  $c_i$  and  $s_i$



## Selection “workflow” before review:

### 1. Initial cuts

- Triggers
- RICH information
- Mass range
- Etc

### 2. BDT

- Combinatorial background
- Optimised for  $\gamma$  sensitivity

### 3. Final cuts

- PID
- Flight significance
- Opening angle
- $K_S^0$  veto

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### 2. BDT

- Combinatorial background
- Optimised for  $\gamma$  sensitivity

### 3. Final cuts

- PID

Minimal change in final results

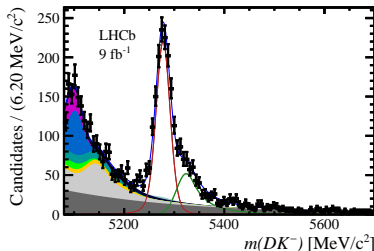
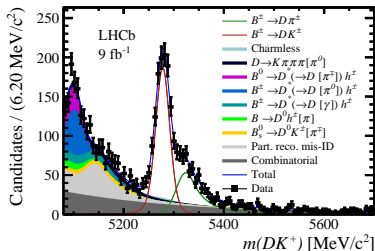
# Summary of LHCb internal systematic uncertainties

Source	$x_-^{DK}$	$y_-^{DK}$	$x_+^{DK}$	$y_+^{DK}$	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
$CP$ violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic $b$ -hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \rightarrow K^- \pi^+ \pi^- \pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \rightarrow p D \pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Fit bias	0.06	0.05	0.13	0.02	0.06	0.13
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57

Numbers for  $\pi^+ \pi^- \pi^+ \pi^-$  are very similar

# Phase-space integrated CP observables

Phase-space integrated study of  $\gamma$ :  
Charged asymmetries measured for  $D \rightarrow K^+ K^- \pi^+ \pi^-$  and  
 $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  in Eur. Phys. J. C **83** 547 (2023)



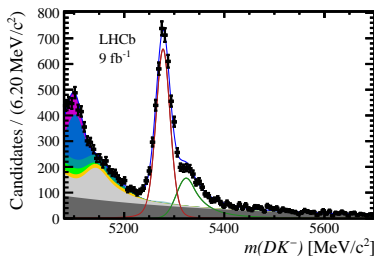
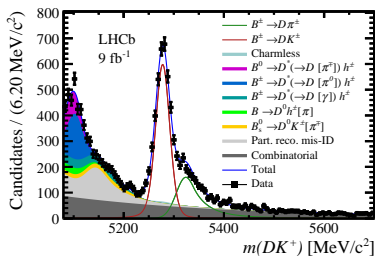
$$D \rightarrow K^+ K^- \pi^+ \pi^-$$

•  $B^\pm \rightarrow [h^+ h^- \pi^+ \pi^-]_D h^\pm$  asymmetries:

- $D \rightarrow K^+ K^- \pi^+ \pi^-$ :  $\mathcal{A} = 0.095 \pm 0.023 \pm 0.002$
- $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ :  $\mathcal{A} = 0.061 \pm 0.013 \pm 0.002$

# Phase-space integrated CP observables

Phase-space integrated study of  $\gamma$ :  
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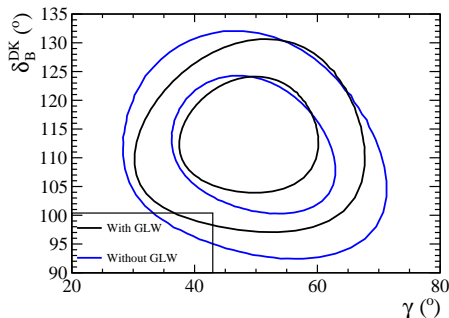
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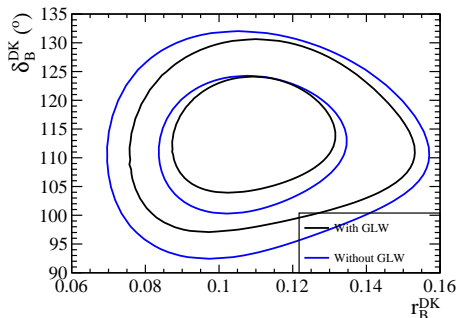
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# Combining phase-space binned and integrated results

We can add phase-space integrated observables as a constraint:



$\gamma$  vs  $\delta_B^{DK}$

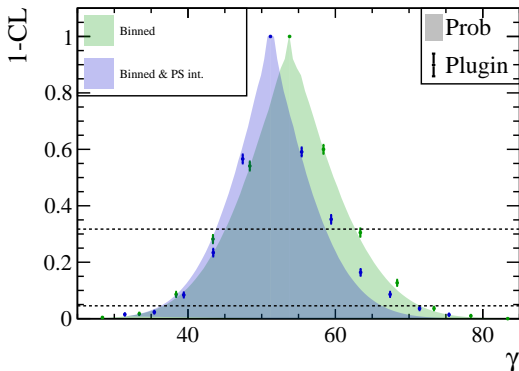


$r_B^{DK}$  vs  $\delta_B^{DK}$

The global asymmetries contain useful information!

# Combining phase-space binned and integrated results

Run Plugin with phase-space integrated constraints:



Final measurement:  $\gamma = (51.2^{+8.9}_{-6.5})^\circ$

- ① Binned model-independent measurement of  $\gamma$  with  $B^\pm \rightarrow [h^+ h^- \pi^+ \pi^-]_D h^\pm$  has been performed:  $\gamma = (54.0^{+10.2}_{-9.5})^\circ$ 
  - External strong-phase inputs from BESIII
- ② Combination with integrated results:  $\gamma = (51.2^{+8.9}_{-6.5})^\circ$
- ③  $3\sigma$  tension in  $D \rightarrow K^+ K^- \pi^+ \pi^-$  has reduced



- We see no showstoppers for this analysis, which will provide important constraints to GammaCombo in the near future
- BESIII results for  $K^+K^-\pi^+\pi^-$  will be available imminently
- We would like this analysis to move to RC
  - You can find the TWiki [here](#)

Thanks for your attention!

# Backup: BESIII preliminary $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ strong-phase results

First binned strong-phase analysis of  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ , which uses the  $2 \times 4$  binning scheme with  $16 \text{ fb}^{-1}$   $\psi(3770)$  data

$$c_1 = -0.28 \pm 0.09 \pm 0.01$$

$$s_1 = -0.68 \pm 0.24 \pm 0.04$$

$$c_2 = +0.83 \pm 0.04 \pm 0.01$$

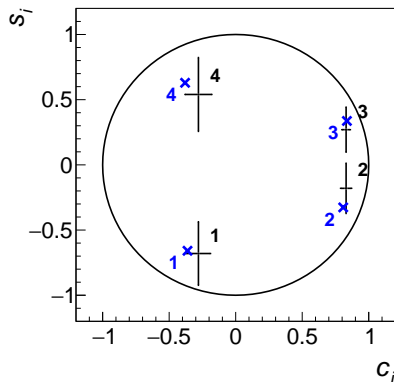
$$s_2 = -0.18 \pm 0.19 \pm 0.03$$

$$c_3 = +0.83 \pm 0.03 \pm 0.01$$

$$s_3 = +0.27 \pm 0.17 \pm 0.03$$

$$c_4 = -0.28 \pm 0.10 \pm 0.01$$

$$s_4 = +0.54 \pm 0.28 \pm 0.04$$



Measured values (black) are consistent and close to LHCb model predictions (blue), so central values are not expected to change much

# Backup: BESIII preliminary $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ strong-phase results

Small differences between model prediction and measurement, but data points are generally close to the unit circle

$$c_1 = +0.12 \pm 0.09 \pm 0.02$$

$$s_1 = -0.42 \pm 0.21 \pm 0.04$$

$$c_2 = +0.74 \pm 0.04 \pm 0.02$$

$$s_2 = -0.39 \pm 0.16 \pm 0.06$$

$$s_3 = -0.25 \pm 0.12 \pm 0.03$$

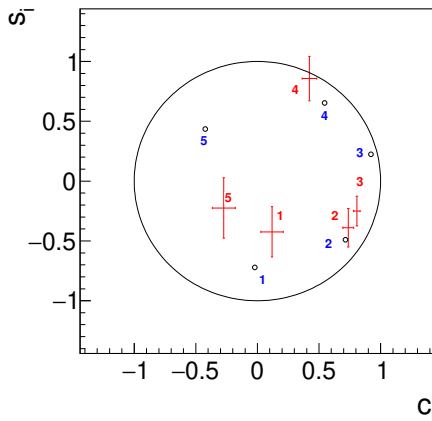
$$c_3 = +0.81 \pm 0.03 \pm 0.01$$

$$c_4 = +0.42 \pm 0.06 \pm 0.02$$

$$s_4 = +0.86 \pm 0.19 \pm 0.07$$

$$c_5 = -0.27 \pm 0.09 \pm 0.03$$

$$s_5 = -0.22 \pm 0.25 \pm 0.08$$



The HyperPlot software is used (binary lookup tree in 5D phase space)

# Backup: BESIII preliminary $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ strong-phase results

- Binned strong-phase analysis of  $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  uses the  $2 \times 5$  “optimal” binning scheme with  $3 \text{ fb}^{-1} \psi(3770)$
- Earlier CLEO-c analysis with  $0.8 \text{ fb}^{-1}$  [JHEP 01 \(2018\) 144](#)
- New BESIII analysis uses a new binning scheme optimised with a BESIII amplitude model [arXiv:2312.02524](#)
  - Amplitude model constructed from a larger data set
  - In principle more sensitive
- Two binning schemes are available:
  - We use the more sensitive “optimal” binning with  $Q = 0.85$
  - The other “equal  $\delta$ ” binning has  $Q = 0.80$

How do we include the  $\pi^+\pi^-\pi^+\pi^-$  mode?

- We have already studied  $B^\pm \rightarrow [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$  for phase-space integrated measurement
  - 1 Different  $D$  daughter PID cuts in stripping
  - 2 No  $D \rightarrow K\pi\pi\pi\pi^0$  background
  - 3 Charmless background recalculated using the sideband
  - 4 Use same BDT
  - 5 No additional peaking backgrounds
- Sort candidates into phase-space bins using BESIII binning scheme
- Can fit separately or simultaneously with  $K^+K^-\pi^+\pi^-$

## Why are $c_i$ and $s_i$ Gaussian constrained?

- Previous BPGGSZ analyses have kept  $c_i$  and  $s_i$  fixed
  - ①  $c_i$  and  $s_i$  uncertainties are added as a systematic through smearing
  - ② Convenient for calculating correlations between different analyses
  - ③ Appropriate when  $c_i$  and  $s_i$  uncertainties are small
- In four-body analyses, uncertainties on  $\gamma$  from  $c_i$  and  $s_i$  are almost the same size as the statistical uncertainty
- Large  $s_i$  uncertainties introduces non-Gaussian uncertainties on  $y_{\pm}$
- $\gamma/\delta_B$  move significantly when fixing  $s_i$  instead of constraining them
- These effects are largest for  $K^+K^-\pi^+\pi^-$ , but are also seen in  $\pi^+\pi^-\pi^+\pi^-$  and in the combined fit

## Summary of free parameters in the CP fit:

$$K^+ K^- \pi^+ \pi^-$$

$$2 \times 2 \times 2 \times 4 = 32 \text{ categories}$$

- 6 CP observables
- 7  $F_i$  parameters
- 8  $c_i$  and  $s_i$  parameters
- 32 combinatorial yields
- 32 low mass yields
- 4 global normalisations
- Total: 89 parameters

$$\pi^+ \pi^- \pi^+ \pi^-$$

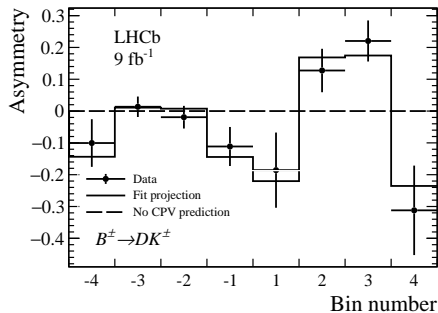
$$2 \times 2 \times 2 \times 5 = 40 \text{ categories}$$

- 6 CP observables
- 9  $F_i$  parameters
- 10  $c_i$  and  $s_i$  parameters
- 40 combinatorial yields
- 40 low mass yields
- 4 global normalisations
- Total: 109 parameters

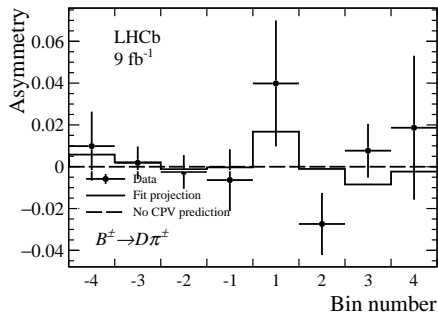
In a combined fit where CP observables are shared, there are  
 $89 + 109 - 6 = 192$  parameters

# Backup: Bin asymmetries

$B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  bin asymmetries



$B^\pm \rightarrow DK^\pm$

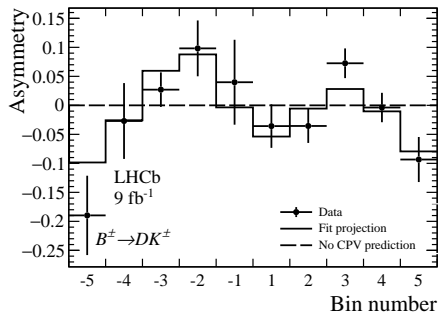


$B^\pm \rightarrow D\pi^\pm$

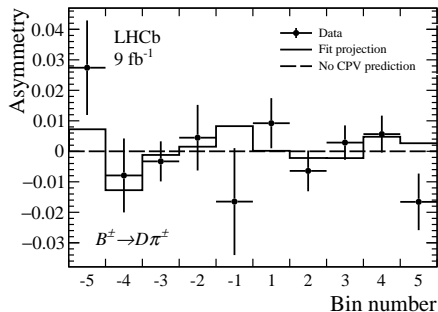


# Backup: Bin asymmetries

$$B^\pm \rightarrow [\pi^+ \pi^- \pi^+ \pi^-]_D h^\pm \text{ bin asymmetries}$$



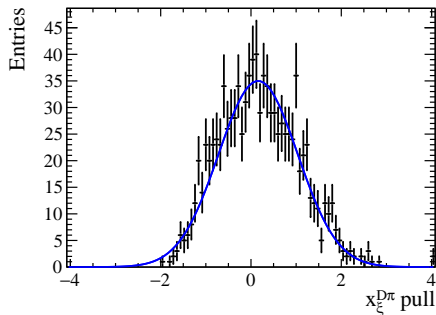
$$B^\pm \rightarrow DK^\pm$$



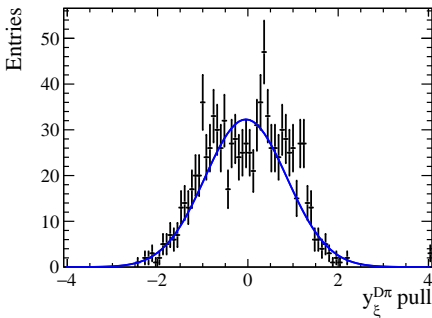
$$B^\pm \rightarrow D\pi^\pm$$

# Backup: CP fit toy studies

In toy studies biases in  $D\pi$  observables are consistent with model-dependent analysis



$x_{\xi}^{D\pi}$

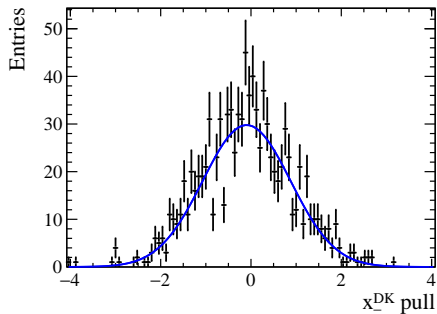


$y_{\xi}^{D\pi}$

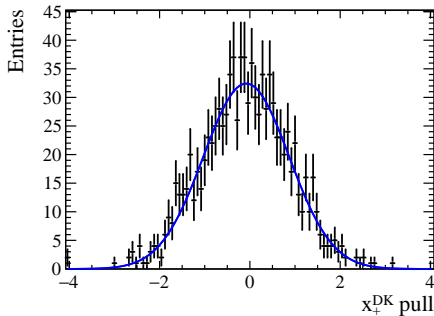
$$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$$

# Backup: CP fit toy studies

Minor biases in  $x_{\pm}^{DK}$  are seen but can be corrected for...



$x_{-}^{DK}$

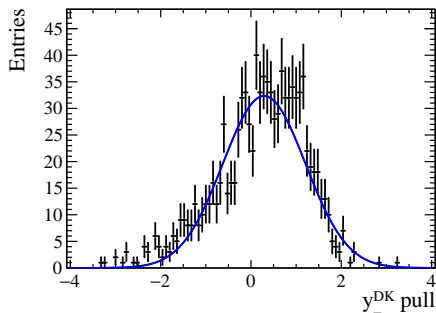


$x_{+}^{DK}$

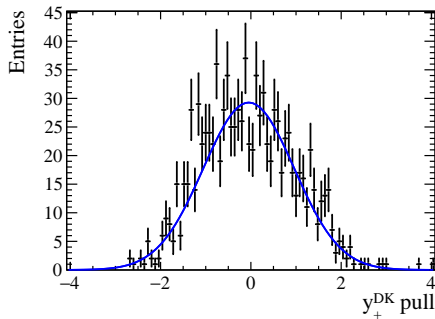
$$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$$

# Backup: CP fit toy studies

...but  $y_{\pm}^{DK}$  pulls are now slightly asymmetric!



$y_-^{DK}$

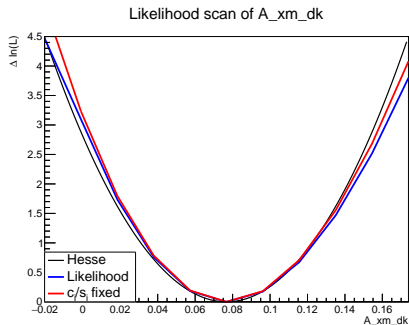


$y_+^{DK}$

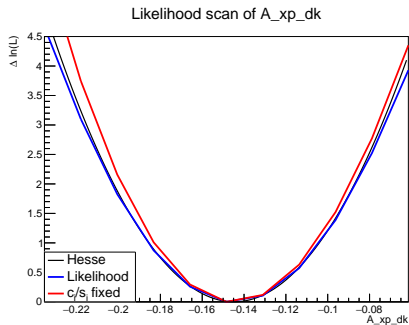
$$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$$

# Backup: Likelihood scan of CP observables

$x_{\pm}^{DK}$  agree well between likelihood scan and Hesse approximation



$x_{-}^{DK}$

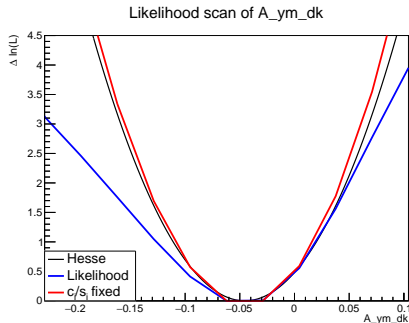


$x_{+}^{DK}$

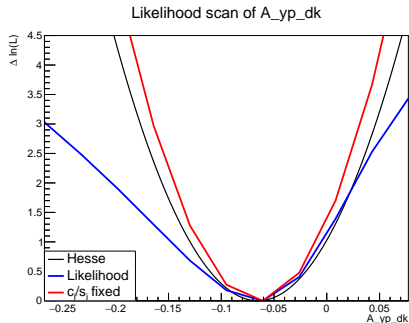
$$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$$

# Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$  diverges from Hesse approximation outside  $1\sigma$



$y_{-}^{DK}$

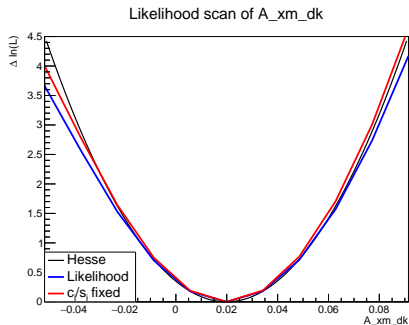


$y_{+}^{DK}$

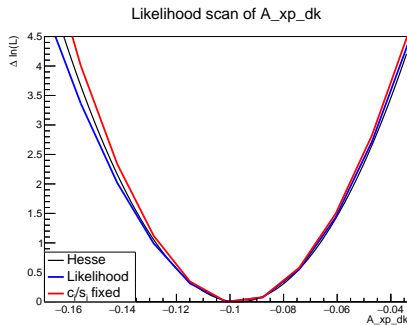
$$D^0 \rightarrow K^+ K^- \pi^+ \pi^-$$

# Backup: Likelihood scan of CP observables

$x_{\pm}^{DK}$  agree well between likelihood scan and Hesse approximation



$x_{-}^{DK}$

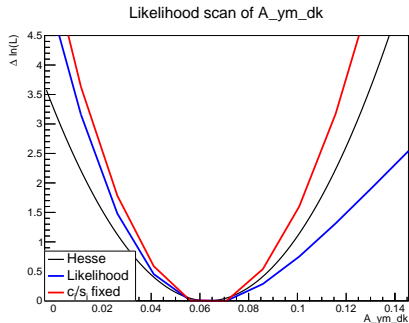


$x_{+}^{DK}$

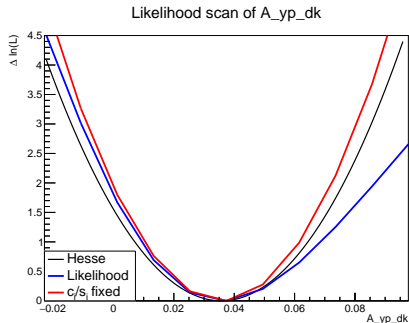
$$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$$

# Backup: Likelihood scan of CP observables

$y_{\pm}^{DK}$  diverges from Hesse approximation outside  $1\sigma$



$y_{-}^{DK}$



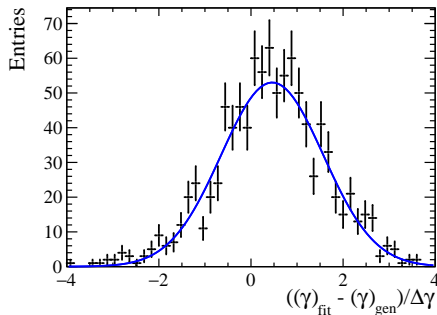
$y_{+}^{DK}$

$$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$$

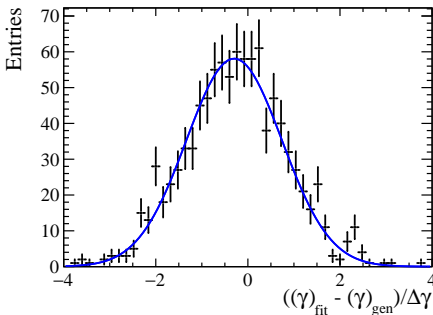


# Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do not expect these to behave very Gaussian...



$K^+ K^- \pi^+ \pi^-$



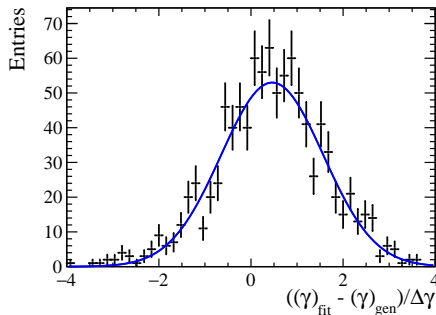
$\pi^+ \pi^- \pi^+ \pi^-$

$\gamma$  pull distributions

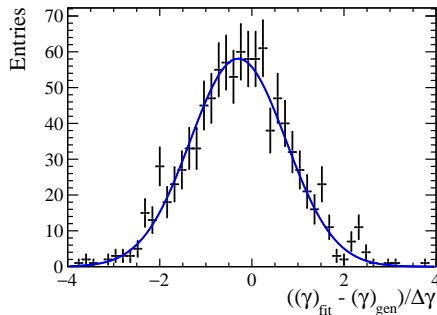
Indeed, small but significant biases are observed!

# Backup: Interpretation toys

We can perform toy studies on the interpretation fit, but we do not expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$



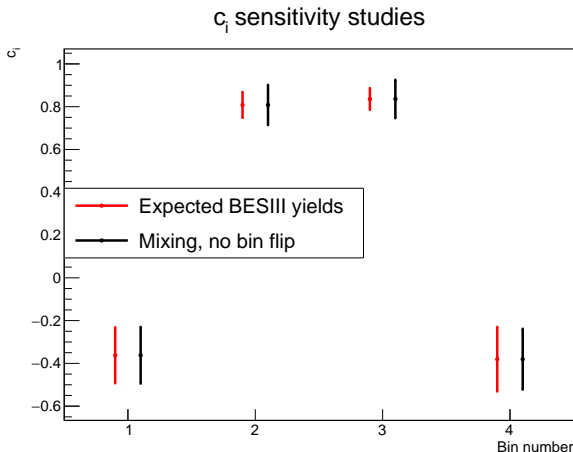
$\pi^+\pi^-\pi^+\pi^-$

$\gamma$  pull distributions

The absolute bias corrections are:

# Backup: Charm mixing studies with multi-body decays

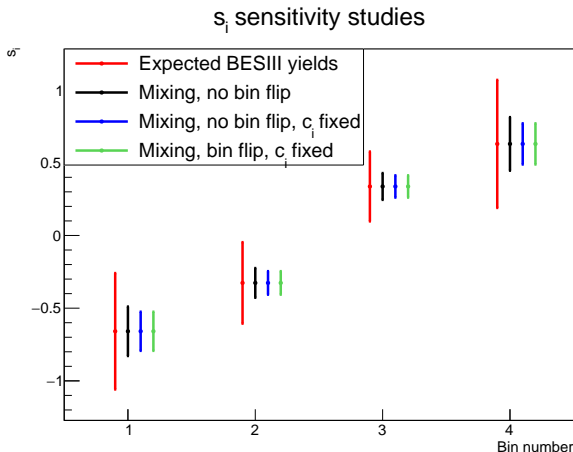
Sensitivity to  $c_i$ : Similar between BESIII and charm mixing at LHCb



- BESIII yields equivalent to  $8 \text{ fb}^{-1}$  of  $\psi(3770)$
- 4 million  $D \rightarrow K^+ K^- \pi^+ \pi^-$  candidates in mixing analysis

# Backup: Charm mixing studies with multi-body decays

Sensitivity to  $s_i$ : Significant improvements expected!



- BESIII yields equivalent to  $8 \text{ fb}^{-1}$  of  $\psi(3770)$
- 4 million  $D \rightarrow K^+ K^- \pi^+ \pi^-$  candidates in mixing analysis