BESIII Charm Meeting
Phase-space binned analysis of strong-phase parameters in $D^0 \to K^+K^-\pi^+\pi^-$

Martin Tat Guy Wilkinson Sneha Malde

University of Oxford

23rd May 2023

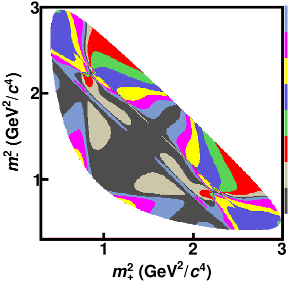
Outline

Introduction

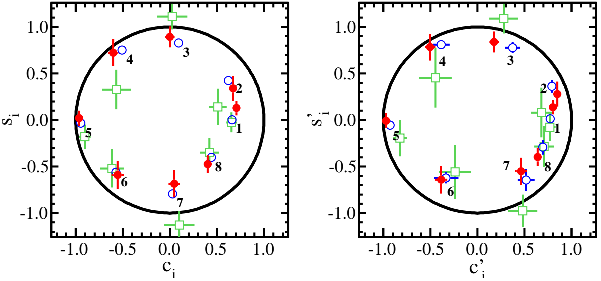
- Aim of analysis: Measure the strong-phase difference between D^0 and $\bar{D}^0 \to K^+K^-\pi^+\pi^-$ using the $8\,\text{fb}^{-1}$ $\psi(3770)$ dataset
- Important input to:
  1. Measurement of the CKM angle γ
  2. Studies of charm mixing and CPV
  3. Strong-phase measurements of other decay modes
- Phase-space integrated measurement: Phys. Rev. D **107** 032009
- Strong phases are diluted when integrated over the 5D phase space $\Longrightarrow$ Perform analysis in bins
- Binned model-dependent analysis of γ has already been performed at LHCb: arXiv:2301.10328

Introduction - Previous strong-phase analyses

- Golden mode for strong-phase analysis: $D^0 \to K^0_{S,L}\pi^+\pi^-$
- Measurements of c_i and s_i performed in 2×8 bins:
  - CLEO: Phys. Rev. D **82** 112006
  - BESIII: Phys. Rev. D **101** 112002

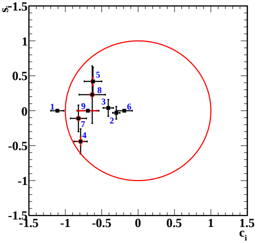


(a) Equal δ binning scheme

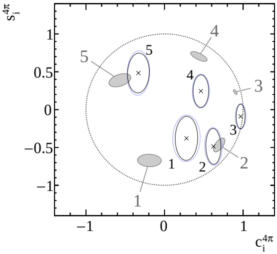


(b) c_i and s_i for (left) $K_S\pi\pi$ and (right) $K_L\pi\pi$

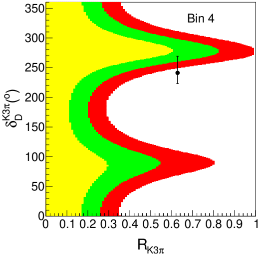
- Four-body decays have a 5D phase space, but their binning scheme may be defined analogously
- Many different strategies for 5D binning schemes:
  - $D \to K_S^0\pi^+\pi^-\pi^0$: JHEP **2018** 82 (2018)
  - $D \to \pi^+\pi^-\pi^+\pi^-$: JHEP **2018** 144 (2018)
  - $D \to K^-\pi^+\pi^-\pi^+$: JHEP **2021** 164 (2021)



(a) $K_S\pi\pi\pi^0$

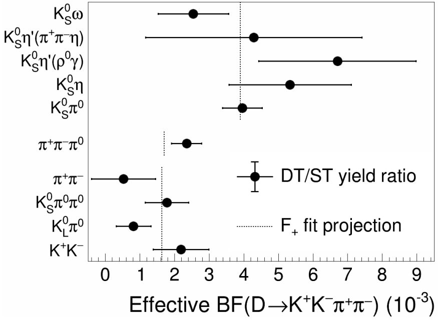


(b) 4π

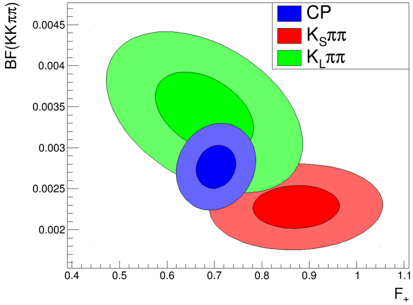


(c) $K\pi\pi\pi$

- The phase-space integrated strong phase of $D \to K^+K^-\pi^+\pi^-$ was previously measured: Phys. Rev. D **107** 032009
- The asymmetry in the branching fraction measured using CP even and CP odd tags is sensitive to the CP-even fraction F_+
  - $c_i = 2F_+ - 1$ is the amplitude-average cosine of the strong phase



(a) BF asymmetry



(b) F_+ combination

Binning scheme

Strong phases are determined from yield asymmetries

- When integrating over a phase-space region, variations in strong phases dilute the asymmetries
- We require a binning scheme to minimise the dilution and enhance asymmetry effects

How to bin a 5-dimensional phase space?

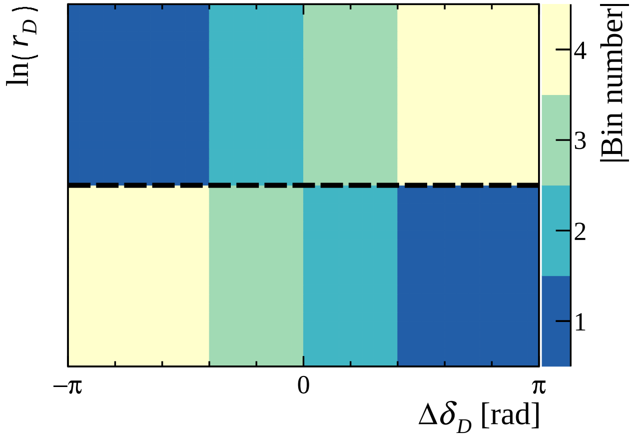
- Generate C++ code for LHCb amplitude model using AmpGen[1]
- For each D event, calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- Bin along δ_D and r_D
  - Bin boundaries in δ_D are moved to maximise sensitivity to γ

[1] AmpGen by Tim Evans

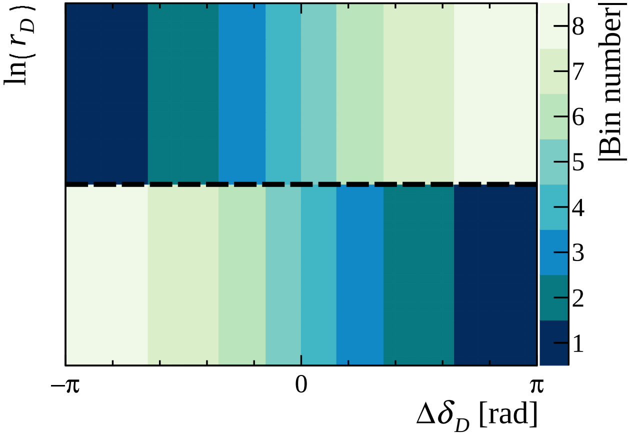
Binning scheme



Bins $i < 0$ on top, $i > 0$ below

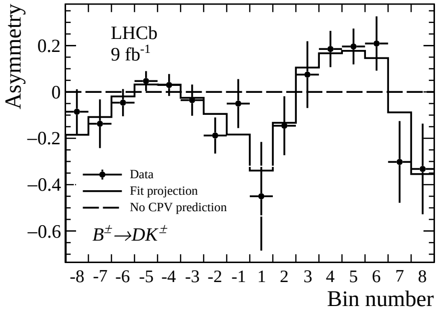
Binning scheme used in this analysis

Binning scheme

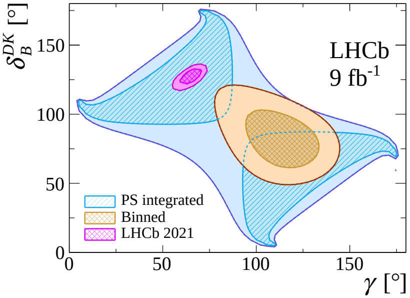


Bins $i < 0$ on top, $i > 0$ below

Possible binning scheme for future analysis with $20\,\text{fb}^{-1}$ dataset



(a) Bin asymmetries in $B^{\pm} \to DK^{\pm}$



(b) Measurement of γ at LHCb

$$\gamma = (116^{+12}_{-14})^{\circ}$$

This result is currently model independent, and needs external inputs from BESIII to become a proper model-independent measurement!

Formalism and measurement strategy

$D\bar{D}$ pair from $\psi(3770)$ is prepared in a $\mathcal{C} = -1$ state

- D mesons are "quantum correlated"
- The decay of $D \to K^+K^-\pi^+\pi^-$ is correlated by the CP content of the tag mode

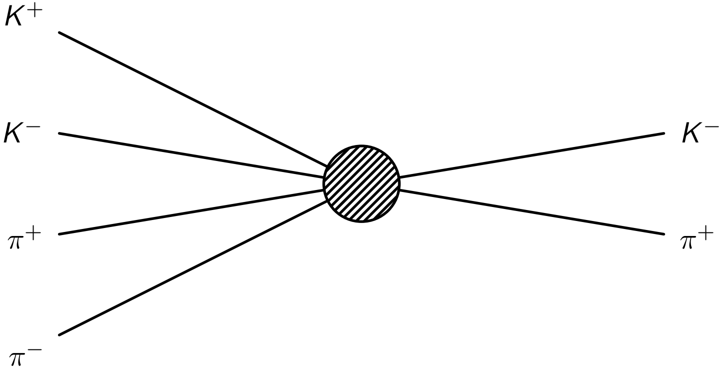
$$|D\bar{D}\rangle = \frac{1}{\sqrt{2}}\big(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle\big)$$

- Equivalently, in terms of CP eigenstates:

$$|D\bar{D}\rangle = \frac{1}{\sqrt{2}}\big(|D_-\rangle|D_+\rangle - |D_+\rangle|D_-\rangle\big)$$

Formalism and measurement strategy

- Tag mode can be a flavour tag
  - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $Ke\nu$

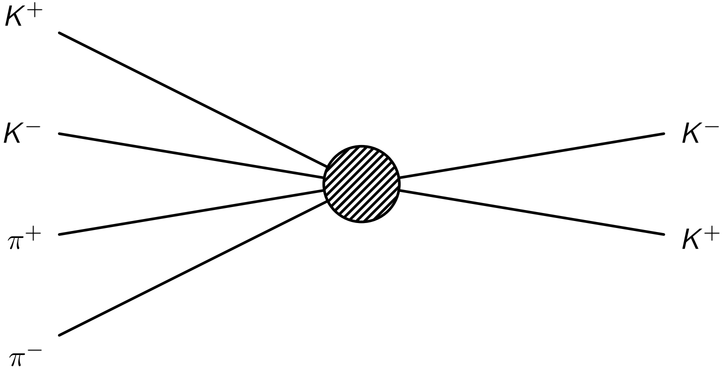


Use flavour tags to measure fraction of $D^0 \to KK\pi\pi$ decays in bin i:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times K_i$$

Formalism and measurement strategy

- Tag mode can be a CP even tag
  - KK (fully and part. reco $KK\pi\pi$), $\pi\pi$, $\pi\pi\pi^0$, $K_S\pi^0\pi^0$, $K_L\pi^0$

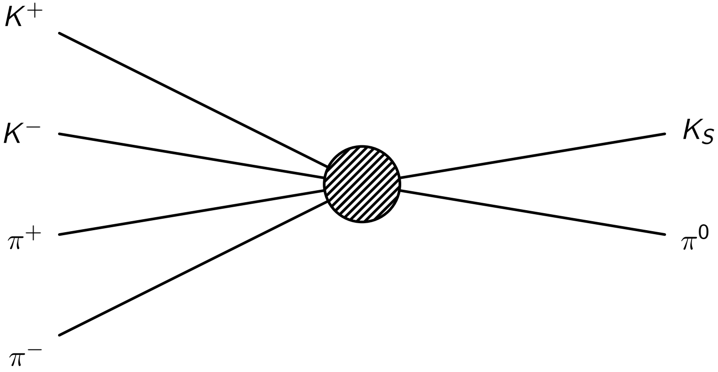


$D \to K^+K^-$, which is CP even, forces $D \to K^+K^-\pi^+\pi^-$ to be CP odd:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times (K_i + K_{-i} - 2\sqrt{K_i K_{-i}} c_i)$$

Formalism and measurement strategy

- Tag mode can be a CP odd tag
  - $K_S\pi^0$ (fully and part.reco $KK\pi\pi$), $K_S\eta$, $K_S\eta'(\pi\pi\eta, \pi\pi\gamma)$, $K_S\pi\pi\pi^0$

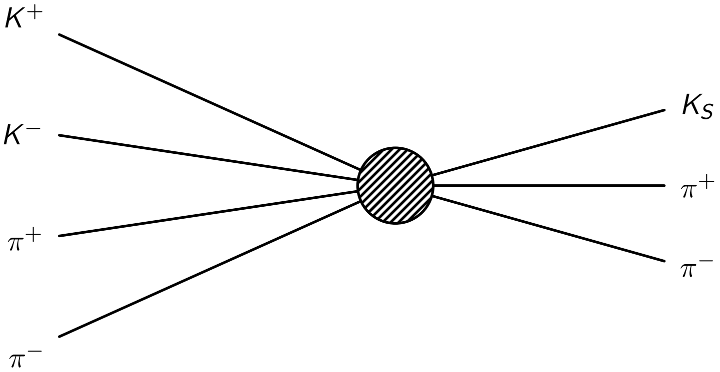


$D \to K_S^0\pi^0$, which is CP odd, forces $D \to K^+K^-\pi^+\pi^-$ to be CP even:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B} \times \left(K_i + K_{-i} + 2\sqrt{K_i K_{-i}}c_i\right)$$

Formalism and measurement strategy

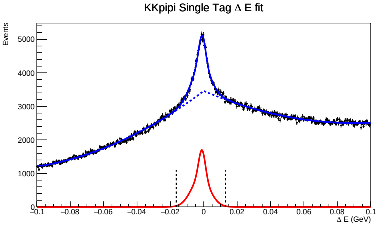
- Tag mode can be a self-conjugate multi-body tag
  - $K_S\pi\pi$ (fully and part.reco $KK\pi\pi$), $K_L\pi\pi$



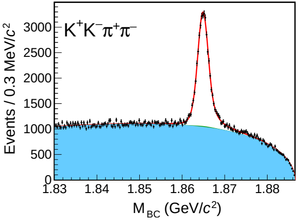
$D \to K^0_S\pi^+\pi^-$ has different strong phases in different bins of phase space:

$$\frac{N^{\mathrm{DT}}_{ij}}{N^{\mathrm{ST}}} = \mathcal{B} \times \left(K_i K'_{-j} + K_{-i} K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)\right)$$

- Double-tag analysis: Select $D \to KK\pi\pi$ events tagged with flavour, CP and multi-body tags
- Selection more or less identical to previous double-tag analyses
- $D \to KK\pi\pi$ selection:
  - 4 good charged tracks
  - 3σ window around signal peak in ΔE
  - Asymmetric $K_S KK$ veto for $m(\pi^+\pi^-) \in [477, 507]$ MeV
  - Flight significance cut at 2



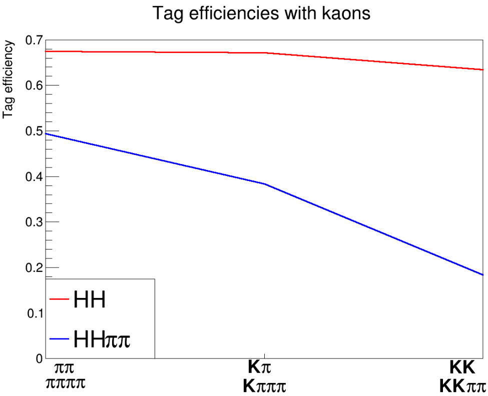
(a) ΔE window



(b) Single tag yield: $29\,227 \pm 268$

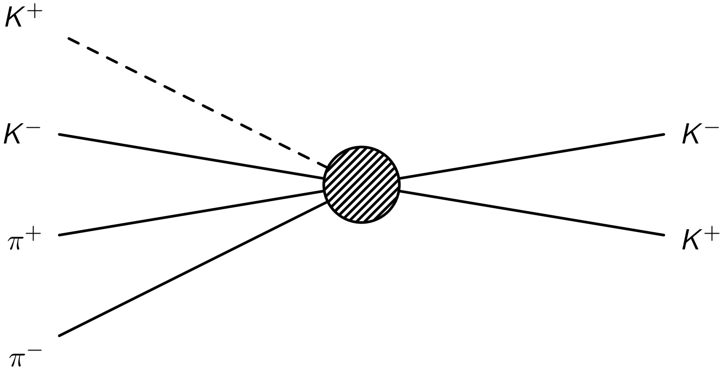
Partially reconstructed $D^0 \to K^+K^-\pi^+\pi^-$

- The reconstruction efficiency of $D \to KK\pi\pi$ is less than 20%
- For comparison, the efficiency of $D \to \pi\pi\pi\pi$ is around 50%!
- Poor kaon tracking efficiency at low momentum



Partially reconstructed $D^0 \to K^+K^-\pi^+\pi^-$

- Solution: Only reconstruct 3 of the charged D daughters
  - Presence of missing kaon is inferred from the missing momentum
  - Yields are similar to fully reconstructed sample
  - Large, but non-peaking background from $D \to K\pi\pi\pi\pi^0$

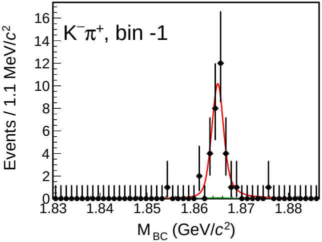


Use this technique with the K^+K^-, $K_S^0\pi^0$ and $K_S^0\pi^+\pi^-$ tags

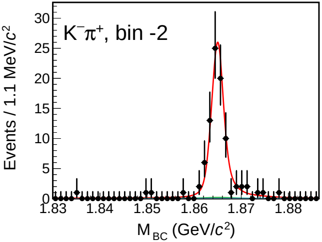
Double tag fits

- Fit strategy: Only fit signal side M_{BC} because of low statistics
- Fit model:
  - Signal: PDF from signal MC, convolved with a Gaussian
  - Flat background: Argus PDF
  - Peaking background: Shape and efficiency from MC, correct for quantum correlation
  - Simple sideband subtraction for correct signal but wrong tag event
- Fit all bins simultaneously
  - Shape is floated and shared across all bins
  - Yield of signal and combinatorial background is floated in each bin

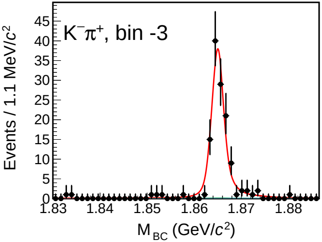
Double tag fit of $KK\pi\pi$ vs $K\pi$



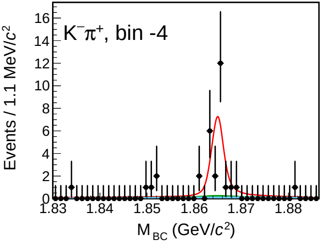
(a) Bin -1 yield: $32.4^{+6.2}_{-5.5}$



(b) Bin -2 yield: $82.7^{+9.7}_{-9.0}$

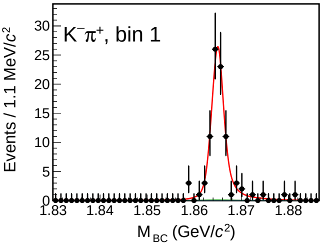


(c) Bin -3 yield: $120.3^{+11.6}_{-10.9}$

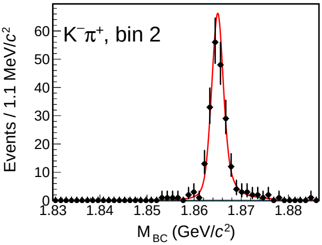


(d) Bin -4 yield: $22.4^{+5.2}_{-4.6}$

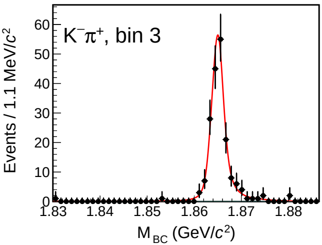
Double tag fit of $KK\pi\pi$ vs $K\pi$



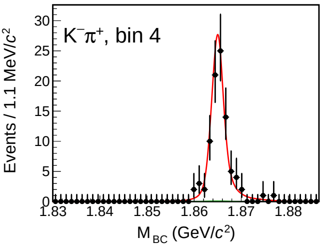
(a) Bin 1 yield: $84.5^{+9.8}_{-9.1}$



(b) Bin 2 yield: $211.2^{+15.4}_{-14.8}$

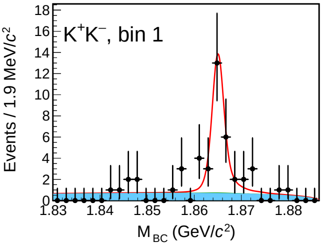


(c) Bin 3 yield: $181.0^{+14.0}_{-13.3}$

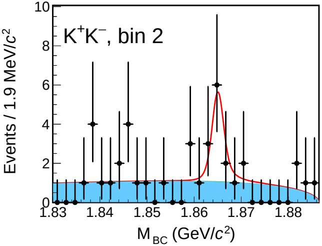


(d) Bin 4 yield: $88.6^{+9.7}_{-9.0}$

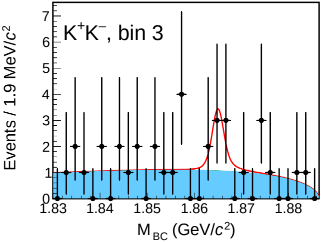
Double tag fit of $KK\pi\pi$ vs KK



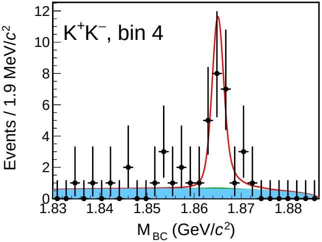
(a) Bin 1 yield: $25.3^{+6.2}_{-5.5}$



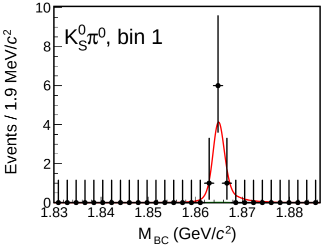
(b) Bin 2 yield: $8.8^{+4.0}_{-3.3}$



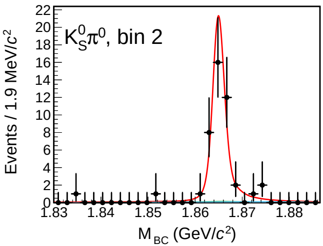
(c) Bin 3 yield: $4.5^{+3.3}_{-2.6}$



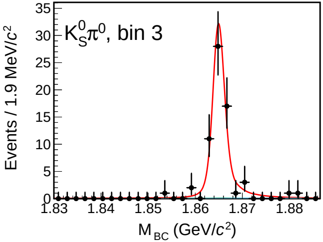
(d) Bin 4 yield: $21.1^{+5.5}_{-4.8}$



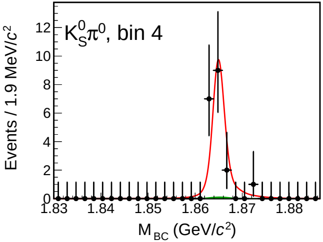
(a) Bin 1 yield: $7.9^{+3.1}_{-2.5}$



(b) Bin 2 yield: $40.4^{+6.8}_{-6.3}$

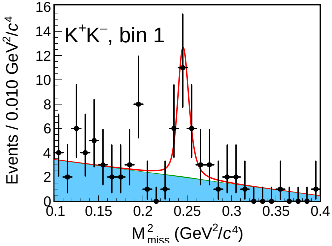


(c) Bin 3 yield: $61.1^{+8.3}_{-7.8}$

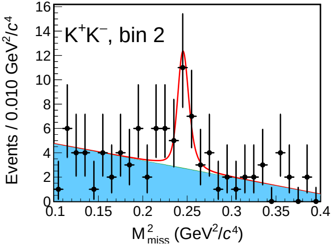


(d) Bin 4 yield: $18.3^{+4.5}_{-3.9}$

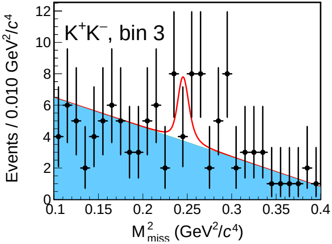
Double tag fit of partially reconstructed $KK\pi\pi$ vs KK



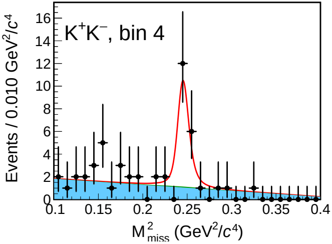
(a) Bin 1 yield: $19.6^{+5.8}_{-5.1}$



(b) Bin 2 yield: $17.7^{+6.1}_{-5.3}$

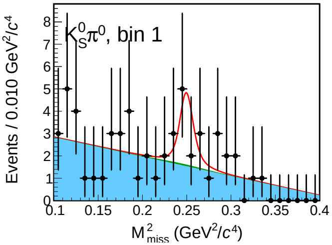


(c) Bin 3 yield: $7.5^{+6.6}_{-5.7}$

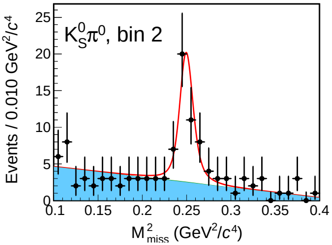


(d) Bin 4 yield: $17.3^{+5.1}_{-4.4}$

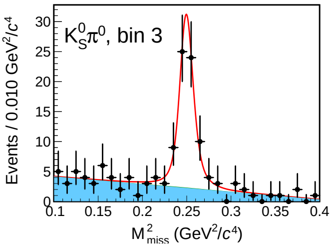
Double tag fit of partially reconstructed $KK\pi\pi$ vs $K_S\pi^0$



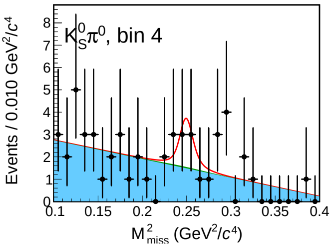
(a) Bin 1 yield: $7.2^{+4.3}_{-3.5}$



(b) Bin 2 yield: $39.3^{+8.3}_{-7.3}$



(c) Bin 3 yield: $64.4^{+9.6}_{-9.0}$



(d) Bin 4 yield: $4.8^{+3.8}_{-3.0}$

Likelihood fit

How to put this all together to determine c_i and s_i?

1. In the past, the strategy has been to measure normalised K_i first, using flavour tags:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i, \quad \sum_i K_i = 1$$

2. However, the K_i must be corrected for efficiencies and bin migrations, in addition to Doubly Cabbibo Suppressed decays:

$$f_i = \left(1 + r_D^2 \frac{K_{-i}}{K_i} - 2\sqrt{\frac{K_{-i}}{K_i}}\left(c_i \cos(\delta_D) + s_i \sin(\delta_D)\right)\right)^{-1}$$

3. Finally, with the K_i fixed, c_i and s_i are determined from CP and multi-body tags:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i + K_{-i} \mp 2\sqrt{K_i K_{-i}} c_i$$
$$\frac{N_{ij}^{\mathrm{DT}}}{N^{\mathrm{ST}}} \propto K_i K'_{-j} + K_{-i} K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}}(c_i c'_j + s_i s'_j)$$

Likelihood fit

We propose a simpler, but more powerful strategy:

1. Treat all K_i, c_i and s_i as free parameters
2. Include the $D \to KK\pi\pi$ branching fraction $\mathcal{B}$ as a free parameter
3. Fit flavour, CP and multi-body tags simultaneously

Master equations:

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} \left(K_{-j} + r_D^2 K_j - 2 r_D \sqrt{K_j K_{-j}} \left(c_j \cos(\delta_D) + s_j \sin(\delta_D) \right) \right)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} (K_j + K_{-j} \mp 2 \sqrt{K_j K_{-j}} c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ijkl} \left(K_k K'_{-l} + K_{-k} K'_l - 2 \sqrt{K_k K_{-k} K'_l K'_{-l}} (c_k c'_l + s_k s'_l) \right)$$

ϵ_{ij} are combined efficiency and bin migration matrices

The master equations use the free parameters $\mathcal{B}$, K_i, c_i and s_i to make a prediction $\hat{N}^{\mathrm{DT}}$ to the measured double tag yields N^{DT}

Likelihood fit

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\big(c_j\cos(\delta_D) + s_j\sin(\delta_D)\big)\Big)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\big(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\big)$$

Ordinarily, we would construct a Gaussian (log)likelihood function $\implies$ Obtain $\mathcal{B}$, K_i, c_i and s_i by minimising the following function:

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j$$

[1] ρ are correlation coefficients

Likelihood fit

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} \left(K_{-j} + r_D^2 K_j - 2 r_D \sqrt{K_j K_{-j}} \left(c_j \cos(\delta_D) + s_j \sin(\delta_D) \right) \right)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} (K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}} c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ijkl} \left(K_k K'_{-l} + K_{-k} K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}} (c_k c'_l + s_k s'_l) \right)$$

Our DT yields are very small, so their uncertainties are asymmetric $\implies$ Approximate covariance matrix from the asymmetric uncertainties[2]:

$$-\ln(\mathcal{L}) = \frac{1}{2} \sum_{\mathrm{Tag}} \sum_{ij} (V^{-1})_{ij} (N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij} \sigma_i \sigma_j, \quad \sigma = \sqrt{\sigma_- \sigma_+ - (\sigma_+ - \sigma_-)(N^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}$$

[2] arXiv:physics/0406120

Likelihood fit

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j, \quad \sigma = \sqrt{\sigma_-\sigma_+ - (\sigma_+ - \sigma_-)(N^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}$$

- The above likelihood has good coverage for flavour and CP tags...
- ... but not for multi-body decays
  - Bins with $\sigma_- \approx 0$ make the fit unstable
  - Fit convergence was found to be less than 60%
- In multi-body decays, use the full unbinned likelihood directly
  - Fit convergence improves to over 95%
  - Much slower, but much more accurate

Parameterisation of K_i

One last technical detail...

- 8 K_i parameters, but $\sum_i K_i = 1$, so only 7 are independent
- We could set $K_4 = 1 - \sum_{i \neq 4} K_i$, but such a parameterisation is unstable due to strong correlations
- Use a recursive fraction parameterisation (JHEP **2021** 169 (2021))
- Recursive fractions R_i are defined as:

$$R_i \equiv \begin{cases} K_i, & i = -4 \\ K_i / \sum_{j \geq i} K_j, & -4 < i < +4 \\ 1, & i = +4. \end{cases}$$

Parameterisation of K_i

Visualisation of this parameterisation:

1. Definition of a_1'

$$\underbrace{a_1}_{\equiv a_1'} + \underbrace{a_2 + a_3 + ...}_{\equiv 1 - a_1'} = 1$$

2. Eliminate a_1

$$a_2 + a_3 + a_4 + ... = 1 - a_1'$$

3. Definition of a_2'

$$\underbrace{\frac{a_2}{1 - a_1'}}_{\equiv a_2'} + \underbrace{\frac{a_3 + a_4 + ...}{1 - a_1'}}_{\equiv 1 - a_2'} = 1$$

Repeat this procedure until the last coefficient, which is 1

Input-output checks

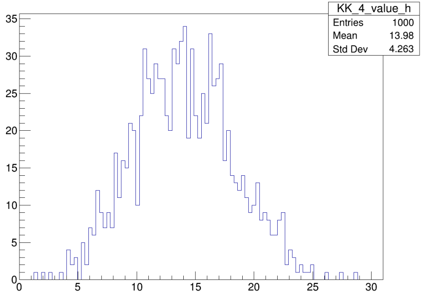
Input-output checks are performed by generating toy datasets

1. Check fit convergence
2. Check error coverage
3. Correct any biases in fitted parameters

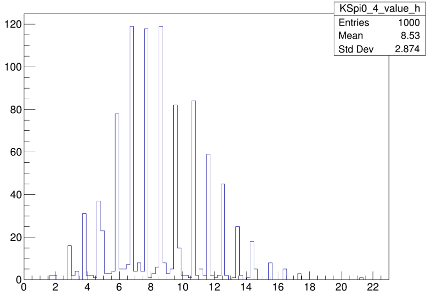
Unfortunately, in this analysis we cannot simply generate Poisson-distributed DT yields:

1. Asymmetric uncertainties
2. Large background-to-signal
3. Multi-body tags with low yields require a full unbinned likelihood

Solution: Generate toy datasets for each double tag fit



(a) Input yield: 13.9

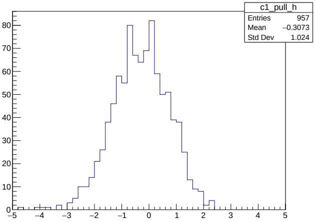


(b) Input yield: 8.6

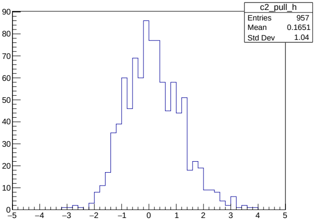
Figure 12: Fitted yields in toy datasets for the (left) KK and (right) $K_S\pi^0$ tags

1. No biases are observed
2. Distributions are asymmetric $\implies$ uncertainties are asymmetric
3. KK uncertainty is non-Poisson because of large backgrounds from $q\bar{q}$
4. $K_S\pi^0$ has small backgrounds, so observed yields Poisson distributed

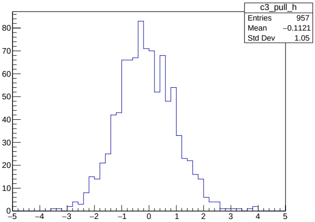
Input-output checks



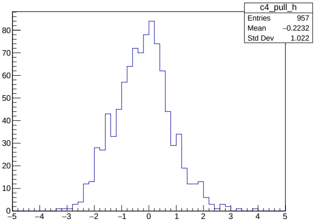
(a) c_1 pulls



(b) c_2 pulls

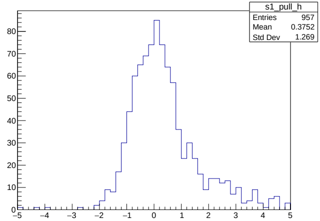


(c) c_3 pulls

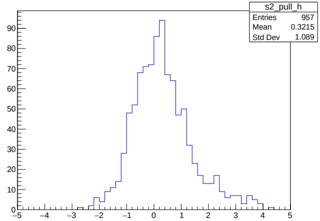


(d) c_4 pulls

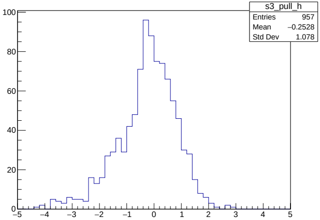
Input-output checks



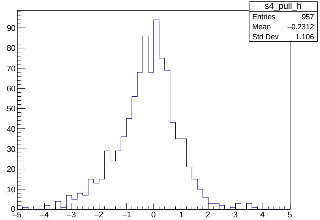
(a) s_1 pulls



(b) s_2 pulls



(c) s_3 pulls



(d) s_4 pulls

What do the toy fits tell us?

1. Both c_i and s_i pulls has asymmetric tails
2. Effect is small in c_i and a small bias correction will be sufficient
3. s_i has a small overcoverage, but a large tail, so special care is required

Fit results

$$c_1 = -0.40 \pm 0.13$$
$$s_1 = -0.18 \pm 0.32$$
$$c_2 = 0.79 \pm 0.06$$
$$s_2 = -0.21 \pm 0.24$$
$$c_3 = 0.85 \pm 0.06$$
$$s_3 = 0.53 \pm 0.24$$
$$c_4 = -0.36 \pm 0.13$$
$$s_4 = -0.38 \pm 0.31$$

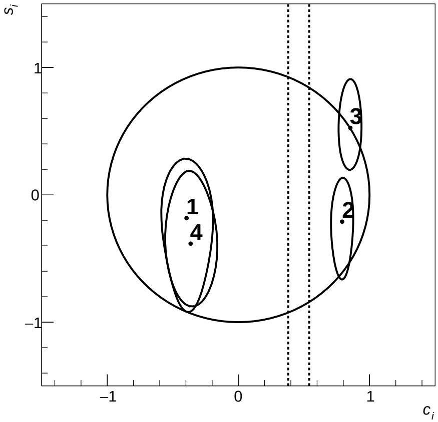
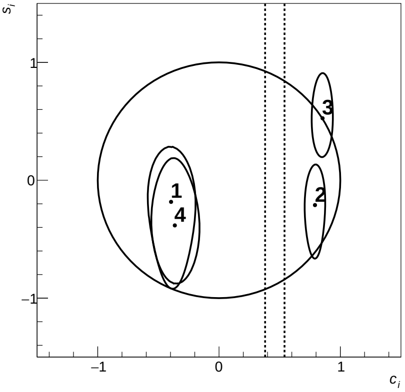
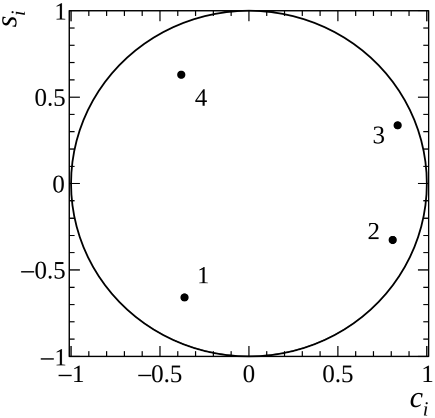


Figure 15: Contours of c_i vs s_i, corresponding to $\Delta\log(\mathcal{L}) = 2.30$, or 68% confidence level. Dashed lines show F_+ measurement.



(a) Data



(b) LHCb model

Figure 16: Comparison between model and model-independent measurements

$\delta_{K\pi}$ measurement

- K_i, which are constrained by flavour tags, are free parameters
- Corrections for DCS decays, which depend on the strong phases δ_D, are part of the fit
- We could instead treat δ_D as a free parameter, and make a simultaneous measurement
- In this analysis we can measure $\delta_{K\pi}$ with negligible reduction in sensitivity to c_i and s_i

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\big(c_j\cos(\delta_D) + s_j\sin(\delta_D)\big)\Big)$$

- Free parameters: $r_D\cos(\delta_{K\pi})$ and $r_D\sin(\delta_{K\pi})$

$\delta_{K\pi}$ measurement

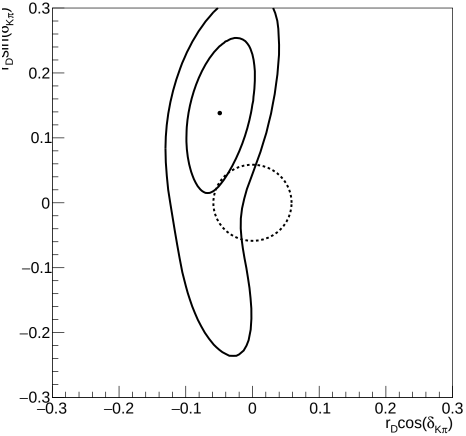


Figure 17: Contours of $r_D \cos(\delta_{K\pi})$ vs $r_D \sin(\delta_{K\pi})$, corresponding to $\Delta \log(\mathcal{L}) = 2.30$ and 6.18, or 68% and 95% confidence level. Dashed line indicates measured value of r_D.

Branching fraction measurement

In addition, we have another nuisance parameter: The $D^0 \to K^+K^-\pi^+\pi^-$ branching fraction

Current PDG value:

$$\mathcal{B} = (2.47 \pm 0.11) \times 10^{-3}$$

Fit result:

$$\mathcal{B} = (2.765 \pm 0.046) \times 10^{-3}$$

Much better precision than the current world average!

Do we have the necessary estimates of tracking and PID systematics to also publish this measurement?

Summary

- A phase-space binned measurement of the $D^0 \to K^+K^-\pi^+\pi^-$ strong-phases has been performed
  - Analysis uses a 2×4 binning scheme
  - Measurement will improve significantly with full $20\,\mathrm{fb}^{-1}$ dataset
  - New strategy: Treat c_i, s_i and K_i on an equal footing
  - Additionally, $\delta_D^{K\pi}$ can also be determined
  - The $D^0 \to K^+K^-\pi^+\pi^-$ branching fraction more than 2 times more precise than the current PDG value
- Next steps:
  1. Perform systematics studies
  2. Write up MEMO

Thank you!