BESIII Charm Meeting
Phase-space binned analysis of strong-phase parameters in $D^0 \to K^+K^-\pi^+\pi^-$

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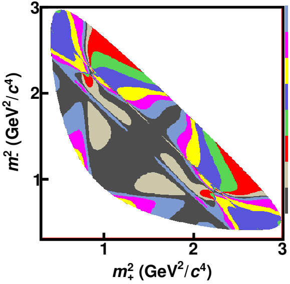
Outline

Introduction

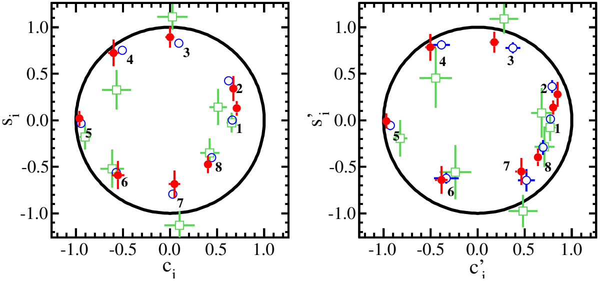
- Aim of analysis: Measure the strong-phase difference between D^0 and $\bar{D}^0 \to K^+K^-\pi^+\pi^-$ using the $8\,\mathrm{fb}^{-1}$ $\psi(3770)$ dataset
- Important input to:
  1. Measurement of the CKM angle γ
  2. Studies of charm mixing and CPV
  3. Strong-phase measurements of other decay modes
- Phase-space integrated measurement: Phys. Rev. D **107** 032009
- Strong phases are diluted when integrated over the 5D phase space $\implies$ Perform analysis in bins
- Binned model-dependent analysis of γ has already been performed at LHCb: arXiv:2301.10328

Introduction - Previous strong-phase analyses

- Golden mode for strong-phase analysis: $D^0 \to K^0_{S,L}\pi^+\pi^-$
- Measurements of c_i and s_i performed in 2×8 bins:
  - CLEO: Phys. Rev. D **82** 112006
  - BESIII: Phys. Rev. D **101** 112002



(a) Equal δ binning scheme

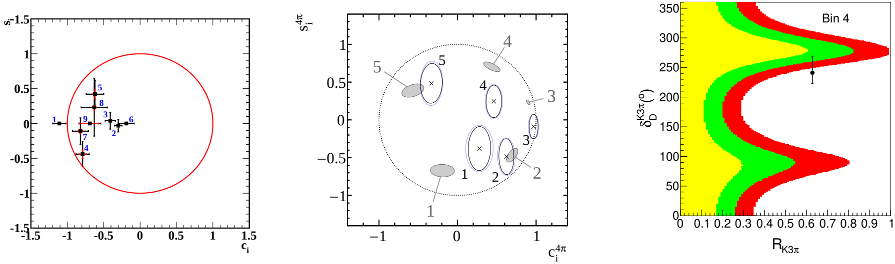


(b) c_i and s_i for (left) $K_S\pi\pi$ and (right) $K_L\pi\pi$

Introduction - Previous strong-phase analyses

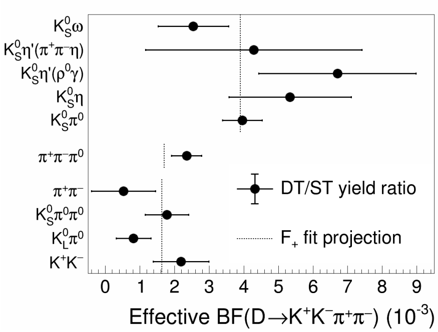
- Four-body decays have a 5D phase space, but their binning scheme may be defined analogously

- Many different strategies for 5D binning schemes:
  - $D \to K_S^0\pi^+\pi^-\pi^0$: JHEP **2018** 82 (2018)
  - $D \to \pi^+\pi^-\pi^+\pi^-$: JHEP **2018** 144 (2018)
  - $D \to K^-\pi^+\pi^-\pi^+$: JHEP **2021** 164 (2021)

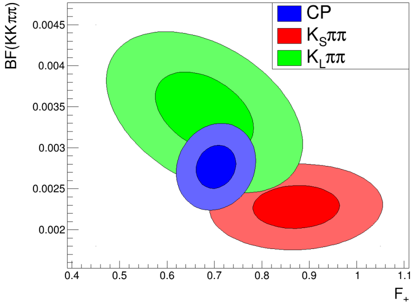


(a) $K_S\pi\pi\pi^0$ **(b)** 4π **(c)** $K\pi\pi\pi$

- The phase-space integrated strong phase of $D \to K^+K^-\pi^+\pi^-$ was previously measured: Phys. Rev. D **107** 032009
- The asymmetry in the branching fraction measured using CP even and CP odd tags is sensitive to the CP-even fraction F_+
  - $c_i = 2F_+ - 1$ is the amplitude-average cosine of the strong phase



(a) BF asymmetry



(b) F_+ combination

Binning scheme

Strong phases are determined from yield asymmetries

- When integrating over a phase-space region, variations in strong phases dilute the asymmetries
- We require a binning scheme to minimise the dilution and enhance asymmetry effects

How to bin a 5-dimensional phase space?

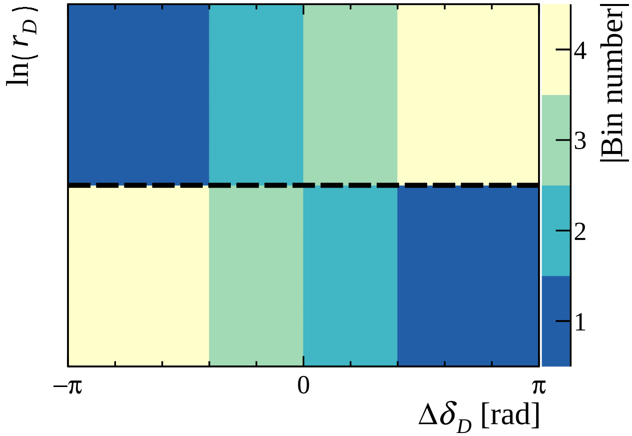
- Generate C++ code for LHCb amplitude model using AmpGen[1]
- For each D event, calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along δ_D and r_D
  - Bin boundaries in δ_D are moved to maximise sensitivity to γ

[1] AmpGen by Tim Evans

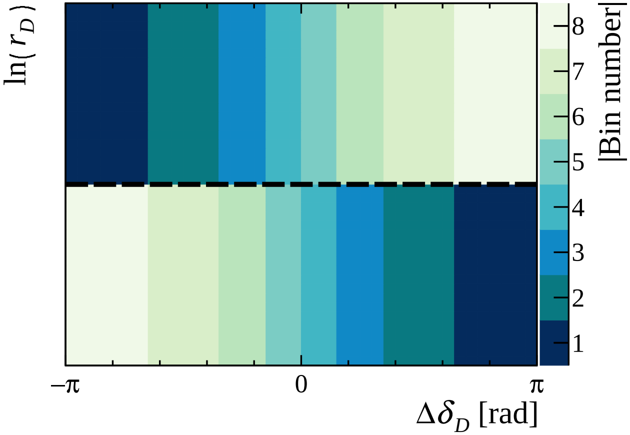
Binning scheme



Bins $i < 0$ on top, $i > 0$ below

Binning scheme used in this analysis

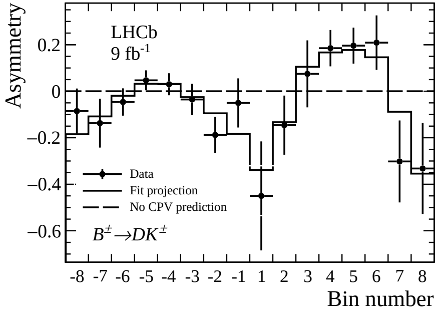
Binning scheme



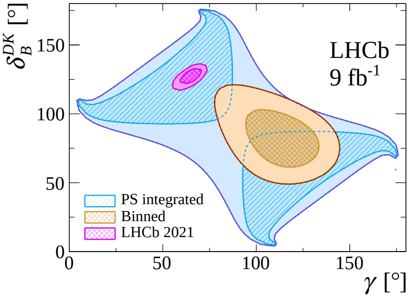
Bins $i < 0$ on top, $i > 0$ below

Possible binning scheme for future analysis with $20\,\mathrm{fb}^{-1}$ dataset

Model-dependent measurement at LHCb



(a) Bin asymmetries in $B^{\pm} \to DK^{\pm}$



(b) Measurement of γ at LHCb

$$\gamma = (116^{+12}_{-14})^{\circ}$$

This result is currently model independent, and needs external inputs from BESIII to become a proper model-independent measurement!

Formalism and measurement strategy

$D\bar{D}$ pair from $\psi(3770)$ is prepared in a $\mathcal{C} = -1$ state

- D mesons are "quantum correlated"
- The decay of $D \to K^+K^-\pi^+\pi^-$ is correlated by the CP content of the tag mode

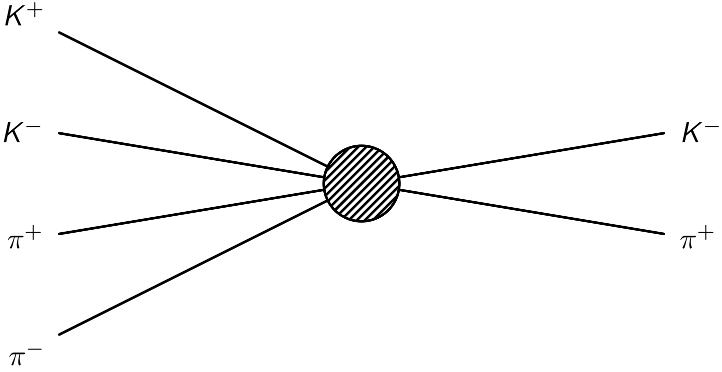
$$|D\bar{D}\rangle = \frac{1}{\sqrt{2}}\big(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle\big)$$

- Equivalently, in terms of CP eigenstates:

$$|D\bar{D}\rangle = \frac{1}{\sqrt{2}}\big(|D_-\rangle|D_+\rangle - |D_+\rangle|D_-\rangle\big)$$

Formalism and measurement strategy

- Tag mode can be a flavour tag
  - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $Ke\nu$

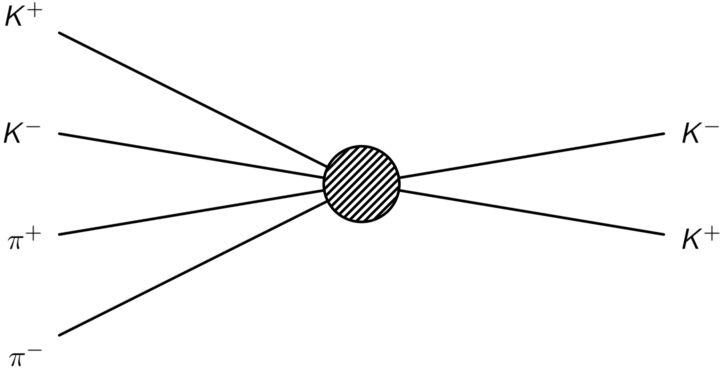


Use flavour tags to measure fraction of $D^0 \to KK\pi\pi$ decays in bin i:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times K_i$$

Formalism and measurement strategy

- Tag mode can be a CP even tag
  - KK (fully and part. reco $KK\pi\pi$), $\pi\pi$, $\pi\pi\pi^0$, $K_S\pi^0\pi^0$, $K_L\pi^0$

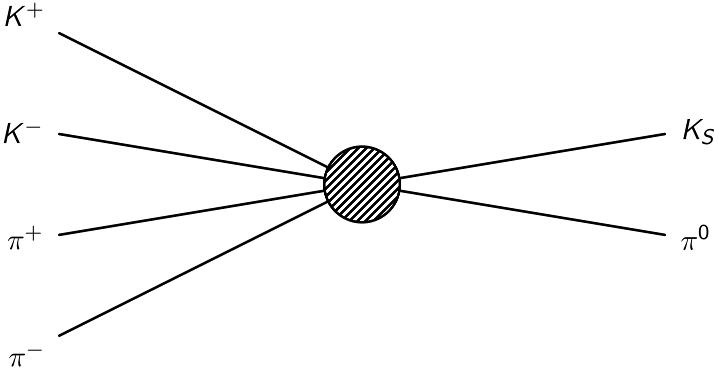


$D \to K^+K^-$, which is CP even, forces $D \to K^+K^-\pi^+\pi^-$ to be CP odd:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times (K_i + K_{-i} - 2\sqrt{K_i K_{-i}} c_i)$$

Formalism and measurement strategy

- Tag mode can be a CP odd tag
  - $K_S\pi^0$ (fully and part.reco $KK\pi\pi$), $K_S\eta$, $K_S\eta'(\pi\pi\eta,\pi\pi\gamma)$, $K_S\pi\pi\pi^0$

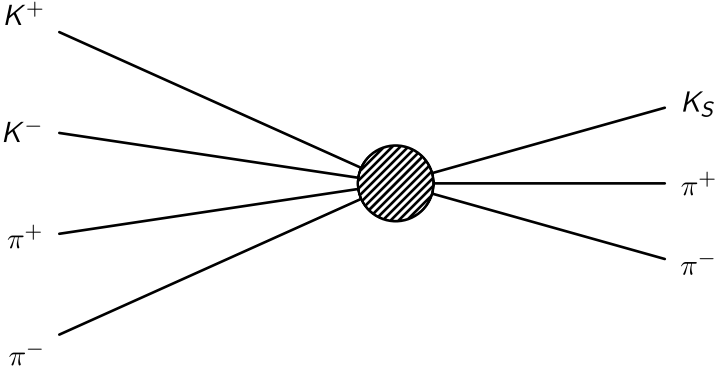


$D \to K^0_S\pi^0$, which is CP odd, forces $D \to K^+K^-\pi^+\pi^-$ to be CP even:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times \left(K_i + K_{-i} + 2\sqrt{K_iK_{-i}}c_i\right)$$

Formalism and measurement strategy

- Tag mode can be a self-conjugate multi-body tag
  - $K_S\pi\pi$ (fully and part.reco $KK\pi\pi$), $K_L\pi\pi$

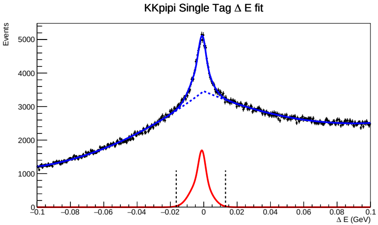


$D \to K_S^0\pi^+\pi^-$ has different strong phases in different bins of phase space:

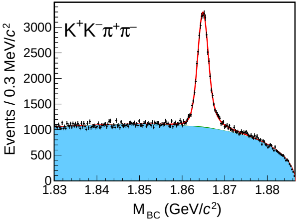
$$\frac{N_{ij}^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times \left(K_i K'_{-j} + K_{-i}K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)\right)$$

Selection

- Double-tag analysis: Select $D \to KK\pi\pi$ events tagged with flavour, CP and multi-body tags
- Selection more or less identical to previous double-tag analyses
- $D \to KK\pi\pi$ selection:
  - 4 good charged tracks
  - 3σ window around signal peak in ΔE
  - Asymmetric $K_S KK$ veto for $m(\pi^+\pi^-) \in [477, 507]$ MeV
  - Flight significance cut at 2



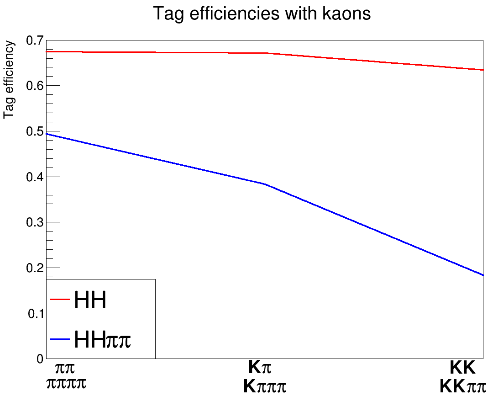
(a) ΔE window



(b) Single tag yield: $29\,227 \pm 268$

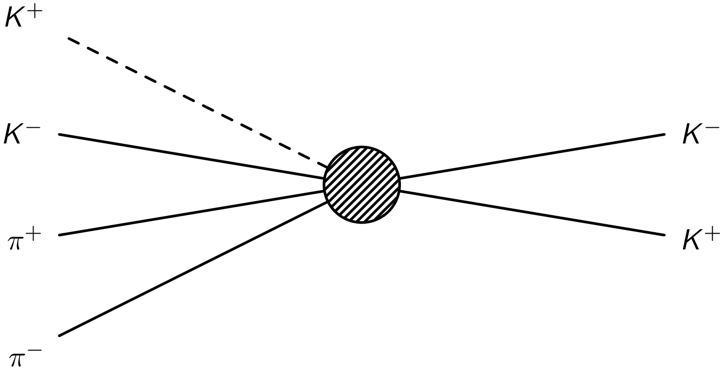
Partially reconstructed $D^0 \to K^+K^-\pi^+\pi^-$

- The reconstruction efficiency of $D \to KK\pi\pi$ is less than 20%
- For comparison, the efficiency of $D \to \pi\pi\pi\pi$ is around 50%!
- Poor kaon tracking efficiency at low momentum



Partially reconstructed $D^0 \to K^+K^-\pi^+\pi^-$

- Solution: Only reconstruct 3 of the charged D daughters
  - Presence of missing kaon is inferred from the missing momentum
  - Yields are similar to fully reconstructed sample
  - Large, but non-peaking background from $D \to K\pi\pi\pi\pi^0$

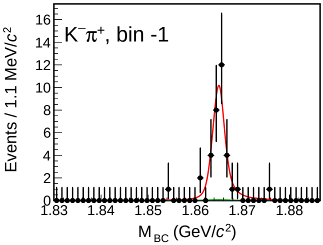


Use this technique with the K^+K^-, $K_S^0\pi^0$ and $K_S^0\pi^+\pi^-$ tags

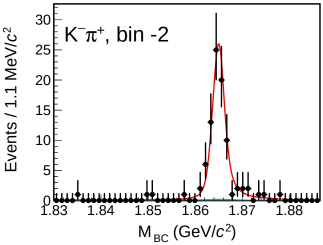
Double tag fits

- Fit strategy: Only fit signal side m_{BC} because of low statistics
- Fit model:
  - Signal: PDF from signal MC, convolved with a Gaussian
  - Flatackground: Argus PDF
  - Peaking background: Shape and efficiency from MC, correct for quantum correlation
  - Simple sideband subtraction for correct signal but wrong tag event
- Fit all bins simultaneously
  - Shape is floated and shared across all bins
  - Yield of signal and combinatorial background is floated in each bin

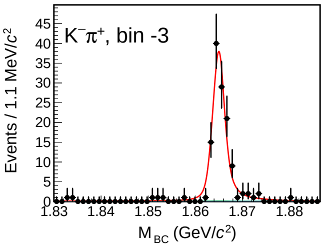
Double tag fit of $KK\pi\pi$ vs $K\pi$



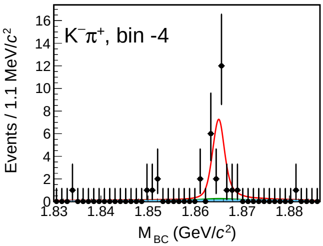
(a) Bin -1 yield: $32.4^{+6.2}_{-5.5}$



(b) Bin -2 yield: $82.7^{+9.7}_{-9.0}$

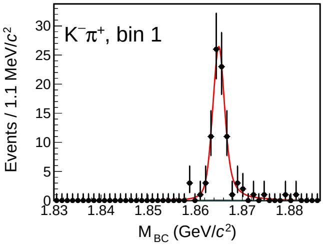


(c) Bin -3 yield: $120.3^{+11.6}_{-10.9}$

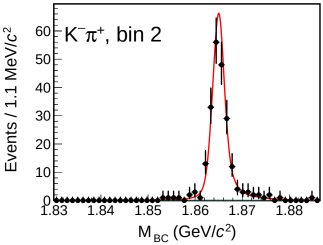


(d) Bin -4 yield: $22.4^{+5.2}_{-4.6}$

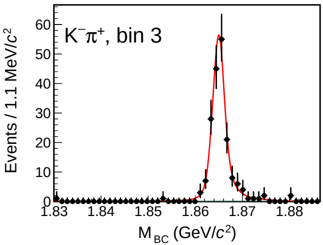
Double tag fit of $KK\pi\pi$ vs $K\pi$



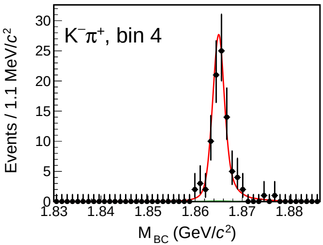
(a) Bin 1 yield: $84.5^{+9.8}_{-9.1}$



(b) Bin 2 yield: $211.2^{+15.4}_{-14.8}$

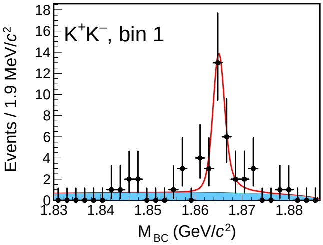


(c) Bin 3 yield: $181.0^{+14.0}_{-13.3}$

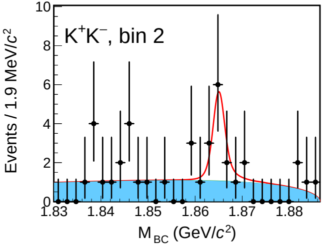


(d) Bin 4 yield: $88.6^{+9.7}_{-9.0}$

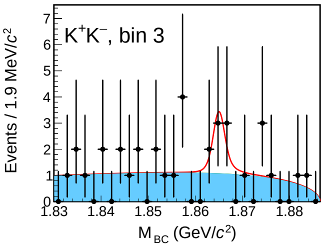
Double tag fit of $KK\pi\pi$ vs KK



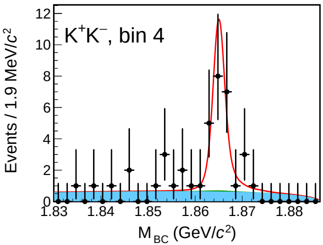
(a) Bin 1 yield: $25.3^{+6.2}_{-5.5}$



(b) Bin 2 yield: $8.8^{+4.0}_{-3.3}$

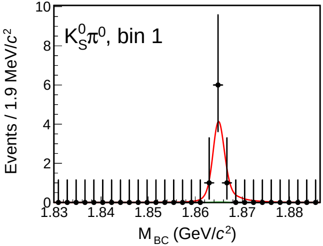


(c) Bin 3 yield: $4.5^{+3.3}_{-2.6}$

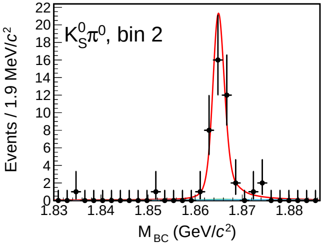


(d) Bin 4 yield: $21.1^{+5.5}_{-4.8}$

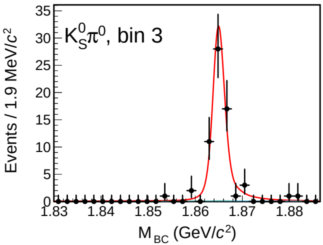
Double tag fit of $KK\pi\pi$ vs $K_S\pi^0$



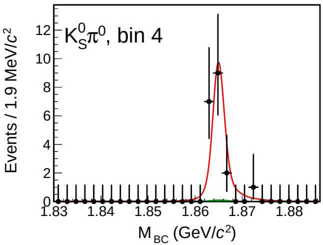
(a) Bin 1 yield: $7.9^{+3.1}_{-2.5}$



(b) Bin 2 yield: $40.4^{+6.8}_{-6.3}$

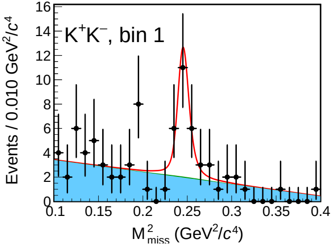


(c) Bin 3 yield: $61.1^{+8.3}_{-7.8}$

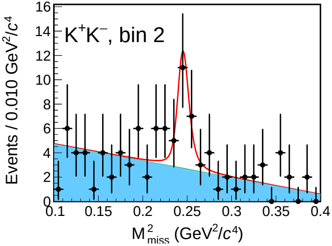


(d) Bin 4 yield: $18.3^{+4.5}_{-3.9}$

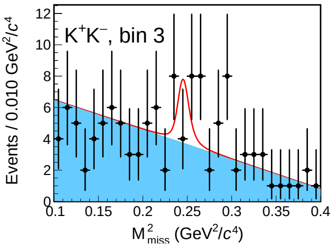
Double tag fit of partially reconstructed $KK\pi\pi$ vs KK



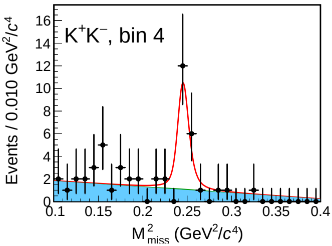
(a) Bin 1 yield: $19.6^{+5.8}_{-5.1}$



(b) Bin 2 yield: $17.7^{+6.1}_{-5.3}$

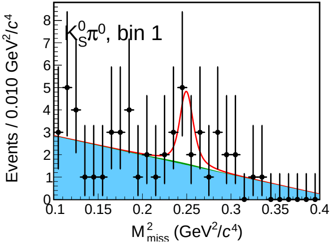


(c) Bin 3 yield: $7.5^{+6.6}_{-5.7}$

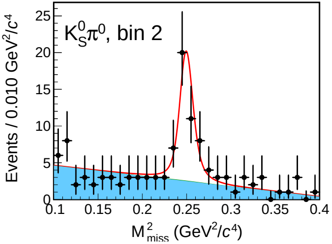


(d) Bin 4 yield: $17.3^{+5.1}_{-4.4}$

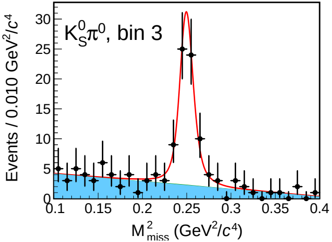
Double tag fit of partially reconstructed $KK\pi\pi$ vs $K_S\pi^0$



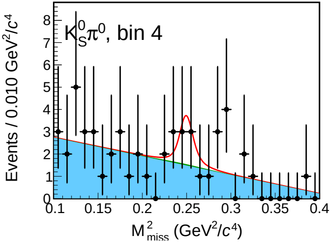
(a) Bin 1 yield: $7.2^{+4.3}_{-3.5}$



(b) Bin 2 yield: $39.3^{+8.3}_{-7.3}$



(c) Bin 3 yield: $64.4^{+9.6}_{-9.0}$



(d) Bin 4 yield: $4.8^{+3.8}_{-3.0}$

Likelihood fit

How to put this all together to determine c_i and s_i?

1. In the past, the strategy has been to measure normalised K_i first, using flavour tags:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i, \quad \sum_i K_i = 1$$

2. However, the K_i must be corrected for efficiencies and bin migrations, in addition to Doubly Cabbibo Suppressed decays:

$$f_i = \left(1 + r_D^2 \frac{K_{-i}}{K_i} - 2\sqrt{\frac{K_{-i}}{K_i}}(c_i \cos(\delta_D) + s_i \sin(\delta_D))\right)^{-1}$$

3. Finally, with the K_i fixed, c_i and s_i are determined from CP and multi-body tags:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i + K_{-i} \mp 2\sqrt{K_i K_{-i}} c_i$$

$$\frac{N_{ij}^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i K'_{-j} + K_{-i} K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)$$

Likelihood fit

We propose a simpler, but perhaps powerful strategy:

1. Treat all K_i, c_i and s_i as free parameters
2. Include the $D \to KK\pi\pi$ branching fraction $\mathcal{B}$ as a free parameter
3. Fit flavour, CP and multi-body tags simultaneously

Master equations:

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + K_j - 2\sqrt{K_j K_{-j}}(c_j \cos(\delta_D) + s_j \sin(\delta_D))\Big)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\big(K_k K'_{-l} + K_{-k} K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\big)$$

ϵ_{ij} are combined efficiency and bin migration matrices

The master equations use the free parameters $\mathcal{B}$, K_i, c_i and s_i to make a prediction $\hat{N}^{\mathrm{DT}}$ to the measured double tag yields N^{DT}

Likelihood fit

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + K_j - 2\sqrt{K_j K_{-j}}\big(c_j\cos(\delta_D) + s_j\sin(\delta_D)\big)\Big)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\big(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\big)$$

Ordinarily, we would construct a Gaussian (log)likelihood function $\Longrightarrow$ Obtain $\mathcal{B}$, K_i, c_i and s_i by minimising the following function:

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j$$

[1] ρ are correlation coefficients

Likelihood fit

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + K_j - 2\sqrt{K_j K_{-j}}(c_j\cos(\delta_D) + s_j\sin(\delta_D))\Big)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l))$$

Our DT yields are very small, so their uncertainties are asymmetric $\Longrightarrow$ Approximate covariance matrix from the asymmetric uncertainties[2]:

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j, \quad \sigma_i = \sqrt{\sigma_-\sigma_+ - (\sigma_+ - \sigma_-)(N_i^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}$$

[2] arXiv:physics/0406120

Likelihood fit

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j, \quad \sigma_i = \sqrt{\sigma_-\sigma_+ - (\sigma_+ - \sigma_-)(N_i^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}$$

- The above likelihood has good coverage for flavour and *CP* tags...
- ... but not for multi-body decays
  - Bins with $\sigma_- \approx 0$ make the fit unstable
  - Fit convergence was found to be less than 60%
- In multi-body decays, use the full unbinned likelihood directly
  - Fit convergence improves to over 99%
  - Much slower, but much more accurate

Parameterisation of K_i

One last technical detail...

- There are 8 K_i parameters, but since $\sum_i K_i = 1$, only 7 are independent
- We could set $K_4 = 1 - \sum_{i \neq 4} K_i$, but such a parameterisation is unstable
- Use a recursive fraction parameterisation (JHEP **2021** 169 (2021))
- Recursive fractions R_i are defined as:

$$R_i \equiv \begin{cases} K_i, & i = -4 \\ K_i / \sum_{j \geq i} K_j, & -4 < i < +4 \\ 1, & i = +4. \end{cases}$$

Parameterisation of K_i

Visualisation of this parameterisation:

1. Definition of c_1'

$$\underbrace{c_1}_{\equiv c_1'} + \underbrace{c_2 + c_3 + \ldots}_{\equiv 1 - c_1'} = 1$$

2. Eliminate c_1

$$c_2 + c_3 + c_4 + \ldots = 1 - c_1'$$

3. Definition of c_2'

$$\underbrace{\frac{c_2}{1 - c_1'}}_{\equiv c_2'} + \underbrace{\frac{c_3 + c_4 + \ldots}{1 - c_1'}}_{\equiv 1 - c_2'} = 1$$

Repeat this procedure until the last coefficient, which is 1

Fit results

Branching fraction measurement

Summary

- A phase-space binned measurement of the $D^0 \to K^+K^-\pi^+\pi^-$ strong-phases has been performed
  - Analysis uses a 2×4 binning scheme
  - Measurement will improve significantly with full $20\,\mathrm{fb}^{-1}$ dataset
  - New strategy: Treat c_i, s_i and K_i on an equal footing
  - Additionally, $\delta_D^{K\pi}$ can also be determined
  - The $D^0 \to K^+K^-\pi^+\pi^-$ branching fraction is almost 3 times more precise than the current PDG value
- Next steps:
  1. Perform systematics studies
  2. Write up MEMO

Thank you!

Backup

Tag modes

- Flavour tags:
  - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $Ke\nu$
- CP even tags:
  - $\underline{KK}$, $\pi\pi$, $\underline{\pi\pi\pi^{0}}^{\ddagger}$, $K_S\pi^0\pi^0$, $K_L\pi^0$
- CP odd tags:
  - $\underline{K_S\pi^0}$, $K_S\eta$, $K_S\pi\pi\pi^{0\S}$, $K_S\eta'_{\pi\pi\eta}$, $K_S\eta'_{\rho\gamma}$
- Self-conjugate tags:
  - $\underline{K_S\pi\pi}$, $K_L\pi\pi$

Underlined tags are also reconstructed with a missing kaon in $KK\pi\pi$

‡Mostly CP even

§Predominantly CP odd