BESIII Summer Collaboration Meeting
Measurement of the CP even fraction F_+ in $D^0 \to K^+K^-\pi^+\pi^-$

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Outline

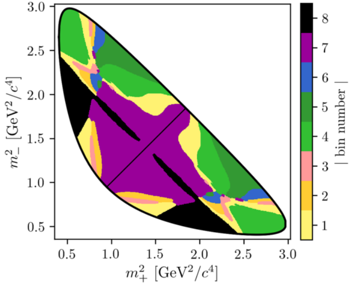
1. Introduction and motivation
2. Strategy of strong-phase analysis
3. Fit of single tag yields
4. Fit of double tag yields
   - Fit strategy
   - CP tags
   - $K_{S,L}\pi\pi$ tags
5. Reweighting of $KK\pi\pi$ model
6. F_+ combination
7. Summary and conclusion

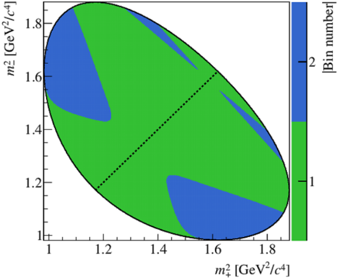
Introduction

- Perform strong-phase analysis of $D^0 \to K^+K^-\pi^+\pi^-$
- This analysis: Phase space integrated analysis
  - $2.93\,\text{fb}^{-1}$ $\psi(3770)$ data from 2010-2011
  - Measure CP even fraction F_+
- Future analysis: Binned phase space analysis
  - Expect $\approx 20\,\text{fb}^{-1}$ $\psi(3770)$ data from 2010-2011 and 2021-2023
  - Measure amplitude-averaged cosine and sine of strong-phase c_i and s_i
  - Plan to analyse 2021-2022 data initially

Introduction to GGSZ analysis of γ

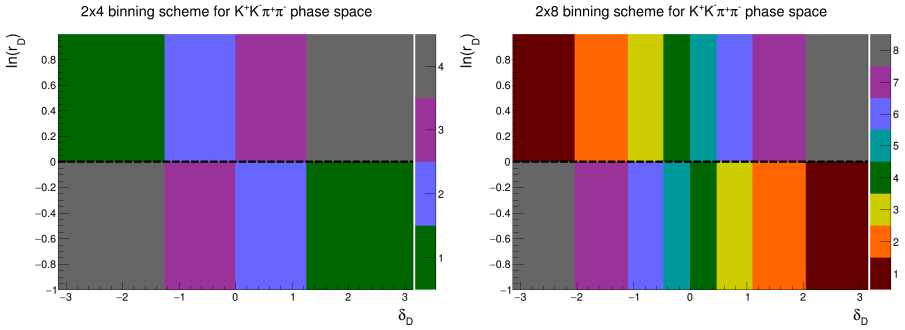
- Main motivation: Measure γ in $B^\pm \to DK^\pm$ with self-conjugate multi-body D decay
  - Model independent measurement: Bins of D decay phase space
  - External inputs: Measure c_i and s_i at BESIII
  - Poor binning reduces statistical sensitivity $\to$ No bias!
- J. High Energ. Phys. 2021, 169 (2021): $B^\pm \to Dh^\pm$, $D \to K_S^0 h^+ h^-$
  - Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^\circ$





Introduction to GGSZ analysis of γ

- Our aim: Analyse $B^{\pm} \to [K^+K^-\pi^+\pi^-]_D h^{\pm}$ in bins of phase space
  - Develop binning scheme using LHCb model JHEP 02 (2019) 126
  - Simultaneously analyse c_i/s_i at BESIII and γ at LHCb
  - Expected precision $\Delta\gamma \approx 12^\circ$ with LHCb Run 1+2

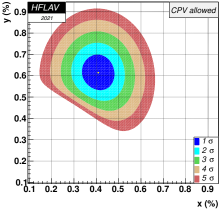


(a) 2×4 bins **(b)** 2×8 bins

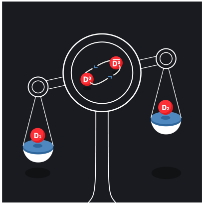
Figure 1: Binning scheme for $D^0 \to K^+K^-\pi^+\pi^-$

Introduction to D mixing and CP-violation

- $D \to KK\pi\pi$ can also be used to study D-mixing
- Phase space integrated analysis:
  - Physical Review D, 91(9), 2015
  - Measure mixing parameter y_{CP} and CP-violation parameter A_Γ in self-conjugate multi-body decays
- Analysis in bins of phase space:
  - Phys. Rev. Lett. 127, 111801 (2021)
  - Bin-flip analysis of $D \to K_S\pi\pi$
  - Measure mixing parameters x, y and CP-violation parameters $|q/p|$, φ



(a) Source: HFLAV



(b) Source: LHCb outreach

Motivation for F_+ measurement

- F_+ describes the CP content of a self-conjugate multi-body decay
  - $F_+ = 1$ (0) for CP even (odd) final states
- F_+ can be measured with current $3\,\mathrm{fb}^{-1}$ dataset
  - First model independent measurement of $F_+^{KK\pi\pi}$!
  - Useful to test agreement with LHCb model prediction: $F_+ = 0.736$
- Important input to quasi-GLW analysis of the CKM angle γ
  - Current GLW modes: KK, $\pi\pi$, $\pi\pi\pi\pi$
  - Minimal effort to include $KK\pi\pi$ in GLW analyses $\implies$ More statistics
- Other F_+ measurements:
  - $D^0 \to \pi^+\pi^-\pi^+\pi^-$ JHEP 01 (2018) 144
  - $D^0 \to K_S\pi^+\pi^-\pi^0$ JHEP 01 (2018) 82
  - $D^0 \to h^+h^-\pi^0$ Physics Letters B 747 (2015)
  - Measurements are from CLEO-c, BESIII analyses ongoing

Strategy for strong-phase analysis

1. Select double tags of $KK\pi\pi$ vs flavour, CP and self-conjugate tags
2. Normalise double tag yields by the corresponding single tag yields
3. Measure flavour tag yields K_i
4. Fit c_i with CP tags and c_i+s_i with self-conjugate tags:

c_i/s_i analysis: Bins of $KK\pi\pi$ phase space

CP: $M_i \propto \left(K_i + K_{-i} - 2c_i\sqrt{K_i K_{-i}}(2F_+^{\mathrm{tag}} - 1)\right)$

Self-conjugate: $M_{ij} \propto \left(K_i K'_{-j} + K_{-i}K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_{-j}\right)$

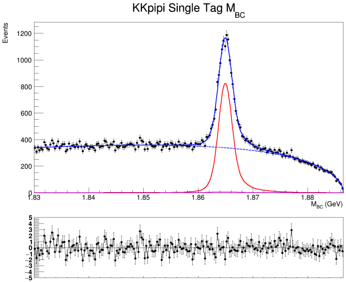
F_+ analysis: Sum over all $KK\pi\pi$ phase space bins

CP: $M \propto \left(1 - 2(2F_+^{KK\pi\pi} - 1)(2F_+^{\mathrm{tag}} - 1)\right)$

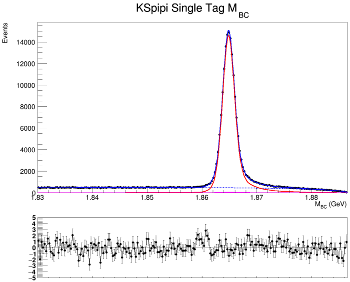
Self-conjugate: $M_j \propto \left(K'_j + K'_{-j} - 2c'_j\sqrt{K'_j K'_{-j}}(2F_+^{KK\pi\pi} - 1)\right)$

Single tag fits

- Fit strategy: Fit m_{BC}
- Fit model:
  - Signal: PDF from signal MC, convoluted with single or double Gaussian
  - Flat background: Argus PDF
  - Peaking background shape and yield fixed
    - Fit shape to dedicated MC samples
    - Calculate yield from ratios of efficiencies and branching fractions



(a) $KK\pi\pi$

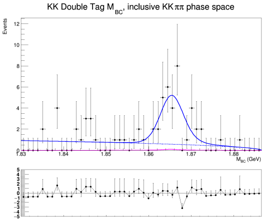


(b) $K_S\pi\pi$

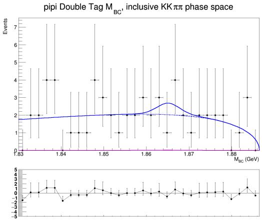
Double tag fits

- Fit strategy:
  - Fully reconstructed tags: Only fit signal m_{BC} because of low statistics
  - Partially reconstructed tags: Fit missing mass squared m^2_{miss}
- Fit model:
  - Signal: PDF from signal MC, convoluted with single Gaussian
  - Background: Argus PDF
  - Peaking backgrounds fixed, with quantum correlation accounted for
  - Simple sideband subtraction for correct signal but wrong tag event
- For tags with multiple bins, perform a simultaneous fit of all bins
  - Shape is floated and shared across all bins
  - Yield of signal and combinatorial background is floated in each bin

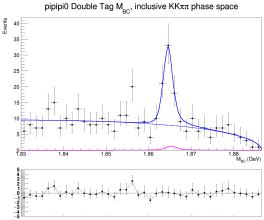
Double tag fits



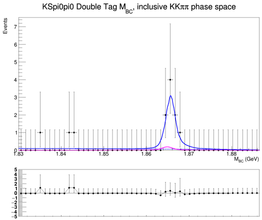
(a) KK



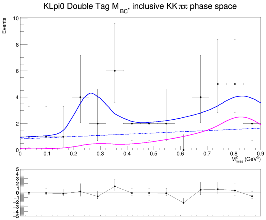
(b) $\pi\pi$



(c) $\pi\pi\pi^0$



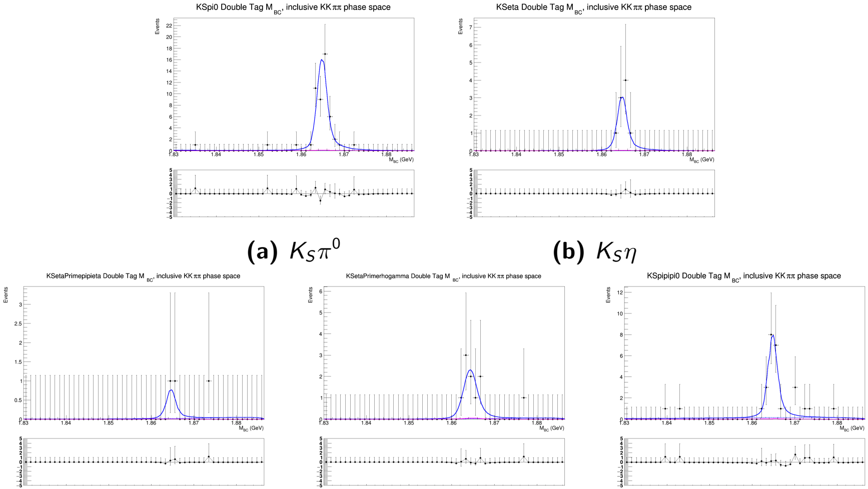
(d) $K_S\pi^0\pi^0$



(e) $K_L\pi^0$

Figure 4: Double tag fits of CP even tags

Double tag fits



(a) $K_S\pi^0$ **(b)** $K_S\eta$

(c) $K_S\eta'(\pi\pi\eta)$ **(d)** $K_S\eta'(\rho\gamma)$ **(e)** $K_S\omega$

Figure 5: Double tag fits of CP odd tags

CP double tag yields and efficiencies

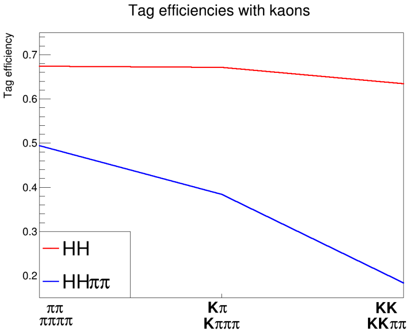
Tag mode	Double tag yield	Double tag efficiency (%)
K^+K^-	28 ± 10	14.52 ± 0.06
$\pi^+\pi^-$	2 ± 4	15.02 ± 0.06
$K_S\pi^0$	48 ± 7	6.87 ± 0.04
$K_S\pi^0\pi^0$	8.0 ± 2.8	2.873 ± 0.026
$K_L\pi^0$	7 ± 5	5.29 ± 0.04
$K_S\eta$	8.9 ± 3.0	5.72 ± 0.04
$K_S\eta'_{\pi\pi\eta}$	2.2 ± 1.6	2.024 ± 0.021
$K_S\eta'_{\rho\gamma}$	8.7 ± 3.0	3.295 ± 0.027
$\pi^+\pi^-\pi^0$	53 ± 10	7.66 ± 0.04
$K_S\omega$	9 ± 3	2.234 ± 0.022

F_+ measurement with $K_{S,L}\pi\pi$ tags

- With $K_S\pi\pi$, increase sensitivity through binning of $K_S\pi\pi$ phase space

$$M_j \propto \left(K'_j + K'_{-j} - 2\sqrt{K'_j K'_{-j}}c'_j(2F_+^{KK\pi\pi} - 1)\right)$$

- Problem: $KK\pi\pi$ reconstruction efficiency is too low $\rightarrow$ Low yields!



- Likely explanation: Softer kaons $\rightarrow$ Kaons get stuck inside tracker

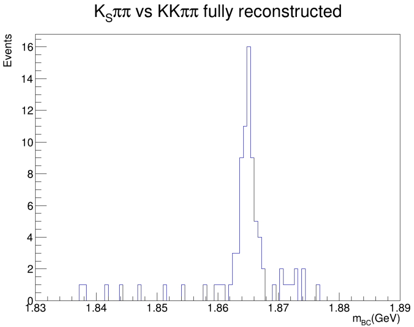
F_+ measurement with $K_{S,L}\pi\pi$ tags

- Solution: Partially reconstructed $KK\pi\pi$
- Strategy:
  1. Reconstruct $D \to K_S\pi\pi$
  2. Require 3 remaining good tracks consistent with $K\pi\pi$
  3. Use missing mass to reconstruct missing kaon

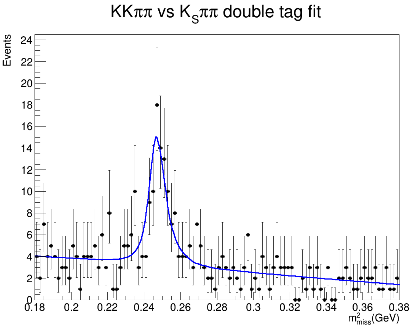
Mode	Inclusive yield	Double tag efficiency
$K_S\pi\pi$ (fully reconstructed)	69 ± 9	6.56 ± 0.04
$K_S\pi\pi$ (partially reconstructed)	91 ± 15	7.01 ± 0.04
$K_L\pi\pi$ (partially reconstructed)	158 ± 15	7.25 ± 0.04

Partially reconstructed $KK\pi\pi$ vs $K_S\pi\pi$

- Main challenge with partially reconstructed $KK\pi\pi$: $K\pi\pi\pi\pi^0$
- Require no π^0 candidates



(a) Fully reconstructed



(b) Partially reconstructed

Figure 6: $KK\pi\pi$ vs $K_S\pi\pi$

Double tag fits

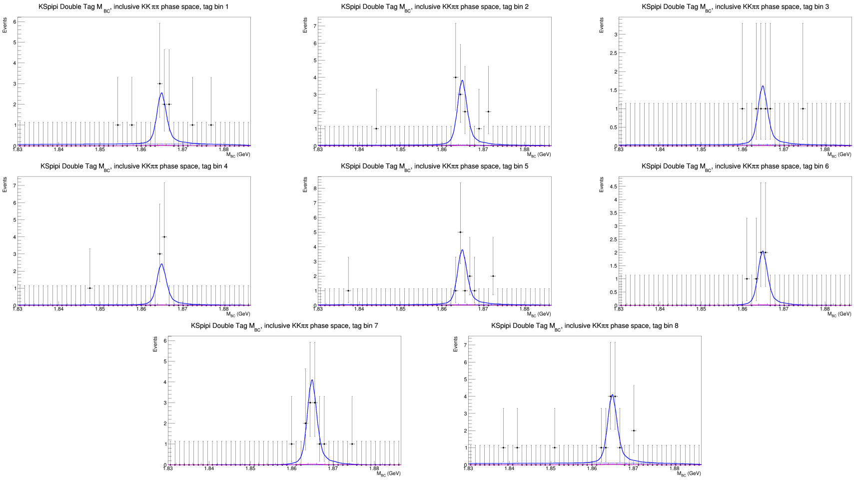


Figure 7: $KK\pi\pi$ vs $K_S\pi\pi$ simultaneous fit

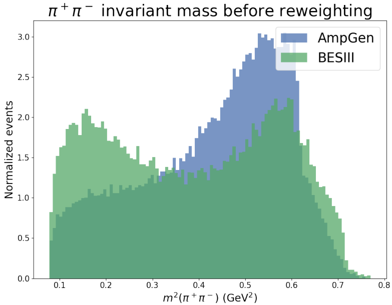
Efficiency corrections

- All yields must be corrected for efficiency
- Problem: BESIII simulation uses a very old $KK\pi\pi$ model in EvtGen
- Solution: Reweight BESIII simulation to look like the LHCb model
  - Use Python hep_ml Gradient Boosted Reweighter
  - Variables:
    1. $m^2(K^+K^-)$
    2. $m^2(K^+\pi^-)$
    3. $m^2(K^-\pi^+)$
    4. $m^2(\pi^+\pi^-)$
    5. $m^2(K^+K^-\pi^+)$

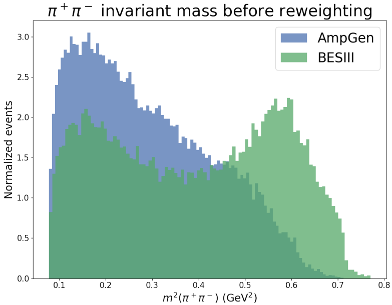
Quantum correlated LHCb model

- Problem with naive reweighting:
  - LHCb model assumes a pure $D^0 \to K^+K^-\pi^+\pi^-$ decay
  - No quantum correlations
  - Example: If tag is $D \to KK$, the $D \to KK\pi\pi$ decay will be CP odd!
  - Quantum correlations will affect phase space distribution $\implies$ Efficiencies could change
- Solution: Separate reweighters for CP even/odd $D \to K^+K^-\pi^+\pi^-$
  - CP even tags: Use efficiencies after reweighting to CP odd model
  - CP odd tags: Use efficiencies after reweighting to CP even model
  - $K_{S,L}\pi\pi$ tags: Do a weighted average of the two efficiencies

Before weighting to CP even/odd models



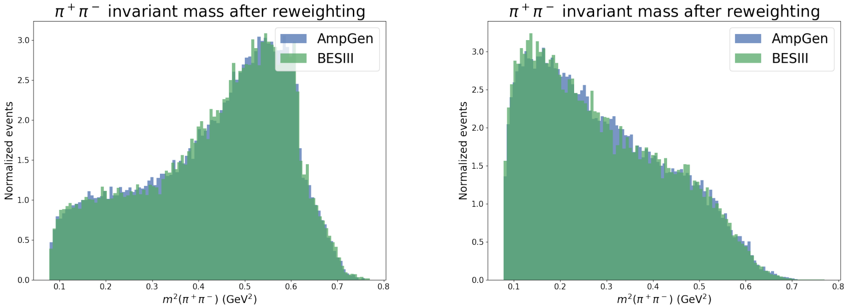
(a) CP even



(b) CP odd

Figure 8: $m^2(\pi^+\pi^-)$ before reweighting

After weighting to CP even/odd models



(a) CP even

(b) CP odd

Figure 9: $m^2(\pi^+\pi^-)$ after reweighting

No reweighting	Naive reweighting	CP even model	CP odd model
18.0%	19.0%	18.1%	21.9%

F_+ measurement with CP tags

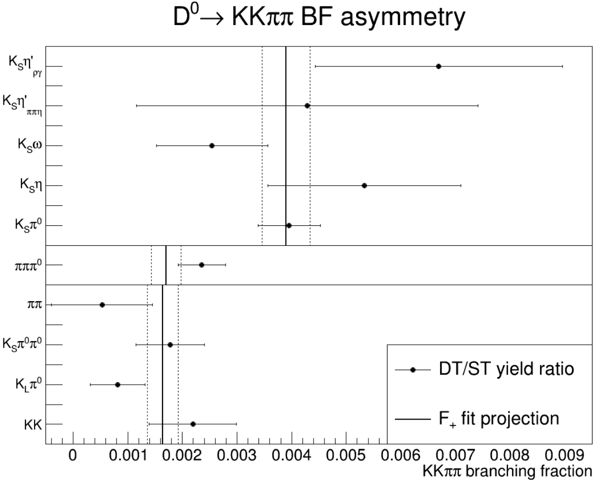
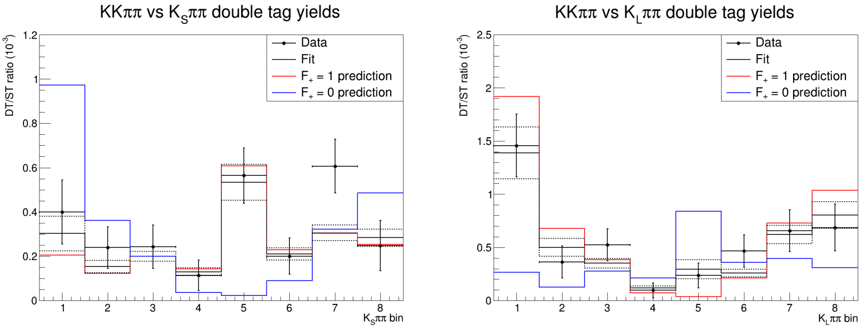


Figure 10: F_+ combination of CP tags

Fit result: $F_+ = 0.703 \pm 0.042$, $\chi^2 = 1.4$

F_+ measurement with $K_{S,L}\pi\pi$ tags



(a) Result: $F_+ = 0.872 \pm 0.091$, $\chi^2 = 1.3$ **(b)** Result: $F_+ = 0.679 \pm 0.103$, $\chi^2 = 0.8$

Figure 11: F_+ combination of $K_S\pi\pi$ (left) and $K_L\pi\pi$ (right)

F_+ combination

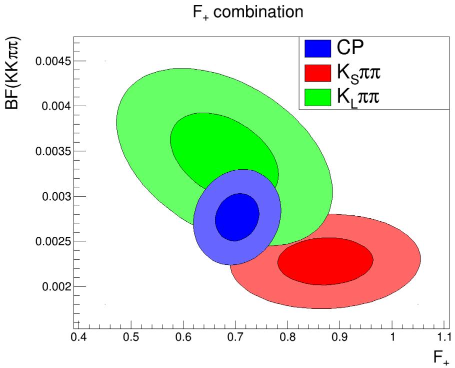


Figure 12: F_+ combination

- Observe large anti-correlation in $K_L\pi\pi$ because $F_+^{K_L\pi\pi} \approx 0.354$
  - Yield of $K_L\pi\pi$ is <u>twice</u> as large as that of $K_S\pi\pi$
  - Fractional bin yields <u>and</u> total yield contains information about F_+
  - When $K_L\pi\pi$ BF is available, combine all tags!

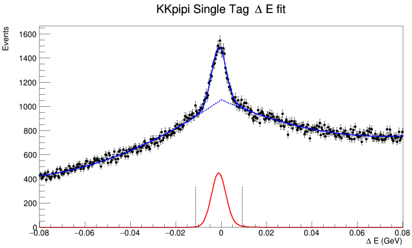
- First model-independent measurement of CP even fraction in $D^0 \to K^+K^-\pi^+\pi^-$
  - $F_+ = 0.730 \pm 0.040 \pm 0.017$
  - Statistics dominated!
  - Very consistent with model prediction: $F_+ = 0.736$
- Valuable input to:
  - γ measurement with GLW method
  - D-mixing and CPV analyses
- Future $\psi(3770)$ data will allow us to perform a binned analysis

Thank you!

Backup

Selection

- Selection of charged and neutral particles follow standard track and shower requirements
- Require flight significance > 2 for K_S
- K_S veto for $KK\pi\pi$ and $\pi\pi\pi^0$ tags
- ΔE cut of 3σ
- ΔE fit for 4-body modes allows a non-smooth background at $\Delta E = 0$



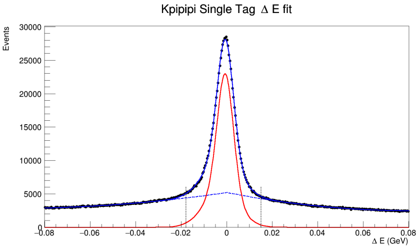


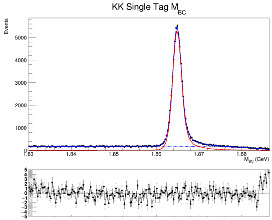
Figure 13: Double Gaussian signal and Chebychev polynomial background

Tag modes

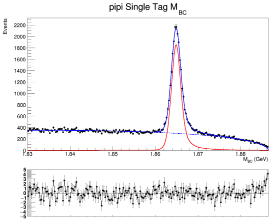
- Flavour tags:
  - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $\underline{Ke\nu}$
- CP even tags:
  - KK, $\pi\pi$, $\pi\pi\pi^0$ (mostly CP even), $K_S\pi^0\pi^0$, $K_L\pi^0$
- CP odd tags:
  - $K_S\pi^0$, $K_S\eta$, $K_S\omega$, $K_S\eta'_{\pi\pi\eta}$, $K_S\eta'_{\rho\gamma}$
- Self-conjugate tags:
  - $K_S\pi\pi$, $K_L\pi\pi$

Underlined tags have not been finalized yet

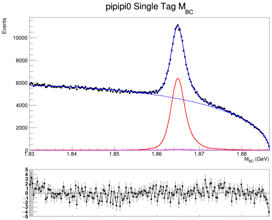
Single tag fits



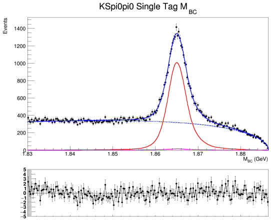
(a) KK



(b) $\pi\pi$



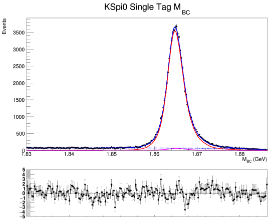
(c) $\pi\pi\pi^0$



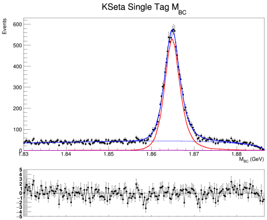
(d) $K_S\pi^0\pi^0$

Figure 14: Single tag fits of CP even tags

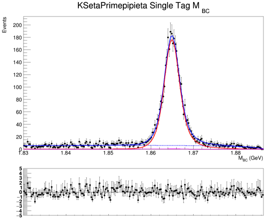
Single tag fits



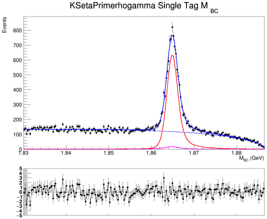
(a) $K_S\pi^0$



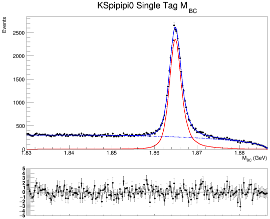
(b) $K_S\eta$



(c) $K_S\eta'(\pi\pi\eta)$



(d) $K_S\eta'(\rho\gamma)$



(e) $K_S\omega$

Figure 15: Single tag fits of CP odd tags

Single tag yields and efficiencies

Tag mode	Single tag yield	Single tag efficiency (%)
$K^+K^-\pi^+\pi^-$	10642 ± 156	19.02 ± 0.09
K^+K^-	56303 ± 262	63.41 ± 0.11
$\pi^+\pi^-$	20386 ± 179	67.41 ± 0.10
$K_S\pi^0$	67876 ± 278	38.18 ± 0.11
$K_S\pi^0\pi^0$	22392 ± 229	14.35 ± 0.08
$K_L\pi^0$	47595 ± 1653	27.83 ± 0.23
$K_S\eta$	9308 ± 113	31.78 ± 0.10
$K_S\eta'_{\pi\pi\eta}$	3213 ± 62	12.81 ± 0.07
$K_S\eta'_{\rho\gamma}$	8283 ± 116	20.80 ± 0.09
$\pi^+\pi^-\pi^0$	107504 ± 602	36.65 ± 0.11
$K_S\omega$	22068 ± 217	14.50 ± 0.08
$K_S\pi^+\pi^-$	161914 ± 440	36.40 ± 0.11
$K_S\pi^+\pi^-$ part. reco.	161914 ± 440	36.40 ± 0.11
$K_L\pi^+\pi^-$	223141 ± 2146	46.1 ± 0.3

Double tag fits

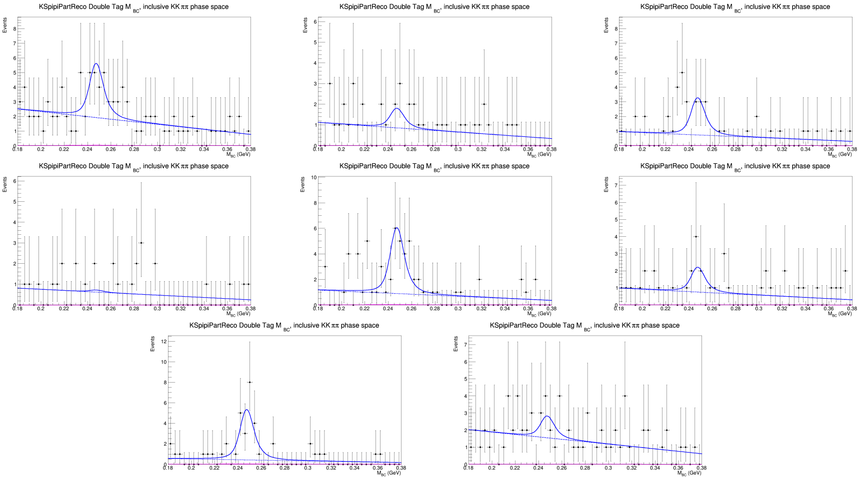


Figure 16: Partially reconstructed $KK\pi\pi$ vs $K_S\pi\pi$ simultaneous fit

Double tag fits

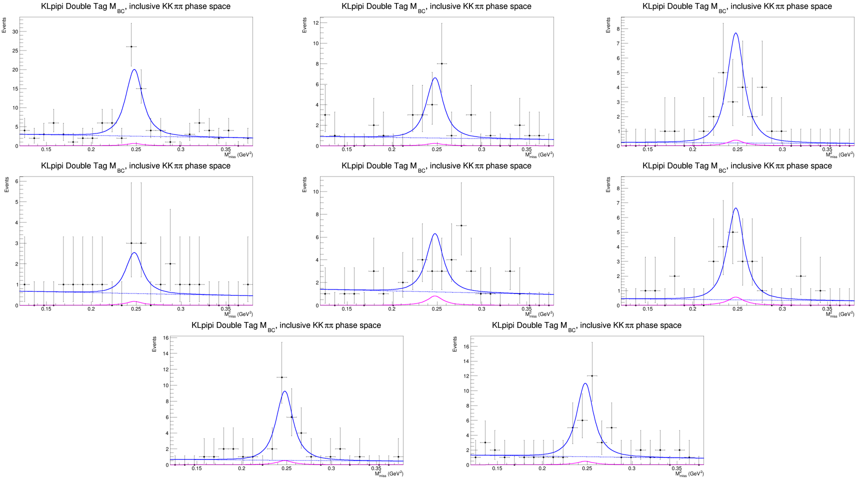
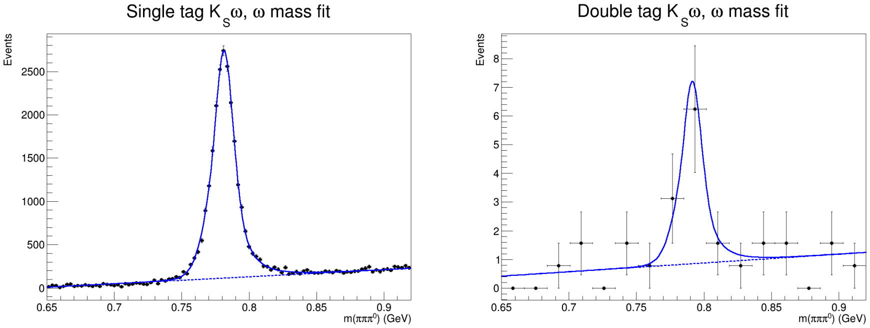


Figure 17: $KK\pi\pi$ vs $K_L\pi\pi$ simultaneous fit

- $K_S\omega$ has CP-even contamination from non-resonant $K_S\pi\pi\pi^0$
  - $F_+(K_S\pi\pi\pi^0) = 0.238 \pm 0.012 \pm 0.012$ from CLEO
- From m_{BC} fit, subtract flat background using sPlot and fit $\pi\pi\pi^0$



(a) Single tag **(b)** Double tag

Peaking backgrounds

- Strategy for fixing peaking backgrounds:
  1. Generate dedicated MC sample
  2. Obtain retention rate of peaking background
  3. Fit background with appropriate shape (Gaussian, Crystal Ball, ...)
  4. Use BFs from PDG to fix background-to-signal ratio

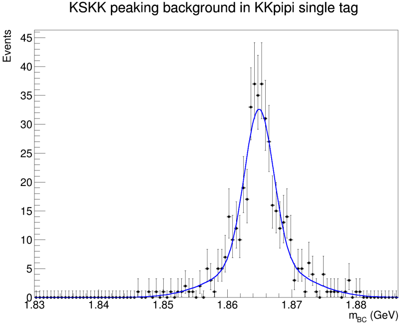
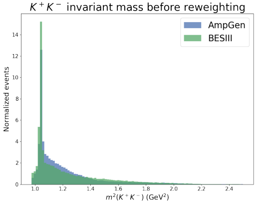


Figure 19: Double Gaussian fit of $K_S KK$ background in $KK\pi\pi$ single tag fit

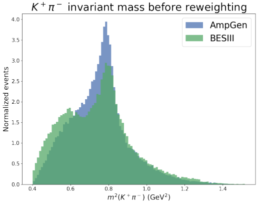
Quantum correlation in peaking backgrounds

- Strategy for peaking backgrounds with different CP:
  - Correct using $F_+^{KK\pi\pi}$ from LHCb model
- Strategy for $K_S KK$ background in $KK\pi\pi$
  - $F_+^{K_S KK} = 0.524 \pm 0.018$ from Phys. Rev. D **102**, 052008
  - Use dedicated MC to find retention in each $K_S KK$ bin
  - K_S veto removes more $K_S\phi$ than $K_S a(980)^0 \implies$ Calculate effective F_+ for $K_S KK$ to $KK\pi\pi$ background

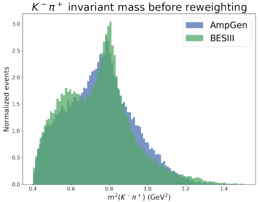
Naive efficiency correction



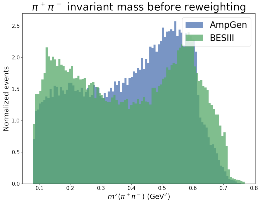
(a) $m^2(K^+K^-)$



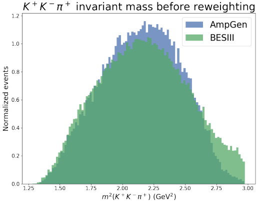
(b) $m^2(K^+\pi^-)$



(c) $m^2(K^-\pi^+)$

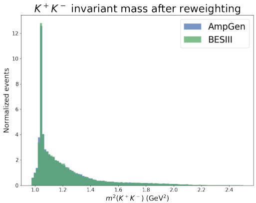


(d) $m^2(\pi^+\pi^-)$

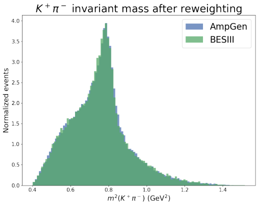


(e) $m^2(K^+K^-\pi^+)$

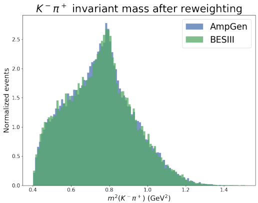
Figure 20: Before reweighting



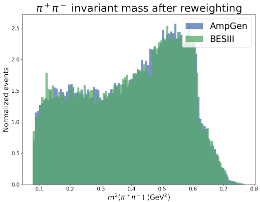
(a) $m^2(K^+K^-)$



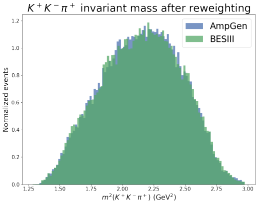
(b) $m^2(K^+\pi^-)$



(c) $m^2(K^-\pi^+)$



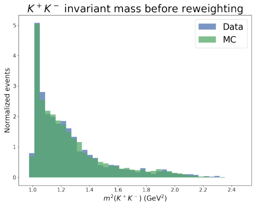
(d) $m^2(\pi^+\pi^-)$



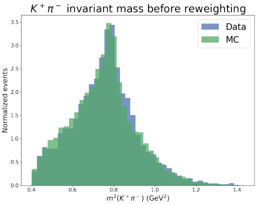
(e) $m^2(K^+K^-\pi^+)$

Figure 21: After reweighting

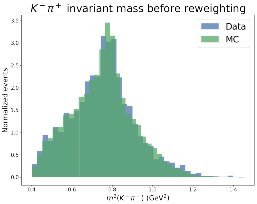
Does the naive reweighting work?



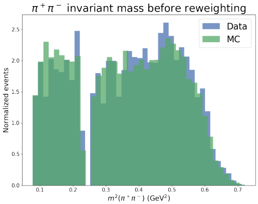
(a) $m^2(K^+K^-)$



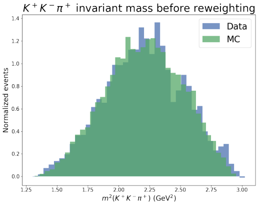
(b) $m^2(K^+\pi^-)$



(c) $m^2(K^-\pi^+)$



(d) $m^2(\pi^+\pi^-)$

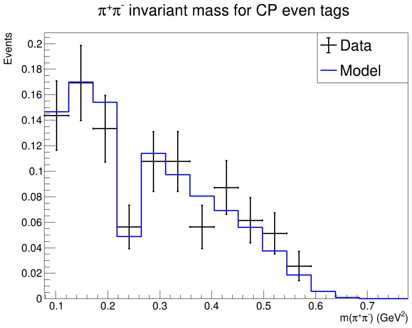


(e) $m^2(K^+K^-\pi^+)$

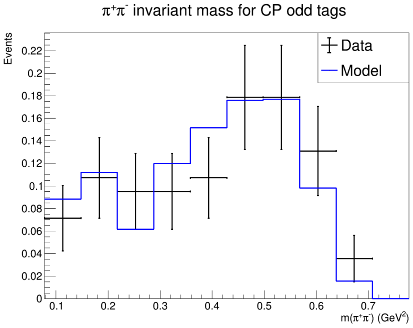
Figure 22: Single tag $D \to KK\pi\pi$ in data and MC after reweighting

Agreement between quantum correlated data and model

- Note: LHCb model knows nothing about quantum correlations
- $D^0/\bar{D}^0$ amplitudes simply combined to obtain CP even/odd models
- Important question: Can the model describe quantum correlated double tag data at all? Answer: Yes!



(a) Double tags of $KK\pi\pi$ vs CP even



(b) Double tags of $KK\pi\pi$ vs CP odd

Figure 23: $m^2(\pi^+\pi^-)$ in double tags, compared with CP even/odd LHCb models

Systematics

Tag-specific systematic uncertainties, in units of 10^{-2}

Source	CP tags	$K_{S,L}\pi\pi$ tags
Efficiency	0.1	0.4
External inputs	0.3	0.8
Peaking backgrounds	0.2	0.3
$K_L^0\pi^0$ ST yield	2.1	N/A
Efficiency factorisation	0.6	N/A
Total	2.2	0.9

Common systematic uncertainties, in units of 10^{-2}

Source	Common systematic
Efficiency reweighting	1.5
K_S^0 veto	0.8