

BESIII Physics & Software Meeting

Measurement of the CP even fraction F_+ in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$

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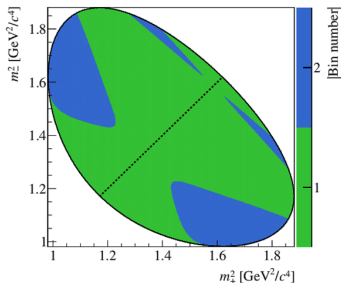
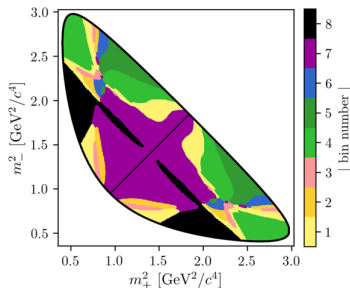
Outline

- 1 Introduction and motivation
- 2 Strategy of strong-phase analysis
- 3 Selection
- 4 Fit of single tag yields
- 5 Fit of double tag yields
 - Fit strategy
 - CP tags
 - $K_{S,L}\pi\pi$ tags
- 6 Reweighting of $KK\pi\pi$ model
- 7 F_+ combination
- 8 Systematics
- 9 Summary and conclusion

- Perform strong-phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$
- This analysis: Phase space integrated analysis
 - 2.93 fb^{-1} $\psi(3770)$ data from 2010-2011
 - Measure CP even fraction F_+
- Future analysis: Binned phase space analysis
 - Expect $\approx 20 \text{ fb}^{-1}$ $\psi(3770)$ data from 2010-2011 and 2022-2023
 - Measure amplitude-averaged cosine and sine of strong-phase c_i and s_i

Introduction to GGSZ analysis of γ

- Main motivation: Measure γ in $B^\pm \rightarrow DK^\pm$ with self-conjugate multi-body D decay
 - Model independent measurement: Bins of D decay phase space
 - External inputs: Measure c_i and s_i at BESIII
 - Poor binning reduces statistical sensitivity \rightarrow No bias!
- J. High Energ. Phys. 2021, 169 (2021): $B^\pm \rightarrow Dh^\pm$, $D \rightarrow K_S^0 h^+ h^-$
 - Single most precise measurement: $\gamma = (68.7_{-5.1}^{+5.2})^\circ$



Introduction to GGSZ analysis of γ

- Our aim: Analyse $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$ in bins of phase space
 - Develop binning scheme using LHCb model [JHEP 02 \(2019\) 126](#)
 - Simultaneously analyse c_i/s_i at BESIII and γ at LHCb
 - Expected precision $\Delta\gamma \approx 12^\circ$ with LHCb Run 1+2

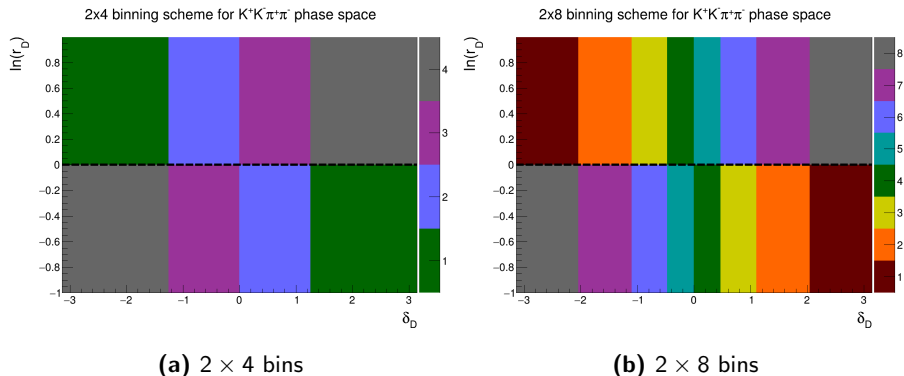
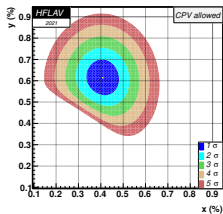


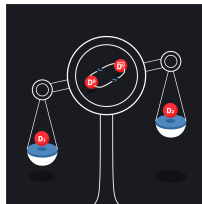
Figure 1: Binning scheme for $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$

Introduction to D mixing and CP-violation

- $D \rightarrow KK\pi\pi$ can also be used to study D -mixing
- Phase space integrated analysis:
 - [Physical Review D, 91\(9\), 2015](#)
 - Measure mixing parameter y_{CP} and CP-violation parameter A_F in self-conjugate multi-body decays
- Analysis in bins of phase space:
 - [Phys. Rev. Lett. 127, 111801 \(2021\)](#)
 - Bin-flip analysis of $D \rightarrow K_S\pi\pi$
 - Measure mixing parameters x, y and CP-violation parameters $|q/p|, \varphi$



(a) Source: [HFLAV](#)



(b) Source: [LHCb outreach](#)

Motivation for F_+ measurement

- F_+ describes the CP content of a self-conjugate multi-body decay
 - $F_+ = 1$ (0) for CP even (odd) final states
- F_+ can be measured with current 3 fb^{-1} dataset
 - First model independent measurement of $F_+^{KK\pi\pi!}$
 - Useful to test agreement with LHCb model prediction: $F_+ = 0.736$
- Important input to quasi-GLW analysis of the CKM angle γ
 - Current GLW modes: $KK, \pi\pi, \pi\pi\pi\pi$
 - Minimal effort to include $KK\pi\pi$ in GLW analyses \implies More statistics
- Other F_+ measurements:
 - $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ [JHEP 01 \(2018\) 144](#)
 - $D^0 \rightarrow K_S\pi^+\pi^-\pi^0$ [JHEP 01 \(2018\) 82](#)
 - $D^0 \rightarrow h^+h^-\pi^0$ [Physics Letters B 747 \(2015\)](#)
 - Measurements are from CLEO-c, BESIII analyses ongoing

Strategy for strong-phase analysis

- 1 Select double tags of $KK\pi\pi$ vs flavour, CP and self-conjugate tags
- 2 Normalise double tag yields by the corresponding single tag yields
- 3 Measure flavour tag yields K_i
- 4 Fit c_i with CP tags and $c_i + s_i$ with self-conjugate tags:

c_i/s_i analysis: Bins of $KK\pi\pi$ phase space

$$\text{CP: } M_i \propto (K_i + K_{-i} - 2c_i\sqrt{K_i K_{-i}}(2F_+^{\text{tag}} - 1))$$

$$\text{Self-conjugate: } M_{ij} \propto (K_i K'_j + K_{-i} K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_{-j}))$$

F_+ analysis: Sum over all $KK\pi\pi$ phase space bins

$$\text{CP: } M \propto (1 - 2(2F_+^{KK\pi\pi} - 1)(2F_+^{\text{tag}} - 1))$$

$$\text{Self-conjugate: } M_j \propto (K'_j + K'_{-j} - 2c'_j\sqrt{K'_j K'_{-j}}(2F_+^{KK\pi\pi} - 1))$$

Selection

- Selection of charged and neutral particles follow standard track and shower requirements
- Require flight significance > 2 for K_S
- K_S veto for $KK\pi\pi$ and $\pi\pi\pi^0$ tags
- ΔE cut of 3σ
- ΔE fit for 4-body modes allows a non-smooth background at $\Delta E = 0$

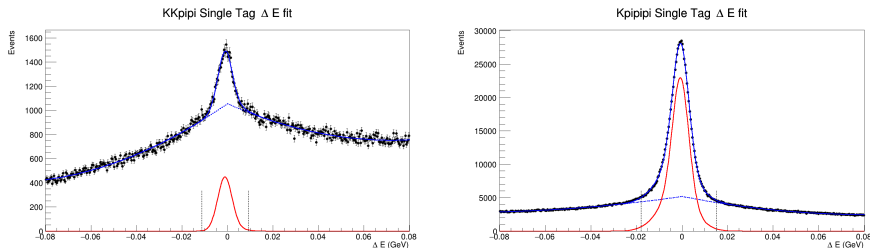
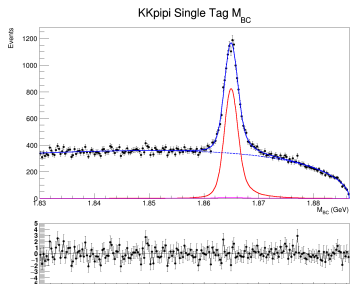


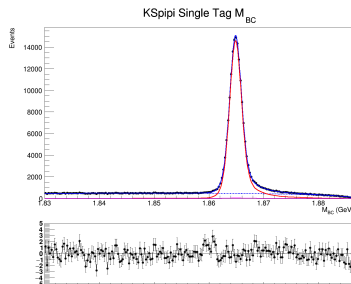
Figure 3: Double Gaussian signal and Chebychev polynomial background

Single tag fits

- Fit strategy: Fit m_{BC}
- Fit model:
 - Signal: PDF from signal MC, convoluted with single or double Gaussian
 - Flat background: Argus PDF
 - Peaking background shape and yield fixed
 - Fit shape to dedicated MC samples
 - Calculate yield from ratios of efficiencies and branching fractions



(a) $KK\pi\pi$



(b) $K_S\pi\pi$

Single tag fits

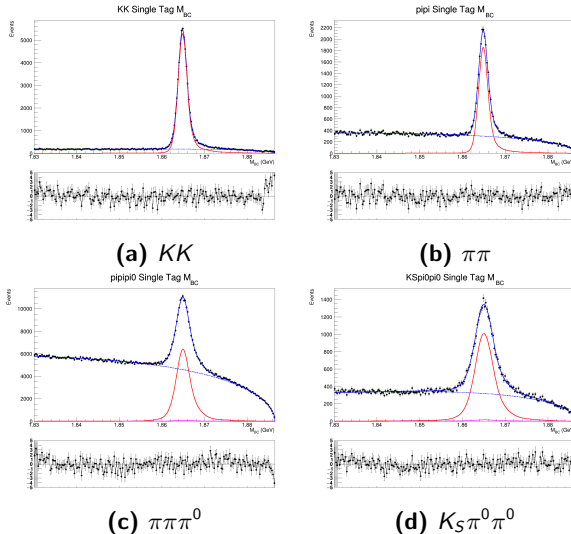
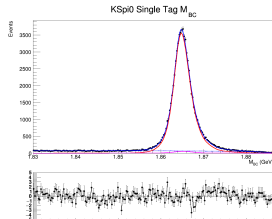
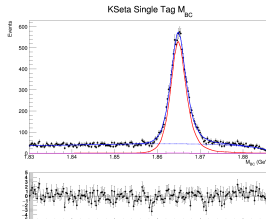


Figure 5: Single tag fits of CP even tags

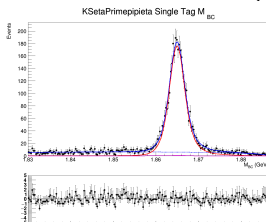
Single tag fits



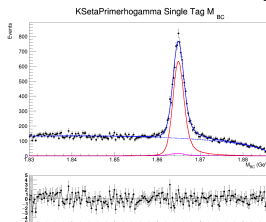
(a) $K_S\pi^0$



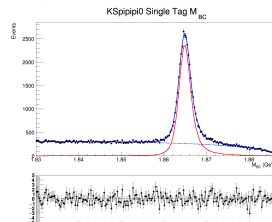
(b) $K_S\eta$



(c) $K_S\eta'(\pi\pi\eta)$



(d) $K_S\eta'(\rho\gamma)$



(e) $K_S\omega$

Figure 6: Single tag fits of CP odd tags

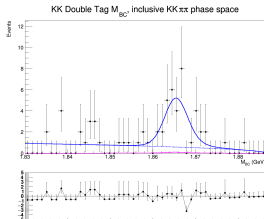
Single tag yields and efficiencies

| Tag mode | Single tag yield | Single tag efficiency (%) |
|-------------------------------|-------------------|---------------------------|
| $K^+ K^- \pi^+ \pi^-$ | 10642 ± 156 | 19.02 ± 0.09 |
| $K^+ K^-$ | 56303 ± 262 | 63.41 ± 0.11 |
| $\pi^+ \pi^-$ | 20386 ± 179 | 67.41 ± 0.10 |
| $K_S \pi^0$ | 67876 ± 278 | 38.18 ± 0.11 |
| $K_S \pi^0 \pi^0$ | 22392 ± 229 | 14.35 ± 0.08 |
| $K_L \pi^0$ | 47595 ± 1653 | 27.83 ± 0.23 |
| $K_S \eta$ | 9308 ± 113 | 31.78 ± 0.10 |
| $K_S \eta'_{\pi\pi\eta}$ | 3213 ± 62 | 12.81 ± 0.07 |
| $K_S \eta'_{\rho\gamma}$ | 8283 ± 116 | 20.80 ± 0.09 |
| $\pi^+ \pi^- \pi^0$ | 107504 ± 602 | 36.65 ± 0.11 |
| $K_S \omega$ | 22068 ± 217 | 14.50 ± 0.08 |
| $K_S \pi^+ \pi^-$ | 161914 ± 440 | 36.40 ± 0.11 |
| $K_S \pi^+ \pi^-$ part. reco. | 161914 ± 440 | 36.40 ± 0.11 |
| $K_L \pi^+ \pi^-$ | 223141 ± 2146 | 46.1 ± 0.3 |

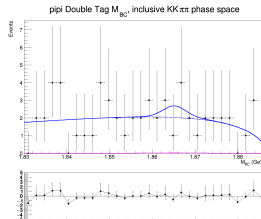
Double tag fits

- Fit strategy:
 - Fully reconstructed tags: Only fit signal m_{BC} because of low statistics
 - Partially reconstructed tags: Fit missing mass squared m_{miss}^2
- Fit model:
 - Signal: PDF from signal MC, convoluted with single Gaussian
 - Background: Argus PDF
 - Peaking backgrounds fixed, with quantum correlation accounted for
 - Simple sideband subtraction for correct signal but wrong tag event
- For tags with multiple bins, perform a simultaneous fit of all bins
 - Shape is floated and shared across all bins
 - Yield of signal and combinatorial background is floated in each bin

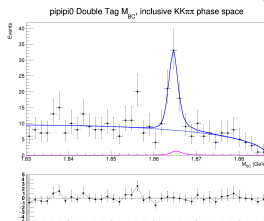
Double tag fits



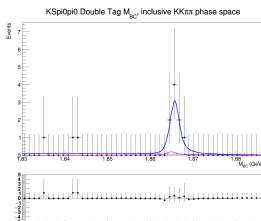
(a) KK



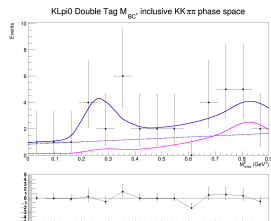
(b) $\pi\pi$



(c) $\pi\pi\pi^0$



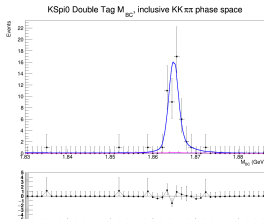
(d) $K_S\pi^0\pi^0$



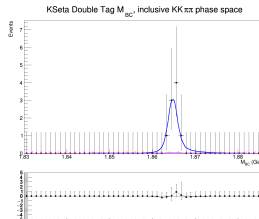
(e) $K_L\pi^0$

Figure 7: Double tag fits of CP even tags

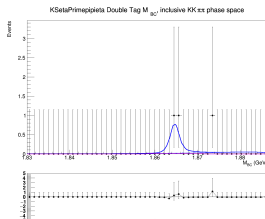
Double tag fits



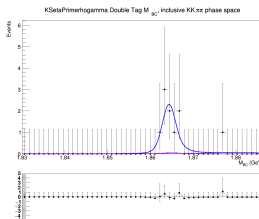
(a) $K_S\pi^0$



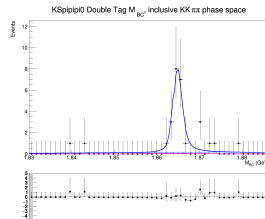
(b) $K_S\eta$



(c) $K_S\eta'(\pi\pi)$



(d) $K_S\eta'(\rho\gamma)$



(e) $K_S\omega$

Figure 8: Double tag fits of CP odd tags

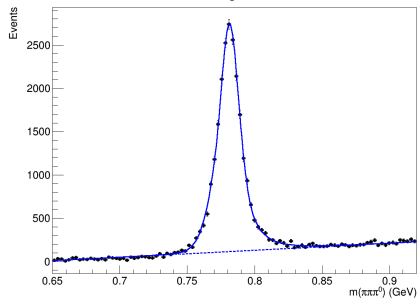
CP double tag yields and efficiencies

| Tag mode | Double tag yield | Double tag efficiency (%) |
|-------------------------|------------------|---------------------------|
| K^+K^- | 28 ± 10 | 14.52 ± 0.06 |
| $\pi^+\pi^-$ | 2 ± 4 | 15.02 ± 0.06 |
| $K_S\pi^0$ | 48 ± 7 | 6.87 ± 0.04 |
| $K_S\pi^0\pi^0$ | 8.0 ± 2.8 | 2.873 ± 0.026 |
| $K_L\pi^0$ | 7 ± 5 | 5.29 ± 0.04 |
| $K_S\eta$ | 8.9 ± 3.0 | 5.72 ± 0.04 |
| $K_S\eta'_{\pi\pi\eta}$ | 2.2 ± 1.6 | 2.024 ± 0.021 |
| $K_S\eta'_{\rho\gamma}$ | 8.7 ± 3.0 | 3.295 ± 0.027 |
| $\pi^+\pi^-\pi^0$ | 53 ± 10 | 7.66 ± 0.04 |
| $K_S\omega$ | 9 ± 3 | 2.234 ± 0.022 |

Non-resonant background in $K_S\omega$

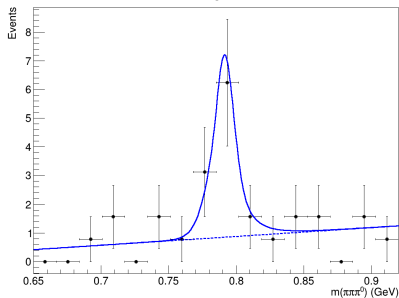
- $K_S\omega$ has CP-even contamination from non-resonant $K_S\pi\pi\pi^0$
 - $F_+(K_S\pi\pi\pi^0) = 0.238 \pm 0.012 \pm 0.012$ from CLEO
- From m_{BC} fit, subtract flat background using sPlot and fit $\pi\pi\pi^0$

Single tag $K_S\omega$, ω mass fit



(a) Single tag

Double tag $K_S\omega$, ω mass fit



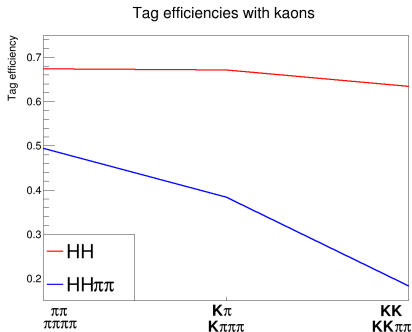
(b) Double tag

F_+ measurement with $K_{S,L}\pi\pi$ tags

- With $K_S\pi\pi$, increase sensitivity through binning of $K_S\pi\pi$ phase space

$$M_j \propto (K'_j + K'_{-j} - 2\sqrt{K'_j K'_{-j}} c'_j (2F_+^{KK\pi\pi} - 1))$$

- Problem: $KK\pi\pi$ reconstruction efficiency is too low \rightarrow Low yields!



- Likely explanation: Softer kaons \rightarrow Kaons get stuck inside tracker

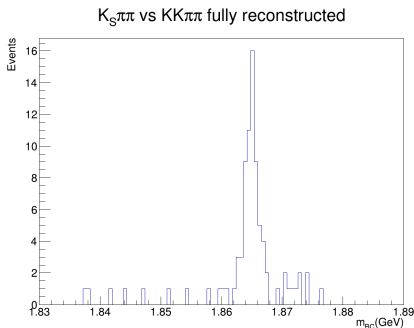
F_+ measurement with $K_{S,L}\pi\pi$ tags

- Solution: Partially reconstructed $KK\pi\pi$
- Strategy:
 - ① Reconstruct $D \rightarrow K_S\pi\pi$
 - ② Require 3 remaining good tracks consistent with $K\pi\pi$
 - ③ Use missing mass to reconstruct missing kaon

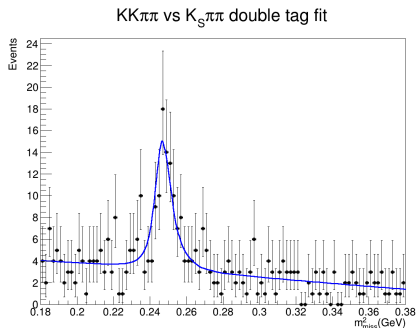
| Mode | Inclusive yield | Double tag efficiency |
|---------------------------------------|-----------------|-----------------------|
| $K_S\pi\pi$ (fully reconstructed) | 69 ± 9 | 6.56 ± 0.04 |
| $K_S\pi\pi$ (partially reconstructed) | 91 ± 15 | 7.01 ± 0.04 |
| $K_L\pi\pi$ (partially reconstructed) | 158 ± 15 | 7.25 ± 0.04 |

Partially reconstructed $KK\pi\pi$ vs $K_S\pi\pi$

- Main challenge with partially reconstructed $KK\pi\pi$: $K\pi\pi\pi\pi^0$
- Require no π^0 candidates



(a) Fully reconstructed



(b) Partially reconstructed

Figure 10: $KK\pi\pi$ vs $K_S\pi\pi$

Double tag fits

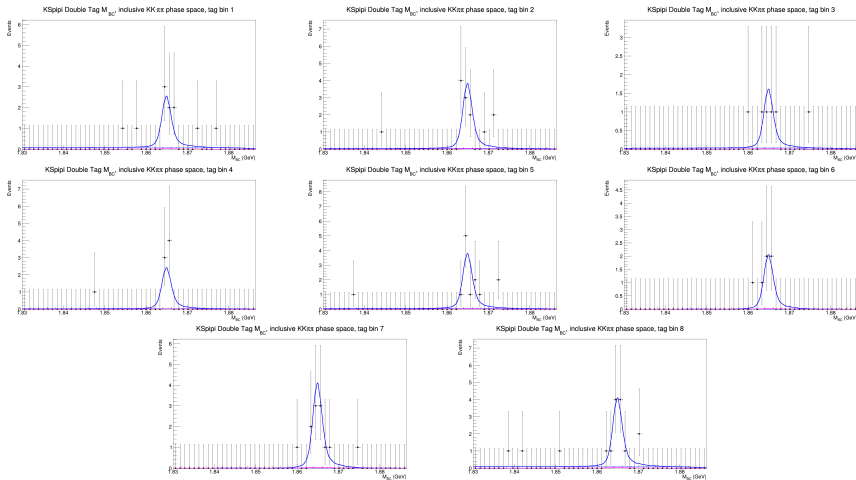
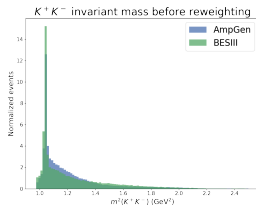


Figure 11: $KK\pi\pi$ vs $K_S\pi\pi$ simultaneous fit

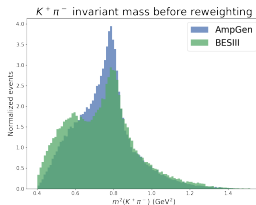
Efficiency corrections

- All yields must be corrected for efficiency
- Problem: BESIII simulation uses a very old $KK\pi\pi$ model in EvtGen
- Solution: Reweight BESIII simulation to look like the LHCb model
 - Use Python `hep_ml` Gradient Boosted Reweighter
 - Variables:
 - 1 $m^2(K^+K^-)$
 - 2 $m^2(K^+\pi^-)$
 - 3 $m^2(K^-\pi^+)$
 - 4 $m^2(\pi^+\pi^-)$
 - 5 $m^2(K^+K^-\pi^+)$

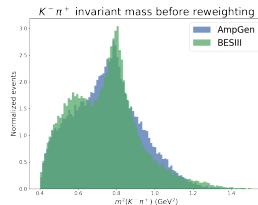
Naive efficiency correction



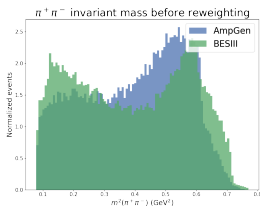
(a) $m^2(K^+ K^-)$



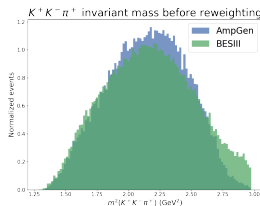
(b) $m^2(K^+ \pi^-)$



(c) $m^2(K^- \pi^+)$



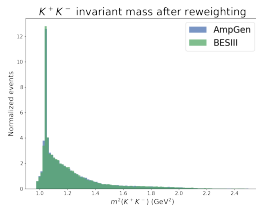
(d) $m^2(\pi^+ \pi^-)$



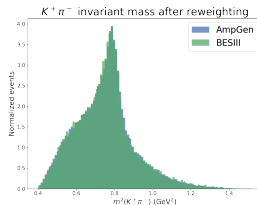
(e) $m^2(K^+ K^- \pi^+)$

Figure 12: Before reweighting

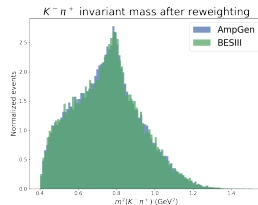
Naive efficiency correction



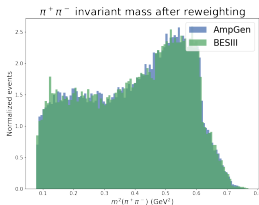
(a) $m^2(K^+ K^-)$



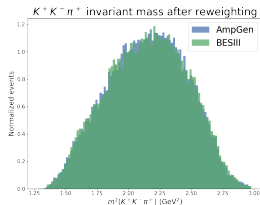
(b) $m^2(K^+ \pi^-)$



(c) $m^2(K^- \pi^+)$



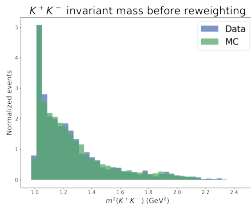
(d) $m^2(\pi^+ \pi^-)$



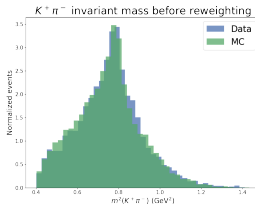
(e) $m^2(K^+ K^- \pi^+)$

Figure 13: After reweighting

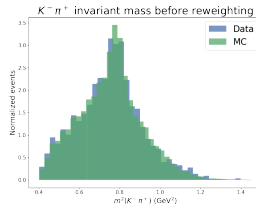
Does the naive reweighting work?



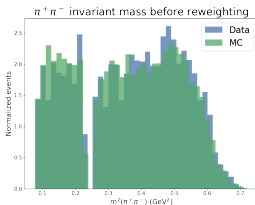
(a) $m^2(K^+K^-)$



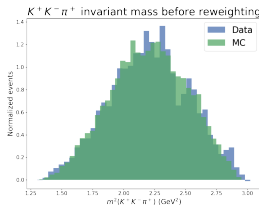
(b) $m^2(K^+\pi^-)$



(c) $m^2(K^-\pi^+)$



(d) $m^2(\pi^+\pi^-)$



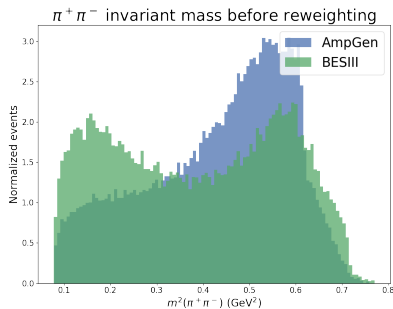
(e) $m^2(K^+K^-\pi^+)$

Figure 14: Single tag $D \rightarrow KK\pi\pi$ in data and MC after reweighting

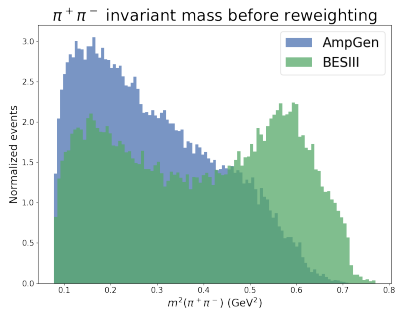
Quantum correlated LHCb model

- Problem with naive reweighting:
 - LHCb model assumes a pure $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ decay
 - No quantum correlations
 - Example: If tag is $D \rightarrow KK$, the $D \rightarrow KK\pi\pi$ decay will be CP odd!
 - Quantum correlations will affect phase space distribution \implies Efficiencies could change
- Solution: Separate reweighters for CP even/odd $D \rightarrow K^+ K^- \pi^+ \pi^-$
 - CP even tags: Use efficiencies after reweighting to CP odd model
 - CP odd tags: Use efficiencies after reweighting to CP even model
 - $K_{S,L}\pi\pi$ tags: Do a weighted average of the two efficiencies

Before weighting to CP even/odd models



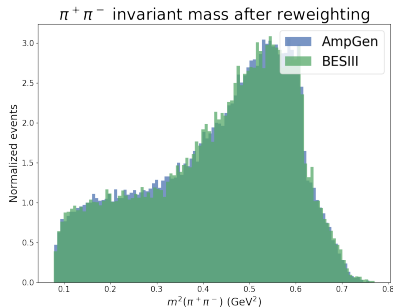
(a) CP even



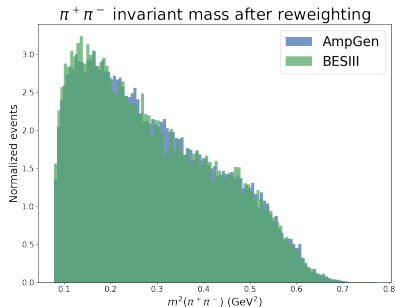
(b) CP odd

Figure 15: $m^2(\pi^+\pi^-)$ before reweighting

After weighting to CP even/odd models



(a) CP even



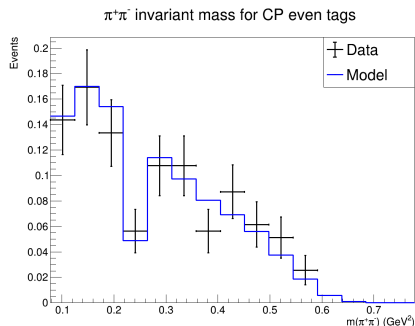
(b) CP odd

Figure 16: $m^2(\pi^+\pi^-)$ after reweighting

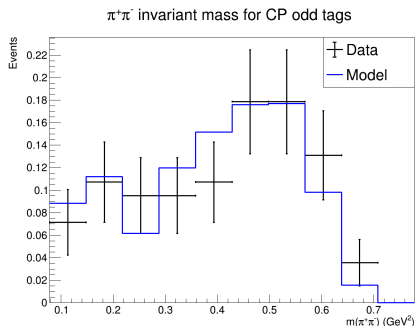
| No reweighting | Naive reweighting | CP even model | CP odd model |
|----------------|-------------------|---------------|--------------|
| 18.0% | 19.0% | 18.1% | 21.9% |

Agreement between quantum correlated data and model

- Note: LHCb model knows nothing about quantum correlations
- D^0/\bar{D}^0 amplitudes simply combined to obtain CP even/odd models
- Important question: Can the model describe quantum correlated double tag data at all? Answer: Yes!



(a) Double tags of $KK\pi\pi$ vs CP even



(b) Double tags of $KK\pi\pi$ vs CP odd

Figure 17: $m^2(\pi^+\pi^-)$ in double tags, compared with CP even/odd LHCb models

F_+ measurement with CP tags

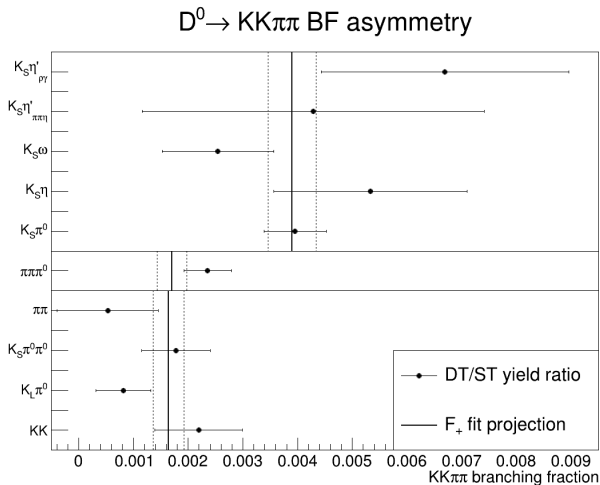
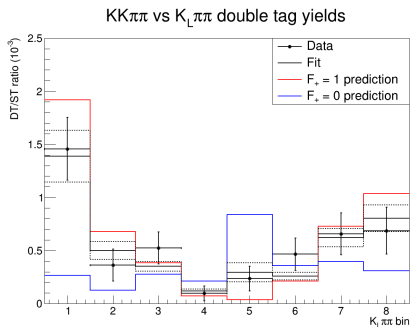
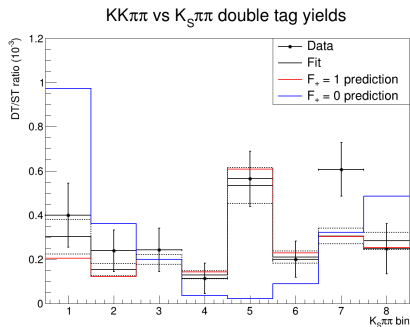


Figure 18: F_+ combination of CP tags
 Fit result: $F_+ = 0.703 \pm 0.042$, $\chi^2 = 1.4$

F_+ measurement with $K_{S,L}\pi\pi$ tags



(a) Result: $F_+ = 0.872 \pm 0.091$, $\chi^2 = 1.3$ **(b)** Result: $F_+ = 0.679 \pm 0.103$, $\chi^2 = 0.8$

Figure 19: F_+ combination of $K_S\pi\pi$ (left) and $K_L\pi\pi$ (right)

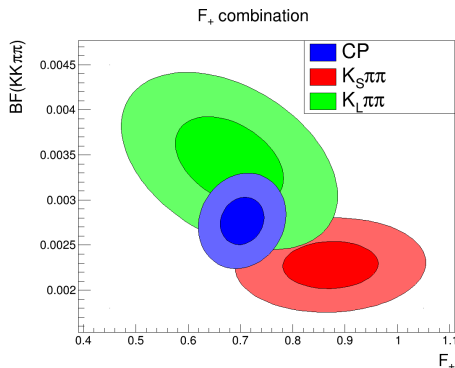


Figure 20: F_+ combination

- Observe large anti-correlation in $K_L\pi\pi$ because $F_+^{K_L\pi\pi} \approx 0.354$
 - Yield of $K_L\pi\pi$ is twice as large as that of $K_S\pi\pi$
 - Fractional bin yields and total yield contains information about F_+
 - When $K_L\pi\pi$ BF is available, combine all tags!

Tag-specific systematic uncertainties, in units of 10^{-2}

| Source | CP tags | $K_{S,L}\pi\pi$ tags |
|--------------------------|---------|----------------------|
| Efficiency | 0.1 | 0.4 |
| External inputs | 0.3 | 0.8 |
| Peaking backgrounds | 0.2 | 0.3 |
| $K_L^0\pi^0$ ST yield | 2.1 | N/A |
| Efficiency factorisation | 0.6 | N/A |
| Total | 2.2 | 0.9 |

Common systematic uncertainties, in units of 10^{-2}

| Source | Common systematic |
|------------------------|-------------------|
| Efficiency reweighting | 1.5 |
| K_S^0 veto | 0.8 |

- First model-independent measurement of CP even fraction in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$
 - Analysis is now ready for review
 - $F_+ = 0.730 \pm 0.040 \pm 0.017$
 - Statistics dominated!
 - Very consistent with model prediction: $F_+ = 0.736$
- Valuable input to:
 - γ measurement with GLW method
 - D -mixing and CPV analyses
- Future $\psi(3770)$ data will allow us to perform a binned analysis

Thank you!

Backup

Tag modes

- Flavour tags:
 - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $Ke\nu$
- CP even tags:
 - KK , $\pi\pi$, $\pi\pi\pi^0$ (mostly CP even), $K_S\pi^0\pi^0$, $K_L\pi^0$
- CP odd tags:
 - $K_S\pi^0$, $K_S\eta$, $K_S\omega$, $K_S\eta'_{\pi\pi\eta}$, $K_S\eta'_{\rho\gamma}$
- Self-conjugate tags:
 - $K_S\pi\pi$, $K_L\pi\pi$

Underlined tags have not been finalized yet

Double tag fits

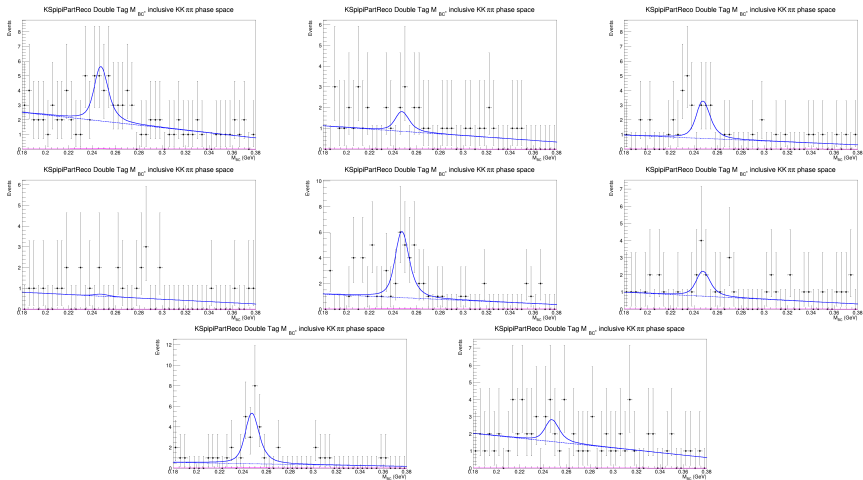


Figure 21: Partially reconstructed $KK\pi\pi$ vs $K_5\pi\pi$ simultaneous fit

Double tag fits

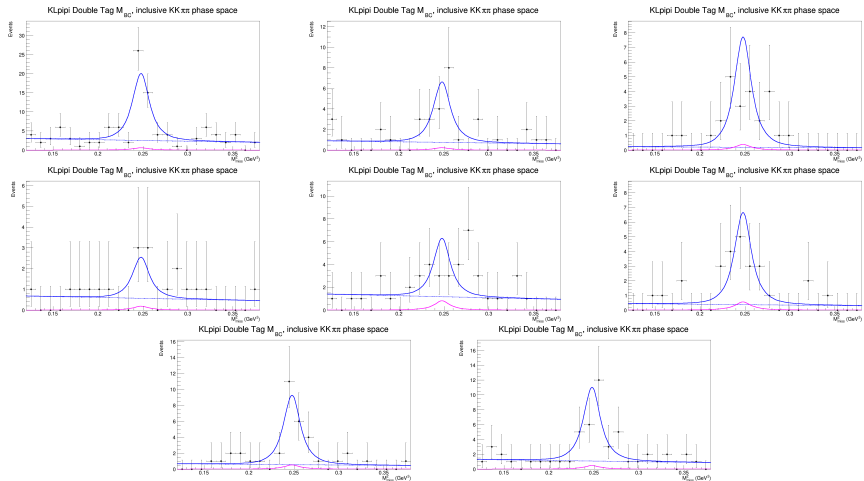


Figure 22: $KK\pi\pi$ vs $K_L\pi\pi$ simultaneous fit

Peaking backgrounds

- Strategy for fixing peaking backgrounds:
 - 1 Generate dedicated MC sample
 - 2 Obtain retention rate of peaking background
 - 3 Fit background with appropriate shape (Gaussian, Crystal Ball, ...)
 - 4 Use BFs from PDG to fix background-to-signal ratio

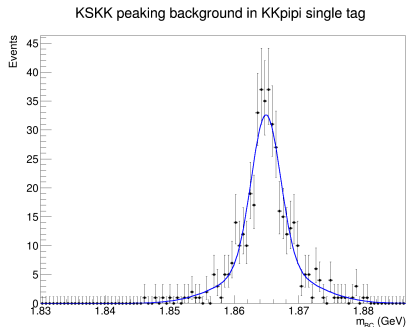


Figure 23: Double Gaussian fit of $K_S KK$ background in $KK\pi\pi$ single tag fit

Quantum correlation in peaking backgrounds

- Strategy for peaking backgrounds with different CP:
 - Correct using $F_+^{KK\pi\pi}$ from LHCb model
- Strategy for $K_S KK$ background in $KK\pi\pi$
 - $F_+^{K_S KK} = 0.524 \pm 0.018$ from [Phys. Rev. D **102**, 052008](#)
 - Use dedicated MC to find retention in each $K_S KK$ bin
 - K_S veto removes more $K_S \phi$ than $K_S a(980)^0 \Rightarrow$ Calculate effective F_+ for $K_S KK$ to $KK\pi\pi$ background